Maintain Power Transmission and Efficiency Tracking Using Variable Capacitors for Dynamic WPT Systems

Junda Zhu, Sami Barmada *, Antonino Musolino and Luca Sani

Department of Energy, Systems, Territory and Construction Engineering, University of Pisa, 56126 Pisa, Italy; junda.zhu@phd.unipi.it (J.Z.); antonino.musolino@unipi.it (A.M.); luca.sani@unipi.it (L.S.)

* Correspondence: sami.barmada@unipi.it

Abstract: This study introduces a new method for real-time efficiency tracking and stable output power of Dynamic Wireless Power Transfer (DWPT) systems using variable capacitors. A preliminary detailed discussion and an analysis of the DWPT system are carried out to show how the system can optimize power transmission and efficiency when the relative positions of transmitter and receiver change using a dynamic real-time control of the variable capacitors belonging to the compensation networks. This paper shows a detailed model of the DWPT system, including magnetic coupling analysis, circuit dynamics analysis, and efficiency characteristics analysis, in order to modify the control input values as needed. By utilizing a group optimization strategy, the transmission efficiency can be quickly maximized without using a position detection module. Simulation results demonstrate the effectiveness of the proposed method under various dynamic conditions, achieving significant improvements in energy efficiency and transmission reliability of the DWPT system. This research provides a powerful method to increase the overall performances of DWPT systems, which will help the development of future wireless charging technology.

Keywords: dynamic wireless power transfer; dynamic compensation network; group optimization

1. Introduction

Dynamic Wireless Power Transfer (DWPT) technology is increasing its popularity, showing a potential impact for urban electric buses and long-distance electric vehicles [1–5]. The DWPT system supplies electrical power while electric vehicles are in motion, resulting in battery capacity reduction and relief of people’s anxiety about mileage. At present, DWPT for electric vehicles is undergoing development and testing, with many research institutions and corporations creating prototypes and conducting pilot projects. The main challenge is maintaining an almost constant power transmission and high efficiency despite the dynamic characteristics of DWPT systems (characterized by changing distances, speeds, and load requirements); for these reasons, DWPT systems have not yet been applied in practice [6]. As is well-known among researchers active in the topic, changing the relative position between transmitter and receiver affects the magnetic coupling and causes large fluctuations in the efficiency and transmitted power. The aim of this paper is to propose an innovative solution to this problem.

The work performed so far to maintain stable power transmission and efficiency tracking in DWPT systems can be divided into three categories:

1. Improved magnetic coupling or repeater [7–14]. The authors of [12] provide a novel coupling structure that enables a stable mutual coupling between the transmitter array and the receiver. This structure is particularly suitable for applications involving DWPT. A segmented dynamic wireless charging system is proposed in [13], where a “twin transmitting” coil was proposed to effectively mitigate power fluctuations.
Reference [14] presents a DWPT system that utilizes detuning repeater to deliver a consistent power output while minimizing the number of inverters.

2. Compensation network or autonomous frequency control [15–22]. Papers [20,21] propose a multi-emitter driven by a scalable N-Legged Converter (NLC) to reduce the size and weight of pickup coils (fixed to the bottom of an electric vehicle chassis) and improve the overall efficiency. Paper [22] utilizes the Tunable Self-Oscillating Switching (TSOS) approach to achieve the optimal operating point in terms of maximum power transfer, efficiency, and Zero-Voltage Switching (ZVS).

3. Inverter and auxiliary converters [23–28]. Paper [27] discusses a DWPT of electric vehicles able to determine the charging area through wireless communication and use multi-excitation units in the charging area to improve power and efficiency. In [28], the authors propose a charging strategy based on phase shift control of a controllable rectifier circuit and an efficiency optimization strategy based on dynamic equivalent impedance matching, which improves the transmission efficiency of the DWPT system while achieving Constant Voltage (CV) and Constant Current (CC).

In this paper, we propose the use of a variable capacitor to modify the compensation circuit of the transmitting coils, aiming to obtain steady output power while maintaining a high efficiency, taking into account the system complexity and with a limited increase in the production costs. Variable capacitors have been proposed in the literature to ensure stable power transmission in Wireless Power Transfer (WPT) system [29–34]; however, most articles are limited to static WPT or one-to-one transmitter and receiver coil systems. A more sophisticated use is proposed in [33,34], where a Dual-Switch Control Capacitor (DSCC) allows for the achievement of selective power transmission and output voltage regulation in a one-to-three WPT system.

Here, we focus on the dynamic variable capacitance adjustment of two transmitting coils to one receiving coil in order to achieve constant power transfer and high efficiency operations while the receiver coil moves from a position aligned to the first transmitter to a position aligned to the other. The two transmitters are fed using the same inverter. This design scenario is the building block of a DWPT system in which two transmitters are always energized and follow the receiver position.

Section 2 is dedicated to the description of the details about the proposed DWPT system, including a descriptions of the magnetic performances and of the compensation networks’ behavior. Section 3 describes how a Switched Controlled Capacitor can improve the performance of a WPT system. Section 4 goes into the details of the control strategy of the proposed DWPT system, while Section 5 describes the results of the simulations performed with a full model (digital twin).

2. Dynamic Wireless Power Transmission System

2.1. DWPT System Overview

There are two types of magnetic couplers used in DWPT systems: multiple short-individual loops, and long-track-loops. Long-track-loop coils are sensitive to large leakage magnetic fields with high reactive power and significant power losses, which is why several short individual coils are commonly used instead. Figure 1a shows a schematic diagram of the DWPT being commonly proposed for electric vehicles.
Figure 1. Dynamic wireless power transfer for electric vehicles. (a) Block diagram of dynamic wireless charging of electric vehicles. (b) Simplified DWPT system.

