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Assessment of Communication Resource Allocation by the Transmission Control Protocol for the Target Virtual Connection under Competitive Conditions

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Abstract: The mathematical framework presented in this article focuses on the controlled-transmission protocol's asynchronous process of bandwidth allocation for the target virtual connection implemented under competition for communication resources. The studied process is formalized as a two-dimensional discrete Markovian chain, taking into account the distributions of queue lengths of TCP data fragments from competing client nodes. Such a chain describes the dynamics of filling the stack of transmitted but unacknowledged data fragments of the investigated end device. Distributions of the chain states were found for various ratios of the target virtual-connection bandwidth, transmission-protocol parameters, and communication-channel characteristics. Analytical dependencies for computing the performance of the target virtual connection for different operating modes were obtained. The results of experiments conducted based on the obtained analytical constructions showed that the performance of the virtual connection with a selective repeat mode is mainly determined by the data-loss intensity, the queue size distribution in transit nodes, and the ratio between the protocol window size and the route length.

Keywords: transmission protocol; virtual connection; logical channel; communication resource allocation; bandwidth; competitive access

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1. Introduction and State of the Art

1.1. Relevance of the Research

Transmitting measurement packets from Internet of Things (IoT) devices over Internet Protocol (IP) networks leads to significant channel loading, even with a small total data-transmission rate from devices [1]. Of course, if only a small number of sensors are planned to be used, the additional network load can be disregarded. However, the development of IoT systems and other similar applications tends to involve increasingly diverse devices and, consequently, an increase in their quantity at a single location. Transmitting measurements from hundreds of sensors can significantly impact network performance if the peculiarities of data transmission over networks are not taken into account during the design stage.

Unification is an important criterion considered by engineers when designing production systems. Choosing a transport protocol for transmitting data collected by a sensor network to the cloud should also be carried out with this criterion in mind. The most common network protocol is the Transmission Control Protocol (TCP) [2,3]. TCP utilizes a "congestion window"—the number of packets that the end device's stack has to send or receive before entering a wait state for feedback signals. The larger this window size, the higher the bandwidth. Slow-start and congestion-avoidance algorithms [4] dictate the congestion window size. The congestion window's maximum size is contingent upon the buffer size allocated by the transmission protocol for the corresponding port. Each port has a default buffer value, which can be adjusted using system library calls before opening the port.

To achieve maximum data-packet transmission speed, it is important to use the optimal buffer size for the TCP port corresponding to the virtual connection. If the buffers are too small, the TCP congestion window may fail to fully open, thereby inhibiting the client from operating at its maximum capacity. If the buffers are too large, the client may overwhelm the server, causing the server to simply drop packets and disable the congestion window. Therefore, tuning the parameters of the transmission protocol for the efficient implementation of the target virtual connection is a relevant task, especially when the communication channel is congested by other connections, which is a typical scenario for information interaction between a sensor network and an edge server.

1.2. State of the Art

In recent times, several research teams have advocated for the adoption of statistical methodologies for the control of traffic on the transport OSI layer. A recurring theme among these methods is the extraction of application-specific traffic traits from training datasets. These characteristics often encompass statistics gleaned from entire data flows or connections, as well as attributes associated with individual packets. For instance, the average packet length represents one such flow-derived statistic, while the initial packet length in a connection exemplifies an attribute at the packet level. A comparative analysis of these diverse methodologies has been presented in surveys [4,5]. Below, we provide an outline of the prevailing traffic-control strategies employing Markovian models.

References [6,7] proposed the estimation of individual hidden Markovian models (HMMs) for each application, taking into account inter-arrival times and packet lengths. Within an HMM framework, the output of a state is not deterministic but follows a random distribution based on the probability distribution for emissions associated with that state. While the states' output is observable, the transitions between states remain concealed. For a thorough introduction to HMMs, interested readers are directed to tutorials in [7,8].

Reference [9] focuses on TCP connections and employs left–right HMMs featuring a significant number of states and probability distributions of discrete emissions. In contrast, reference [10] utilizes ergodic HMMs consisting of four to seven states. These models incorporate gamma-distributed emission probabilities to analyze unidirectional UDP and TCP traffic from clients to a set of servers, with a disregard for packets lacking a payload. In both cases, traffic control assigns fresh connections to the application that yields the highest likelihood through its HMM.

Our study is inspired by the work of the research teams in [11,12], who utilize observable ergodic Markovian models that are employed to identify abnormalities in TCP connections. In a studied Markovian model, each state emits a distinct symbol, facilitating the deduction of state transitions directly from an observation series. In the context of [13,14], the Markovian model generates the TCP flag-combination sequences observed in packets transmitted by the client to a server within the TCP connection. Thus, each state corresponds to a particular arrangement of TCP flags, and each transition represents the receipt of a new packet within the existing TCP connection. The transition matrix is derived from anomaly-free training data. During the detection phase, anomalies are identified by computing the a posteriori probability and comparing it against a predefined lower threshold.

In their approach, reference [15] employs distinct Markovian models for different applications, which are differentiated by their well-known port numbers. Building upon this concept, reference [16] adopts and extends the modeling strategy introduced by [15] specifically for managing TCP connections. The training phase remains consistent across both studies; researchers estimate separate Markovian models for different applications using corresponding training datasets. However, in the classification phase, a deviation arises. While [17] solely focuses on the calculation of a posteriori probabilities for a given observed connection within each Markovian model, ref. [18] expands upon this by considering both directions of the TCP connection. Moreover, ref. [19] incorporates payload lengths rather than TCP flag combinations in their analysis. Following this, the connection is assigned to the application linked with the Markovian model that exhibits the highest posterior probability.

It is worth noting separately that existing models of asynchronous control procedures for individual transit nodes and transmission protocols [5,20,21] do not account for the utilization of shared network resources, such as the individual inter-node channel bandwidths resulting from the presence of other virtual connections, leading to the formation of queues in the transit-node buffers. Eliminating such a limitation is the starting point of our research.

1.3. Main Attributes of the Research

The ***object*** of the research is the controlled-transmission protocol's asynchronous process of allocating bandwidth to the target virtual connection, which is implemented under conditions of competition for communication resources.

The ***subject*** of the research includes teletraffic theory, recovery theory, Markovian chains theory, and functional analysis methods.

The ***aim*** of the research is to analytically formalize the relationship among the target virtual-connection bandwidth, the transport-protocol parameters, and the communication-channel characteristics, for which competing clients contend for resources.

To achieve the stated aim, it is necessary to sequentially solve the following ***tasks***:

- Define the parameter space comprehensively to provide an adequate description of the research object (Section 2.1);
- Formalize the concept of controlling information transfer in the logical channel with an arbitrary number of transit nodes (Section 2.2);
- Formulate the mathematical framework for assessing the communication resource allocation by the transmission protocol to the target virtual connection in a loaded communication channel (Section 2.3);
- Conduct a numerical experiment to validate the efficacy of the developed mathematical framework (Section 3).

In summary, the ***main contribution*** of the research is the development of a mathematical framework focused on the controlled-transmission protocol's asynchronous process of allocating bandwidth to the target virtual connection under conditions of competition for communication resources. The investigated process is formalized as a two-dimensional discrete Markovian chain, taking into account the distributions of queue lengths of TCP data fragments from competing client nodes. This chain describes the dynamics of filling the stack of transmitted but unacknowledged data fragments of the investigated end device. The research results include findings of state distributions of the Markovian chain for various ratios of the target virtual-connection bandwidth, transmission-protocol parameters, and characteristics of the communication channel (including window size, number of hops in the route, waiting period for feedback signals, probabilities of data fragment distortion in transit nodes, and queue lengths in the buffers of the latter). Analytical dependencies for computing the performance of the target virtual connection for different operating modes have been derived.

2. Materials and Methods

2.1. Statement of the Research

Bandwidth capacity is the most crucial characteristic of a transmission protocol that controls the parameters of the logical data-transmission channel between end devices. The actual value of this characteristic is determined not only by the speed and dependability of a specific logical channel but also by the intensity of information flows in other logical channels that share the same physical medium with the investigated virtual connection. Evaluating this indirect impact is proposed by analyzing the length of queues of data fragments (segments (OSI transport layer)—packets (OSI network layer)—frames (OSI data link layer)) formed by the investigated virtual connection at transit nodes.

