Fixed-Time Formation Tracking Control of Multiple Unmanned Surface Vessels Considering Lumped Disturbances and Input Saturation

Bowen Sui, Jianqiang Zhang *, Yan Li, Yuanyuan Zhang and Zhong Liu

Article

1. Introduction

The characteristics of unmanned surface vessels include low expenses, great flexibility, and a high degree of autonomy and intelligence. They offer special abilities for carrying out a variety of activities in challenging maritime situations without endangering human life [1]. When completing complex tasks, a formation composed of multiple USVs is more capable and adaptable than a single USV. Therefore, the formation control of multiple USVs has lately drawn a lot of interest as a key technology for these applications [2]. One major issue for multiple USV systems is formation tracking control, which requires the USVs to maintain a specified formation while concurrently following a reference trajectory. Numerous relevant study projects on the formation tracking control of USVs have been conducted, and numerous fruitful research findings have been published [3–5].

On the one hand, the dynamics of USVs are highly nonlinear and strongly coupled, which makes the formation control of multi-USVs quite challenging. This is further complicated by the uncertainties of model parameters and the complex ocean environment. To deal with model parameter uncertainties and ocean disturbances, a variety of algorithms have been developed, mainly based on neural network approximation algorithms [6–8] and disturbance-observer-based algorithms [9–11]. The neural-network-based technique uses the neural network’s excellent approximation capabilities to assess model parameter uncertainties and ocean disturbances. Due to the existence of approximate residuals in the neural network algorithm, it can only obtain globally uniformly bounded tracking performance rather than asymptotically stable performance. In [12], a trajectory tracking...
controller is proposed for underactuated surface vehicles, considering uncertainties and unmeasurable velocities. The system uncertainties are estimated by RBF neural network, and simulation results show the effectiveness of the proposed method. Ref. [13] provides finite-time trajectory tracking control schemes for marine surface vessels that are influenced by dynamic uncertainties and unknown time-varying disturbances. Neural networks are applied to reconstruct the vehicle’s dynamic uncertainties, and the sum of the upper bound of approximation error and external unknown disturbances is estimated by designing an adaptive law. Subsequently, rigorous theoretical analyses are provided to prove that, owing to the developed finite-time trajectory tracking control strategies, all the signals of the closed-loop trajectory tracking control system are bounded, and that the actual trajectory of MSVs can track the reference trajectory in finite time. In the algorithm based on the disturbance observer, the parameter uncertainties and ocean disturbances are estimated by the disturbance observer, and the accurate estimation of model uncertainties and ocean disturbances is realized. In [14], a non-singular terminal sliding model controller is proposed for the trajectory tracking control of the underactuated USV, with a nonlinear disturbance observer which is designed to measure complex environmental disturbances, such as wind, waves, and currents. Exploratory simulations were carried out and the results show that the proposed controller is effective and robust for the trajectory tracking of underactuated USVs in the presence of environmental disturbances. The method of designed trajectory tracking for an underactuated unmanned surface vehicle in the presence of ocean disturbances is addressed in [15]. The nonlinear disturbance observer is designed to obtain the estimated values of unknown disturbances in the ocean. The inherent robustness of the controller and estimates of the observer are used to resist and compensate for disturbances. Finally, the simulation experiments of linear trajectory and sinusoidal trajectories are carried out to prove the effectiveness and reliability of the control algorithm designed.

On the other hand, the convergence rate is a significant statistic that reflects the response characteristics of the cooperative formation control of multiple USVs. The majority of USV formation control systems in the past were asymptotically stable, which means that under the control of the asymptotic stability controller, the system state equilibrium position cannot converge in a finite time to the equilibrium position [16]. The rate at which the system state converges to the equilibrium point varies between the controllers for asymptotic stability and finite time. The finite-time controller converges more quickly [6,17]. However, the convergence performance of the finite-time stable control depends on the initial state of the system. It is difficult to obtain the initial value of the system in practical engineering, which limits the application of this method. The most important aspect of the fixed-time control theory [18] is that it not only increases the speed of system convergence but also eliminates the system’s starting state’s laziness by simply using the controller’s design parameters as a factor when calculating system convergence time. A multivariable, fixed-time leader–follower formation control approach is presented in [19] for a group of nonholonomic mobile robots. The proposed algorithm can estimate numerous uncertainties, and an experimental simulation confirms the program’s efficacy. In [20], an event-triggered, fixed-time multiple stratospheric airship formation trajectory tracking controller is designed, composed of the airship leader trajectory tracking controller and the airship follower formation tracking controller. The formation control of underactuated USVs with unknown dynamics and ocean disturbances is discussed in [21], and a novel fixed-time sliding mode controller is introduced. Ref. [22] investigates the multiple unmanned surface vehicle systems with intermittent actuator defects and fixed-time fuzzy formation tracking control issues. Ref. [23] offers a fixed-time controller approach for controlling the formation of surface vehicles based on a fixed-time strategy, a finite-time disturbance observer, and a leader–follower algorithm. The fixed-time formation-containment control is examined in [24] for multi-agent systems with model uncertainties and external disturbances. However, the input saturation problem has not been discussed. Therefore, some research is urgently needed to fill this gap given the dual requirements of anti-interference capability and the convergence rate of the system.
Motivated by the above observations, we present a unique fixed-time formation trajectory tracking control scheme for multiple unmanned surface vessels affected by uncertain dynamics, environmental ocean disturbances, as well as input saturation. The main advantage and motivation of our paper are twofold: (1) An adaptive super-twisting lumped disturbance observer is developed to achieve fast and accurate reconstruction and compensation of lumped disturbance containing external ocean perturbations and uncertain model dynamics. Subsequently, the stability analysis of the whole observer is proven by Lyapunov’s arguments. (2) In addition, command filter technology is used to cope with the differential explosion problem of the backstepping approach. Integrated with fixed-time theory and a leader–follower algorithm, as well as an anti-saturation auxiliary system, a fixed-time formation controller is designed for the follower USVs. The proposed formation tracking control scheme improves the convergence rate of the system, and input saturation of the intermediate control law is avoided.