The DWPT system is powered by the grid through a DC power bus. Multiple full-bridge high-frequency inverter circuits fed by the DC bus are connected in parallel, performing a DC/AC conversion resulting in a phase shift modulated square wave at the fundamental frequency of 85 kHz. Due to the filtering properties of the compensation networks, the Fundamental Harmonic Approximation (FHA) method is used to analyze the DWPT system. As shown in Figure 1a, the proper switches are activated based on the receiving coil’s location to energize the transmitting coils in correspondence to the electric vehicle. Figure 1b illustrates the simplified model of the DWPT system, consisting of two transmitting coils and one receiving coil, including the circuitry on the vehicle and the battery. This model will be deeply analyzed and discussed in the subsequent sections.

2.2. Magnetic Coupler Design

The evaluation of the magnetic field generated by the coils is crucial in the design of DWPT systems. According to the paths of the magnetic flux lines, the coils can be classified into two types based on their characteristics: unipolar, and bipolar. An appropriately built coil can enhance the system’s power transfer, efficiency, and tolerance to misalignment. Common structures include Circular Pads (CP), Rectangular Pads (RP), Double D (DD) pads, Bipolar pads (BP), and DD-Quadrature (DDQ). The circular pads feature a simple layout, and are suitable for static WPT systems. Rectangular pads are applicable in long-track-loop DPWT systems; however, they are neither cost-effective nor energy-efficient because of their significant magnetic field leakage. The DD coil is a bipolar coil configuration with excellent tolerance to horizontal misalignment in transverse direction with respect to the motion, and is an excellent candidate for various short-individual DWPT systems. The DDQ coil, derived from the DD coil, has improved tolerance to horizontal misalignment, but requires two synchronous input power sources, limiting its practical applications.

The multiple short-individual WPT system can be schematically represented as in Figure 1b, illustrating the relationship between two short-individual transmitting DD coils and one receiving coil. Additional losses may occur if the two DD coils are mutually coupled, as in cases of the overlapping of the two transmitters. The additional losses are reduced if the two transmitting coils are far apart, but this will result in significant fluctuations of the transmitted power. A trade-off between these two instances has to be considered.

To save computational time during the numerical simulations, a rectangular slab replaces the array of ferrite bars in the 3D model (see Figure 1a), while the current in the conductor is assumed to be uniformly distributed in the cross-section. This hypothesis is supported by the choice of litz wire for the manufacturing of the coil. The mutual inductance between the coils can be readily determined using three-dimensional Finite Element Method (3D-FEM) software, ANSYS Maxwell suite 2021 R1 (Ansys, Canonsburg, Penn., United States, USA); the results of the simulations are reported in Figure 2b, showing the
mutual induction coefficients as a function of the position of the receiver; the 0 of the quantity on the horizontal axis corresponds to the alignment between the transmitting coil 1 (TX1) and receiving coil (RX1). The variation of the self-inductance of the transmitter is negligible due to the high distance between the transmitter and receiver, while the receiver is always facing shields and ferrite places, so its self inductance variation is even lower.

![Diagram](image)

**Figure 2.** The single DD coils: (a) architecture; (b) the mutual inductance between the TX and RX coils.

Table 1 shows that the cross-coupling coefficient between two adjacent transmitting coils is two orders of magnitude smaller than the major coupling coefficients (i.e., those between the single transmitter and the receiver), a characteristic that was imposed by design for reasons related to the reduction in additional losses, as stated before.

<table>
<thead>
<tr>
<th>Offset Direction L(mm)</th>
<th>Coupling Coefficient (TX1-RX1)</th>
<th>Coupling Coefficient (TX2-RX1)</th>
<th>Coupling Coefficient (TX1-TX2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−50</td>
<td>0.313</td>
<td>0.010</td>
<td>−0.019</td>
</tr>
<tr>
<td>0</td>
<td>0.364</td>
<td>0.016</td>
<td>−0.017</td>
</tr>
<tr>
<td>50</td>
<td>0.314</td>
<td>0.034</td>
<td>−0.015</td>
</tr>
<tr>
<td>100</td>
<td>0.203</td>
<td>0.083</td>
<td>−0.014</td>
</tr>
<tr>
<td>150</td>
<td>0.099</td>
<td>0.178</td>
<td>−0.014</td>
</tr>
<tr>
<td>200</td>
<td>0.041</td>
<td>0.291</td>
<td>−0.015</td>
</tr>
<tr>
<td>250</td>
<td>0.019</td>
<td>0.364</td>
<td>−0.017</td>
</tr>
<tr>
<td>300</td>
<td>0.011</td>
<td>0.332</td>
<td>−0.019</td>
</tr>
<tr>
<td>350</td>
<td>0.008</td>
<td>0.225</td>
<td>−0.023</td>
</tr>
<tr>
<td>400</td>
<td>0.007</td>
<td>0.117</td>
<td>−0.026</td>
</tr>
</tbody>
</table>

This paper adopts the improved DD coils proposed in [35,36], which conjugates high tolerance with respect to horizontal (transverse) misalignment, and minimal interference between adjacent coils in the two horizontal directions (see Figure 3).
Both TX and RX are constituted by the series connection of three DD coils on a row in a transverse direction with respect to the motion. Referring to the transmitter, the current in the central DD coil (TX1_1) is assumed to produce the flux according to the central red arrows (see Figure 4), while the lateral DD coils (series connected forming together the TX1_2) produce fluxes directed as the orange arrows. The fluxes produced by the transmitting coils TX1_2 pass through TX1_1 and cancel each other due to the geometrical symmetry and the orientation of the currents in the coils of TX1_1. Computations confirmed that the cross-coupling coefficient between TX1_1 and TX1_2 is significantly lower than the coupling between TX1 and RX1 or TX2 and RX2 (about three orders of magnitude), as shown in Table 2. The receiving coil is characterized by a similar construction, and hence exhibits the same behavior. Moreover, if we consider that the connection of the three transmitting coils can be easily reconfigured, we can envisage the possibility to transfer power to a different arrangement of receiving coils, achieving a good degree of interoperability.

