

Article

Enabling Scalable and Manufacturable Large-Scale Antenna Arrays Through Hexagonal Subarray Implementation over Goldberg Polyhedra

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Abstract

We introduce a scalable and manufacturable approach to conformal large-scale antenna arrays, leveraging Goldberg Polyhedra configurations with hexagonal subarrays to enable cost-effective, high-performance beam steering. Planar array designs face challenges in phase control and beam deformation when steering away from the broadside, leading to increased beamwidth and degraded angular resolution. Our near-spherical Goldberg structures offer a fabrication-friendly, periodic architecture that supports industrial scalability while enabling efficient 360° digital beamforming with minimal distortion. Simulation results confirm significant reductions in sidelobe levels and improved energy concentration, providing enhanced multibeam capabilities and simplified digital beamforming (DBF) control. This approach paves the way for next-generation radar and satellite systems requiring precise directional control, minimal interference, and robust, flexible beam steering performance.

Keywords: large-scale antenna array; Goldberg Polyhedra; subarray; beam steering; electromagnetic simulation; industry; manufacturing



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1. Introduction

Large Array Antennas (LAAs) are now a focal point in electromagnetic research due to their potential in applications such as wireless communications, radar, and space systems. Reconfigurable systems have spurred interest in LAAs, which promise unprecedented performance. Achieving optimal performance remains a challenge due to phase control inefficiencies, widened beamwidth, and increased sidelobe levels—especially when steering beams away from the broadside direction [1,2]. Historically, spherical arrays were set aside primarily due to significant manufacturing challenges and complex signal processing requirements. The high costs and intricate design parameters often led to compromised performance in real-world scenarios. In contrast, conformal arrays have gained traction for their inherent ability to conform to various surfaces, offering improved integration. Their flexibility and ease of integration made them a hot topic.

Amid these developments, quasi-spherical arrays emerge as a promising alternative. By combining the desirable attributes of spherical geometries with enhanced manufacturability and streamlined operational parameters, quasi-spherical arrays offer a compelling pathway forward. They aim to strike a balance between the complex electromagnetic

behavior of true spherical arrays and the practical constraints of current fabrication technologies. For a quasi-spherical array to be considered both feasible and promising, several key requirements must be met. Paramount among these is the need for a manufacturing process that is not only cost-effective but also capable of producing high-precision elements in scalable configurations. Additionally, parametric array factors provide an essential tool for predicting and optimizing the array's radiation pattern, ensuring that the design can meet stringent performance criteria while keeping manageable operational complexities.

Equally critical is the comprehensive assessment of electromagnetic (EM) compatibility and overall system response through detailed simulation studies. Only by rigorously validating the system in simulated environments can designers ensure that the proposed array meets the desired performance metrics under real-world conditions [3]. This validation process is heavily influenced by the selection of the type of radiating elements—whether they are patch antennas, Vivaldi antennas, horn antennas, or other alternatives—as each type offers distinct advantages and performance characteristics that directly impact the overall efficiency and effectiveness of the array.

State-of-the-art designs include structures such as the GEODA platform developed at the Universidad Politécnica de Madrid (UPM) and the Escuela Técnica Superior de Ingenieros de Telecomunicación (ETSIT), as well as large defence infrastructures like the United States radar installations in Alaska, which employ truncated-pyramidal geometries to maximise environmental robustness and operational reliability. Although these systems demonstrate the engineering advantages of pyramid-based architectures, they are characterised by extremely high development and deployment costs, largely driven by their highly specialised materials, construction methods, and mission-specific subsystems. Consequently, their design principles cannot be readily extrapolated to broader civil or industrial applications, as they lack the modularity, scalability, and cost-efficiency required for general-purpose adoption. Such projects therefore remain single-use, bespoke solutions whose limited transferability renders them economically unviable beyond their narrowly defined operational contexts.

