Abstract: In the article, an unknown system dynamics estimator-based anti-disturbance attitude funnel control technique is considered for quadrotors to ensure tracking performance when experiencing parametric uncertainties and external perturbations. To reinforce the anti-disturbance ability, an unknown system dynamics estimator was established by constructing the filtering operation-based invariant manifold, resulting in a more concise design framework, lower computational consumption and an asymptotic error convergence. Additionally, a funnel control policy was employed to regulate angle-tracking errors within a minor overshoot, a faster convergence time and a lower steady-state error by devising the funnel variables, where an exponential decaying function was used to construct the funnel boundary. The great improvements beyond the available quadrotor control policies are related to satisfied disturbance mitigation and performance guarantees as a priority despite uncertainties. The error arguments comprising of angle and angular rate for quadrotors were ultimately uniformly bounded and the angles controlling the deviations were restricted to the funnel boundary. Finally, the simulations and experiments verified the superiority of the proposed control technique in terms of reduced control actions and higher precision, as well as shorter settling time.

Keywords: unknown system dynamics estimator; funnel control; anti-disturbance; quadrotor
perturbations. In Ullah et al. [13], a radial basis function neural network-based robust adaptive control was introduced to eliminate the performance degradation caused by the unknown system dynamics. With the aid of the above observers, excellent disturbance identification can be ensured and the robustness can be improved. However, certain defects including the unexpected chattering phenomenon in [5–8] and complex argument selections impose restrictions on engineering practices to a certain extent. In addition, it is generally difficult to make a compromise between the peak phenomenon and the convergence speed of estimation errors by using ESO [9–11]. On the other hand, the tedious iteration calculations in [12–14] caused by weight updating inevitably increase the computational burden and deteriorate control response. Notably, by filtering the available system states and introducing the invariant manifold theory, the unknown system dynamics estimator (USDE) is designed to identify the unknown disturbances. In Wang et al. [15], by designing an USDE-based sliding mode control policy for servo mechanisms, a fast convergence and strong anti-disturbance ability could be obtained. In Na et al. [16], an USDE-based motion control was proposed for robotic systems to address the internal unknown dynamics and environmental disturbances. Compared to the aforementioned disturbance observers [5–14], the USDE possesses a convenient parameter tuning rule and a satisfied estimation accuracy; however, the existing USDE-based outcomes are oriented towards simple integral-chain systems, which cannot be directly applied to the quadrotor attitude with obvious coupling and multiple variables, yielding to a more complicated controller development and stability analysis. Additionally, the key metrics, i.e., overshoot, convergence time and steady-state errors, are not actively considered during the above control developments.

To regulate transient and steady-state profiles obeying the predetermined performance boundary, a funnel control (FC) was exploited in Ilchmann et al. [17] by replacing conventional tracking errors with a funnel variable by devising a funnel boundary. For instance, in Berger et al. [18], by regulating the tracking errors within the predefined funnel boundary, an FC-based tracking control with low complexity was considered for nonlinear systems. In Xu et al. [19], an FC-based feedback control was designed for hydraulic systems subject to model uncertainties, resulting in remarkable tracking performance. In Wang et al. [20], by introducing a novel funnel variable to FC, a differentiator-based adaptive control rule was implemented for servosystems to realize an asymptotic convergence. In Zahedi et al. [21], an FC-based sliding mode control policy was introduced for nonlinear system to govern the transient and steady-state convergence. To enable the tracking errors to converge to steady-state with the constraint conditions taken into consideration, an FC-based asymptotic tracking control method was proposed for a class of nonlinear systems in Verginis et al. [22]. Evidently, by exploiting a monotonous funnel function-based funnel variable into the controller design process, the tracking profiles are governed to the predefined funnel region with a satisfied overshoot, convergence time and precision. However, the previous literature [17–22] is merely applied to enforce the anti-disturbance performance for a class of simple single input and single output systems, the existing results cannot be easily extended to the complex system model, such as quadrotor dynamics characterized by strong coupling and multiple inputs and multiple outputs. Furthermore, there is no relevant experimental verification for FC-based quadrotor attitude control. Thus, it is still an open field to develop an FC-based tracking control policy to preassign quadrotor attitude errors.