<table>
<thead>
<tr>
<th>Offset Direction L(mm)</th>
<th>Coupling Coefficient (TX1-RX1)</th>
<th>Coupling Coefficient (TX2-RX1)</th>
<th>Coupling Coefficient (TX1-TX2)</th>
<th>Coupling Coefficient (TX1_1-TX1_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.362</td>
<td>0.014</td>
<td>−0.014</td>
<td>−0.0002</td>
</tr>
<tr>
<td>100</td>
<td>0.334</td>
<td>0.030</td>
<td>−0.013</td>
<td>−0.0003</td>
</tr>
<tr>
<td>200</td>
<td>0.269</td>
<td>0.072</td>
<td>−0.012</td>
<td>−0.0002</td>
</tr>
<tr>
<td>300</td>
<td>0.191</td>
<td>0.141</td>
<td>−0.012</td>
<td>−0.0002</td>
</tr>
<tr>
<td>400</td>
<td>0.113</td>
<td>0.221</td>
<td>−0.012</td>
<td>−0.0003</td>
</tr>
<tr>
<td>500</td>
<td>0.052</td>
<td>0.296</td>
<td>−0.013</td>
<td>−0.0003</td>
</tr>
<tr>
<td>660</td>
<td>0.014</td>
<td>0.362</td>
<td>−0.015</td>
<td>−0.0003</td>
</tr>
</tbody>
</table>
2.3. Compensation Network Analysis

In the remainder of the manuscript, the word misalignment indicates a movement in the horizontal direction transverse to the motion, while with the word travel, it is meant to refer to the movement in the motion direction. In addition, the attribute “off-centered” is used for a receiver with a displacement from the correspondent transmitter without specifying if its position is due to a misalignment or to a travel state. When traveling with zero misalignment, a receiver leaves one transmitter and approaches the nearby one; in these cases, the word transition is used.

The transmission power and efficiency of a DWPT system will always vary in contrast to a Static WPT due to changes in the coupling coefficient (often the mutual induction $M$ is used in place of the coupling coefficient). Therefore, selecting the suitable compensation circuits is essential for an optimal operation of the whole DWPT system. Previous papers have discussed various compensation networks, such as basic first-order circuits (S-S, S-P, P-S, P-P) and higher-order circuits (LCL-LCL, LCC-LCC). Additionally, hybrid-order circuits include LCL-S, LCC-S, LCL-P, and LCC-P configurations. All of these compensation networks operate at resonant frequency, and the ones analyzed in this work are shown in Figure 5.

![Figure 5. Usual compensation networks topologies: (a) LCL-LCL; (b) LCC-S; (c) LCC-LCC; (d) LCC-S.](image)

Basic first-order circuits typically require fewer components and have excellent transmission efficiency, but they are not capable of dealing with misalignment between transmitting and receiving. Without a receiving coil or when the receiver is highly off-centered, it is not unusual to create harmful overcurrent on the transmitting device. Double LCL and double LCC topology circuits are commonly utilized for their ability to adapt to receiver displacements and load variations. However, the addition of inductive and capacitive components increases the circuit’s complexity and energy consumption. In the LCC topology, a constant primary current is maintained regardless of mutual inductance and load, requiring a smaller resonant inductance than the LCL topology, but requires an additional capacitor. Hybrid-order topological circuits combine the advantages of higher-order topological circuits while reducing the necessary number of components [37].

In addition to that, it is essential to maintain a constant current (CC) and voltage (CV) output for Wireless Power Transfer Electric Vehicle (WPT-EV) systems when charging lithium batteries. Every compensating circuit possesses distinct output characteristics that are useful for charging processes with varying loads. The output properties are typically analyzed using L-shaped or T-shaped resonant networks in numerous studies [38,39]. We outline four typical compensation circuits, as displayed in Table 3 below. CV or CC conversion can be achieved with a single-stage WPT converter by using appropriate compensation and operating frequency selection control. Since a DC-DC conversion circuit is
added to the back end of the WPT-EV system of this design, it is appropriate to choose the compensation network with constant voltage output, namely LCL-S or LCC-S.

Table 3. Compensation network for constant output.

<table>
<thead>
<tr>
<th>Compensation Networks</th>
<th>Constant Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCL-LCL</td>
<td>Constant Current (CC)</td>
</tr>
<tr>
<td>LCL-S</td>
<td>Constant Voltage (CV)</td>
</tr>
<tr>
<td>LCC-LCC</td>
<td>Constant Current (CC)</td>
</tr>
<tr>
<td>LCC-S</td>
<td>Constant Voltage (CV)</td>
</tr>
</tbody>
</table>

2.4. Comparison of LCC-S and LCL-S Compensation Network Circuits

We aim to identify a compensation network capable of preserving the system’s output power and efficiency while the coupling coefficient varies. This study examines each component of the two compensation networks through numerical calculations, and shows their effect in terms of output power and efficiency.

We start our analysis with the LCL-S topology, evaluating the transmitted power and the efficiency as a function of the coupling coefficient $k = M/\sqrt{L_tL_r}$ in the interval 0.05 to 0.4 (self and mutual inductance values of the pads are obtained using the 3D-FEM simulations).

