An Enhanced DV-Hop Localization Scheme Based on Weighted Iteration and Optimal Beacon Set

Tianfei Chen 1,2,3, Shuaixin Hou 1,2,3, Lijun Sun 1,2,3, and Kunkun Sun 1,2,3

Abstract: Node localization technology has become a research hotspot for wireless sensor networks (WSN) in recent years. The standard distance vector hop (DV-Hop) is a remarkable range-free positioning algorithm, but the low positioning accuracy limits its application in certain scenarios. To improve the positioning performance of the standard DV-Hop, an enhanced DV-Hop based on weighted iteration and optimal beacon set is presented in this paper. Firstly, different weights are assigned to beacons based on the per-hop error, and the weighted minimum mean square error (MMSE) is performed iteratively to find the optimal average hop size (AHS) of beacon nodes. After that, the approach of estimating the distance between unknown nodes and beacons is redefined. Finally, considering the influence of beacon nodes with different distances to the unknown node, the nearest beacon nodes are given priority to compute the node position. The optimal coordinates of the unknown nodes are determined by the best beacon set derived from a grouping strategy, rather than all beacons directly participating in localization. Simulation results demonstrate that the average localization error of our proposed DV-Hop reaches about 3.96 m, which is significantly lower than the 9.05 m, 7.25 m, and 5.62 m of the standard DV-Hop, PSO DV-Hop, and Selective 3-Anchor DV-Hop.

Keywords: WSN; node localization; DV-Hop; AHS

1. Introduction

As the key technology of the internet of things (IoT), WSN has brought enormous changes to our society. WSN comprises some sensor nodes with the ability for mutual communication and data transmission [1,2]. The sensor nodes can monitor temperature, noise, pressure, humidity, and so on. At present, the wireless sensor network has been extensively applied in many fields, such as military reconnaissance, health care, urban transportation, precision agriculture, and forest fire monitoring [3–7]. For most applications of WSN, it is necessary to know the specific position of the event so as to make the corresponding countermeasures quickly. Therefore, how to obtain the precise position information of sensor nodes for WSN has critical research significance.

The simplest and most direct way is to add a global positioning system (GPS) device to each sensor node. Then, the accurate position coordinates of all nodes can be obtained directly. However, the deployment of GPS for each sensor in WSN has certain limitations [8,9], such as: (1) in indoor environments, the signal transmission and line of sight (LoS) can be disturbed by obstacles, thus, further affecting the positioning performance; (2) the purchase of GPS equipment is a significant expense, especially for large-scale networks, which undoubtedly increase the deployment costs of WSN; (3) the energy consumption of GPS devices to sensor nodes is also a problem that cannot be ignored. Since GPS devices require sensors to power them for operation, this will significantly reduce the lifespan of sensors.
At present, the most common and feasible way is to install GPS devices on some nodes (called beacon nodes) and compute the position of unknown nodes through the existing beacon information and mutual communication between sensors.

To obtain the precise unknown node coordinates, some scholars have proposed different node localization schemes. These schemes can divide into two types: range-free and range-based types [10,11]. At present, the range-based scheme includes: received signal strength indicator (RSSI) [12,13], time difference of arrival (TDOA) [14,15], time of arrival (TOA) [16,17], and so on. The range-free scheme includes: centroid algorithm [18,19], approximate point in triangle (APIT) [20,21], distance vector hop (DV-Hop) [22,23], etc. The range-based scheme has high positioning accuracy, but it usually needs additional hardware to determine the angle and actual distance between nodes, which will increase the overhead of the network. On the contrary, the range-free scheme does not demand extra tools and has the characteristics of easy implementation, low cost, and suitable for large-scale networks.

The standard DV-Hop is the extensively applied positioning algorithm, which possesses all the strengths of the range-free scheme. Nevertheless, due to the complexity of the network and the limitations of the algorithm itself, its positioning accuracy is poor and cannot meet the needs of some applications that require higher accuracy [24]. To ameliorate the positioning performance of standard DV-Hop, different improved positioning algorithms based on standard DV-Hop have emerged, one after another. These improved algorithms mainly include two improvements: On the one hand, the precision of the AHS of sensor nodes is enhanced. This will further decrease the distance error between nodes and improve the precision of node localization. On the other hand, the unknown node coordinates computed by the standard DV-Hop are further optimized through some mathematical tools or intelligent optimization algorithms.

In standard DV-Hop, the AHS of beacon nodes is just determined by the unbiased estimation criterion. Recently, some scholars have pointed out that it is more reasonable to calculate the AHS using MMSE or weighted MMSE. However, the extant literature does not ascertain the optimal iterative number of weightings in finding the AHS of beacons. Meanwhile, different weighting methods get different results. During the estimation distance phase, most of the literature is devoted to improving the accuracy of distance by enhancing the AHS of unknown nodes. However, the AHS of unknown nodes is derived by referring to the AHS of beacons, so the AHS of unknown nodes will inevitably deviate from the real AHS. In the positioning stage, the beacons with large distance errors to unknown nodes are not excluded in most studies, and all beacons are directly employed to calculate the location of unknown nodes. Therefore, the positioning result will be seriously affected by these inferior beacons. Although part of the literature demonstrates that better localization can be achieved by using partial beacon nodes, the number of beacon nodes required to locate each unknown node is fixed.

To fill the gaps in the existing literature, an enhanced DV-Hop scheme based on weighted iteration and optimal beacon set is proposed in this study. The major objectives and contributions are as follows:

1. To determine the optimal number of iterations and enhance the AHS of beacons, a weighted iterative strategy based on the MMSE criterion is presented. Different weights are assigned to beacons according to the per-hop error between beacons. In calculating the optimal AHS of beacons, the number of iterations required is determined adaptively according to the variation in the AHS error. In this way, the impact of beacon AHS on the subsequent steps is minimized as much as possible.

