

Article

Kinematic Analysis of a Wheeled-Leg Small Pipeline Robot Turning in Curved Pipes

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Abstract: A wheeled-leg pipeline robot suitable for operation in small pipes is proposed to address the challenges of detecting the condition of pipelines, such as solution corrosion and crack defects, which cannot be conducted externally due to the pre-buried pipe system embedded in other structures. Inspired by existing pipeline robots, the proposed robot employs a mechanical structure with six wheeled legs arranged in an alternating pattern. To analyze the motion state of the pipeline robot turning in curved pipes, kinematic analysis based on geometry is conducted to figure out the kinematic characteristics of the robot navigating in curved pipes. The relationship between the motion trajectories of each contact wheel and the posture angle of the robot in the pipeline is the focal point. Additionally, a turning method preventing wheel slippage is proposed specifically for this type of robot. Finally, an experiment with the pipeline robot navigating in the curved pipeline is implemented and demonstrates successful passing through curved pipes with an inner diameter of 120 mm as well as a turning radius of 240 mm, with the effectiveness of the kinematic analysis validated.

Keywords: pipeline robot; wheel-legged structure; kinematic analysis; curved pipes



Citation: Wang, J.; Mo, Z.; Cai, Y.; Wang, S. Kinematic Analysis of a Wheeled-Leg Small Pipeline Robot Turning in Curved Pipes. *Electronics* **2024**, *13*, 2170. <https://doi.org/10.3390/electronics13112170>

Academic Editor: Giuseppe Menga

Received: 18 April 2024

Revised: 30 May 2024

Accepted: 30 May 2024

Published: 2 June 2024



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1. Introduction

Pipeline systems play a crucial role in socio-economic development by facilitating the remote transmission of oil and natural gas, as well as water and gas supply in urban planning. Over time, these pipeline systems inevitably age, and the failure to timely replace damaged pipelines results in significant economic losses, as highlighted in reports from Japan and the United States [1]. There are also special occasions when the quality of the pipeline is extremely important, such as ship-buried pipeline systems, where space constraints within the vessel necessitate embedding pipelines within the hull structure, and then these pipelines are constantly immersed in seawater, susceptible to corrosion from marine organisms and seawater, which can lead to catastrophic failures or potentially compromise the safety of the entire vessel. Therefore, designing a pipeline robot specifically tailored for detecting the condition of these types of pipeline systems is of utmost importance. Such a robot would be able to promptly identify erosion and defects in the pipelines, providing invaluable insights into maintaining the safety and integrity of the vessel.

Pipeline robots are generally divided into external inspection robots and internal inspection robots. External inspection robots need to be capable of grasping and climbing along the pipeline [2,3]. Whereas internal pipeline robots can be mainly categorized into the following types: PIGs (pipeline inspection gauges) [4], wheeled-type pipeline robots [5–12], tracked-type pipeline robots, helical-type pipeline robots [13,14], snake-like type robots [15–17], inchworm-like pipeline robots [18–20]. In summary, except for PIGs, which rely on fluid flow for propulsion, all other types of internal pipeline inspection robots are equipped with motors or air pumps as their own power sources.

Each type of pipeline robot has its own specific advantages and disadvantages. However, the commercialization of wheeled-type and tracked-type pipeline robots is relatively mature. For example, a pipeline robot developed by Jun from the Harbin Institute of Technology in China features a six-wheeled-leg design suitable for 195mm pipelines [5]. They provided the slip rate and optimal motor output power for this robot operating in curved pipes. Kim from Sungkyunkwan University in South Korea proposed a multi-axis differential gear mechanism [6], in which a single motor is designed as the power input and multiple sets of wheels as the output. This differential mechanism automatically allocates driving force for each set of wheels, making it highly efficient for traveling in straight and curved pipelines. Kwon from Hanyang University in South Korea proposed pipeline robots with double-row wheels [7] and triple-row wheels [8], respectively. The supporting mechanism (also known as the wall-press mechanism) used in these robots has a wide range of variations and is suitable for pipelines with a diameter of 100 mm. The wheels of the double-row wheel mechanism [7] have one degree of freedom for steering, making it suitable for turning in T-shaped pipelines. However, the limited number of motors used for traction resulted in a limited traction force. Subsequently, Kwon adopted a framework similar to [8] and replaced the wheeled mobile mechanism with tracks to construct a triple-row tracked pipeline robot [9]. Likewise, Zheng from Beijing Institute of Technology in China referenced a similar double-row structure as in [7] and developed a pipeline robot using a tracked mobile mechanism [10], enhancing its obstacle-surmounting capability to some extent. Jang from Sungkyunkwan University in South Korea introduced a four-row tracked pipeline robot suitable for medium-sized pipelines with a diameter of 500 mm [11], which is characterized by modular design and robust power. Vadapalli from the Robotics Research Center at the International Institute of Information Technology in India adopted a differential mechanism similar to that of a car to introduce a triple-row-track pipeline robot [12], which prevented the robot from track slippage during turns in the curved pipelines.

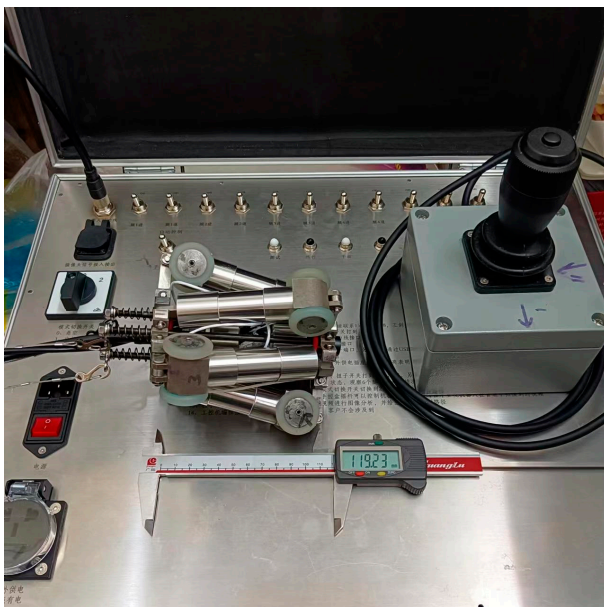
Considering the application scenario, it is proposed to use a six-legged locomotion mechanism, applicable to pipelines with a diameter limit of 120 mm. The focus of this paper is on the kinematic analysis of small pipeline robots navigating curved pipelines. Due to the diversity of current robot structures and the complexity of their interactions with curved pipelines, in-depth research on the kinematics of pipeline robots overcoming obstacles in curved pipelines is relatively limited. Representative studies include: Brown from the University of Manchester conducted kinematic analysis of a three-row wheeled pipeline robot's turning in curved pipes [21]. He estimated the robot's posture and turning radius by setting a whisker-like structure. Roh from Sungkyunkwan University in Korea performed kinematic analysis of a three-row wheeled pipeline robot's steering in curved pipes [22]. He utilized the symmetry of the robot's front and rear structure as well as the parameter equation of the curved pipeline to solve for the robot's posture angle, the distance from each wheel's contact point to the robot's center, and the required rotational velocity of each wheel to prevent slippage during operation. Kwon from Hanyang University in Korea conducted kinematic analysis of a three-row tracked pipeline robot [23], and he established a linear relationship between the rotational speeds of three tracks and the robot's rotational and forward movements, enabling the calculation of each motor's input angular velocity. He also studied the rotational performance of the robot in various directions by using the eigen values of the linear relationship matrix. Kakogawa from Ritsumeikan University in Japan conducted kinematic analysis of a spiral-type pipeline robot's turning in curved pipes [24]. Also, he established the spatial trajectory of each wheel's center based on homogeneous coordinate transformations in robotics and analyzed kinematic equations under scenarios with and without additional mechanisms. Yan from the North University of China studied the kinematics of a similar spiral-type pipeline robot turning in curved pipes [25], using a method similar to that in [24] involving homogeneous coordinate transformations. However, he only considered scenarios with the spiral mechanism, rather than including additional structures.