In this paper, we present an innovative approach to build spherical conformal arrays using Goldberg Polyhedra configurations. These designs offer a scalable and reproducible method for creating quasi-spherical arrays that maintain optimal performance across various operating conditions (i.e., mobile communications, radar, electronic warfare, etc.) and simplified steering. By employing hexagonal and pentagonal subarrays, the design ensures quasi-uniform coverage, simplifies manufacturing, and enhances beam steering efficiency. The contributions include:

- Assessment of Goldberg Polyhedra to build spherical conformal arrays using replicable structures at scale [4] with physical identical manufacturable subarrays, where only vertices of subarrays (hexagons/pentagons) have antennas.
- Formulation of an activation function to control the single beam forming from sets of subarray structures.
- Computation of the general form of the array factor for Goldberg Polyhedra (GP) structures for single beam forming.
- Assessment of border degradation on the factor array under several circumstances (multibeam, windowing, etc.) with mathematical and electromagnetic approaches.
- Assessment of the Goldberg Polyhedra to build spherical conformal arrays using replicable structures at scale with physical manufacturable subarrays.
- Computation of the radiation pattern via CST Studio Suite corresponding to the Goldberg Polyhedra structure using Vivaldi antennas in the SHF band, to validate the expected performance.
- Evaluation of manufacturing errors and their impact on system robustness.

The remainder of the paper is organized as follows: In Section 2, we discuss prior contributions approximating spherical conformal arrays, to present our approximation based on the Goldberg Polyhedra in Section 3. Section 4 describes the proposed GP inner structure of activated set of antennas. Section 5 discusses our approach toward the activation function. The materials and methods used to evaluate this array structure are listed in Section 6, while the results of our experiments are described in Section 7. Conclusions are summarized in Section 8.

2. Conformal Structures

Recent advancements in fabrication technologies have opened up promising avenues for the development of conformal structures in antenna array applications [5]. Past efforts to design and implement geoids, as well as flexible and reconfigurable arrays, have demonstrated the limitations of constructing semi-spherical structures of various dimensions using antennas. These attempts have proven to be neither practical nor feasible, which explains their limited implementation to date. Projects such as Geodesic Dome Phased Array Antenna (GDPA) were conceived as isolated developments that demanded significant investment in resources and time, yielding only partial results and persistent beamforming issues that required complex calibration processes [6,7]. It has been assessed that such systems are neither scalable nor replicable in other contexts, thereby preventing the application of economies of scale and rendering them financially unprofitable [6].

As an alternative to these configurations, the focus has shifted towards flexible and reconfigurable antennas capable of adapting their frequency, pattern, and polarization [8]. These systems utilize non-conventional materials and innovative design approaches to achieve conformality while maintaining electromagnetic performance [9]. This paradigm shift not only confirms the feasibility of conformal structures but also opens the door to explore a wide range of approximations and synthesis methodologies tailored to the unique challenges of array design.

One significant advantage of these new conformal configurations lies in their potential for scalability and regularity. By designing arrays that are replicable and configurable [10], it becomes possible to accelerate the transition of conformal systems to industrial-scale applications. This approach directly addresses long-standing limitations observed in earlier implementations, where semi-spherical geometries were essentially custom-built and lacked generalizability [6].

Furthermore, certain conformal layouts inherently offer properties that facilitate operation and calibration. Prior studies have highlighted the challenges involved in calibrating 3D arrays [7], often requiring ongoing adjustments and complex compensation schemes. However, structural regularity and intelligent geometric constraints can mitigate many of these complications. Despite this, the integration of reconfigurable elements continues to pose significant hurdles in terms of mechanical stability and maintaining consistent electronic performance under bending and deformation [6,9].

In parallel, new deterministic synthesis methods have been proposed to address these issues [11]. The Auxiliary Array Pattern formulation stands out for enabling the design of singly curved and elliptically shaped arrays with reduced computational complexity [12,13]. Nevertheless, the extension of such methods to fully spherical configurations remains an open challenge. Realizing this goal will depend on progress in both fabrication protocols and the theoretical models that describe array behavior under mechanical deformation.