Let us examine the process of information exchange between end devices, implemented through a symmetric duplex logical channel and routed through a set of transit nodes. The number of hops in the investigated logical channel is characterized by the indicator L. Feedback regarding the data-transfer performance is conducted through the same route but in reverse order. Every l-th hop, $l = \overline{1, L}$, is characterized by the probability of information message distortion $C_k(l)$, $k = \overline{0, K}$, and the probability of feedback signal distortion $C_r(l)$. Accordingly, the dependability of the successful transmission of an information message through the logical channel is characterized by the probability $D_k = \prod_{l=1}^{L}(1 - C_k(l))$, $k = \overline{0, K}$, while the dependability of the successful transmission of a feedback signal through the logical channel is characterized by the probability $D_r = \prod_{l=1}^{L}(1 - C_r(l))$. We assume that the end devices always have information to send. Information messages are divided into fragments of equal length, for which the processing duration by transit nodes is constant. Feedback signals are transmitted in a stream opposite to the stream of information-message transmission. The selective acknowledgement procedure for information exchange is implemented according to RFC 2883 [22].

Let us introduce a probability function s_k, $k = \overline{0, K}$, which characterizes the event when each fragment of an information message propagated through the investigated logical channel encounters a queue of k data fragment lengths at the transit node along its route, where K is the transit-node stack-buffer capacity.

We characterize the size of the transmission-protocol window by the value V, and then the duration of waiting for an acknowledgement of successful data transfer is determined as $T > V$. Time T counting starts before the transmission of the first fragment of the information message and is fixed for all its subsequent fragments within the interval V. After sending the next data fragment, the transmission protocol copies it into the stack of sent but unacknowledged data and initiates the countdown T. Once the stack size reaches V, the transmission protocol suspends the transmission of data fragments and enters a waiting state until it receives a feedback signal or until the end of the period T. Upon receiving a feedback signal from the server for the corresponding fragment, the client removes it from the stack of sent but unacknowledged data. If the time-limited period T for the sent fragment elapses but the client does not receive the corresponding feedback signal, then this data fragment is re-sent, and the countdown T for it is restarted.

Let us generalize the term "round-trip delay", equal to d, as the sum of periods for the following: the preparation of the data fragment for transmission, the duration of the data fragment in the logical channel, and the analysis of the received data fragment by the server. In turn, the waiting period for the client to receive a feedback signal regarding the sent data fragment will be interpreted as a stochastic variable distributed geometrically with parameter D_r and delay duration d.

The process of functioning of the stack of sent but unacknowledged data on the client's side is controlled by the transmission protocol. Based on the description provided above, it can be stated that an adequate model for this process would be a two-dimensional discrete Markovian chain defined in the space of stochastic parameters such as period T and stack capacity $K + 1$. The value of the period T should be chosen so that the client has enough time to receive a feedback signal regarding the sent data fragment. Taking into account the introduced concept of round-trip delay d and the probable delay of a data fragment in the queues of transit nodes of the logical channel, we can characterize the approximate duration of the period T by the inequality $T \geq 2L + k$. Following the same logic, we characterize the time for the client to wait for a feedback signal regarding the sent data fragment by the inequality $t \geq 2L + k$. As we see, determining the parameters T and t requires an assessment of the value of the parameter k, which characterizes the determined duration of queues before the stream with data fragments being transmitted and before the corresponding stream with feedback signals.

Let us examine the discrete Markovian chain (n, k) for the process of transmitting an information message consisting of n data fragments, which, when propagating through the logical channel, upon the next hop, enters a queue of length k data fragments. States $\left(n \in \left\{\overline{0, V+k}\right\}, k \in \left\{\overline{0, K}\right\}\right)$ characterize the number of sent data fragments whose reception has not been acknowledged and the period since the initiation of the transmission of the information message, of which these fragments are a part. States $\left(n \in \left\{\overline{V+k+1, T-1}\right\}, k \in \left\{\overline{0, K}\right\}\right)$ characterize the periods when the transmission of data fragments is suspended until receiving a feedback signal regarding the successful receipt of the information message consisting of V data fragments.

We characterize the set of states of the Markovian chain (n, k) with corresponding probabilities $Q(n, k)$, $n = \overline{0, V-1}$, $k = \overline{0, K}$. Thus, for a queue of zero length, the stack with sent but unacknowledged data fragments will be in a state $(2L-1, 0)$ with probability s_r. Further growth of the stack is characterized by probability $s_r(1-D_r)$. Upon receiving a feedback signal, the investigated process, which is in state $\left(n \in \left\{\overline{2L-1+k, V-1}\right\}, k \in \left\{\overline{0, K}\right\}\right)$, can react by either sending fragments of a new information message (positive feedback signal) or by retransmitting distorted data fragments (negative feedback signal).

The event where the propagation of an information message through the logical channel may be suspended due to the presence of a queue at any transit node of the routed path (transition of the investigated process from state $\left(n \in \left\{\overline{0, V-2}\right\}, k = 0\right)$ to state d$\left(n \in \left\{\overline{0, V-2}\right\}, k \in \left\{\overline{1, K}\right\}\right))$ is characterized by probability s_k. Accordingly, we characterize the dynamics of the Markovian chain by defining analytical expressions for the probability q_{nk}^{ij} of the transition of the investigated process from state (n, k) to state (i, j):

$$q_{nk}^{ij} = \begin{cases} s_r \forall n = \overline{0, 2L-2}, k = 0, i = n+1, j = 0; \\ s_r(1-D_r) \forall n = \overrightarrow{2L-1, V-2}, k = 0, i = n+1, j = 0; \\ s_j \forall n = \overline{0, V-2}, k = 0, i = n, j = \overline{1, K}; \\ s_r D_r \forall n = \overline{2L-1, V-1}, k = 0, i = 2L-1, j = 0; \\ n = \overline{V, V+2L-2}, k = 0, i = V+2L-2-n, j = 0; \\ n = \overline{V+2L-1, T-2}, k = i = j = 0; \\ 1 \forall n = T-1, k = \overline{0, K}, i = j = 0; \\ n = \overline{0, 2L-2+k}, k = \overline{1, K}, i = n+1, j = k, \\ 1-D_r \forall n = \overline{2L-1+k, V-2}, k = \overline{1, K}, i = n+1, j = k; \\ D_r \forall n = \overline{2L-1+k, V-1+k}, k = \overline{1, K}, i = 2L-1, j = 0; \\ n = \overline{V+k, V+k+2L-2}, k = \overline{1, K}, i = V+k+2L-2-n, j = 0; \\ n = \overline{V+k+2L-1, T-2}, k = \overline{1, K}, i = j = 0. \end{cases} \quad (1)$$

In the context of our study, we will interpret the bandwidth of the controlled-transmission protocol of the logical channel B as the ratio of the average data volume sent between the moments of receiving two consecutive feedback signals $\overline{W}$ to the average interval between the moments of receiving such signals $\overline{I}$. Interpreting this definition in terms of round-trip delay d, we obtain an analytical expression $B(V, T) = \overline{W}/\overline{I}$.

Since independent feedback signals arrive at the client with periodicity d (provided that these signals have not been distorted in the logical channel of length L between the end devices), the interval I is a stochastic variable distributed geometrically with parameter D_r; hence, $\overline{I} = 1/D_r$. When formalizing the value of $\overline{W}$, let us consider that with probability s_k, $k = \overline{0, K}$, each data fragment on its route through the logical channel will encounter a queue of length k, increasing the amount of transmitted information by the quantity $\lfloor 1/(k+1) \rfloor$.