The remainder of this article is structured as follows. Section 2 presents the problem formulation and preliminaries. The main findings of this work are introduced in Section 3. Section 4 carries out the simulation and analyzes the simulation results. Finally, the concluding remarks are given in Section 5.

2. Preliminaries and Problem Statement

2.1. Kinematic and Dynamic Model of USV

Consider N surface vessels as the USV systems. The dynamics of the kth (k = 1, ⋯, N) USV are depicted as follows [25]:

\[ \dot{\eta}_k = J(\psi_k)v_k \]

where \( \eta_k = [x_k, y_k, \psi_k]^T \) represents the position vector of the kth USV in the earth-fixed inertial frame (OXYZ); \( v_k = [u_k, v_k, r_k]^T \) denotes the velocity vector of the kth USV in the body-fixed frame (OXkYkZk). The transformation matrix \( J(\psi) \) from the inertial coordinate system to the USV body coordinate system can be written as follows:

\[
J(\psi_k) = \begin{bmatrix}
\cos(\psi_k) & -\sin(\psi_k) & 0 \\
\sin(\psi_k) & \cos(\psi_k) & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad M_k = \begin{bmatrix}
m_{11,k} & 0 & 0 \\
0 & m_{22,k} & m_{23,k} \\
0 & m_{32,k} & m_{33,k}
\end{bmatrix},
\]

\[ C_k(v_k) = \begin{bmatrix}
0 & 0 & -c_{1,k} \\
0 & 0 & c_{2,k} \\
c_{1,k} & -c_{2,k} & 0
\end{bmatrix}, \quad D_k(v_k) = \begin{bmatrix}
d_{11,k} & 0 & 0 \\
0 & d_{22,k} & d_{23,k} \\
0 & d_{32,k} & d_{33,k}
\end{bmatrix}
\]

represent the inertia matrix, Coriolis matrix, and hydrodynamic damping matrix, respectively. The matrices above are given in the following format with model uncertainties to describe the dynamics of USV more accurately:

\[ M_k = M_{0,k} + \Delta M_k, C_k = C_{0,k} + \Delta C_k, D_k = D_{0,k} + \Delta D_k \]

where \( M_{0,k}, C_{0,k}, \) and \( D_{0,k} \) are nominal matrices. \( \Delta M_k, \Delta C_k, \) and \( \Delta D_k \) demonstrate model parameter uncertainties.

\( \tau_k = [\tau_{k,u}, \tau_{k,v}, \tau_{k,r}]^T \) illustrates the control forces and moments. \( \tau_{w,k} = [\tau_{w,k,u}, \tau_{w,k,v}, \tau_{w,k,r}]^T \) depicts the ocean disturbances. Additionally, the command control input \( \tau_{k,c} \), determined by the control law, is subject to input saturation given the physical limitations of the vessel propulsion system, which are represented as follows:

\[
\tau_k = \begin{cases}
\tau_{k,u}^+ & \text{if } \tau_{k,u} > \tau_{k,u}^+ > 0 \\
\tau_{k,v} & \text{if } \tau_{k,u} \leq \tau_{k,v} \leq \tau_{k,v}^+, \quad i = u, v, r \quad k = 1, 2 \\
\tau_{k,c}^- & \text{if } \tau_{k,c} < \tau_{k,c}^- < 0
\end{cases}
\]
where $\tau_k = [\tau_{k,x}, \tau_{k,y}, \tau_{k,\theta}]^T$, $\tau_k^+ > 0$ and $\tau_k^- < 0$ are the maximum and the minimum control force or moment. Let $\Delta \tau_k = \tau_k - \tau_k^-$ be the actual control input minus the command control input.

2.2. Virtual Leader–Follower Algorithm

This study presents a virtual leader–follower algorithm and adds the idea of a virtual reference USV to the conventional leader–follower method. The main design idea is to assume that the virtual USV can track the leader USV in real time. Hence, formation control is realized if the established control law causes the following USV to arrive at the position of the virtual reference USV within the required time.

Let $\eta_l = [x_l, y_l, \psi_l]^T$, $\eta_{v,k} = [x_{v,k}, y_{v,k}, \psi_{v,k}]^T$, and $\eta_k = [x_k, y_k, \psi_k]^T$ indicate the leader USV’s position and heading, the virtual reference USV’s position and heading, and the following USV’s position and heading, respectively. The formation configuration is determined by the distance $\rho$ and the relative orientation $\dot{\theta}$ between the virtual reference USV and the leader USV, with the following expression:

$$
\begin{pmatrix}
    x_{v,k} \\
    y_{v,k} \\
    \psi_{v,k}
\end{pmatrix} =
\begin{pmatrix}
    x_l \\
    y_l \\
    \psi_l
\end{pmatrix} +
\begin{pmatrix}
    \cos(\psi_l) & -\sin(\psi_l) & 0 \\
    \sin(\psi_l) & \cos(\psi_l) & 0 \\
    0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    \rho_k \cos(\theta_k) \\
    \rho_k \sin(\theta_k) \\
    0
\end{pmatrix}
$$

Equation (5) can be written in a compact vector form:

$$
\eta_{v,k} = \eta_l + J(\psi_l)\phi_k
$$

where $\phi_k = [\rho_k \cos(\theta_k), \rho_k \sin(\theta_k), 0]$.