Once these protocols are capable of reliably producing such structures, the impact will be considerable. The proposed direction is technically viable, as Active Electronically Scanned Array (AESA) systems function correctly with small pointing angles, and the physical technology required to fabricate subarrays already exists. This perspective supports

the design of a system intended for industrial mass production that represents a significant advance in the democratization of this currently high-cost technology. The system must be replicable, configurable, and scalable to suit the specific requirements of each application. In short: we aim for structures that offer similar properties but are attainable.

3. Goldberg Structure

Goldberg Polyhedra (GP) provide an efficient and practical framework for the construction of near-spherical arrays. These mathematical structures resemble a spherical cage [14] composed of flat hexagonal faces and a minimal number of pentagonal ones, arranged periodically across the surface [15]. This arrangement ensures high replicability, configurability, and suitability for industrial-scale production. Goldberg structures are defined by three key geometric properties: each face is either a pentagon or a hexagon, three faces meet at each vertex, and the overall configuration exhibits rotational icosahedral symmetry. This symmetry guarantees that hexagonal substructures rotate periodically to form the overall GP shape. Additionally, Euler's polyhedron formula dictates that any GP will contain exactly twelve pentagonal faces. Both pentagons and hexagons in these configurations are regular in shape [16].

Such near-spherical structures offer a practical and feasible solution for array design using current technologies. Their geometry ensures that, for any desired beam steering direction, there exists a portion of the structure whose normal vector is sufficiently aligned with that direction. In particular, the beam can be formed using any transducer located at a point on the sphere where the angle between the local normal and the pointing vector is below a fixed threshold. This allows for the identification of a virtual set of antennas for any steering direction, enabling the creation of an array factor with controlled beam deformation for such near-spherical arrays.

This is especially beneficial for wide-angle coverage scenarios—such as in radar or satellite communications—where the system must operate effectively even during angular boundary scans. These scans, which occur near the edge of the array's operational angular range, pose the greatest challenge, not the alignment itself. The near-spherical configuration of GP thus enables the formation of beam patterns with controlled deformation under far-field conditions and across the full angular range.

Our objective is to cover all the vertices of the GP—not the faces themselves—with the fewest number of hexagonal and pentagonal faces. Each hexagon inherently covers six vertices, and each vertex is shared by three faces: either two hexagons and one pentagon, or three hexagons. In our implementation, the physical subarrays are composed of hexagons and pentagons, with antennas positioned at the vertices. This guarantees complete and non-redundant coverage of all vertices [17]. A hexagonal subarray contains six antennas, while a pentagonal subarray contains five.

This methodology highlights the replicability and scalability of the polyhedral structure. By this, we refer to the ability to begin with a simple, small-scale case (e.g., GP(1,1) with a limited number of vertices) and scale up to more complex configurations GP(m,n), where antennas are placed on the minimal set of hexagonal and pentagonal subarrays necessary to cover all vertices. Each antenna's location is predefined and consistent across instances of the same class.

A comprehensive review of icosahedral Goldberg Polyhedra showed that only two different sets satisfy the previous requirements: GP($m,0$) with $\{m \in \mathbb{Z} \mid m = 3k, k \in \mathbb{Z}\}$, e.g., GP(3,0), and GP(m,n) with $m = n$, e.g., GP(4,4). Figure 1 depicts GP(3,0) to the left, and GP(4,4) to the right, where the former ideas are captured. The grey pentagons and hexagons are the only ones that must be placed to cover all the vertices and to avoid contiguous faces, a requirement only fulfilled by these two sets of GPs. In the industrial implementation,

only the grey faces will be physical subarrays of antennas; all the other faces do not require to be manufactured as with our method their vertices are already covered.

