A Modified High-Selective Frequency Selective Surface Designed by Multilevel Green’s Function Interpolation Method

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Abstract: A compact high-selective band-pass frequency selective surface (FSS) with the unit cell less than \( \lambda/7 \) is presented. For the simulation of the structure, the multilevel Green’s function interpolation method (MLGFIM) using Floquet theory is adopted to accelerate the calculation of the complex unit cell. The radial basis function (RBF)-QR method is used in the interpolation, which makes the shape parameter in the RBF function not required to be retested for different periodicity. In this design, with an aperture coupling structure between the top and bottom layers patterned by triangular patches and meander lines, the FSS has two transmission zeros (TZs) on both sides of the pass-band and achieves a steep roll-off rate of 192 dB/GHz. Consequently, the FSS has high selectivity and out-of-band suppression, besides profiting from the low profile and symmetric geometry, this FSS exhibits good angular and polarization stabilities. The prototype of the proposed FSS is fabricated and good performance is obtained.

Keywords: high-selective; frequency selective surface; aperture coupling; transmission zero (TZs); multilevel Green’s function interpolation method (MLGFIM)

1. Introduction

Frequency Selective Surfaces (FSSs) are widely used in microwave and millimeter-wave areas. In [1], B. A. Munk describes an FSS as a periodic structure that transmits or reflects electromagnetic waves in a specific frequency range. FSSs play an important role in the design of modern radio communications, radar systems, and antennas by controlling the transmission and reflection characteristics of electromagnetic waves to achieve selective modulation of signals in a specific frequency range [2]. A typical FSS is made of periodic units using slot or patch elements that always perform as spatial filters, polarizers, and sensors.

Recently, many FSSs for 5G applications have been published, such as a kind of conformal, polarization insensitive high angular incident angle (AIA) stable broadband bandstop mm-Wave FSS [3], a textile-based tri-band FSS [4], and miniaturized conformal dual-band mm-wave FSS [5]. All of them can be used in 5G (N258, N257, N260, and N261) band electromagnetic interference (EMI) shielding. The FSSs can also be applied to enhance the antenna performance, an asymmetric low radar cross section (RCS) FSS-based antenna applying in a tri-band shared-aperture array is proposed [6]. Ref. [7] uses FSS to achieve a low mutual coupled four-port multiple-input multiple-output (MIMO) antenna for X-band communications. Ref [8] proposes a compact coplanar waveguide (CPW)-fed antenna with a single-layer FSS load, designed for ultra-wideband applications and achieving very high gain. With the development of flexible material, flexible FSS is used for a conformal design. A mechanically constructed flexible double-layer adjustable FSS is
developed in [9] that can synchronously modulate the resonant frequencies of two filter bands through mechanical stretching.

However, many FSSs in the above applications do not have high selectivity (sharp roll-off), and sharp roll-off can improve the ability of anti-jamming. Cascaded periodic surfaces, or the technique of inducing transmission zeros (TZs), are applied to realize high selectivity. But they are not preferred in many applications [10], due to the prominent profile of cascaded periodic surfaces. In [11], the technology of substrate integrated waveguide (SIW) cavity was employed to achieve a quasi-elliptic band-pass response, introducing two transmission zeros close to the skirts of the pass-band, which makes the roll-off sharp. An aperture-coupled resonators (ACRs)-based FSS with two TZs has been reported in [12], and high selectivity of FSS was achieved. Previously, an FSS based on a complementary compact microstrip resonant cell (CCMRC) [13] has been published in [14], two TZs at the skirt of pass-band, and angular and frequency stability are realized. However, the TZ at the high frequency is not close to the pass-band, so the roll-off rate at the high-frequency pass-band is not high.

