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Energy-Spectrum Efficiency Trade-Off in UAV-Enabled Mobile Relaying System with Bisection-PSO Algorithm

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Abstract: Unmanned aerial vehicle (UAV)-enabled mobile relaying is regarded as an important wireless connectivity component in areas without infrastructure coverage due to its rapid response, strong mobility, and low cost. This paper studies a delay tolerant UAV-enabled mobile relaying system and adopts the load-carry-and-deliver paradigm. The UAV is employed to assist in the information transmission from a ground transmitter to a ground receiver with their direct link blocked. Two kinds of UAV flight trajectories are proposed in this system, i.e., a straight line and circular trajectory. Suppose that the UAV employs time-division duplexing (TDD)-based decode-and-forward (DF) relaying. This paper then aims to maximize the spectrum efficiency (SE) and energy efficiency (EE) in of the UAV-enabled relaying system by jointly optimizing the time allocation, flight speed, and the flying radius of the circular trajectory. Then, we develop an efficient algorithm by leveraging the bisection method and particle swarm optimization (PSO) algorithm. Simulation results show the superiority of the proposed algorithm as compared to other benchmark schemes. In addition, numerical results show that, when the communication distance is 1000 m, the SE and EE performance of the circular trajectory is better than the SLF trajectory when the obstacle height is greater than 300 m. Thus, the height of the obstacle between the communication nodes and the trade-off between the SE and EE should be taken into account when we design the optimal trajectory of the UAV-enabled mobile relaying system.

Keywords: unmanned aerial vehicle; mobile relaying; spectrum efficiency; energy efficiency

1. Introduction

1.1. Background and Related Works

The development of UAV technology has greatly improved its applicability [1,2]. The applications of UAVs include traffic control, border patrolling, forest fire monitoring, agriculture mapping, emergency services, etc. [3–5]. Compared with traditional terrestrial communication, which mainly depends on the ground communication infrastructure, UAV-enabled communication has the advantages of low cost and strong mobility, especially in emergency communication settings such as disaster relief or battlefields [6,7]. Employing UAVs as mobile relays in communication systems has many advantages [8–10]. Firstly, UAVs have fast reactions, flexibility and survivability, which are particularly suitable for critical missions. Secondly, compared with terrestrial links, the characteristics of air-ground (AG) channels enable appealing line-of-sight (LoS) connections between UAVs and ground users, which can provide higher link capacity. Recently, several important works have been conducted on UAV-enabled mobile relaying systems. Trajectory planning and power allocation is one of the effective ways to improve the SE of UAV relay systems. In [11], the UAV flies between source and destination nodes, and the trajectory of UAV and the transmit power of source/relay node are optimized to maximize the system throughput. A multi-hop UAV relaying system was proposed in [12], where the average throughput from the source to the destination was maximized by jointly optimizing the resource allocation to each hop and the trajectories of these UAVs. Considering the reliability of the LoS communication,
the authors in [13] studied a UAV-enabled relaying system with multiple ground users (MUs). The sum rate was maximized by jointly optimizing the UAVs’ 3D trajectory and power allocation. Antenna systems are very important in wireless communication [14,15]. Antenna and beamforming optimization are very important components in the structure of UAV-enabled mobile relaying systems to transmit and receive signals [16–19]. A novel miniature ultrawideband antenna was proposed in [16], which can improve performance in UAV-enabled wireless systems. The authors in [17] investigated a simple method based on the metasurface concept to suppress the mutual coupling between the antenna arrays, which enable it to be applicable for a multiple-input single-output (MIMO) UAV-enabled network. The trajectory and beamforming design were jointly optimized in [18,19] to enhance the SE in a cognitive multiple-input single-output (MISO) UAV-enabled network and UAV swarm-enabled communication network, respectively. In term of the secure transmission, a UAV swarm-enabled relaying system was proposed in [20], where the rotary wing UAVs were always hovering in a fixed position. Then, the beamforming and bandwidth allocation were optimized under both the amplify-and-forward (AF) and decode-and-forward (DF) protocols.

Considering the limited energy of UAVs, energy efficiency (EE) becomes another important performance metric in UAV-enabled communication systems [21]. In [22], an energy efficient cooperative relaying for wireless sensor networks is investigated, which minimizes the energy consumption via optimizing the transmission schedule of the UAVs. The work in [23] designed an energy-efficient UAV relaying network, which jointly optimizes the UAV’s trajectory, speed, and acceleration. To maximize the EE in a UAV-enabled cognitive radio network, the authors in [24,25] optimized the sensing performance, the resource allocation, and the UAV positions based on a circular trajectory. By considering the quality of service (QoS) requirements of the users, a UAV-enabled mobile relay network where the fixed-wing UAV followed a circular trajectory was proposed in [26], and the user scheduling and power allocation were optimized to maximize the EE by considering the QoS requirements of the users. An energy-efficient fixed-wing UAV-enabled full-duplex relaying system was proposed in [27], where the UAV followed a straight trajectory and the flight speed was optimized. The authors in [28] presented UAV spectrum sensing in straight flight and proposed the spectrum sensing optimization method to maximize the effective throughput of the UAV under the constraint of interference throughput.

