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Simulation Analysis of Phase Jitter in Differential Sampling of AC Waveforms Based on the Programmable Josephson Voltage Standard

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Abstract: The effect of phase jitter on differential sampling using the programmable Josephson voltage standard (PJVS) system is studied in this paper. A phase jitter model is established for the measured signal, and compensation coefficients for phase jitter removal are derived for three different post-processing methods based on the discrete Fourier transform algorithm (DFT). Based on our analysis, the phase jitter compensation coefficients are determined by the phase jitter angle distribution and harmonic order. Furthermore, after analyzing and simulating various common distributions, the phase jitter compensation coefficients have been verified. The simulation shows that when the standard deviation of the phase jitter angle is 20 ns, and the frequency of the measuring waveform is 3.46 kHz, the influence of the phase jitter is 1×10^{-7} . The results of the simulation indicate that, in the differential sampling of AC waveforms using a PJVS system, phase jitter is one of the error terms for an uncertainty budget that cannot be neglected, particularly as the frequency of the measured waveforms increases.

Keywords: phase jitter; differential sampling; precision measurement; compensation coefficient



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1. Introduction

With rapid advancements in Josephson quantum voltage and its quantization benefits, quantum standards are gradually replacing traditional physical standards in the field of electrical metrology [1–3]. Josephson effect-based quantum voltage standards have been widely developed and applied. In recent years, the differential sampling of AC waveforms based on PJVS have become popular [4]. The differential sampling method based on PJVS was first introduced to measure AC voltage by the Physikalisch-Technische Bundesanstalt (PTB) in 2007 [5]. The PJVS system synthesized the quantized step of the stepwise approximated waveforms as a reference voltage; the differential signal between the signal being measured and its reference voltage was measured by a sampler. After data post-processing, the waveform information for the measured signal can be obtained. This method reduces the gain and reference error of the sampler, and the uncertainty is one order of magnitude lower than that of direct sampling [5,6]. The National Institute of Standards and Technology (NIST) experimentally verified the feasibility of AC voltage measurement by the differential sampling method based on PJVS for the first time with two sets of PJVS systems in 2008 [7]. Over the past decade, several national metrology institutions (NMIs) such as PTB [5,8], NIST [7,9,10], Korea Research Institute of Standard and Science (KRISS) [11–13], National Institute of Advanced Industrial Science and Technology (AIST) [14,15] and the National Institute of Metrology (NIM) [16] have conducted research on differential sampling systems based on PJVS to measure AC voltages. The process of differential sampling based on PJVS

for AC waveform measurement is shown in Figure 1. Commonly used samplers by NMIs include Keysight 3458A [7,9–13,15,17], NI PXI cards 5922 [8,12,13,18,19] and 4461 [18,19], and Fluke 8588A [13]. Various countries have different post-processing algorithms to meet different application requirements [20]. This paper focuses on analyzing the post-processing algorithm based on the DFT algorithm. These systems are being developed as a replacement for traditional methods for AC voltage measurement, such as thermal converters (TCs) [17,21,22]. It is worth investigating the factors that influence differential sampling systems based on PJVS to minimize the impact on measurement uncertainty.

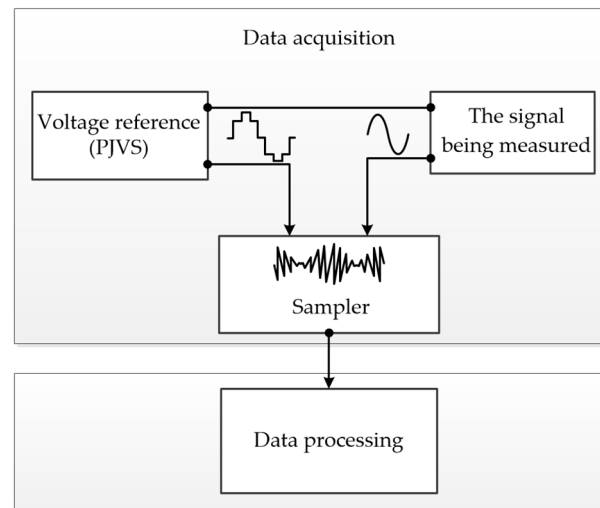


Figure 1. Flow chart of differential sampling of AC waveforms based on PJVS.

Phase jitter is a crucial factor that affects the accuracy of the differential sampling system based on PJVS. In light of previous research on the phase jitter of factors affecting the difference sampling system based on PJVS [11], the following issues have been observed:

- (1) Most studies rely solely on simulations or lack sufficient theoretical analyses, let alone theoretical derivation of compensation coefficient, making it difficult to evaluate type B measurement uncertainty;
- (2) Assume that phase jitter remains constant over a measurement cycle;
- (3) When using Taylor expansion as a linear hypothesis, only the first order can be analyzed; thus, only type A measurement uncertainty can be analyzed;
- (4) Assume that phase jitter follows uniform or specific distributions.