2.3. Relative Lemmas and Assumptions

**Lemma 1** [26]. If a continuous radially bounded function $V(x)$ exists such that any solution $x(t)$ satisfies

1. $V(x) = 0 \iff x = 0$;
2. If any $x(t)$ satisfies the inequality:

$$
\dot{V}(x) \leq -\mu_1 V^\alpha(x) - \mu_2 V^\beta(x)
$$

where $\mu_1, \mu_2, \alpha$, and $\beta$ are positive constants, $0 < \alpha < 1$, $\beta > 1$. Then, the system is globally fixed time and the settling time is bounded by $T$:

$$
T \leq T_{\text{max}} := \frac{1}{\mu_1(1-\alpha)} + \frac{1}{\mu_2(\beta-1)}
$$

3. If any $x(t)$ satisfies the inequality:

$$
\dot{V}(x) \leq -\mu_1 V^\alpha(x) - \mu_2 V^\beta(x) + \theta
$$

The system is actually stable in fixed time and the settling time $T$ is bounded

$$
T \leq T_{\text{max}} := \frac{1}{\mu_1 \theta(1-\alpha)} + \frac{1}{\mu_2 \theta(\beta-1)}
$$
Lemma 2 [27]. If $\delta_1, \delta_2, \cdots, \delta_M \geq 0$, we have

$$\sum_{i=1}^{n} \delta_i^k \geq \left( \sum_{i=1}^{n} \delta_i \right)^k, 0 < k < 1$$

$$\sum_{i=1}^{n} \delta_i^k \geq n^{1-k} \left( \sum_{i=1}^{n} \delta_i \right)^k, 1 < k < \infty$$

Assumption 1 [28]. The position, heading, and velocity of the leader USV can be obtained by following USVs.

Assumption 2 [28]. The trajectory $\xi$ of the leader USV is smooth and bounded, and its first and second derivatives $\dot{\xi}, \ddot{\xi}$ exist and are bounded.

3. Controller Design

At first, an adaptive super-twisting lumped disturbance observer is put forward to estimate the total disturbance of the system; then, based on the observer, in conjunction with the backstepping approach, command filter technique, as well as anti-saturation auxiliary system, a fixed-time convergence formation tracking controller is designed.

3.1. Adaptive Super-Twisting Lumped Disturbance Observer (ASTLDO)

Integrate Equations (1)–(3), the USV dynamics can be reformulated as follows:

$$\begin{align*}
\dot{\eta}_k &= f(\psi_k)v_k \\
M_{0,k}(v_k)\dot{v}_k + C_{0,k}(v_k)v_k + D_{0,k}(v_k)v_k = \tau_k + \tau_{d,k}
\end{align*}$$

where $\tau_{d,k} = \tau_{w,k} - \Delta M_k(v_k)\dot{v}_k - \Delta C_k(v_k)v_k - \Delta D_k(v_k)v_k$.

Assumption 3 [29]. $\tau_{d,k}$ in Equation (12) is bounded, i.e., $\|\tau_{d,k}\| \leq \Xi < \infty$, where $\Xi$ is a constant; $\|\cdot\|$ is the 2-norm of matrix or vector.

Remark. The environment disturbances are always considered as slowly varying and have finite energy. As a result, the disturbances acting on USV can be viewed as unknown finite change rates and bounded signals. In summary, assumption 3 is reasonable.

To facilitate the design of the disturbance observer, the auxiliary variables are defined as $\zeta_{1,k}(t) = \eta_k(t), \zeta_{2,k}(t) = \dot{\eta}_k(t)$. Equations (1) and (2) can be translated as follows:

$$\begin{align*}
\dot{\zeta}_{1,k}(t) &= \zeta_{2,k}(t) \\
\dot{\zeta}_{2,k}(t) &= S(r_k)\zeta_{2,k}(t) + f(\psi_k)M_{0,k}^{-1}(v_k)(-C_{0,k}(v_k)v_k - D_{0,k}(v_k)v_k) \\
&\quad + \tau_{d,k}(t) + f(\psi_k)M_{0,k}^{-1}(v_k)\tau_k
\end{align*}$$

Furthermore, let $F_k = S(r_k)\zeta_{2,k}(t) + f(\psi_k)M_{0,k}^{-1}(v_k)(-C_{0,k}(v_k)v_k - D_{0,k}(v_k)v_k)$, $G_k = f(\psi_k)M_{0,k}^{-1}(v_k)$ and Equation (13) can be simply rewritten as follows:

$$\begin{align*}
\dot{\zeta}_1(t) &= \zeta_2(t) \\
\dot{\zeta}_2(t) &= F + G\tau + \tau_d(t)
\end{align*}$$

Let $\zeta_2(t) = \sigma(t)$, $\tau_d(t) = \chi(t)$

The subsequent equation is obtained as follows:

$$\begin{align*}
\dot{\sigma}(t) &= F + G\tau + \chi(t) \\
\dot{\chi}(t) &= h(t)
\end{align*}$$
where $h(t)$ is the time derivative of the lumped disturbance $\chi(t)$.

Subsequently, a novel super-twisting disturbance observer is proposed [30].

$$
\dot{\tilde{\chi}}(t) = \tilde{\chi}(t) - \chi(t)
$$

$$
\tilde{\tilde{\chi}}(t) = \tilde{\chi}(t) - \chi(t)
$$

$$
\dot{\tilde{\sigma}}(t) = \tilde{\sigma}(t) - \sigma(t)
$$

$$
\dot{\tilde{\chi}} = F + G\tau - K_1(\tilde{\sigma}) + \tilde{\chi}(t)
$$

$$
\dot{\tilde{\chi}} = -K_2(\tilde{\sigma})
$$

where $\tilde{\sigma}(t)$ and $\tilde{\tilde{\chi}}(t)$ are the estimation of $\sigma(t)$, $\chi(t)$. Each element in vectors $\Phi_1(\tilde{\sigma}) = [\phi_{11}, \phi_{12}, \phi_{13}]^T$ and $\Phi_2(\tilde{\sigma}) = [\phi_{21}, \phi_{22}, \phi_{23}]^T$ is defined as follows:

