Curriculum Considerations for Developing Mathematical Talent in Elementary Students

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Abstract: To effectively nurture the talents of elementary students with mathematical talent they need to be challenged with an advanced curriculum that is designed with their specific needs in mind. The purpose of this paper is to describe an effective model for developing such a curriculum and explore effective, research-based examples of curriculum and instruction using this model. The paper begins with a review of literature covering a broadened definition of mathematical talent that showcases the need for a more complex and multi-dimensional curriculum to nurture their talents. Next, evidence-based curriculum considerations for talented students at the elementary level are explored. Essential characteristics of mathematics curriculum for talented students are detailed including a focus on advanced content, conceptual development and higher-level processes akin to those used by mathematicians. Accompanying research-based instructional strategies to increase challenge, complexity, and creativity are outlined. Examples at the primary and elementary grade levels are provided to illustrate effective, research-based curriculum and instruction that develop mathematical talent in advanced students.

Keywords: talent development; mathematical ability/aptitude; advanced learning; content-based curriculum; instructional strategies; problem solving; creative thinking

1. Introduction

Mathematics is one of the essential core subjects in the curriculum of every student’s education. When one thinks about teaching mathematics to children, what comes to mind? Often we think about how to work with numbers, such as how to count, how to order numbers, how to compute with whole numbers, fractions and decimals. But there is so much more to mathematics, and it is important to recognize this in order to identify and develop mathematical talent.

Researchers have explored this idea over the years. Conducting seminal work in the field, Krutetskii, a Russian psychologist, was interested in finding out just what mathematical talent looked like. He observed many students as they actually worked on problems. From his studies, he found that students who had high mathematical ability were of three types: those with an “algebraic cast of mind” who reasoned abstractly; those with a “geometric” mind who had a keen sense of spatial visualization and abilities; and those who had a combination of the two types [1]. Krutetskii also found that gifted students actually saw the world through a mathematical lens, a “mathematical cast of mind”.

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The National Council of Teachers of Mathematics (NCTM) has issued a position paper on gifted mathematics students affirming these characteristics [9].

Students with exceptional mathematical promise include those who demonstrate patterns of focused interest; are eager to try more difficult problems or extensions or to solve problems in different, creative ways; are particularly good at explaining complex concepts to others or demonstrate in other ways that they understand mathematical material deeply; and/or are strongly interested in the material.

It is important to note that Krutetskii concluded from his extensive interviews and observations of children that neither speed nor facility in computation nor the ability to memorize formulas were necessary requirements for mathematical talent. Nor does NCTM mention speed or ability to compute easily as an indication of talent. However, often we recognize students who come up with an answer quickly and can compute easily as the only students demonstrating mathematical talent. We need to look beyond this. First, we need to provide opportunities for students to demonstrate higher-level problem solving across a variety of mathematical domains. This means using curriculum and instruction that focus on much more than procedural knowledge, i.e., how to compute and use algorithms. Second, we need to recognize deeper conceptual understanding as well as critical and creative thinking in solving challenging problems as evidence of mathematical talent. Thus, students need a challenging, advanced curriculum focused on complex problem solving with ample opportunities for creative problem posing. Hand-in-hand with this, inquiry-based strategies must be included for teachers to encourage the development of these higher-level thinking processes. This is especially true at the elementary level where the emphasis on passing standardized assessments often gets in the way of focusing on students who are well beyond mastery of grade-level math concepts in the regular curriculum and need so much more. The purpose of this paper is to describe an effective model for developing such a curriculum and explore effective, research-based examples of curriculum and instruction using this model.

2. Curriculum Development

Not only is the regular mathematics curriculum not advanced enough for talented students, but “enrichment worksheets”, which are often provided with a standard mathematics textbook program to meet the needs of talented students, are also not the solution. They usually are not challenging enough, nor do they involve complex problem solving. What is needed and appropriate is an advanced curriculum that is created specifically to nurture students with mathematical talent. This is especially important to start early at the elementary level so that mathematically talented students are motivated and prepared to take advanced course work at the secondary level including AP and university courses.

