

Article

Improved Implementation of Chua's Circuit on an Active Inductor and Non-Autonomous System

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Abstract: Chua's circuit is a well-established model for studying chaotic phenomena and is extensively implemented in fields like encrypted communication. However, a traditional Chua's circuit has large volume, high component precision requirements and limited adjustable parameter range, which are not conducive to application. In order to solve these problems, we propose an improved implementation of Chua's circuit on an active inductor and non-autonomous system. First, we adopt the strategy of using active inductors instead of traditional passive inductors, achieving the miniaturization of the circuit and improving the accuracy of inductance. In addition, we present the theory of substituting non-autonomous systems for classical autonomous systems to reduce the requirements for the accuracy of components and improve the robustness of the circuit. Lastly, we connect the extension resistor in parallel with Chua's diode to optimize circuit structure, thereby increasing the range of the adjustable parameter. Based on the three improvements above, experiments have shown that the average maximum error tolerance of components of our improved design has been increased from 1.88% to 7.38% when generating a single vortex, and from 4.73% to 12.61% when generating a double vortex, compared with the traditional Chua's circuit. The range of the adjustable parameter has been increased by 195.83% and 36.98%, respectively, when generating a single vortex and double vortex. In summary, our improved circuit is more practical than the traditional Chua's circuit and has good application value.

Keywords: Chua's circuit; active inductor; non-autonomous system; robustness; range of the adjustable parameter



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1. Introduction

Chua's circuit is widely recognized as a significant example among typical chaotic signal generation circuits. In comparison to other nonlinear circuits such as a Van der Pol oscillator and Colpitts oscillator, Chua's circuit is the simplest design in structure, since it has only five circuit elements. However, Chua's circuit is dynamically the most complex among all nonlinear circuits and systems, described by a 21-parameter family of continuous odd-symmetric piecewise-linear vector fields [1]. Due to characteristics of nonlinearity, determinism, and sensitivity to initial conditions, Chua's circuit is widely used in fields such as encrypted communication, random number generation, and sensors [2–4]. Therefore, studying how to improve the design to make Chua's circuit more favorable to application is of great significance.

Currently, the integration of a circuit is an important trend in electronics. When designing integrated circuits, the traditional passive inductor used in Chua's circuit has large volume and is accompanied by parasitic series equivalent resistance [5], making it difficult to achieve integration. To address this issue, Demirkol et al. proposed a method of using

active components to form active inductors which can theoretically achieve miniaturization and precise control [6]. This theory has been accepted by most scholars and a variety of active inductors have been designed. For example, Xiaofei Duan et al. from the Chinese Academy of Sciences introduced a lossless active inductance type and integrated it into Chua's circuit [7]. However, their design suffered from poor waveform quality, even resulting in incomplete double vortex generation. Among numerous active inductors, the Riordan gyrator is widely recognized due to its simple structure and stable performance. For instance, Francisco et al. designed a floating meminductor emulator on a Riordan gyrator, and Ananda et al. proposed a high-frequency meminductor emulator using a Riordan gyrator, both with good performance [8,9]. For these reasons, we adopted the strategy of using active inductors instead of traditional passive inductors and the Riordan gyrator was adopted in this improved implementation of Chua's circuit.

Efforts have been made to improve the circuit structure in multiple aspects, especially focusing on Chua's diodes, to advance the application of Chua's circuit [10,11]. For example, Elwakil et al. combined attractive features of the current feedback op amp (CFOA) operating in both voltage and current modes to construct the active three-segment voltage-controlled nonlinear resistor. This reduces the component count and extends the chaotic spectrum to higher frequencies [12]. By replacing the Chua's diode with a first-order hybrid diode circuit, Bocheng Bao et al. presented a fourth-order modified Chua's circuit which exhibits complicated nonlinear phenomena including self-excited attractors, coexisting self-excited attractors, hidden attractors, and coexisting hidden attractors [13]. Additionally, Fei Yu et al. constructed a compound hyperbolic tangent–cubic nonlinear function in a canonical Chua's circuit, and the proposed multi-scroll Chua's circuit also exhibited rich dynamic behaviors, like coexisting multiple attractors, transient period, intermittent chaos, and offset boosting [14]. While previous studies have primarily focused on modifying the circuit to generate diverse dynamic behaviors, little attention has been given to improve circuit reliability, which is quite crucial for practical applications.

In applications, Chua's circuit manifests in two forms. The first form involves selecting resistors with different resistance values through switch to generate different signals, as is shown in Figure 1a [15,16]. Despite straightforward operation, this method suffers from poor robustness as Chua's circuit is extremely sensitive to initial conditions and has high requirements for component accuracy. The second form involves changing the value of the resistor R by sliding the rheostat, resulting in a single and double vortex, as is shown in Figure 1b [17]. Although this design is robust, the application is limited by the small range of the adjustable resistor R .

Due to the defects in the structure of the traditional Chua's circuit, there are weaknesses in the above two forms. To study this issue, Yeong-Chan Chang et al. made an attempt by using a hybrid adaptive-robust tracking control scheme to study nonlinear chaotic Chua's circuits involving plant uncertainties and external disturbances [18], but it did not fundamentally improve circuit robustness.

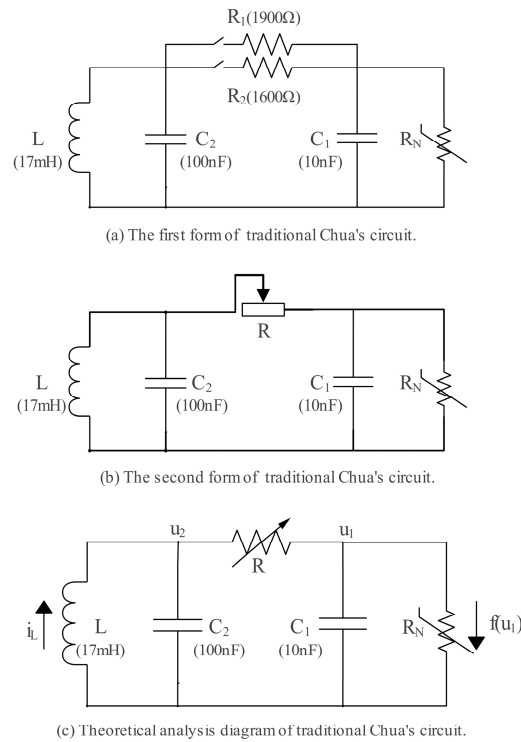


Figure 1. Traditional Chua's circuit.

In order to compensate for the structural defects of the traditional Chua's circuit, an improved implementation to the structure of Chua's circuit is proposed in this paper. The main improvements we proposed are summarized as follows:

- We adopt the strategy of using active inductors instead of traditional passive inductors. Since the volume of the op amp and resistors are much smaller than that of the inductor coils, this design enables the miniaturization and integration of circuits. Meanwhile, the accuracy of the inductor is greatly improved because the inductance value can be controlled precisely by adjusting the resistance value.
- We present the theory of substituting non-autonomous systems for classical autonomous systems. According to the state equation of the circuit and the chaotic conditions of a system, the excitation current source we added can reduce the requirements for the accuracy of components and improve the robustness of the circuit.
- We connect the extension resistor in parallel with Chua's diode to optimize the circuit structure, increasing the range of the adjustable parameter.

In summary, the improvements above compensate for the defects of the Chua's circuit structurally and improve the reliability. As a result, the shortcomings of the two forms of Chua's circuit can be perfectly solved. For ease of presentation and evaluation of the robustness of the circuit, we defined several concepts in this paper [19,20].