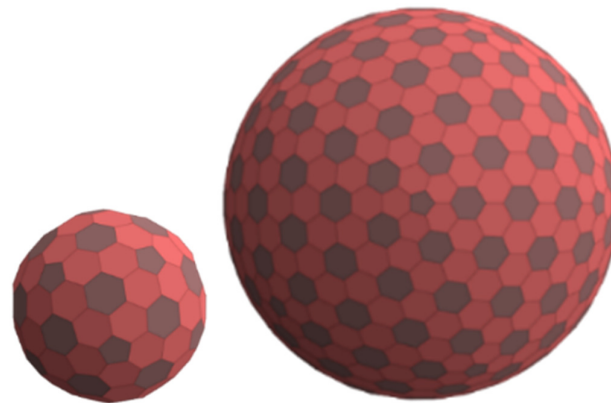


Figure 1. Hexagonal distribution of subarrays in the Goldberg substructures in grey. The physical design omits all the red faces and keep the grey subarrays. GP(3,0) is to the left; GP(4,4) is to the right. All the grey faces have the same dimensions and, therefore, the GP(3,0) is smaller than the GP(4,4).

4. Physical Array Structure

By allocating antennas at the vertices of one face in grey in Figure 1 we have one physical subarray (hexagonal or pentagonal). By applying that subarray structure in all the GP($m,0$) or GP(m,m) grey faces, one can form the array polyhedron that covers all the vertices.

Based on Schein and Gayed's patent [4], one can compute the position of all vertices for a desired GP(m,n) [18]. This patent employs an icosahedron to generate the GP as we will explain step by step. Figure 2 depicts how m and n determine the dimensions of the icosahedron's faces. Note that the values of m and n not only define the dimensions of the icosahedron's faces but also directly influence the geometric layout and curvature of the antenna array during the final spherification.

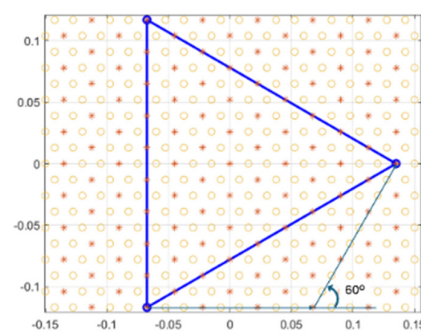


Figure 2. Construction of an icosahedron face. An equilateral triangle is constructed from a hexagonal grid: the center of each hexagon is the asterisk in orange, and the vertices of the hexagonal grid are the circles in yellow. Its bottom edge spans $m = 3$ hexagons in the horizontal direction, and then $n = 3$ hexagons at 60° ; thus, the final GP will be a GP(3,3).

By positioning the former identical faces, one can produce the icosahedral cage in Figure 3. On the final icosahedron, composed in Figure 3, the elements at the vertices of the icosahedron generate pentagonal faces, and all the other elements generate hexagons among them, even the edges of the icosahedron.

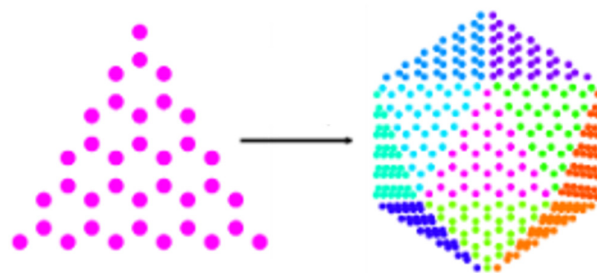


Figure 3. Icosahedron face and icosahedral cage construction with hexagonal grid for the G(2,2) structure (1620 nodes). All the icosahedral cage faces match the planar triangular substructure, leading to an identical three-dimensional diagram on each face. The 3D diagrams of adjacent faces are connected in a triangle mapping, generating an untied and closed geometry.

Finally, we project the icosahedral cage into a spherical cage. The radius to perform the spherification is the distance from the centre of the icosahedron's further points, coinciding with its vertices. All the other points are projected into a sphere by elongating its normal vectors—from the origin of the sphere up to the radius of the spherical cage.