Referring to existing locomotion mechanisms, a six-legged wheeled pipeline robot is developed in this paper. Unlike the symmetrical structures of wheeled pipeline robots mentioned in [21,22], this robot requires consideration of more posture angles in curved pipes. Six variables are selected to describe the spatial position and posture of the robot, and then, by using homogeneous coordinate transformations, expressions for the centers of the six wheels with respect to these variables are derived. And then, based on the knowledge of analytic geometry and the selected structural parameters of the pipeline, a mathematical model of the curved pipe is founded. Finally, six constraints are formulated based on the relationships between the six wheel centers and the pipe wall. These constraints can be used to solve the unknown variables, such as the kinematic posture of the pipeline robot in curved pipes. By gradually changing one variable, such as the rolling of the robot in the pipeline, and using the same method to establish constraints, the remaining five variables can be determined, thus completing the kinematic analysis of the entire pipeline robot.

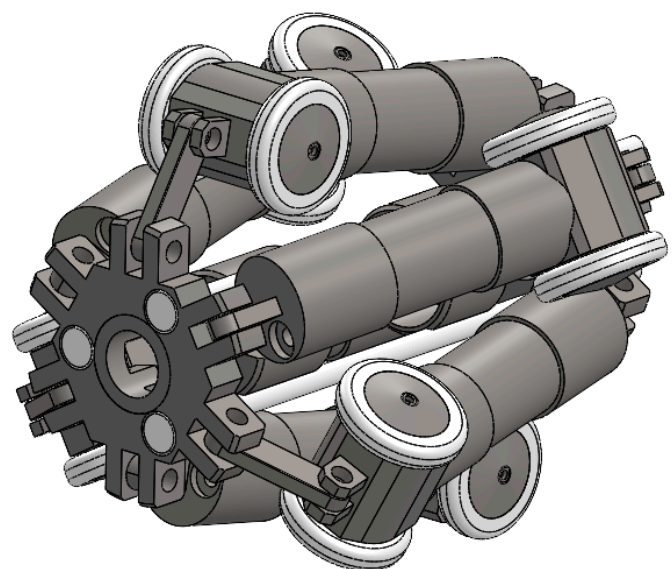
The arrangement of the paper is as follows: In Section 2, an introduction to the structure and working principles of the proposed pipeline robot is given. This section primarily outlines the central framework structure, locomotion mechanism, and support mechanism of the proposed pipeline robot. In Section 3, kinematic analysis of the proposed pipeline robot is carried out, which includes simplification of the robot's structure, establishment of mathematical models for curved pipes, pose transformation of the pipeline robot, inverse solving of pose variables through constraints, and optimization of wheel speeds to prevent slippage. Section 4 shows real experiments of the robot's rotation in curved pipes. In Section 5, a conclusion can be drawn.

2. The Mechanical Structure of the Proposed Six-Legged Pipeline Robot

The design of the pipeline robot's structure is primarily aimed at facilitating its movement within the pipeline, requiring the coordinated cooperation of various mechanical modules. As shown in Figure 1a,b, the pipeline robot proposed in this paper, which is named π -II, includes the central framework mechanism, locomotion mechanism, and support mechanism.



(a)



(b)

Figure 1. π -II: a small pipeline robot with six wheeled legs: (a) Physical model of the proposed pipeline robot; (b) 3-D model of the proposed pipeline robot.

2.1. The Central Framework Mechanism of the Pipeline Robot

The central framework mechanism primarily provides a structural base for the entire robot, integrating the locomotion and support mechanisms. As depicted in Figure 2, the pipeline robot proposed in this paper features a central framework mechanism consisting mainly of front and rear covers and three long bolts. The front and rear covers serve as attachment points for the six-legged modules, with six hinge points evenly distributed around the circumference of each cover. The rear cover can be moved along the long bolts and is equipped with compression springs to provide a certain pre-tension force.

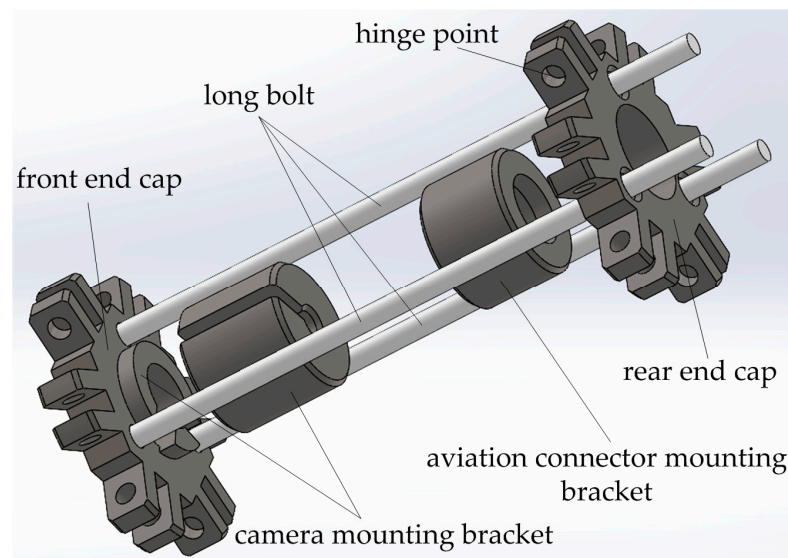


Figure 2. The central framework mechanism of the proposed pipeline robot.

Due to space limitations within the pipeline, the central framework mechanism needs to assist in accommodating various other functionalities, such as carrying inspection sensors. Common examples include underground CCTV visual systems [26] or CCD cameras [27], as well as leak detection or ultrasonic inspection systems [28], which are typically compact to fit onto the robot. As shown in Figure 2, fixed brackets for cameras or other sensors can be installed within the central framework mechanism. If waterproofing is required, aviation plug mounting bases can be set up to protect the power lines of miniature motors.

2.2. The Locomotion Mechanism of the Pipeline Robot

The locomotion mechanism primarily provides the robot with the power to move forward. As shown in Figure 3, this paper refers to existing solutions [5] and further improves the structure to adapt to pipeline diameters as small as 120 mm. Additionally, considering the potential future use of ultrasonic sensors, the design incorporates waterproof capabilities. Combining Figures 1–3, it can be observed that the locomotion mechanism of this pipeline robot employs six identical modular structures, referred to as wheeled-leg units, which are articulated in an alternating pattern on the front and rear end caps. Each leg encapsulates a miniature DC motor, which is enclosed within three sleeves: upper, middle, and lower sleeves.

These sleeves are threaded together and secured with thread-locking adhesive for waterproofing and fixation. Additionally, the upper sleeve features a support rod for articulation with the front or rear end cap, while the head of the lower sleeve is articulated with a connecting rod. The other end of the connecting rod can be articulated with either the front or rear end cap.