As a result, we can express $\overline{W}=\sum\limits_{k=0}^{K}\frac{1}{k+1}\left(\sum\limits_{z=2L-1+k}^{V+2L-2+k}\overline{z}Q(z,k)+\sum\limits_{z=2L-1+k}^{T-1}\overline{V}Q(z,k)\right)$, where $\overline{z}=(z-2L-k+2)D_k$, $\overline{V}=VD_k$, $k=\overline{0,K}$, in the context of interpreting the average number of successfully sent data fragments in the selective retransmission procedure for distorted data fragments. Therefore, we express the bandwidth of the investigated data-transmission process is represented as a function of such parameters as the window size V and the period T:

$$B(V,T)=D_fD_r\sum_{k=0}^{K}\frac{1}{k+1}\left(\sum_{z=2L-1+k}^{V+2L-2+k}(z-2L+2-k)Q(z,k)+V\sum_{z=2L-1+k}^{T-1}Q(z,k)\right). \quad (2)$$

2.2. The Concept of Controlling Information Transfer in a Logical Channel with an Arbitrary Number of Transit Nodes

Based on the provided transition probabilities (1), we form a system of equilibrium equations governing the process of information transfer in a logical channel with several hops $L=1$, window size $V\geq 1$, and period $T>V$, $T\geq K+2$:

$$Q(0,0)=D_r\left(s_r\sum_{n=V}^{T-2}Q(n,0)+\sum_{k=1}^{K}\sum_{n=V+k}^{T-2}Q(n,k)\right)+\sum_{k=0}^{K}Q(T-1,k), \quad (3)$$

$$Q(1,0)=s_rQ(0,0)+D_r\left(s_r\sum_{n=1}^{V-1}Q(n,0)+\sum_{k=1}^{K}\sum_{n=k+1}^{V-1+k}Q(n,k)\right), \quad (4)$$

$$Q(n,0)=s_r(1-D_r)Q(n-1,0),\ n=\overline{2,T-1}, \quad (5)$$

$$Q(0,k)=s_kQ(0,0),\ k=\overline{1,K}, \quad (6)$$

$$Q(n,k)=s_kQ(n,0)+Q(n-1,k),\ n=\overline{1,k+1},\ k=\overline{1,K}, \quad (7)$$

$$Q(n,k)=s_kQ(n,0)+(1-D_r)Q(n-1,k),\ n=\overline{k+2,T-2},\ k=\overline{1,K}, \quad (8)$$

$$Q(T-1,k)=(1-D_r)Q(T-2,k),\ k=\overline{1,K}. \quad (9)$$

Now, we derive the analytical solution for the system of equilibrium (Equations (3)–(9)). We transform expression (5) into the form $Q(n,0)=Q(1,0)(s_r(1-D_r))^{n-1}$, $n=\overline{1,T-1}$. Based on the obtained expression and considering expressions (6) and (7) for $n=\overline{0,k+1}$, $k=\overline{1,K}$, we write the following:

$$Q(n,k)=s_k\left(Q(0,0)+\frac{1-(s_r(1-D_r))^n}{1-s_r(1-D_r)}Q(1,0)\right). \quad (10)$$

Let us substitute expression (10) into expressions (8) and (9). For $k\geq 1$, we obtain

$$Q(n,k)=s_r(1-D_r)^{n-1}\left(Q(1,0)\left(\frac{D_rs_r^{k+1}}{(1-s_r)(1-s_r(1-D_r))}-\frac{s_r^n}{1-s_r}+\frac{1}{(1-s_r(1-D_r))(1-D_r)^k}\right)+\frac{Q(0,0)}{(1-D_r)^k}\right),\ n=\overline{k+1,T-2}; \quad (11)$$

$$Q(T-1,k)=s_k(1-D_r)^{T-2}\left(Q(1,0)\left(\frac{D_rs_r^{k+1}}{(1-s_r)(1-s_r(1-D_r))}-\frac{s_r^{T-2}}{1-s_r}+\frac{1}{(1-s_r(1-D_r))(1-D_r)^k}\right)+\frac{Q(0,0)}{(1-D_r)^k}\right). \quad (12)$$

We substitute expressions (11) and (12) into (4) and obtain the desired expression for calculating $Q(1,0)$:

$$Q(1,0)=(AQ(0,0))/(1-D_r)^{V-1}, \quad (13)$$

where $A = \frac{(1-s_r)(1-s_r(1-D_r))\left(1-(1-s_r)(1-D_r)^{V-1}\right)}{(1-s_r)\left(1-s_r+D_r s_r^V\right)+s_r D_r\left(1-s_r^{V-1}\right)\sum\limits_{k=1}^{K} s_k(s_r(1-D_r))^k}$.

Finally, after normalization, we find the working expression for calculating the probability (3):

$$Q(0,0) = (1-D_r)^{V-1} \Big/ \left((1-D_r)^{V-1}\left(\frac{3-2(1+s_r-D_r)+s_r(1-D_r)}{D_r} + \sum_{k=1}^{K} s_k\left(k - \frac{(1-D_r)^{T-k-1}}{D_r}\right)\right) + \right.$$
$$+A\left(\frac{2-(1+s_r-D_r)(1-2s_r(1-D_r))-s_r(1-D_r)(3+s_r(1-D_r))}{D_r(1+s_r(1-D_r))^2} - \frac{(1-s_r)(s_r(1-D_r))^{T-2}}{1-s_r(1-D_r)} + \right.$$
$$\left.\left.+\sum_{k=1}^{K} s_k\left(\frac{k}{1-s_r(1-D_r)} - \frac{(1-D_r)^{T-k-1}}{D_r(1-s_r(1-D_r))} + \frac{(s_r(1-D_r))^{k+1}}{(1-s_r(1-D_r))^2} - \frac{s_r^{k+1}(1-D_r)^{T-1}}{(1-s_r)(1-s_r(1-D_r))}\right)\right)\right).$$

Let us investigate how the analytical form of solutions (12) and (13) will change in response to the parameterization of a series of typical situations that occur during the deployment of the studied process.

Suppose the queue for sending is absent, that is, $s_r = 1$. In this case, we obtain

$$Q(0,0) = \frac{D_r(1-D_r)^{V-1}}{1+\left(D_r-(1-D_r)^{T-V}\right)(1-D_r)^{V-1}},$$

$$Q(n,0) = \frac{D_r(1-D_r)^{n-1}}{1+\left(D_r-(1-D_r)^{T-V}\right)(1-D_r)^{V-1}} \forall n = \overline{1, T-1},$$

$$Q(n,k) = 0 \;\; \forall n = \overline{0, T-1}, k = \overline{1, K}.$$

Now let us characterize the case where the information message always enters the queue before going on the route, i.e., $s_r = 0$. For such a situation, we will obtain

$$Q(0,0) = \frac{D_r(1-D_r)^{V-1}}{1+D_r\left(1+\overline{K}\right)+\left(D_r-\sum\limits_{k=1}^{K} s_k(1-D_r)^{T-V-k}\right)(1-D_r)^{V-1}}, \tag{14}$$

$$Q(1,0) = \frac{Q(0,0)\left(1-(1-D_r)^{V-1}\right)}{(1-D_r)^{V-1}}, \tag{15}$$

where $\overline{K} = \sum\limits_{k=1}^{K} k s_k$.

If the information message enters a queue of deterministic length, i.e., $s_k = 1, k \geq 1$, then expression (15) will take the form

$$Q(0,0) = \frac{D_r(1-D_r)^{V-1}}{1+\left(D_r(1-D_r)^{T-V-k}\right)(1-D_r)^{V-1}}$$

If the window size is unlimited, i.e., $V = \infty$, then, correspondingly, the period T becomes unlimited too. As a result, the initial state $(0,0)$ becomes irreversible, i.e., $(0,0)$, and the probability (15) can be characterized by the expression

$$Q(1,0) = D_r(1-s_r(1-D_r))^2 / \big(2-(1+s_r-D_r)(1-2s_r(1-D_r)) -$$
$$-s_r(1-D_r)(3+s_r(1-D_r)) + \overline{K}D_r(1-s_r(1-D_r)) + D_r\sum_{k=1}^{K} s_k(s_r(1-D_r))^{k+1}\big). \tag{16}$$

If we supplement the preamble to expression (16) with the fact that the investigated virtual connection always competes for communication resources, that is, $s_r = 0$, then we will obtain

$$Q(1,0) = D_r / \left(1 + D_r \left(1 + \overline{K}\right)\right). \tag{17}$$

If we assume the impossibility of distorting the feedback signal, i.e., $D_r = 1$, then with the configuration settings of the transmission protocol of type $T \geq K + 2$, $V = 1$, the set of states (n, k) is defined by the $n = \overline{0, k+1}$, $k = \overline{0, K}$, values and is characterized by probabilities

$$Q(0,0) = \left(3 - s_r^2 + (1 + s_r)\overline{K}\right)^{-1},\ Q(n,k) = s_k Q(0,0). \tag{18}$$

If we introduce a change in the preamble to expression (18), i.e., $V \geq 2$, then states with indices $n = \overline{1, k+1}$, $k = \overline{0, K}$, will be characterized by the non-zero probabilities

$$Q(1,0) = \left(2 - s_r + \overline{K}\right)^{-1},\ Q(n,k) = s_k Q(1,0). \tag{19}$$

The analytical material provided above characterizes the process of information transfer in a logical channel with several hops, i.e., $L = 1$, window size $V \geq 1$, and period $T > V$, $T \geq K + 2$. Generalizing this mathematical apparatus is of great practical importance for the scenario where the path of the analyzed virtual connection is laid through an arbitrary number of transit nodes, i.e., the number of hops $L > 1$. Generalizing such a scenario is accompanied by objective complications; a similar system of equilibrium equations (Equations (3)–(9)) formulated for $L > 1$ with $T > V$ does not have a solution. A solution was successfully determined in two cases: (1) for $T = V + 1$, $V \geq 2L + K$; (2) for $s_r = 0$.