When using a large number of antennas, the projections minimally distort the distance between elements, and the slight deformation of the hexagons at each face of the icosahedron is negligible. In digital arrays, the goal is to ensure accurate positioning of points rather than merely adjusting their layout. By determining these positions after projecting hexagonal subarrays, curvature distortions can be corrected, enabling precise antenna feeding and effective signal processing. Calibration references can compensate for spacing discrepancies, maintaining alignment integrity. Figure 4 depicts the final projection of the GP(2,2), for which we have a mathematical and replicable method for positioning the sensors at the vertices of the cage following a GP distribution.

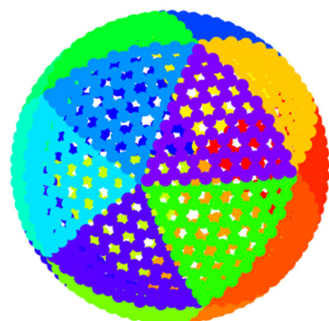


Figure 4. Spherical cage distribution with hexagonal grid for the G(2,2) structure in Figure 3 and the same 1620 elements.

This projection process, together with the regularity and symmetry of GP structures, naturally leads to a consistent method for defining the physical subarrays to be placed across the array. The regular distribution of pentagons and hexagons across the surface enables a structured approach to identify and orient each subarray while ensuring complete coverage and scalability.

We derived generalizable steps from specific cases to ensure our method's applicability across both classes of GP. First, we consider the arrangement of pentagonal faces. Their positions are predetermined and well-documented [16]. Additionally, the surface angle measures 60° , while the central angle—the angle between adjacent vertices of the icosahedron—is 63.44° [16].

Based on the preceding considerations and extensive experimentation [19], we propose the following structured method to obtain the position of the centres and the orientation

of the physical subarrays in the structure, to be manufactured and connected to create the array:

1. Initialization: Select one of the twelve pentagonal faces. This pentagonal face marked with a red star in Figure 5 serves as the starting point for our example.
2. First movement: Proceed to an adjacent hexagonal face moving perpendicularly to any edge of the pentagon. Such movements are denoted as “1” in Figure 5. During this initial step, one could not encounter another pentagon due to the GP structure.
3. Second movement: The direction of the subsequent movement is relative to the prior one. Rotate the movement direction by 60° (clockwise or counterclockwise). The new direction is perpendicular to the edge of the current hexagon. This movement to an adjacent face along the perpendicular direction is denoted as “2” in Figure 5.
4. Cover the full structure: Subsequent movements are determined by the type of face reached in the previous step: Hexagonal face reached: If we have reached and activated a hexagonal face at the end of the former second movement, we must repeat step (3) twice, and then again, this one, (4). Pentagonal face reached: If we have reached and activated a pentagonal face at the end of the former second movement, we must repeat steps (2) and (3), and then again, this one, (4).

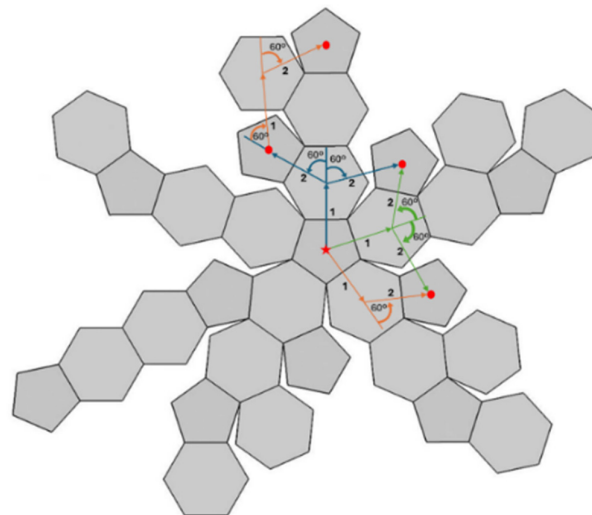


Figure 5. Unfolded truncated icosahedron, i.e., GP(1,1). The procedure to determine the physical subarrays (red dots and star) to be placed on the physical structure is illustrated starting from the pentagon with the red star, taking two (“1” and “2”) steps with a 60° difference determines the physical subarrays needed to cover all the vertices (antennas) of the structure (array).