On the other hand, there are various FSS analysis techniques, such as the finite element method (FEM), finite-difference time-domain (FDTD), method of moment (MoM), and multilevel fast multipole algorithm (MLFMA). The advantages and disadvantages of various FSS computation techniques and their scope of application are summarized in [15]. Among them, MLFMA is justified as one of the most efficient computation methods. However, this method is particularly suitable for solving the free-space problem, but it is not easy to extend this method for periodic structures because of the kernel-dependent property of the MLFMA. At the same time, it cannot strictly prove error control [16]. A kernel-independent algorithm, multilevel Green’s function interpolation method (MLGFIM) [16]-[17] based on periodic Green’s function interpolation is applied for fast simulation of the complex periodic unit. This method has been proven to reduce the computational complexity from $O(N^3)$ of MoM to $O(N\log N)$. Some strategies, e.g., the interpolation method, near and interaction list definitions, should be modified to make the MLGFIM fit for FSS simulation. Based on the simulated data and artificial neural network (ANN), the FSS design procedure could be accelerated. An inverse design method based on the design method of ANN and gradient for high degree-of-freedom FSS design is proposed, which greatly reduces the design time [18].

In this paper, a modified FSS with high selectivity and out-of-band rejection is presented. The designed FSS features two TZs on either side of the pass-band to enhance frequency selectivity. By utilizing aperture-coupled resonators, it can further improve the frequency selectivity and out-of-band rejection, and slightly tune the bandwidth. The structure is simulated by MLGFIM, the radial basis function (RBF)-QR interpolation method is used in this method, which makes the shape parameter in interpolation function not required to be retested for different periodicity. The designed FSS shows the distinct advantages of high-frequency selectivity and out-of-band rejection, low profile, small periodicity (less than $\lambda_0/7$), and stable angle stability. The testing results are included to demonstrate the practical performance of this design.

2. Design of High-Selectivity FSS

Physically, a unit cell of an FSS can be regarded as an equivalent resonance circuit when illuminated by an incident wave. The metal strips of the unit cell can be treated as equivalent inductance when the E-field of the incident wave is parallel to the metal strips. Also, the slot of the unit cell can be considered as capacitive when the E-field of the incident wave is perpendicular to the slot. Hence, the resonant frequency is determined by the formula $f = 1/(2\pi\sqrt{LC})$, where $L$ and $C$ represent the equivalent inductance and capacitance of the unit cell, respectively. According to filter theory, the parallel $LC$ resonators can generate transmission zeros. When two transmission zeros are positioned close to the two sides of the pass-band, they can improve the sharpness of the filter response in the transitions between the pass-band and the stop bands.
The previous FSS with two metal layers can realize high selectivity with two TZs [14]. The unit cell is inspired by CCMRC. This element consists of two metal layers and a dielectric substrate. The metal shapes on the top layer and bottom layer are the same. Each unit cell on the top and bottom layers is composed of four triangular patches, connected at the corners of the square loop by metallic meander lines. The resonance frequency can be adjusted by using meander wires of different lengths [14]. However, the maximum transmission response of the stop-band on the low frequency side is about −8.3 dB. The TZ on the right side is at some distance from the pass-band, which deteriorates the sharpness of the filter response. Hence, adding a coupling aperture between the top layer and the bottom layer can solve this problem. Figure 1 shows the topology of the FSS with coupling aperture interlayer. The resonance frequency can be tuned by using different lengths of meander lines. With a dielectric constant of 2.65 and loss tangent of 0.001, F4B-2 substrates are selected for the design. The dimensions of the designed FSS are listed in Table 1.

![Figure 1. Topology of the proposed FSS. (a) 2 x 2 unit cell structure of the proposed FSS. (b) Unit cell of the FSS. (c) aperture coupling interlayer.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>D</th>
<th>a₁</th>
<th>a₂</th>
<th>a₃</th>
<th>b₁</th>
<th>b₂</th>
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<td>0.8</td>
<td>0.4</td>
<td>1.15</td>
<td>0.48</td>
<td>0.8</td>
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<table>
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<tr>
<th>Parameter</th>
<th>c₁</th>
<th>c₂</th>
<th>c₃</th>
<th>w</th>
<th>h₁, h₂</th>
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<tr>
<td>Value (mm)</td>
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<td>2.6</td>
<td>1.4</td>
<td>0.2</td>
<td>1.24</td>
</tr>
</tbody>
</table>

Since FSSs are essentially resonators, they behave as coupled resonator waveguides to the incident plane wave when stacked together [19]. A coupling aperture can be added to control the coupling strength between the top and bottom resonators. With different sizes of r, the frequency response of the FSS will be varied.