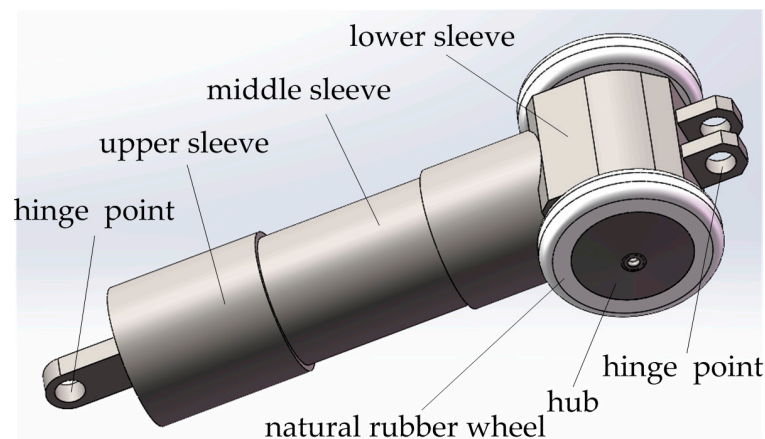


Figure 3. A wheeled-leg unit that constructs the locomotion mechanism.

The locomotion module utilizes a pair of bevel gears to transmit the output of the miniature DC motor to the wheels, as depicted in Figure 4a. The bevel gears are fixed to both the output shaft of the motor reducer and the output shaft of the locomotion module by threading, respectively. The hub is securely attached to the output shaft of the locomotion module. The wheels are coated with natural rubber material using an encapsulation process, and O-rings are used for sealing between the lower sleeve and the hub. Waterproof adhesive is applied between the hub and the output shaft of the locomotion module to prevent liquid from entering the motor from the lower sleeve side. Furthermore, a through-hole is created in the upper sleeve to allow the power supply for the motor to pass through. Additionally, grommets can be arranged around the through-hole, and the wires are passed through the grommet with waterproof adhesive applied between them to prevent liquid from entering the motor from the upper sleeve side.

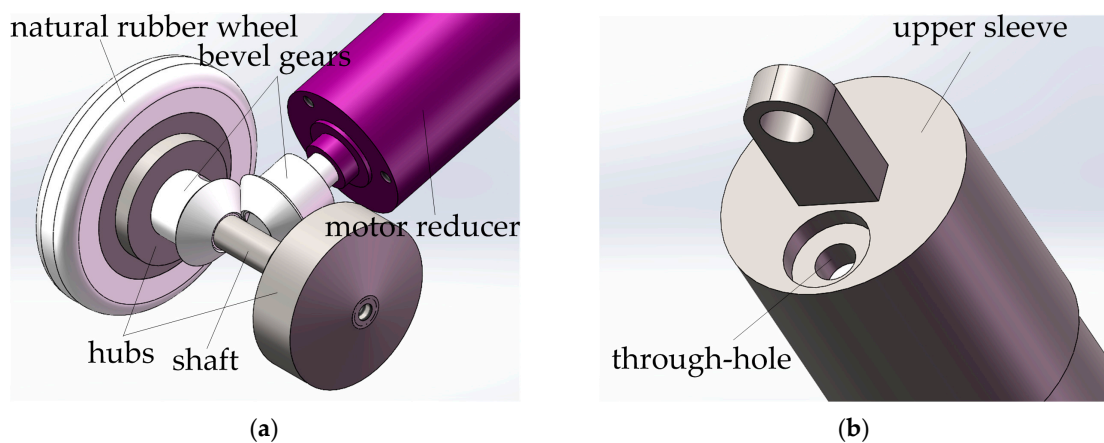


Figure 4. Wheel-leg power output structure and waterproof measures: (a) Power output chain of the wheeled leg; (b) through-hole of the upper sleeve.

2.3. The Support Mechanism of the Pipeline Robot

The support mechanism, also referred to as the wall-pressure mechanism in some literature, of the proposed pipeline robot consists of components of both the central framework and the locomotion mechanism that serve as the backbone. As illustrated in Figure 5, it primarily comprises front and rear end caps, links, lower sleeve, middle sleeve, and upper sleeve, long bolts, springs, and nuts. While only two symmetric support mechanism modules are shown in Figure 5, the rest are identical. Here is how it operates: When the pipeline robot is placed inside the pipeline, the springs with a certain rigidity compress, causing the rear end cap to move towards the front end cap relative to the long bolt. This compression

presses the wheels in a radial direction, causing them to make tight contact with the inner wall of the pipeline. As a result, the inner wall exerts a certain amount of pressure on the wheels. When the control motor drives the wheels, this pressure generates friction, which translates into traction, and propels the pipeline robot forward. The magnitude of the traction force can be adjusted by positioning the nuts, thus adjusting the pressure exerted by the inner wall. This allows the motor to provide an appropriate amount of traction to propel the entire pipeline robot and any associated loads, such as cables, forward.

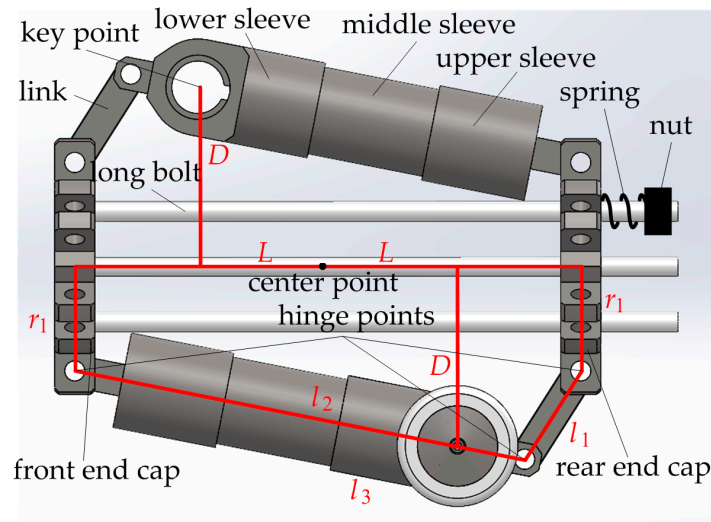


Figure 5. Side view diagram of the support mechanism.

The function relationship between L and D can be obtained from Figure 5 as follows:

$$L = f(D) = \frac{2l_2 - l_3}{2l_2} \sqrt{(l_2)^2 - (D - r_1)^2} - \frac{1}{2l_2} \sqrt{(l_1 l_2)^2 - [l_3(D - r_1)]^2}, \quad (1)$$

where $r_1 \leq D \leq \frac{l_2(r_1 + l_1)}{l_3}$, as r_1 is the distance from the center of the front or rear end cap to the point where they articulate with the upper sleeve or link, l_1 is the distance between the two articulation points of the link, l_2 is the distance from the articulation point of the upper sleeve to that of the lower sleeve in the same wheeled leg, and l_3 is the distance from the articulation point of the upper sleeve to the key point of the same wheeled leg. All of them are measurable structural parameters. D is the distance from the key point to the centerline of the robot, and $2L$ is the projection length between the key points of two staggered legs in the direction of the centerline of the robot. These two are variables.