Each face reached at the end of the second movement is part of the built polyhedron structure and contributes to the coverage of the overall vertices (antennas). This step must be repeated until all the faces have been reached.

Figure 5 illustrates the steps on the 2D net of a truncated icosahedron, GP(1,1). The corresponding movements and angles are indicated. It is important to note that, due to the distortion inherent in representing a 3D object in 2D, there may be apparent misalignments between the movement directions and the edges of the faces, which should always be perpendicular, as well as the 60° rotations. This figure serves solely as a visual representation of the procedural steps and does not accurately depict the true 3D structure.

After defining which physical subarrays to place on the unfolded icosahedron, we apply this arrangement to the 3D structure. As two classes of Goldberg Polyhedra have been identified to meet the requirements outlined in Section 3—namely, GP(m,n) with $m = n$ (e.g., GP(3,3)), and GP($m,0$) with $\{m \in \mathbb{Z} \mid m = 3k, k \in \mathbb{Z}\}$ (e.g., GP(6,0))—we will first

demonstrate how the proposed method is applied to a GP(3,0), followed by an application to a GP(4,4), representing examples from both classes.

Figure 6 illustrates the sets movements, each in different colours. These movements follow the structured method previously proposed. As noted earlier, both clockwise and counterclockwise rotations must be performed to ensure complete structure coverage. Numbers “1” and “2” represent the first and second movement; remember that the first movement must always lead to a hexagonal face. The curved arrows represent the 60° rotation. All the straight arrows are perpendicular to the edge of the faces.

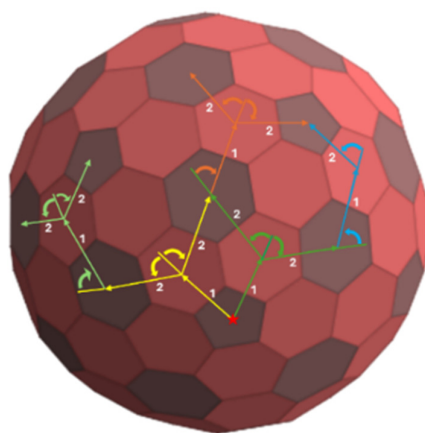


Figure 6. GP(3,0) printed with the overlaid sequence of our procedure to determine the physical subarrays (dark red faces) that need to be placed on the physical structure. Starting from the pentagon with the red star, taking two (“1” and “2”) steps with a 60° difference determine the physical subarrays to cover all the vertices (antennas).

Concerning the Goldberg structure, it was noted that structures of the class GP(m,n) with $m = n$ (e.g., GP(3,3)), also exhibit the property of replicability. The iterative method described earlier successfully determines the physical subarrays in this second class of Goldberg Polyhedra; specifically in Figure 7 for GP(4,4).

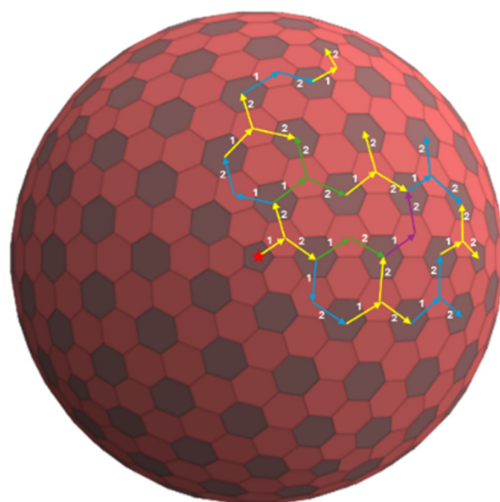


Figure 7. GP(4,4) depicting the procedure to determine the physical subarrays (dark red faces) that need to be placed on the physical structure. Starting from the pentagon with the red star, taking two (“1” and “2”) steps with a 60° difference determines the physical subarrays needed to cover all the vertices (antennas) of the structure (array).