$$
\phi_{11}(\tilde{\sigma}_i) = \mu_{11}|\tilde{\sigma}_i|^{1/2}\text{sgn}(\tilde{\sigma}_i) + \mu_{21}\tilde{\sigma}_i
$$

$$
\phi_{21}(\tilde{\sigma}_i) = \frac{1}{2}\mu_{11}^2|\tilde{\sigma}_i|^{1/2}\text{sgn}(\tilde{\sigma}_i) + \frac{3}{2}\mu_{12}|\tilde{\sigma}_i|^{1/2}\text{sgn}(\tilde{\sigma}_i) + \mu_{22}\tilde{\tilde{\sigma}}_i
$$

where $\mu_{1i}, \mu_{2i} > 0, i = 1, 2, 3$. $K_1 = \text{diag}(k_{11}, k_{12}, k_{13})$ and $K_2 = \text{diag}(k_{21}, k_{22}, k_{23})$ denote the positive definite gain matrices of the observer. Each element of $K_1$ and $K_2$ is represented as follows:

$$
k_{1i}(t) = \begin{cases} \omega_i\sqrt{\frac{2}{\epsilon_i^2}} & \text{otherwise} \\ 0 & \text{if } |\tilde{\sigma}_i| < \delta_i \end{cases}
$$

$$
k_{2i}(t) = 2\epsilon_i k_{1i}(t) + \beta_i + 4\epsilon_i^2
$$

**Theorem 1.** The proposed disturbance observer ASTLDO (16) can precisely observe and compensate the lumped disturbances for the system (12) with lumped disturbances by the designed adaptive laws (18) and (19) so that the observation error converges to zero in a finite time.

**Proof.** The observer error dynamics are provided by the following equation. □

$$
\dot{\tilde{\sigma}} = \tilde{\tilde{\chi}}(t) - K_1(\tilde{\sigma}) - h(t)
$$

$$
\dot{\tilde{\chi}} = -K_2(\tilde{\sigma}) - h(t)
$$

Define the following auxiliary variables:

$$
s_{1i} = \tilde{\sigma}_i
$$

$$
s_{2i} = \tilde{\tilde{\sigma}}_i
$$

Equation (20) can be rewritten as follows:

$$
\dot{s}_{1i} = -k_{1i}[|\mu_{11}|s_{1i}|^{1/2}\text{sgn}(s_{1i}) + \mu_{2i} s_{1i}] + s_{2i}
$$

$$
\dot{s}_{2i} = -k_{2i}[\frac{1}{2}\mu_{11}^2|s_{1i}|^{1/2}\text{sgn}(s_{1i}) + \frac{3}{2}\mu_{12}|s_{1i}|^{1/2}\text{sgn}(s_{1i}) + \mu_{22}s_{1i}] + h_i(t)
$$

Without losing generality, the system (22) can be simplified to the following expression:

$$
\dot{s}_1 = -k_{1}[|\mu_1|s_1|^{1/2}\text{sgn}(s_1) + \mu_2 s_1] + s_2
$$

$$
\dot{s}_2 = -k_{2}[\frac{1}{2}\mu_1^2|s_1|^{1/2}\text{sgn}(s_1) + \frac{3}{2}\mu_1\mu_2|s_1|^{1/2}\text{sgn}(s_1) + \mu_2^2 s_1] + h(t)
$$

To prove the stability of Equation (23), the following Lyapunov function is constructed as follows:

$$
V(s_1, s_2, k_1, k_2) = V_0(\cdot) + \frac{1}{2\xi_1}(k_1 - k_1^*)^2 + \frac{1}{2\xi_2}(k_2 - k_2^*)^2
$$

where $\xi_1, \xi_2, k_1^*$ and $k_2^*$ are positive constants. $V_0(\cdot)$ is selected as follows:

$$
V_0(s_1, s_2, k_1, k_2) = \xi^T P\xi
$$
where $\xi^T = [\phi_1(s_1), s_2]$, $P = P^T = \begin{bmatrix} \beta + 4\varepsilon^2 & -2\varepsilon \\ -2\varepsilon & 1 \end{bmatrix} > 0$.

Since $P$ is a positive definite matrix, $\beta$ and $\varepsilon$ can be set to any positive number. Obviously, $V_0(\cdot)$ satisfies the following inequality:

$$\lambda_{\min}(P)\|\xi\|^2_2 \leq V_0(s, k) \leq \lambda_{\max}(P)\|\xi\|^2_2$$

(26)

where $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ are the minimum and maximum eigenvalues of matrix $P$, respectively. $\|\xi\|^2_2 = \mu_1^2|s_1| + 2\mu_1\mu_2|s_1|^{3/2} + \mu_2^2s_2^2 + s_2^2$ is the Euclidean norm of $\xi$ and the next inequality is satisfied as well.

$$|\phi_1(s_1)| \leq \|\xi\|_2 \leq \frac{V_0^{1/2}(\xi)}{\lambda_{\min}^{1/2}(P)}$$

(27)

To facilitate the calculation, the time derivative of $V(\cdot)$ can be figured out initially and then the derivative of $V(\cdot)$ is obtained. The specific calculation process can be divided into two steps.