An equivalent circuit model is proposed in Figure 2 to illustrate the mechanism behind the generation of TZs. In this design, the loop traces between the unit cells are represented by inductor L₂, the gap between the triangular patches can be modeled by capacitor C₁, and the metallic meandering lines are modeled by inductor L₁. For both of the
transmission lines, the characteristic impedance is $Z_T$. In addition, the inductor $L_3$ denotes the coupling aperture between the top layer and the bottom layer. The coupling effect between the two inductors ($L_2$) is represented by a mutual coupling coefficient $K_1$, and correspondingly, the coupling effect between $L_1$ is represented by a mutual coupling coefficient $K_2$. Furthermore, mutual coupling coefficients $K_{11}$ and $K_{21}$ represent the coupling effect between $L_2$ and $L_3$, and $L_1$ and $L_3$, respectively. By adjusting these parameters, two transmission zeros near the skirt of the pass-band can be realized.

![Figure 2. Equivalent circuit mode of the proposed FSS.](image)

3. The Analysis of the FSS Using MLGFIM

The structure of the FSS is complicated and thus the simulation will be time-consuming. The Floquet theorem enables the analysis of FSSs to be confined to a single unit cell; however, the simulations are still time-consuming when there are fine features in the unit cell, especially for the optimization of the size of the geometry. In this paper, we use an integral equation-based fast algorithm, i.e., MLGFIM [17], to accelerate the computation. Because the periodic Green’s function is required for the unit cell simulation, the kernel-independent MLGFIM is a good candidate for the efficient integral equation solver with the complex Green’s function kernel.

Consider the periodic FSS structure shown in Figure 3.

![Figure 3. 2-D pictorial representation of a periodic FSS.](image)

Using VSIE, the total scattered electric field can be expressed as the sum of the field scattered from the surface of the conducting objects and the volume of the dielectric objects, viz.,

$$E^{scat}(r) = E_{x}^{scat}(r) + E_{y}^{scat}(r)$$

where

$$E_{x}^{scat}(r) = -i k J \int d r' \left[ J_s(r') + \frac{1}{k_0^2} \nabla \cdot J_s(r') \nabla \right] G(r, r'),$$

$$(* = S \ or \ V)$$

and $J$, $k$, $r$, $r'$, $\eta$, $G(r, r')$ and $E^{scat}(r)$ are the current density, wavenumber, observation point, source point, impedance of the host medium, periodic Green’s function, and
scattering electric field, respectively. (2) is the mixed potential integral equation [20], which describes the scattered electric field excited by the surface (* = S) and volume (* = V) current. For the field point in dielectric region or on the conducting surface, the integral equations are respectively written as

$$E^{\text{inc}}(\mathbf{r}) = \frac{-D(\mathbf{r})}{\varepsilon(\mathbf{r})}E^{\text{inc}}_v(\mathbf{r}) - E^{\text{inc}}_s(\mathbf{r}), \quad \mathbf{r} \text{ in } V$$

and

$$E^{\text{inc}}(\mathbf{r}) |_{\text{in}} = -E^{\text{inc}}_v(\mathbf{r}) |_{\text{in}} - E^{\text{inc}}_s(\mathbf{r}) |_{\text{in}}, \quad \mathbf{r} \text{ on } S$$