2. To make the estimated distance between unknown nodes and beacons more reliable, the calculation scheme of the distances is redesigned. In this study, the distances are calculated by referring to the optimal AHS obtained in the previous step, instead of using the AHS of unknown nodes. Consequently, this not only strengthens the accuracy of estimated distance, but also simplifies the calculation procedure of the DV-Hop algorithm.
3. To select the superior beacon nodes suitable for localization for each unknown node, a grouping strategy is introduced in this paper. All the beacon nodes are divided into different combinations according to their distance from the unknown node. The location of unknown nodes is determined only by the combination of beacon nodes with minimum localization error. Meanwhile, the potential errors present when linearizing the set of distance equations are effectively handled.

The remainder of the article is arranged as follows. Section 2 introduces the related work of the DV-Hop. In Section 3, an overview of the standard DV-Hop and its error source is presented. Our proposed algorithm is described in Section 4. In Section 5, the simulation and result analysis are conducted. Lastly, we conclude this article in Section 6.

2. Related Works

Although the standard DV-Hop has the features of low cost, easy operation and strong practicability, its poor localization precision limits the application to some scenarios. To enhance the node localization precision, various improved DV-Hops have been put forward by scholars. Depending on the purpose of improvements, it mainly includes three aspects: the AHS of beacon nodes, the AHS of unknown nodes, and the optimization for multilateral localization.

The standard DV-Hop employs unbiased estimation to calculate the AHS of beacons, which inevitably has a large error in AHS. Shen et al. provided a correction strategy to modify the AHS of beacons based on the difference between estimated and true distance between beacons [25]. In [26], the author indicates that the localization error usually follows Gaussian distribution. Therefore, it is more reasonable to apply the MMSE instead of unbiased estimation when calculating the AHS of beacons. However, the interaction between beacon nodes is not taken into account. In [27], Mehrabi et al. proposed a once-weighted MMSE criterion to compute the AHS. According to the hop count between beacons, all beacons are assigned different weights to balance the internal influence.

In standard DV-Hop, the AHS of unknown nodes is simply determined by the AHS of the closest beacon, which is not enough to reflect the network attributes. Therefore, more beacons should participate in the calculation of AHS. In [28], the AHS of unknown nodes is decided by the mean of AHS of all its neighboring beacons within its hop size. In [29], the AHS of partial beacons is utilized, and different weights are assigned to beacons based on the hop values. The effect of the distant beacon on unknown nodes is weakened by a smaller weight. The AHS of unknown nodes is derived after weighted normalization of the AHS of the nearest three beacons. To further ascertain the specific number of beacons involved, a connectivity-weighted scheme is presented by Chen et al. [30]. Both the network connectivity and the distances between nodes are considered, and the simulations demonstrate the optimal proportion of beacons that participated in the calculation is 10%. After that, the distance between unknown nodes and beacons is gained by the product of AHS of unknown nodes and the hop count between them.

During the multilateral positioning phase, to reduce the potential errors in solving equation systems, an enhanced localization algorithm named PERLA is introduced by Kumar et al. [31]. The distance equation composed of the closest beacon is divided by the other equations to minimize the error propagation. Moreover, the estimated position of unknown nodes is further refined by the redundant information that exists in solving equations. In [32], Gui et al. proposed a selective 3-Anchor DV-Hop, in which the coordinates of unknown nodes are only determined by the best three beacon nodes derived from all the combinations. However, this undoubtedly increases the time complexity of positioning. In [33], the author pointed out that the better positioning accuracy of unknown nodes can be achieved by using only the nearest five beacons to locate. In recent years, the boost of intelligent optimization algorithms has played a certain role in improving the positioning accuracy of standard DV-Hop. The multilateral localization is transformed into a constraint problem, and the intelligent optimization algorithms are employed to search for the optimal position of unknown nodes in the monitoring area, such as the squirrel.
search algorithm [28], genetic algorithm [34], modified artificial bee colony algorithm [30], and particle swarm optimization [24].

Although some algorithms take into account the interaction between beacons [27], different weight allocation schemes will still produce different results. Furthermore, it is still worth exploring how many times the weighted process is required for the optimal result. Inspired by this, we proposed a weighted iterative strategy to calculate the optimal AHS of beacons. Meanwhile, most algorithms are dedicated to improving the AHS of unknown nodes to amend the distance accuracy, ignoring the fact that it is also possible to calculate the distance referring to the AHS of beacons. Additionally, despite the existing algorithms proving that better positioning accuracy can be obtained by partial beacons [32,33], the number of beacons localized to each unknown node is fixed. This encourages us to find the optimal beacon node set for positioning each unknown node. Therefore, a grouping strategy is presented in this paper to extract the best beacon set suitable for localization.

3. Standard DV-Hop Algorithm

3.1. Overview of Standard DV-Hop

The standard DV-Hop was first proposed by Dragos Niculescu [35], which has always drawn the attention of researchers. The specific execution process of the standard DV-Hop principally comprises the following three phases:

Phase 1: Calculation for the minimum hop count between sensor nodes. All beacon nodes first send a package via flooding to their neighbor nodes. This packet includes its specific position and the hop values initialized to 0. The neighbor nodes save only the package from the same node with the minimum hop count and increase the value by 1. Then, the package will forward to their other neighbor nodes. By the broadcasting strategy, all sensor nodes will get the coordinates of each beacon and the minimum hop count to all beacons.

Phase 2: Computation for the AHS of sensor nodes. The AHS of beacon nodes can be determined by Equation (1) based on the unbiased estimation method. For a particular beacon, the ratio of the sum of the Euclidean distances between this beacon and the other beacons to the sum of hops between them is considered directly as AHS.

\[
AHS_i = \frac{\sum_{j=1}^{N_a} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{\sum_{j=1}^{N_a} h_{ij}}
\]

where, \( N_a \) denotes the number of beacon nodes. \((x_i, y_i)\) and \((x_j, y_j)\), individually represent the coordinates of the beacon node \( i \) and beacon node \( j \). \( h_{ij} \) is the minimum hop count between beacon node \( i \) and beacon node \( j \). \( AHS_i \) indicates the AHS of beacon node \( i \).