In this paper, the influence of phase jitter with an arbitrary distribution on the differential sampling system based on PJVS is modeled. The paper also summarizes three post-processing methods that are used in different NMIs and analyzes the phase jitter with these methods. A model for phase jitter compensation is established, and the simulation results are validated with three common distributions. The results show consistency with the existing research.

2. Phase Jitter Model

This section presents a theoretical analysis of phase jitter in the differential sampling system based on PJVS, and the phase jitter model is established. Additionally, the effect of phase jitter on different data post-processing methods used by different NMIs is analyzed, and the compensation coefficient is provided.

Figure 1 shows that the sources of phase jitter in a differential sampling system based on PJVS mainly include voltage reference (PJVS), the waveform to be measured, and the sampler. As the differential sampling is performed on the quantum-flat portion of the PJVS, and all the trigger and clock signals of the differential sampling systems are provided by the same clock source via optical isolation, the phase jitter from the voltage reference can

be negligible. The actual sampling value of the general signal caused by the sampling jitter of the sampler can be expressed as follows:

$$v_{ln} = x(t_n + \Delta t_{s_ln}), \quad (1)$$

where $n = 0, 1, \dots, N - 1$, N is the number of PJVS steps per period, $l = 0, 1, \dots, L - 1$, L is the number of measured periods, and Δt_{s_ln} is the phase jitter angle caused by sampling jitter from the sampler at the n th step of the l th period. The phase jitter between different samplings is independent and identically distributed. In contrast to other studies [11], the signal to be measured is not simplified to a sine wave in this paper. However, sinusoidal signals are used to illustrate the sampling process for clarity in the graphics. Similarly, the actual sampling value due to phase jitter from the general waveform to be measured can be expressed as follows:

$$v_{ln} = x(t_n + \Delta t_{w_l}), \quad (2)$$

where Δt_{w_l} is the phase jitter angle caused by the waveform to be measured at the l th period. To sum up, the actual sampling voltage value resulting from phase jitter in each part of the differential sampling system can be expressed as follows:

$$v_{ln} = x(t_n + \Delta t_{s_ln} + \Delta t_{w_l}). \quad (3)$$

After further simplification, Equation (3) can be expressed as follows:

$$v_{ln} = x(t_n + \Delta t_{ln}), \quad (4)$$

where Δt_{ln} is the phase jitter angle at the n th step of the l th period.

In regard to the post-processing of the sampling value, different NMIs have proposed various methods. The methods can be mainly divided into the following. One method is to obtain the effective voltage by directly adding the square sum of the sampling values obtained by each step and then taking the square root. For example, this post-processing method can be adopted when the AC waveform is measured by differential sampling using a high-speed sampler such as NI 5922 [8]. The other method is using the DFT algorithm to extract waveform information (effective value, phase, harmonics, etc.). The DFT algorithm is usually used in the differential sampling system with an integrating sampler (such as Keysight 3458A) for post-processing. The integrating sampler generates an integrated sampling value by integrating each step for a period of time. This process is equivalent to a “rectangular window” operation for each step. Therefore, it is necessary to calculate the effective value after compensating for each harmonic in the frequency domain. Hence, the first algorithm cannot be used in this case. If the DFT algorithm is used to calculate the waveform information, it can be divided into the following:

- (1) A discrete Fourier transform algorithm is applied first to get the information on fundamentals and harmonics, and then an average is calculated to get the RMS of the waveform to be measured [15]; this post-processing method is called “DFT-Average” in this paper;
- (2) An average is calculated first to receive one voltage per step of the waveform to be measured, and then the DFT algorithm is applied to get the information of fundamentals and harmonics of the waveform to be measured [16]; this post-processing method is called “Average-DFT” in this paper;
- (3) Due to the limitation of the sampling principle of samplers such as integral samplers, an integrated sampling is used first to obtain one voltage per step of the waveform to be measured, and then the DFT algorithm is applied to get the RMS of the waveform to be measured [9,10,12]; this post-processing method is called “Integrate-DFT” in this paper.

The impact of phase jitter on PJVS systems with different data post-processing methods is analyzed as follows.

2.1. Impact of Phase Jitter on the “DFT-Average” Method

The principle of the “DFT-Average” is shown in Figure 2, which can be described as follows. The number of sampling points that are used for processing at each step is M with transients deleted. The sampling points are represented by different-colored arrows at each step of the waveform in Figure 2. There are N steps per period, and a total of L periods are sampled. In order to simplify this, six samples of a PJVS reference waveform are chosen in this paper (which means $N = 6$, $L = 1$ in this example). After adding each measured difference to the corresponding PJVS step voltages (v_{nm}), a DFT algorithm is applied to the sampled values at the same position of different steps. Then, a total of M results of RMS for the AC waveform to be measured can be obtained in each period. Finally, all results are averaged to obtain the final results of RMS for the AC waveform to be measured. For one measurement period, the voltage of the fundamental and each harmonic obtained by the “DFT-Average” method can be expressed as follows:

$$\begin{aligned} V_k &= \frac{1}{M} \sum_{m=0}^{M-1} DFT_k(v_{nm}) \\ &= \frac{1}{M} \sum_{m=0}^{M-1} \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}kn} x(t_n + \Delta t_{mn}), \end{aligned} \quad (5)$$

where k is harmonic order, $k = 0, 1, \dots$; when $k = 0$, it is the DC offset and $k = 1$ is the RMS value of the fundamental.