Inspired by the above statements, an unknown system dynamics estimator-based anti-disturbance attitude FC policy was exploited for quadrotors undergoing total disturbance; the highlights of our paper are generalized as follows:

- Distinguishing it from the existing sliding mode observer [5–8]-based control strategies, where chattering cannot be efficaciously surmounted by raising the deployment of the sign function, the unknown system dynamics estimator (USDE) is presented for quadrotor attitude dynamics to identify the unmeasurable perturbations by utilizing an invariant manifold principle. In addition, differing from previous function approximator-based control schemes [12–14], by using simple filtering calculations for
the available quadrotor states, a more concise design frame and a lower calculation burden could be guaranteed.

- In contrast to the conventional controllers \([23,24]\), where the error decaying process of the quadrotors is governed by some unknown terms and only ultimately uniformly bounded (UUB) results are delivered, here, by predefining the funnel bound-based funnel variables, an FC was constructed to govern the transient and steady-state profiles as a priority, such that the overshoot, convergence time and steady-state errors could be regulated within the appropriate range. Significant efforts were devoted to ensure system stability and error convergence analysis, which is more challenging than the existing FC-based controllers without considering uncertainties and USDE-related outcomes with no performance constraints. More importantly, several real-time validations concerning the suggested schemes on quadrotor platform were conducted for the first time.

2. Problem Statements

As seen in Figure 1, an inertial-fixed coordinate \(\{E\}\) and a body-fixed coordinate \(\{B\}\) were firstly established to promote the description of quadrotor attitude dynamics, where \(F_i\) represents the rotor thrusts produced by the rotor actuations. By resorting to \([25,26]\), the mathematical attitude model with external disturbances is typically expressed as

\[
\begin{align*}
\dot{\Theta} &= \Gamma \omega \\
J \dot{\omega} &= -\omega \times J \omega + F + d
\end{align*}
\]  

(1)

where \(\Theta = [\phi, \theta, \psi]^T\), and \(\phi, \theta, \psi\) represent the roll, pitch and yaw angles in frame \(\{E\}\). \(\omega = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^T\), \(\dot{\phi}, \dot{\theta}, \dot{\psi}\) represent the angular rates in frame \(\{B\}\). The definite positive diagonal matrix \(J = \text{diag}(J_\phi + J_{\phi}^* + J_\theta + J_{\theta}^* + J_\psi + J_{\psi}^*)\), where \(J_\phi, J_\theta, J_\psi\) stand for nominal moments of inertia and \(J_{\phi}^*, J_{\theta}^*, J_{\psi}^*\) are defined as the inertia moment uncertainties. \(F = [F_\phi, F_\theta, F_\psi]^T\) is control input vector. \(d = [d_\phi, d_\theta, d_\psi]^T\) describes the external environmental disturbances in attitude kinetics. Furthermore, the matrix relative to the available states in attitude loop is expressed as

\[
\Gamma = \begin{pmatrix}
1 & \tan \theta \sin \phi & \tan \theta \cos \phi \\
0 & \cos \phi & -\sin \phi \\
0 & \sec \theta \sin \phi & \sec \theta \cos \phi
\end{pmatrix}
\]

(2)

In line with (1), it can be further derived as

\[
\ddot{\Theta} = \tilde{\Gamma} \dot{\omega} + \Gamma (J_\Theta)^{-1} (-\omega \times J_\Theta \dot{\omega} + d - J_\Theta^{*} \dot{\omega}) + (J_\Theta)^{-1} \Gamma F
\]

(3)

where \(J_\Theta = \text{diag}(J_\phi, J_\theta, J_\psi), J_\Theta^* = \text{diag}(J_{\phi}^*, J_{\theta}^*, J_{\psi}^*)\).
Giving the definition of $\Delta = \hat{\Gamma} \omega + \Gamma (J_\Theta)^{-1} (-\omega \times J \omega + d - J_\Theta \dot{\omega})$ and $u = (J_\Theta)^{-1} \Gamma F$, it can be simplified as follows:

$$
\begin{cases}
\dot{\Omega}_1 = \Omega_2 \\
\dot{\Omega}_2 = u + \Delta
\end{cases}
$$

where $\Omega_1 = \Theta = [\Omega_{1,\phi}^T, \Omega_{1,\theta}^T, \Omega_{1,\psi}^T]^T$, $\Omega_2 = \Gamma \omega = [\Omega_{2,\phi}^T, \Omega_{2,\theta}^T, \Omega_{2,\psi}^T]^T$.