Therefore, the system of equilibrium equations defined concerning the parameters $\langle T, V, K \rangle$ for $L > 1$ with $T = V + 1$, $V \geq 2L + K$, has the form

$$Q(0,0) = \sum_{k=0}^{K} Q(V,k);\ Q(n,0) = s_r Q(n-1,0),\ n = \overline{1, 2L-2};$$

$$Q(2L-1,0) = s_r Q(2L-2,0) + \sum_{n=2L-1}^{V-1} s_r D_r Q(n,0) + \sum_{k=1}^{K} \sum_{n=2L-1+k}^{V-1} D_r Q(n,k);$$

$$Q(n,0) = s_r (1 - D_r) Q(n-1,0),\ n = \overline{2L, V};$$

$$Q(0,k) = s_k Q(0,0),\ k = \overline{1, K};$$

$$Q(n,k) = Q(n-1,k) + s_k Q(n,0),\ n = \overline{1, 2L-1+k},\ k = \overline{1, K};$$

$$Q(V,k) = (1 - D_r) Q(V-1,k),\ k = \overline{1, K}.$$

The analytical solution to such an equilibrium system has been found, which is formed by the sequence of the following relationships:

$$Q(n,0) = Q(0,0) s_r^n,\ n = \overline{1, 2L-2};$$

$$Q(n,0) = Q(2L-1,0)(s_r(1 - D_r))^{n-2L+1},\ n = \overline{2L-1, V};$$

$$Q(n,k) = Q(0,0) s_k \frac{1 - s_r^{n+1}}{1 - s_r},\ n = \overline{0, 2L-2},\ k = \overline{1, K};$$

$$Q(n,k) = Q(0,0)s_k \frac{1-s_r^{2L-1}}{1-s_r} + Q(2L-1,0)s_k \frac{1-(s_r(1-D_r))^{n-2L+2}}{1-s_r(1-D_r)},\ n = \overline{2L-1, 2L-1+k},\ k = \overline{1,K};$$

$$\begin{aligned} Q(n,k) = Q(0,0)s_r \tfrac{1-s_r^{2L-1}}{1-s_r}(1-D_r)^{n-2L-k+1} + Q(2L-1,0)s_r(1-D_r)^{1-2L+1} \times \\ \times \left(\tfrac{1}{(1-D_r)^k (1-s_r(1-D_r))} - \tfrac{s_r^{n-2L+2}}{1-s_r} + \tfrac{s_r^{k+1} D_r}{(1-s_r)(1-s_r(1-D_r))} \right), \end{aligned} \quad n = \overline{2L+k, V-1},\ k = \overline{1,K};$$

$$\begin{aligned} Q(V,k) = Q(0,0)s_r \tfrac{1-s_r^{2L-1}}{1-s_r}(1-D_r)^{V-2L-k+1} + Q(2L-1,0)s_r(1-D_r)^{V+2L+1} \times \\ \times \left(\tfrac{1}{(1-D_r)^k (1-s_r(1-D_r))} - \tfrac{s_r^{V-2L+1}}{1-s_r} + \tfrac{s_r^{k+1} D_r}{(1-s_r)(1-s_r(1-D_r))} \right), \end{aligned} \quad ,\ k = \overline{1,K};$$

$$Q(2L-1,0) = (EQ(0,0))/(1-D_r)^{V-2L+1},$$

where

$$\begin{aligned} Q(0,0) = (1-D_r)^{V-2L+1} \Bigg((1-D_r)^{V-2L+1} \left(2L + \tfrac{1-s_r^{2L-1}}{1-s_r} \sum_{k=1}^{K} s_r \left(k - \tfrac{(1-D_r)^{V-2L+2-k}}{D_r} \right) + \tfrac{1-(1+D_r)s_r^{2L-1}}{D_r} \right) + \\ + E \Bigg(\tfrac{2-(1+s_r-D_r)(1-2s_r(1-D_r)) - s_r(1-D_r)(3+s_r(1-D_r))}{D_r(1-s_r(1-D_r))^2} - \tfrac{(1-s_r)(s_r(1-D_r))^{V+2L+1}}{1-s_r(1-D_r)} + \\ + \sum_{k=1}^{K} s_k \left(\tfrac{k}{1-s_r(1-D_r)} - \tfrac{(1-D_r)^{V-2L-k+2}}{D_r(1-s_r(1-D_r))} + \tfrac{(s_r(1-D_r))^{k+1}}{(1-s_r(1-D_r))^2} - \tfrac{s_r^{k+1}(1-D_r)^{V-2L+2}}{(1-s_r)(1-s_r(1-D_r))} \right) \Bigg) \Bigg)^{-1}, \end{aligned}$$

$$E = \frac{(1-s_r(1-D_r)) \left(1 - s_r - \left(1 - s_r^{2L-1}\right)(1-D_r)^{V-2L+1} \sum_{k=1}^{K} \tfrac{s_k}{(1-D_r)^k} \right)}{\sum_{k=1}^{K} s_k \left(D_r s_r^{k+1} + \tfrac{1-s_r}{(1-D_r)^k} \right)}$$

Note that for $L = 1$, the above-demonstrated analytical solutions coincide with similar expressions presented at the beginning of the section for $V = T - 1$.

We apply the obtained analytical constructions to analyze typical instances of the studied process for which the initial conditions $L > 1$, $T = V + 1$, $V \geq 2L + K$ hold.

Given the condition $s_r = 1$ (assuming the absence of a queue on the client's side), for the probability $Q(0,0)$, we obtain

$$Q(0,0) = \frac{D_r(1-D_r)^{V-1}}{1 + (2LD_r - 1)(1-D_r)^{V-2L+1}},\ Q(2L-1,0) = \frac{Q(0,0)}{(1-D_r)^{V-2L+1}}.$$

If, when propagating through a logical channel traversing $L > 1$ transit nodes, the information message is inevitably queued, i.e., $s_r = 0$, then we characterize the states of the corresponding Markovian chain by the following relationships:

$$\begin{gathered} Q(0,0) = \frac{\sum_{k=1}^{K} s_k (1-D_r)^{V-2L-k+1}}{1 + \overline{K} + \frac{1}{D_r} + \left(2L - \frac{1}{D_r}\right) \sum_{k=1}^{K} s_k (1-D_r)^{V-2L-k+1}}; \\ Q(2L-1,0) = Q(0,0) \left(1 - \sum_{k=1}^{K} s_k (1-D_r)^{V-2L-k+1} \right) \Big/ \sum_{k=1}^{K} s_k (1-D_r)^{V-2L-k+1}; \\ Q(n,0) = 0,\ n = \left\{ \overline{1, 2L-2}, \overline{2L, V} \right\};\ Q(n,k) = s_k Q(0,0),\ n = \overline{0, 2L-2},\ k = \overline{1,K}; \\ Q(n,k) = s_k (Q(0,0) + Q(2L-1,0)),\ n = \overline{2L-1, 2L-1+k},\ k = \overline{1,K}; \\ Q(n,k) = s_k (Q(0,0) + Q(2L-1,0))(1-D_r)^{n-2L-k+1},\ n = \overline{2L+k, V},\ k = \overline{1,K}. \end{gathered} \tag{20}$$

If the window size V is not limited when configuring the transmission protocol ($V = \infty$), then the states (n,k), $n = \overline{0, 2L-2}$, $k = \overline{0,K}$, are irreversible: $Q(n,k) = 0$, $n = \overline{0, 2L-2}$, $k = \overline{0,K}$. The state $(2L-1, 0)$ is characterized by a probability, analytically described by expression (16). This indicates the independence of this state concerning the value of a parameter such as the length of the route laid for the investigated virtual connection.