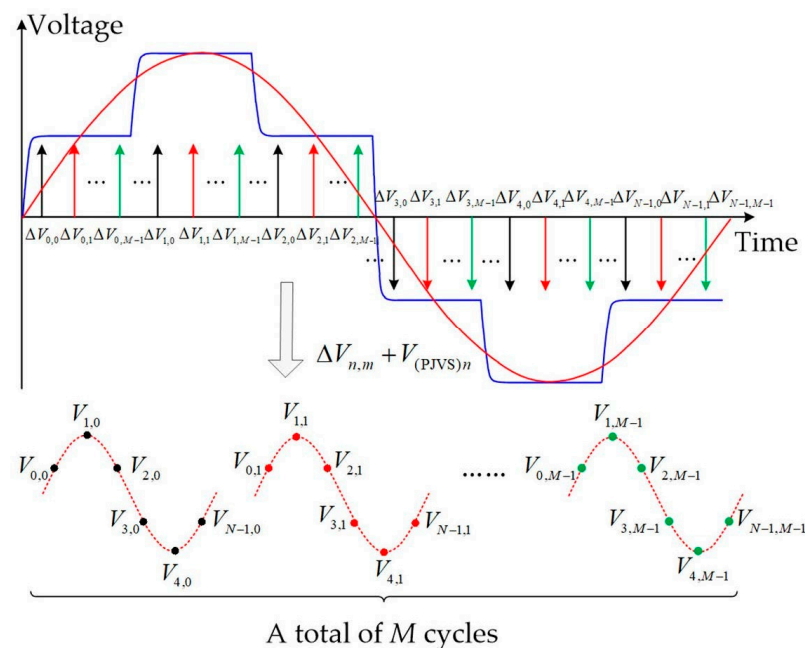


Figure 2. (Color online) Schematic of the “DFT-Average” method. (Blue) PJVS stepwise approximated waveforms. (Red) Waveform to be measured (sine wave as an example). $\Delta V_{n,m}$ represents the voltage difference of m th point on the n th step, where $n = 0, 1, \dots, N-1$, $m = 0, 1, \dots, M-1$; N is the step number of the waveform, and M is the number of sampling points that are used for processing at each step, which are represented by different-colored arrows at each step of the waveform. $V_{(PJVS)n}$ represents the n th step voltage of PJVS.

Taking the measured average value of L periods, Equation (5) can be derived as follows:

$$\bar{V}_k = \frac{1}{L} \sum_{l=0}^{L-1} \frac{1}{M} \sum_{m=0}^{M-1} \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}kn} x(t_n + \Delta t_{mn}), \quad (6)$$

where Δt_{mn} is a random variable at the m th point on the n th step with arbitrary distribution, and the distribution is expressed as $u(\Delta t)$. When L is large enough, (6) can be expressed as follows:

$$\begin{aligned}
 \overline{V}_k &= \frac{1}{M} \sum_{m=0}^{M-1} \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}kn} \int u(\Delta t) x(t_n + \Delta t_{mn}) d\Delta t \\
 &= \frac{1}{M} \sum_{m=0}^{M-1} DFT_k[u(t) \otimes x(t_n + \Delta t_{mn})] \\
 &= \frac{1}{M} \sum_{m=0}^{M-1} DFT_k[u(t)] \cdot DFT_k[x(t_n + \Delta t_{mn})] \\
 &= DFT_k[u(t)] \cdot \frac{1}{M} \sum_{m=0}^{M-1} DFT_k[x(t_n + \Delta t_{mn})].
 \end{aligned} \tag{7}$$

It can be concluded that the waveform to be measured with phase jitter is equivalent to the waveform to be measured without phase jitter multiplied by the compensation coefficient $DFT_k[u(t)]$. It is difficult to assess the impact of phase jitter on the outcome of a single application of the DFT algorithm. However, when multiple DFT operations are averaged, the effect of phase jitter on a waveform is equivalent to convolving the original waveform with the probability density function of the phase jitter in the time domain, which is also equivalent to directly multiplying the original waveform with the probability density function of the phase jitter in the frequency domain.

2.2. Impact of Phase Jitter on the “Average-DFT” Method

The process of the “Average-DFT” method is as follows. An average amplitude is obtained at each step by averaging M sampling points, which is then added to its corresponding PJVS step voltage. There are N average amplitudes obtained per period, which are represented by different-colored points at the reconstructed waveform. Finally, the DFT algorithm is applied to obtain the RMS of the AC waveform to be measured. The principle of the “Average-DFT” algorithm is shown in Figure 3.