### 1.2. Motivation and Contributions

As mentioned above, trajectory design is significant both in the EE and SE enhancement and can be can be divided into 3D trajectories and 2D trajectories. Specifically, most 3D trajectory planning usually assumes that the UAV can fly freely in 3D space [13]. Conversely, 2D trajectory planning can be specifically classified as circle-based [24–26], straight line-based [27,28], and arbitrary trajectories [11,12,22] within the limits of UAV maneuverability. However, the existing works on UAV-enabled mobile relaying systems often neglect the influence of the actual terrain. It is worth noting that in practice, UAV trajectory planning is affected by obstacle heights. In emergency communication scenarios, we often do not have detailed information about the terrain in the communication scenario. In this case, how to quickly design the UAV trajectory and improve system performance is a problem which should be further studied. At the same time, considering the limited energy of the UAV, a great deal of research has been devoted to improve the EE of UAV-enabled communication systems [21–28]. It is worth noting that, due to the influence of UAV energy consumption, the maximization of EE and SE of UAV often cannot achieved simultaneously. There is less analysis of the constraint relationship between the EE and SE. Thus, the trade-off between EE and SE also needs to be studied. This paper considers a delay-tolerant scenario, where the UAV is employed to establish a two-hop time division duplex (TDD) and decode-and-forward (DF) wireless relaying transmission link. We aim to investigate the fundamental trade-off between the EE and SE for a UAV-enabled mobile relaying system, and reveal the influence of terrain factors on the UAV trajectory design.
For ease of elaboration, we assume that the UAV follows two kinds of simple trajectories, i.e., straight-line flight (SLF) with constant speed and a circular trajectory, in which the UAV’s energy consumption can be formulated in closed form [14]. The time allocation of information receiving/transmitting, flight speed and circular radius of the UAV are jointly optimized to maximize the EE and SE. The contributions of this article can be summarized as follows:

- A UAV-enabled mobile relaying communication system in the presence of mountain obstacles is proposed in this paper. In order to enhance the SE and EE of this system, we formulate both the EE and SE maximization problems of SLF and a circular trajectory by jointly optimizing the flight speed, time allocation, and circular radius.
- According to the special structure of the problem, an efficient algorithm is proposed, a more accurate optimal solution interval is given by mathematical analysis, and the effectiveness of the proposed algorithm is verified by simulation results.
- This paper provides in-depth simulations under various obstacle dimension. Simulation results demonstrate the influence of obstacle dimension on UAV trajectory design and provide insights regarding the restrictive relationship between EE and SE in UAV-enabled mobile relaying systems. Furthermore, this paper reveals the impacts of obstacle dimensions on UAV trajectory design and analyzes the EE-SE trade-off in UAV-enabled mobile relaying systems, which gives an additional design dimension.

1.3. Organization

The rest of this paper is organized as follows. In Section 2, we consider a fixed-wing UAV with a circular trajectory and SLF trajectory. The UAV flies between S and D to complete the information transmission periodically. In Section 3, an efficient algorithm is proposed to obtain the maximum EE and SE of the UAV-enabled mobile relaying system. In Section 4, the simulation results are provided. The results show that the circular trajectory and SLF trajectory have different advantages with increasing height of mountain obstacles. Meanwhile, there is a trade-off between the EE and SE under the circular trajectory. Finally, conclusions are drawn in Section 5.

2. System Model and Problem Formulation

As shown in Figure 1, we consider a wireless communication model where transmitter S is intended to communicate with the receiver D, which are separated by L meters. It is assumed that the direct link between S and D is blocked by mountains with height H. Therefore, we establish a two-hop wireless relaying transmission link by using a UAV as relay to maintain the continuous and reliable LoS communication. We propose two schemes to solve this problem as follows.

![Figure 1. UAV-enabled communication system with mountain obstacles.](image)

2.1. SLF Trajectory with Constant Speed on the Top of the Mountain

As shown in Figure 2, the UAV flies on the top of the mountain and periodically sends information between S and D. Without loss of generality, we consider a three-dimensional Cartesian coordinate system with S and D located at (0, 0, 0) and (L, 0, 0), respectively.
Figure 2. UAV-enabled communication system under SLF flight trajectory.

We assume that the UAV flies in a straight line with fixed altitude $H_S$ and constant speed $v$, the length of the straight line is $L$. In practice, the value of $H_S$ is selected to satisfy the minimum altitude of terrain avoidance without the need of extra energy consumption due to descending/ascending operations. Therefore, the UAV’s round trip period is $T = 2L/v$. Note that in practice, the total operating time $T_{tot}$ is usually much larger than the UAV’s round trip period $T$, and the consecutive communication process can be split into many single periods. Therefore, the analysis for each individual period is similar. Considering that the UAV can convert the transceiver relationship to exchange information during the round trip, we only need to consider a half round trip period $t \in (0, T/2)$ due to the symmetry. The trajectory of a UAV projected on the horizontal plane is expressed as $Q(t) = [X(t), 0] \in R^{2 \times 1}$, where $0 \leq t \leq T/2$. Therefore, the time-varying distance between the UAV and $S$ can be expressed as $D(t)^2 = H_S^2 + Q(t)^2, 0 \leq t \leq T/2$. We assume that the Doppler effect due to mobility can be perfectly compensated [11]. The impact of multipath fading and a non-LoS link will be considered in our future work. Therefore, the communication channel follows the free-space path loss model [29], which can be expressed as

$$h(t) = \beta_0 d(t)^{-2} = \frac{\beta_0}{H_S^2 + D(t)^2}, 0 \leq t \leq T/2$$

where $\beta_0$ denotes the channel power gain at the reference distance $d_0 = 1$ m. We assume that the UAV adopts the DF strategy and the size of the data buffer is large enough [13]. The time allocation index of the UAV relay is defined as $u(t)$, where $0 \leq t \leq T/2$.