3. Kinematic Analysis of Pipe Robot Turning in Curved Pipes

3.1. Simplification of the Mechanism of the Proposed Pipeline Robot

As seen in Figure 3, in the pipeline robot proposed in this paper, one motor drives the two wheels on each side. When the robot moves inside the pipe, it needs to consider the contact between these two wheels and the inner wall of the pipe, complicating the problem. To enable analysis with reasonable complexity, it is possible to simplify this mechanical structure, even at the cost of some accuracy. As illustrated in Figure 6, when the robot moves straight within the pipe, the two wheels on one leg contact the inner wall, and assuming no deformation of the rubber wheels, the contact trajectory between the wheels and the inner wall of the pipe is a straight line parallel to the pipe's center. Therefore, we can replace the two wheels with a virtual wheel of negligible width and place it in the middle of the output shaft of the locomotion mechanism. By this way, each leg has only one point of contact with the inner wall of the pipe, i.e., a point on the virtual wheel. When conducting dynamic analysis on curved pipes, the same equivalent single virtual wheel is also utilized. It is assumed that at least one point will come into contact with the inner wall

of the curved pipe. Furthermore, analyzing the kinematics of robots in curved pipes can be equivalently performed by examining the intersection between the center of the virtual wheel and the virtual pipe. For descriptive convenience, the center of the virtual wheel can be referred to as a key point.

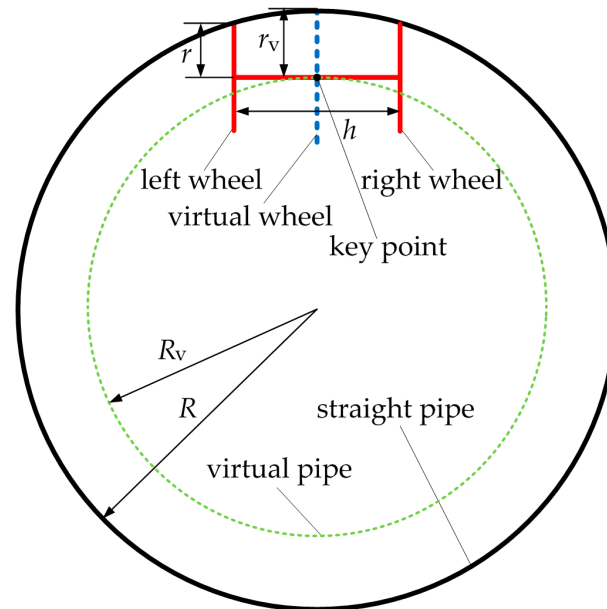


Figure 6. Dimension simplification of the proposed pipeline robot in straight pipe.

As shown in Figure 6, the radius of the virtual pipe can be calculated as follows:

$$R_v = \sqrt{R^2 - \frac{h^2}{4}} - r, \quad (2)$$

Therefore, the radius of the virtual wheel can be calculated as:

$$r_v = R - R_v = R + r - \sqrt{R^2 - \frac{h^2}{4}}, \quad (3)$$

When analyzing the kinematics of the robot in curved pipe, the curved pipe can be equivalently replaced by a virtual curved pipe denoted as R_v .

3.2. The Mathematical Model of the Curved Pipe

As shown in Figure 7, considering the virtual curved pipe with a radius of R_v and a radius of curvature of the pipe centerline of a , it can be set to the global reference system 0 at the center of curvature of the pipe centerline, with the z-axis oriented vertically upwards.

The coordinates of a point on the pipe can be denoted as (x, y, z) . Then, this point should satisfy the equation:

$$\left(\sqrt{x^2 + y^2} - a \right)^2 + (z)^2 = (R_v)^2, \quad (4)$$

Equation (4) represents the mathematical model of the curved pipe, which is the key point on the surface of a tour. If the selected point is located inside the pipe, Equation (4) changes from equality to inequality, meaning that the value on the left side is less than the value on the right side.

Considering the motion of the pipeline robot within the curved pipe, as seen in Figure 7, the pipe shown is a virtual curved pipe, denoted as R_v , rather than the real one, denoted as R . In this scenario, the contact points between the pipeline robot and the virtual pipe are

the six key points of the legs, as illustrated in Figure 6. A local reference system 1 can be bound at the center of the pipeline robot, with its origin coordinates marked as (x_0, y_0, z_0) in the global reference system. The y -axis points in the forward direction towards the center of the front end cap, with the oyz plane containing the key point of the first wheeled leg. The posture of the local reference system 1 relative to the global reference system 0 can be gained through a series of transformations. Initially, the local reference system 1 aligns its three axes with those of the global reference system 0. Then, it rotates by an angle θ around the y -axis, followed by a pitch angle φ around the x -axis of the transformed coordinate system, and finally, a yaw angle ψ around the z -axis of the transformed coordinate system. The diameter of the circle formed by a set of three key points of the legs hinged on the front or rear end cap can be set as $2D$, with a distance of $2L$ between the two circles formed by the two sets of key points of the legs. D and L maintain a certain functional relationship. Due to the symmetrical structure of the curved pipe, the distance traveled by the pipeline robot within the pipe does not affect the kinematic analysis of the pipeline robot. Hence, we can simply consider the scenario where $y_0 = 0$, meaning the center of the pipeline robot remains at $(x_0, 0, z_0)$. Consequently, there are six variables $(x_0, z_0, \theta, \varphi, \psi, D)$ determining the pose of the pipeline robot, referred to as kinematic variables. Additionally, the six key points of the pipeline robot's legs satisfy the constraint Equation (4), resulting in six constraints that uniquely determine the pose of the pipeline robot.

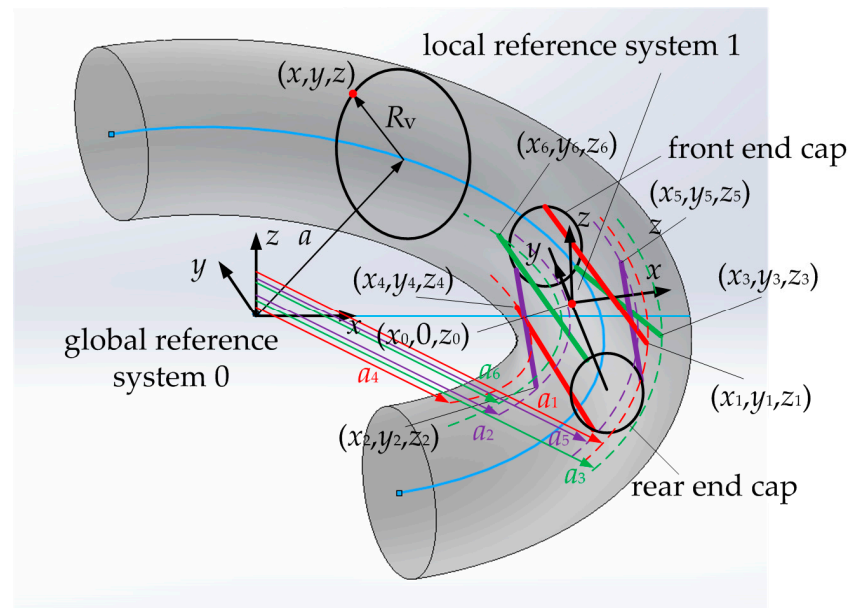


Figure 7. Schematic diagram of mechanism simplification for the proposed pipeline robot moving in curved pipe.