With a specific focus on mathematics, a curriculum model for advanced students has been developed [10]. This model comprises four components: (a) emphasis on complex and creative problem solving, (b) connections with and across mathematics and other disciplines and across a wide range of contexts including real-world, authentic problems, (c) an inquiry-based approach focused on processes and skills used by mathematicians in their work, and (d) appropriate pacing. It is important to note that the model highlights the importance of instructional processes as a vital component of an advanced curriculum. All four components are equally important to provide a curriculum that is both challenging and in-depth. This model is also aligned with the Integrated Curriculum Model [11], an exemplary model for the design of gifted curriculum developed by Dr. Joyce VanTassel-Baska and her colleagues at the College of William and Mary.

2.1. Advanced Content

The first two components of the model focus on content development. In the mathematics community, there has been significant research on the important concepts that all students should learn throughout their schooling and how these should be developed across grade levels. The Mathematics Content Standards outlined in the Common Core
State Standards [12] and the NCTM Principles and Standards for School Mathematics [13] outline these concepts. In addition, mathematics curriculum needs to be grounded in research-based evidence of how students come to learn important mathematical concepts. There needs to be coherence in the development of content that helps students gain a deep understanding, including how concepts are connected within the discipline [14].

Along with a strong conceptual focus, curriculum for talented mathematics students must provide more. These students need advanced content that highlights:

- complexity with an emphasis on abstract thinking and challenging material;
- depth with a focus on understanding the underlying structure of concepts; and
- opportunities to develop and demonstrate creativity such as creating new problems to investigate and novel solutions to problems.

Researchers at the University of Connecticut used these guiding principles to develop two curriculum series for talented elementary students and conducted longitudinal research studies on their effectiveness in terms of advanced mathematics achievement under the auspices of two federal grants [2,3]. Project M$^3$: Mentoring Mathematical Minds was a five-year project funded by the U.S. Department of Education under a Javits research grant. These units were written and field-tested with multiple cohorts of students from diverse populations in grades 3–6. Additional units have been added across the grade levels and new editions have been published. Project M$^2$: Mentoring Young Mathematicians was a five-year National Science Foundation award to develop, field-test, and publish advanced curriculum for students in kindergarten through second grade. Both series have also been awarded the NAGC Distinguished Curriculum Award for several years. Research results from both programs have shown highly significant gains from pre-to-post unit assessments as well as on norm-referenced, above-grade level standardized testing. There have also been highly significant gains over comparison groups of students from the same schools with the same demographics and ability levels [2,3,15,16]. The following illustrations from these research-based curricula are used to highlight the various components of effective curriculum to nurture mathematical talent in young students.

### 2.2. Examples of Advanced Content

At the primary level, a second grader may be thought to be gifted in mathematics because he or she can solve equations. But often in reality the student has memorized a procedure for solving an equation such as $2x + 4 = 30$. They cannot describe why this procedure makes sense or explain why it works.

Contrast this to the development of algebraic thinking in *Shopping at the Bazaar: Connecting Number and Algebra with the Meerkats* [17], one of the units in the primary series Project M$^2$: Mentoring Young Mathematicians. Based on the work of Carpenter and colleagues [18], the investigations in this unit help students gain an in-depth understanding of equality, develop flexibility in relational thinking, and explore the commutative property and properties of zero. Students look for patterns and make generalizations, connecting algebraic reasoning and number concepts. These foundational concepts facilitate a deeper understanding of equations and their solutions. Students end the unit using the mathematics they have learned by solving a variety of situational complex problems while shopping at the African bazaar. They also have an opportunity to create problems of their own (a higher-level learning task) to share and solve.

At the upper elementary level, an example of challenging, advanced content can be found in *Record Makers and Breakers: Analyzing Graphs, Tables and Equations* from Project M$^3$: Mentoring Mathematical Minds [19]. This unit is devoted to the study of patterns and relationships among quantities which are major algebra concepts that form an essential foundation for more advanced mathematics including algebra and calculus. Students learn about algebra as a style of mathematical thinking for formalizing patterns of change. Their investigations, discussions and written reflections provide opportunities for advancing mathematical understanding and are an important prelude to the formal study of algebra.
In the culminating lesson, students consolidate their learning about analysis of change and its effect on functions. They analyze reading rates of students who are trying to set a school record for reading the most books. In the problem described below, the challenge level increases when they are asked to create original scenarios with specific parameters rather than being presented with a scenario to analyze. The focus is on depth of understanding and complexity. They need to integrate all their learning throughout the unit and make connections among various representations of linear functions. They are given this task:

