Enhanced Gain Difference Power Allocation for NOMA-Based Visible Light Communications

Xian Zhong, Pu Miao * and Xiaoqing Wang

School of Electronic and Information Engineering, Qingdao University, Qingdao 266071, China; 2021020652@qdu.edu.cn (X.Z.); 2021020681@qdu.edu.cn (X.W.)
* Correspondence: mpvae@qdu.edu.cn

Abstract: With the escalating demand for high-data-rate wireless services, visible light communication (VLC) technology has emerged as a promising complement to traditional radio frequency wireless networks. To further enhance the achievable rate and error performance in non-orthogonal multiple access-based VLC downlinks, an efficient power allocation scheme named enhanced gain difference power allocation (EGDPA) is proposed for a multiple-input multiple-output VLC system. The power factors are determined by considering users’ channel gains and utilizing the residual allocation principle, which focuses on the remaining power available after allocating it to the previous users. In addition, the impacts of the user distribution and transmission power are investigated, and the performance metrics in terms of achievable data rate, energy efficiency, and bit error rate are also analytically presented. Simulation results demonstrate that energy efficiency can be significantly improved and the achievable data rate gain can be enhanced by at least 6.25% with the proposed EGDPA scheme as compared with other traditional methods, confirming its superiority and validity for efficient multi-user accessing.

Keywords: visible light communication (VLC); non-orthogonal multiple access (NOMA); power allocation

1. Introduction

Wireless data traffic has grown exponentially with the increase in mobile applications and emerging services [1]. However, the limited spectrum of existing radio frequency (RF) is progressively becoming congested, and the available RF resources cannot fully satisfy the specific communication requirements for high spectral and energy efficiency scenarios [2]. Recently, visible light communication (VLC) has been regarded as a potential supplementary technology to traditional RF wireless networks [3] due to its many advantages, such as an abundant and unlicensed spectrum, low cost, low power consumption, and enhanced security characteristics [4]. One of the main disadvantages of VLC is the limited modulation bandwidth of the employed light-emitting diodes (LEDs) [5], where the 3 dB bandwidth has only 5~10 MHz. To improve the achievable data rate, extensive studies on methods such as advanced optical modulation [6,7], channel equalization [8], multiple access schemes [9], and multiple-input multiple-output (MIMO) [10] have been carried out based on intensity modulation and direct detection (IM/DD) architecture.

Non-orthogonal multiple access (NOMA) is one of the key-enabler technologies in 5G networks and has attracted increasing attention from the academic and industrial communities owing to its high spectrum efficiency, user fairness, strong reliability, and massive connectivity [11]. Unlike orthogonal multiple access (OMA) techniques, multiple users can be simultaneously served with the same time–frequency resources by using NOMA, which is more suitable for massive connectivity. Briefly, the power domain resources are used at the transmitter to distinguish and superimpose transmission for different users, while successive interference cancellation (SIC) is performed at the receiver to detect the signals for each user. NOMA can be integrated with VLC since it performs well at a high signal-to-noise ratio (SNR), which is a typical feature guaranteed by VLC systems [12]. By pairing
users and employing appropriate LEDs, the performance of NOMA over traditional OMA can be enhanced accordingly [13]. Closed-form expressions for the bit error rate (BER) of NOMA-VLC systems with on-off keying (OOK) and L-ary pulse position modulation have been derived, considering perfect and imperfect channel state information [14]. The results of [15] demonstrated that the performance metrics (BER, sum rate, and outage probability) in a multi-user NOMA-VLC system can be affected by the number of users, signal type, and shadowing under different half-angles of LEDs and signal reflection path conditions.

Due to the intercluster interference in the SIC procedure, users who have poor channel conditions should employ higher power to decode their useful information. How to reasonably allocate the limited power to each user plays a significant role in NOMA [16]. Several studies on efficient power allocation including fixed allocation, fractional transmit allocation, strategy design [17–20], heuristics [21,22], and indirect methods based on mathematical theory [23,24] have been proposed in NOMA-VLC systems. In [17], a gain ratio power allocation (GRPA) strategy was proposed, which was reliant on the user’s gain in comparison to that of the first sorted user based on the decoding order. The corresponding BER performance of this strategy was found to outperform the fixed power allocation method. For MIMO-VLC networks, a normalized gain difference power allocation (NGDPA) method that relies on the channel gain difference to determine the power allocation coefficients was proposed to increase the total rate in [18]. In addition, multiple LEDs were utilized to enhance the performance of the communication system and improve the data transfer rate, capacity, and robustness. An improved fractional strategy (IFS) revising the power factors within GRPA was proposed in [19], where the constraints of the proposed strategy were rigorously explained through the proposed asymptotic and compact throughput bound. In [22], the nonlinear marine predator algorithm was applied to solve the fair power allocation problem, optimizing the sum rate efficiently and allowing for quick convergence. By utilizing the derived lower bound of the achievable rate and semidefinite relaxation technology, optimal power allocation schemes for static and mobile users were derived in [24].

However, the negative impact of residual user interference on system performance during SIC implementation has not been fully considered in the above literature. In [25], adjustable superposition coding and SIC decoding schemes were proposed to alleviate the influence of error propagation by adjusting the relative bit rate of each user. A convolutional neural network-based demodulator for NOMA-VLC was presented in [26], aiming to achieve joint signal compensation and recovery. The experimental results demonstrated that this receiver exhibited improved robustness against linear and non-linear distortions compared to receivers using SIC and joint detection. A modified SIC decoder was proposed to improve the symbol error rate performance of the three-user uplink/downlink NOMA by assuming channel gains, and the joint influence of the SNR and channel gains on the symbol error rate was also analyzed in [27]. Based on the above analysis, we can see that most existing works have focused primarily on designing signal detection algorithms at the receiver to improve error performance. However, the residual interference is not well mitigated, which may lead to significant performance degradation in the achievable data rate and BER performance.