After that, each beacon node broadcasts its AHS to the networks. The unknown nodes just store the AHS of the beacon node first received and take it as its AHS. It can ensure that the AHS of unknown nodes will be the same as the AHS of the closest beacon node. Subsequently, the distance between unknown nodes and beacon nodes is computed by Equation (2).

\[
d_{ui} = AHS_u \times h_{ui}
\]

where, \( AHS_u \) and \( h_{ui} \), respectively, imply the AHS of unknown node \( u \) and the minimum hop count to beacon node \( i \). \( d_{ui} \) represents the estimated distance between unknown node \( u \) and beacon \( i \).

Phase 3: Calculation for the estimated coordinates of unknown nodes. The position of unknown nodes can be obtained by the multilateral method or the maximum likelihood
estimation method when the distance between nodes has been determined. Based on the existing information, we can get the following equations.

\[
\begin{align*}
\sqrt{(x_u - x_1)^2 + (y_u - y_1)^2} &= d_{u1} \\
\sqrt{(x_u - x_2)^2 + (y_u - y_2)^2} &= d_{u2} \\
&\vdots \\
\sqrt{(x_u - x_{N_u})^2 + (y_u - y_{N_u})^2} &= d_{uN_u}
\end{align*}
\]

(3)

After linearization, we have:

\[
\begin{align*}
x_1^2 + y_1^2 - x_{N_u}^2 - y_{N_u}^2 - 2x_u(x_1 - x_{N_u}) - 2y_u(y_1 - y_{N_u}) \\
= d_{u1}^2 - d_{uN_u}^2 \\
x_2^2 + y_2^2 - x_{N_u}^2 - y_{N_u}^2 - 2x_u(x_2 - x_{N_u}) - 2y_u(y_2 - y_{N_u}) \\
= d_{u2}^2 - d_{uN_u}^2 \\
\vdots \\
x_{N_u-1}^2 + y_{N_u-1}^2 - x_{N_u}^2 - y_{N_u}^2 - 2x_u(x_{N_u-1} - x_{N_u}) - 2y_u(y_{N_u-1} - y_{N_u}) \\
= d_{u(N_u-1)}^2 - d_{uN_u}^2
\end{align*}
\]

(4)

We can convert the above equations into the matrix form of \( AX = B \). Among them:

\[
A = 2 \begin{bmatrix}
(x_1 - x_{N_u}) & (y_1 - y_{N_u}) \\
(x_2 - x_{N_u}) & (y_2 - y_{N_u}) \\
&\vdots \\
(x_{N_u-1} - x_{N_u}) & (y_{N_u-1} - y_{N_u})
\end{bmatrix}
\]

(5)

\[
B = \begin{bmatrix}
x_1^2 - x_{N_u}^2 + y_1^2 - y_{N_u}^2 + d_{uN_u}^2 - d_{u1}^2 \\
x_2^2 - x_{N_u}^2 + y_2^2 - y_{N_u}^2 + d_{uN_u}^2 - d_{u2}^2 \\
\vdots \\
x_{N_u-1}^2 - x_{N_u}^2 + y_{N_u-1}^2 - y_{N_u}^2 + d_{uN_u}^2 - d_{u(N_u-1)}^2
\end{bmatrix}
\]

(6)

\[
X = \begin{bmatrix}
x_u \\
y_u
\end{bmatrix}
\]

(7)

Finally, the unknown node coordinates are determined by Equation (8) based on the least square method. A similar method is also employed in the control system theory [36].

\[
X = (A^T A)^{-1} A^T B
\]

(8)

3.2. Error Analysis of Standard DV-Hop

As shown in the network topology displayed in Figure 1, five unknown nodes and three beacon nodes are unevenly distributed in a certain region. The direct connection between nodes means that they can communicate with each other directly. Following the execution principle of standard DV-Hop, the AHS of beacon Q1 can be calculated: \((30 + 60)/(4 + 5) = 10\) m. Then, we can obtain the approximate distance from beacon Q1 to beacon Q2 and beacon Q3, 40 m and 50 m based on Equation (2), respectively, which have a significant difference from the actual distance in the network. Therefore, there is a large deviation in the AHS of beacons calculated by the standard method.
where, $AHS_{init}^i$ represents the initial AHS of beacon node $i$. $Dis_{ij}$ denotes the actual distance between beacon $i$ and beacon $j$. $\eta_{init}^i$ is the error of the initial AHS of beacon node $i$.

Figure 1. The network topology diagram.

At the same time, all unknown nodes adopt the AHS of the closest beacon node as its AHS. Hence, the unknown U1 and beacon Q1 have an equal AHS. Since the unknown U1 has the same hop count to beacon Q1 and beacon Q2, they should have the same estimated distance. However, as we can see from Figure 1, the actual distances from U1 to Q1 and Q2 are not equal. As a result, the AHS of the closest beacon node cannot exactly represent the AHS of unknown nodes. In addition, it also can be observed that the beacon node is far away from U1, which also has larger hop values. There may be much deviation in the estimated distance, which will further influence the node localization accuracy.

4. Our Proposed DV-Hop Scheme

4.1. Weighted Iterative Strategy for the AHS of Beacon Nodes

During the positioning stage of unknown nodes, if the AHS of each beacon node is not accurate, it will increase the error of the results obtained in subsequent steps. Therefore, it is necessary to select an accurate way of computing the AHS. Because the sensor nodes are randomly deployed in the detection area, the nodes should have different influences on each other in positioning. The proposed scheme firstly adopts the MMSE criterion to compute the initial AHS of beacons and then assigns different weights based on the per-hop error to each beacon to balance the influence of the beacon with a large per-hop error. The process of calculating the AHS and assigning weights will continue until the end condition is met. At last, the AHS with the smallest error can be defined as the final AHS of beacons.