Control objective: This work aims to realize an unknown system dynamics estimator-based anti-disturbance attitude funnel control for quadrotors, which is capable of precisely driving the angles $\phi, \theta, \psi$ to follow the pregiven reference in the face of external disturbances and parametric uncertainties.

3. Main Results

The flow chart of proposed controller structure is shown in Figure 2, a USDE-based anti-disturbance attitude funnel control for quadrotors is proposed to realize an accurate tracking performance experiencing parametric uncertainties and external perturbations, where the USDE is capable of accurately estimating the unknown disturbances with a lower calculation burden, and an FC is introduced to govern the attitude tracking errors within the predefined funnel boundary along the whole response curves.

![Figure 2. Flow chart of the proposed controller structure.](image)

3.1. Unknown System Dynamics Estimator Design

According to [27,28], the following filtering manipulations for the available states $\Omega_2$ and $u$ are introduced:

$$
\begin{cases}
k \Omega_2^f + \Omega_2^f = \Omega_2, \Omega_2^f(0) = [0, 0, 0]^T \\
k u^f + u^f = u, u^f(0) = [0, 0, 0]^T
\end{cases}
$$

where $u = [u_\phi, u_\theta, u_\psi]^T$, $k$ represents the filtering argument to be adjusted. $\Omega_2^f, u^f$ denote the auxiliary filtered variables.

**Lemma 1** [29]: The auxiliary variable $\eta = k^{-1} (\Omega_2 - \Omega_2^f) - (u^f + \Delta)$ is bounded and satisfies the following condition, given that the filtering constant holds $k \in (0, +\infty)$.

$$
\lim_{k \to 0} \left\{ \lim_{t \to \infty} \left[ \frac{1}{k} \left( \Omega_2 - \Omega_2^f \right) - \left( u^f + \Delta \right) \right] \right\} = 0
$$
It is worth emphasizing that the auxiliary variable $\eta$ is an invariant manifold, revealing a mapping between the lumped disturbances $\Delta$ and filtered variables $\Omega^2, u^f$. Thus, a USDE can be constructed as
\[
\hat{\Delta} = \frac{1}{k} (\Omega_2 - \Omega^2_f) - u^f
\]  
(7)
where $\hat{\Delta} = [\hat{\Delta}_\phi, \hat{\Delta}_\theta, \hat{\Delta}_\psi]^T$ defines the estimation of lumped disturbances.

**Assumption 1** [30,31]: The differential of unknown perturbations is deemed to be bounded, and there exists an unknown constant holding $\delta \geq \| \Delta \|$.

**Theorem 1**: The disturbance estimation errors $\tilde{\Delta} = \Delta - \hat{\Delta}$ can be governed to converge to the following small residual regions:
\[
\| \tilde{\Delta} \| \leq \sqrt{\tilde{\Delta}_i^2(0)e^{-t/k} + k^2\delta^2}
\]  
(8)

**Proof**: According to (4), it can be derived by imposing a first-order filtering $1/(ks + 1)$:
\[
\frac{1}{ks + 1} \dot{\Omega}_2 = \frac{1}{ks + 1} u + \frac{1}{ks + 1} \Delta
\]  
(9)
where $s$ represents a Laplace operator.

Considering the quadrotor attitude Model (1) and (4) and filtering operation (5), we have
\[
\Omega^2_f = \frac{1}{k} (\Omega_2 - \Omega^2_f) = u^f + \Delta^f
\]  
(10)
where $\Delta^f = \Delta/(ks + 1)$.

Thus, we can further obtain $\Delta^f = \hat{\Delta}$, and the estimation errors can be derived as
\[
\tilde{\Delta} = \Delta - \Delta^f = \frac{ks}{ks + 1} \Delta
\]  
(11)
whose derivation with respect to time is derived as
\[
\dot{\tilde{\Delta}} = \dot{\Delta} - \dot{\Delta}^f = \dot{\Delta} - \frac{\Delta - \Delta^f}{k}
\]  
(12)