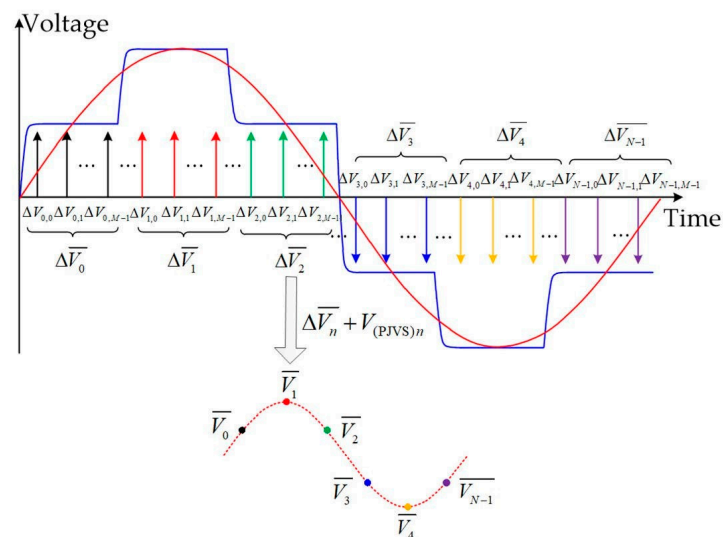


Figure 3. Schematic of the “Average-DFT” method. $\Delta \overline{V}_n$ is the average difference voltage of the n th step. $\overline{V}_n = \Delta \overline{V}_n + V_{(PJVS)_n}$ is the average voltage by adding $\Delta \overline{V}_n$ to the corresponding PJVS step voltages.

The voltage of the fundamental and each harmonic obtained by the “Average-DFT” method can be expressed as follows:

$$\begin{aligned}
 V_k &= DFT_k \left(\frac{1}{M} \sum_{m=0}^{M-1} v_{nm} \right) \\
 &= \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}kn} \frac{1}{M} \sum_{m=0}^{M-1} x(t_n + \Delta t_{mn}).
 \end{aligned} \tag{8}$$

After the number of L measurement periods, Equation (8) can be derived as follows:

$$\overline{V}_k = \frac{1}{L} \sum_{l=0}^{L-1} \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}kn} \frac{1}{M} \sum_{m=0}^{M-1} x(t_n + \Delta t_{mn}) \tag{9}$$

Generally, L is large enough. Equation (9) can be expressed as follows:

$$\begin{aligned}
 \overline{V}_k &= \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}kn} \int u(\Delta t) \frac{1}{M} \sum_{m=0}^{M-1} x(t_n + \Delta t_{mn}) d\Delta t \\
 &= DFT_k \left[u(t) \otimes \frac{1}{M} \sum_{m=0}^{M-1} x(t_n + \Delta t_{mn}) \right] \\
 &= DFT_k[u(t)] \cdot DFT_k \left[\frac{1}{M} \sum_{m=0}^{M-1} x(t_n + \Delta t_{mn}) \right]
 \end{aligned} \tag{10}$$

The conclusion is consistent with the “DFT-Average” method. According to the formula, the effect of phase jitter can be minimized by the compensation coefficient $DFT_k[u(t)]$.

2.3. Impact of Phase Jitter on the “Integrate-DFT” Method

As previously mentioned, the “Integrate-DFT” method is based on the working principle of an integrating sampler, for instance, Keysight 3458A. The principle of the “Integrate-DFT” algorithm is shown in Figure 4. After the delay time of the sampler (which is represented by t_d in Figure 4), each step of the waveform gets an integral value of the difference voltage. There are N points per period to carry out the DFT algorithm by adding each integral value of the difference voltage to the corresponding PJVS step voltages. Then, the reconstructed waveform of the original waveform can be obtained by the DFT algorithm. The differential voltage of each step obtained by the integral sampler can be expressed as follows:

$$v_n = \frac{1}{t_i} \int_{\frac{n}{Nf} + t_d}^{\frac{n}{Nf} + t_d + t_i} x(t_n + \Delta t_n) dt \tag{11}$$

where f is the frequency of the waveform, t_d is the delay time per step and t_i is the integral time per step, Δt_n is a random variable with the arbitrary distribution. Adding the difference voltage to the corresponding PJVS step voltage, the voltage is represented by V_n' . When the DFT algorithm is applied to V_n' , the amplitude of the fundamental and each harmonic can be expressed as follows:

$$V(k) = \sum_{n=0}^{N-1} V_n' \cdot e^{-j\frac{2\pi}{N}kn} \tag{12}$$

The derivation process is consistent with the “Average-DFT” method, and when L is large enough, (12) can be derived as follows:

$$\begin{aligned}
 \overline{V}_k &= \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}kn} \int_{\frac{n}{Nf} + t_d}^{\frac{n}{Nf} + t_d + t_i} u(\Delta t) \frac{1}{T_s} \int_{\frac{n}{Nf} + t_d}^{\frac{n}{Nf} + t_d + t_i} x(t_n + \Delta t_n) dt_n d\Delta t \\
 &= DFT_k \left[u(t) \otimes \frac{1}{T_s} \int_{\frac{n}{Nf} + t_d}^{\frac{n}{Nf} + t_d + t_i} x(t_n + \Delta t_n) dt \right] \\
 &= DFT_k[u(t)] DFT_k \left[\frac{1}{T_s} \int_{\frac{n}{Nf} + t_d}^{\frac{n}{Nf} + t_d + t_i} x(t_n + \Delta t_n) dt \right].
 \end{aligned} \tag{13}$$

Based on the derivation presented above, it can be concluded that the effect of phase jitter on the final measurement results remains consistent with the three post-processing methods. In all cases, the results are expressed as the DFT operation results of the phase jitter distribution. Therefore, the compensation coefficients of the phase jitter can be calculated as follows:

$$C_k = DFT_k[u(t)], \quad (14)$$

which is determined by the distribution of the phase jitter angle and the harmonic order.