The detailed process of our proposed scheme for AHS is described as follows:

**ST1:** Get the initial AHS and its error. Assuming all beacons have the same influence on each other at first, the initial AHS of each beacon can be obtained by Equation (9) according to the MMSE criterion [26]. At the same time, the actual distance between beacon nodes is known in advance, and the estimated distance can be replaced by the product of AHS and hop count. Therefore, the error in the initial AHS can be obtained by Equation (10).

$$AHS_{init}^i = \frac{\sum_{j \neq i}^{N_a} Dis_{ij} \cdot h_{ij}}{\sum_{j \neq i}^{N_a} h_{ij}^2}$$

$$\eta_{init}^i = \frac{\sum_{j \neq i}^{N_a} |Dis_{ij} - AHS_{init}^i \cdot h_{ij}|}{N_a - 1}$$

where, $AHS_{init}^i$ represents the initial AHS of beacon node $i$. $Dis_{ij}$ denotes the actual distance between beacon $i$ and beacon $j$. $\eta_{init}^i$ is the error of the initial AHS of beacon node $i$. 
**STEP 2:** Calculate the per-hop error between beacons. The per-hop error between beacons can be obtained by the following equation.

\[ EH_{ij} = \left| \frac{\text{Dis}_{ij} - AHS_i^* \cdot h_{ij}}{h_{ij}} \right| \]  

(11)

where, \( EH_{ij} \) is the per-hop error from beacon \( i \) to beacon \( j \). \( AHS_i^* \) represents the initial AHS or the AHS obtained by iteration of beacon node \( i \).

**STEP 3:** Obtain the weighted AHS. The mutual influence between beacons can be reflected by the per-hop error. If the error of each hop between beacon node \( i \) and beacon node \( j \) is large, beacon node \( j \) will be assigned a smaller weight value to weaken the influence on the AHS of beacon node \( i \). Otherwise, beacon node \( j \) will be given a bigger weight. After that, the AHS can be updated by the weighted MMSE criterion. The calculation equation of the weights and the weighted AHS is given below.

It is worth mentioning that since the differences between the estimated distance and actual distance cannot be completely cleared, the denominator of Equation (12) will not be zero.

\[ \alpha_{ij} = \left( \frac{1}{EH_{ij}} \right)^2 \]  

(12)

\[ AHS_i^{(m)} = \frac{\sum_{j \neq i} N_a \cdot \alpha_{ij} \cdot \text{Dis}_{ij} \cdot h_{ij}}{\sum_{j \neq i} N_a \cdot \alpha_{ij} \cdot h_{ij}^2} \]  

(13)

where, \( \alpha_{ij} \) is the weight value for beacon \( j \) when calculating the AHS of beacon \( i \). \( AHS_i^{(m)} \) is the AHS of beacon \( i \) obtained by the \( m \)-th iteration.

**STEP 4:** Compute the error of weighted AHS. The evaluation criteria of weighted AHS are the same as Equation (10). It is worth mentioning that this scheme is also used in the work of finite-time-scaled consensus through parametric linear iterations [37].

\[ \eta_i^{(m)} = \frac{\sum_{j \neq i} N_a \left| \text{Dis}_{ij} - AHS_i^{(m)} \cdot h_{ij} \right|}{N_a - 1} \]  

(14)

where, \( \eta_i^{(m)} \) indicates the error of AHS obtained in the \( m \)-th iteration of beacon node \( i \).

**STEP 5:** Determine the optimal AHS. If the value of \( \eta_i^{(m)} \) is less than the \( \eta_i^{(m-1)} \), go back to STEP 2 and recalculate the per-hop error using \( AHS_i^{(m)} \). Hence, a new AHS of beacon node \( i \) can be computed by Equation (13) again. According to the above process, the weighted iterative calculation is repeatedly performed on the AHS of the beacon node. For a certain iteration, if the value of \( \eta_i^{(m)} \) is bigger than the \( \eta_i^{(m-1)} \), the iteration stops, and the AHS obtained in the \( (m-1) \)-th iteration is adopted as the optimal AHS of beacon \( i \).

Through the weighted iterative calculation, each beacon node can find its optimal AHS, which can effectively reduce the cumulative error for locating unknown nodes. It is worth noting that, in a particular case, the initial AHS of the beacon node is sufficiently accurate, and the weighted iterative will not be required. For most beacon nodes, the iterative process will execute until the optimal solution is encountered. The weighted iterative process of computing the optimal AHS of beacon nodes is shown in Figure 2.
4.2. Calculation of Estimated Distance between Nodes

All unknown nodes take the AHS of the closest beacon node as its AHS in the standard DV-Hop and use the product of its AHS and the hop count as the estimated distance. However, the AHS of unknown nodes in real networks may differ significantly from the AHS of the nearest beacon node, which will exacerbate the estimated distance deviation and further influence the localization precision of unknown nodes.

Since the relatively accurate AHS of beacon nodes has been obtained, the procedure of computing the AHS of unknown nodes can be cut out. We can adopt the product of the optimal AHS of beacon nodes and the least hop count to unknown nodes as the estimated distance. Equation (15) is applied to find the estimated distance between nodes. By this method, not only the calculation process of the DV-Hop algorithm is simplified, but also the distance deviation between nodes is further reduced.

\[
d_{ui} = AHS_{i}^{best} \times h_{ui} \tag{15}
\]

where, \(AHS_{i}^{best}\) is the optimal AHS of beacon node \(i\) obtained by weighted iteration strategy.

4.3. Determining Optimal Beacon Set for Unknown Nodes

The standard DV-Hop and other existing DV-Hops generally arrange all beacon nodes to calculate the coordinates of unknown nodes. However, as the hop count between
nodes within the communication range is treated as one hop, this causes a certain distance deviation between nodes and beacon nodes farther away from unknown nodes usually have large hop values, which means that there may be a big deviation in the estimated distance and true distance. If there is a beacon node with a large distance error to the unknown node in the multilateral positioning process, it will notably reduce the localization precision of the unknown nodes. This inspires us to propose a new scheme to determine the optimal beacon nodes for locating each unknown node. Meanwhile, in linearizing the distance equations, the first \( n - 1 \) equations will subtract the last equation. However, if there is a large error in the last equation, the results will be significantly biased. To reduce the potential error in the set of equations, we can let each distance equation be defined as the last equation in turn. If the positioning process involves \( n \) beacon nodes, we can get \( n \) candidate coordinates of the unknown nodes. Then, the result with the lowest error is taken as the optimal solution.