Using the Galerkin method, the integral equation can be converted to $N \times N$ dense matrix equations, $\mathbf{\tilde{A}} \mathbf{x} = \mathbf{b}$. After decomposing the entries of matrix $\mathbf{\tilde{A}}$ into scalar form, each part has the same expression, viz.:  

$$\tilde{A}_{ij} = \int_{i} d\mathbf{r} \int_{j} d\mathbf{r}' \tau_i(\mathbf{r}) \varphi_j(\mathbf{r}') G(\mathbf{r}, \mathbf{r}')$$

where $\tau_i(\mathbf{r})$ and $\varphi_j(\mathbf{r}')$ are respectively related to the weighting function and the basis function. Ewald’s transformation [21], which efficiently combines both spatial and spectral formulations of the periodic Green’s function, is applied to speed up the evaluation of the periodic Green’s function. Besides, since the unit cells are actually connected, the periodic boundary condition must be applied. Suppose there are two half-basis functions on the two boundaries along the $\rho_a$-direction (shown in Figure 3). According to the Floquet theorem, which relates the currents at different unit cells, we can obtain the relation of the currents corresponding to the half-basis functions, which can be written as [17]

$$\mathbf{J}_1 = \mathbf{J}_2 \cdot \exp \left( ik_0 \hat{k}_0^a \cdot \mathbf{p}_a \right)$$

where

$$\hat{k}_0^a = -\sin \theta' \cos \phi' \hat{x} - \sin \theta' \sin \phi' \hat{y}$$

In this case, these two half-basis functions can be combined into a complete basis function to satisfy the periodic boundary condition (6). Afterward, we can solve for the integral Equations (3) and (4).

To implement MLGFIM, the definitions of neighbor and interaction list are required to be modified accordingly as [17] for periodic structures comparing with aperiodic-based MLGFIM. By using the Green’s function interpolation method, the Green’s function can be expressed as:

$$G(\mathbf{r}, \mathbf{r}') = \sum_{l=1}^{K_l} \sum_{n=1}^{K_n} \omega_{n,l}(\mathbf{r}) \omega_{n,l}(\mathbf{r}') G(\mathbf{r}_{n,l}, \mathbf{r}'_{n,l})$$

where $\mathbf{r}_{n,l}$ and $\mathbf{r}'_{n,l}$ are the $i$-th and $j$-th interpolation points in field group $m$ and source group $n$, $\omega_{n,l}(\mathbf{r})$ and $\omega_{n,l}(\mathbf{r}')$ are the $i$-th and $j$-th interpolation functions, respectively, and $K_l$ is the number of interpolation points at level $l$. Substituting (8) into (5), then the MLGFIM will be implemented. Because the RBF used as an interpolation function in MLGFIM is very sensitive to the shape parameter $c$, this parameter requires to be retested when the interpolated function changes. This is not a problem for free-space Green’s function interpolation, because the Green’s function is only related to the relative locations of source point and field point. However, the value of periodic Green’s function is not only related to the relative locations, but also related to the periodicities $\rho_a$ and $\rho_b$. Hence,
for each periodic structure, the shape parameter should be retested. This problem can be remedied by using RBF-QR. Consider a function \( f(\mathbf{r}) \) in an influence domain that has a set of \( N \) arbitrarily distributed nodes with corresponding values \( \{ f(\mathbf{r}_i) \}_{i=1}^N \). Applying the RBF \( \varphi(|\mathbf{r} - \mathbf{r}_i|) \), the approximation function \( f_a(\mathbf{r}) \) is obtained:

\[
f_a(\mathbf{r}) = \left[ \varphi_1 \cdots \varphi_N \right] \Phi^{-1} \left[ \begin{array}{c} f(\mathbf{r}_1) \\ \vdots \\ f(\mathbf{r}_N) \end{array} \right]
\]

(9)

where \( \varphi_i = \varphi(|\mathbf{r} - \mathbf{r}_i|) \) and the entries of matrix \( \Phi \) are \( \Phi_{ij} = \varphi(|\mathbf{r}_i - \mathbf{r}_j|) \). For the Gaussian RBF, we use Taylor expansion and spherical-Chebyshev expansion, and obtain:

\[
\varphi_i = \exp\left(-c(|\mathbf{r} - \mathbf{r}_i|)^2\right) = \sum_{p=0}^{\infty} \sum_{m=0}^{\infty} d_{j,m,v} f_{j,m,v}(\mathbf{r}_i) \Phi_{j,m,v}(\mathbf{r})
\]

