



Energy-Oriented Management in Water Distribution Networks: A Mixed-Integer Nonlinear Programming Approach [†]

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[†] Presented at the 6th International Conference on Efficient Water Systems (EWaS6), Thessaloniki, Greece, 11–14 May 2026.

Abstract

Pressure management in water distribution networks requires the integration of hydraulic modeling and discrete device allocation, naturally leading to mixed-integer nonlinear programming (MINLP) formulations. This study presents a MINLP problem for the joint placement of pressure-reducing valves and pumps as turbines, using the Global Excess Power index to quantify energy inefficiencies related to excess pressure and leakage. The formulation incorporates nonlinear hydraulic equations, leakage–pressure relationships, and operational constraints. Application to a real network illustrates model behavior and highlights the computational challenges associated with energy-oriented pressure control in practical water distribution systems.

Keywords: deterministic optimization; pressure reducing valves; pumps as turbines; mixed-integer nonlinear programming; pressure management; global excess power

1. Introduction

Water distribution networks (WDNs) are increasingly required to operate under stringent environmental and economic constraints. Excess pressure within the system not only increases leakage rates but also results in unnecessary energy dissipation, reducing overall operational efficiency [1]. For this reason, pressure management has become a key strategy in the design and operation of modern WDNs, typically implemented through pressure-reducing valves (PRVs) and, more recently, through energy recovery devices such as pumps operating as turbines (PATs) [2]. The integration of pressure control and energy recovery within a unified optimization framework introduces significant modeling challenges. Hydraulic behavior is inherently nonlinear, as head losses depend on discharge through nonlinear relationships, while leakage is strongly influenced by pressure levels. In addition, decisions regarding the installation and configuration of control devices are discrete in nature. The combination of these aspects leads to mixed-integer nonlinear programming (MINLP) formulations, characterized by strong variable coupling and nonconvexity [3]. This study introduces a mixed-integer nonlinear programming (MINLP) model for the joint allocation of pressure-reducing valves (PRVs) and pumps operating as turbines (PATs) in water distribution networks. The Global Excess Power (GEP) index is adopted as an indicator of energy inefficiency to guide the optimization process. The aim is to analyze the structure of the proposed formulation and evaluate



Academic Editors: Vasilis Kanakoudis, Evangelos Keramaris, Francesco De Paola, Habib Muhammetoglu and Theodora Papamitsou

Published: 8 July 2026

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its implications in terms of hydraulic coherence, solution characteristics, and practical applicability to real-world water distribution systems.

2. Problem Formulation

The optimization problem addresses the joint placement of pressure-reducing valves (PRVs) and pumps operating as turbines (PATs) in a water distribution network, with the objective of minimizing the Global Excess Power (GEP), which quantifies the rate of excess stream power relative to the minimum required delivered power. The objective function can be expressed as:

$$\text{GEP} = \sum_{n=1}^N \left(\frac{\gamma q_0 \Delta h_n}{P_{\text{MIN}}} + \frac{\gamma q_{\text{leak}}^{h_0} \Delta h_n}{P_{\text{MIN}}} + \frac{\gamma q_{\text{leak}}^{\Delta h} (z_n + h_0 + \Delta h_n)}{P_{\text{MIN}}} \right) \quad (1)$$

where q_0 , h_0 and z_n are, respectively, the total water demand, the minimum required pressure and the elevation of the n -th node ($n = 1..N$). Moreover, Δh_n is the excess pressure at the node; γ is the specific weight of water; $q_{\text{leak}}^{h_0}$ and $q_{\text{leak}}^{\Delta h}$ represent the leakage flow rate related to the minimum and excess pressure, respectively. Finally, P_{MIN} is the minimum delivered power, expressed as:

$$P_{\text{MIN}} = \sum_{n=1}^N \gamma q_0 (z_n + h_0) \quad (2)$$

The objective function accounts for the excess power absorbed by delivered demand and leakage under surplus pressure conditions. Since leakage and nodal excess pressure depend on hydraulic heads, the objective is strongly coupled with the network hydraulic state and inherently nonlinear. More details about GEP formulation can be found in [4]. Hydraulic mass and energy balance equations are included as constraints to ensure physical consistency. Operational constraints impose limits on head drops, device installation exclusivity per pipe, and upper bounds on the number of installed devices. Environmental requirements, expressed through minimum carbon savings associated with energy recovery, are introduced as bounded constraints, further shaping the feasible solution space. The formulation includes continuous hydraulic variables (nodal heads and pipe discharges) and binary variables representing device installation and configuration. The coexistence of discrete decisions and nonlinear hydraulic relationships leads to a mixed-integer nonlinear programming (MINLP) problem.