To find the optimal beacon nodes suitable for localization, a grouping strategy is introduced based on the estimated distance between unknown nodes and beacon nodes. Generally, the farther the beacon node is from the unknown node, the larger the estimated distance error may be in between. Furthermore, the minimum number of beacon nodes for locating an unknown node is three. Based on the above constraints, all beacons can be divided into \( N_a - 2 \) different sets, denoted as \( \text{ANSET}_{k>2} \) \( (3 \leq k \leq N_a) \). \( \text{ANSET}_{k>2} \) represents the \( k \) beacon nodes closest to the unknown node. Subsequently, each beacon set is employed to calculate the coordinates of the unknown node based on the least-square method, respectively. Moreover, to avoid the hidden errors in the linearization process of the distance equations, the distance equation constructed by each beacon node is, in turn, used as the last equation to calculate the position. For a certain beacon set, if the ANSET contains \( k \) beacon nodes, \( k \) candidate coordinates of the unknown node will be yielded. Then, from the \( k \) candidate coordinates, the one with the smallest error is selected as the optimal coordinates of this ANSET. Finally, the final position of the unknown node is determined after comparing the optimal coordinates generated by different ANSETs.

After getting the candidate coordinates for each ANSET, an evaluation indicator is needed to calculate the quality of the solution position. Although the exact position of unknown nodes is unknown during positioning, the coordinates of each beacon are known in advance. Meanwhile, the estimated distance between unknown nodes and beacons is obtained in the previous steps. Therefore, the corresponding error for each candidate coordinate can be evaluated by Equation (16).

\[
\gamma = \frac{1}{N_u} \sum_{i=1}^{N_u} \left( \sqrt{(x_u^i - x_i)^2 + (y_u^i - y_i)^2} - d_{ui} \right)^2 
\]

where, \((x_u^i, y_u^i)\) is the candidate coordinates of unknown node \( u \).

To better explain our proposed approach, the network structure depicted in Figure 3 is utilized as an example. All sensor nodes are unevenly distributed in the area of 50 m \( \times \) 50 m. The communication range of sensors is 20 m. Assuming that the unknown node \( U \) (the true coordinate is (24.14, 25.73)) is to be located, three different ANSETs can be formed individually according to the distance between unknown nodes and beacons. Then, the corresponding candidate coordinates and errors, respectively, are calculated using the least-squares method and Equation (16). Table 1 displays the calculation results. Comparing the error of the optimal candidate coordinates obtained by different ANSET, the candidate coordinates (26.43, 24.29) are selected as the final coordinates of \( U \). From this example, we can see that the best position of node \( U \) is only determined by the part of beacons, as of Q2, Q3, Q4, and Q1.
Table 1. The estimated coordinates and errors for unknown node $U$.

<table>
<thead>
<tr>
<th>ANSET</th>
<th>Beacon Nodes</th>
<th>Candidate Coordinate</th>
<th>Error</th>
<th>Optimal Coordinate</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANSET&lt;3&gt;</td>
<td>Q4, Q3, Q1</td>
<td>(16.79, 27.02)</td>
<td>61.79</td>
<td>(16.79, 27.02)</td>
<td>61.79</td>
</tr>
<tr>
<td>ANSET&lt;3&gt;</td>
<td>Q1, Q4, Q3</td>
<td>(16.79, 27.02)</td>
<td>61.79</td>
<td>(16.79, 27.02)</td>
<td>61.79</td>
</tr>
<tr>
<td>ANSET&lt;3&gt;</td>
<td>Q1, Q3, Q4</td>
<td>(16.79, 27.02)</td>
<td>61.79</td>
<td>(16.79, 27.02)</td>
<td>61.79</td>
</tr>
<tr>
<td>ANSET&lt;4&gt;</td>
<td>Q2, Q3, Q4, Q1</td>
<td>(26.43, 24.29)</td>
<td>34.70</td>
<td>(26.43, 24.29)</td>
<td>34.70</td>
</tr>
<tr>
<td>ANSET&lt;4&gt;</td>
<td>Q1, Q2, Q4, Q3</td>
<td>(32.46, 27.33)</td>
<td>49.40</td>
<td>(32.46, 27.33)</td>
<td>49.40</td>
</tr>
<tr>
<td>ANSET&lt;4&gt;</td>
<td>Q1, Q3, Q2, Q4</td>
<td>(33.94, 29.05)</td>
<td>66.70</td>
<td>(33.94, 29.05)</td>
<td>66.70</td>
</tr>
<tr>
<td>ANSET&lt;4&gt;</td>
<td>Q1, Q3, Q4, Q2</td>
<td>(35.14, 27.65)</td>
<td>62.23</td>
<td>(35.14, 27.65)</td>
<td>62.23</td>
</tr>
<tr>
<td>ANSET&lt;5&gt;</td>
<td>Q5, Q3, Q4, Q2, Q1</td>
<td>(15.07, 18.25)</td>
<td>37.66</td>
<td>(32.57, 24.36)</td>
<td>35.69</td>
</tr>
<tr>
<td>ANSET&lt;5&gt;</td>
<td>Q1, Q5, Q4, Q2, Q3</td>
<td>(32.57, 24.36)</td>
<td>35.69</td>
<td>(32.57, 24.36)</td>
<td>35.69</td>
</tr>
<tr>
<td>ANSET&lt;5&gt;</td>
<td>Q1, Q3, Q5, Q2, Q4</td>
<td>(20.18, 22.93)</td>
<td>43.18</td>
<td>(20.18, 22.93)</td>
<td>43.18</td>
</tr>
<tr>
<td>ANSET&lt;5&gt;</td>
<td>Q1, Q3, Q4, Q5, Q2</td>
<td>(21.68, 21.02)</td>
<td>38.18</td>
<td>(21.68, 21.02)</td>
<td>38.18</td>
</tr>
<tr>
<td>ANSET&lt;5&gt;</td>
<td>Q1, Q3, Q4, Q2, Q5</td>
<td>(14.31, 17.66)</td>
<td>36.06</td>
<td>(14.31, 17.66)</td>
<td>36.06</td>
</tr>
</tbody>
</table>

4.4. The Complete Process of Our Proposed DV-Hop

The pseudocode of our proposed DV-Hop is exhibited in Algorithm 1. Stage1 is the same part as the standard DV-Hop. Our innovations primarily focus on Stage2, Stage3, and Stage4, which are the biggest differences from the standard DV-Hop.