- The resistor connected in parallel with Chua's diode is termed the extension resistor due to its functional role.
- The maximum error tolerance Y_i of component y_i is defined as $Y_i = \frac{\Delta m_{y_i}}{m_{y_i}} \times 100\%$. m_{y_i} is the standard value of component y_i and Δm_{y_i} is the maximum allowable deviation of component y_i .
- The average maximum error tolerance X of components $y_1, y_2, \dots, y_n$ is defined as $X = \frac{\sum_{i=1}^n Y_i}{n}$.
- In the context of the "range of the adjustable parameter", the parameter refers specifically to the adjustable resistor R connecting capacitors C_1 and C_2 , and the range refers specifically to the resistance that can create chaos in a circuit.

It is important to note that the excitation current source is considered as part of the signal generation circuit in this paper. However, some scholars suggest that when the amplitude of the excitation current source significantly exceeds what is used in this paper, it can be regarded as the coupling of two systems with the chaotic generation circuit, and changing the amplitude can produce different types of sudden oscillations [21].

The remainder of the paper is organized as follows. Section 2 reviews the traditional Chua's circuit. Section 3 introduces this improved implementation of Chua's circuit on an active inductor and non-autonomous system. Section 4 compares two circuits theoretically. Section 5 conducts experimental testing and comparison on the two circuits. Conclusions are drawn in Section 6.

2. Traditional Chua's Circuit

A traditional Chua's circuit consists of a passive inductor L , two capacitors C_1 and C_2 , an adjustable resistor R , and Chua's diode R_N [22,23], as is shown in Figure 1c. The Chua's diode R_N is shown in Figure 2.

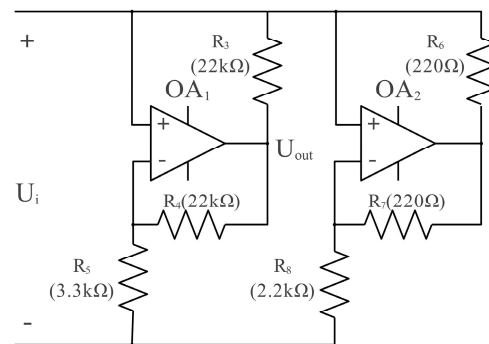


Figure 2. Chua's diode R_N .

According to Kirchhoff's Current Law and Kirchhoff's Voltage Law, the state equation of the circuit can be described as

$$\begin{aligned} \frac{du_1}{dt} &= \frac{u_2 - u_1}{RC_1} - \frac{f(u_1)}{C_1} \\ \frac{du_2}{dt} &= \frac{u_1 - u_2}{RC_2} + \frac{i_L}{C_2} \\ \frac{di_L}{dt} &= -\frac{u_2}{L} \end{aligned} \quad (1)$$

$$f(u_1) = G_b u_1 + 0.5(G_a - G_b)[|u_1 + B_P| - |u_1 - B_P|], \quad (2)$$

where $f(u_1)$ is the current flowing through the Chua's diode, G_a is the inner interval conductivity, G_b is the outer interval conductivity, and B_P is the turning point voltage of the inner and outer intervals. The self-excited circuit system described in (1) and the nonlinear resistor described in (2) are sources of chaos in Chua's circuit. Moreover, parameters in Figures 1 and 2 are presented according to the specifications proposed by American electrical engineer Leon O. Chua in 1983. Since then, this specification has been used in most practical applications of Chua's circuit [24–27].

3. Improved Implementation of Chua's Circuit on an Active Inductor and Non-Autonomous System

This improved implementation of Chua's circuit adopts the strategy of using active inductors instead of traditional passive inductors. Among numerous active inductors, we took into account the structure, performance, and stability, finally choosing the Riordan gyrator. Additionally, we present the theory of using non-autonomous systems instead of classical autonomous systems, and the excitation current source is added. Lastly, the extension resistor is connected in parallel with the Chua's diode. This improved implementation of Chua's circuit is shown in Figure 3.

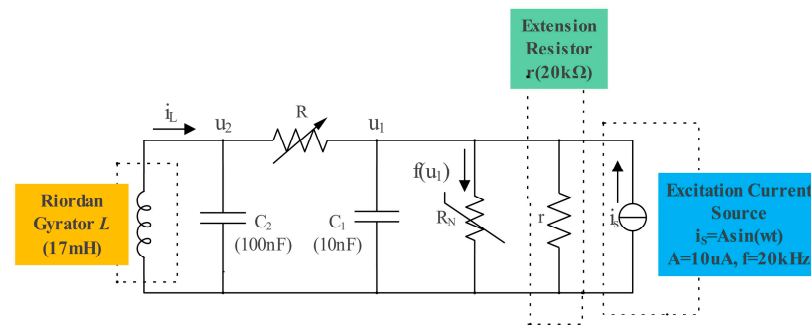


Figure 3. Improved implementation of Chua's circuit on an active inductor and non-autonomous system.

According to Kirchhoff's Current Law and Kirchhoff's Voltage Law, the state equation of the circuit can be described as

$$\begin{aligned} \frac{du_1}{dt} &= \frac{u_2 - u_1}{RC_1} - \frac{f(u_1)}{C_1} + \frac{i_s - \frac{u_1}{r}}{C_1} \\ \frac{du_2}{dt} &= \frac{u_1 - u_2}{RC_2} + \frac{i_L}{C_2} \\ \frac{di_L}{dt} &= -\frac{u_2}{L} \end{aligned} \quad (3)$$

where L refers to the equivalent inductance value of the Riordan gyrator. Please kindly note that the non-autonomous system described in (3) is the source of the circuit's chaotic phenomena, and is different from the autonomous system of traditional Chua's circuit. Parameters of the extension resistor and the excitation current source will be discussed in the following part.

3.1. Extension Resistor r

The extension resistor r is a linear resistor parallel to the Chua's diode. It plays a crucial role in altering the range of the adjustable parameter, thereby inducing chaos in the circuit by modifying the circuit's state equation. To determine the most suitable value of extension resistor r , the sine excitation current source i_s was selected to conduct a series of experiments, with an amplitude of 10 μ A and a frequency of 20 kHz. According to the experiment results, the range of the adjustable parameter that causes chaos in the circuit with the varied extension resistor r is shown in Figure 4.

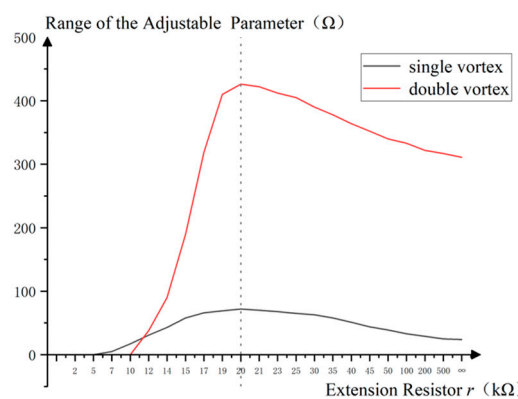


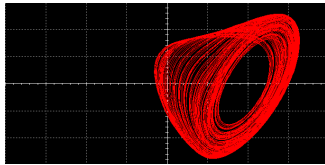
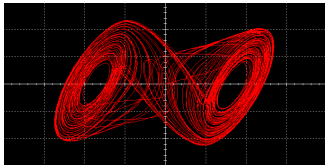
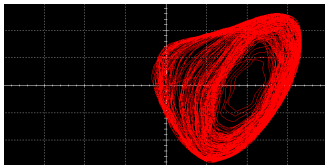
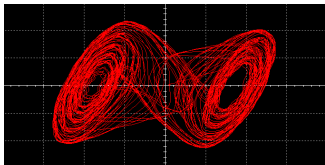
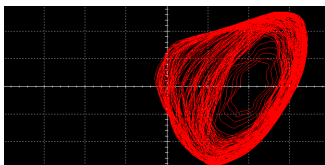
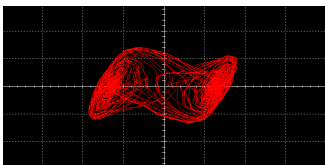
Figure 4. Range of the adjustable parameter for different values of the extension resistor r .