3. Results

The proposed GEP-based optimization framework has been applied to a rural water distribution network located in the Blackstairs region, Ireland. The system comprises 138 pipes and 126 nodes and is supplied by an elevated reservoir providing a constant hydraulic head of 291 m. Due to the mountainous source configuration, a large portion of the network operates at elevations significantly lower than the supply level. This topographic condition generates substantial excess pressure throughout the system, making pressure control measures particularly relevant. The network was first analyzed under baseline conditions, assuming the absence of any pressure management strategy (Scenario 0). Performance indicators were evaluated for this uncontrolled configuration. Figure 1 illustrates the spatial distribution of the performance indicator at node scale across the network. The node-scale indicator exhibits a clear dependence on ground elevation. Nodes located in lower-altitude areas operate under higher pressure conditions and therefore show larger values of excess power relative to the minimum required energy. This spatial trend reflects the significant elevation difference between the supply reservoir and downstream portions of the system. The aggregated network-level indicator, expressed by the Global Excess

Power (GEP), is equal to 1.34. This value indicates that approximately 134% of the minimum required delivered power is dissipated as excess within the system, highlighting the potential for pressure regulation and energy recovery interventions.

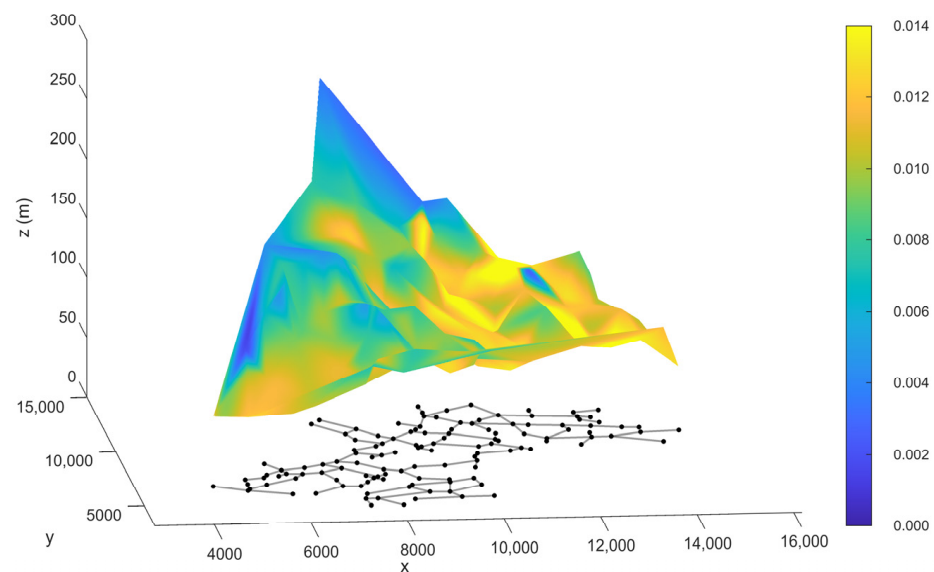


Figure 1. Spatial distribution of excess power indicators at node scale in Scenario 0.

The optimization procedure has been implemented in A Mathematical Programming Language (AMPL) and solved using the global optimization solver SCIP [5]. Different lower bounds on emission savings (C_R), representing the environmental benefit associated with energy recovery and reduced system inefficiencies, have been imposed to investigate the influence of environmental constraints on the resulting configurations. The optimal solutions obtained under varying C_R bounds are summarized in Table 1. For each configuration, the Net Present Value (NPV) has been evaluated a posteriori to assess economic performance. Table 1 shows that solution (a) achieves the minimum GEP value (0.312), corresponding to a 77% reduction compared to Scenario 0. This configuration involves the installation of 10 PRVs and no PATs, leading to a water saving of 153 m³/day and an NPV of 103 k€. As no energy recovery devices are installed, both energy production and emission savings are null. The configuration with the highest economic return is solution (b), characterized by an NPV of 113 k€. This solution includes nine PRVs and one PAT, producing 80 kWh/day and achieving emission savings of 7.6 tonCO₂/year. The corresponding GEP value (0.337) reflects a 75% improvement in energy efficiency compared to the uncontrolled scenario. The maximum emission savings are obtained in solution (c), with five PRVs and five PATs. This configuration produces 143 kWh/day, achieves emission savings of 13.6 tonCO₂/year, and reduces the GEP to 0.413 (69% improvement), with an associated NPV of 88 k€.