$$\begin{cases} u(t) = 1, & \text{transmission from S to R} \\ u(t) = 0, & \text{transmission from R to D} \end{cases}$$

Furthermore, we have the following result.

**Lemma 1.** Suppose that $u^*(t)$ is the optimal time allocation function; then, we have that $d_{SR}(t_1) < d_{SR}(t_2)$ when $u^*_s(t_1) = 1$ and $u^*_s(t_2) = 0$.

**Proof of Lemma 1.** Please refer to Appendix A.

Intuitively, Lemma 1 indicates that the transmission method shown in Figure 2 can achieve the maximum average throughput in delay tolerant communication scenarios. As
illustrated in Figure 2, the SLF trajectory is divided into two parts, with the left part for receiving information from transmitter S and the right part for forwarding information to receiver D, i.e.,

\[
\begin{align*}
    u(t) = 1, & \quad (0 < X(t) < \xi) \quad \text{transmission from S to R} \\
    u(t) = 0, & \quad (\gamma < X(t) < L - \xi) \quad \text{transmission from R to D}
\end{align*}
\] (3)

Supposing that the transmission power of S and R are \( P_S \) and \( P_R \), respectively, we define the instantaneous rate from S to R and R to D as \( R_{SR}^S(t) \) and \( R_{RD}^S(t) \).

According to Lemma 1, the average transmission rate for S-R and R-D links can be expressed as

\[
\bar{R}_{SR}^S(t) = \frac{2}{T} \int_0^{T/2} R_{SR}^S(t) \, dt = \frac{1}{L} \int_0^\xi \log_2 \left( 1 + \frac{P_S \gamma_0}{x^2 + H_S^2} \right) \, dx
\] (4)

\[
\bar{R}_{RD}^S(t) = \frac{2}{T} \int_0^{T/2} R_{RD}^S(t) \, dt = \frac{1}{L} \int_0^{L-\xi} \log_2 \left( 1 + \frac{P_R \gamma_0}{x^2 + H_S^2} \right) \, dx
\] (5)

where \( \sigma^2 \) is the noise power and \( \gamma_0 = \beta_0 / \sigma^2 \) is the reference signal-to-noise ratio (SNR) at the reference distance \( d_0 \). Note that the communication capacity of the downlink between the UAV and the receiver will not exceed the channel capacity of the uplink between the transmitter and the UAV during the relaying process, and the following information causal constraint should be satisfied \[11\]:

\[
\int_0^t R_{RD}^S(\tau) \, d\tau < \int_0^t R_{SR}^S(\tau) \, d\tau, \quad 0 \leq t \leq T
\] (6)

Firstly, we aim to optimize the time allocation \( u(t) \) to maximize the SE of the mobile relaying system. The problem is formulated as follows:

\[
\text{(OP1)} \quad \max_{u(t)} \bar{R}_{SD}^S = \frac{1}{T} \int_0^T \bar{R}_{RD}^S(t) \, dt \quad \text{subject to}
\]

\[
\int_0^t R_{RD}^S(\tau) \, d\tau \leq \int_0^t R_{SR}^S(\tau) \, d\tau, \quad 0 \leq t \leq T
\] (7b)

\[
u(t) \in \{0, 1\}
\] (7c)

Considering the limited energy of UAV, EE is also an important performance metric. The total energy consumption of the UAV consists of two components. The first one is communication-related energy, the other is propulsion energy, which is used to remain aloft and support its mobility. In fact, the energy associated with communication is usually much smaller than the propulsion energy of the UAV, for example, a few watts versus hundreds of watts \[30,31\]. Therefore, the communication energy is ignored in this article. The UAV’s propulsion power consumption can be modeled as \[22\]

\[
P_{SLF}(v) = c_1 v^3 + \frac{c_2}{v}
\] (8)

For the extreme case of \( v = 0 \), we have \( P_S \to \infty \), which means that the fixed-wing UAV must keep non-zero forward speed, so we do not consider the relay method of UAV hovering. Therefore, the average EE of the mobile relay system in time period T can be expressed as

\[
\Xi_S = \frac{\bar{R}_{SD}^S}{\bar{P}_{SLF}}
\] (9)
According to (8), the problem of EE maximization can be formulated as

\[
\text{(OP2)} \quad \max_{u(t),u} \mathbb{E}_S
\]

s.t. \[
\int_0^t R_{RD}^S(\tau)d\tau \leq \int_0^t R_{SR}^S(\tau)d\tau, \forall 0 \leq t \leq T
\]

\[u(t) \in \{0, 1\}\] (10a, 10b, 10c)