Create a scenario involving three students reading who each start with a different number of pages read. Each reads at a constant rate and all end with the same number of pages read at the end of 15 days. Explain why your scenario works using a graph, table, and equation. (p. 250)

These examples have been presented to show how the curriculum model is put into practice when developing high-level mathematics activities. The activities illustrate advanced content at different elementary grades from proven, evidence-based curriculum series with a focus on depth of conceptual understanding and critical and creative problem solving.

2.3. An Inquiry-Based Approach Using Higher-Level Processes

Not only must we consider what advanced students are learning but equally important is how they are engaged in learning the concepts. George Polya, a well-respected mathematician, believed that the only difference between the work of a professional mathematician and a talented student of mathematics was in the degree of sophistication they use [4]. Polya believed that students are capable of mathematical creativity just as mathematicians are, with each operating at their own level of understanding. A hallmark of gifted education promoted by many experts in the field is the idea of students working similar to practicing professionals in the field as they explore the discipline [11,20–22]. Students are encouraged to use the same investigative methods that practitioners in the field use to seek answers to their questions and make contributions to their field. This is indeed an inquiry-based approach to learning.

In mathematics, this practice refers to the student as a young mathematician using similar problem-solving strategies and mathematical tools that a mathematician would use to delve into the mathematics being explored and to create something original. The NCTM processes that include problem solving, reasoning and proof, communication, representations, and connections [13] and the CCSS Standards for Mathematical Practice [12] embody the principles of practicing mathematicians. In addition, these processes rest on the strands of mathematical proficiency specified in the National Research Council’s report, *Adding It Up* [23]: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition. Lastly, when students are engaged in these processes, they are developing the 21st Century Skills of critical thinking, creativity, collaboration, and communication [24].

As a model for developing and using mathematics curriculum, the Standards for Mathematical Practice [12] provide a practical and comprehensive set of eight practices. These practices, listed below, are what students should be using on a daily basis as they do mathematics:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.
In addition, NAGC, in collaboration with NCTM and the National Council of Supervisors of Mathematics, has proposed a ninth practice focused on creativity which is especially relevant for gifted learners: “solve problems in novel ways and pose new mathematical questions of interest to investigate [25]”. The Standards for Mathematical Practice included very important critical-thinking skills but left out one of the most important practices that distinguish mathematicians, the ability to create new mathematics. In this regard, researchers have studied the effects of using problem posing with students to focus on depth of conceptual understanding at the elementary level [26], advanced problem solving at the secondary level [27] and the development of mathematical creativity [28].

Researchers and teachers have found that a focus on these practices with talented elementary students has helped develop high-level reasoning and conceptual understanding in mathematically talented students, as evidenced in classroom discussions and on written assessments [2,3,15,16].

### 2.4. Appropriate Pacing

The last component, pacing, while not a feature of a curriculum per se, is especially important for gifted students. It focuses on how quickly students can move through the curriculum. Advanced students need an accelerated pace when it is evident they already have learned skills currently being taught in the classroom. Added to that, they often learn new skills faster with increased understanding than their peers in the regular mathematics program. Emphasizing this point, the National Mathematics Advisory Panel [29] recommends that mathematically gifted students can learn mathematics much faster, with no harm to their learning, and should be allowed to do so (p. 53). Since talented students need more than the regular grade-level curriculum, it seems like an easy solution to move students to the next grade level during mathematics class or offer the next grade-level textbook curriculum. It can be a good start and practical since it is relatively easy to accomplish. But, in fact, this by itself is not a solution.

It is also important to recognize that there is a range of mathematical talent from students who learn mathematics skills and procedures and make sense of concepts quickly and easily to extremely precocious students who can, at an early age, generalize and make abstractions, demonstrate creative ways of solving problems, come up with new problems to solve and eagerly tackle complex problems and persevere in solving them. Certainly, precocious elementary students need to be advanced well beyond their grade level. But the quality of the curriculum they use needs to be considered as well. The bottom line is curriculum for talented students needs to go beyond grade level (sometimes several grade levels) and be differentiated qualitatively to provide an appropriate and equitable mathematics program for these students [30–32].