Figure 6 shows the equivalent circuit of the LCL-S compensated WPT system, where $V_{in}$ stands for the input power supply voltage, $I_{in}$ stands for the input current, while $I_t$ and $I_r$ are the currents in the transmitting and receiving coils, respectively. $L_{t1}$ and $C_{t1}$ constitute the primary side of the LCL compensation network, $L_t$ and $L_r$ denote the transmitting and receiving coil inductances, respectively, while $M$ is the mutual induction coefficient between $L_t$ and $L_r$, and $C_r$ is the series resonant compensation capacitance of the secondary side. $R_L$ represents the load, while $R_t$ and $R_r$ are the parasitic resistances of the transmitting and receiving coils, respectively.

In this case, the varying parameters are the operating frequency and the values of the components of the matching networks, namely $L_{t1}$, $C_{t1}$, and $C_r$. The results reported in Table 4 are obtained analytically by recurring to the Kirchhoff laws applied to the equivalent circuit.

Figure 6. LCL-S compensation networks.

The loop analysis of the circuit in Figure 6 yields the following:

$$
\begin{align*}
\begin{bmatrix}
    j\omega L_{t1} + \frac{1}{j\omega C_{t1}} & -\frac{1}{j\omega C_{t1}} & 0 \\
    -\frac{1}{j\omega C_{t1}} & \frac{1}{j\omega C_{t1}} + j\omega L_t + R_t & -j\omega M \\
    0 & -j\omega M & j\omega L_r + \frac{1}{j\omega C_r} + R_L + R_r
\end{bmatrix}
\begin{bmatrix}
    I_{in} \\
    I_t \\
    I_r
\end{bmatrix}
= \begin{bmatrix}
    V_{in} \\
    0 \\
    0
\end{bmatrix}
\end{align*}
$$

(1)
Table 4. The output power and efficiency of LCL-S under changing parameters.

<table>
<thead>
<tr>
<th>Varying Parameter</th>
<th>Output Power</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$L_{t1}$</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$C_{t1}$</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$C_r$</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>

Similarly, we considered the LCC-S compensation shown in Figure 7, whose governing equations are as follows:

\[
\begin{bmatrix}
  j\omega L_{t1} + \frac{1}{j\omega C_{t1}} & -\frac{1}{j\omega C_{t1}} & 0 \\
  -\frac{1}{j\omega C_{t1}} & j\omega L_t + R_t & j\omega M \\
  0 & -j\omega M & j\omega L_r + \frac{1}{j\omega C_r} + R_L + R_r \\
\end{bmatrix}
\begin{bmatrix}
  i_{in} \\
  i_t \\
  i_r \\
\end{bmatrix} = \begin{bmatrix}
  V_{in} \\
  0 \\
  0 \\
\end{bmatrix}
\]

(2)
The changing parameters are the components of the matching networks \((L_{t1}, C_{t1}, C_t,\) and \(C_r)\) and the operating frequency. The related results are reported in Table 5.

![Diagram](image-url)

**Figure 7.** LCC-S compensation networks.

**Table 5.** The output power and efficiency of LCC-S under changing parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Output Power</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td><img src="image-url" alt="Graph" /></td>
<td><img src="image-url" alt="Graph" /></td>
</tr>
<tr>
<td>(L_{t1})</td>
<td><img src="image-url" alt="Graph" /></td>
<td><img src="image-url" alt="Graph" /></td>
</tr>
<tr>
<td>(C_{t1})</td>
<td><img src="image-url" alt="Graph" /></td>
<td><img src="image-url" alt="Graph" /></td>
</tr>
</tbody>
</table>
Looking at the curves reported in Tables 4 and 5, we see that using both the matching networks, it is possible to find values of the capacitor $C_{t1}$ that make it possible to maintain a stable and highly efficient transferred power as the coupling coefficient varies. This offers theoretical support to the possibility of maintaining a steady power output in a DWPT system. The transmission efficiency has a similar behavior, allowing for it to maintain high values, as we can observe by considering the envelope of the curves in the plot in the third row of Table 5.

In the remaining part of the paper, we use the LCC-S matching circuits because $L_{t1LCC-S} \ll L_{t1LCL-S}$, while maintaining the same input voltage and the same load parameters, i.e., the LCC-S compensated system exhibits higher transmitted power. Moreover, a smaller inductor implies a reduction in component cost, occupied space, and power losses.

3. Analysis of SCC and Single Transmitting and Receiving Coil Model

3.1. Switched Controlled Capacitor (SCC)

Variable capacitors, schematically represented in Figure 8a, were initially referenced in paper [40] and have been extensively utilized in power electronic systems. As shown in Figure 8b, when the MOSFETs (or their diodes) parallel-connected to the capacitor are in conductive state, they effectively short-circuit the capacitor, resulting in an equivalent capacitance value approaching infinity. If the MOSFETs are always open, the equivalent capacitance remains the same as the original capacitor’s value.
Figure 8. Switched Controlled Capacitor: (a) Schematic diagram; (b) current and voltage waveforms; (c) relationship between $\alpha$ and angle $\delta$.

Figure 8b assumes that the current passing through the entire SCC follows a sine wave pattern. During the $0-\pi$ interval, when the current is in the positive half cycle, turning on MOSFET1 allows for the current to flow through MOSFET1 and the body diode within MOSFET2, thus short-circuiting the capacitor, and when MOSFET1 is turned off, the current flows into the capacitor, indicating that the capacitor is operational. During the $\pi-2\pi$ interval, when the current is in the negative half cycle, turning on MOSFET2 causes the capacitor to discharge due to the voltage across it being larger than zero. When the voltage reaches zero, the current flows via MOSFET2 and the body diode within MOSFET1, causing the capacitor to be short-circuited once more. It is worth noting that before turning on the MOSFET, the current passes via its body diode, allowing for the MOSFET to achieve zero voltage turn-on, known as soft switching.