In this paper, a new multi-user power allocation scheme named enhanced gain difference power allocation (EGDPA) is proposed to mitigate the adverse effect of residual interference and then improve the achievable data rate and detection performance. The allocation factors are determined by considering users’ channel gains and utilizing the residual allocation principle, which focuses on the remaining power available after allocating to the previous user rather than the initially assigned power. Moreover, the corresponding achievable data rate, energy efficiency, and error probability are analyzed to characterize the impact of power allocation factors on NOMA design. Simulation results show that the proposed scheme can provide a satisfactory sum rate, energy efficiency, and error performance compared with the OMA scheme or traditional NOMA power allocation strategies, which validates the effectiveness of the proposed scheme.
The rest of this paper is organized as follows. In Section 2, the system model for MIMO-NOMA-VLC is presented. In Section 3, the EGDPA scheme is proposed. The system sum achievable data rate, energy efficiency, and error probability are also evaluated in Section 3. Simulation results are included in Section 4, followed by conclusions summarized in Section 5.

2. System Model

As illustrated in Figure 1, we consider a MIMO-VLC system comprising \( I \) LEDs and \( K \) users. Therein, each user is equipped with \( J \) photodetectors (PDs). The coverage area radius for the cell is \( R \) and \( U_1 \) is positioned at the center of the cell. We assume that all users are arranged in straight lines, and the distance between the central user and the edge user is denoted as \( r \), while the distance between the central user and \( U_k \) is denoted as \( r_k \). We define \( L_i \) as the \( i \)-th LED transmitter, and \( D_j \) as the \( j \)-th PD. In addition, a DC bias \( I_{DC} \) is always added to the signal to obtain the non-negative waveform \( x_i \) to drive \( L_i \), which can be expressed by

\[
x_i = \sum_{k=1}^{K} \sqrt{\mu_{i,k} P_i s_{i,k}} + I_{DC}, \quad (1)
\]

where \( P_i \) is the electrical power of the emitter, \( \mu_{i,k} \) represents the normalized power allocation factor at the \( i \)-th LED transmitter for \( U_k \), and \( s_{i,k} \) denotes the zero-mean OOK modulated signal prepared for \( U_k \) at \( L_i \). To maintain constant total electrical power, the power allocation factors should satisfy \( \sum_{k=1}^{K} \mu_{i,k} = 1 \).

After VLC channel transmission, the optical signal is captured by the PD at \( U_k \) and then converted to an electrical current based on optical-electrical conversion. Since the DC signal does not convey any useful information, it is always eliminated from the received signal before demodulation. Therefore, the received signal for \( U_k \) can be given by

\[
y_k = \gamma_{oe} P_0 \zeta H_k x + n_k, \quad (2)
\]
where $\gamma_{oe}$ is the optical-electrical responsivity of the PD, $P_s$ represents the output optical power of the emitter, $\zeta$ denotes the modulation index, the channel matrix for $U_k$ is denoted as $H_k \in \mathbb{C}^{L \times M}$, and the vector $x$ represents the transmitted symbol vector from all LEDs. Additionally, $n_k$ denotes the additive noise vector with zero mean and variance $\sigma_n^2$, which comprises shot noise and thermal noise [13,28] and can be expressed by

$$\sigma_n^2 = \sigma_{\text{shot}}^2 + \sigma_{\text{thermal}}^2 = \left(2qI_b g_s I_2 B + 2q\gamma_{oe}P_s B H\right) + \left(\frac{8\pi k T}{G} \eta A_{PD} I_2 B^2 + \frac{16\pi K T}{G_m} \eta^2 A_{PD} I_3 B^3 \right),$$

where $q$ is the electronic charge, $I_b = 5100$ $\mu$A represents background noise, and the equivalent bandwidth of noise is denoted as $B$. $k$ is the Boltzmann constant, $G = 10$ denotes the open-loop voltage, $\eta = 112$ $\text{pF}/\text{cm}^2$ denotes the input capacitance of the PD, $\Gamma = 1.5$ represents the field-effect transistor (FET) channel noise factor, and $g_m$ captures the FET transconductance [14].

In this paper, we only focus on the LOS component in VLC systems because the power of the NLOS links is relatively lower than that of the LOS components. Considering the Lambertian radiation of LEDs, the LOS channel gain between $L_i$ and $D_j$ for $U_k$ can be formulated by

$$h_{ji,k} = \frac{(n_0)_{g_s} \bar{g}_s(\psi_{ji,k})}{2\pi d_{ji,k}} \cos^2 \psi_{ji,k}, \quad 0 \leq \psi_{ji,k} \leq \psi_C,$$

where the Lambertian order of the LED is $n_0 = -1/\log_2(\cos(\phi/2))$ and $\phi_{1/2}$ denotes the semi-angle of the LED. $A_{PD}$ captures the area of detection of the PD, and $d_{ji,k}$ represents the distance between $L_i$ and $D_j$ for $U_k$. $\psi_{ji,k}$ and $\psi_C$ are the irradiance angle and the incident angle of the optical link for $U_k$, respectively, while $\psi_C$ is the field of view (FOV). $T_s$ denotes a constant of an optical filter gain, and $g_s(\psi_{ji,k})$ represents the gain of an optical concentrator for $U_k$ with a refractive index $n$, which is given as

$$g_s(\psi_{ji,k}) = \begin{cases} \frac{n^2}{\sin^2(\psi_C)}, & 0 \leq \psi_{ji,k} \leq \psi_C, \\ 0, & \psi_{ji,k} > \psi_C. \end{cases}$$

By substituting $d_{ji,k}$ in (4), the LOS channel gain can be expressed as

$$h_{ji,k} = \frac{\Omega_{ji,k}(\bar{n}_0 + 1)L n_0 + 1}{(n_k^2 + L)^{\frac{g_s(\psi_{ji,k})}{2}}}$$

where $\Omega_{ji,k} = \frac{A_{PD} T_s g_s(\psi_{ji,k})}{2\pi}$, $L$ is the height of the light source. The channel gain primarily relies on the distance between the user and the LED, assuming a constant LED height and $\Omega_{ji,k}$ values.