If there is information regarding the length of the queue into which the investigated virtual connection will enter; e.g., $s_k = 1$, $k > 0$, then the range of states of the Markovian chain, which determines the bandwidth for the investigated virtual connection, can be characterized by probabilities

$$Q(2L-1,0) = D_r/(1+D_r(k+1)),\ Q(n,k) = D_r(1-D_r)^{n-2L-k+1}/(1-D_r(k+1)),\ n > 2L-1+k,\ k > 0.$$

If the routed path for propagating feedback signals is characterized by the parameter $D_r = 1$, and the configuration settings of the transmission protocol imply that $V \geq K + 2L$, then the set of significant states (n,k) of the Markovian chain is bounded by ranges $n = \overline{2L-1, 2L-1+k}$, $k = \overline{0,K}$, and characterized by probabilities

$$Q(2L-1,0) = 1/\left(2 - s_r + \overline{K}\right),\ Q(n,k) = s_k Q(2L-1,0). \tag{21}$$

Finally, let us characterize the situation where the information message always enters the queue while propagating through the logical channel, i.e., $s_r = 0$, and the configuration settings of the transmission protocol satisfy the conditions $T > V + K$, $V \geq 2L$. In this formulation, the system of equilibrium equations in question takes the following form

$$Q(0,0) = \sum_{k=1}^{K}\left(Q(T-1,k) + \sum_{n=V+2L+k-2}^{T-2} D_r Q(n,k)\right);$$

$$Q(n,0) = \sum_{k=1}^{N} D_r Q(V++2L+k-2-n,k),\ n = \overline{1,2L-2},\ k = \overline{1,K};$$

$$Q(2L-1,0) = \sum_{k=1}^{K}\sum_{n=2L-1+k}^{V-1} D_r Q(n,k);$$

$$Q(0,k) = s_k Q(0,0),\ k = \overline{1,K};$$

$$Q(n,k) = Q(n-1,k) + s_k Q(n,0),\ n = \overline{1,2L-1},\ k = \overline{1,K};$$

$$Q(n,k) = Q(n-1,k),\ n = \overline{2L,2L-1+k},\ k = \overline{1,K};$$

$$Q(n,k) = (1-D_r)Q(n-1,k),\ n = \overline{2L+k,T-1},\ k = \overline{1,K}.$$

The analytical solutions for such a system of equilibrium equations are presented below:

$$Q(n,0) = D_r Q(0,0)/(1-D_r)^n,\ n = \overline{1,2L-2};$$

$$Q(2L-1,0) = Q(0,0)\left(1-(1-D_r)^{V-2L+1}\right)/(1-D_r)^{V-1};$$

$$Q(n,k) = s_k Q(0,0)/(1-D_r)^n,\ n = \overline{0,2L-2},\ k = \overline{1,K};$$

$$Q(n,k) = s_k Q(0,0)/(1-D_r)^{V-1},\ n = \overline{2L-1,2L+k-1},\ k = \overline{1,K};$$

$$Q(n,k) = s_k Q(0,0)(1-D_r)^{n-2L-k+1}/(1-D_r)^{V-1},\ n = \overline{2L+k,T-1},\ k = \overline{1,K};$$

$$Q(0,0) = \frac{D_r(1-D_r)^{V-1}}{1+D_r\left(\overline{K}+1\right)+(1-D_r)^{V-2L+1}\left(1-(1-D_r)^{2L-1}\right)-\sum\limits_{k=1}^{K} s_k(1-D_r)^{T-2L-k+1}}.$$

When $L = 1$, the obtained solution converges to the form of (15), and when $s_r = 0$, it converges to the form of (21). Finally, when $V = \infty$, we obtain

$$Q(n,k) = 0,\ n = \overline{0, 2L-2},\ k = \overline{0, K};$$

$$Q(2L-1,0) = D_r/\left(1+D_r\left(1+\overline{K}\right)\right);$$

$$Q(n,k) = s_k D_r/\left(1+D_r\left(1+\overline{K}\right)\right),\ n = \overline{2L-1, 2L+k-1},\ k = \overline{1, K};$$

$$Q(n,k) = \frac{s_k D_r(1-D_r)^{n-2L-k+1}}{1+D_r\left(1+\overline{K}\right)},\ n \geq 2L+k,\ k = \overline{1, K}.$$

2.3. Estimation of the Volume of Communication Resources Allocated by the Transmission Protocol to the Target Virtual Connection in a Congested Communication Channel

In Section 2.2, probability–time characteristics for the target virtual connection under $V \geq 2L$ were formalized. Applied interest involves an analytical formalization of the situation in which $V < 2L$, i.e., the communication resources of the physical channel are not fully utilized by competing virtual connections. We represent the equilibrium equations system characterizing the dynamics of filling the stack of sent data fragments awaiting acknowledgement in conditions where $V < 2L$ as follows:

$$Q(0,0) = \sum_{n=2L-2+V}^{T-2} s_r D_r Q(n,0) + \sum_{k=1}^{K} \sum_{n=2L-2+k+V}^{T-2} D_r Q(n,k) + \sum_{k=0}^{K} Q(T-1,k),\ V = \overline{1, 2L-1};$$

$$Q(n,0) = s_r Q(n-1,0) + s_r D_r Q(2L-2+V-n,0) + \sum_{k=1}^{K} D_r Q(2L-2+k+V-n,k),\ n = \overline{1, V-1},\ V = \overline{1, 2L-1};$$

$$Q(n,0) = s_r Q(n-1,0),\ n = \overline{V, 2L-1},\ V = \overline{1, 2L-1};$$

$$Q(n,0) = s_r(1-D_r)Q(n-1,0),\ n = \overline{2L, T-1};$$

$$Q(0,k) = s_r Q(0,0),\ k = \overline{1, K};$$

$$Q(n,k) = Q(n-1,k) + s_k Q(n,0),\ n = \overline{1, 2L-1+k},\ k = \overline{1, K};$$

$$Q(n,k) = (1-D_r)Q(n-1,k) + s_k Q(n,0),\ n = \overline{2L+k, T-2},\ k = \overline{1, K};$$

$$Q(T-1,k) = (1-D_r)Q(T-2,k),\ k = \overline{1, K}.$$

Let us assume that the feedback signals, in the process of their propagation, do not undergo distortions, i.e., $D_r = 1$. Taking this condition into account, let us formalize the solution of the above-mentioned equilibrium equations system defined for $V < 2L$. Obviously, under these assumptions, the states (n,k), $n = \overline{0, V-2}$, $k = \overline{0, K}$, are irreversible, meaning that the equation $Q(n,k) = 0$ is satisfied for them. For the remaining permissible states, we obtain

$$
\begin{gathered}
Q(n,0)=Q(V-1,0)s_r^{n-V+1},\ n=\overline{V-1,2L-1},\ V=\overline{1,2L-1};\\
Q(n,k)=s_kQ(V-1,0)\sum_{i=0}^{n-V+1}s_r^i=s_kQ(V-1,0)\frac{1-s_r^{n-V+2}}{1-s_r},\ n=\overline{V-1,2L-1},\ k=\overline{1,K},\ V=\overline{1,2L-1};\\
Q(n,k)=s_kQ(V-1,0)\frac{1-s_r^{2L-V+1}}{1-s_r},\ n=\overline{2L-1,2L-1+k},\ k=\overline{1,K},\ V=\overline{1,2L-1}.
\end{gathered} \tag{22}
$$

Applying the normalization condition to (22), we obtain

$$
Q(V-1,0)=1\Bigg/\left(2L-V+2-s_r^{2L-V+1}+\overline{K}\frac{1-s_r^{2L-V+1}}{1-s_r}\right). \tag{23}
$$

With $L=1$, $V=1$, expression (23) converges to form (18).