The equivalent capacitance value is determined by the conduction times of the MOSFETs. It is important to monitor the zero-crossing point of the current at both ends of the capacitor in real-time in practical applications, which necessitates an extra detecting circuit.

Figure 8c reports the relationship between the offset angle $\delta$ ($\pi/2 \leq \delta < \pi$) and the ratio $\alpha$ between the equivalent capacitance of the variable capacitance seen at the terminals and the capacitance of the real capacitor $C_{t1}$.

The analytical expression of the curve is:

$$\alpha C_{t1} = \frac{\pi}{2\pi - 2\delta + \sin 2\delta} C_{t1}$$

(3)
3.2. The LCC-S Compensation Network with SCC

The equivalent circuit of the LCC-S compensating WPT system is depicted in Figure 9a, which differs from Figure 7 only because the capacitor $C_{t1}$ is now considered variable, and assuming the value $\alpha C_{t1}$. The input impedance has been determined in [41].

![Figure 9a](image1.png)

Figure 9. The LCC-S circuit with variable capacitor: (a) circuit model diagram; (b) simplified model diagram.

The relationship between the component values is as follows:

$$\omega L_{t1} = \frac{1}{\omega C_{t1}}$$  \hspace{1cm} (4)

$$C_t = \frac{1}{\omega^2 (L_t - L_{t1})}$$  \hspace{1cm} (5)

$$\omega L_r = \frac{1}{\alpha C_r}$$  \hspace{1cm} (6)

Considering $X_t = \omega L_{t1}$ and inserting $\alpha$ into the above Equations, we can obtain the following Equations:

$$\frac{1}{j\omega \alpha C_{t1}} = -j \frac{1}{\alpha} X_t$$  \hspace{1cm} (7)

$$j\omega L_t + \frac{1}{j\omega C_t} = j\omega L_{t1} = jX_t$$  \hspace{1cm} (8)

$$j\omega L_r + \frac{1}{j\omega C_r} = jX_r$$  \hspace{1cm} (9)

According to (7)–(9), the LCC-S compensation network can be redrawn as shown in Figure 9b.

With the new notation, it is easy to obtain the following:

$$\begin{bmatrix} (1 - \frac{1}{\alpha}) jX_t & -\frac{1}{\alpha} jX_t & 0 \\ -\frac{1}{\alpha} jX_t & (1 - \frac{1}{\alpha}) jX_t + R_t & -j\omega M \\ 0 & -j\omega M & jX_r + R_L + R_r \end{bmatrix} \begin{bmatrix} I_{L1} \\ I_t \\ I_r \end{bmatrix} = \begin{bmatrix} V_{in} \\ 0 \\ 0 \end{bmatrix}$$  \hspace{1cm} (10)

Then, the input current and input impedance are as follows:

$$I_{in} = \frac{[Y + j \left(1 - \frac{1}{\alpha}\right) X_t (R_L + R_r)] V_{in}}{(\frac{2}{\alpha} - 1) X_t^2 (R_L + R_r) + j \left(1 - \frac{1}{\alpha}\right) X_t Y}$$  \hspace{1cm} (11)

$$Z_{in} = \frac{\left(\frac{2}{\alpha} - 1\right) X_t^2 (R_L + R_r) + j \left(1 - \frac{1}{\alpha}\right) X_t Y}{Y + j \left(1 - \frac{1}{\alpha}\right) X_t (R_L + R_r)}$$  \hspace{1cm} (12)
The output power and efficiency are as follows:

\[
P_{\text{out}} = \frac{\omega^2 M^2 R_L V_{\text{in}}^2}{[(2 - \alpha)X_t(R_L + R_r) + j(\alpha - 1)Y]^2}
\]

\[
\eta = \frac{\left[\frac{2}{\alpha} - 1\right]X_t^2(R_L + R_r) + j\left(1 - \frac{1}{\alpha}\right)X_t Y}{[Y + j\left(1 - \frac{1}{\alpha}\right)X_t(R_L + R_r)][(2 - \alpha)X_t(R_L + R_r) + j(\alpha - 1)Y]^2}
\]

where:

\[
Y = (R_t(R_L + R_r) + \omega^2 M^2)
\]

In the trivial case in which \(\alpha = 1\), the capacitance of capacitor \(C_{t1}\) remains constant yielding the following:

\[
Z_{\text{in}} = \frac{X_t^2 R_{L+R}}{R_t(R_L + R_r) + \omega^2 M^2}
\]

\[
P_{\alpha} = \frac{M^2 V_{\text{in}}^2 R_L}{(L_{t1}(R_L + R_r))^2}
\]

\[
\eta = \frac{\omega^2 M^2 R_L}{Y(R_L + R_r)}
\]

The results of the above analysis are reported in Table 6. Due to the power MOSFET’s switching capability limitation, the inverter cannot accomplish ZVS operation when the input impedance angle \(\theta\) is less than or equal to zero (i.e., the phase of the current is positive). At the same time, to minimize the reactive current of the resonant circuit, \(\theta\) must not be too large [42]. The ideal condition should be to keep \(\theta\) close to a small positive reference value to guarantee that the input impedance angle is slightly larger than zero, allowing for the previous full-bridge inverter circuit to operate in the ZVS state. Typically, a large coupling coefficient ensures that \(\theta\) is greater than zero. If the coupling coefficient decreases further, it is not guaranteed that \(\theta\) will always be greater than zero. In Table 6, the values of \(Z_{\text{in}}\) illustrates that a higher value of \(\alpha\) results in a broader range of coupling coefficient values where the impedance angle is higher than zero.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Absolute</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_{\text{in}})</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
3.3. Preliminary Simulation for the Improved Circuit Model

The above subsection reports some results obtained considering a simple equivalent circuit of the WPT system with SCC at a steady condition. We now show the results of the simulations considering a more accurate model reported in Figure 10a and composed by a DC voltage supply, a full-bridge inverter, an LCC-S matching circuit, a rectifier bridge, a buck converter, and a load. The simulation of $C_{11}$ follows the control strategy of the previously proposed variable capacitor.