Considering the estimation errors $\tilde{\Delta}$, a Lyapunov function is designed as
\[
V_1 = \frac{1}{2} \tilde{\Delta}^T \tilde{\Delta}
\]  
(13)
Calculating the derivative of (13), it yields
\[
\dot{V}_1 = -\frac{1}{k} \tilde{\Delta}^T \dot{\Delta} + \dot{\tilde{\Delta}}^T \dot{\Delta}
\]  
\[
\leq -\frac{1}{k} \tilde{\Delta}^T \dot{\Delta} + \frac{1}{2k} \tilde{\Delta}^T \tilde{\Delta} + \frac{k\delta^2}{2}
\]  
(14)
\[
= -\frac{1}{k} V_1 + \frac{k\delta^2}{2}
\]
Integrating (14) over time, it yields
\[
V_1(t) \leq V_1(0)e^{-t/k} + \frac{k^2\delta^2}{2}
\]  
(15)
In conclusion, it can be derived as
\[ ||\Delta|| = \sqrt{2V_1(t)} \leq \sqrt{\Delta^2(0)e^{-t/k}} + k^2\delta^2, \]
\[ i = \phi, \theta, \psi \]
\[ (16) \]

Obviously, when time approximates to be infinite, the error dynamics \( \Delta \) can be ensured to converge to the origin in an exponential sense, given that the filtering constant \( k \to 0 \). □

3.2. Controller Design

In this section, giving the definition of angle tracking error as \( e = \Omega_1 - r \), where \( e = [e_\phi, e_\theta, e_\psi]^T \). \( r = [\phi_d, \theta_d, \psi_d]^T \) denotes the reference signal. Then, to regulate the error \( e \) within the predetermined funnel envelope \( -\theta_l(t) < e_i(t) < \theta_l(t) \), a funnel function is selected as \[ (32) \]
\[ \theta_l(t) = ae^{-ht} + \beta, i = \phi, \theta, \psi \]
\[ (17) \]
where \( a, \beta, l \) are the design parameters and satisfy \( a \geq \beta > 0, |e_i(0)| < \theta_l(0) = a + \beta \).

Therefore, the funnel variable \( \zeta_i(t) \) is selected as
\[ \zeta_i(t) = \frac{e_i(t)}{\theta_l(t) - |e_i(t)|} \]
\[ (18) \]
The differential of (18) can be derived as
\[ \dot{\zeta}_i = \frac{\dot{e}_i(\theta_l - |e_i|) - (\frac{\dot{\theta}_l - |\dot{e}_i|}{\theta_l - |e_i|})e_i}{(\theta_l - |e_i|)^2} \]
\[ (19) \]
and it can be further rewritten as
\[ \dot{\zeta}_i = \frac{1}{\theta_l - |e_i|} \left( \dot{e}_i - \frac{(\dot{\theta}_l - |\dot{e}_i|)e_i}{\theta_l - |e_i|} \right) = \chi_i(\dot{e}_i + \mu_i) \]
\[ (20) \]
where \( \chi_i = 1/(\theta_l - |e_i|), \mu_i = -\frac{(\dot{\theta}_l - |\dot{e}_i|)e_i}{(\theta_l - |e_i|)} \).

To stabilize the funnel variable \( \zeta_i(t) \), the virtual control input \( \tau_i \) is constructed as
\[ \tau_i = -k_{\Theta,i}^{} \zeta_i(t) + \dot{r}_i - \mu_i, i = \phi, \theta, \psi \]
\[ (21) \]
where \( k_{\Theta,i} \) denotes the subsystem gain that needs to be regulated.

Then, the angular rate errors can be described as
\[ z = \Omega_2 - \tau \]
\[ (22) \]
where \( z = [z_\phi, z_\theta, z_\psi]^T \), \( \tau = [\tau_\phi, \tau_\theta, \tau_\psi]^T \).

According to the attitude Model (4), the differential of (22) with respect to time can be computed as
\[ \dot{z} = u + \Delta - \dot{\tau} \]
\[ (23) \]
Substituting the disturbance estimation (7) into (23), the control input results in
\[ u_i = -k_{\omega,i}^{} \dot{z}_i + \dot{r}_i - \dot{\Delta}_i \]
\[ (24) \]
where \( k_{\omega,i} \) represents the controller gain to be governed.