(10)

where \( p = \text{mod}(j, 2) \) and \( j_{\text{trunc}} \) is the number of terms of the truncated Taylor expansion. Other coefficients in function (10) are defined as

\[
d_{j,m,v} = 2^{3p+4m+1} \frac{((j+p+2m)/2)!}{((j-p-2m)/2)!((j+1+p+2m)!}
\]

(11)

\[
g_{j,m,v}(\mathbf{r}_i) = t_{j-2m,v} e^{-c|\mathbf{r}_i|/2} Y_{j+2m+p}^v(\theta, \phi) z F_j(\rho_{j,m,v} c^2 R_i^2)
\]

(12)

\[
T_{j,m,v}(\mathbf{r}) = e^{-c|\mathbf{r}|/2} R^{2m} Y_{2m+p}^v(\theta, \phi) T_{j-2m}(R)
\]

(13)

where

\[
\begin{align*}
Y_{\mu}^v(\theta, \phi) = & P^v_{\mu}(\cos \theta) \cos(v \phi), v = 0, \ldots, \mu \\
Y_{\mu}^v(\theta, \phi) = & P^v_{\mu}(\cos \theta) \sin(v \phi), v = 1, \ldots, \mu
\end{align*}
\]

(14)

\[
\mathbf{r} = R(\sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z})
\]

(15)

\[
\rho_{j,m} = \left[ (j-2m+1)/2, (j-2m+2)/2 \right]
\]

(16)

\[
\sigma_{j,m} = \left[ (j-2m+1, (j-2m+2)/2, (j+2m+3)/2 \right]
\]

(17)

and \( y_0 = 0.5, y_0 = 1(v > 0), t_0 = 0.5, t_{i-2m} = 1(j > 2m) \). \( P^v_{\mu}(x) \), \( T_j(x) \) and \( z F_j(x) \) are the normalized associated Legendre function, Chebyshev polynomial of the first kind, and hypergeometric function, respectively. Then we obtain the following relation using the expansion (10)

\[
\begin{bmatrix}
\varphi_1 \\
\varphi_2 \\
\vdots \\
\varphi_N
\end{bmatrix} =
\begin{bmatrix}
g_{0,0,0}(\mathbf{r}_1) & g_{0,0,-1}(\mathbf{r}_1) & \cdots & g_{0,0,(j-2m+2)/2}(\mathbf{r}_1) \\
\vdots & \vdots & \ddots & \vdots \\
g_{0,0,0}(\mathbf{r}_N) & g_{0,0,-1}(\mathbf{r}_N) & \cdots & g_{0,0,(j-2m+2)/2}(\mathbf{r}_N)
\end{bmatrix}
\begin{bmatrix}
d_{0,0} & 0 & \cdots & 0 \\
0 & d_{1,0} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & d_{j-2m+1,(j-2m+2)/2}
\end{bmatrix}
\begin{bmatrix}
T^{(0)}_{0,0}(\mathbf{r}) \\
T^{(1)}_{0,1}(\mathbf{r}) \\
\vdots \\
T^{(j-2m+1)}_{0,(j-2m+2)/2}(\mathbf{r})
\end{bmatrix}
\]

(18)

or
\[ \Psi = \tilde{\mathbf{R}} \mathbf{d} \cdot \mathbf{T}(\mathbf{r}) = \tilde{\mathbf{Q}} \cdot \mathbf{R} \cdot \mathbf{d} \cdot \mathbf{T}(\mathbf{r}) = \tilde{\mathbf{Q}} \begin{bmatrix} \tilde{\mathbf{R}}_1 & \tilde{\mathbf{R}}_2 \end{bmatrix} \begin{bmatrix} \mathbf{d}_1 & 0 \\ 0 & \mathbf{d}_2 \end{bmatrix} \cdot \mathbf{T}(\mathbf{r}) \]  