3. Proposed Power Allocation Scheme

In this section, we propose an enhanced gain difference power allocation (EGDPA) scheme that is based on the differences in channel gains among all users. Then, based on the residual allocation principle, the corresponding analytical expressions in terms of achievable data rate, energy efficiency, and error probability are presented accordingly.

3.1. Allocation Principle Formulation

A block diagram of a MIMO-VLC system with the proposed power allocation scheme is depicted in Figure 2. At the transmitter, the data of all users are superimposed in the power domain according to an allocation strategy and then combined with a DC signal to drive the LEDs. After passing through the VLC channel, the user captures the received
signals by using J PDs. According to MIMO demultiplexing and the SIC procedure, the signals are finally demodulated into useful data for each user. With the dynamic adjustment of the power allocation factors, the transmission rate for the MIMO-VLC system employing NOMA can be enhanced. It is worth noting that a constant channel gain for fixed transmitter and receiver positions can be obtained according to (6). The GRPA scheme ranks the user channel gains and calculates the power allocation coefficients based on the numerical channel gain relationship between adjacent users in the channel gain ranking. As for GRPA, the relationship between the power factors assigned to \( U_k \) and \( U_{k+1} \) at \( L_i \) can be described by

\[
\mu_{i,k} = \left( \frac{h_{1,k+1} + h_{2,k+1}}{h_{1,1} + h_{2,1}} \right)^{k+1} \mu_{i,k+1}.
\] (7)

In the single-cell NOMA-VLC scenario, the primary problem is multi-user interference, where the power assigned to the demodulated user is supposed to surpass the total power allocated to the previously demodulated users, in particular at a high SNR. The problem can be alleviated by a modified fixed-power allocation (MFPA) scheme, which relies on the remaining power after the allocation to the previous user rather than the power initially assigned to them. The corresponding allocated power can be described as

\[
P_k = \begin{cases} 
(1 - \alpha)^{k-1} P_t, & k = 1, 2, \ldots, K - 1 \\
(1 - \alpha) P_t, & k = K 
\end{cases}.
\] (8)

where \( \alpha \) is the fixed power factor of the scheme. Equation (8) is referred to as the residual power principle.

In this paper, by combining (7) and (8), an efficient power allocation scheme with enhanced gain difference power allocation (EGDPA) is proposed to further enhance the achievable data rate. The main idea is that the allocation factors are determined by the users’ channel gain differences, which can offer higher flexibility than fixed allocation factors and is better suited to the practical needs. Then, the residual allocation principle is employed to further reduce residual interference from previously demodulated users to the intended users. In particular, the power allocation factor for \( U_k \) at \( L_i \) can be formulated as

\[
\mu_{i,k} = \begin{cases} 
\alpha_{i,1}, & k = 1 \\
\prod_{q=1}^{k-1} (1 - \alpha_{i,q}) \alpha_{i,k}, & k = 2, \ldots, K - 1 \\
\prod_{q=1}^{K} (1 - \alpha_{i,q}), & k = K 
\end{cases}.
\] (9)

where \( b_{i,k} \) can be expressed by

\[
b_{i,k} = \left( \frac{h_{1,k+1} + h_{2,k+1} - h_{1,k+1} - h_{2,k+1}}{h_{1,1} + h_{2,1}} \right)^{k} b_{i,k+1},
\] (10)

and \( \alpha_{i,k} \) is represented as

\[
\alpha_{i,k} = \frac{b_{i,k}}{1 + \sum_{k=2}^{K} b_{i,k}}.
\] (11)

Comparing with (7) and (8), we find that the adjacent power allocation factors formulated by (9) exhibit smaller differences, which results in less residual interference from the previous user to the subsequent user in the SIC process.

The proposed power allocation algorithm is shown in Algorithm 1. With I LEDs and K users, the computational complexity required for \( b_{i,k} \) can be approximated as \( O(IK) \) based on (10) and lines 2–6 in Algorithm 1. According to (11), \( \alpha_{i,k} \) can be calculated using the results of (10) without additional complexity. Based on (9), the computational complexity for \( \mu_{i,k} \) can be approximated as \( O(IK^2) \). Therefore, the overall computational complexity
of the proposed algorithm can be approximately denoted as $O(IK^2)$. The computational complexity of GRPA is comparable to the proposed algorithm, which can be estimated as $O(IK^2)$ due to the calculation of $\mu_{i,k}$, as shown in (7). The calculation process of NGDPA is similar to that of GRPA, with a required computational complexity of $O(IK^2)$. As described in [25], the computational complexity of MFPA can be approximately expressed as $O(IK)$ in this paper. In summary, the proposed algorithm, in comparison to MFPA, exhibits slightly increased computational complexity. However, with the advancements in computing power, this additional complexity can be easily handled. Furthermore, the computational complexity of the proposed algorithm aligns with that of the GRPA and NGDPA methods.