To help educators assess what students have already mastered and provide time to use advanced curriculum with them, curriculum compacting (Reis, Renzulli, et al., 2016) is a proven tool. Many gifted education experts [25,32,33] recommend that teachers use preassessments to determine pacing for advanced mathematics students. Once students demonstrate their understanding of skills and concepts that will be taught in a particular unit of instruction or, in some circumstances, an entire curriculum for a given school year, teachers can then help students advance to the next level of learning. There are a variety of ways to do this depending on the school structure and the student’s level of mathematical talent. Flexible grouping and cluster grouping provide opportunities within the regular classroom for talented students to receive appropriate differentiated curriculum and instruction. Special classes which include pull-out programs, gifted mathematics classes, honors classes, as well as specialized STEM schools, help provide a spectrum of settings in which to develop a range of mathematical giftedness in students.

There is a caveat, however. It is important to realize that the pace of instruction can be accelerated if the student is learning a new procedure or skill, since advanced students can often master these faster than other students. However, when working on complex, advanced problems, students need to be encouraged to actually slow down to
use critical thinking to analyze the problem and have time to discuss their ideas just as mathematicians do. This involves changing the mindset of students who think being good at mathematics means only that they can find a solution quickly and easily. Rather, it is in the productive struggle with challenging tasks that students come to gain deeper understanding of the mathematics. Grappling with problems that do not present an easy solution or an obvious method of solving helps students learn from their errors and makes them stronger mathematicians. They need to be encouraged to do this.

3. Instructional Considerations

3.1. Mathematical Discussions

Having talented students work on complex problems and advanced content by themselves in a quiet corner is sometimes viewed as an appropriate way of meeting their needs. However, this is not encouraging the practices and processes highlighted above and falls short of developing student mathematicians. Rather, teachers should strive to create a community of learners using purposeful discussion. In fact, this should be the basis for teaching and learning mathematics in the classroom. Students who have had opportunity to talk at length with their peers and teachers about solving problems tend to persist longer in trying to make sense of a new problem. Listening to their peers helps them clarify and revise their thinking. They also learn to consider problems from different perspectives and appreciate a variety of ways to solve problems. Researchers in Projects M³ and M² have found this discussion is key to high-level thinking and learning [28]. To help teachers create rich discussion, Chapin and colleagues [34] have created a series of talk moves for mathematics classes which are used in Projects M³ and M². The talk moves appear simple and hence are easy to use; yet they are very powerful in generating high-level discussions. They include the following:

- revoicing what a student says and verifying whether it was accurate;
- asking a student to repeat or rephrase what another student has said;
- asking a student to say more, i.e., elaborate on an idea they just stated;
- asking a student to agree or disagree with an idea and explain why; and
- asking a student to add on to what another student has stated.

Coupled with the “talk tools” of wait time, partner talk, and scribing, the classroom conversation is elevated to a new level focused on high-quality reasoning and creative thinking. Note that all talk moves are equally as appropriate and effective in a kindergarten class as they are in a high-school class of advanced mathematics students.

3.2. Mathematical Writing

If mathematicians did not write down their discoveries, i.e., their new theorems and proofs, we would never learn about them or build upon them. Developing students’ written communication is equally as important as engaging them in rich mathematical discussions. In our research, we have found that advanced students sometimes come up with insightful solutions to problems or even post new ones but have difficulty putting their thoughts in writing. They need to step back and think metacognitively which in itself is a high-level process. Student journal writing is incorporated in every lesson in Projects M³ and M² curriculum units and is focused on analytical problem solving, essential questions and creative thinking. This writing contained in a Student Mathematician’s Journal affords students the opportunity to record their thinking and communicate with their peers and their teachers. Journals can also be a useful tool for establishing a dialog between teachers and students and serve as formative assessments of learning.