Algorithm 1 The Pseudo Code for Our Proposed DV-Hop

Input: Monitoring region; total number of sensor nodes; total number of beacon nodes; communication range of sensors;
Stage 1: All sensor nodes obtain the coordinates of each beacon and the minimum hop count to each beacon by flooding strategy;
Stage 2: Calculate the optimal AHS of each beacon;

for $i = 1$: Number of beacons do
  calculate the initial AHS of the beacon node by Equation (9);
  determine the error of initial AHS by Equation (10);
  while
    calculate the per-hop error between beacons based on Equation (11);
    obtain the weighted AHS according to Equations (12) and (13);
    determine the error of weighted AHS by Equation (14);
    if the error of new AHS > error of previous iteration obtained AHS then
      take the pervious iteration obtained AHS as the optima AHS;
      break;
    end if
  end while
end for
Algorithm 1 Cont.

Stage 3: Estimate the distance between unknown nodes and beacon nodes;
for \( u = 1: \text{number of unknowns} \) do
  calculate the estimated distance from \( u \) to each beacon by Equation (15);
end for
Stage 4: Determine the coordinates of unknowns by optimal beacon nodes;
for \( u = 1: \text{number of unknowns} \) do
  for \( s = 1: \text{number of ANSET} \) do
    let each beacon in ANSET as the last equation in turn;
    calculate the candidate coordinates by the least square method;
    compute the error of each candidate coordinates using Equation (16);
    select the candidate coordinates with minimal error as the optimal position of this ANSET.
  end for
  compare the optimal position of each ANSET and take the smallest error as the final coordinates.
end for
Output: The position of each unknown node.

5. Simulation and Results Analysis

5.1. Experimental Setup and Evaluation Criteria

To prove the performance capability of our proposed algorithm, simulation experiments are conducted based on MATLAB R2015a. The hardware configuration of the computer is as follows: 16 GB running memory, I5 processor, and windows 10 operating system. The parameter settings of the simulation environment are displayed in Table 2, which are consistent with the reference [10]. The beacon nodes are randomly selected from all the sensor nodes. In addition, to avoid the contingency of experimental results, all results are the average of 100 independent experiments under the same simulation conditions.

1. The precision of the AHS of each beacon node is determined as below:

\[
AHS_{Error_i} = \frac{\sum_{j \neq i} |AH S_{best}^i \cdot h_{ij} - D i s_{ij}|}{R \cdot (N_a - 1)} \tag{17}
\]

where, \( R \) represents the communication radius of sensors.

2. The average normalized distance error of the network is computed by Equation (18).

\[
ANDE = \frac{\sum_{u=1}^{N_u} \sum_{j=1}^{N_a} |d_{uj} - D i s_{uj}|}{R \cdot N_a \cdot N_u} \tag{18}
\]

where, \( N_u \) denotes the total number of unknown nodes. \( D i s_{uj} \) indicates the actual distance between unknown node \( u \) and beacon node \( j \).

3. The average normalized localization error of the network is defined as:

\[
error_u = \sqrt{(x_{u, true} - x_{u, est})^2 + (y_{u, true} - y_{u, est})^2} \tag{19}
\]

\[
ANLE = \frac{\sum_{u=1}^{N_u} error_u}{R \cdot N_u} \tag{20}
\]

where, \((x_{u, est}, y_{u, est})\) and \((x_{u, true}, y_{u, true})\) are, respectively, the estimated coordinates and the actual coordinates of unknown node \( u \). ANLE is the average normalized localization error of the whole network.
4. The standard deviation of average normalized localization error can be calculated by the following equation.

\[
\text{error}_{\text{network}} = \frac{\sum_{u=1}^{N_u} \sqrt{(x_{u,\text{true}} - x_{u,\text{est}})^2 + (y_{u,\text{true}} - y_{u,\text{est}})^2}}{N_u}
\]

\[
\text{SDE} = \sqrt{\frac{\sum_{u=1}^{N_u} (\text{error}_u - \text{error}_{\text{network}})^2 / N_u}{R}}
\]

where, \( \text{error}_{\text{network}} \) is the mean localization error of all unknown nodes. SDE represents the standard deviation of localization error and the positioning stability.

Table 2. Simulation environment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node deployment</td>
<td>random</td>
</tr>
<tr>
<td>Monitoring region</td>
<td>100 m \times 100 m</td>
</tr>
<tr>
<td>Sensor nodes</td>
<td>100</td>
</tr>
<tr>
<td>Beacon nodes</td>
<td>30</td>
</tr>
<tr>
<td>Communication radius</td>
<td>30 m</td>
</tr>
</tbody>
</table>

5.2. Evaluation for AHS of Beacon Nodes

Under the simulation settings mentioned in Section 5.1, the precision of the AHS of beacon nodes obtained by our proposed algorithm is compared with the standard DV-Hop, Chen’s algorithm [26], and Mehrabi’s algorithm [27]. To get the AHS of beacon nodes, Chen’s algorithm uses the MMSE criterion, while Mehrabi’s algorithm adopts the once-weighted MMSE criterion. Figure 4 illustrates that the precision of the Chen’s algorithm is superior to the standard DV-Hop and Mehrabi’s algorithm. In contrast, the AHS accuracy of beacon nodes obtained by Mehrabi’s algorithm is relatively poor compared to standard DV-Hop. The experimental results expose that MMSE is more reasonable than unbiased estimation in AHS calculation, but the accuracy of AHS will markedly decrease if the weights are not properly allocated. In addition, the AHS accuracy of beacon nodes calculated by our proposed scheme performs better than the other three algorithms, which indicates our weight allocation and iterative strategy can effectively improve the AHS precision of beacons. In more detail, the average normalized errors of AHS of the whole network are, respectively, 0.2063, 0.2056, 0.2079, and 0.2045 for the standard DV-Hop, Chen’s algorithm, Mehrabi’s algorithm, and the proposed DV-Hop.