Figure 2. Block diagram of a MIMO-VLC system with the proposed power allocation scheme.

Algorithm 1 Enhanced gain difference power allocation

**Input:** Number of LEDs I, Number of PDs J, Number of users K

1. Compute channel gains $h_{i,j,k}$ and sort decoding order in ascending order based on $h_k$
2. for $i=1$ to $I$ do
3. \hspace{1em} \textbf{for} $k=1$ to $K$ do
4. \hspace{2em} Calculate $b_{i,j,k}$ based on (10)
5. \hspace{1em} \textbf{end for}
6. \hspace{1em} \textbf{end for}
7. for $i=1$ to $I$ do
8. \hspace{1em} \textbf{for} $k=1$ to $K$ do
9. \hspace{2em} Calculate $\alpha_{i,k}$ based on (11)
10. \hspace{2em} if $k = K$ then
11. \hspace{3em} $\mu_{i,k} = \prod_{q=1}^{K} (1 - \alpha_{i,q})$
12. \hspace{2em} else
13. \hspace{3em} if $k = 1$ then
14. \hspace{4em} $\mu_{i,k} = \alpha_{i,k}$
15. \hspace{3em} else
16. \hspace{4em} $\mu_{i,k} = \prod_{q=1}^{k-1} (1 - \alpha_{i,q}) \alpha_{i,k}$
17. \hspace{2em} \textbf{end if}
18. \hspace{2em} \textbf{end if}
19. \hspace{2em} \textbf{end for}
20. \hspace{1em} \textbf{end for}
In NOMA-VLC systems with multiple LEDs, decoding is arranged based on the aggregate channel gains from all LEDs to mitigate interference among users. Specifically, the user with the poorer channel condition would be assigned more power in the SIC decoding schemes [11]. For simplicity, within the same LED, the power allocation factor for $U_k$ can be equivalently represented as $\mu_k$, and the equivalent channel gain can be expressed as $h_k = \sum_{j=1}^{I} h_{j,k}$. The channel gains of users can be arranged in ascending order, given as

$$h_1 \leq \ldots \leq h_k \leq \ldots \leq h_K,$$

where the first and last users are regarded as the weakest and strongest, respectively. Therefore, to ensure the quality of the edge user, the decoding order follows an increasing order of the channel gains. Therefore, the power allocated to each user can be ordered as

$$\mu_1 \geq \ldots \geq \mu_k \geq \ldots \geq \mu_K.$$  

When SIC is used, the residual interference from users of superior decoding order can be regarded as noise. According to (2), the interference and noise at $U_k$ are represented as

$$N_k = \kappa \sum_{u=1}^{k-1} h_k^2 P_u + \sum_{v=k+1}^{K} h_k^2 P_v + \sigma^2_{\text{noise}},$$

where $\kappa$ is a constant factor denoting the degree of residual interference, which falls within the range [0,1]. A smaller value of $\kappa$ indicates a better decoding performance for SIC. When $\kappa$ is equal to 0, this represents perfect decoding of the user information with no residual. When $\kappa$ is not equal to 0, this signifies imperfect decoding of the user information with some residual remaining. For $P_k = \mu_k P_t$, the signal-to-interference-plus-noise ratio (SINR) for $U_k$ can be expressed by

$$\gamma_k = \frac{(h_k R_k)^2}{\kappa \sum_{y=1}^{k-1} (h_k R_y)^2 + \sum_{v=k+1}^{K} (h_k R_v)^2 + \sigma^2_{\text{noise}}}, \quad 1 \leq k \leq K,$$

where $\sigma^2 = \sigma^2_{\text{noise}} / P_t$. Therefore, the corresponding achievable rate for $U_k$ can be derived as

$$R_k = \begin{cases} \frac{B}{T} \log_2 \left( 1 + \frac{(h_k R_k)^2}{\kappa \sum_{y=1}^{k-1} (h_k R_y)^2 + \sum_{v=k+1}^{K} (h_k R_v)^2 + \sigma^2_{\text{noise}}} \right), & 1 \leq k < K, \\ \frac{B}{T} \log_2 \left( 1 + \frac{(h_k R_k)^2}{\kappa \sum_{y=1}^{k-1} (h_k R_y)^2 + \sigma^2_{\text{noise}}} \right), & k = K. \end{cases}$$

where $B$ is the modulation bandwidth. It should be noted that (16) is conditioned on the fact that $U_k$ can detect all messages from $U_j$, for $\forall \ j \leq k$. The rate at which $U_k$ detects the messages sent to $U_j$ is denoted as $R_{k\rightarrow j}$ and the target rate for $U_j$ is $R_j$. This condition can be formulated as

$$R_{k\rightarrow j} = \begin{cases} \frac{B}{T} \log_2 \left( 1 + \frac{(h_k R_j)^2}{\kappa \sum_{y=1}^{k-1} (h_k R_y)^2 + \sum_{v=k+1}^{K} (h_k R_v)^2 + \sigma^2_{\text{noise}}} \right) \geq R_j, & j \leq k, j \neq K, \\ \frac{B}{T} \log_2 \left( 1 + \frac{(h_k R_j)^2}{\kappa \sum_{y=1}^{k-1} (h_k R_y)^2 + \sigma^2_{\text{noise}}} \right) \geq R_j, & j = k = K. \end{cases}$$

If (17) is satisfied, it can be inferred that the perfect SIC in the decoding chain can be achieved. Otherwise, communication interruption will occur at $U_k$. It is assumed that each user has not specified any requirements for the target data rate but instead strives to maximize their communication performance using the allocated power (i.e., $\tilde{R}_j = R_j$).