The proposed method also fulfils the objective of minimizing the number of physical subarrays. This reduction brings significant economic benefits, as it allows for efficient

coverage with antennas of all the vertices of the Goldberg Polyhedra. Physical subarrays do not overlap, ensuring replicability and reconfigurability. The iterative process, as demonstrated in the 2D net and extended to the 3D structure, successfully set the necessary physical subarrays while maintaining the set geometric constraints. Moreover, the method utilizes only two types of subarrays—pentagonal and hexagonal—which simplifies the manufacturing process, reducing complexity and making it more cost-effective for industrial production. By ensuring that all vertices are covered while maintaining the structural integrity of the polyhedron, our approach not only meets the design requirements but also optimizes the production process, confirming the practical viability of the method for large-scale applications.

Once the topology of our solution is set, the next step is the computation of the array factor as a function of the selected Goldberg structure.

5. Activation Function

Following the symmetry in GP structures, we introduce an activation function to manage the DBF capabilities. For a given steering direction (θ, φ) , in azimuth and elevation, this function ensures that the steering direction is covered from the activated elements.

To determine the activation function for a standard behaviour, we proceed as follows: Attending to the antennas' half power beamwidth (HPBW) (considering the type: patch, horn, Vivaldi, etc.), we activate those for which the angle between the desired steering direction and the antenna's pointing direction (matching the vector that connects the centre of the structure and the point on the surface of the sphere where the antenna sits) is smaller than the antenna HPBW. Figure 8 depicts the activation function general idea for a very directive horn antenna.

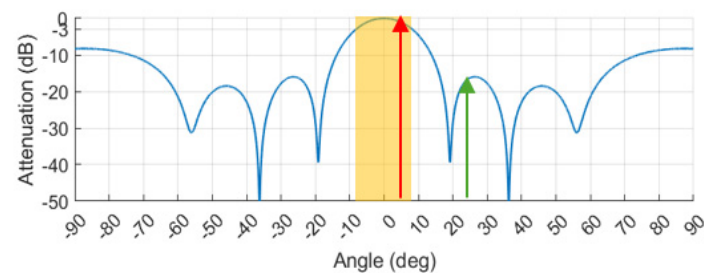


Figure 8. The red arrow determines the direction of arrival of a signal for a specific horn placed at the vertex of the GP; the angle in this case 5° , and our horn is activated. Any signal received or transmitted inside the orange region will activate this specific horn of the array. The same signal will have different incidence angles for each horn in the array. The green arrow that its outside of the orange region will not activate this horn but could activate other horns.

The activation function must evaluate the dot product, where both vectors desired pointing and antenna's vectors, have norm one. Let $\mathbf{d}$ be the desired pointing vector (normalized), $\mathbf{v}$ be the antenna vector (normalized), θ_0 be the angle between $\mathbf{d}$ and $\mathbf{v}$, θ_{HPBW} be the half-power beamwidth angle. As the vectors are normalized for angle θ_0 between $\mathbf{d}$ and $\mathbf{v}$ smaller than θ_{HPBW} , the cosine is greater. Hence, the threshold:

$$\mathbf{d} \cdot \mathbf{v} = \cos \theta_0 \geq \cos(\theta_{HPBW}) \quad (1)$$

for angle θ_0 between $\mathbf{d}$ and $\mathbf{v}$ is smaller than θ_{HPBW} , cosine is greater than $\cos(\theta_{HPBW})$. The activation function can then be defined as:

$$f(\mathbf{d}, \mathbf{v}, \theta_{HPBW}) = \begin{cases} 1 & \text{if } \mathbf{d} \cdot \mathbf{v} \geq \cos(\theta_{HPBW}) \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

As an example, for an array of patch antennas, we set a maximum angle of 45° from the desired steering direction (90° being the full HPBW). This situation is depicted in Figure 9 for the GP(2,2) array structure in Figure 4.