(19)

where \( \tilde{\mathbf{R}}_1 \) is upper triangular and both \( \tilde{\mathbf{R}}_1 \) and \( \mathbf{d}_1 \) are \( N \times N \). Finally, the equation (9) can be rewritten as:

\[ f_\omega (\mathbf{r}) = \mathbf{X}'(\mathbf{r}) \begin{bmatrix} \mathbf{T}(\mathbf{r}_1) & \mathbf{T}(\mathbf{r}_2) & \ldots & \mathbf{T}(\mathbf{r}_n) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{T}' \end{bmatrix}^{T} \begin{bmatrix} \mathbf{R} \end{bmatrix} \]  

(20)

where

\[ \mathbf{X}(\mathbf{r}) = \begin{bmatrix} \mathbf{d}_1 & \mathbf{d}_2 \end{bmatrix} \begin{bmatrix} \mathbf{Q}_1^{-1} & \mathbf{Q}_2^{-1} \end{bmatrix} \cdot \mathbf{T}(\mathbf{r}) = \begin{bmatrix} \mathbf{T} \end{bmatrix} \begin{bmatrix} \mathbf{R} \end{bmatrix} \]  

(21)

Because the RBF-QR method is insensitive to the shape parameter \( c \), it is not required to retest the shape parameter when the interpolated function changes. Figure 4 shows the maximum interpolation error for the periodic Green’s function interpolation. In this figure, the periodicities of the structure are set as \( \rho_0 = \rho_1 = 20 \, \text{mm} \) and the interpolation box size in MLGFIM is assumed to be \( l_0 = \lambda/4 \). From the interpolation result using conventional RBF in Figure 4, it is observed that when the parameter \( c \) is small, the interpolation errors are large because of the ill-conditioning issue. Increasing the shape parameter alleviates the conditioning problem, but when \( c > 12 \), the interpolation errors increase again, due to the smoothness of RBF. Differently from the conventional RBF method, the results using RBF-QR method are very stable.

![Figure 4](image_url)  

**Figure 4.** Maximum interpolation error for periodic Green’s function interpolation.

With the modified MLGFIM, the proposed FSS can be simulated quickly. For the design of the proposed FSS, there are 12 parameters to be optimized. To sweep these parameters, many iterations of electromagnetic simulation are required. For each simulation, there is only about 8 s for the matrix solution at one frequency. The solution time is much quicker than commercial software. After parameter sweep, the structure is designed and 11 parameters among them are listed in Table 1. To show the effect of the coupling aperture interlayer, Figure 5 shows the full-wave simulation results of transmission responses of the proposed FSS with different coupling aperture dimensions of \( \mathbf{r} \). As the coupling aperture dimension decreases, the TZ on the right side moves toward the left, and the transmission response of the stop-band rapidly declines. Moreover, this method can...
slightly change the bandwidth of the FSS. When \( r = 3.8 \) mm, it is observed that two TZs close to the narrow pass-band are achieved. The two TZs at the frequency of 4.74 GHz and 5.46 GHz make the FSS achieve high selectivity. The transitions between the pass-band and the stop bands have steep roll-offs, about 389 dB/GHz on the low frequency side and 192 dB/GHz on the high frequency side.

![Transmission response graph](image)

**Figure 5.** Transmission responses of the proposed FSS with different coupling aperture dimensions.

And, the roll-off rate is defined as

\[
\zeta = \frac{\alpha_{\text{max}} - \alpha_{\text{min}}}{f_2 - f_1}
\]

where \( \zeta \) is measured in dB/GHz, \( \alpha_{\text{min}} \) is the −3 dB attenuation point, \( \alpha_{\text{max}} \) is the −10 dB attenuation point, \( f_1 \) is the −10 dB stop-band frequency and \( f_2 \) is the −3 dB cutoff frequency. The insertion loss within the pass-band is kept below 1.1 dB at its maximum. The electrical size of one unit at the pass-band is about \( \lambda_0/7 \), where \( \lambda_0 \) represents the free-space wavelength corresponding to the resonant frequency.