According to Fourier decomposition, the voltage $V_{in}$ in time domain can be expressed as follows:

$$V_{in}(t) = \frac{4}{\pi} V_{DC} \sin\frac{\varphi}{2} \sum_{n=1,3,5...}^{\infty} \frac{\sin n\omega t - \frac{\pi - \varphi}{2}}{n}$$  \hspace{1cm} (19)

The fundamental voltage of the inverter is the following:

$$V_{in}(t) = \frac{4}{\pi} V_{DC} \sin\frac{\varphi}{2} \sin (\omega t - \frac{\pi - \varphi}{2})$$  \hspace{1cm} (20)

while its root-mean-square (rms) value is as follows:

$$V_{in} = \frac{2\sqrt{2}}{\pi} V_{DC} \sin\frac{\varphi}{2}$$  \hspace{1cm} (21)
Figure 10. Under different $\alpha$ values, the LCC-S circuit changes with the coupling coefficient: (a) simulation schematic; (b) simulation result statistics.

The results of the simulations are reported in Figure 10b. We assumed a coupling coefficient ($k$) which varies between 0.15 and 0.4, whereas $\delta$ denotes the delay angle in the previous SCC (ref. Figure 8c). The simulation findings demonstrate that adjusting the input angle $\delta$, which changes the value of the variable capacitor $C_{t1}$, may maintain constant output power and excellent transmission efficiency despite variations in the coupling coefficient.

4. Analysis of Two Transmitting Coils and Single Receiving Coil Model for DWPT

4.1. Equivalent Models of Two Transmitter Coils and Single Receiver Coils

We start our analysis by using a simplified model of the DWPT system, as shown in Figure 11. The parallel connection of two transmitters with LCC-compensating topology to the same inverter can reduce the use of components. The parameters $\alpha$ and $\beta$ represent the tunable capacitance variation range in the two LCC topologies.
The governing equations of the circuit in Figure 11 are as follows:

\[
\begin{pmatrix}
(1 - \frac{1}{\alpha})X_t & -\frac{1}{\alpha}X_t & 0 & 0 & 0 \\
-\frac{1}{\alpha}X_t & (1 - \frac{1}{\beta})X_t & \frac{1}{\beta}X_t & 0 & -j\omega M_1 \\
0 & 0 & (1 - \frac{1}{\beta})X_t & \frac{1}{\beta}X_t & 0 \\
0 & 0 & -\frac{1}{\beta}X_t & (1 - \frac{1}{\beta})X_t & \frac{1}{\beta}X_t \\
0 & -j\omega M_1 & 0 & -j\omega M_2 & X_t + (R_L + R_t)
\end{pmatrix}
\begin{pmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4 \\
I_5
\end{pmatrix} =
\begin{pmatrix}
V_{in} \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\] (22)

According to the uniformity, we can simplify this Equation by the following mathematical expression:

\[
\begin{pmatrix}
Z_{T1} & 0 & M \\
0 & Z_{T2} & M^T \\
M^T & Z_R + R_L
\end{pmatrix}
\begin{pmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4 \\
I_5
\end{pmatrix} =
\begin{pmatrix}
V_{in} \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\] (23)

where

\[
Z_{T1} = \begin{pmatrix}
(1 - \frac{1}{\alpha})X_t & -\frac{1}{\alpha}X_t \\
-\frac{1}{\alpha}X_t & (1 - \frac{1}{\beta})X_t + R_t
\end{pmatrix}
\] (24)

\[
Z_{T2} = \begin{pmatrix}
(1 - \frac{1}{\beta})X_t & \frac{1}{\beta}X_t \\
-\frac{1}{\beta}X_t & (1 - \frac{1}{\beta})X_t + R_t
\end{pmatrix}
\] (25)

\[
M = \begin{pmatrix}
0 & -j\omega M_1 \\
0 & 0
\end{pmatrix}
\] (26)

4.2. Transfer Characteristics under Variable Capacitance

Solving the matrix equation yields the output power, as follows:

\[
P_{out} = I_{out}^2 R_L
\] (27)
where:

\[ I_{\text{out}} = \frac{j\omega M_1}{(R_L + R_r)} l_2 + \frac{j\omega M_2}{(R_L + R_r)} l_4 \]

\[ \cong \frac{-\omega M_1 V_{\text{in}}}{j(\alpha - 1)(R_L + R_r) + \omega^2 M_2^2 + (2 - \alpha)X_L(R_L + R_r)} \]

\[ + \frac{-\omega M_2 V_{\text{in}}}{j(\beta - 1)(R_L + R_r) + \omega^2 M_2^2 + (2 - \beta)X_L(R_L + R_r)} \] (28)

This expression for \( I_{\text{out}} \) is a consequence of the presence of two LCC compensation networks fed by an ideal voltage generator; this makes the currents \( l_2 \) and \( l_4 \) on the transmitting coils largely independent on the load near the resonant frequency.

The expression of \( P_{\text{out}} \) can be considered as a function of the four variables \( M_1, M_2, \alpha, \beta \). \( M_1 \) and \( M_2 \) vary, as in Table 1 or Table 2, whose entries were obtained using an FEM simulation. As a consequence, the control strategy of \( \alpha, \beta \) in order to achieve a constant value of \( P_{\text{out}} \), with

\[ P_{\text{out}} = f(M_1, M_2, \alpha, \beta) \] (29)

while maintaining a good efficiency is the main goal of this section.