As is shown in Figure 4, when the resistance of the extension resistor r approaches 20 kΩ, the range of the adjustable parameter for both the single vortex and the double vortex reaches the maximum. Therefore, $r = 20$ kΩ was determined. It is also notable that there will be no double vortex when the resistance value of r is less than 10 kΩ and chaos will not appear when it is less than 5 kΩ.

3.2. Excitation Current Source i_S

The excitation current source i_S can reduce the requirements for the accuracy of the components and improve circuit robustness. We selected the extension resistor $r = 20 \text{ k}\Omega$ and compared the chaotic signals under different types of excitation current sources; sine wave, square wave, and triangular wave, whose amplitude $A = 10 \text{ }\mu\text{A}$ and frequency $f = 20 \text{ kHz}$, as is shown in Table 1.

Table 1. Chaotic signals under different types of excitation current sources.

	Single Vortex	Double Vortex
Sine Wave		
Square Wave		
Triangular Wave		

The results confirm that the chaotic signal is the smoothest when sine wave is used, which is quite important for applications such as encrypted communications. For chaotic encrypted communication systems, a smoother chaotic signal offers great advantages in terms of synchronization ease, noise resistance, spectral utilization, hardware simplicity, transmission efficiency, and BER performance [28–30]. Therefore, we chose the sine wave in our experiment. Next, we will discuss parameters of the sine wave excitation current source.

In order to determine the amplitude A and frequency f , we performed a variation in amplitude for different frequencies and measured the average maximum error tolerance X of components L , C_1 , and C_2 for corresponding conditions, as is shown in Figure 5.

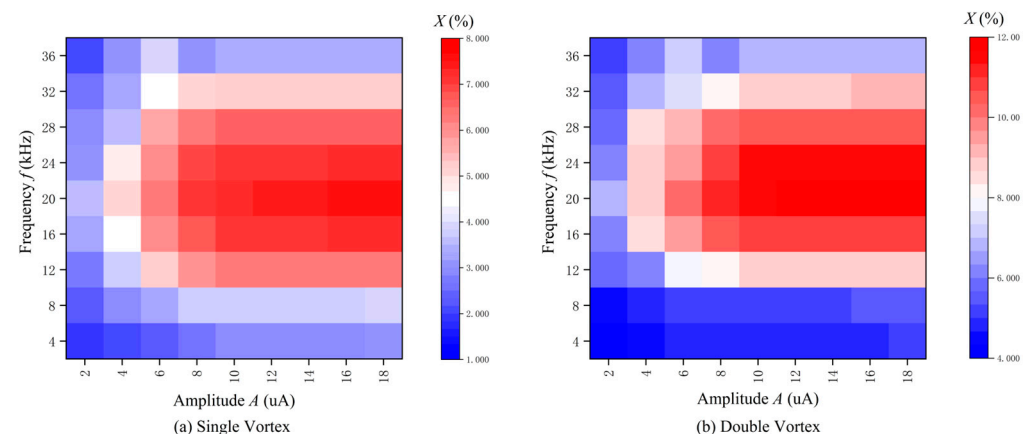


Figure 5. Average maximum error tolerance X of components L , C_1 , and C_2 for different amplitudes and frequencies.

From Figure 5, it can be easily observed that the larger the amplitude A , the greater the average maximum error tolerance X of components L , C_1 , and C_2 becomes. However, there is no significant increase in tolerance when $A > 10 \mu\text{A}$, and the chaotic signal waveform is no longer smooth and will transform into polygons when the amplitude A is larger. The amplitude $A = 10 \mu\text{A}$ is determined based on the above reasons. On the other hand, X reaches its maximum when the frequency f is around 20 kHz, so the frequency $f = 20 \text{ kHz}$ is determined.

4. Theoretical Comparison

Compared with the traditional Chua's circuit, this improved implementation of Chua's circuit has three improvements in structure: one is the change from passive inductor to active inductor, another is the improvement from an autonomous system to a non-autonomous system, and the other is the addition of the extension resistor. Since the latter two improved theoretical analyses require the Lyapunov exponent, they are put together for theoretical comparisons.

4.1. Change from Passive Inductor to Active Inductor

The inductance of traditional passive inductors depends on the distribution of the surrounding dielectrics and the number, the size, and the shape of the wound coil. In practice, not only is it bulky and difficult to adjust, but it is also accompanied by parasitic resistance, which can cause additional voltage drops and power losses, resulting in large errors.

The strategy of using active inductors instead of traditional passive inductors can be a good solution to the above problems. The Riordan gyrator consists of two operational amplifiers, four resistors, and a capacitor, with a simple structure and high stability [8,31], as is shown in Figure 6.

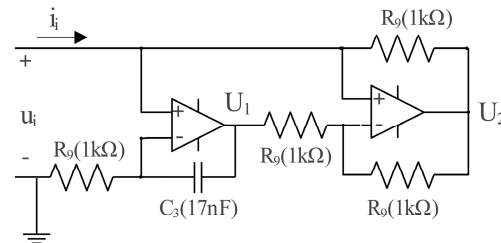


Figure 6. Riordan gyrator.

According to the characteristics of the operational amplifier, the state equation of the Riordan gyrator can be described as (4). Based on (4), we can derive the input impedance Z of the circuit, as shown in (5), and the equivalent inductance value L , as shown in (6).

$$\begin{aligned} \frac{u_i - U_2}{R_9} &= \frac{U_1 - u_i}{R_9} \\ i_i &= \frac{u_i - U_2}{R_9} \\ U_1 &= \left(1 + \frac{1}{j\omega R_9 C_3}\right) u_i \end{aligned} \quad (4)$$

$$Z(j\omega) = \frac{u_i}{i_i} = j\omega R_9^2 C_3 \quad (5)$$

$$L = R_9^2 C_3 \quad (6)$$

According to (4)–(6), the inductance value of the Riordan gyrator in this improved implementation of Chua's circuit can be precisely controlled by adjusting the value of the resistors. Moreover, this design eliminates parasitic resistance, with high accuracy and reliability, and its small size further facilitates integration. In this circuit (Figure 6), $L = 17 \text{ mH}$, therefore, R_9 could be derived as $1 \text{ k}\Omega$ and C_3 could be derived as 17 nF .

4.2. Improvement from an Autonomous System to a Non-Autonomous System and the Addition of an Extension Resistor

A traditional Chua's circuit has high component precision requirements and limited adjustable parameter range, especially when generating a single vortex. According to the state equation of the circuit and the chaotic conditions of a system, the theory of substituting non-autonomous systems for classical autonomous systems can reduce the requirements for the accuracy of the components, and the extension resistor parallel to the Chua's diode can increase the range of the adjustable parameter effectively. To determine whether a system is chaotic and prove the above theory, we introduced the Lyapunov exponent.

Chaotic systems exhibit a trademark "sensitive dependence on initial conditions" (SDIC). This means that very small perturbations will cause the resulting trajectories to separate exponentially quickly. Lyapunov exponents directly measure SDIC by quantifying the exponential rates at which neighboring orbits on an attractor diverge (or converge) as the system evolves in time. Dissipative deterministic systems that exhibit at least one positive Lyapunov exponent are defined as "chaotic". Therefore, to determine whether the circuit system is chaotic, we need to calculate the maximum Lyapunov exponent in the system [32–34].