Based on the found solutions (18), let us formulate expressions for estimating the bandwidth level of the target virtual connection, which is propagated by a logical channel with characteristics $L=1$, $D_r=1$, and controlled by the transmission protocol in the context of establishing the value of V and during a sufficiently long period T (according to expression (2)):

$$
\begin{gathered}
B(V=1,T)=D_r\left(s_r+(1+s_r)\sum_{k=1}^{K}\frac{s_k}{k+1}\right)\Bigg/\left(3+\overline{K}(1+s_r)-s_r^2\right),\\
B(V\geq 2,T)=D_r\left(1+\sum_{k=1}^{K}\frac{s_k}{k+1}\right)\Bigg/\left(2+\overline{K}-s_r\right).
\end{gathered} \tag{24}
$$

Based on the found solutions (15), let us formulate expressions for estimating the target virtual-connection bandwidth level, which is propagated by a logical channel with characteristics $L=1$, $s_r=0$, and controlled by the transmission protocol in the context of establishing the value of $V\geq 1$ and during a sufficiently long period T (according to expression (2)):

$$
B(V,T)=D_k\frac{D_r^2\left(1-(1-D_r)^{V-1}\right)+\sum_{k=1}^{K}\frac{s_k}{k+1}\left(1-(1-D_r)^{V}-VD_r(1-D_r)^{T-k-1}\right)}{1+D_r\left(1+\overline{K}\right)+(1-D_r)^{V-1}-(1-D_r)^{V}-\sum_{k=1}^{K}s_k(1-D_r)^{T-k--1}},\ k=\overline{1,K}.
$$

Based on the found solutions (20), let us formulate expressions for estimating the bandwidth level of the target virtual connection, which is propagated by a logical channel with characteristics $L\geq 2$, $s_r=0$, and controlled by the transmission protocol in the context of establishing the value of $V\geq 2L+K$ and $T=V+1$. Taking into account (2), we obtain

$$
B(V\geq 2L+1,T=V+1)=D_k\frac{D_r^2\left(1-\sum_{k=1}^{K}s_r(1-D_r)^{V-2L-k+1}\right)+\sum_{k=1}^{K}\frac{s_k}{k+1}(1-D_r(V-2L-k+2))\left(1-(1-D_r)^{V-2L-k+2}\right)}{1+D_r\left(1+\overline{K}\right)+(2LD_r-1)\sum_{k=1}^{K}s_k(1-D_r)^{V-2L+k+1}},
$$

Let us formalize the adaptation of the above expression for the case when the parameters of the transmission protocol control are defined as $V\geq 2L$ and $T=V+K$ (the parameters of the virtual connection remain unchanged):

$$
B(V\geq 2L,T>V+K)=D_k\frac{D_0^2\left(1-(1-D_r)^{V-2L+1}\right)+\sum_{k=1}^{K}\frac{s_k}{k+1}\left(1-(1-D_r)^{V}-VD_r(1-D_r)^{T-2L-k+1}\right)}{1+D_r\left(1+\overline{K}\right)+(1-D_r)^{V-2L+1}-(1-D_r)^{V}-\sum_{k=1}^{K}s_k(1-D_r)^{T-2L-k+1}}.
$$

If $D_r=1$, $V\geq 2L$ are satisfied, then the transitions between states of the Markovian chain describing the corresponding virtual connection are defined by expression (21). In such a situation, the transmission protocol allocates bandwidth to support the virtual connection defined by expression (24) regardless of the value of the parameter L. However, if the conditions $s_k = 1$, $k > 0$ are also met, then $L > 1$ yields $B(V,T) = D_k/(k+1)$, $k = \overline{1,K}$.

Finally, if the investigated virtual connection is characterized by parameters $D_r = 1$, $V < 2L$, then, relying on decisions (22) and (23), the volume of communication resources that the transmission protocol allocates to support such a connection is characterized by the expression

$$B(V,T) = D_k \frac{s_r^{2L-V} + (1-s^r)^{-1}\left(1 - s_r^{2L-V+1}\right)\sum_{k=1}^{K} s_k/(k+1)}{2L - V + 2 - s_r^{2L-W+1} + \overline{K}(1-s_r)^{-1}\left(1 - s_r^{2L-V-1}\right)},\ k = \overline{1,K}. \tag{25}$$

For the case of $s_k = 1$, $k > 0$, expression (25) converges to form $B(V,T) = D_k/((k+1)(2L-V+2+k))$, and when the equalities $V = 1$, $L = 1$, $s_r = 1$ are met, based on expression (25), we denote it as $B(V=1,T) = D_k/2$. Therefore, the volume of communication resources that the transmission protocol is capable of allocating to support a virtual connection with characteristics $V \geq 2L$ or $V < 2L$ can be estimated by expressions (23) or (24), respectively. It should be noted that the distribution of communication resources occurs on a competitive basis among virtual connections, among which we allocate the target one.

3. Results and Discussion

Let us move on to the stage of numerical modeling, which is aimed at demonstrating the functionality of the proposed mathematical framework. When conducting experiments, it is important not only to demonstrate the capabilities of our development but also to justify its adequacy. In this context, it is productive to implement the authors' concepts in the framework of a well-established, verified software environment. Guided by the specifics of our research, we have chosen MATLAB 9.11/Simulink as the basis for conducting experiments. This environment provides an impressive toolkit for the efficient modeling and optimization of complex dynamic systems. As a source of data, we have chosen another popular integrated development environment for automation systems, namely TIA Portal.

There are known examples [23,24] of the connection between MATLAB/Simulink and TIA Portal through the Open Platform Communications (OPC) server, in which these environments exchange data packets at a maximum frequency of 10 packets per second. This limitation determines the maximum sampling frequency. For modeling and operating fast-acting systems characterized by a wide bandwidth, it is necessary to control the sampling frequency over a significantly broader range. The speed of direct data transmission/reception through a controlled-transmission-protocol logical channel is tens of times higher than the speed of information exchange through the OPC server. However, for TCP communication with TIA Portal controllers acting as TCP clients, the MATLAB/Simulink model must operate in server mode. There are no ready-made solutions for such a configuration in MATLAB libraries, which necessitates custom development under these initial conditions:

- TIA Portal and Simulink environments are connected by a controlled TCP data-transmission channel;
- the Simulink model exchanges data with TIA Portal clients through a TCP server developed using MATLAB tools;
- the server exchanges data at speeds below the bandwidth of the TCP channel;
- TIA Portal ensures the single launch of the corresponding software module (client) each time it receives target data from the Simulink server.

For collaborative modeling, the TIA Portal environment controller (or its model) must receive data about the state of the Simulink model at each modeling step and return control inputs to the models. The time for the collaborative modeling directly depends on the bandwidth of the data-transmission channel. This is precisely the parameter that the entire mathematical apparatus presented in Section 2 was oriented towards controlling.

Testing the bandwidth of the TCP channel with the structure "TIA Portal (client)"—"virtual Siemens Ethernet adapter"—"MATLAB (server)" demonstrated the following results:

- The frequency of one-way byte-by-byte data transmission using MATLAB 9.11 *fwrite* commands is 3000 bps;
- The frequency of byte exchange (with *fwrite* and *fread* commands used at each step) is approximately 300 transfers per second;
- When exchanging data in Double (LReal) format, where each number consists of 8 bytes and *fwrite* and *fread* commands are used, the frequency practically does not differ from the byte exchange frequency.

Thus, the use of the TCP channel allows TIA Portal and MATLAB to collaborate at a maximum sampling frequency of about 300 Hz.

To check the sequence of implementation and the quality of collaborative modeling, two control loops were constructed using identical unstable second-order analog objects with identical discrete PID controllers. The first "etalon" loop is implemented solely using Simulink. The controller in the second loop is built on the TIA Portal PLC platform, while the controlled object is simulated using Simulink. The discrepancy signal reception and the transmission of the control action computed by the PLC are carried out through a TCP channel. The corresponding Simulink model is presented below in Figure 1.