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Electronics 2024, 13, 776
**Lemma 1.** The rate at which $U_k$ detects the messages sent to $U_j$ is always higher than the achievable rate of $U_j$.

**Proof.** As for $j = k$, the rate at which $U_k$ detects the messages sent to $U_j$ equals the achievable rate for $U_j$ based on (16) and (17). As for $j < k$, according to (16) and (17), $R_{k \rightarrow j}$ can be expressed as

$$R_{k \rightarrow j} = \frac{B}{2} \log_2 \left(1 + \frac{(h_{j} \mu_{j})^2}{\kappa \sum_{p=1}^{j-1} (h_{k} \mu_{p})^2 + \sum_{o=j+1}^{K} (h_{j} \mu_{o})^2 + \sigma^2} \right),$$

and $R_j$ is calculated by

$$R_j = \frac{B}{2} \log_2 \left(1 + \frac{(h_{j} \mu_{j})^2}{\kappa \sum_{p=1}^{j-1} (h_{j} \mu_{p})^2 + \sum_{o=j+1}^{K} (h_{j} \mu_{o})^2 + \sigma^2} \right).$$

To simplify the subsequent analysis, let $S_{k \rightarrow j}$ and $S_j$ replace $R_{k \rightarrow j}$ and $R_j$ according to (19) and (18), respectively. This can be represented as

$$S_{k \rightarrow j} = \frac{(h_{j} \mu_{j})^2}{\kappa \sum_{p=1}^{j-1} (h_{k} \mu_{p})^2 + \sum_{o=j+1}^{K} (h_{j} \mu_{o})^2 + \sigma^2},$$

$$S_j = \frac{(h_{j} \mu_{j})^2}{\kappa \sum_{p=1}^{j-1} (h_{j} \mu_{p})^2 + \sum_{o=j+1}^{K} (h_{j} \mu_{o})^2 + \sigma^2}.$$

We arrange all users according to (12); thus, we have $h_j \leq h_k$. Based on the aforementioned analysis, the comparison of $R_{k \rightarrow j}$ and $R_j$ is comparable to that of $S_{k \rightarrow j}$ and $S_j$. Consequently, the difference between $S_{k \rightarrow j}$ and $S_j$ can be expressed as

$$S_{k \rightarrow j} - S_j = \frac{(h_{j} \mu_{j})^2}{\kappa \sum_{p=1}^{j-1} (h_{k} \mu_{p})^2 + \sum_{o=j+1}^{K} (h_{j} \mu_{o})^2 + \sigma^2} - \frac{(h_{j} \mu_{j})^2}{\kappa \sum_{p=1}^{j-1} (h_{j} \mu_{p})^2 + \sum_{o=j+1}^{K} (h_{j} \mu_{o})^2 + \sigma^2}.$$

Let $S'$ denote (22), which can be further represented by

$$S' = (h_{j} \mu_{j})^2 \kappa \sum_{p=1}^{j-1} (h_{k} \mu_{p})^2 + \sum_{o=j+1}^{K} (h_{j} \mu_{o})^2 + \sigma^2 - (h_{j} \mu_{j})^2 \kappa \sum_{p=1}^{j-1} (h_{j} \mu_{p})^2 + \sum_{o=j+1}^{K} (h_{j} \mu_{o})^2 + \sigma^2.$$

Therefore, the rate difference can be expressed by

$$S' = [(h_{j} \mu_{j})^2 - (h_{j} \mu_{j})^2] \sigma^2 \geq 0.$$

The proof is completed. □

Consequently, each user can achieve a data rate determined by (16), and the total system rate is a summation of all users’ data rates. Furthermore, it can be observed from (16) that $R_k$ can be increased with the value of $h_k$ for a given power allocation factor. If all users possess equal power allocation factors, the user with superior channel conditions will attain a higher data rate. Moreover, to enhance fairness among users, the channel allocation factor for users with inferior channel conditions is augmented while the power allocation factor for users with superior channel conditions is reduced.

Accordingly, the energy efficiency can be calculated by
where $P_{\text{max}}$ represents the maximum transmit power of the LED. Hence, considering that $P_{\text{max}}$ is constant, the problem of maximizing energy efficiency can be reduced to the problem of maximizing achievable rates.

3.2. Error Probability

For simplicity, only two users are considered in the NOMA-based MIMO-VLC system. It is assumed that the symbols of both users are mutually independent and equiprobable when evaluating the error probability. Considering the OOK modulation, the BER for $U_1$ (i.e., distant user) can be formulated as

$$P_{e_1} = \frac{1}{4} \{2Q(\gamma_1 \mu_2) + Q(\gamma_1 (\mu_2 - 2 \mu_1)) + Q(\gamma_1 (2 \mu_1 + \mu_2))\},$$

where $\gamma_1 = \gamma_0 h_2/\sigma_n$, $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-u^2/2)\,du$. Let $\Theta_1$ and $\Theta_2$ represent the successful and erroneous demodulation of $U_i$ ($i = 1, 2$), respectively. Thus, the BER for $U_2$ (i.e., near user) can be calculated as

$$P_{e_2} = P(\Theta_2, \Theta_1) + P(\Theta_2, \Theta_1)$$

$$= (1 - P(\Theta_2, \Theta_1))P(\Theta_2|\Theta_1) + P(\Theta_2, \Theta_1)P(\Theta_1),$$

where $P(\Theta_2, \Theta_1)$ represents the joint probability when $U_1$ correctly decodes its signal, whereas $U_2$ has incorrect decoding. Similarly, $P(\Theta_2, \Theta_1)$ denotes the joint probability when both $U_1$ and $U_2$ obtain the incorrect decoding at the same time. $P(\Theta_2|\Theta_1)$ and $P(\Theta_2, \Theta_1)$ are the conditional probabilities as $U_2$ decodes its signal incorrectly on the conditions that the correct and incorrect decoding of $U_1$ are achieved, respectively. Replacing $\gamma_1$ with $\gamma_2$ in (26), the BER for decoding the $U_1$’s signal at $U_2$ can be given as

$$P_2(\Theta_1) = \frac{1}{4} \{2Q(\gamma_2 \mu_2) + Q(\gamma_2 (\mu_2 - 2 \mu_1)) + Q(\gamma_2 (2 \mu_1 + \mu_2))\},$$

where $\gamma_2 = \gamma_0 h_1/\sigma_n$.