Step 1: Noting that $\phi_2(s_1) = \phi_1'(s_1)\phi_1(s_1)$, where $\phi_1'(s_1) = \mu_1 \frac{1}{2|s_1|^\varepsilon} + \mu_2 > 0$, and introducing $L = \frac{h(t)}{\phi_1(s_1)\phi_1'(s_1)}$. Then, the time derivative is as follows:

$$V_0 = \xi^T P\xi + \xi^T P\xi = 2\xi^T P\xi = 2\xi^T P \begin{bmatrix} \phi_1'(-k_1\phi_1(s_1) + s_2) \\ -k_2\phi_2(s_1) - h(t) \end{bmatrix}$$

$$= 2\xi^T P \begin{bmatrix} \phi_1'(s_1) - k_1\phi_1(s_1) + s_2 \\ \phi_1'(s_1)\phi_1(s_1) - k_2 - L \end{bmatrix} = \phi_1'(s_1) 2\xi^T P \begin{bmatrix} -k_1 \\ -k_2 - L \\ 0 \end{bmatrix} \xi$$

(28)

where $Q = \begin{bmatrix} 2(k_2 \beta + 4\varepsilon^2) - 2\beta(k_2 - 1) \\ -2k_1\varepsilon + k_2 + L - \beta - 2\varepsilon^2 \\ 4\varepsilon \end{bmatrix}$.

By selecting the observer gain $k_2 = 2k_1\varepsilon + \beta + 4\varepsilon^2$, we can obtain

$$Q - 2\varepsilon I = \begin{bmatrix} 2k_1\beta - 4\varepsilon(\beta + 4\varepsilon^2) - 2\varepsilon^{1/2}L \\ L \end{bmatrix}$$

(29)

The matrix $Q$ will be positive definite with a minimal eigenvalue $\lambda_{\min}(Q) \geq 2\varepsilon$ if

$$k_1 > \delta_0 + \frac{8\varepsilon^2}{4\varepsilon^2} + \frac{\varepsilon(2\beta + 4\varepsilon^2 + L) + 1}{2\beta}$$

(30)

Then, the time derivative of $V_0(\cdot)$ can be rewritten as follows:

$$V_0 = -\phi_1'(s_1)\xi^T Q\xi \leq -2\varepsilon\phi_1'(s_1)\xi^T\xi = -2\varepsilon \left( \mu_1 \frac{1}{2|s_1|^\varepsilon} + \mu_2 \right) \xi^T\xi$$

(31)

Integrating Equations (27) and (31), the time derivative of $V_0(\cdot)$ can be represented as follows:

$$V_0 \leq -\frac{\varepsilon^{1/2}(P)}{\lambda_{\max}(P)}\mu_1 V_0^{1/2} - \frac{2\varepsilon}{\lambda_{\max}(P)}\mu_2 V_0 \leq -\gamma V_0^{1/2}$$

(32)

where $\gamma = \mu_1 \varepsilon^{1/2}(P) \lambda_{\max}(P)$. 
Step 2: The time derivative of Equation (24) can be calculated as follows:

\[
\dot{V} = \dot{V}_0(\cdot) + \frac{1}{\lambda_1}(k_1 - \dot{k}_1)\dot{k}_1 + \frac{1}{\lambda_2}(k_2 - \dot{k}_2)\dot{k}_2 \\
\leq -\gamma V_0^{1/2} + \frac{1}{\lambda_1}(k_1 - \dot{k}_1)\dot{k}_1 + \frac{1}{\lambda_2}(k_2 - \dot{k}_2)\dot{k}_2 \\
= -\gamma V_0^{1/2} - \frac{\omega_1}{\sqrt{2\lambda_1}}|k_1 - \dot{k}_1| - \frac{\omega_2}{\sqrt{2\lambda_2}}|k_2 - \dot{k}_2| + \frac{1}{\lambda_1}(k_1 - \dot{k}_1)\dot{k}_1 + \\
\frac{1}{\lambda_2}(k_2 - \dot{k}_2)\dot{k}_2 + \frac{\omega_1}{\sqrt{2\lambda_1}}|k_1 - \dot{k}_1| + \frac{\omega_2}{\sqrt{2\lambda_2}}|k_2 - \dot{k}_2| \tag{33}
\]

The first three terms of Equation (33) can be synthesized as follows using the inequality.

\[
\sqrt{x^2 + y^2 + z^2} \leq |x| + |y| + |z|
\]

\[
-\gamma V_0^{1/2} - \frac{\omega_1}{\sqrt{2\lambda_1}}|k_1 - \dot{k}_1| - \frac{\omega_2}{\sqrt{2\lambda_2}}|k_2 - \dot{k}_2| \leq -\pi V^{1/2} \tag{34}
\]

where \(\pi = \min(\gamma, \omega_1, \omega_2)\).

Assuming that there exist positive constants \(k_1^*\) and \(k_2^*\) such that \(k_1 - k_1^* < 0\) and \(k_2 - k_2^* < 0\) are satisfied \(\forall t \geq 0\). Then, the time derivative of \(V(\cdot)\) can be rewritten as follows:

\[
\dot{V} \leq -\pi V^{1/2} - |k_1 - k_1^*|(\frac{1}{\lambda_1}\dot{k}_1 - \frac{\omega_1}{\sqrt{2\lambda_1}}) - |k_2 - k_2^*|(\frac{1}{\lambda_2}\dot{k}_2 - \frac{\omega_2}{\sqrt{2\lambda_2}}) + \theta \tag{35}
\]

where \(\theta = -|k_1 - k_1^*|(\frac{1}{\lambda_1}\dot{k}_1 - \frac{\omega_1}{\sqrt{2\lambda_1}}) - |k_2 - k_2^*|(\frac{1}{\lambda_2}\dot{k}_2 - \frac{\omega_2}{\sqrt{2\lambda_2}})\).