2.2. Circular Trajectory with Constant Speed at a Certain Altitude

As shown in Figure 3, in order to solve the problem that the mountain blocks the LoS communication, we assume that the UAV follows a simple circular trajectory with altitude \(H_C\) and speed \(v\) [18]. In practice, \(H_S\) should be higher than the height of mountain to avoid collision, but \(H_C\) only need to satisfy the condition of LoS. Similar to Lemma 1, we suppose the flight arc length is \(\iota\), when \(\iota < \kappa\), the UAV R receives the data from S, and then sends the data to D.

\[
h_{SR}(t) = \beta_0 d_{SR}^2(t)
\]

\[
h_{RD}(t) = \beta_0 d_{RD}^2(t)
\]

where \[
\begin{align*}
d_{SR}^2(t) &= \left(\frac{r}{\iota} - \cos\left(\frac{\iota}{r}\right)\right)^2 + \left(r \sin\left(\frac{\iota}{r}\right)\right)^2 + H_C^2 \\
d_{RD}^2(t) &= \left(\frac{r}{\iota} - \cos\left(\pi - \frac{\kappa}{r}\right)\right)^2 + \left(r \sin\left(\pi - \frac{\kappa}{r}\right)\right)^2 + H_C^2
\end{align*}
\]

Define the instantaneous rate from S to R and R to D as \(R_{SR}^C(t), R_{RD}^C(t)\), respectively. The average rates from S to R and from R to D are

\[
R_{SR}^C = \frac{1}{T} \int_0^T R_{SR}^C(t)dt = \frac{1}{\pi r} \int_0^{\pi} \log_2 \left(1 + \frac{P_S\gamma_0}{r^2 - rL \cos(\iota/r) + \frac{L^2}{4} + H_C^2}\right) d\iota
\]

\[
R_{RD}^C = \frac{1}{T} \int_0^T R_{RD}^C(t)dt = \frac{1}{\pi r} \int_0^{\pi-r} \log_2 \left(1 + \frac{P_R\gamma_0}{r^2 - rL \cos(\iota/r) + \frac{L^2}{4} + H_C^2}\right) d\iota
\]
The SE optimization problem is expressed as

\[(\text{OP3}) \quad \max_{u(t),r} R_{SD}^C = \frac{1}{T} \int_0^T R_{RD}^C(t)dt \quad (15a)\]

\[\text{s. t.} \quad \int_0^t R_{RD}^C(\tau)d\tau \leq \int_0^t R_{SR}^C(\tau)d\tau, \forall 0 \leq t \leq T \quad (15b)\]

\[u(t) \in \{0, 1\} \quad (15c)\]

According to [22], the UAV’s propulsion power consumption can be modeled as

\[\bar{P}_C(r, v) = \left(c_1v^3 + \frac{c_2}{g^2r^2}\right)v^3 + \frac{c_2}{v} \quad (16)\]

Then, the EE of the UAV system is

\[\Xi_C = \frac{R_{SD}^C}{\bar{P}_C} \quad (17)\]

Similar to OP3, we formulate the EE maximization problem as follows:

\[(\text{OP4}) \quad \max_{u(t),r,v} \Xi_C \quad (18a)\]

\[\text{s. t.} \quad (14b), (14c) \quad (18b)\]

3. Problem Formulation

In this section, we investigate the optimal solutions to the SE/EE maximization problems of SLF trajectory and circular trajectory.

3.1. Slf with Constant Speed Over the Top of the Mountain

The information causal constraint (6) is one of the main challenges to solve OP1, which essentially involves infinite numbers of non-convex constraints. To address this issue, we firstly consider the relaxed problem OP1.1, where we only consider the constraint for, i.e.,

\[(\text{OP1.1}) \quad \max_{u(t)} R_{SD}^S = \frac{1}{T} \int_0^T R_{RD}^S(t)dt \quad (19a)\]

\[\text{s. t.} \quad \int_0^t R_{RD}^S(\tau)d\tau \leq \int_0^t R_{SR}^S(\tau)d\tau, \forall 0 \leq t \leq T \quad (19b)\]

\[u(t) \in \{0, 1\} \quad (19c)\]

Considering the continuity of sending and receiving information in time \(T\), we define \(T_1 = \xi/v\).

1. When \(0 < t < T_1\), \(\int_0^t R_{RD}^S(t)dt = 0\), constraint (6) can be guaranteed;
2. When \(T_1 < t \leq T\), the UAV has completed the reception of information from S, \[
\left\{\begin{array}{l}
\int_0^t R_{SD}^S(t)dt = \int_0^{T_1} R_{SD}^S(t)dt, \\
\int_0^t R_{SR}^S(t)dt < \int_0^T R_{SR}^S(t)dt
\end{array}\right., \text{if } \int_0^T R_{RD}^S(t)dt \geq \int_0^T R_{RD}^S(t)dt, \text{and constraint (6) can also be satisfied.}
\]