Then, we calculate the Lyapunov exponents in this improved implementation of Chua's circuit. We can obtain (7) by deforming (3).

$$\begin{aligned} C_1 \frac{dx}{dt} + f(x) &= \frac{y-x}{R} + i_S \\ C_2 \frac{dy}{dt} + \frac{y-x}{R} &= z \\ L \frac{dz}{dt} &= -y \end{aligned} \quad (7)$$

$$f(x) = G_b x + 0.5(G_a - G_b)[|x + B_P| - |x - B_P|]$$

By assuming $x = f_1(i_S)$, $y = f_2(i_S)$, $z = f_3(i_S)$ and taking a point i_0 of i_S , the process of calculating the Lyapunov exponents is as follows:

$$a_0 = i_0, b_0 = i_0, c_0 = i_0 \quad (8)$$

$$a_1 = f_1(a_0), b_1 = f_2(b_0), c_1 = f_3(c_0) \quad (9)$$

.....

$$a_{n-1} = f_1(a_{n-2}), b_{n-1} = f_2(b_{n-2}), c_{n-1} = f_3(c_{n-2}) \quad (10)$$

The Lyapunov exponents in this improved implementation of Chua's circuit are

$$\begin{aligned} \lambda_1 &= \lim_{r \rightarrow \infty} \frac{1}{r} \sum_{k=0}^{r-1} \ln |f'_1(a_k)| \\ \lambda_2 &= \lim_{r \rightarrow \infty} \frac{1}{r} \sum_{k=0}^{r-1} \ln |f'_2(b_k)| \\ \lambda_3 &= \lim_{r \rightarrow \infty} \frac{1}{r} \sum_{k=0}^{r-1} \ln |f'_3(c_k)| \end{aligned} \quad (11)$$

We assume that

$$\lambda_{new} = \max\{\lambda_1, \lambda_2, \lambda_3\} \quad (12)$$

Similar to the above process, the Lyapunov exponents of the traditional Chua's circuit are

$$\begin{aligned} \lambda_4 &= \lim_{r \rightarrow \infty} \frac{1}{r} \sum_{k=0}^{r-1} \ln |f'_4(a_k)| \\ \lambda_5 &= \lim_{r \rightarrow \infty} \frac{1}{r} \sum_{k=0}^{r-1} \ln |f'_5(b_k)| \\ \lambda_6 &= \lim_{r \rightarrow \infty} \frac{1}{r} \sum_{k=0}^{r-1} \ln |f'_6(c_k)| \end{aligned} \quad (13)$$

We assume that

$$\lambda_{old} = \max\{\lambda_4, \lambda_5, \lambda_6\} \quad (14)$$

To answer the problem of why this design can reduce the requirements for the accuracy of components and increase the range of the adjustable parameter, we will calculate the maximum Lyapunov exponent.

4.2.1. Robustness

When generating a single vortex, the maximum Lyapunov exponent of the circuit with 5% error in components L , C_1 , and C_2 , respectively, is shown in Table 2. According to the analysis of Table 2, the maximum Lyapunov exponent of the traditional Chua's circuit could be less than zero when there is a 5% error in components L , C_1 , and C_2 , respectively, resulting in no chaos being generated. By comparison, the maximum Lyapunov exponents of this improved implementation of Chua's circuit are all greater than zero, which means that the chaotic signal can still be generated stably.

Table 2. The maximum Lyapunov exponent of the circuit under different parameter errors (single vortex).

	$\min\{\lambda_{old}(+5\%), \lambda_{old}(-5\%)\}$	$\min\{\lambda_{new}(+5\%), \lambda_{new}(-5\%)\}$
No Error	0.41	0.57
$L \pm 5\%$	−0.73	0.10
$C_1 \pm 5\%$	−0.85	0.04
$C_2 \pm 5\%$	−0.42	0.35

When generating a double vortex, the maximum Lyapunov exponent of the circuit with a 5% error in components L , C_1 , and C_2 , respectively, is shown in Table 3. According to Table 3, the maximum Lyapunov exponent of the traditional Chua's circuit may be less than zero and cannot generate chaos when there is a 5% error in C_1 . In contrast, the maximum Lyapunov exponents of this improved implementation of Chua's circuit are all greater than zero, and the circuit can still generate chaos.

Table 3. The maximum Lyapunov exponent of the circuit under different parameter errors (double vortex).

	$\min\{\lambda_{old}(+5\%), \lambda_{old}(-5\%)\}$	$\min\{\lambda_{new}(+5\%), \lambda_{new}(-5\%)\}$
No error	0.66	0.68
$L \pm 5\%$	0.03	0.38
$C_1 \pm 5\%$	−0.23	0.10
$C_2 \pm 5\%$	0.18	0.49

Based on the calculations above, this improved implementation of Chua's circuit has lower requirements for the accuracy of components and higher robustness to the deviation of components' values than the traditional Chua's circuit.

4.2.2. Range of Adjustable Parameters

We changed the value of the adjustable resistor and calculated the maximum Lyapunov exponent for different R corresponding systems, as is shown in Figure 7. The result shows that $\lambda_{old} > 0$ is a sufficient but unnecessary condition for $\lambda_{new} > 0$. Therefore, this improved Chua's circuit extends the range of chaotic operation, reducing the inconvenience and facilitating the application of this circuit.

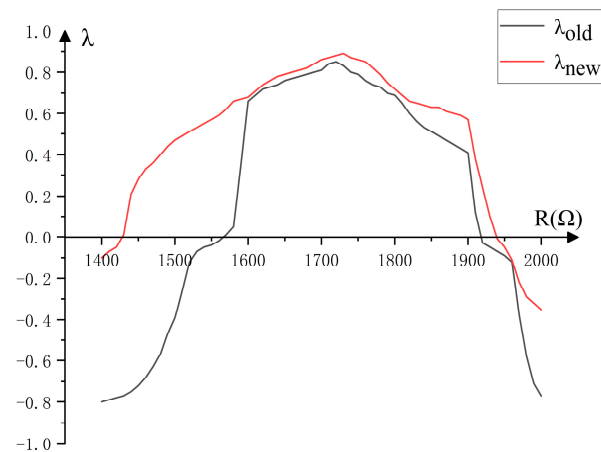


Figure 7. Maximum Lyapunov exponent in the system for different resistors R .

5. Experimental Comparison

In the experiment, the digital oscilloscope we used was a GDS-1104R made by GWINSTEK, in Suzhou, China, with a bandwidth of 100 MHz and a sampling rate of 1 GS/s, and the signal generator we used was a MFG-2260MRA made by GWINSTEK, in Suzhou, China. Circuit components were manufactured by RISYM, in Shenzhen, China. Regarding how the oscilloscope is connected to the final circuit, channel A and channel B of the oscilloscope measured the circuit nodes with voltages u_2 and u_1 , respectively. Then, we set the oscilloscope mode to $\frac{u_2}{u_1}$, displaying the waveforms. The external test circuits of traditional Chua's circuit and the improved implementation of Chua's circuit are shown in Figure 8a and Figure 8b, respectively.

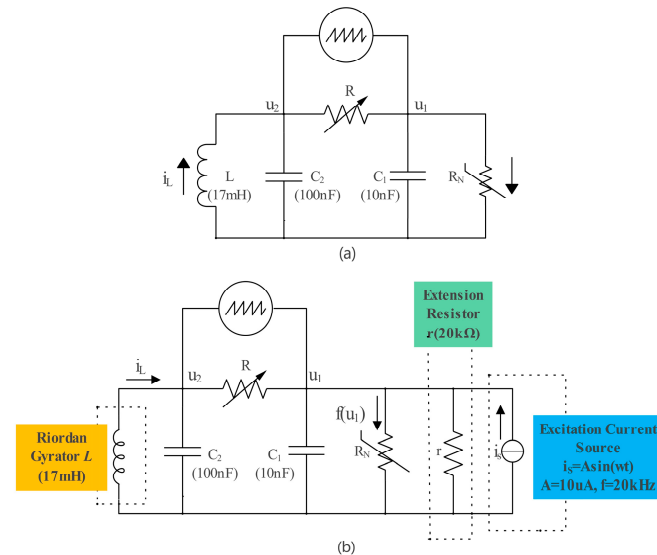


Figure 8. External test circuit.