4. Stability Analysis

Based on (21), the differential of funnel variables (18) can be derived as
\[ \dot{\zeta}_i = -k_{\Theta,i}^{} \zeta_i + \chi_i z_i \]
\[ (25) \]
Similarly, the derivation of angular rate errors $\mathbf{z} = [\mathbf{z}_\phi, \mathbf{z}_\theta, \mathbf{z}_\psi]^T$ can be derived by invoking (24):

$$\dot{\mathbf{z}}_i = -k_{\mathbf{\omega},i} \mathbf{z}_i + \tilde{\Delta}_i$$

(26)

**Theorem 2**: For the attitude dynamics (4), USDE (7), and control laws (21), (24), all the error dynamics can guarantee the UUB results, especially, and the angle errors $e_i(t)$ can be regulated to converge to a predefined funnel boundary when the initial angle errors satisfy $-\vartheta_i(0) < e_i(0) < \vartheta_i(0)$, $i = \phi, \theta, \psi$, on condition that the following restrictions on controller gains hold:

$$\kappa_1 = \lambda_{\min}(k_{\mathbf{\Theta}}) - \frac{1}{2} \lambda_{\max}(\chi) > 0,$$

$$\kappa_2 = \lambda_{\min}(k_{\mathbf{\omega}}) - \frac{1}{2} \lambda_{\max}(\chi) - \frac{1}{2} > 0$$

(27)

**Proof**: Considering the entire attitude kinetics, a Lyapunov function is selected as

$$V = \frac{1}{2} \sum_{i=\phi,\theta,\psi} (\mathbf{z}_i^2 + \mathbf{z}_{\psi}^2)$$

(28)

The derivation of (28) can be derived by combining with (25) and (26):

$$\dot{V} = \sum_{i=\phi,\theta,\psi} \left( \mathbf{z}_i \dot{\mathbf{z}}_i + \mathbf{z}_i \dot{\mathbf{z}}_i \right)$$

$$= \sum_{i=\phi,\theta,\psi} \left( \mathbf{z}_i \left(-k_{\mathbf{\omega},i} \mathbf{z}_i + \dot{\chi}_i \mathbf{z}_i + \mathbf{z}_i \right) \right)$$

$$= \sum_{i=\phi,\theta,\psi} \left( -k_{\mathbf{\omega},i} \mathbf{z}_i^2 + \mathbf{z}_i \dot{\chi}_i \mathbf{z}_i - k_{\mathbf{\omega},i} \mathbf{z}_i^2 + \mathbf{z}_i \tilde{\Delta}_i \right)$$

(29)

Utilizing Young’s inequality [2], it is derived as

$$\sum_{i=\phi,\theta,\psi} |\chi_i \mathbf{z}_i| \leq \frac{1}{2} \lambda_{\max}(\chi) \left( \|\mathbf{z}\|_2^2 + \|\mathbf{z}\|_2^2 \right)$$

$$\sum_{i=\phi,\theta,\psi} |\mathbf{z}_i \mathbf{\tilde{\Delta}}_i| \leq \frac{1}{2} \|\mathbf{z}\|_2^2 + \frac{1}{2} \|\mathbf{\tilde{\Delta}}\|_2^2$$

(30)

where $\chi = \text{diag}\{\chi_\phi, \chi_\theta, \chi_\psi\}$, $\zeta = [\zeta_\phi, \zeta_\theta, \zeta_\psi]^T$. $\lambda_{\max}$ denotes the maximum value of the matrix. □

Thus, (29) can be computed as

$$\dot{V} \leq -\lambda_{\min}(k_{\mathbf{\Theta}}) \|\mathbf{\zeta}\|_2^2 + \frac{1}{2} \lambda_{\max}(\chi) \|\mathbf{\zeta}\|_2^2 + \frac{1}{2} \lambda_{\min}(\chi) \|\mathbf{z}\|_2^2$$

$$-\lambda_{\min}(k_{\mathbf{\omega}}) \|\mathbf{z}\|_2^2 + \frac{1}{2} \|\mathbf{z}\|_2^2 + \frac{1}{2} \|\mathbf{\tilde{\Delta}}\|_2^2$$

(31)

where $\lambda_{\min}$ represents the minimum value of a matrix.