From Figure 7, it can be seen that the coordinates of the six key points of the legs in the local reference system 1 are as follows:

$$\begin{cases} [x_1^1, y_1^1, z_1^1]^T = [D \cos(90^\circ), -L, D \sin(90^\circ)]^T \\ [x_2^1, y_2^1, z_2^1]^T = [D \cos(90^\circ + 120^\circ), -L, D \sin(90^\circ + 120^\circ)]^T \\ [x_3^1, y_3^1, z_3^1]^T = [D \cos(90^\circ + 240^\circ), -L, D \sin(90^\circ + 240^\circ)]^T \\ [x_4^1, y_4^1, z_4^1]^T = [D \cos(-90^\circ), L, D \sin(-90^\circ)]^T \\ [x_5^1, y_5^1, z_5^1]^T = [D \cos(-90^\circ + 120^\circ), L, D \sin(-90^\circ + 120^\circ)]^T \\ [x_6^1, y_6^1, z_6^1]^T = [D \cos(-90^\circ + 240^\circ), L, D \sin(-90^\circ + 240^\circ)]^T \end{cases}, \quad (5)$$

We can substitute Equation (1) into Equation (5) to express the coordinates of the six key points solely in terms of the variable D .

3.3. Homogeneous Coordinate Transformation for the Proposed Pipeline Robot

The homogeneous coordinate transformation matrix from global reference system 0 to local reference system 1 needs to be obtained first, and then, combining with the coordinates of the six key points of the wheeled legs of the robot in local reference system 1, the coordinates of the six key points in global reference system 0 can be obtained.

The transformation process of the local reference system 1 from the global reference system 0 can be described by the following steps:

$$\begin{aligned}
 T_1^0 &= \text{Trans}(x_0, 0, z_0) \cdot \text{Rot}(y, \theta) \cdot \text{Rot}(x, \varphi) \cdot \text{Rot}(z, \psi) \\
 &= \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi & 0 \\ 0 & \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \psi & -\sin \psi & 0 & 0 \\ \sin \psi & \cos \psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos \theta \cos \psi + \sin \theta \sin \varphi \sin \psi & -\cos \theta \sin \psi + \sin \theta \sin \varphi \cos \psi & \sin \theta \cos \varphi & x_0 \\ \cos \varphi \sin \psi & \cos \varphi \cos \psi & -\sin \varphi & 0 \\ -\sin \theta \cos \psi + \cos \theta \sin \varphi \sin \psi & \sin \theta \sin \psi + \cos \theta \sin \varphi \cos \psi & \cos \theta \cos \varphi & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned} \quad (6)$$

Taking Equation (5) and Equation (6) into account, we can obtain the coordinates of the six key points in global reference system 0 as follows:

$$\begin{cases} \begin{bmatrix} x_1^0 & y_1^0 & z_1^0 & 1 \end{bmatrix}^T = T_1^0 \begin{bmatrix} x_1^1 & y_1^1 & z_1^1 & 1 \end{bmatrix}^T \\ \begin{bmatrix} x_2^0 & y_2^0 & z_2^0 & 1 \end{bmatrix}^T = T_1^0 \begin{bmatrix} x_2^1 & y_2^1 & z_2^1 & 1 \end{bmatrix}^T \\ \begin{bmatrix} x_3^0 & y_3^0 & z_3^0 & 1 \end{bmatrix}^T = T_1^0 \begin{bmatrix} x_3^1 & y_3^1 & z_3^1 & 1 \end{bmatrix}^T \\ \begin{bmatrix} x_4^0 & y_4^0 & z_4^0 & 1 \end{bmatrix}^T = T_1^0 \begin{bmatrix} x_4^1 & y_4^1 & z_4^1 & 1 \end{bmatrix}^T \\ \begin{bmatrix} x_5^0 & y_5^0 & z_5^0 & 1 \end{bmatrix}^T = T_1^0 \begin{bmatrix} x_5^1 & y_5^1 & z_5^1 & 1 \end{bmatrix}^T \\ \begin{bmatrix} x_6^0 & y_6^0 & z_6^0 & 1 \end{bmatrix}^T = T_1^0 \begin{bmatrix} x_6^1 & y_6^1 & z_6^1 & 1 \end{bmatrix}^T \end{cases}, \quad (7)$$

So the coordinates of the i -th ($i = 1, 2, 3, 4, 5, 6$) key point (x_i^0, y_i^0, z_i^0) in global reference system 0 are entirely functions of the kinematic variables $(x_0, z_0, \theta, \varphi, \psi, D)$. Additionally, the i -th ($i = 1, 2, 3, 4, 5, 6$) key point lies on the inner wall of the pipe, satisfying the constraint given by Equation (4):

$$\begin{cases} \left(\sqrt{(x_1^0)^2 + (y_1^0)^2} - a \right)^2 + (z_1^0)^2 = (R_V)^2 \\ \left(\sqrt{(x_2^0)^2 + (y_2^0)^2} - a \right)^2 + (z_2^0)^2 = (R_V)^2 \\ \left(\sqrt{(x_3^0)^2 + (y_3^0)^2} - a \right)^2 + (z_3^0)^2 = (R_V)^2 \\ \left(\sqrt{(x_4^0)^2 + (y_4^0)^2} - a \right)^2 + (z_4^0)^2 = (R_V)^2 \\ \left(\sqrt{(x_5^0)^2 + (y_5^0)^2} - a \right)^2 + (z_5^0)^2 = (R_V)^2 \\ \left(\sqrt{(x_6^0)^2 + (y_6^0)^2} - a \right)^2 + (z_6^0)^2 = (R_V)^2 \end{cases}, \quad (8)$$

Thus, from the six constraint equations, the values of the six kinematic variables can be solved backwards.

3.4. Inverse Solution of a Kinematic Variable

Structural parameters can be measured in advance, as shown in Table 1.

Table 1. Structural parameters of the proposed pipeline robot.

Parameter	$R/(\text{mm})$	$a/(\text{mm})$	$r/(\text{mm})$	$h/(\text{mm})$	$r_1/(\text{mm})$	$l_1/(\text{mm})$	$l_2/(\text{mm})$	$l_3/(\text{mm})$
Value	60	240	15	31	26	26	97	114

Substituting the structural parameters from Table 1 into Equation (8), solving this equation system is quite challenging. So the numerical method of MATLAB, namely “fsolve”, can be utilized, with initial values preset for the solution. Here, $(x_0, z_0, \varphi, \psi, D)$ can be set as (240 mm, 0 mm, 0° , 0° , 30 mm) as initial inputs for “fsolve”, while θ , which indicates the rolling angle of the robot inside the pipeline, can be set for “fsolve” to range from 0° to 360° with a certain step size many times. This will yield a set of non-arrayed kinematic variables. Once solved, the obtained set of kinematic variables can be sorted in ascending order of θ , providing the kinematic variable values for the robot at different θ values. Figure 8 illustrates the variation in other kinematic variables as θ changes from 0° to 360° . Figure 8a indicates that the center of the robot remains essentially stationary but is not on the centerline of the pipeline, which aligns with the actual scenario of the curved pipe. Figure 8b shows that when the robot rolls inside the pipe, it needs to adapt to the size constraints of the curved pipe, resulting in corresponding variations in pitch and yaw. However, these variations are very small and exhibit a certain regularity, resembling a sine wave. Additionally, the amplitudes of the pitch and yaw angles are equal, with a certain phase difference. Figure 8c demonstrates how the proposed robot rolls inside the curved pipe, with the virtual pipe radius remaining constant. This means that the radial motion of the robot is constrained to a circle with a certain radius, and the distance between the front and rear end caps of the robot remains unchanged. In reality, due to slight disturbances in the pitch and yaw angles, the virtual pipe radius is not strictly constant, as depicted in Figure 8d, showing extremely small variations. Although these variations can be negligible, it is worth noting that, due to the slight tilt of the robot inside the pipeline, the circles formed by the three adjacent key points on the same side are not perpendicular to the centerline of the curved pipe. Consequently, the radius $D = 43.1486$ mm formed by the key points is slightly larger than the calculated virtual radius $R_V = 42.9634$ mm, which is consistent with the actual scenario.