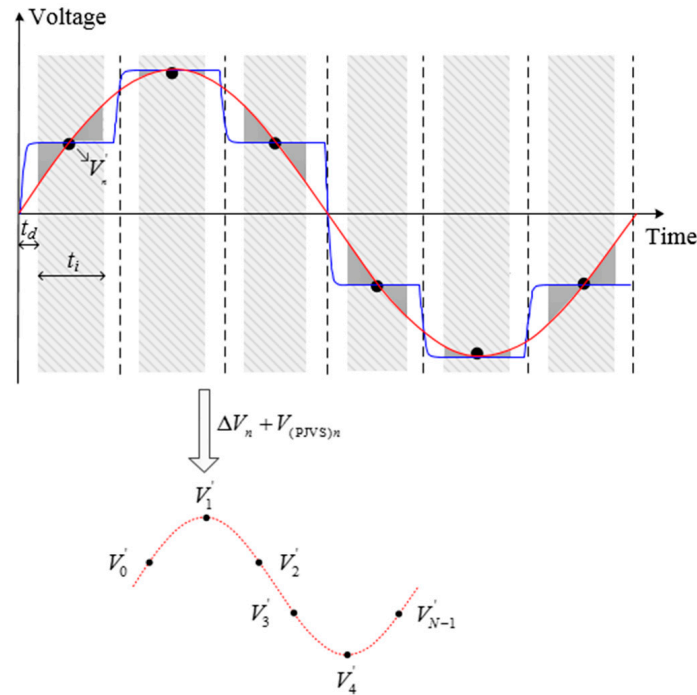


Figure 4. Schematic of the “Integrate-DFT”. The gray rectangular boxes are sampling windows.

3. Simulation of the Phase Jitter

3.1. Compensation Coefficients of Phase Jitter following Some Common Distributions

It is derived that the compensation coefficients for phase jitter have a relationship with the distribution of the phase jitter angle in Section 2. To examine the impact of different phase jitter distributions on the final measurement results, this section simulates and verifies compensation coefficients for several common distributions.

According to (14), if the phase jitter follows a normal distribution with a mean of 0 and a standard deviation of σ , the compensation coefficient can be derived as follows:

$$C_{k,\text{norm}} = e^{-\pi s^2}, \quad s = \frac{\sigma}{T_k} \sqrt{2\pi}, \quad (15)$$

where T_k is the period of the harmonic wave. If the phase jitter follows a triangular distribution with a mean of 0 and a period of τ_{tri} , the compensation coefficient can be derived as follows:

$$C_{k,\text{tri}} = \sin^2(\pi s), \quad s = \frac{\tau_{tri}}{T_k}, \quad (16)$$

where $\tau_{tri} = \sqrt{6}\sigma$, σ is the standard deviation of phase jitter following a triangulate distribution. Equation (16) can be derived as follows:

$$C_{k,\text{tri}} = \sin^2\left(\sqrt{6}\pi \frac{\sigma}{T_k}\right). \quad (17)$$

If the phase jitter follows a rectangular distribution with a mean of 0 and a period of $\tau_{rect.}$, the compensation coefficient can be derived as follows:

$$C_{k,rect} = \text{sinc}(\pi s), \quad s = \frac{\tau_{rect.}}{T_k}, \quad (18)$$

where $\tau_{rect.} = 2\sqrt{3}\sigma$, σ is the standard deviation of the phase jitter following a rectangle distribution. Thus, (18) can be derived as follows:

$$C_{k,rect} = \text{sinc}\left(2\sqrt{3}\pi \frac{\sigma}{T_k}\right). \quad (19)$$

The compensation coefficients of phase jitter follow the three distributions summarized in Table 1. As shown in the table, the coefficient is determined by the ratio of the standard deviation and the harmonic period. This ratio (σ/T_k) is named the normalized standard deviation in the following discussion.

Table 1. Compensation coefficients of different distributions.