According to Theorem 1, we have

$$\lim_{t \to \infty} \|\mathbf{\tilde{\Delta}}\| \leq k \delta$$

(32)

It can be derived as

$$\dot{V} \leq -\left( \lambda_{\min}(k_{\mathbf{\Theta}}) - \frac{1}{2} \lambda_{\max}(\chi) \right) \|\mathbf{\zeta}\|_2^2$$

$$-\left( \lambda_{\min}(k_{\mathbf{\omega}}) - \frac{1}{2} \lambda_{\max}(\chi) - \frac{1}{2} \right) \|\mathbf{z}\|_2^2 + \frac{1}{2} k^2 \delta^2$$

(33)
Thus, it can be integrated as
\[ \dot{V} \leq -\kappa V + \sigma \] (34)
where \( \kappa = \min\{2\kappa_1, 2\kappa_2\} > 0, \sigma = k_2^2\delta^2/2. \)
Solving the inequality (34), one has
\[ 0 \leq V(t) \leq V(0)e^{-\kappa t} + \frac{\sigma}{\kappa}(1 - e^{-\kappa t}) \] (35)

From (35), note that the error dynamics \( \zeta, z \) are bounded and the upper bound can be computed when time goes to infinity:
\[ \|\zeta\| \leq \sqrt{2\sigma/\kappa} \]
\[ \|z\| \leq \sqrt{2\sigma/\kappa} \] (36)

Then, it can be rewritten as
\[ \xi_i^2 \leq 2\sigma/\kappa < \epsilon, i = \phi, \theta, \psi \] (37)
where \( \epsilon \) is an unknown positive constant.

By incorporating the funnel variable \( \xi_i(t) \) in (18), it can be derived as
\[ \frac{e_i^2}{(\theta_i(t) + |e_i(t)|)(\dot{\theta}_i(t) - |e_i(t)|)} \leq \frac{e_i^2}{(\dot{\theta}_i(t) - |e_i(t)|)} = \epsilon \] (38)

Then, it can be further calculated as
\[ e_i^2 \leq \epsilon (\theta_i^2 - e_i^2) < \epsilon \theta_i^2 - \epsilon e_i^2 + \theta_i^2 \] (39)

It follows that
\[ (1 + \epsilon)e_i^2 < (1 + \epsilon)\theta_i^2 \] (40)

Therefore, we have
\[ -\theta_i(t) < e_i(t) < \theta_i(t), \forall t \geq 0, i = \phi, \theta, \psi \] (41)

5. Simulations
5.1. Simulation Validation for Effectiveness

The initial angles are set as \( \phi = 10\text{deg}, \theta = 20\text{deg}, \psi = -15\text{deg}, \) and the reference is given as follows
\[ \begin{cases} 
\phi_d = 15\sin(0.25t) \\
\theta_d = 15\cos(0.25t) \\
\psi_d = 40 
\end{cases} \] (42)

To demonstrate the practicability and effectiveness for the presented control schemes, MATLAB/SIMULINK-based numerical simulation consequences were accomplished and the sampling rate was selected as 1 kHz, including S-function based attitude Model (1) and (4), M-file based funnel control Schemes (17) and (18) and control Law (21) and (24). In addition, according to [2], the selections of related arguments are listed in Table 1, including the moment of inertia, inertia moment uncertainties, and environmental disturbances. In particular, the designed controller arguments are collected in Table 2.
To embody the practicability and effectiveness for the designed control schemes, simulations are offered in Figures 3–5. Figure 3 gives the angle tracking curves under the proposed control scheme, and it is intuitively presented that the angle states can be governed to be UUB. In Figure 4, it shows that the USDE can promptly observe and capture the unmeasurable perturbations with improved accuracy. Figure 5 exhibits the control actions that steer quadrotor attitude dynamics.

**Table 1.** Parameters of quadrotor attitude dynamics.

<table>
<thead>
<tr>
<th>Section</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of inertia</td>
<td>$I_\phi = 0.021, I_\theta = 0.021, I_\psi = 0.039$</td>
</tr>
<tr>
<td>Inertia moment uncertainties</td>
<td>$I^\phi_\phi = 0.012, I^\phi_\theta = 0.012, I^\phi_\psi = 0.012$</td>
</tr>
<tr>
<td>Environmental disturbances</td>
<td>$d_\phi = 1 + 0.5 \cos(2t), d_\theta = 1 + 0.4 \sin(t), d_\psi = 1 + \cos(1.5t)$</td>
</tr>
</tbody>
</table>

**Table 2.** Parameters of the controllers.

<table>
<thead>
<tr>
<th>Section</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>USDE</td>
<td>$k = 0.01$</td>
</tr>
<tr>
<td>Funnel control</td>
<td>$\alpha = 6, \beta = 0.3, \eta = 0.4$</td>
</tr>
<tr>
<td>Control law</td>
<td>$k_{\Theta,\phi} = k_{\Theta,\theta} = k_{\Theta,\psi} = 1.2, k_{\omega,\phi} = k_{\omega,\theta} = k_{\omega,\psi} = 5$</td>
</tr>
</tbody>
</table>

Figure 3. Attitude tracking response.