Figure 2 illustrates the relationship between Net Present Value (NPV) and emission savings C_R . Due to the imposed lower bounds on C_R , the optimal solutions align along two distinct trends. This behavior is associated with a discrete change in turbine configuration: when the emission savings target exceeds 7.6 tonCO₂/year, a shift in turbine size occurs, leading to a different investment–return structure. Solution (a), characterized by the minimum GEP value (0.312), lies on the lower trend and corresponds to $C_R = 0$, as no turbines are installed ($N_T = 0$). In contrast, solutions (b) and (c), with GEP values of 0.337 and 0.413, respectively, belong to the upper trend, where energy recovery contributes to emission savings. An intermediate configuration is represented by solution (d), with GEP equal to 0.324. This solution achieves emission savings between 6.4 and 7.6 tonCO₂/year

and exhibits an NPV (approximately 90 k€) comparable to that of solution (c), highlighting the trade-off between environmental targets and economic performance.

Table 1. Different solutions of the optimization procedure.

Solution	N_T	N_V	Water Saving (m ³ /Day)	Energy Production (kWh/Day)	GEP	NPV (k€)	C_R (tonCO ₂ /year)
0	-	-	-	-	1.340	-	-
a	0	10	153	0	0.312	103	0
b	1	9	149	80	0.337	113	7.6
c	5	5	135	143	0.413	88	13.6
d	4	6	151	68.2	0.324	88	6.4
e	1	9	148	19.3	0.348	97	1.8
f	2	8	148	38.6	0.340	95	3.6
g	3	7	146	114	0.354	105	10.7
h	4	6	150	135	0.341	104	12.7

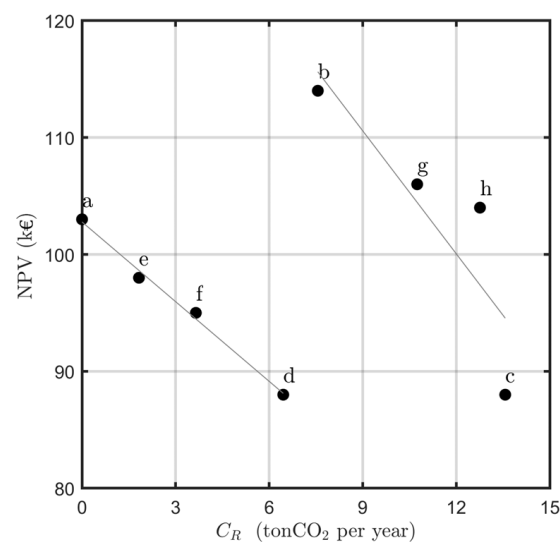


Figure 2. Trend of NPVs depending on the emission savings C_R for the obtained solutions.

4. Conclusions

This study presented a mixed-integer nonlinear programming (MINLP) formulation for the joint placement of pressure-reducing valves and pumps operating as turbines in water distribution networks, adopting the Global Excess Power (GEP) index as an energy-oriented performance measure. The model integrates nonlinear hydraulic relationships, leakage–pressure coupling, and discrete installation decisions within a unified optimization framework. Application to a real rural network highlighted the significant potential for reducing excess energy through pressure management strategies. The results showed that substantial improvements in energy efficiency can be achieved under different environmental constraints, while maintaining economic feasibility. The analysis also revealed clear trade-offs among GEP minimization, emission savings, and Net Present Value, reflecting the influence of discrete device configurations on system performance. From a modeling perspective, the interaction between binary variables and nonlinear hydraulic equations leads to a nonconvex optimization problem, whose solution structure is sensitive to imposed environmental targets. The proposed approach provides water utilities with a flexible framework to balance economic performance, environmental objectives, and operational efficiency.

Author Contributions: Conceptualization, M.C.M., A.C. and O.F.; methodology, M.C.M., A.C. and O.F.; software, M.C.M. and R.M.; M.C.M., A.C. and O.F.; formal analysis, O.F. and R.M.; investigation, M.C.M. and R.M.; resources, A.C.; data curation, M.C.M., A.C., O.F. and R.M.; writing—original draft preparation, M.C.M., A.C., O.F. and R.M.; writing—review and editing, M.C.M., A.C., O.F. and R.M.; visualization, M.C.M., A.C., O.F. and R.M.; supervision, A.C. and O.F.; project administration, A.C.; funding acquisition, A.C. All authors have read and agreed to the published version of the manuscript.

Funding: This work is supported by the “Network 4 Energy Sustainable Transition—NEST” project (MUR project code PE00000021, Concession Decree No. 1561 of 11 October 2022), in the framework of the NextGenerationEu PNRR plan (CUPE63C22002160007).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available upon request from the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

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