Figure 9 demonstrates how the orientation angles (φ, ψ) of the pipeline robot vary with the rolling angle θ . It can be observed that although the robot rolls at different angles inside the pipeline, its posture of head remains essentially unchanged according to the pipe’s centerline.

Figure 10 illustrates the relationship between the coordinates of Key Point 1, (x_1^0, y_1^0, z_1^0) , in global reference system 0 and the rolling angle θ of the robot. The x and z coordinates can be regarded as the cosine and sine components of a circle, similar to the variation of coordinate components when the robot rotates about a point. Additionally, since only case $y_0 = 0$, where the robot’s center is fixed along the y -direction, is considered, the pose of the robot undergoes only minor fluctuations, resulting in little change in the y -coordinate value of Key Point 1 (Figure 10a). In the three-dimensional coordinate system with the y -direction coordinates magnified, the overall variation in Key Point 1 can be observed (Figure 10b).

Similarly, all six key points’ coordinates are plotted in a three-dimensional coordinate system, where the rolling angle θ of the robot ranges from 0° to 115° , as shown in Figure 11. Key points 1, 2, and 3 lie on one trajectory, while key points 4, 5, and 6 lie on another trajectory. The symbol “*” represents the initial positions of the trajectory for some key point, corresponding to $\theta = 0^\circ$, while the symbol “+” represents the final positions of the trajectory for the same key point, corresponding to $\theta = 115^\circ$. If θ reached 360° from 0° , only two trajectory loops would be visible.

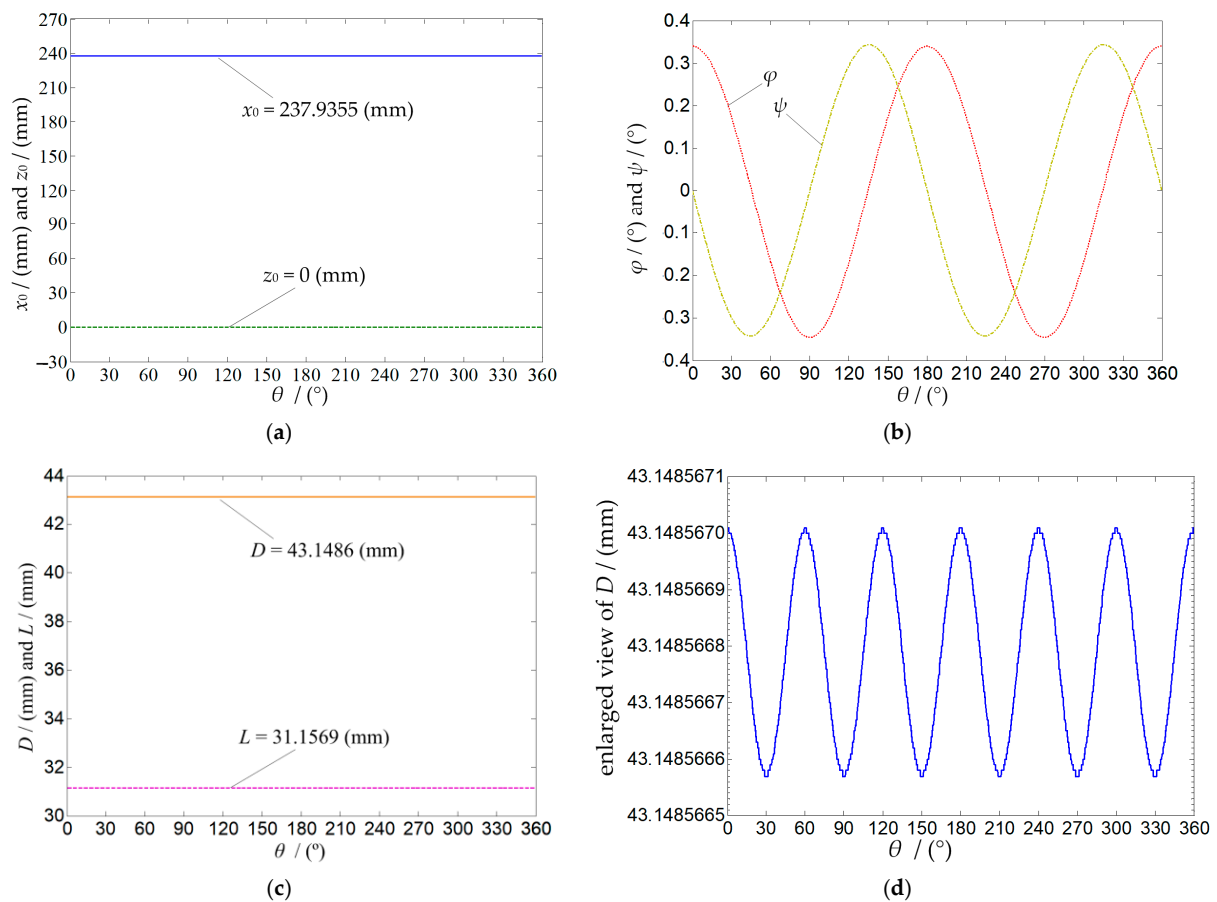


Figure 8. Variations of other parameters with θ : (a) Changes in parameters x_0 and z_0 ; (b) Changes in parameters φ and ψ ; (c) Changes in parameters D and L ; (d) Small variation of D .

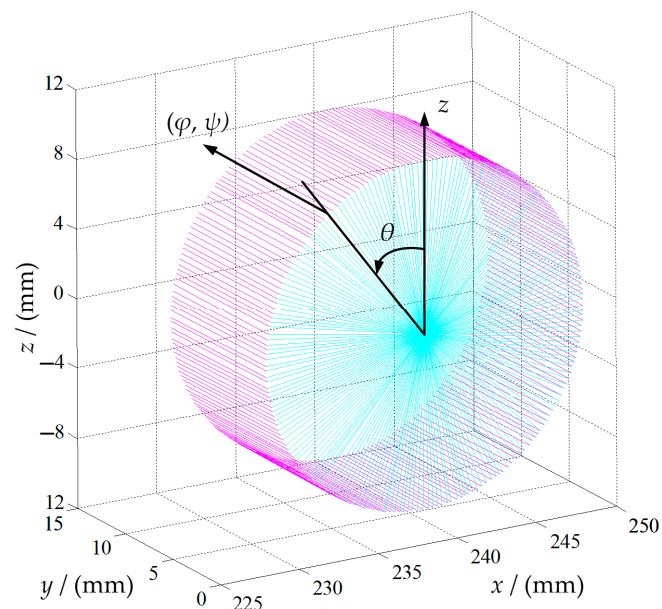


Figure 9. Azimuth angle of (θ, φ, ψ) .

The distance errors between each key point obtained through a numerical solution and the virtual pipe inner wall obtained through ideal assumptions, as they vary with the rolling angle θ , can be calculated. This is shown in Figure 12, where each of the six colors corresponds to one of the six key points. It can be observed that none of the errors

exceed 2.5×10^{-5} mm, indicating that the obtained kinematic inverse solution satisfies the constraints (8).