As for the evaluation of $P_{e_2}$, the decoding process is initially applied to the far user and subsequently followed by the implementation of SIC. After the successful execution of SIC at $U_1$, the error probability for $U_2$ can be derived as

$$P(\Theta_2|\Theta_1) = Q(\gamma_2 \mu_1).$$

When the signal of $U_1$ is incorrectly decoded at $U_2$, the joint error probability of $U_2$ can be derived as

$$P(\Theta_2, \Theta_1) = \frac{1}{4} \{Q(\gamma_2 \mu_2)Q(\gamma_2 (\mu_1 + 2 \mu_2)) + Q(\gamma_2 \mu_2)Q(\gamma_2 (\mu_1 - 2 \mu_2))$$

$$+ Q(\gamma_2 (\mu_1 - 2 \mu_2))Q(\gamma_2 (\mu_2 - 2 \mu_1)) + Q(\gamma_2 (2 \mu_1 + \mu_2))Q(\gamma_2 (\mu_1 + 2 \mu_2))\}.$$

By substituting (28)–(30) into (27), the error probability for $U_2$ can be achieved.

4. Simulations

A NOMA-based MIMO-VLC system with $I = 2$ and $J = 2$ is considered for the simulations by employing various power allocation strategies. The architecture of the system is depicted in Figure 1, where the LED spacing is 1 m and the PD spacing is 4 cm.
The receiving plane has a height of 0.85 m above the floor, and the cell radius $R$ is 4 m. Let $\phi_C$ and $\phi_{1/2}$ be fixed to $72^\circ$ and $50^\circ$, respectively, and the modulation index $\zeta$ be 0.5. The refractive index $n$ is 1.5, the transmitted optical power $P_o$ is 10 W, whilst the modulation bandwidth $B$ is configured to 10 MHz. Regarding the PDs, the detection area and the optical-electrical responsivity are $1 \text{ cm}^2$ and 0.53 A/W, respectively. For brevity, the main simulation parameters are listed in Table 1. We assume that the users are uniformly distributed with stationary positions. Additionally, the OMA strategy with equal power allocation is evaluated here for performance comparison.

### Table 1. Simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell radius ($R$)</td>
<td>4 m</td>
</tr>
<tr>
<td>Height of room ($L_1$)</td>
<td>3 m</td>
</tr>
<tr>
<td>Height of receiving panel ($L_2$)</td>
<td>0.85 m</td>
</tr>
<tr>
<td>LED spacing</td>
<td>1 m</td>
</tr>
<tr>
<td>PD spacing</td>
<td>4 cm</td>
</tr>
<tr>
<td>Semi-angle of the LED ($\phi_{1/2}$)</td>
<td>50$^\circ$</td>
</tr>
<tr>
<td>Optical filter gain ($T_s$)</td>
<td>1</td>
</tr>
<tr>
<td>Refractive index ($n$)</td>
<td>1.5</td>
</tr>
<tr>
<td>Modulation index ($\zeta$)</td>
<td>0.5</td>
</tr>
<tr>
<td>Optical-electrical responsivity ($\gamma_{oe}$)</td>
<td>0.53 A/W</td>
</tr>
<tr>
<td>FOV ($\psi_C$)</td>
<td>72$^\circ$</td>
</tr>
<tr>
<td>Active area ($A_{PD}$)</td>
<td>$1 \text{ cm}^2$</td>
</tr>
<tr>
<td>Modulation bandwidth ($B$)</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Transmitted optical power ($P_o$)</td>
<td>10 W</td>
</tr>
</tbody>
</table>

#### 4.1. Comparisons of Achievable Rates with Different User Numbers

For $K = 2$, Figure 3 demonstrates the achievable data rate provided by each LED at different relative distances. It can be seen that the data rates of the two methods exhibit minimal difference when the relative distance is below 1.6 m. As the relative distance ranges from 1.6 m to 2.4 m, the data rates of each LED are rapidly diminished. Nonetheless, we find that the reduction in data rates is relatively moderate when using the proposed EGDPA. For instance, as the distance grows from 2 m to 2.4 m, the rate for $L_2$ using the proposed EGDPA reduces from 56.9 Mbit/s to 54.9 Mbit/s, while that of NGDPA drops more visually from 55.6 Mbit/s to 53.1 Mbit/s. When the relative distance exceeds 2.8 m, the rate of $L_2$ rebounds and stabilizes at about 57.7 Mbit/s while that of $L_1$ declines greatly due to its poor channel conditions. The proposed scheme achieves better results relative to the NGDPA method, albeit by a small margin.