Thus, OP1.1 and OP1 have the same optimal solutions. Noting that, if the constraint (6) is satisfied with an inequality, we can always adjust the time allocation to increase the objective value. Thus, at the optimal solutions to OP1.1, constraint (6) must be satisfied with an equality. Based on [32], OP1.1 can be rewritten as

\[\text{OP1.1} \quad R_{SD}^S(\xi) = \max_{0<\xi<L} \left\{\min \left\{R_{SR}^S(\xi), R_{RD}^S(\xi)\right\}\right\} \quad (20)\]
It is easily derived that $R_{SR}^S(\xi)$ monotonically increases with $\xi$ for all $\xi > 0$, and $R_{SD}^S(\xi)$ monotonically decreases with $\xi$ for all $0 < \xi < L$. Suppose that $\xi^*$ satisfies $R_{SR}^S(\xi^*) = R_{RD}^S(\xi^*)$. If $\xi > \xi^*$, $R_{SR}^S(\xi^*) > R_{SR}^S(\xi)$, then hence, $R_{SD}^S(\xi) = R_{SR}^S(\xi^*) < R_{SR}^S(\xi^*) = R_{SD}^S(\xi^*)$, and vice versa. Thus, the optimal solution to OP1.1 is equivalent to solving $R_{SR}^S(\xi^*) = R_{RD}^S(\xi^*)$. When $P_S = P_R$, the maximum value of SE is obtained when $\xi = L/2$. If $P_S \neq P_R$, and the maximum value of SE can be efficiently obtained by the Bisection algorithm, which is summarized in Algorithm 1.

Algorithm 1 Optimal time allocation algorithm.

1: Input: $L, H, P_R, P_S, \xi_0$
2: Initialize: $\xi_1 = 0, \xi_2 = L$, tolerance $\Delta > 0$.
3: Update: $\bar{\xi} = (\xi_1 + \xi_2)/2$
4: If: $\bar{R}_{RD}(\bar{\xi}) < R_{SR}(\bar{\xi})$, let $\xi = \bar{\xi}$.
5: Else: let $\xi_2 = \bar{\xi}$.
6: Repeat: step 3–5 calculate until $|\xi_2 - \xi_1| \leq \Delta$
7: Output: $\xi^* = \bar{\xi}$

For the SLF scheme, $R_{SD}^S$ is only related to $\xi$, and $P_{SLF}$ is only related to $v$, so the problem of maximizing EE only needs to optimize $v$ to get the minimum value of $P_{SLF}$, and the optimal value of $\xi$ is the same as $\xi^*$. From (8), we obtain that, with optimal speed $v^* = (c_2/(3c_1))^{1/4}$ [21], the corresponding minimum energy consumption is $P_{min} = (3^{-3/4} + 3^{1/4})c_1^{1/4}c_2^{3/4}$ [21].

3.2. Circular Flight at a Certain Altitude

The Optimization of SE

Similar to the optimization method in Section 3.1, OP3 can be rewritten as

\[
\max_{u(t), r, \rho} R_{SD}^C = \frac{1}{T} \int_0^T R_{RD}^C(t)dt \tag{OP3.1}
\]

\[
\text{s.t. } \int_0^T R_{RD}^C(\tau)d\tau \leq \int_0^T R_{SR}^C(\tau)d\tau, \forall 0 \leq \tau \leq T \tag{21b}
\]

\[
u(t) \in \{0, 1\} \tag{21c}
\]

Note that both $R_{SR}^C$ and $R_{RD}^C$ are independent of $v$. Therefore, this problem is essentially a joint optimization problem of radian $\kappa$ and radius $r$. To solve the problem of OP2, we will firstly optimize $\kappa$ with fixed value of $r$. The optimal $\kappa^*(r)$ can be obtained via the same method in Section 3.1, and the details are omitted here for brevity. With the optimal radian $\kappa^*(r)$ obtained, the objective function of OP2 is reduced to an un-variate function with respect to, which is denoted as

\[
R_{SD}^C(r) = R_{RD}^C(\kappa^*(r), r) \tag{22}
\]

Using $\tau/r^* = \bar{\theta}$, (12) can be rewritten as

\[
R_{RD}^C = \frac{1}{T} \int_0^T R_{RD}^C(t)dt = \frac{1}{\pi} \int_0^{\pi} \log_2 \left(1 + \frac{P_s\gamma_0}{r^2 - rL\cos(\theta) + \frac{\mu}{4} + Hc^2} \right) d\theta \tag{23}
\]

Now, we have the following result.

**Lemma 2.** Suppose that $r^*$ is the optimal radius for OP3. Thus, we have $0 < r^* < L/2$. 
Algorithm 2 Bisection-PSO SE optimization algorithm.

1: Generate an initial population $O = \{i_1, i_2, \ldots, i_N\}$, where $O$ is composed of $N$ random particles, and set the maximum number of iterations $M$, and $m = 1$.

2: While: $m < M$

3: For: each particle $i$,

   initialize velocity $v_i$ and position $r_i$ for particle $i$,

   calculate fitness value $R_{SD}(\kappa^*(r_i), r_i)$ according to the objective function, where is calculated by Algorithm 1.