To validate the circuit model, the full-wave simulation and the equivalent circuit model (shown in Figure 2) results are compared. In this comparison, \( r \) is chosen as 3.8 mm. And in this case, the parameters in Figure 2 are: \( C_1 = 0.991 \) pF; \( L_1 = 1.856 \) nH; \( L_2 = 9.995 \) nH; \( L_3 = 0.119 \) nH; \( K_1 = -0.192 \); \( K_{11} = 0.803 \); \( K_2 = -0.095 \); \( K_{21} = 0.139 \); \( Z_0 = 377 \) Ω. For the selection of the characteristic impedance \( Z_T \), we have added the frequency response for the different impedances \( Z_T \) as shown in Figure 6. The comparison shows that the pass-band moves to higher frequency and the bandwidth becomes narrower as the characteristic impedance \( Z_T \) increases. The frequency response is close to the full-wave simulation when \( Z_T \) is equal to 50 Ω, and the results are compared in Figure 7. At the low frequency, the full-wave simulation and the equivalent circuit simulation results are basically the same, and the TZ is perfectly coincident. However, there is some deviation between the two curves at the high frequency. The deviation is because the parasitic parameters of the layout cannot be fully represented by the equivalent circuits. Besides the main LC components described in Figure 2, there still exist other parasitic components, e.g., the capacitance between different metal layers. It is difficult to fully extract all parasitic parameters, which will also make the equivalent circuit very complex. Because the main components are extracted in the equivalent circuit and the two curves are close, it can validate the circuit model of the proposed FSS with the coupling aperture interlayer.
Figure 6. Comparison of frequency response for different characteristic impedances Zt.

Figure 7. Comparison of the transmission coefficient between the equivalent circuit model and full-wave simulation.

Figure 8 demonstrates the current distribution on the surface of the metal structure at transmission zero frequencies 4.74 GHz and 5.46 GHz and pass-band frequency 4.86 GHz, where \( r \) of the coupling aperture dimension is 3.8 mm. As shown in Figure 8a, at the lower TZ frequency of 4.74 GHz, the current direction in the L1–C1 path consisting of triangular patches and meander lines is opposite in the higher and lower layers, while the current direction in the L2–L3 path consisting of the outer loop path and the coupling aperture is also opposite. When the electromagnetic wave passes through this FSS structure, the induced current generated is in the opposite direction, which generates a reverse electromagnetic field, so the passage of the incident electromagnetic wave is inhibited. Thus, the frequency of 4.74 GHz behaves as a stop-band frequency. Similarly, Figure 8c shows that at the higher TZ frequency of 5.46 GHz, the direction of the current in the outer loop path is also opposite in the upper and lower layers of the FSS, which inhibits the passage of the incident electromagnetic wave. The frequency of 5.46 GHz exhibits a stop-band frequency as well. The presence of the coupling aperture layer allows the opposite current direction to greatly improve the rejection of the blocking band. On the contrary, as shown in Figure 8b, all the paths of the upper, lower, and middle layers have the same current direction at the frequency of 4.86 GHz, so is electromagnetically enhanced. The frequency of 4.86 GHz exhibits the pass-band frequency.
Figure 8. Current distribution in (a) 4.74 GHz, (b) 4.86 GHz, and (c) 5.46 GHz (The arrow represents the direction of current flow).

Figure 9 illustrates the performance of the proposed FSS under various incident angles and polarization states. It is observed that the proposed FSS demonstrates a remarkable stability across a wide range of incident angles up to 45° for both polarizations. The deviation is merely 0.4% for TE polarization waves, when the incident angle increases to 45°. The insertion loss is increased for the TE polarization waves as the angle of incidence angle increases. Sharp harmonics appear in the low frequency stop-band for the oblique incidence TE polarization waves. But it is below −20 dB, so it can be ignored. For the TM
polarization, the bandwidth is increased slightly as the incidence angle increases. The center frequency has 0.7% deviation for TM polarization of 45° incident angle.