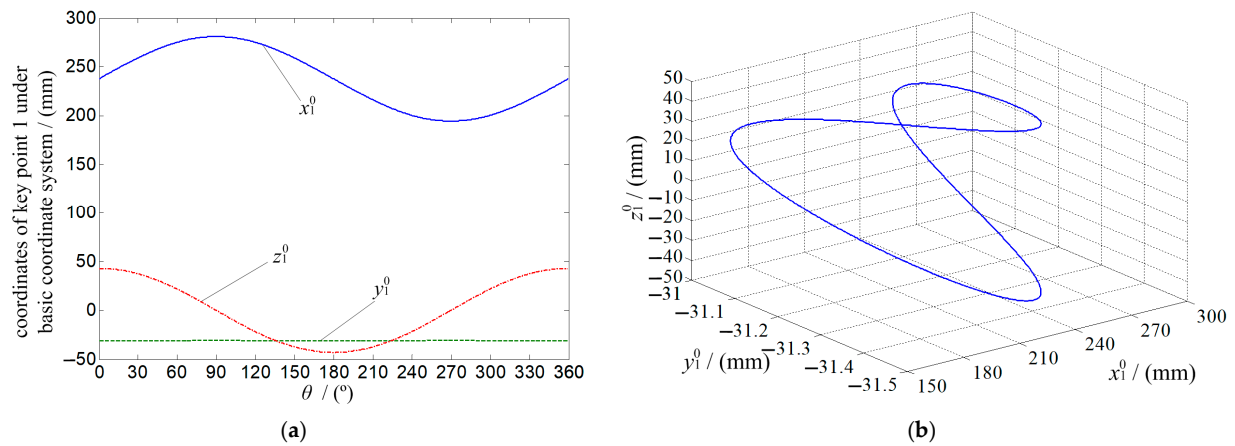


Figure 10. Relationship between the three coordinates of Key Point 1 and the rolling angle θ : (a) Relationship curves in a plane coordinate system; (b) The curve of Key Point 1 in space according to the variation in rolling angle θ .

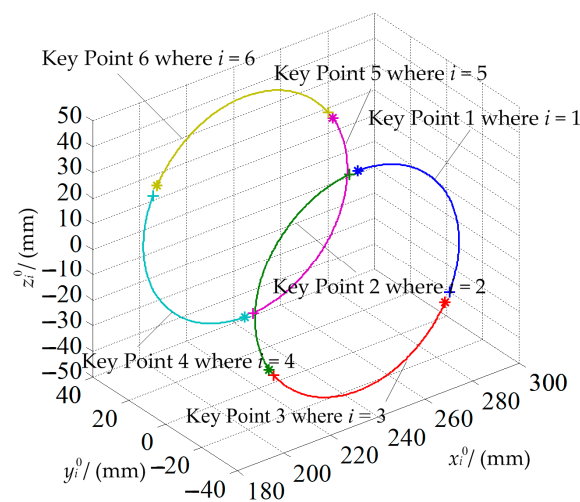


Figure 11. The trajectories of all the key points in space are formed by the variation of θ .

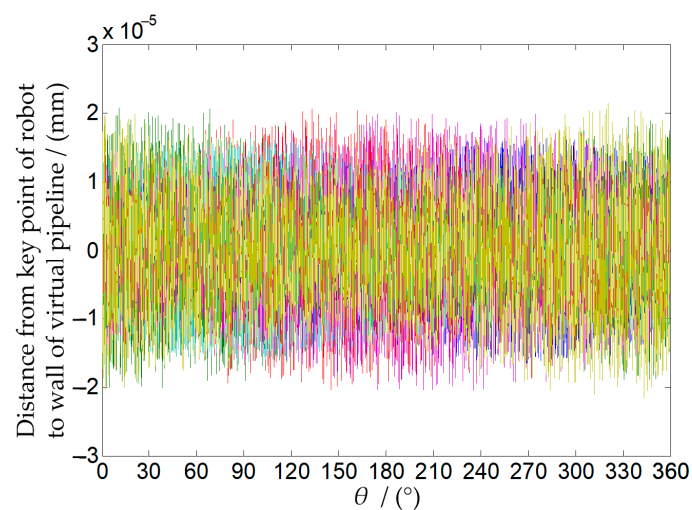


Figure 12. Error of each key point relative to the inner wall of the virtual pipe while varying with rolling angle θ .

3.5. Optimization of the Wheel Rotation Speed of Each Leg of the Proposed Pipeline Robot

Here, the purpose of speed optimization is to avoid situations where the robot's wheels slip relative to the inner wall of the curved pipe. Drawing inspiration from the theory of circular motion, the linear velocity of the wheels is proportional to the turning radius. From Figure 7, we have:

$$\frac{v_1^0}{a_1} = \frac{v_2^0}{a_2} = \frac{v_3^0}{a_3} = \frac{v_4^0}{a_4} = \frac{v_5^0}{a_5} = \frac{v_6^0}{a_6} \leq \frac{v_{\max}^0}{a_{\max}}, \quad (9)$$

where a_i and v_i ($i = 1, 2, \dots, 6$) represent the rotational radius and speed of each key point of the pipeline robot relative to the z^0 axis in the global reference system 0. $a_{\max} = a + R_V$ denotes the maximum rotational radius, which corresponds to the maximum speed $v_{\max}^0 = \omega_{\max} \cdot r_V$ that the micro motor can provide. The turning radius of Key Point i ($i = 1, 2, \dots, 6$) can be determined by the following equation.

$$(a_i) = \sqrt{(x_i^0)^2 + (y_i^0)^2} \quad (10)$$

Figure 13 shows the variation of the turning radius of the whole key points of the pipeline robot as the rolling angle θ of the robot increases in the curved pipe. They exhibit a sinusoidal pattern with the same amplitude and frequency, each differing by a phase angle of 60 degrees. This graph also illustrates that in order to prevent the wheels of the pipeline robot from slipping, it is necessary to ensure that the output speeds of the micro DC motors have a ratio of sinusoidal functions.

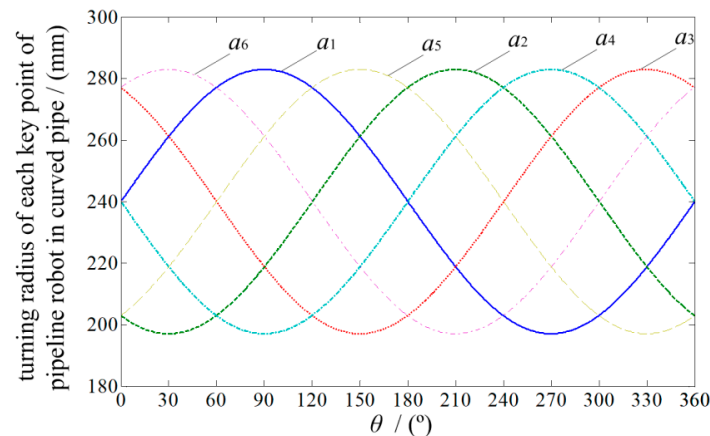


Figure 13. Curvature radius of the whole key points after velocity optimization.

4. Turning Experiment of Pipeline Robot π -II in the Curved Pipe

As shown in Figure 14, a turning operation test was conducted for a pipeline robot in a curved pipe. The pipe is made of 304 stainless steel, with an inner diameter of 120 mm and a turning radius of 240 mm.