5.1. Robustness When Generating a Single Vortex

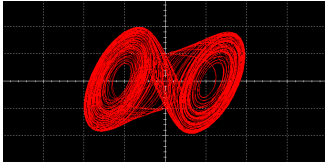
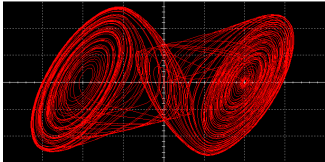
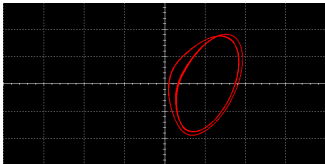
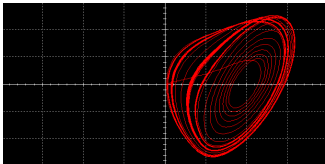
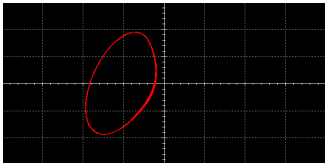
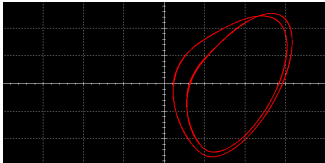
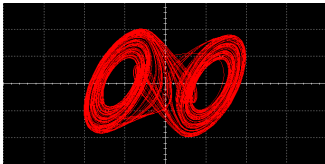
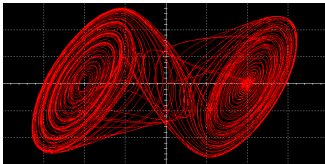
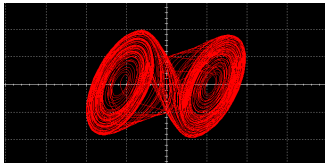
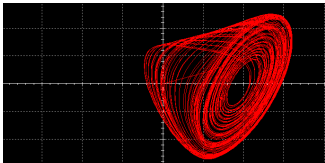
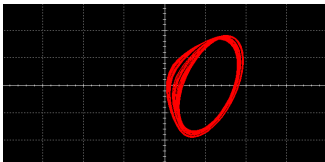
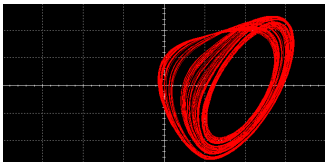
5.1.1. Test Case 1 ($R = 1890 \Omega$)

When a single vortex is generated stably, the maximum error tolerance Y of components L , C_1 , and C_2 , respectively, are shown in Table 4. When there is a 5% error in components L , C_1 , and C_2 , respectively, signals generated by the circuit are shown in Table 5.

Table 4. The maximum error tolerance γ of components L , C_1 , and C_2 , respectively (single vortex, $R = 1890 \Omega$).

	Traditional Chua's Circuit	Improved Implementation of Chua's Circuit
L	1.22%	3.91%
C_1	0.76%	1.80%
C_2	3.00%	6.25%
Average (X)	1.66%	3.99%

Table 5. Signals when there is a 5% error in components L , C_1 , and C_2 , respectively (single vortex, $R = 1890 \Omega$).

		Traditional Chua's Circuit	Improved Implementation of Chua's Circuit
L	+5%		
	-5%		
C_1	+5%		
	-5%		
C_2	+5%		
	-5%		

5.1.2. Test Case 2 ($R = 1900 \Omega$)

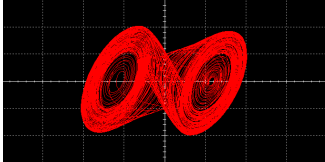
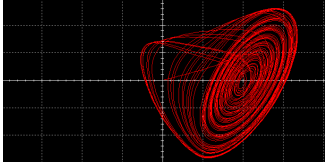
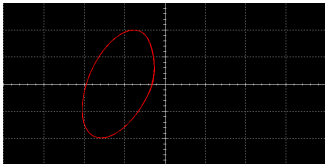
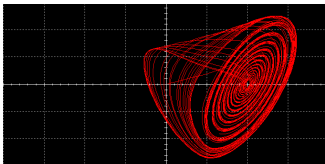
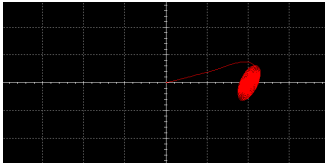
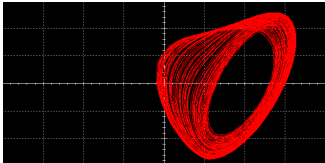
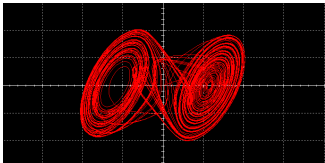
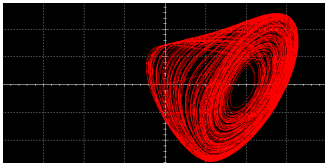
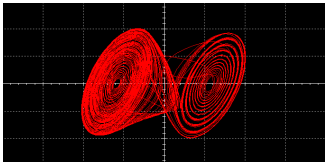
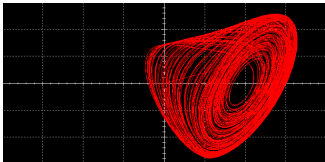
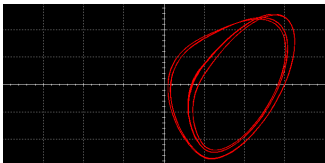
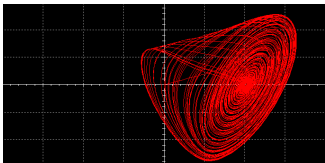
When a single vortex is generated stably, the maximum error tolerance γ of components L , C_1 , and C_2 , respectively, is shown in Table 6. When there is a 5% error in

components L , C_1 , and C_2 , respectively, signals generated by the circuit are shown in Table 7.

Table 6. The maximum error tolerance γ of components L , C_1 , and C_2 , respectively (single vortex, $R = 1900 \Omega$).

	Traditional Chua's Circuit	Improved Implementation of Chua's Circuit
L	1.76%	6.97%
C_1	1.12%	5.27%
C_2	2.75%	9.90%
Average (X)	1.88%	7.38%

Table 7. Signals when there is a 5% error in components L , C_1 , and C_2 , respectively (single vortex, $R = 1900 \Omega$).