We have found that developing writing in mathematics to present a solution to a complex problem, to explain and defend reasoning, and to pose new problems or a novel solution is challenging for teachers. Many elementary students, particularly younger students, are just beginning to develop reading and writing skills, and explaining and defending their thinking is a complex skill to verbalize, let alone write down. And not all students who are mathematically talented are also talented in reading and writing. In
fact, some have reading and writing challenges, including English learners. Because we believe that written communication fosters many higher-level thinking processes, rather than look for alternatives to writing, we found several supports helpful. First and foremost, the verbal discourse we presented above is invaluable as a prelude to writing. Teachers and students discuss the mathematics and solutions to problems prior to writing. This allows students to hear the ideas of others, defend their own ideas and further clarify their thinking before putting their responses in writing. In addition, writing should be developed over a series of lessons starting with creating a class exemplar response, to working with partners and finally, to individual writing. Teachers also are encouraged to provide written feedback for journal writing to establish a dialogue between teacher and student with the purpose of improving written responses. It is also important to emphasize that writing does not always mean putting sentences down on paper. Students should be encouraged to use pictures, tables, charts and diagrams to help them explain their thinking. An emphasis on understanding the mathematical language appropriate to the content is important as well and should be imbedded in the curriculum. Of course, different students need different supports, and some may need none at all. But being able to explain and defend their thinking in writing strengthens conceptual understanding and problem solving and encourages logical thinking and precision.

The combination of mathematical discussion and writing as instructional strategies has been used in implementing the Projects M³ and M² advanced curriculum units in numerous classrooms over many years. As previously stated, research results have shown highly significant achievement gains from pre to post testing and over comparison groups. These assessments include complex, open-ended questions that require students to explain and defend their reasoning [2,3,15,16].

4. Concluding Comments

Educators at all grade levels, but especially in the primary and elementary formative years, need to be talent scouts. They need to be cognizant of the different types of mathematical ability. This means they need to look beyond strong computational skills and recognize insightful sense-making ability, a deep interest in solving and creating challenging problems, quantitative reasoning, and/or spatial reasoning, as further evidence of mathematical talent.

Once students are identified, educators need to provide challenging, differentiated curriculum and instruction to nurture the gifts and talents of these students. This must be qualitatively different from the standard textbook curriculum and move beyond enrichment worksheets, logic puzzles and fun mazes. The curriculum should be a coherent development of advanced concepts and include an emphasis on abstract thinking and challenging material, a focus on the underlying structure of the mathematics, and opportunities to develop and demonstrate creativity.

How the curriculum is taught is just as important as what is taught. The focus should be on an inquiry-based approach using the mathematical practices that mathematicians use on a regular basis. These include persevering in solving challenging problems, using high-level reasoning, defending their thinking, using math models and tools appropriately, and looking for patterns and structure. These practices are deepened and enhanced by activities in which students and teachers talk, write, and use mathematical language and symbols together, while working to convey and clarify their mathematical thinking. Mathematical discourse both in classroom discussions and also in journal writing enhance depth of understanding and help to increase flexibility and creativity in mathematical thinking.

In order to improve mathematics curriculum and instruction for talented elementary students, teachers need to be given opportunities to work with talented students within or outside of the regular classroom. In addition, they need to use advanced curriculum and instructional strategies that are challenging and develop talented students. To do this effectively, administrators need to support teachers by providing professional learning experiences to enhance their mathematical background if needed and opportunities to
investigate appropriate curriculum materials in order to facilitate their success in teaching talented students.

In conclusion, our mathematically talented students will be the next generation of leaders in an ever-increasing global and technological society. We must recognize and nurture their talents beginning in their early years. We cannot afford to fail them.

**Funding:** Funding for the curriculum grant projects described in this article were provided by the following: Project M3: Mentoring Mathematical Minds was funded by the U.S. Department of Education under grant no, S206A0200061; Project M2: Mentoring Young Mathematicians was funded by the National Science Foundation (NSF) under grant no. DRL-0733189.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** No new data were created or analyzed in this study. Data sharing is not applicable to this article.

**Conflicts of Interest:** The author declares no conflicts of interest. The opinions, conclusions, and recommendations expressed in this article are those of the author and do not necessarily reflect the position or policies of the U.S. Department of Education or the National Science Foundation.

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