In essence, the dynamic and algebraic equations described in (18) through (19) will be used to raise the adaptive gains \(k_1\) and \(k_2\) until condition (30) is met. In such a case, the matrix \(Q\) will be positive and definite, and the convergence in a finite time will be guaranteed by (28). The prior finding ensures that \(\tilde{\sigma}\) and \(\tilde{\chi}\) will eventually converge to zero over a limited time, at which point adaptive benefits \(k_1\) and \(k_2\) will cease to increase, leading to the emergence of \(k_1 = 0\). This completes the proof. \(\square\)

3.2. Design of Fixed-Time Formation Tracking Controller

Due to the actuator’s limitations, attaining an overly large control law value in practical engineering applications is typically challenging. To address the issue of input saturation, the following adaptive auxiliary system is developed [28]:

\[
\begin{cases}
\dot{\lambda}_1 = -C_1\lambda_1 + J^T(\psi)\lambda_2 \\
\dot{\lambda}_2 = -C_2M^{-1}\lambda_2 + M^{-1}\Delta\tau
\end{cases} \tag{36}
\]

where \(\lambda_1\) and \(\lambda_2\) are the auxiliary vectors. \(C_1\) and \(C_2\) represent the designed parameter matrices. \(\Delta\tau = \tau - \tau_c\) is the difference between the command control input and the actual control input. To avoid the instability of the trajectory tracking control system caused by too large \(\Delta\tau\), it is necessary to select \(C_1\) and \(C_2\) large enough. At the same time, it can also make the USV with the required thrust out of the saturation region, and the auxiliary variable quickly converges to zero to achieve the control performance when the saturation is not triggered.

By Assumptions 1 and 2, the super-twisting lumped disturbance observer and the control law are created by combining the backstepping method and fixed-time control so that the follower USVs can reach the position of the virtual reference USV in a fixed time and the preset formation tracking configuration is achieved.

(1) Design of kinematic controller

Define the errors of position and heading for the following USVs:

\[
\gamma_1 = \eta_f - \eta_v - \lambda_1 \tag{37}
\]
Taking the time derivative of $\gamma_1$, we obtain
\[ \dot{\gamma}_1 = \dot{\eta}_f - \dot{\eta}_v - \dot{\lambda}_1 = J(\psi_f)v_f - J(\psi_l)v_l - J(\psi_t)S(\rho) + \dot{\lambda}_1 \]  
(38)

where $S(\rho) = \begin{bmatrix} 0 & -r_i & 0 \\ r_i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $r_i$ indicates the yaw angular velocity of the leader USV.

The kinematic control law is selected as follows:
\[ a_{\text{KF}} = J^T(\psi_f)[-\varphi(\gamma_1) + J(\psi_l)v_l + J(\psi_t)S(\rho) + \dot{\lambda}_1] \]  
(39)

where $\varphi(\gamma_1) = \mu_1 \gamma_1^\alpha + \mu_2 \gamma_1^\beta$. $\alpha$ and $\beta$ are constant parameters. $\mu_1$ and $\mu_2$ are the positive definite diagonal matrices to be designed.

Construct the following Lyapunov function to verify the convergence of $\gamma_1$.
\[ V_1 = \frac{1}{2} \gamma_1^T \gamma_1 \]  
(40)

Integrating Equations (32) and (33), the time derivative of $V_1(\cdot)$ can be obtained as follows:
\[
\dot{V}_1 = \gamma_1^T \gamma_1 = 
\gamma_1^T \left\{ J(\psi_f) (J^T(\psi_f)[-\varphi(\gamma_1) + J(\psi_l)v_l + J(\psi_t)S(\rho) + \dot{\lambda}_1] 
- J(\psi_l)v_l - J(\psi_t)S(\rho) - \dot{\lambda}_1) \right\} 
\]  
(41)
\[
= -\gamma_1^T \varphi(\gamma_1) - \Lambda_1(\gamma_1^T \gamma_1)^{(\alpha+1)/2} - \Lambda_2(\gamma_1^T \gamma_1)^{(\beta+1)/2} 
\leq -\lambda_{\text{min}}(\Lambda_1)(2V_1)^{(\alpha+1)/2} - \lambda_{\text{min}}(\Lambda_2)(2V_1)^{(\beta+1)/2}
\]

According to Lemma 1, $\gamma_1$ is stable in a fixed time.

The command filter approach is utilized to generate the new variables $a_{\text{KF}}^d$ and $\dot{a}_{\text{KF}}^d$ to prevent the differential explosion brought on by the repetitive derivation of the virtual control law [31].
\[ \dot{h} = f \ell_1 \]  
(42)
\[ \dot{\ell}_1 = -2 h f \ell_1 - f(h - a_{\text{KF}}) \]

where $a_{\text{KF}}$ represents the input signal of the filter. $h$ and $\ell_1$ denote the output signal of the filter. $f > 0$ and $\ell_1 \in [0, 1]$ are the designed filter gains and depict the filter frequency and damping ratio, respectively. Moreover, $h_1(0) = a_{\text{KF}}(0)$, $\ell_1(0) = 0$.

(2) Design of dynamic controller

Define the velocity error as follows:
\[ \gamma_2 = v_f - a_{\text{KF}}^d - \lambda_2 \]  
(43)

By figuring out the time derivative of $\gamma_2$, we obtain
\[ \dot{\gamma}_2 = M_0^{-1}(-C_0(v_f)v_f - D_0(v_f)v_f + \tau_f) + \tau_d \]  
(44)

Furthermore, the following control law is formulated:
\[ \tau_f = M_0[-\varphi(\gamma_2) + \dot{\tau}_d] + C_0(v_f)v_f + D_0(v_f)v_f + a_{\text{KF}}^d + \dot{\lambda}_2 \]  
(45)

where $\varphi(\gamma_2) = \Lambda_3 \gamma_2^\delta + \Lambda_4 \gamma_2^\beta + \Lambda_5 \gamma_2$, $\Lambda_i \in \mathbb{R}^{3 \times 3}, (i = 1, 2, 3, 4, 5)$ are positive definite diagonal matrices satisfying $\lambda_{\text{min}}(\Lambda_3) > 1/2$. 