		Traditional Chua's Circuit	Improved Implementation of Chua's Circuit
L	+5%		
	−5%		
C_1	+5%		
	−5%		
C_2	+5%		
	−5%		

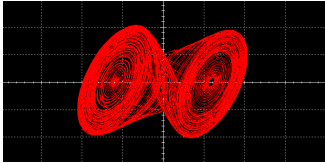
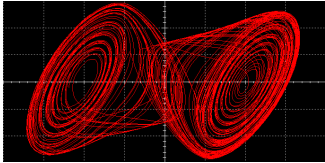
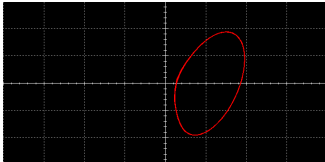
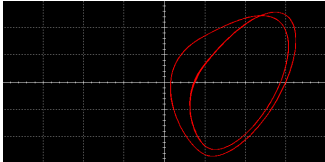
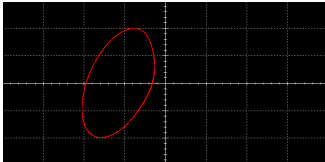
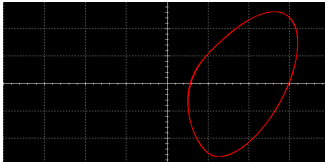
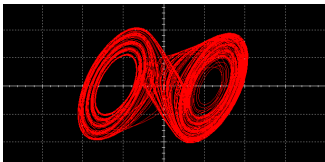
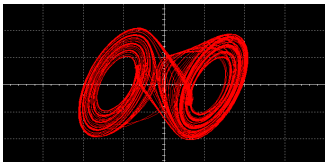
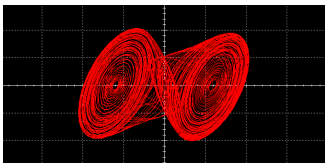
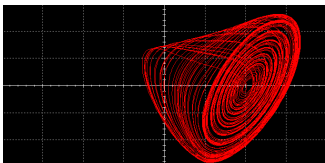
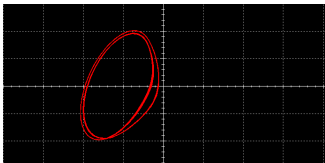
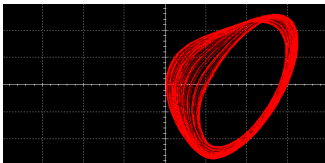
5.1.3. Test Case 3 ($R = 1910 \, \Omega$)

When a single vortex is generated stably, the maximum error tolerance Y of components L , C_1 , and C_2 , respectively, is shown in Table 8. When there is a 5% error in components L , C_1 , and C_2 , respectively, signals generated by the circuit are shown in Table 9.

Table 8. The maximum error tolerance Y of components L , C_1 , and C_2 , respectively (single vortex, $R = 1910 \, \Omega$).

	Traditional Chua's Circuit	Improved Implementation of Chua's Circuit
L	0.72%	2.12%
C_1	0.28%	0.63%
C_2	2.41%	5.25%
Average (X)	1.14%	2.67%

Table 9. Signals when there is a 5% error in components L , C_1 , and C_2 , respectively (single vortex, $R = 1910 \, \Omega$).

		Traditional Chua's Circuit	Improved Implementation of Chua's Circuit
L	+5%		
	−5%		
C_1	+5%		
	−5%		
C_2	+5%		
	−5%		

5.1.4. Brief Summary

The results above verify that the single vortex generated by the traditional Chua's circuit could evolve into a double vortex or degenerate into a periodic motion in many component error cases, but the improved implementation of Chua's circuit can still generate a stable single vortex under the same conditions. The average maximum error tolerance X of components L , C_1 , and C_2 for different adjustable resistors R is shown in Figure 9.

According to Figure 9, the average maximum error tolerance X of components L , C_1 , and C_2 in the improved implementation of Chua's circuit is about three times that in the traditional Chua's circuit and is increased from 1.88% to 7.38% under typical parameters when generating a single vortex.

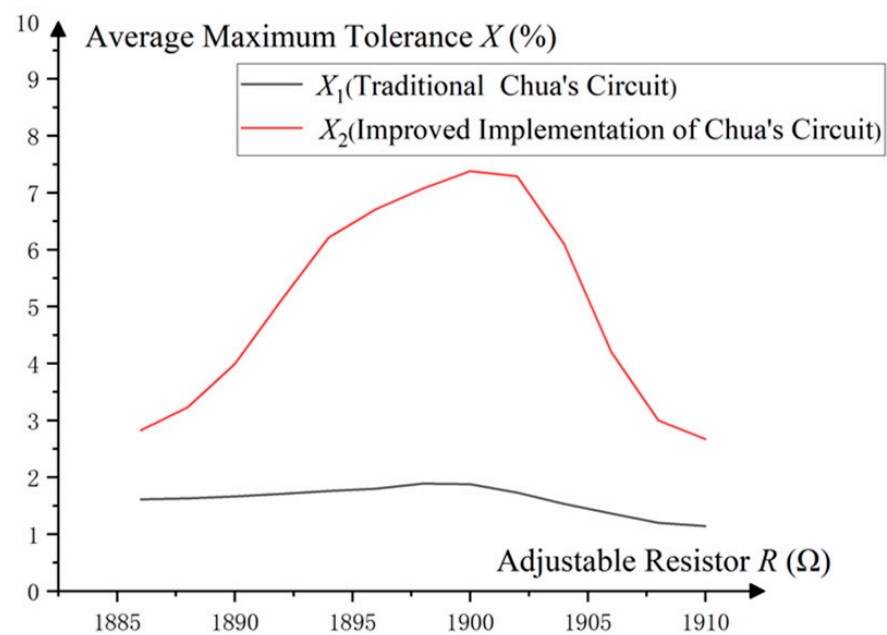


Figure 9. Average maximum error tolerance X of components L , C_1 , and C_2 for different adjustable resistors R .

5.2. Robustness When Generating a Double Vortex

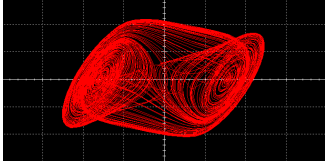
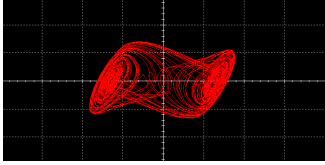
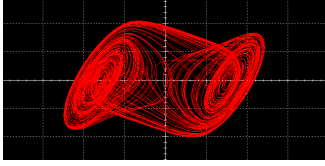
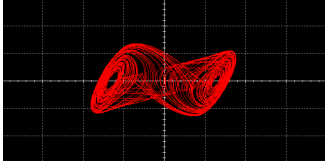
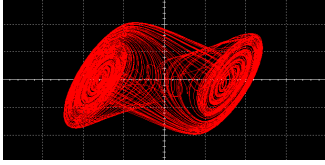
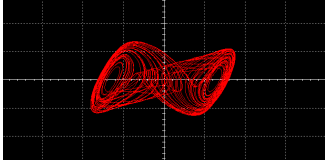
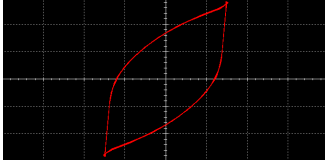
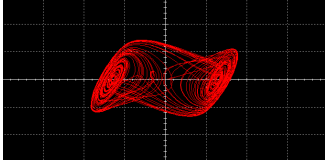
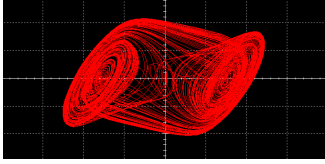
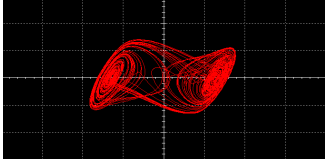
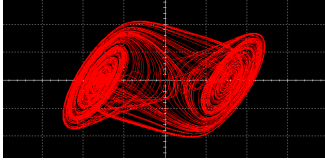
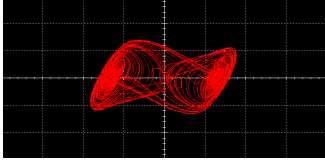
5.2.1. Test Case 1 ($R = 1600 \Omega$)

When a double vortex is generated stably, the maximum error tolerance Y of components L , C_1 , and C_2 , respectively, is shown in Table 10. When there is a 5% error in components L , C_1 , and C_2 , respectively, signals generated by the circuit are shown in Table 11.