5.3. Accuracy of Estimated Distance

To lessen the computational effort and enhance the precision of the estimated distance, the calculation process for the AHS of unknown nodes is cut out. The proposed scheme is compared with the standard DV-Hop, Zhao’s algorithm [29], and Chen’s algorithm [30], in terms of the estimated distance precision. Both the Zhao’s algorithm and Chen’s algorithm employ a weighted approach when finding the AHS of unknown nodes. Figure 5 depicts the distribution of distance error from unknown nodes to beacon nodes obtained by various positioning algorithms under 100 independent simulations. ANDE is the average normalized distance error between unknown nodes and the beacons, which is defined in Equation (18). The red line and the green circle in the blue box represent the median and mean of the experimental results, respectively. As shown in Figure 5, the Zhao’s algorithm and Chen’s algorithm are slightly superior to the standard DV-Hop. The results reveal that the distance accuracy can be improved to a certain degree by the weighting method. Furthermore, it can easily find that the proposed algorithm achieves a markedly lower median and lower mean values than the other three algorithms. More precisely, the mean ANDE of the standard DV-Hop, Zhao’s algorithm, Chen’s algorithm, and the proposed one are 0.2178, 0.2152, 0.2142, and 0.2122, respectively. Therefore, it is possible to
effectively decrease the estimated distance deviation from the actual distance by removing the procedure of calculating the AHS of unknown nodes.

![Figure 4. Precision comparison of the AHS of each beacon node.](image)

**Figure 4.** Precision comparison of the AHS of each beacon node.

![Figure 5. Comparison for the distance precision between unknown nodes and beacon nodes.](image)

**Figure 5.** Comparison for the distance precision between unknown nodes and beacon nodes.

### 5.4. Positioning Performance

In this section, to thoroughly investigate the positioning performance of our proposed scheme, three typical DV-Hop schemes are selected as benchmark algorithms, including standard DV-Hop [35], PSO DV-Hop [24], and Selective 3-Anchor DV-Hop [32]. All beacon nodes are used directly for locating unknown nodes in standard DV-Hop and PSO DV-Hop, while only the best three beacons are employed in Selective 3-Anchor DV-Hop. Meanwhile, the parameter settings of these benchmarks are the same as in the original paper.

#### 5.4.1. Normalized Localization Error

Based on the simulation environment described in Table 2, the localization error of each unknown node obtained by benchmarks and the proposed one is plotted in Figure 6. The blue squares and red circles represent the unknown nodes and beacon nodes, respectively. The positioning deviation between the estimated location and actual location of unknown
nodes is depicted as a black straight line. The longer the line, the larger the positioning error of the unknown node. Figure 6 visually illustrates which and where unknown nodes are positioned precisely, and which has a larger deviation. Detailed data statistics of localization error are summarized in Table 3.

![Diagram](image)

**Figure 6.** Normalization localization error of each unknown node: (a) Standard DV-Hop; (b) PSO DV-Hop; (c) Selective 3-Anchor DV-Hop; (d) Proposed DV-Hop.

**Table 3.** Summary of positioning results of different DV-Hop algorithms.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Min Error</th>
<th>Max Error</th>
<th>Avg Error</th>
<th>Error &gt; R/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard DV-Hop</td>
<td>0.0381R</td>
<td>1.0373R</td>
<td>0.3017R</td>
<td>10</td>
</tr>
<tr>
<td>PSO DV-Hop</td>
<td>0.0234R</td>
<td>0.7634R</td>
<td>0.2415R</td>
<td>3</td>
</tr>
<tr>
<td>Selective 3-Anchor DV-Hop</td>
<td>0.0228R</td>
<td>0.7256R</td>
<td>0.1874R</td>
<td>1</td>
</tr>
<tr>
<td>Proposed DV-Hop</td>
<td>0.0070R</td>
<td>0.6340R</td>
<td>0.1320R</td>
<td>1</td>
</tr>
</tbody>
</table>

The many long black lines presented in Figure 6a mean that the positioning error of standard DV-Hop is larger. Meanwhile, the black lines in Figure 6a to Figure 6d get progressively shorter. As listed in Table 3, we can find that the average error, maximum error, minimum error, and the number of errors exceeding R/2 of standard DV-Hop are significantly higher than the other comparison algorithms. It also shows that the proposed scheme has a better positioning performance. More precisely, the average positioning error of standard DV-Hop, PSO DV-Hop, Selective 3-Anchor DV-Hop, and the proposed DV-Hop are close to 9.05 m, 7.25 m, 5.62 m, 3.96 m, respectively. Compared with the standard
DV-Hop, PSO DV-Hop, and Selective 3-Anchor DV-Hop, the average localization precision of the proposed one is optimized by 56.25%, 45.34%, 29.56%, individually.

5.4.2. Effect of Node Density

In this experiment, to investigate the impact of node density on ANLE and SDE, the number of sensors in the detected area increases from 100 to 200 with 20 as a step size. The other simulation parameter remains fixed, as depicted in Table 2. The simulation results, as shown in Figure 7, indicate that there is no significant impact on ANLE and SDE by increasing the total number of nodes in the network. The results suggest that the node density is not the key factor affecting the positioning accuracy of DV-Hop algorithms when the beacon ratio and communication radius are kept constant. Obviously, it can be observed that the standard DV-Hop has the worst positioning accuracy and stability. Furthermore, in terms of ANLE and SDE, the results also demonstrate that the proposed DV-Hop has an evident improvement over the other three benchmarks, no matter what scenario. From the results, the mean ANLE in different node densities of the standard DV-Hop, PSO DV-Hop, Selective 3-Anchor DV-Hop, and the proposed DV-Hop is 0.2941, 0.2400, 0.1837, and 0.1423 with an average SDE of 0.1648, 0.1471, 0.0993, and 0.0795, respectively.