   Update the global fitness value $R_{min}^* = \max \{R_{SD}(\kappa^*(r_i), r_i) \}, i \in O$

   Update the optimal population particle $r_i$ and the global particle $r^*$

4: $m = m + 1$

5: End while

3.3. The Optimization of EE

With fixed $r$, from (14), we obtain that with the optimal speed $v_{opt}^*(r) = (\frac{c_2}{3}(c_1 + c_2(2r^2)))^{1/4}$ [21], the corresponding minimum energy consumption is $P_{min}^*(r) = (3^{-3/4} + 3^{1/4})c_2^{3/4} \left( c_1 + \frac{c_2}{2r^2} \right)^{1/4}$ [21]. Hence, OP4 can be rewritten as

$$\begin{align}
\text{(OP4.1)} & \quad \max_{r, u(t)} \mathcal{Z}_C = \frac{R_{SD}(r, u(t))}{P_{min}^*(r)} \\
\text{s.t.} & \quad \int_0^T R_{RD}^C(\tau)d\tau \leq \int_0^T R_{SR}^C(\tau)d\tau \\
& \quad u(t) \in \{0, 1\}
\end{align}$$

It is noted that the denominator of $\mathcal{Z}_C$ is independent of time allocation, and the constraints of OP4.1 and OP3 are identical. Thus, the optimal time allocation can be solved by Algorithm 1, and then OP4.1 can be reduced as a unitary function on $r$. It is difficult to obtain the closed expression for an optimal radius. However, thanks to the structure of OP4.1, we have the following result.

Lemma 3. It is assumed that $r^*_EE$ is the optimal radius for OP4; we thus have $r^*_EE > r^*$.

Proof of Lemma 3. Lemma 3 can be proved by contradiction. Suppose that $r^*$ is the optimal radius of OP3. When $r < r^*$, we have $R_{SD}(\beta, r) < R_{SD}(\beta, r^*_SE)$ and $P_C(r) > P_C(r^*_SE)$. Thus, $\mathcal{Z}_C(r) < \mathcal{Z}_C(r^*_SE)$. This demonstrates that the optimal solution can be obtained in the interval $[r^*_SE, L]$. □

We can also use the bisection-PSO search methods to get the optimum and the corresponding radian $\kappa$; the details are omitted for brevity.
3.4. The Trade-Off between EE and SE

Based on the above analysis, the value of $r$ will affect both the EE and the SE, and they may not reach the maximum value at the same time. In this section, we introduce an influence factor $p (0 \leq p \leq 1)$ and formulate a balancing problem as follows:

\[
\text{(OP5)} \quad \max_{r, u(t)} Y_C = p R_{SD}^C(\mu(t), r) + (1 - p) \Xi_C(\mu(t), r)
\]

subject to:

\[
\int_0^T R_{RD}^C(\tau) d\tau \leq \int_0^T R_{SR}^C(\tau) d\tau
\]

\[
u(t) \in \{0, 1\}
\]

where $R_{SD}^C(\mu(t), r)$ and $\Xi_C(\mu(t), r)$ are normalized, respectively. The objective function of OP5 can be rewritten as $\frac{R_{SD}^C(r, \mu(t))(p \Xi_{\text{min}}(r) + (1-p))}{\Xi_{\text{min}}(r)}$. It is easy to know that the denominator of (22) is independent of time allocation, and OP5, OP4 and OP3 have the same constraints. Thus, the time allocation optimization problem is equivalent to OP3.

Thanks to the particular structure of OP5, we propose an efficient solution to optimize the trade-off between EE and SE. To this end, we give the following result:

1. Suppose that $r^*_\rho$ is the optimal radius for OP5. If $r^*_\rho < r^*$, we have $R_{SD}^C(r^*_\rho) < R_{SD}^C(r^*_SE)$ and $\Xi_C(r^*_\rho) < \Xi_C(r^*_SE)$, so $r^*_\rho$ must satisfy $r^*_\rho > r^*$.

2. Based on Lemma 2, we know that $R_{SD}^C$ decreases monotonically when $r > L/2$. Suppose that the optimal radius $r^*_\rho$ for OP5 satisfies $r^*_\rho > r^*_EE$. When $r^*_EE > L/2$, we have the following conclusions:

\[
R_{SD}^C(r^*_\rho) < R_{SD}^C(r^*_EE)
\]

\[
\Xi_C(r^*_\rho) < \Xi_C(r^*_EE)
\]

Based on the above analysis, we propose an efficient optimization Algorithm 3 for the trade-off between EE and SE.

**Algorithm 3** Optimization algorithm for the trade-off between EE and SE.

1. If $r^*_EE > L/2$

   Solve OP5 by Algorithm 2 and replace the objective function with (22). The search range of the radius is $(r^*_SE, r^*_EE)$.

2. Else

   Solve OP5 by Algorithm 2 and replace the objective function with (22). The search range of the radius is $(r^*_SE, L)$.