Looking at the results reported at the third row of Table 5, we see that the power that can be transmitted to a receiver aligned with the transmitter is about 10 kW, with an efficiency higher than 0.9 when the coupling coefficient is higher than 0.1.

Numerical calculations performed on the system in Figure 11 for different values of \( M_1 \) and \( M_2 \) (correspondent to intermediate position of the receiver traveling on the two transmitter) and for a number of couples \((\alpha, \beta)\) with \( 1 < \alpha < 2 \) and \( 1 < \beta < 2 \) have been shown, as follows:

- \( P_{\text{out}} > 10kW \); the normalized \( P = \frac{P_{\text{out}}}{P_{\text{max}} - P_{\text{min}}} \) is shown by the red surface.
- Efficiency \( \eta \geq 85\% \), shown by the green surface.
- Input impedance angle \( \theta > 0 \); normalized \( \theta = \frac{\theta}{\theta_{\text{max}} - \theta_{\text{min}}} \) is shown in magenta.

The following Table 7 shows the relationship between the output power, transmission efficiency, and angle of the input impedance as a function of the variables \( \alpha, \beta \) when the relative distance between the two transmitting coils and the receiving one is changed, and \( M_1 \) and \( M_2 \) vary accordingly.

To simplify observation, we will assign a value of zero to all surfaces that do not match the conditions for output power, transmission efficiency, and impedance angle.

We see that the transmitted power always satisfies the constraint; the same is not true for the efficiency and input angle. Candidate values of the couple \((\alpha, \beta)\) are in the regions shown in yellow, where the three surfaces are simultaneously not zero.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Output</th>
<th>Parameters</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = -50mm )</td>
<td><img src="image1.png" alt="Image 1" /></td>
<td><img src="image2.png" alt="Image 2" /></td>
<td>( L = 0mm )</td>
</tr>
</tbody>
</table>
Figure 12a illustrates the results obtained by satisfying all the conditions of $\alpha$ and $\beta$. In Figure 12b, red, green, blue, cyan, magenta, yellow, black, and purple are used to represent the first to eighth areas that meet the conditions, respectively.

The ideal selection point in the chosen area is symmetrical. When the receiving coil is close to the first transmitting coil, the variable capacitor in the LCC compensation
network for the second transmitting coil needs to be adjusted to a larger angle. This means that the variable capacitor value should change significantly to provide appropriate compensation, ensuring overall output power stability and efficiency improvement. The same outcome is achieved in the opposite case.

4.3. Group Optimization

In this subsection, we introduce the utilization of the group optimization technique, in order to being able to maintain a consistent output power and efficiency without the need of measuring the receiver position, i.e., without the need of position sensors.

As shown before, in order to maintain a constant power output while $M_1$ and $M_2$ change with distance, $\alpha, \beta$ are the only variables that can be controlled. Furthermore, the symmetry shown in Figure 12b shows that $\alpha$ and $\beta$ have a negative correlation. As a consequence, we discretize the interval of the allowed values of $\alpha$ and $\beta$ as follows:

$$\alpha = \{1, \Delta, 2\Delta, 3\Delta, \ldots, 2\}$$  (30)

$$\beta = \{2, \ldots, 3\Delta, 2\Delta, 1\}$$  (31)

$$\Delta = 1/26$$  (32)

The group optimization algorithm consists of the following steps: initialize $\alpha$ and $\beta$ at the values of $4\Delta$, $13\Delta$, and $22\Delta$, and compute their output power and efficiency. Then, identify the group with the highest value among the three and the maximum efficiency and proceed with a second computation (see Figure 13). The optimum value can be achieved after three successive calculations.

![Figure 13. Group optimization diagram.](image)

The flowchart reported in Figure 14 gives the details of the adopted procedure.
Figure 14. Detailed flow chart for group optimization.
The algorithm requires three variables as input: output power, system efficiency, and input impedance angle at the matching circuit input terminal. The results of the algorithm are the angles of the control input for the SCCs. The approach is integrated into Simulink, utilizing the system module of MATLAB for conducting the required operations.

In this strategy, the output power is the most crucial parameter to consider, followed by the efficiency, and lastly the input impedance angle. The optimum group is primarily chosen based on the output power, considering the outcomes of the first testing. Three points are measured in the first stage, two points in the second, and two points in the third stage. To find the best configuration, a total of seven points can be measured.

5. Simulation and Verification

5.1. Digital Twin for Co-Simulation

As known Digital Twins (DT) provide numerous benefits in the design process of complex devices, and can also be of great aid during the device operation. The DT of the proposed system has been created combining different tools, as shown in Figure 15.

![Digital Twin](image)

**Figure 15.** Digital Twin of the system for co-simulation.

The physical coils are modeled in the Electromagnetic module of the commercial FEM code ANSYS, which is then integrated into the circuit model after order reduction. MATLAB Simulink is finally utilized to implement the previously proposed group optimization algorithm and enable real-time control.

5.2. Simulation Results

The group optimization process is shown in full in Figure 16. The WPT system needs to reach an almost stable operation point during the T0–T1 stage. In the T1–T4 stage, according to the group optimization process, three tests were conducted separately to obtain the best group. The group was then tested again in the T4–T8 stage. Three tests were attempted, and the best outcomes were attained.
According to the previously described group optimization calculation, joint simulation calculations were performed on six typical position test points. From Figure 16, it can be seen that only a total of seven (3+2+2) test times are needed to obtain the optimal SCC control input angle.

Figure 17 reports the change in all parameters in the optimization process, including input parameters and output parameters: Figure 17a is the change of $\alpha$ and $\beta$ at each stage, and Figure 17b is the value of output power and output efficiency.