![Figure 7](image-url)

**Figure 7.** The effect of node density on ANLE and SDE: (a) normalized localization error; (b) localization stability.

5.4.3. Effect of Beacon Ratio

To evaluate how the beacon ratio affects the ANLE and SDE, the number of sensor nodes is kept constant at 100 and all sensor nodes possess the same communication radius of 30 m. Figure 8 displays the node localization error and stability of different algorithms as the number of beacon nodes incrementally changes from 15 to 45. We can see from Figure 8a that the positioning error of each algorithm presents a descending tendency as the ratio of beacon nodes gradually increases. The results imply that a higher beacon ratio is conducive to ameliorating the positioning precision to some degree. At the same time, the curve declining rate of our proposed and the two enhanced DV-Hop is significantly faster than the standard DV-Hop. For test scenarios, when the number of beacons varied from 15 to 45, the ANLE was reduced by 13.56%, 32.57%, 29.89%, and 29.94% for the standard DV-Hop, PSO DV-Hop, Selective 3-Anchor DV-Hop, and the proposed DV-Hop, individually. This is due to the fact that the increase in the beacon ratio resulted in a promotion in the precision of the AHS of beacons. Therefore, the accuracy of the estimated distance between beacon nodes and unknown nodes is improved, which can further reduce the node localization error. In addition, it can be concluded that our proposed algorithm has a comparative lower positioning error and better stability in the different scenarios.
5.4.4. Effect of Communication Range

To verify how the communication radius affects the ANLE and SDE, the communication radius changes from 20 m to 50 m, and other parameters are the same as mentioned in Section 5.1. Figure 9 shows the simulation results. It displays that the localization error of each DV-Hop scheme drops sharply at first and then gradually tends to be stable as the communication range expands. Specifically, the ANLE of the standard DV-Hop, PSO DV-Hop, Selective 3-Anchor DV-Hop, and the proposed one declines notably from 0.4479, 0.3019, 0.2956, and 0.2814 to 0.2929, 0.2432, 0.1932, and 0.1523 as the communication radius changes from 20 m to 30 m. The main reason for this phenomenon is that when the node communication range is small, the connectivity of the network is reduced, and the hop count between nodes is increased. This will lead to a huge deviation in the estimated distance between unknown nodes and beacon nodes. Consequently, all unknown nodes have a lower positioning accuracy. However, the network structure gradually returns to normal with the increase in the communication range, and, thus, the positioning performance of all algorithms is improved. Moreover, the proposed DV-Hop is superior to other DV-Hop algorithms in terms of positioning precision and stability.

Figure 8. The effect of beacon ratio on ANLE and SDE: (a) normalized localization error; (b) localization stability.

Figure 9. The effect of communication range on ANLE and SDE: (a) normalized localization error; (b) localization stability.
6. Conclusions

The position information of sensors plays a critical role in the WSN area. This study aims at improving the positioning performance of the standard DV-Hop to obtain the specific location of sensors in the detection region. The unbiased estimation method is replaced by the weighted MMSE criterion to iteratively calculate the optimal AHS of beacon nodes. The scheme to estimate the distance between nodes is redesigned. Moreover, the optimal beacons suitable for localization are found by a grouping strategy, rather than just using all beacons to locate directly. Simulation results prove that the proposed DV-Hop has better positioning performance, with an average positioning error as low as 3.96 m. Compared with standard DV-Hop, PSO DV-Hop, and Selective 3-Anchor DV-Hop, the average localization error of the proposed DV-Hop is reduced by 56.25%, 45.34%, and 29.56%, respectively.

Our future work will focus on the following directions: (1) Our current research is mainly aimed at two-dimensional scenarios; how to further improve and extend the proposed DV-Hop to three-dimensional scenarios is our primary task. (2) Due to the weighted iteration and grouping strategy, the computational complexity of the proposed one is slightly higher than that of the standard DV-Hop. On the premise of ensuring positioning accuracy and stability, how to reduce the complexity of the algorithm is still worth further research.

Author Contributions: Conceptualization, T.C.; Methodology, and Formal Analysis, T.C. and S.H.; Writing—original draft preparation, S.H. and L.S.; review and editing, S.H. and L.S.; Visualization, T.C. and K.S.; Supervision, L.S. and K.S.; funding acquisition, L.S. and T.C. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by The National Natural Science Foundation of China (grant number: 62173127, 61803146, 61973104), The Scientific and Technological Innovation Leaders in Central Plains (grant number: 224200510008), The Henan Excellent Young Scientists Fund (grant number: 212300410036), The Program for Science and Technology Innovation Talents in Universities of Henan Province (grant number: 21HASTIT029), The Training Program for Young Backbone Teachers in Universities of Henan Province (grant number: 2019GJS089), The Innovative Funds Plans of Henan University of Technology (grant number: 2020ZKCJ06), The Zhengzhou Science and Technology Collaborative Innovation Project (grant number: 21ZZXTCX06), The Cultivation Program of Young Backbone Teachers in Henan University of Technology (grant number: 21420080), and The Open Fund from Research Platform of Grain Information Processing Center in Henan University of Technology (grant number: KFJJ2020107).

Data Availability Statement: The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:
- WSN: wireless sensor networks
- DV-Hop: distance vector hop
- MMSE: minimum mean square error
- AHS: average hop size
- IoT: internet of things
- GPS: global positioning system
- LoS: line of sight
- RSSI: received signal strength indicator
- TDOA: time difference of arrival
- TOA: time of arrival
- APIT: approximate point in triangle
References


37. Shang, Y. Finite-time scaled consensus through parametric linear iterations. *Int. J. Syst. Sci.* 2017, 48, 2033–2040. [CrossRef]