During the operation of the pipeline robot in a curved pipe, the robot's roll angle θ is continuously reset, and the robot's 26 different posture states are then measured using an inclinometer by spreading the roll angle θ between 0° and 360° . Figure 15a shows the experimental test condition of the pitch angle φ changing relative to the roll angle θ , which is approximately 10 times the relation proposed by the ideal kinematic model in Figure 8b. The actual tilt curve is not a cosine-like curve but is irregularly distributed around 0° . The cause of these two issues is two-fold: one is the presence of manufacturing errors in the curved pipe and the pipeline robot; the assumptions of virtual pipes and wheels in Figure 6 cannot strictly hold, meaning there might be cases where not all of the robot's six wheel-legs contact the pipe's wheels. Secondly, the wheels are made of rubber and can deform, deviating from treating the entire robot as a rigid body when positioning different postures in the curved pipe. Nevertheless, the real experimental curve changes within a very small amplitude range, which to a certain extent, validates the reasonableness of

the kinematic model of the pipeline robot turning into the curve presented in this paper. Figure 15b shows the real experimental curve of the yaw angle ψ changing with the roll angle θ , differing by about 15 times from the ideal curve due to the same reasons but also changing within a relatively small range, thus partially validating the effectiveness of the proposed kinematic model.

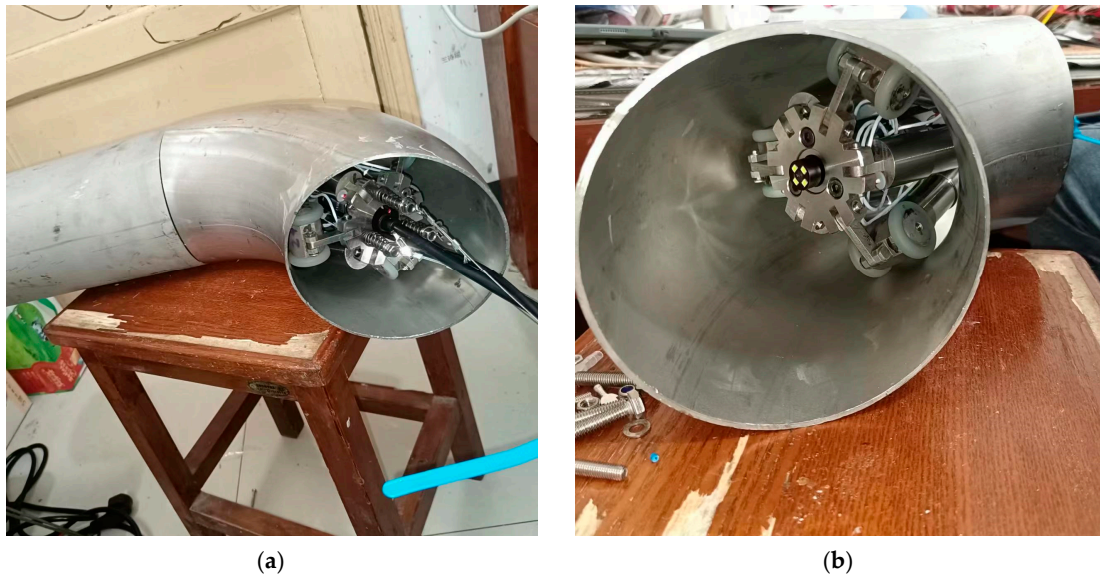


Figure 14. Experiment of pipeline robot π -II turning in the curved pipe: (a) Rear view of the robot when turning in the curved pipe; (b) Front view of the robot when turning in the curved pipe.

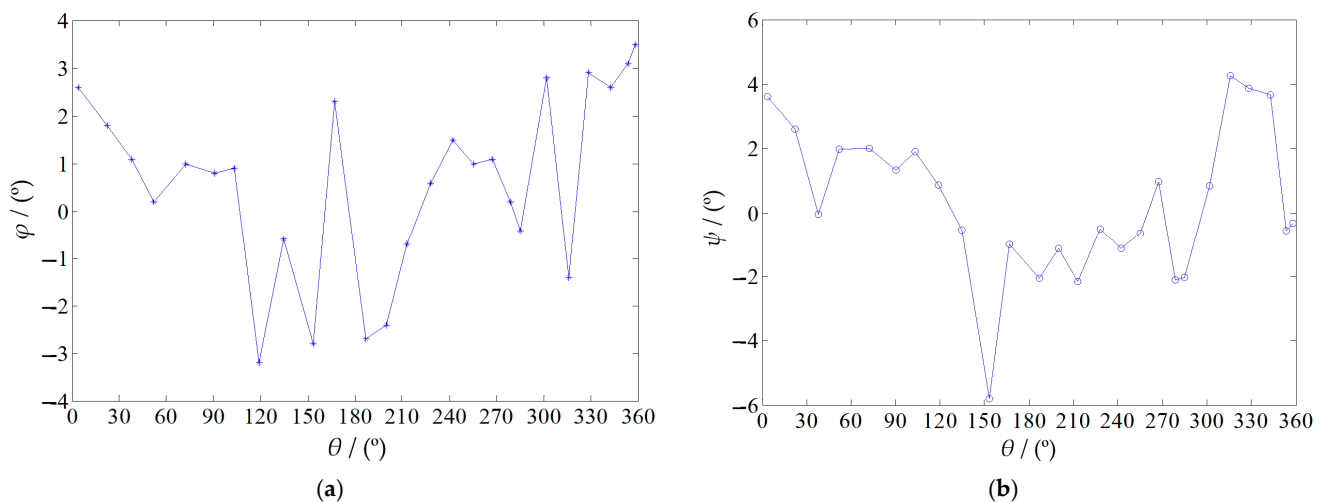


Figure 15. Testing curves of pipeline robot π -II in different postures in the curved pipe: (a) The curve of the robot's pitch angle φ changing with the roll angle θ ; (b) The curve of the robot's yaw angle ψ changing with the roll angle θ .

5. Conclusions

The paper presents a six-legged interlaced small pipeline robot system consisting mainly of a central frame mechanism, a mobile mechanism, and a support mechanism. The relevant structural design indicates that the proposed robot system can operate within a curved pipe with an inner diameter of 120 mm and a turning radius of 240 mm. To address the turning in such curved pipes, a kinematic model for the pipeline robot during turning is proposed using relevant analytical geometry knowledge and the homogeneous transformation method in robotics. By solving six kinematic variables using six constraint

equations constructed from the contact between the pipeline robot's wheels and the inner wall of the pipe, mainly the curves of the other five variables changing with the roll angle, the final real experiments validate the effectiveness of the proposed kinematic model. Additionally, the sources of error between real and theoretical data are analyzed, laying the groundwork for subsequent obstacle avoidance control and internal detection of pipeline robots.

In future work, the structure of the pipeline robot will be optimized, the weight will be reduced by choosing more appropriate material, and research on the kinematics of turning in more challenging "T"-shaped pipes will be conducted.

Author Contributions: Conceptualization, J.W.; Formal analysis, J.W. and Z.M.; Funding acquisition, J.W. and Y.C.; Investigation, Z.M. and S.W.; Methodology, J.W.; Project administration, J.W.; Resources, J.W. and Y.C.; Software, J.W.; Supervision, J.W., Y.C. and S.W.; Writing—original draft, J.W.; Writing—review and editing, Z.M. and Y.C. All authors have read and agreed to the published version of the manuscript.

Funding: This research was partially funded by the Specific Research Project of Guangxi for Research Bases and Talents (AD20159061) and the Specific Research Project of Guangxi for Research Bases and Talents (AD20159078).

Data Availability Statement: The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest: The authors declare no conflicts of interest.

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