The subsequent Lyapunov function is developed to confirm the convergence of $\gamma_2$.

$$V_2 = \frac{1}{2} \gamma_2^T \gamma_2$$  \hspace{1cm} (46)

According to Equation (22) and Young’s inequality, the time derivative of Equation (24) can be obtained as follows:

$$\dot{V}_2 = \frac{1}{2} \dot{\gamma}_2^T \left[ M_0^{-1} [-C_0(v_f) v_f - D_0(v_f) v_f] + M_0^{-1} [-M_0(\varphi(\gamma_2) + \tilde{v}_{d,k}) + C_0(v_f) v_f]ight.

+ D_0(v_f) v_f + \dot{\alpha}_d - \dot{\lambda}_2 + \tau_{d,k} - \dot{\alpha}_d - \dot{\lambda}_2 \right] = \frac{1}{2} \dot{\gamma}_2^T [-\varphi(\gamma_2) + \tilde{v}_{d,k} - \tau_{d,k}]

\leq - \Lambda_3 (\gamma_2^T \gamma_2)^{(a+1)/2} - \Lambda_4 (\gamma_2^T \gamma_2)^{(\beta+1)/2} - \frac{1}{a} (2\Lambda_3 - 1) (\gamma_2^T \gamma_2) + \frac{1}{2} \| \tilde{v}_{d,k} \|

\leq - \lambda_{\min}(\Lambda_3) 2^{(a+1)/2} V_2^{(a+1)/2} - \lambda_{\min}(\Lambda_4) 2^{(\beta+1)/2} V_2^{(\beta+1)/2} + \frac{1}{2} \| \tilde{v}_{d,k} \|$$  \hspace{1cm} (47)

From Theorem 1, it is obtained that there exists a positive constant $\kappa$ such that $\tilde{v}_{d,k} \leq \kappa, t \in [0, T_1]$ and $\tilde{v}_{d,k} = 0, \forall t \geq T_1$. From Lemma 1 and Lemma 2, $\gamma_2$ is stable in a fixed time.

The following theorem can be deduced from the investigation described above.

**Theorem 2.** Considering the USVs with models (1) and (2), the super-twisting disturbance observer (10), the kinematics controller (17), and the dynamics controller (23) are constructed under Assumptions 1–3, which can realize the formation tracking error control signal stable in a fixed time.

**Proof.** The stability of the overall formation tracking system is analyzed using the ensuing Lyapunov function.

$$V = V_1 + V_2$$  \hspace{1cm} (48)

According to Equation (19), Equation (25), and Lemma 1, the time derivative of $V(\cdot)$ is obtained as follows:

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \leq

- \lambda_{\min}(\Lambda_3) 2^{(a+1)/2} V_1^{(a+1)/2} - \lambda_{\min}(\Lambda_4) 2^{(\beta+1)/2} V_2^{(\beta+1)/2} -\frac{1}{a} V^{(a+1)/2} - b V^{(\beta+1)/2}$$  \hspace{1cm} (49)

where $a = 2^{(a+1)/2} \lambda_{\min}(\Lambda_1, \Lambda_3), b = 2 \lambda_{\min}(\Lambda_2, \Lambda_4)$. The convergence time $T_2$ satisfies the following inequality.

$$T_2 \leq T_{\text{max}} = \frac{2}{a(1-a)} + \frac{2}{b(1-\beta)}$$  \hspace{1cm} (50)

This completes the proof. □

**4. Simulation Studies**

In this subsection, two sets of simulations are carried out, which consist of three well-known Cyber-Ships II [25], to verify the effectiveness of the designed formation tracking control scheme. USV0 stands for the leader, and USV1 and USV2 are two followers. For the sake of study convenience, it is assumed that all USVs have identical kinematic and dynamic models, and the model parameters are given in Tables 1 and 2 [32].
The following USVs can accurately track the virtual reference trajectory and complete the preset formation configuration. In Figures 2 and 3, the trajectory tracking errors of USV1 and USV2 are displayed, respectively. Figures 4 and 5 present the velocity tracking errors of USV1 and USV2. Figures 2–5 show that the follower USV can track the virtual trajectory under the influence of the controller developed in this article, enabling the leader–follower technique to be used for multi-USV formation tracking control. The kinematic and dynamic controllers of the following USVs are given in Figures 6–9. Due to the constraint of the
fixed-time stability, the actual control effect is enormous, which could be more conducive to practical application. Therefore, the anti-saturation auxiliary is introduced in this work. Figures 8 and 9 show that the proposed anti-saturation auxiliary system can limit the control input within a reasonable range, avoiding the problem that the actuator cannot work due to excessive control input. Figures 10 and 11 are the disturbance observer estimation response curves of the following USVs. From the diagram, the super-twisting disturbance observer errors can quickly converge to zero and accurately estimate the lumped disturbances, which can quickly compensate for the influence of external disturbances on the formation system.

Figure 1. The formation tracking trajectory of the USVs in the XY plane.

Figure 2. The position and heading tracking errors of USV1.

Figure 3. The position and heading tracking errors of USV2.
Figure 1. The formation tracking trajectory of the USVs in the XY plane.

Figure 2. The position and heading tracking errors of USV1.

Figure 3. The position and heading tracking errors of USV2.

Figure 4. The velocity tracking error of USV1.

Figure 5. The velocity tracking error of USV2.

Figure 6. The kinematic controller of USV1.

Figure 7. The kinematic controller of USV2.
Figure 5. The velocity tracking error of USV2.

Figure 6. The kinematic controller of USV1.

Figure 7. The kinematic controller of USV2.

Figure 8. The dynamic controller of USV1.

Figure 9. The dynamic controller of USV2.

Figure 10. The lumped disturbance estimation error of USV1.

Figure 11. The lumped disturbance estimation error of USV2.