Figure 4. Estimation results for lumped disturbances using USDE.
5.2. Simulation Validation for Superiority

To illustrate the superiority of the evolved USDE in offering good estimation profiles, a constrastive investigation with a recently reported SMO [33] and a high-order extended state observer (HESO) [34] was carried out on the MATLAB/SIMULINK platform; it should be pointed that to govern the fairness of comparisons, for each estimator, the related design arguments were finely adjusted to assure the uniform convergence rate. Figure 6 gives the estimation errors between the USDE, and SMO [33], HESO [34]. Combined with the quantitative analysis collected in Table 3, significant jumping appeared in the SMO, which was incurred by using a weak sign function gain to reduce harmful chattering, and although HESO and the designed USDE shared similar estimation accuracy, intricate iterative operations were inevitably imposed on HESO, yielding a mass of calculational burden, while the USDE was capable of identifying the unmeasurable disturbances with a lessened computational consumption.

Table 3. Quantitative analysis of USDE, SMO and HESO.

<table>
<thead>
<tr>
<th>Index</th>
<th>Notation</th>
<th>USDE</th>
<th>SMO</th>
<th>HESO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimation accuracy</td>
<td>$\Delta_\phi$</td>
<td>0.0247</td>
<td>0.0876</td>
<td>0.0311</td>
</tr>
<tr>
<td></td>
<td>$\Delta_\theta$</td>
<td>0.0277</td>
<td>0.0941</td>
<td>0.0292</td>
</tr>
<tr>
<td></td>
<td>$\Delta_\psi$</td>
<td>0.0301</td>
<td>0.0899</td>
<td>0.0371</td>
</tr>
<tr>
<td>Computational burden</td>
<td></td>
<td>5.11%</td>
<td>42.01%</td>
<td>37.13%</td>
</tr>
</tbody>
</table>
To exhibit the superiority of the entire controller in operating quadrotor attitudes, herein the tracking response of the proposed method and the continuous time sliding-mode control (SMC) [33] were taken into account, where controller parameters were both tuned via trial and error until optimal control levels were satisfied. From Figure 7, combining with the quantized results including convergence time, standard derivation and control consumption in Table 4, although the tracking errors with SMC [33] could be enforced within a small vicinity around the origin, the adverse effects, i.e., severe control chattering existed and overlarge actuating expenditure was induced, as demonstrated in Figure 8. Conversely, by employing a funnel boundary-based funnel variable, the tracking performance under the proposed control policy can restrict transient and steady profiles within a user-defined boundary with reduced energy consumption and smooth executing actions.

Figure 7. Comparisons of angle tracking errors between FC and SMC.

Table 4. Contrastive outcomes between proposed method and SMC [33] in simulations.

<table>
<thead>
<tr>
<th>Index</th>
<th>Notation</th>
<th>Proposed Method</th>
<th>SMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convergence time</td>
<td>$e_\phi$</td>
<td>5.818</td>
<td>10.240</td>
</tr>
<tr>
<td></td>
<td>$e_\theta$</td>
<td>3.916</td>
<td>10.238</td>
</tr>
<tr>
<td></td>
<td>$e_\psi$</td>
<td>3.899</td>
<td>11.786</td>
</tr>
<tr>
<td>Tracking accuracy</td>
<td>$e_\phi$</td>
<td>0.0022</td>
<td>0.1087</td>
</tr>
<tr>
<td>(standard deviation)</td>
<td>$e_\theta$</td>
<td>0.0274</td>
<td>0.1117</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<td>116.4</td>
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</table>
bounced control inputs. In addition, to demonstrate the effectiveness of the proposed method, the external environmental disturbances were produced by adding an air blower to generate gusts of 250 Hz. In addition, to demonstrate the effectiveness of the proposed method, the external environmental disturbances were produced by adding an air blower to generate gusts of wind in the vicinity of the experimental testbench.

### 6. Experimental Verifications

#### 6.1. Experimental Validation for Effectiveness

As revealed in Figure 9, some experimental validations were introduced using the Links-UAV test bench fabricated by Beijing Links Co., Ltd., Beijing, China. The rotational angles with respect to the inertial frame were acquired online by onboard accelerometer and gyroscopes with ±3% measurement error. The control method was performed to output the required actuating torques via the installed Pixhawk V3 with a sampling frequency of 250 Hz. In addition, to demonstrate the effectiveness of the proposed method, the external environmental disturbances were produced by adding an air blower to generate gusts of wind in the vicinity of the experimental testbench.