Figure 4 compares the achievable sum rate with different power allocation schemes. The results demonstrate that as the relative distance rises, the sum rate for OMA and GRPA is dramatically decreased. Only a slight performance degradation is introduced in the proposed scheme. When the relative distance is less than 1.6 m, NGDPA, MFPA[25], and the proposed scheme completely overlap. However, after the relative distance exceeds 1.6 m, the rates of NGDPA and MFPA fluctuate below that of the proposed scheme. For instance, for $K = 2$, at a relative distance of 2.4 m, the minimum sum rate is attained by NGDPA, MFPA, and the proposed scheme. However, by utilizing the proposed scheme, a greater sum rate of 106.9 Mbit/s is attained, which still surpasses the 102.7 Mbit/s and 104.4 Mbit/s accomplished by MFPA and NGDPA. For $K = 3$, at relative distances of 2 m and 4 m, the sum rate of NGDPA remains relatively low, achieving approximately 105 Mbit/s and 104.9 Mbit/s, respectively. Nonetheless, the proposed scheme achieves better rates of 111.7 Mbit/s and 109.7 Mbit/s, respectively, at these distances. Compared with alternative methods, the proposed scheme displays notable resistance to interference, indicating the robustness of the system.
Figure 3. Achievable rate for each LED with two users ($K = 2$).

Figure 4. Achievable sum rate with two and three users ($K = 2$ and 3).

The gain in sum rate obtained by the proposed EGDPA when compared to the traditional NGDPA is demonstrated in Figure 5. The results clearly show that the sum rate gain of the proposed EGDPA over the traditional NGDPA increases substantially when the number of users increases from 2 to 3. As compared with NGDPA at $r = 2$ m, the proposed scheme achieves improvements in data rate by 2.12% and 6.25% for $K = 2$ and 3, respectively. Furthermore, when the relative distance is set as 4 m, the proposed scheme achieves a gain of 4.58% for $K = 3$ since the furthest user is at the edge of the cell. The aforementioned analysis substantiates the effectiveness of the proposed scheme.

Figure 6 shows the performance comparison of the achievable sum rate under different numbers of served users. Notably, the GRPA, IFS [19], and NGDPA methods employ a format dependent on the channel gain, while the MFPA method uses a modified fixed allocation format. The figure clearly shows that all schemes can achieve an excellent data rate when serving a small number of users. As $K$ rises, the achievable data rates provided by GRPA, IFS, NGDPA, and MFPA decline sharply, while that of the proposed scheme still achieves rather stable performance. The main reason for this is that the adjacent power
allocation factors of the proposed scheme are more different compared with other schemes when the number of users is increased. Additionally, the proposed EGDPA scheme has improved the sum rate by 25.2% with 30 users.

![Figure 5. Sum rate gain of the proposed scheme over NGDPA.](image1)

Figure 5. Sum rate gain of the proposed scheme over NGDPA.

4.2. The Impact of Residual Interference and Modulation Bandwidth

As shown in Figure 7, we investigated the impact of the residual interference factor \( \kappa \) on the sum rate when using the proposed scheme. The values of \( \kappa \) were set to 0, 0.0001, 0.001, and 0.01. As the value escalated, the rate of the proposed scheme declined. Consequently, it became apparent that the residual interference, which is not fully eliminated during the SIC process, significantly hampers the system performance. For instance, when \( \kappa \) is 0, the sum rate of 10 users in the illuminated area is approximately 113.1 Mbit/s. Nevertheless, as \( \kappa \) rises to 0.0001, 0.001, and 0.01, the sum rate decreases to 109.8 Mbit/s, 94.8 Mbit/s, and 66.1 Mbit/s, respectively.

![Figure 6. Comparison of the achievable sum rate under different numbers of users.](image2)

Figure 6. Comparison of the achievable sum rate under different numbers of users.
Figure 7. Comparison of the achievable data rate for different numbers of users with various values of $\kappa$.

Figure 8 illustrates the impact of the transmission power and modulation bandwidth on the sum rate performance for the proposed scheme. The sum rate of the proposed scheme is positively correlated with transmission power when the modulation bandwidth is fixed. As the signal power increases, the additive power also increases to a lesser extent, resulting in improved SNR and subsequently a higher rate. Furthermore, increasing the modulation bandwidth also leads to an increase in rate with a fixed transmission power. Additionally, with the increase in modulation bandwidth, the impact of transmission power on the rate of the proposed scheme becomes more prominent. For instance, for a required system sum rate of 100 Mbit/s, the power consumption is 6 W at a modulation bandwidth of 10 MHz, whereas it reduces to 1.3 W when the bandwidth is increased to 20 MHz.

Figure 8. Impact of the transmission power and modulation bandwidth for the proposed scheme.

4.3. Comparisons of Energy Efficiency

The performance of energy efficiency in two-user and three-user scenarios is shown in Figure 9. The proposed scheme demonstrates superior energy efficiency in both scenarios when compared to GRPA, NGDPA, and MFPA. Furthermore, the energy efficiency of
the proposed scheme improves markedly as the number of users grows, while those of
the GRPA and NGDPA methods exhibit a decline. As K grows from 2 to 3, the energy
efficiency obtained by GRPA decreases significantly by at least 8.14%. At the same time,
that of NGDPA increases initially but subsequently decreases, e.g., by 3.85% when the
transmission power is 15 W. The main reason for this is that the channel differences are
decreased in the three-user scenario. Nevertheless, the proposed scheme can better utilize
user channel information to address power imbalances among users. Hence, the proposed
scheme is suitable for energy-constrained conditions.

Figure 9. Energy efficiency comparison for different power allocation schemes.