3. End if

   Let $m \leftarrow m + 1$, repeat 2

4. Simulation Results and Discussion

4.1. Simulation Results

In this section, the performance of our proposed design is verified by simulations. The specific setting of the remaining parameters is shown in Table 1.
Table 1. Simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_S$(w)</td>
<td>1</td>
<td>$P_R$(w)</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma^2$(dBm)</td>
<td>-110</td>
<td>$L$(m)</td>
<td>500</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$9.26 \times 10^{-4}$</td>
<td>$c_2$</td>
<td>2250</td>
</tr>
<tr>
<td>$g$(m/s$^2$)</td>
<td>$9.8 \times 10^{-4}$</td>
<td>$\beta_0$(dB)</td>
<td>-50</td>
</tr>
</tbody>
</table>

To illustrate the effectiveness of the proposed designs, we compare the proposed optimization scheme with the other two benchmark schemes, i.e., the exhaustive attack method and the 2D-PSO algorithm. Figure 4 shows that when the size of particles is 20 and the maximum number of cycles is 30, the proposed optimization algorithm outperforms the 2D-PSO algorithm, and the accuracy of the proposed algorithm is close to the exhaustive attack method.

In Algorithm 1, when defining the length of the search interval as $K$ and the accuracy of the algorithm as $\Delta$, the corresponding complexity is given by $O_1(\log_2(K/\Delta))$. In Algorithm 2, when defining the number of particles as $M$ and the maximum number of iterations as $N$, the corresponding complexity is given by $O_2(MN)$. The complexity for exhaustive attack method and 2D-PSO should be $O_3(K/\Delta)$ and $O_4(2MN)$, respectively. Specifically, the complexity of the algorithm for different designs are given in Table 2.

Thus, the proposed algorithm is superior than exhaustive attack algorithm in time complexity. Furthermore, combined with simulation results, the proposed algorithm is more accurate than 2D-PSO for the same number of particles. This is because the more accurate feasible region is obtained by theoretical analysis, which reduces the probability of falling into a local optimum solution.

Figure 5 illustrates the influence of the obstacle’s height on the scheme selection. To show essential insights, we assume that the flight height of the circular trajectory is fixed at $H = 50$ m, and the flight height of the SLF trajectory is affected by the height of the obstacle. The SE and EE performance of the two flight trajectories are shown in Figure 5a,b, respectively. We can see that when the obstacle’s height is low, the SE of SLF is better than that of the circular trajectory. This is because the distance of the SLF trajectory between transceiver nodes is closer than that of circular trajectory. However, when the obstacle’s height increases, circular trajectories outperform the SLF trajectories since circular trajectories do not require flying over the top of the hill.

For ease of presentation, in Figure 5a,b, the intersection of the SLF trajectory and the circular trajectory under the same $L$ are denoted as A and B, respectively. As shown in Figure 5, B will be larger than A with the same $L$, which means that the energy efficiency advantage of the SLF trajectory is greater when the communication distance is reduced.

In Figure 6a, as increases, the gap between optimal radius for SE and EE max design increases. The reason is that the SE decreases with the value of H, and the EE of UAV will be mainly influenced by energy consumption for sufficiently large H. Thus, only considering one performance metric (EE or SE) will sacrifice the other one. Figure 6b shows the optimal radius versus $L$ for different designs. Firstly, it can be seen that, for all values of $L$, the optimal radius $r_{SE}$ for the SE max design satisfies the constraint $r_{SE} < L/2$, and the optimal radius $r_{EE}$ for the EE max design is in accordance with Lemma 2, i.e., $r_{EE} > r_{SE}$. Next, it is also observed that the gap between $r_{EE}$ and $r_{SE}$ decreases with the value of $L$. The reason is that the optimal radius of the SE max design decreases with $L$. On the other hand, smaller trajectory radius means more propulsion power required for the UAV relay. Thus, the small optimal radius of the SE max design is energy inefficient.
Figure 4. Proposed algorithm vs. some benchmark schemes. (a) SE comparison. (b) EE comparison.

Table 2. Time Complexity Comparison.

<table>
<thead>
<tr>
<th>Time Complexity Comparison</th>
<th>EE/SE-Max Design (SLF Trajectory)</th>
<th>SE-Max Design (Circular Trajectory)</th>
<th>EE-Max Design (Circular Trajectory)</th>
<th>EE/SE-Trade off Design (Circular Trajectory)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm 3 (Proposed Algorithm)</td>
<td>$O_2$</td>
<td>$O_2O_1$</td>
<td>$O_2O_1$</td>
<td>$O_2O_1$</td>
</tr>
<tr>
<td>Exhaustive attack Algorithm</td>
<td>$O_3$</td>
<td>$O_3(L/\Delta)(\pi/\Delta)$</td>
<td>$O_3(L/\Delta)(\pi/\Delta)$</td>
<td>$O_3(L/\Delta)(\pi/\Delta)$</td>
</tr>
</tbody>
</table>
Figure 5. SLF trajectory vs. circular trajectory. (a) SE comparison. (b) EE comparison.
Figure 6. The optimal trajectory radius vs. (a) $H$; (b) $L$.

Figure 7a shows the EE of the UAV-enabled mobile relaying system with different designs. It is noted that for small values of $L$, there exists a big gap between the EE max design and the SE max design. Furthermore, the gap between the two designs reduces with the value of $L$. Figure 7b shows the SE of the UAV-enabled mobile relaying system with different designs. It can be seen that there is a significant decrease in SE with increasing $L$ for both two designs. Intuitively, according to Figure 6b, the optimal radius gradually increases with $L$; as a result, the signal experiences more attenuation when $L$ increases.
Next, we studied the SE-EE trade-off for UAV-enabled mobile relaying systems. Figure 8 shows the optimal circular trajectory radius under different values of $p$ when $H = 200$ m. It is shown that there exists a trade-off between the achievable SE and EE in the UAV-enabled mobile relaying system, i.e., in practical applications, we can balance the contradiction between throughput and limited energy of the UAV.