![Links-UAV test bench](image)

**Figure 9.** Links-UAV test bench.

The initial angles were designed as $\phi = -5\text{deg}, \theta = 8\text{deg}$, the reference signal is given as $\phi_d = 15\sin(0.25t), \theta_d = 15\cos(0.25t), \psi_d = 25$. Controller arguments were selected to be the same as Table 2.

Figure 10 gives the tracking response of the proposed control scheme, and it exhibited intuitively that precise tracking could be guaranteed. The filtered angular rate states are shown in Figure 11, illustrating that a relatively smooth filtering variable could be obtained, which is helpful in yielding a chatter-free uncertainty estimate. Figure 12 depicts the bounded control inputs.

![Comparison of control inputs between proposed method and SMC](image)

**Figure 8.** Comparison of control inputs between proposed method and SMC.
6. Experimental Verifications

6.1. Experimental Validation for Effectiveness

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Figure 9. Links-UAV test bench.

The initial angles were designed as $\phi = -20\, \text{deg}$, $\theta = 5\, \text{deg}$, $\psi = 8\, \text{deg}$, the reference signal is given as $\phi = 15\sin(0.25\, t)$, $\theta = 15\cos(0.25\, t)$, $\psi = 25\, \text{deg}$. Controller arguments were selected to be the same as Table 2.

Figure 10 gives the tracking response of the proposed control scheme, and it exhibited intuitively that precise tracking could be guaranteed. The filtered angular rate states are shown in Figure 11, illustrating that a relatively smooth filtering variable could be obtained, which is helpful in yielding a chatter-free uncertainty estimate. Figure 12 depicts the bounded control inputs.

Figure 10. Attitude tracking response.

Figure 11. Filtering operations upon angular rates.

Figure 12. Control inputs.
6.2. Experimental Validation for Superiority

To further illustrate the superiority of the proposed method, several comparisons implemented on the Links-UAV testbench are provided in this subsection. The system parameters and external disturbances were set as the same values to ensure the fairness of comparisons.

From Figure 13, with the aid of FC, angle tracking errors were restricted to the predefined funnel boundary in the whole tracking convergence, combining with the quantitative comparison in Table 5, a brilliant transient and steady-state behavior (i.e., reinforced convergence time and improved tracking accuracy) could be governed by the proposed method without incurring chattering. Figure 14 gives the control inputs using SMC, where remarkable oscillations widely appear as simulations, which is not conducive to the stable operation of quadrotor attitude regulation.

![Figure 13. Comparisons of angle tracking errors with SMC.](image)

Table 5. Quantitative analysis between proposed method and SMC in experiments.

<table>
<thead>
<tr>
<th>Index</th>
<th>Notation</th>
<th>Proposed Method</th>
<th>SMC</th>
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<tbody>
<tr>
<td>Convergence time</td>
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<td>Tracking accuracy</td>
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<tr>
<td>(Standard deviation)</td>
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<td>$e_\phi$</td>
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</table>
7. Conclusions

In this paper, an unknown system dynamics estimator-based anti-disturbance funnel control was proposed for quadrotor attitude dynamics. Based on the invariant manifold principle, a USDE was constructed to identify online the unknown perturbations by establishing the corresponding relationship between filtered dynamics and total perturbations. In addition, by designing a funnel function and funnel variable, a funnel control policy was exploited to govern the angle errors within the predefined region, where the overshoot, convergence time and steady-state accuracy were all restricted to an appropriate range. Finally, according to the results of the simulations and experiments, the USDE was capable of identifying the unmeasurable disturbances with less computational consumption. With the aid of FC technology, the convergence profiles including overshoot, convergence time and steady-state accuracy could be restricted within a better scope, thus the practicability and effectiveness for the proposed control method could be verified.

Note that the proposed control strategy can ensure the UUB results with satisfactory convergence profiles both in simulations and experiments, giving a solution to regulate the quadrotor attitude tracking errors in the face of external disturbances. In future work, we will make further efforts to implement a 6-DOF tracking control on an experimental platform for quadrotors, which is a more valuable and explorative task.

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References