Distribution	Compensation Coefficients	$p = \frac{\sigma}{T_k}$
Normal	$\exp\left[-2\pi^2\left(\frac{\sigma}{T_k}\right)^2\right]$	$\exp[-2\pi^2 p^2]$
Triangulate	$\text{sinc}^2\left(\sqrt{6}\pi \frac{\sigma}{T_k}\right)$	$\text{sinc}^2(\sqrt{6}\pi p)$
Rectangular	$\text{sinc}\left(2\sqrt{3}\pi \frac{\sigma}{T_k}\right)$	$\text{sinc}(2\sqrt{3}\pi p)$

3.2. Simulation for Phase Jitter following Different Distributions

According to Table 1, the relationship between the compensation coefficients of the three different distributions and their relationship with the normalized standard deviation is depicted in Figure 5. The difference between the compensation coefficients of the phase jitter of the three different distributions is less than 1 part in 10^7 when the normalized standard deviation is less than 0.005. For example, when the phase jitter standard deviation is less than 5 μs , even for 1 kHz waveforms to be measured, the difference in the compensation coefficients of the three distributions is less than 1 part in 10^7 . Therefore, the compensation coefficient can be calculated using only the standard deviation of phase jitter without estimating its distribution in the differential sampling system when the normalized standard deviation is less than 0.005.

In order to verify the compensation coefficient in Table 1, a simulation is compared with [11]. In paper [11], researchers analyzed a 50 Hz waveform with a peak voltage of 1 V, containing a random phase jitter that followed a Gaussian distribution. The step number used in the simulation of the phase jitter is 100 ($N = 100$). As shown in Figure 6, the results of the theoretical calculation are consistent with the simulation results in [11]. It is assumed, in [11], that phase jitter existed only at the initial sampling point, and the phase changes of subsequent sampling points were consistent with the first point. This simplification significantly improved the convergence speed of the simulation but failed to accurately reflect phase jitter in actual measurements. Considering the experiment in practice, where the phase jitter of each sampling point is independent, 1,000,000 traces of simulation are used for the simulation results to converge in our experiment.

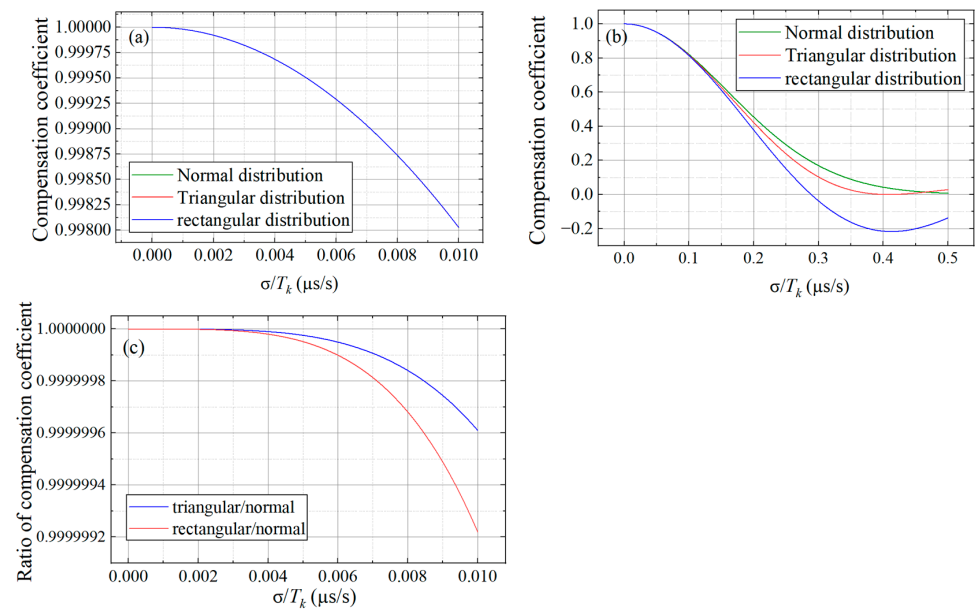


Figure 5. (a) Compensation coefficients with the normalized standard deviation of phase jitter noise smaller than 0.01. The curve trends of the triangular and normal distributions are overlapped by the curve trends of the rectangular distribution. (b) Compensation coefficients with the normalized standard deviation of phase jitter noise smaller than 0.5. (c) Difference of the compensation coefficients between different distributions.

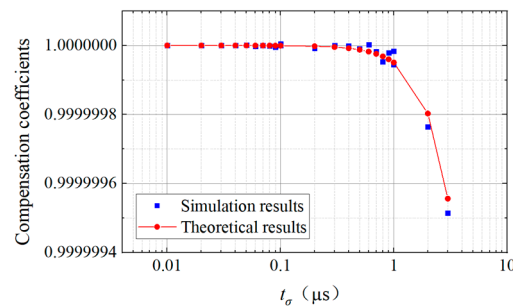


Figure 6. Comparison between the theoretical and simulation results of this paper.