Table 10. The maximum error tolerance Y of components L , C_1 , and C_2 , respectively (double vortex, $R = 1600 \Omega$).

	Traditional Chua's Circuit	Improved Implementation of Chua's Circuit
L	5.88%	10.82%
C_1	2.60%	5.69%
C_2	5.72%	21.31%
Average (X)	4.73%	12.61%

Table 11. Signals when there is a 5% error in components L , C_1 , and C_2 , respectively (double vortex, $R = 1600 \Omega$).

		Traditional Chua's Circuit	Improved Implementation of Chua's Circuit
L	+5%		
	−5%		
C_1	+5%		
	−5%		
C_2	+5%		
	−5%		

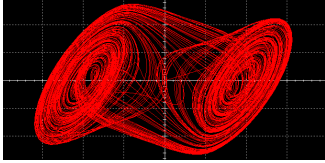
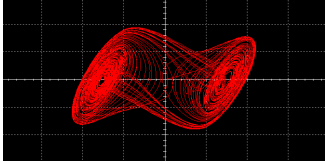
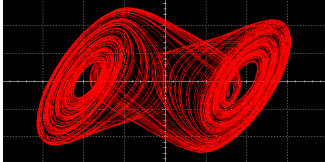
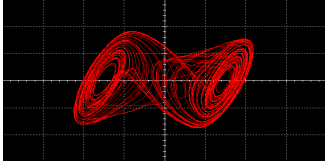
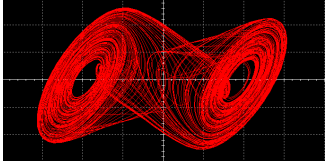
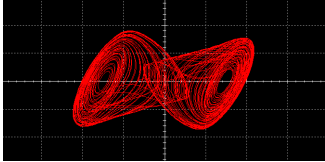
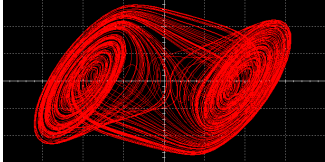
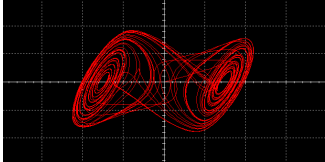
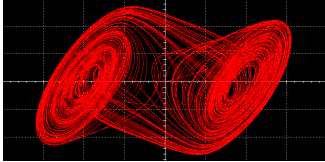
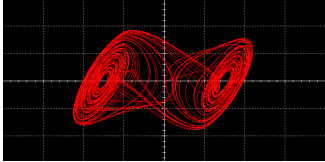
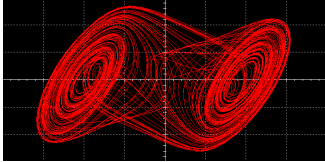
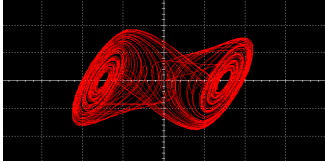
5.2.2. Test Case 2 ($R = 1700 \Omega$)

When a double vortex is generated stably, the maximum error tolerance Y of components L , C_1 , and C_2 , respectively, is shown in Table 12. When there is a 5% error in components L , C_1 , and C_2 , respectively, signals generated by the circuit are shown in Table 13.

Table 12. The maximum error tolerance Y of components L , C_1 , and C_2 , respectively (double vortex, $R = 1700 \Omega$).

	Traditional Chua's Circuit	Improved Implementation of Chua's Circuit
L	7.72%	17.49%
C_1	6.24%	12.03%
C_2	7.31%	26.90%
Average (X)	7.09%	18.81%

Table 13. Signals when there is a 5% error in components L , C_1 , and C_2 , respectively (double vortex, $R = 1700 \Omega$).

		Traditional Chua's Circuit	Improved Implementation of Chua's Circuit
L	+5%		
	−5%		
C_1	+5%		
	−5%		
C_2	+5%		
	−5%		

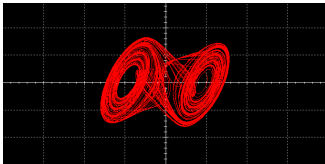
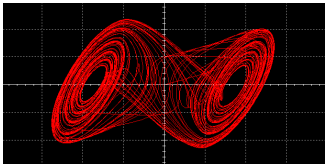
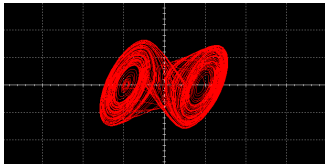
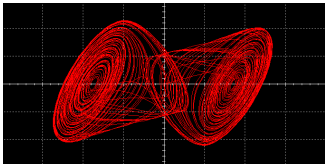
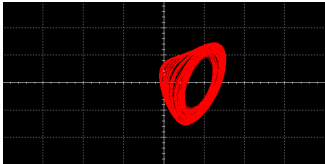
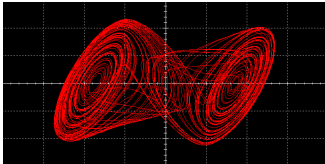
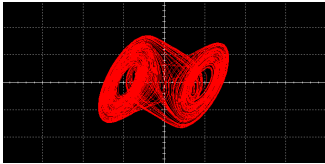
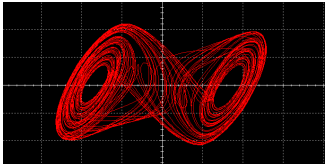
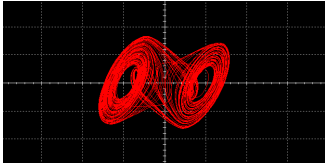
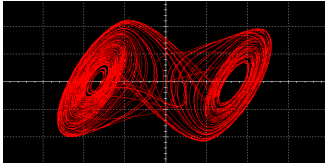
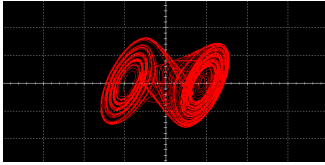
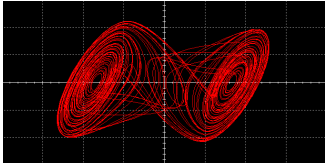
5.2.3. Test Case 3 ($R = 1800 \Omega$)

When a double vortex is generated stably, the maximum error tolerance Y of components L , C_1 , and C_2 , respectively, is shown in Table 14. When there is a 5% error in components L , C_1 , and C_2 , respectively, signals generated by the circuit are shown in Table 15.

Table 14. The maximum error tolerance Y of components L , C_1 , and C_2 , respectively (double vortex, $R = 1800 \Omega$).

	Traditional Chua's Circuit	Improved Implementation of Chua's Circuit
L	5.37%	11.04%
C_1	3.58%	8.55%
C_2	6.92%	15.38%
Average (X)	5.29%	11.66%

Table 15. Signals when there is a 5% error in components L , C_1 , and C_2 , respectively (double vortex, $R = 1800 \, \Omega$).

		Traditional Chua’s Circuit	Improved Implementation of Chua’s Circuit
L	+5%		
	−5%		
C_1	+5%		
	−5%		
C_2	+5%		
	−5%		

5.2.4. Brief Summary

The experiments above indicate that in many component error cases, the double vortex generated by the traditional Chua’s circuit could diverge or degenerate into a single vortex, but the improved implementation of Chua’s circuit can still generate a stable double vortex. The average maximum error tolerance X of components L , C_1 , and C_2 for different adjustable resistors R is shown in Figure 10.