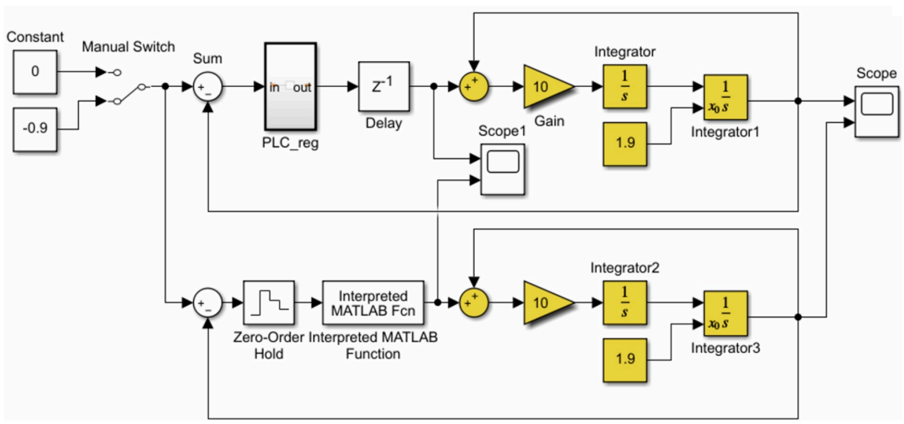

Figure 1. Simulink model of the control loop for an analog unstable second-order object (highlighted in yellow).

It should be noted that Figure 1 mentions two similar control loops. The upper loop (reference) is implemented solely using Simulink. The lower loop yields results when TIA Portal PLC and Simulink collaborate. The aforementioned TCP server is clocked by the "Interpreted MATLAB Function" block.

We took into account that when establishing a TCP connection, the server must be connected first (by switching to the "open" mode) and then the client. If the client switches to the "closed" state while the virtual connection is active, the connection status changes from "ESTABLISHED" to "CLOSE_WAIT" after some time. Upon subsequent attempts to return the client to the "open" mode, MATLAB displays an error message "Unsuccessful open: Connection refused: connect". MATLAB issues the same error message when attempting to connect the client when the server is absent. In such cases, to restore the TCP connection, it is necessary to disconnect both the server and the client (switch them to the "closed" states) and then repeat the connection establishment procedure.

To test TIA Portal network devices, the following MATLAB commands were used:

- To print a list of active real and virtual Ethernet network adapters and their IP addresses in the Command Window: >> *!ipconfig*;
- To determine the local address of the end device: >> *[name,address] = resolvehost("localhost")*;
- To obtain a list of active TCP ports: >> *!netstat -a -n -p TCP*.

We used the designed software construct consisting of “TIA Portal clients”—“Ethernet”–“MATLAB server” to conduct experiments on controlling the transmission-protocol resources of the target virtual connection in a loaded communication channel. The analytical dependencies provided below were obtained using the capabilities of the created software construct, the bandwidth control block of which implements the analytical dependencies presented in Section 2.3.

Figure 2 shows the dependence of bandwidth B on the number of hops L made by the information message (the feedback signal) on the route between end devices. The steady parameters for this study were the transmission-protocol window size $V = 16$, the period $T = 35$ (the duration of waiting for confirmation of successful data transfer), the probability function $s_1 = 1$, and the dependability of spreading the information packet $D_k = 1$, $k = \overline{1,15}$. The trajectories of dependence $B = f(L)$ presented in Figure 2 were formed by varying the dependability of spreading the feedback signal D_r in the range of $\{0.3, 0.5, 0.8, 1\}$ values. Precise definitions regarding the mentioned parameters can be found in Section 2.1.

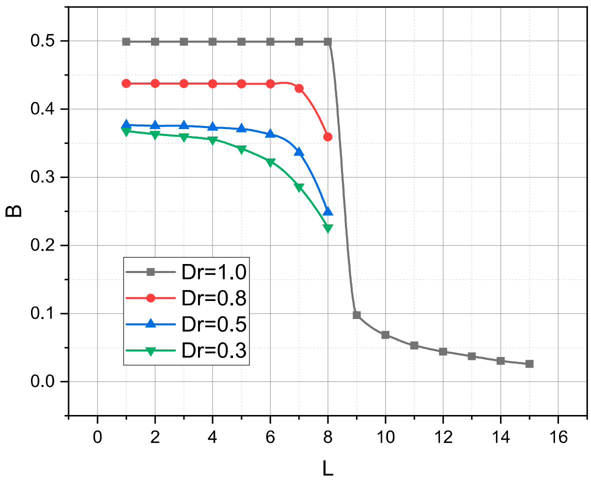

Figure 2. Calculated trajectories of dependence $B = f(L, D_r)$.

From the results presented in Figure 2, it can be seen that the bandwidth of the investigated virtual connection when $V \geq 2L$ practically does not depend on the number of transit nodes on the route of spreading the information message. At the same time, when $V < 2L$ is considered, the latter parameter becomes critically important, especially in combination with the parameter D_r. In particular, when $V < 2L$, $D_r < 1$ are considered, the situation of distortion of the feedback signal becomes deterministic, leading to the overflow of the stack of sent but unconfirmed data fragments on the client’s side ($B = f(L, D_r < 1)$ trajectories).

The fact that there was a significant decrease in the bandwidth of the investigated virtual connection under conditions where $V < 2L$ (see Figure 2) became the basis for further research. Figure 3 shows the trajectories of the dependence $B = f(V, D_r)$ for the steady parameters $T = 30$, $L = 1$, $s_k = 1$, $D_k = 1$, $k = \overline{1,5}$. In contrast, Figure 4 presents the trajectories of the dependence $B = f(V, D_r)$ for the steady parameters $T = 30$, $L = 2$, $s_k = 1$, $D_k = 1$, $k = \overline{1,5}$.

From the results presented in Figures 3 and 4, it can be observed that once the condition $V \geq 2L$ starts to be satisfied, the target virtual-connection bandwidth is determined not by the window size V but by the dependability of spreading the feedback signal D_r. Specifically, the increase in the value of this parameter becomes significant under conditions of increasing L. It is worth noting that the value of the parameter D_r directly determines the queue occupancy both at the client and server sides, as well as at the transit nodes of the route for spreading the information message. Thus, the hypothesis underlying our research that the bandwidth of the target virtual connection in a congested communication channel depends on the stack occupancy, including that of its transit elements, has received empirical confirmation. A visual confirmation of this fact is the displacement along the abscissa axis (while maintaining the character) of the trajectories presented in Figure 4 relative to the position of the trajectories presented in Figure 3, where the value L increases from 1 to 2, respectively. Overall, it can be stated that when $V < 2L$ holds, the bandwidth for the target virtual connection is underutilized; hence, the effective speed of information-message transfer is significantly reduced.

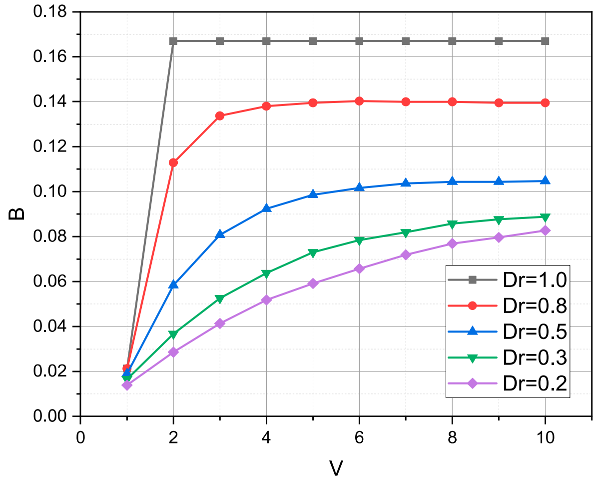

Figure 3. Calculated trajectories of dependence $B = f(V, D_r, L = 1)$.

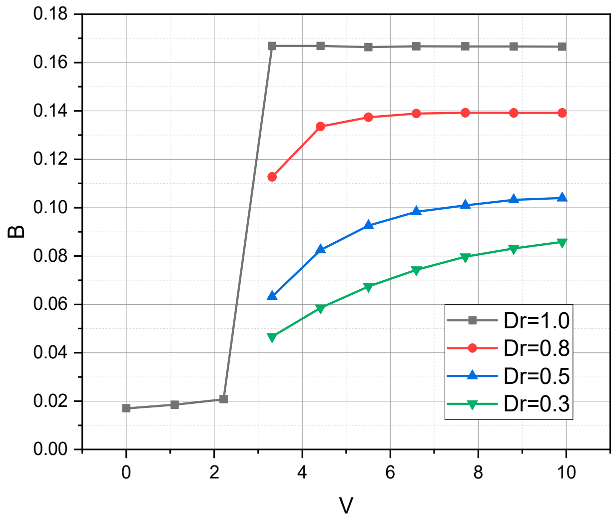

Figure 4. Calculated trajectories of dependence $B = f(V, D_r, L = 2)$.