3.3. Simulation of the Effect of Phase Jitter with Different Waveform Frequencies

As discussed in paper [11], even if the standard deviation of phase jitter angle is 500 ns and the frequency of the waveform is 50 Hz, its impact on measurement results after the DFT algorithm can be ignored. In order to verify the relationship between phase jitter and frequency under this condition, the influence of phase jitter at different waveform frequencies is discussed in this section. The compensation coefficient variation with different waveform frequencies ranging from 50 Hz to 5 kHz is shown in Figure 7a, while the standard deviation of phase jitter is fixed at 500 ns. As discussed in Section 3.2, the difference in compensation coefficients of the three distributions can be ignored. Therefore, we chose the most common normal distribution of noise for simulation in the experiment. It can be observed that when measuring a low frequency waveform of 50 Hz, the impact of phase jitter can be negligible (with an order of 10^{-8}). Therefore, there is no requirement for compensation when the waveform frequency is low. However, the impact of phase jitter becomes more significant as the frequency to be measured increases. The impact of phase jitter reaches 5 parts in 10^6 when the waveform frequency is 1 kHz. The compensation coefficient in Table 1 must be used to compensate for the final result as the waveform frequency increases. Since the standard deviation of phase jitter angle with 500 ns is too large for the PJVS-based differential sampling system, the paper [11] focused on phase jitter of less than 80 ns. Figure 7b–d demonstrate the effect of phase jitter on different

waveform frequencies while keeping the standard deviation of phase jitter angle fixed at 80 ns, 50 ns and 20 ns, respectively. The compensation factor of 0.9999999 is shown in the figure by the red horizontal dotted line. This means that if the compensation factor is higher than this value, then the impact of phase jitter on the measurement of the system will be more than 1×10^{-7} . The red vertical dashed line indicates the waveform frequency at this compensation factor. The results of the simulation show that if the standard deviation of phase jitter angle is 80 ns and the frequency is higher than 890 Hz, the impact of phase jitter will be greater than 1×10^{-7} . Similarly, if the standard deviation of phase jitter angle is 50 ns and the frequency is above 1.4 kHz, the effect of phase jitter will be greater than 1×10^{-7} . Finally, if the standard deviation of phase jitter angle is 20 ns and the frequency is above 3.46 kHz, the impact of phase jitter will be greater than 1×10^{-7} . Therefore, it is essential to compensate for the phase jitter when the waveform frequency increases in a PJVS-based differential sampling system for the jitter values that we have used.

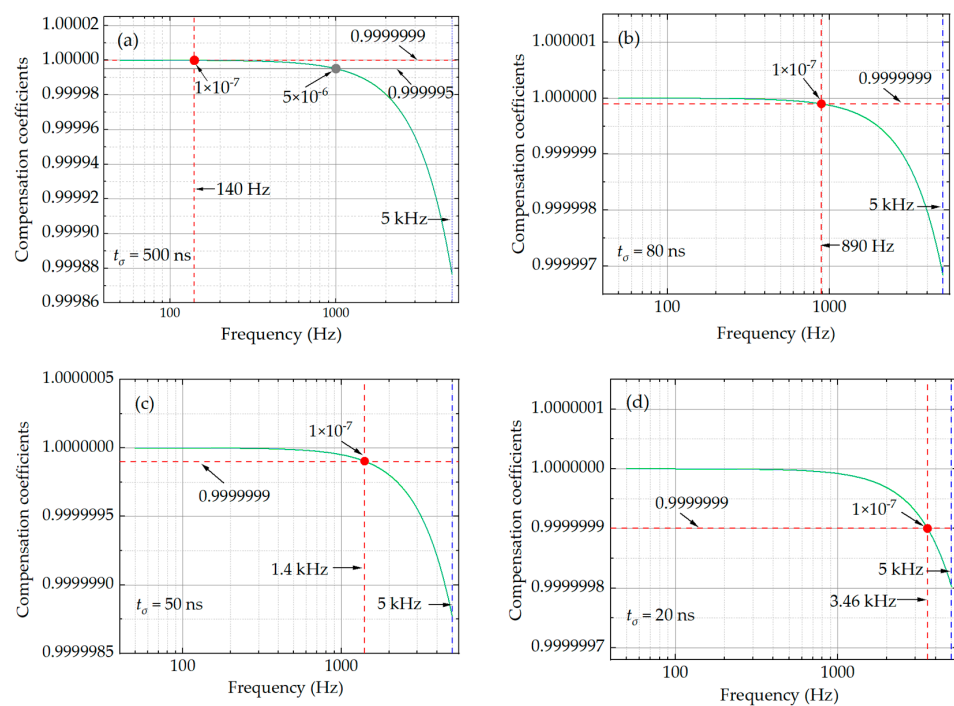


Figure 7. The influence of phase jitter with different waveform frequencies. The waveform frequencies range from 50 Hz to 5 kHz, where the standard deviation of phase jitter is (a) $t_\sigma = 500$ ns, (b) $t_\sigma = 80$ ns, (c) $t_\sigma = 50$ ns, (d) $t_\sigma = 20$ ns. The red horizontal dashed line indicates a compensation factor of 0.9999999, the red vertical dashed line indicates the waveform frequency under this compensation factor, and the blue vertical dashed line indicates a frequency of 5 kHz.