As the distance $L$ varies from 0 mm to 660 mm, the input angle $\alpha$ of SCC varies from a small value to a large value, while the $\beta$ value changes from large to small. The optimization results can ensure that the output power of the system is the set value of 10 kW and the system efficiency is greater than 85% throughout the whole process.

The full-bridge inverter circuit can achieve soft switching, as shown in Figure 18a. The computed waveform for SCC matches the waveform previously analyzed in Figure 18b. It can be concluded that the waveform of SCC is consistent with the previous theory.
The effectiveness of the group optimization underwent a preliminary validation using some static experiments under development on a prototype based on the system described in [43] that was developed for a low voltage battery charge. Because of the low voltage level on the load, a buck regulator was used.

5.3. Loss Power

Losses are estimated by calculating the current and voltage flowing through the component in the steady state, and the results are shown in Figure 19. It is evident that the compensation circuit and magnetic coupler are responsible for the largest part of the losses, accounting for more than 80% of them. Regarding the SCC control module, its unique features allow it to operate in a soft switching condition, meaning a low value of losses are associated with it.
5.4. Comparison with Previous Methods

According to previous referenced articles, we listed some typical methods for DPWT systems in Table 8.

Table 8. Comparison with different methods.

<table>
<thead>
<tr>
<th>References</th>
<th>Methods</th>
<th>ZVS</th>
<th>Optimum Efficiency</th>
<th>Output Power Fluctuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>[8]</td>
<td>Improved magnetic coupling</td>
<td>YES</td>
<td>90.37%</td>
<td>±2%</td>
</tr>
<tr>
<td>[11]</td>
<td>Improved magnetic coupling</td>
<td>YES</td>
<td>91.6%</td>
<td>±4%</td>
</tr>
<tr>
<td>[14]</td>
<td>Detuning Repeater</td>
<td>NO</td>
<td>82%</td>
<td>±8.5%</td>
</tr>
<tr>
<td>[17]</td>
<td>Frequency-tracking</td>
<td>NO</td>
<td>91%</td>
<td>/</td>
</tr>
<tr>
<td>[24]</td>
<td>Composite control strategy</td>
<td>NO</td>
<td>89.5%</td>
<td>±4.4%</td>
</tr>
<tr>
<td>[30]</td>
<td>SCC + Triple-phase-shift</td>
<td>YES</td>
<td>92.5%</td>
<td>/</td>
</tr>
<tr>
<td>[33]</td>
<td>SCC + Enhanced pulse density voltage control</td>
<td>YES</td>
<td>90.57%</td>
<td>/</td>
</tr>
<tr>
<td>Our proposal</td>
<td>SCC+ Group Optimal</td>
<td>YES</td>
<td>89.5%</td>
<td>4.5%</td>
</tr>
</tbody>
</table>

Through a comparison of common techniques used in DWPT systems throughout seven previous articles, we can observe that many approaches have specific goals, such as improvements in transmission power stability, transmission efficiency, and ZVS. The method presented in this study guarantees that the three requirements of ZVS, transmission efficiency, and output efficiency may all be satisfied simultaneously, and appears to be more balanced and effective than earlier approaches.

6. Conclusions

In this paper, a complete model including the magnetic behavior of the coils, the characteristics of the compensation network, and the parametric relationship between transmission power and efficiency is presented. More specifically, a DPWT system with two transmitting coils and one receiving coil has been studied, and a control strategy which allows for constant power and high efficiency is proposed. Moreover, by the use of a group optimization algorithm, the control strategy can be implemented without the use of additional position sensors or complex algorithms for the receiver’s position.
estimation/detection. The accuracy of the optimization algorithm has been preliminarily tested by considering a quasi-static WPT prototype already developed at our laboratories. The proposed technique can be easily extended to a system with several transmitting coils (and one receiver), since the receiver is coupled with no more than two transmitters at once because of the same length of transmitter and receiver pads in the motion direction. The same inverter can be used for energizing a portion of the electrified track with several transmitters.

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Data Availability Statement: The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author/s.

Conflicts of Interest: The authors declare no conflict of interest.

Parameters of the DWPT system:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{DC}$</td>
<td>Input DC Voltage</td>
<td>250 V</td>
</tr>
<tr>
<td>$f$</td>
<td>Fundamental Frequency</td>
<td>85 kHz</td>
</tr>
<tr>
<td>$L_{TX}$</td>
<td>Inductance of transmitter coil</td>
<td>50 µH</td>
</tr>
<tr>
<td>$L_{RX}$</td>
<td>Inductance of receiver coil</td>
<td>50 µH</td>
</tr>
<tr>
<td>$M_{TX-RX}$</td>
<td>Range of Mutual Inductance</td>
<td>(1.2–31.9) µH</td>
</tr>
<tr>
<td>$R_1$</td>
<td>ESR of transmitter coil</td>
<td>0.13 Ω</td>
</tr>
<tr>
<td>$R_2$</td>
<td>ESR of receiver coil</td>
<td>0.13 Ω</td>
</tr>
<tr>
<td>$L_{CI}$</td>
<td>Auxiliary inductor</td>
<td>20 µH</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Parallel capacitor (SCC)</td>
<td>($α$×175) nF</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Series capacitor of transmitter side</td>
<td>117 nF</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Series capacitor of receiver side</td>
<td>70 nF</td>
</tr>
<tr>
<td>$θ$</td>
<td>Input impedance angle</td>
<td>rad</td>
</tr>
<tr>
<td>$δ$</td>
<td>Phase-shift angle</td>
<td>2.97 rad</td>
</tr>
<tr>
<td>$α$</td>
<td>The offset angle for SCC</td>
<td>($π/2−π$) rad</td>
</tr>
</tbody>
</table>

References


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