![Graph](image)

**Figure 9.** Transmission responses of the proposed FSS under different incident angles: (a) TE mode; (b) TM mode.

### 4. FSS Fabrication and Measurement

In order to validate the designed structures in the above section, prototypes of the proposed FSSs are fabricated using the normal PCB technique, and measured using the free-space measurement method. Figure 10 shows the fabricated FSS samples. The three-metal-layer FSS was fabricated on an F4B-2 board. The size of the PCB is 176 × 176 mm². For the three-metal-layer FSS, two PCBs are compressed by plastic screws. The FSSs are measured in a microwave anechoic chamber. Two standard horn antennas are placed on the two sides of the FSSs as the transmitting and receiving antennas. The measurement setup is shown in Figure 11. The measured and simulated transmission coefficients of the FSS using our calculation and CST for the normally incident plane wave are shown in the Figure 12. Our calculation and CST results match very well. The frequency response of the measured results has some shift compared with simulated one. That is because the air gap between two neighboring PCBs makes the resonance frequency slightly shifted. To verify the angle and polarization stability of the proposed FSSs, the transmission coefficients of the FSSs under different incident angles and polarizations are shown in Figure 13. Although the insertion loss for inclined incidence is larger than normal incidence, the resonant frequencies between the different angles and polarizations are very stable.
Figure 10. The fabrication photograph of the proposed three-metal-layers FSS prototype.

Figure 11. Measurement setup.

Figure 12. Measurement versus simulation of the FSS.
Figure 13. Measured transmission curves with different polarizations under 0 degree and 45 degree incidence.

In addition, the performance comparisons with other previously reported FSSs are shown in Table 2. The roll-off rates of their designed FSSs in [19,22,23] are around 8 dB/GHz to 40 dB/GHz, while our results are much better than the above structures, with roll-off rates of 389 dB/GHz at the low frequency and 192 dB/GHz at the high frequency. In practical engineering applications, especially the FSSs applied on radomes, the vast majority of units are at large angle incidence to the incoming wave, so the FSSs must have good angular stability. The angular stability of our design is also better than the other three papers. The deviation is only less than 0.7% when the incident angle increases to 45° for the proposed design. And the other parameters listed in Table 2 of this design is comparable with the other three papers.

Table 2. Performance comparisons for the FSSs.

<table>
<thead>
<tr>
<th>Refs.</th>
<th>Roll-Off Rate (dB/GHz)</th>
<th>Angular Stability</th>
<th>Polarization Stability</th>
<th>Insertion Loss (dB)</th>
<th>Return Loss(dB)</th>
<th>VSWR</th>
<th>Electrical Size (λ₀)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[19]</td>
<td>35.6</td>
<td>Up to 40°</td>
<td>TE incidence deviates TM</td>
<td>&lt;1</td>
<td>27</td>
<td>1.09</td>
<td>1/9</td>
</tr>
<tr>
<td>[22]</td>
<td>8.75</td>
<td>Up to 45°</td>
<td>TE incidence matches well with TM</td>
<td>&lt;1</td>
<td>15</td>
<td>1.43</td>
<td>1/10</td>
</tr>
<tr>
<td>[23]</td>
<td>14.2</td>
<td>Up to 30°</td>
<td>TE incidence matches well with TM</td>
<td>&lt;1.5</td>
<td>6.25</td>
<td>2.90</td>
<td>1/2</td>
</tr>
<tr>
<td>This work</td>
<td>192</td>
<td>Up to 45°</td>
<td>TE incidence matches well with TM</td>
<td>&lt;1.1</td>
<td>14</td>
<td>1.28</td>
<td>1/7</td>
</tr>
</tbody>
</table>

5. Conclusions

A modified band-pass FSS inherits the idea of CCMRC with two TZs was proposed to improve frequency selectivity of band-pass FSSs. The simulation method MLGFIM is modified for the adaption of FSS simulation. The roll-off rate of this design is 192 dB/GHz, and consequently the design achieves high selectivity and out-of-band suppression. The structure also realizes good performance in angular and polarization stabilities. The deviation is only less than 0.7% when the incident angle increases to 45° for the proposed design. An equivalent circuit model is given to provide a physical understanding of the mechanism behind the generation of the TZs. The current distribution on the layout is provided to explain the frequency response. A prototypical sample of the proposed FSS was fabricated and the performance of each design was measured, and the results validate the performance of the design.

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