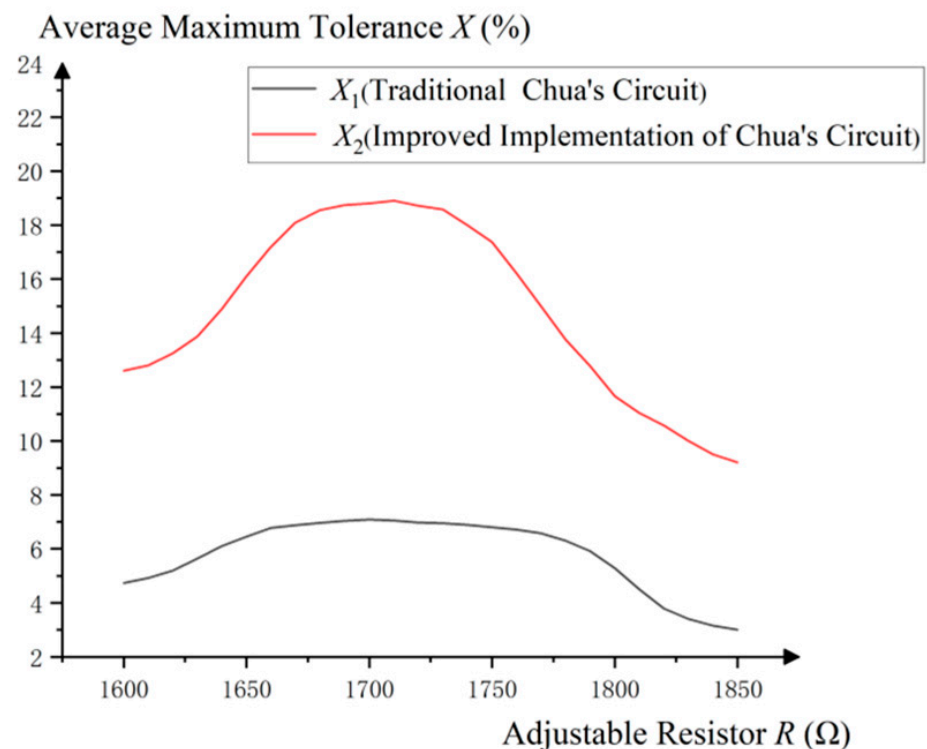


Figure 10. Average maximum error tolerance X of components L , C_1 , and C_2 for different adjustable resistors R .

According to Figure 10, the average maximum error tolerance X of components L , C_1 , and C_2 in the improved implementation of Chua's circuit is about three times that in the traditional Chua's circuit and is increased from 4.73% to 12.61% under typical parameters when generating a double vortex.

5.3. Range of Adjustable Parameters

In Chua's circuit, when the adjustable resistor R is changed from large to small, the circuit produces the following signals in order: unstable bright spot; periodic motion (multiply periodic motion); single vortex; double vortex; divergence. For ease of analysis, we set up the following:

Point 1: Critical point of transition from unstable bright spot to periodic motion (multiply periodic motion).

Point 2: Critical point of transition from periodic motion (multiply periodic motion) to single vortex.

Point 3: Critical point of transition from single vortex to double vortex.

Point 4: Critical point of transition from double vortex to divergence.

The critical points of the traditional Chua's circuit and this improved implementation of Chua's circuit are shown in Table 16. Settings for images above are as shown in Table 17.

According to Table 16, the range of the adjustable parameter R is 24 Ω for generating a single vortex and 311 Ω for generating a double vortex in a traditional Chua's circuit. By comparison, this improved implementation of Chua's circuit generates a single vortex and double vortex with adjustable parameter ranges of 71 Ω and 426 Ω , increasing by 195.83% and 36.98%, respectively, making it more convenient to adjust the resistor R to generate different signals.

Table 16. Critical points of traditional Chua’s circuit and improved implementation of Chua’s circuit.

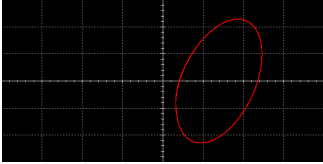
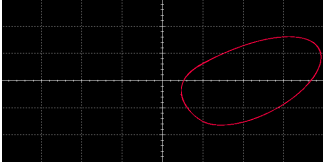
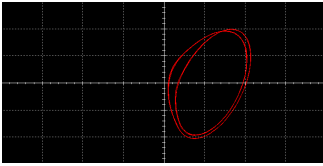
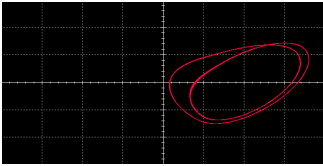
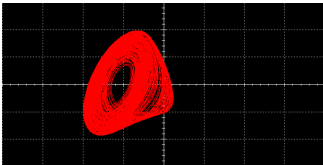
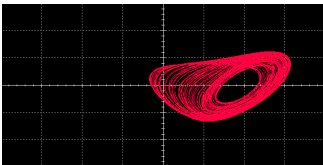
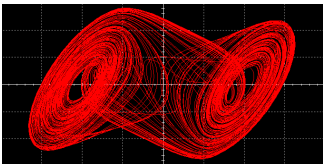
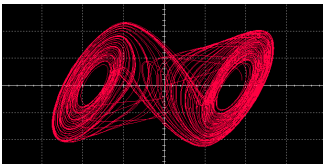
	Traditional Chua’s Circuit	Improved Implementation of Chua’s Circuit
Point 1	1989 Ω	2103 Ω
Periodic Motion		
	(1)	(2)
Multiply Periodic Motion		
	(3)	(4)
Point 2	1910 Ω	1939 Ω
Single Vortex		
	(5)	(6)
Point 3	1886 Ω	1868 Ω
Double Vortex		
	(7)	(8)
Point 4	1575 Ω	1442 Ω

Table 17. Settings for the above images.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Time Base (ms)	50	50	50	50	50	50	50	50
Gradation of Channel A (V/Div)	1	1	1	1	1	1	0.5	0.5
Gradation of Channel B (V/Div)	5	2	5	2	5	2	2	2

5.4. Summary of the Experimental Comparison

In practice, parameter perturbations can be caused by many reasons, such as errors caused by production processes and parameter mismatches caused by environmental temperature and humidity. Corresponding to that, robustness, the ability of a circuit system to resist these factors, is quite important for circuit stability and utility. Experiments proved that the robustness of this improved implementation of Chua’s circuit is significantly better than that of the traditional Chua’s circuit, and the average maximum tolerance for component errors is increased by 5.50% and 7.88%, respectively, under typical parameters when generating a single vortex and double vortex.

At the same time, the range of the adjustable parameter increased by 195.83% and 36.98%, respectively, when generating a single vortex and double vortex, compared with the

traditional Chua's circuit. Notably, the increase in the range of adjustable parameters also means that the circuit can generate richer dynamic behaviors due to its extreme sensitivity to initial values, which is very important in fields such as encrypted communication.

6. Conclusions

As a structurally simple but dynamically complex chaotic circuit, Chua's circuit plays an important role in the study of nonlinear systems and chaos. However, the traditional Chua's circuit has drawbacks such as large volume, high requirements for component accuracy, and limited adjustable parameter range, limiting its application greatly. To address these issues, we adopt the strategy of using active inductors instead of traditional passive inductors, present the theory of substituting non-autonomous systems for classical autonomous systems, and connect the extension resistor in parallel with Chua's diode. With these efforts, the above drawbacks are solved and the utilities of both forms of Chua's circuit are enhanced. Experiments have shown that the average maximum tolerance of component errors has been increased from 1.88% to 7.38% when generating a single vortex and from 4.73% to 12.61% when generating a double vortex. The range of the adjustable parameter has been increased by 195.83% and 36.98%, respectively, when generating a single vortex and a double vortex, compared with the traditional Chua's circuit. In summary, this paper offers valuable insights with significant practical application potential.

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