By definition, the probability function s_k, $k = \overline{0, K}$, characterizes the event when each fragment of the information message, propagated by the investigated logical channel, encounters a queue of k data-fragment lengths at a transit node along its route, where K is the transit-node stack-buffer capacity. Let us explore how the bandwidth B depends on the occupancy of the stack buffers of transit nodes, represented by the value of the parameter K. Figure 5 presents the trajectories of dependence $B = f(K, D_r)$ calculated for the following steady parameter values: $T = 30$, $L = 4$, $V = 1$, $s_K = 1$, $D_k = 1$, $k = \overline{1, K}$.

From the results presented in Figure 5, it can be observed that reducing the dependability D_r value shifts the trajectories of dependence $B = f(K, D_r)$ downwards along the ordinate scale but does not affect the shape of these graphs. As the competition among clients for communication resources increases, leading to the growth of queues in the stacks of communication-channel elements, the speed of information-message transfer decreases exponentially. Such a pronounced dependence between the values of parameters B and

K allows us to recognize the capacity of the stacks of communication-channel elements as critical parameters that largely determine the actual bandwidth of the target virtual connection implemented under competitive conditions.

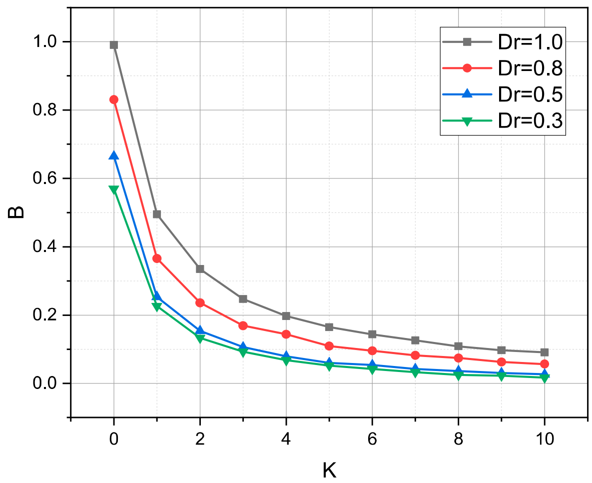

Figure 5. Calculated trajectories of dependence $B = f(K, D_r)$.

Finally, we conclude the series of experiments by presenting the graphs of dependence $B = f(D_r, V)$ calculated for the following steady parameter values: $T = 30$, $L = 4$, $s_k = 1$, $D_k = 1$, $k = \overline{1,5}$.

From the data presented in Figure 6, it is evident that increasing the window size V positively affects the bandwidth B values of the target virtual connection across the entire range of the abscissa. The dependability of spreading the information message and the level of effective information-transfer speed demonstrate a linear dependency.

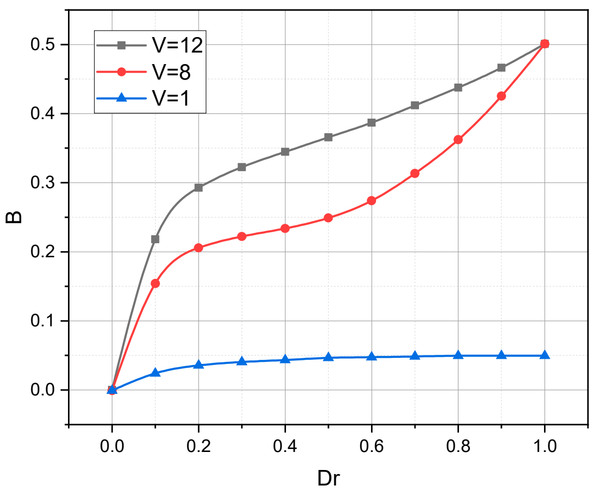

Figure 6. Calculated trajectories of dependence $B = f(D_r, V)$.

Summarizing the results presented in this section, it should be noted that with an unlimited window size V and, therefore, with an unlimited period T, the portion of the communication channel's throughput capacity available to the target virtual connection (2) is analytically characterized by the function $B(n > 0, k > 0) = \frac{D_k\left(1+(k+1)D_r^2\right)}{(k+1)(1+(k+1)D_r)}$. Consequently, when $D_r = 0$ and $D_r = 1$ are applied, we obtain identical values for parameter B, i.e., $B(n > 0, k > 0) = D_k/(k+1)$, and at point $D_r = \left(\sqrt{k+1} - 1\right)/(k+1)$, the bandwidth availability index for the target virtual connection reaches its maximum: $\text{MAX}(B) = D_k\left(2\sqrt{k+2} - 2\right)\Big/(k+1)^2, k > 0$.

4. Conclusions and Future Work

The share of active IoT devices in the infosphere is constantly growing. According to forecasts, by 2030, 75% of all devices will be IoT. In this context, improving the efficiency of communication between IoT devices remains a constantly relevant scientific problem. At the same time, the most important aspect is the optimal control of the parameters of the transmission protocol used for the connection of IoT devices to the internet.

First and foremost, it is worth noting that the use of TCP in IoT device operations provides several advantages:

- Data-transmission reliability: TCP ensures the delivery of data in order and without loss. This is crucial for IoT devices, particularly when transmitting critical information such as equipment status or security;
- Control of information flows: TCP provides a flow control mechanism, allowing for the management of data-transmission speed based on the devices' ability to receive and process these data. This helps avoid network congestion and reduces the likelihood of data loss;
- Connection management: TCP offers mechanisms for establishing and terminating connections between devices, ensuring more reliable and controllable communication. This is especially important in unstable network conditions or when devices are periodically connecting and disconnecting;
- Error-handling terminal: TCP provides mechanisms for detecting and retransmitting lost or corrupted data packets, enhancing transmission reliability in noisy or unreliable environments.

Thus, it is TCP that the authors position as the most suitable transport-layer protocol for facilitating interactions between the cloud and the sensor network for critical infrastructure in a smart city.

The mathematical framework presented in the article is focused on the controlled-transmission protocol's asynchronous process of allocating bandwidth to the target virtual connection under conditions of competition for communication resources. The investigated process is formalized as a two-dimensional discrete Markovian chain, taking into account the distributions of queue lengths of TCP data fragments from competing client nodes. This chain describes the dynamics of filling the stack of transmitted but unacknowledged data fragments of the investigated end device. The research results include findings of state distributions of the Markovian chain for various ratios of the target virtual-connection bandwidth, transmission-protocol parameters, and characteristics of the communication channel (including window size, number of hops in the route, waiting period for feedback signals, probabilities of data-fragment distortion in transit nodes, and queue lengths in the buffers of the latter). Analytical dependencies for computing the performance of the target virtual connection for different operating modes have been derived.

The results of experiments, conducted based on the obtained analytical constructions, have shown that the performance of the virtual connection with a selective repeat mode primarily depends on the queue size distribution in transit nodes, the data-loss intensity, and the relationship between the protocol window size and the route length. Based on the empirical results, it can be argued that with a given window size, the target virtual-connection bandwidth increases with an increase in the acknowledgement period and reaches the theoretical threshold when saturated by the TCP parameter V for values T exceeding the protocol window size by three to five round-trip delays d.

In this article, the authors focused on evaluating the virtual-connection performance implemented in a competitive environment with a selective repeat mode of retransmission. An obvious direction for **further research** is the adaptation of the obtained mathematical framework for the case of a group retransmission mode. In doing so, the authors intend to expand upon the results obtained in the works described in [25,26].

Furthermore, drawing on the thoughtful recommendations of one of our esteemed reviewers, in planning future research, we intend to consider aspects such as:

- Parameter variation: experimenting with a wider range of parameters such as queue sizes, data-loss rates, acknowledgement periods, and protocol window sizes.
- Scenario diversification: expanding the experimental scenarios to encompass a broader range of network conditions and configurations.
- Additional metrics: including additional performance metrics beyond bandwidth, such as latency, jitter, and packet-loss rate.
- Sensitivity analysis: conducting sensitivity analysis to assess the sensitivity of virtual-connection performance to changes in key parameters.
- Real-world testing: whenever feasible, complementing laboratory experiments with real-world testing in live network environments.

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