4.2. Scenario 2

Table 4 displays the initial states for the leader USV and the follower USVs in scenario 2. Moreover, the control parameters and the disturbance observer parameters are given the same as in scenario 1. The formation offsets are set as $\rho_1 = 52\, \pi/4$, $\theta_1 = 7\, \pi/4$; $\rho_2 = 52\, \pi/4$, $\theta_2 = 3\, \pi/4$. The external ocean disturbances acting on the follower USVs are set as follows:
4.2. Scenario 2

Table 4 displays the initial states for the leader USV and the follower USVs in scenario 2. Moreover, the control parameters $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5, f, \chi$, and the disturbance observer parameters $\varepsilon_1, \varepsilon_2, \mu_1, \mu_2, \gamma$ are given the same as in scenario 1. The formation offsets are set as $\rho_1 = 5\sqrt{2}$, $\theta_1 = \frac{7\pi}{4}$; $\rho_2 = 5\sqrt{2}$, $\theta_2 = \frac{3\pi}{4}$. The external ocean disturbances acting on the follower USVs are set as follows:

\[
\begin{align*}
\tau_{w,1u} &= 0.5 \sin(0.02t) + 2.5 \sin(0.02t) (N) \\
\tau_{w,1v} &= 3 \sin(0.03t) + \sin(0.05t) (N) \\
\tau_{w,1r} &= \sin(0.04t) + 2 \sin(0.01t) (N \cdot m) \\
\tau_{w,2u} &= 3 \sin(0.01t) + \sin(0.05t) (N) \\
\tau_{w,2v} &= 1.5 \sin(0.01t) + 1.5 \sin(0.03t) (N) \\
\tau_{w,2r} &= 2 \sin(0.02t) + \sin(0.05t) (N \cdot m)
\end{align*}
\]

Table 4. The model parameters of USV.

<table>
<thead>
<tr>
<th>States</th>
<th>USV0</th>
<th>USV1</th>
<th>USV2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta(0)$</td>
<td>$[0, 0, 0]^T$</td>
<td>$[-3, -10, 0]^T$</td>
<td>$[-3, 10, 0]^T$</td>
</tr>
<tr>
<td>$v(0)$</td>
<td>$[2.5, 0.5, 0.1]^T$</td>
<td>$[0, 0, 0]^T$</td>
<td>$[0, 0, 0]^T$</td>
</tr>
</tbody>
</table>

The formation tracking simulation results are illustrated in Figures 12–18. Figure 12 displays the formation tracking trajectory of USVs, and the following USVs can reach the desired trajectory accurately and quickly. The trajectory tracking errors for USV1 and USV2 are indicated in Figures 13 and 14, respectively. It can be seen from Figures 13 and 14 that the tracking error of the follower USVs can converge quickly and achieve a preset formation in a fixed time. Figures 15 and 16 are the disturbance observer estimation response curves of the following USVs. From the diagrams, the super-twisting disturbance observer errors can quickly converge to zero and accurately estimate the lumped disturbances, which can compensate for the influence of external disturbances on the formation system in finite time. With the aid of the auxiliary anti-winup system (36), it can be seen from Figures 17 and 18 that the control inputs are constrained within a pre-set reasonable range. In summary, according to the simulation results, the fixed-time formation tracking control scheme proposed in this paper can improve the convergence performance of the formation system and be stable for a fixed time, which is of great significance in practical applications.

![Figure 12. The formation tracking trajectory of the USVs in the XY plane.](image-url)
Figure 13. The position and heading tracking errors of USV1.

Figure 14. The position and heading tracking errors of USV2.

Figure 15. The lumped disturbance estimation error of USV1.
5. Conclusions

This article focuses on uncertain model parameters, external ocean disturbances, and input saturation of actuators in the process of fixed-time formation tracking control of multi-USVs systems. A multi-USVs formation tracking control technique is developed by combining backstepping technology, fixed-time theory, and the anti-saturation auxiliary system. Through theoretical proof and simulation research, the following conclusions are obtained:

Figure 16. The lumped disturbance estimation error of USV2.

Figure 17. The dynamic controller of USV1.

Figure 18. The dynamic controller of USV2.
5. Conclusions

This article focuses on uncertain model parameters, external ocean disturbances, and input saturation of actuators in the process of fixed-time formation tracking control of multi-USVs systems. A multi-USVs formation tracking control technique is developed by combining backstepping technology, fixed-time theory, and the anti-saturation auxiliary system. Through theoretical proof and simulation research, the following conclusions are obtained:

1. The proposed adaptive super-twisting disturbance observer is used to approximate the lumped disturbance of the system, which enhances the robustness of the system and ensures that the observation error of the system converges in a fixed time.

2. The multi-USVs formation control strategy based on the ASTLDO is designed to ensure the fixed-time stability of the closed-loop system. The convergence time of the proposed method is not affected by the initial value of the system, which overcomes the difficulty of obtaining the initial state of the system accurately in practical engineering.

3. By introducing the anti-saturation dynamic auxiliary system, the input saturation problem is effectively avoided.

However, since the fixed-time formation control’s convergence time depends on the system’s design parameters, it cannot be arbitrarily altered, which restricts its practical application. Consequently, the prescribed time formation control will be the focus of our future research.

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References


10. Sun, Z.; Sun, H.; Li, P.; Zou, J. Formation Control of Multiple Underactuated Surface Vessels with a Disturbance Observer. J. Mar. Sci. Eng. 2022, 10, 1016. [CrossRef]

22. Wu, W.; Tong, S. Fixed-time formation fault tolerant control for unmanned surface vehicle systems with intermittent actuator faults. *Ocean Eng.* 2023, 281, 114813. [CrossRef]
30. Guerrero, J.; Torres, J.; Creuze, V.; Chemori, A. Adaptive disturbance observer for trajectory tracking control of underwater vehicles. *Ocean Eng.* 2020, 200, 107080. [CrossRef]
32. Liu, H.; Weng, P.; Tian, X.; Mai, Q. Distributed adaptive fixed-time formation control for UAV-USV heterogeneous multi-agent systems. *Ocean Eng.* 2023, 267, 113240. [CrossRef]

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