4.4. Error Probability

The error probability achieved by the proposed scheme in the two-user scenario is
illustrated in Figure 10. As the location of the users is fixed, the system’s error probability
reduces obviously with an increase in the LED optical power. Specifically, U₁ (i.e., the
distant user) is allocated more power, resulting in better error performance. Despite the
better channel conditions of U₂, its BER performance is slightly inferior due to receiving less
power. To a certain extent, this indicates the fairness of the proposed scheme. Furthermore,
as the relative distance between users rises, the error performance improves considerably
when the optical power is determined. For instance, when the error probability equals
$10^{-3}$, the required optical power decreases from 2.7 W to 1.3 W as the relative distance $r$
grows from 0.8 m to 1 m. This is due to the symmetrical geometric positioning of the two
users around $L_2$ when $r$ is 1 m, resulting in power resource savings. Overall, the proposed
scheme achieves a superior BER performance, indicating its reliability.

4.5. Comprehensive Analysis

Table 2 presents a comprehensive comparison of different schemes. The proposed
scheme outperforms other schemes in terms of the sum rate and energy efficiency. For
example, compared to the GRPA scheme, the proposed scheme achieves a maximum
sum rate gain of 36.26%, a maximum sum rate gain of 25.09% compared to the NGDPA
scheme, and a gain of 10.87% compared to the MFPA scheme. Additionally, when the total
transmission power of the three users is 15 W, the proposed scheme achieves an energy
efficiency gain of 23.44% compared to GRPA, 5.68% compared to NGDPA, and 1.71%
compared to MFPA. The proposed scheme’s performance gain increases with the number
of users, as its residual allocation principle effectively reduces inter-user interference caused
by multiple users.
grows from 0.8 \text{m} \text{ to } 1 \text{m}. This is due to the symmetrical geometric positioning of the two users around \( L_2 \) when \( r = 1 \text{m} \), resulting in power resource savings. Overall, the proposed scheme achieves a superior BER performance, indicating its reliability.

Figure 10. Error probability of the proposed scheme with increasing optical power between two users.

Table 2. Comprehensive comparison of different schemes.

<table>
<thead>
<tr>
<th>K = 3</th>
<th>K = 10</th>
<th>K = 15</th>
<th>K = 20</th>
<th>K = 25</th>
<th>K = 30</th>
<th>K = 2</th>
<th>K = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRPA</td>
<td>102.85</td>
<td>88.42</td>
<td>86.61</td>
<td>84.72</td>
<td>84.17</td>
<td>83.27</td>
<td>52.5</td>
</tr>
<tr>
<td>NGDPA</td>
<td>108.74</td>
<td>94.73</td>
<td>93.40</td>
<td>91.77</td>
<td>91.44</td>
<td>90.7</td>
<td>58.5</td>
</tr>
<tr>
<td>MFPA</td>
<td>109.27</td>
<td>112.96</td>
<td>112.77</td>
<td>111.07</td>
<td>107.49</td>
<td>102.08</td>
<td>58.3</td>
</tr>
<tr>
<td>Proposed scheme</td>
<td>109.29</td>
<td>113.2</td>
<td>113.43</td>
<td>113.73</td>
<td>113.71</td>
<td>113.94</td>
<td>59.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>GRPA</th>
<th>NGDPA</th>
<th>MFPA</th>
<th>Proposed scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>The performance gain of the proposed scheme over GRPA</td>
<td>5.62%</td>
<td>26.33%</td>
<td>30.97%</td>
<td>33.39%</td>
</tr>
<tr>
<td>The performance gain of the proposed scheme over NGDPA</td>
<td>0.54%</td>
<td>18.03%</td>
<td>21.45%</td>
<td>23.33%</td>
</tr>
<tr>
<td>The performance gain of the proposed scheme over MFPA</td>
<td>0.004%</td>
<td>0.12%</td>
<td>0.59%</td>
<td>2.11%</td>
</tr>
</tbody>
</table>

* The values (Mbit/s/W) were measured when the transmission power was 15 W.

5. Conclusions

In this paper, an enhanced gain difference power allocation scheme has been proposed to improve the sum rate of a multi-user NOMA-based MIMO-VLC system, which adapts to user channel conditions and efficiently utilizes the gain difference. Efficient power allocation is achieved by utilizing the residual allocation principle, which emphasizes the power that remains available after allocation to the preceding users, rather than the initially assigned power. Furthermore, an assessment of performance metrics such as the achievable data rate, energy efficiency, and BER was conducted. The numerical results demonstrate that the interference in SIC can be effectively alleviated, and the proposed scheme can achieve a significant performance improvement in terms of both sum rate and energy efficiency over the traditional schemes. In addition, the proposed scheme requires more iterative operation than alternative schemes, so the consumption of hardware resources is slightly higher. In the scenario of a random geometric distribution of users, there may be users with the same channel gain. However, the sorting criterion for this case has not been taken into account. In the future, we are planning to extend our proposed EGDPA scheme into multi-cell scenarios with NLOS links. To better accommodate practice, we will consider adaptive SNR requirements and user association mode selection. The proposed EGDPA scheme will be adjusted and optimized to adapt to the various system requirements.
Author Contributions: Conceptualization, P.M.; methodology, P.M.; formal analysis, X.Z.; data curation, X.Z.; writing—original draft preparation, X.Z.; writing—review and editing, X.Z., P.M. and X.W.; validation, X.Z. and X.W.; supervision, P.M.; project administration, P.M.; funding acquisition, P.M. All authors have read and agreed to the published version of the manuscript.

Funding: This research was partially funded by the Shandong Provincial Natural Science Foundation under grant ZR2023MF096, and by the National Natural Science Foundation of China under grant 61801257.

Data Availability Statement: The data presented in this study are available upon request from the corresponding authors.

Conflicts of Interest: The authors declare no conflicts of interest.

References


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