3.4. Simulation of the Effect of Phase Jitter with Frequencies at 50 Hz, 1 kHz and 3.125 kHz

In order to further study the influence of phase jitter on differential sampling systems based on PJVS, this section simulates the influence of the standard deviation of the phase jitter angle ranging from 0 to 500 ns at frequencies commonly used in previous studies, including 50 Hz, 1 kHz, and 3.125 kHz. The simulation results are shown in Figure 8. As shown in Figure 8a, even if the standard deviation of the phase jitter angle is 500 ns, it has a negligible effect on waveforms with a frequency of 50 Hz, which is consistent with previous studies. When the frequency of the waveform is 1 kHz, as shown in Figure 8b, and the standard deviation of the phase jitter angle is more than 72 ns, it has an impact on the PJVS-based differential sampling system of more than 1×10^{-7} . Moreover, if the standard deviation of the phase jitter angle is 226 ns, the influence will increase to 1×10^{-6} . The simulation of the effect of phase jitter in the 0–500 ns range at 3.125 kHz waveform frequency is shown in Figure 8c. The simulation results show that when the

phase jitter is larger than 23 ns, the influence of phase jitter on a PJVS-based differential sampling system is greater than 1×10^{-7} , and when the phase jitter is 73 ns, the influence expands to 1×10^{-6} . The results further indicate that the research presented in this paper has important guiding significance for the differential sampling system based on PJVS, especially when the frequency of the measured waveforms increases.

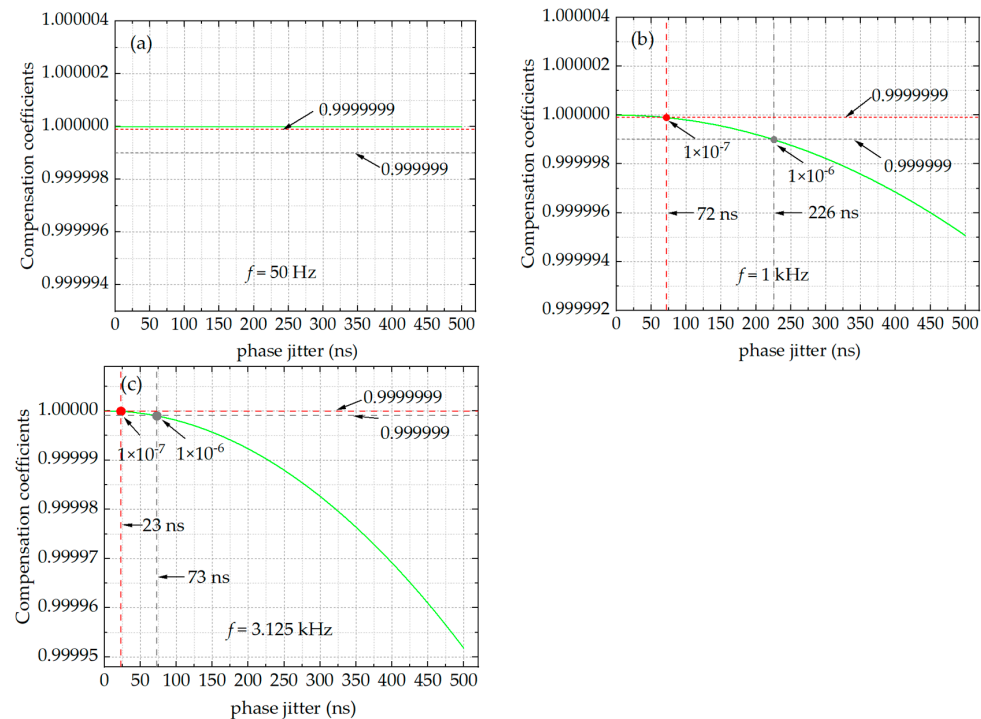


Figure 8. The influence of the phase jitter with frequencies at (a) 50 Hz, (b) 1 kHz and (c) 3.125 kHz. The red horizontal dashed line indicates a compensation factor of 0.9999999, the red vertical dashed line indicates the waveform frequency under this compensation factor, and the gray horizontal dashed line indicates a compensation factor of 0.999999, with the gray vertical dashed line indicating the waveform frequency under this compensation factor.

4. Conclusions

The differential sampling method based on PJVS to measure AC waveform has been one of the hot research projects in recent years. In this paper, the effect of phase jitter on measurement results is analyzed theoretically and verified by simulation. The effect of phase jitter on different post-processing methods used in different NMIs is analyzed in detail. Results show that the effect of phase jitter on the three post-processing methods is consistent. The phase jitter compensation coefficients are determined by the phase jitter angle distribution and harmonic order. In addition, the simulation of phase jitter with three common distributions (normal distribution, triangular distribution and rectangular distribution) has shown that the compensation coefficient can be calculated using only the standard deviation of phase jitter, without estimating its distribution in the differential sampling system. The impact of phase jitter can be negligible when the waveform frequency is low. However, with an increase in waveform frequency, the influence of phase jitter on the measurement results increases obviously. The simulation shows that when the standard deviation of the phase jitter angle is 50 ns, and the frequency of the measuring waveform is 1.4 kHz, the influence of the phase jitter is 1×10^{-7} . Similarly, when the standard deviation of the phase jitter angle is 20 ns, and the frequency of the measuring waveform is 3.46 kHz, the influence of the phase jitter is 1×10^{-7} . Therefore, in the differential sampling system based on PJVS, the compensation of phase jitter is particularly important with an increase in waveform frequency.

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