Figure 9 shows the EE and SE with the corresponding optimal radius $r^*_p$. It is worth noting that when EE is small, sacrificing a smaller value of SE can significantly enhance EE. However, when EE is close to two-thirds of the maximum achievable EE of this UAV-enabled mobile relaying system, the improvement in SE will cause a significant decrease in
EE. Therefore, the study of the SE-EE trade-off in UAV-enabled mobile relaying systems has a great practical significance.

Figure 8. The optimal radius vs. $\rho$.

Figure 9. SE-EE trade-off in UAV-enabled mobile relaying systems.

4.2. Discussion

In this paper, a point-to-point communication in delay-tolerant wireless networks is studied. We propose an efficient algorithm to optimize the EE, SE, and the trade-off under the SLF and circular trajectory. Figure 4 shows that, when the size of particles is 20 and the maximum number of cycles is 30, the proposed optimization algorithm outperforms the 2D-PSO algorithm, and the accuracy of the proposed algorithm is close to that of the exhaustive attack method. Specifically, the proposed algorithm is superior to the exhaustive attack algorithm in time complexity. Furthermore, we obtain a more accurate feasible region by theoretical analysis, and then propose the bisection-PSO algorithm by leveraging the special structure of the problem. Thus, the proposed algorithm can effectively avoid from falling into local optimal solutions and outperforms the 2D-PSO algorithm.
The proposed algorithm has low complexity, and the optimized solution can be obtained quickly with known obstacle heights. At the same time, by studying the trade-off, we can solve the problem that optimizing one of EE and SE performance will have too great a negative impact on the other one.

The algorithm in this paper has some limitations, including that it is only applicable to time-delay tolerant systems. The trajectory planning scheme is based on circular trajectories and straight lines, and lacks the study of arbitrary trajectories. The 3D trajectory design based on obstacle avoidance with specific obstacle information will be our future research direction.

5. Conclusions

In this paper, a UAV-enabled mobile relaying system in the presence of mountain obstacles is investigated. We study two kinds of flight trajectory of UAV, and an efficient algorithm is proposed to optimize the EE and SE under different trajectories. At the end of this paper, the effectiveness of the proposed optimization algorithm is demonstrated by numerical results. When the communication distance is 1000 m, the SE and EE of the circular trajectory is better than the SLF trajectory when the obstacle height is greater than 300 m. Above all, this paper reveals the impacts of obstacle dimensions on UAV trajectory design and analyzes the EE-SE trade-off problem, which gives an additional design dimension in UAV-enabled mobile relaying systems.

Author Contributions: Q.A. researched the literature, built the model, derived the formulas, and carried out the simulations. She also finished the whole manuscript. Y.P. and H.H. (Hang Hu) revised the manuscript. H.H. (Huizhu Han) and Y.H. assisted with the integrity of the entire study. All authors have read and agreed to the published version of the manuscript.

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Appendix A

As shown in Figure A1, the locations of the UAV relay at \( t_1 \) and \( t_2 \) are denoted as \( R_1 \) and \( R_2 \), respectively. The distance between \( S-R_1 \), \( D-R_1, S-R_2 \), \( D-R_2 \) are expressed as \( H_S/\sin(\angle R_1 SD), H_S/\sin(\angle R_1 DS), H_S/\sin(\angle R_2 SD), H_S/\sin(\angle R_2 DS) \). If \( d_{SR_1} > d_{SR_2} \), it can be derived that \( \angle R_1 SD < \angle R_2 SD \), i.e., \( R_1 \) is at the right side of \( R_2 \). Thus, we have the following result:

When \( t_1, t_2 \in [0, T/2] \), if \( d_{SR_1} > d_{SR_2} \), \( d_{RD_1} < d_{RD_2} \) must be satisfied simultaneously.

Based on that, Lemma 1 can be proved by contradiction. Suppose, on the contrary, that \( d_{SR_1} > d_{SR_2} \) when \( u^*(t_1) = 1 \) and \( u^*(t_2) = 0 \). Since \( d_{SR_1} > d_{SR_2} \), \( d_{RD_1} > d_{RD_2} \) must be satisfied simultaneously. Then, we construct a new time allocation function \( \bar{u}(t) \), where \( \bar{u}(t_1) = 0, \bar{u}(t_2) = 1 \). If we define a sufficiently small time slot \( \Delta T \), the distance of the UAV-S and UAV-D can be considered to remain constant during a time slot. Intuitively, \( \bar{u}(t) \) can achieve a higher rate than \( u^*(t) \) due to the shorter communication distance during \([t_1 - \Delta T, t_1]\) and \([t_2 - \Delta T, t_2]\), respectively. It contradicts that \( u^*(t) \) is the optimal time allocation function. Thus, The original hypothesis is invalid, and the proof is thus completed.
Figure A1. Schematic for the proof of Lemma 1.

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