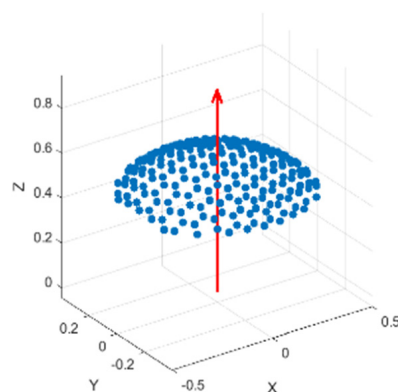


Figure 9. Activated section of the antenna array computed on the same GP(2,2) in Figure 4 attending to the activation function for steering direction (0,0) (red arrow).

Symmetry in the antenna radiation pattern is crucial for achieving uniform beam-forming and maximizing the scan range in an array. A symmetrical pattern like that of a patch antenna, ensures that the contributions from all elements add coherently, leading to a solid main beam (consistent gain and low distortion) and reducing sidelobes. Asymmetric patterns, such as those of Vivaldi antennas, create directional biases. Those biases may limit array's effectiveness in applications requiring broad, uniform coverage and flexible steering. One could choose to work with one type of antenna or the other, or roll the antenna along its axis if it is not symmetrical, etc.

6. Materials and Methods

To assess the performance and scalability of the proposed GP antenna array, various GP(m,n) configurations were simulated, including cases with up to $m = n = 8$, corresponding to approximately 4500 antenna nodes.

The design and simulation of the GP antenna arrays were carried out using MATLAB 2025a and CST Studio Suite 2023. MATLAB was employed for preliminary calculations, including the geometric construction of the GP structure and precise positioning of the array elements. Specifically, the array factor calculation utilized the indexing of vertices in the GP(m,n) configuration based on their orientation vectors and relative spatial positions.

CST Studio Suite was used for high-frequency 3D electromagnetic simulations to model the antenna elements and evaluate critical parameters such as radiation patterns, sidelobe levels (SLLs), and main lobe widening. These simulations provided a realistic validation of the theoretical performance, accounting for practical effects including element coupling and directivity, which are essential for real-world deployment.

The impact of edge degradation on array performance was examined by activating different sets of subarrays along the borders of each polyhedron's face. These tests evaluated the stability and coherence of the array factor near the edges, where signal degradation could potentially compromise overall system performance. Finally, the multi-beam steering capability of the GP antenna array was assessed by configuring multiple virtual sets of active antennas, each steering a beam in a distinct direction. This approach demonstrates the flexibility and robustness of the design for advanced applications requiring simultaneous multi-directional coverage [20].

The practical realisation of the proposed GP-based array may be achieved through a modular fabrication workflow grounded in the repetition of identical planar hexagonal and

pentagonal subarrays. Each subarray is manufactured using standard RF PCB processes (e.g., Taconic substrates, $\lambda/2$ inter-element spacing) and incorporates mechanical alignment features to ensure consistent orientation. These units are subsequently mounted on a lightweight supporting cage that enforces the required GP(m,n) geometry, with the normal vector of each subarray matching its theoretical orientation after spherification. Assembly tolerances, including tilt and pitch, are recorded through photogrammetry or laser-tracking to obtain the true coordinates and orientations of all elements. The RF architecture may rely on per-element or per-subarray digital control to enable DBF operation, with calibration access points integrated into the feeding network. This modular approach ensures that the physical structure remains fully replicable, minimises geometric distortion during assembly, and aligns with the manufacturability goals central to the GP methodology presented in this work.

The measurement of the realised prototype requires a structured validation campaign combining electrical, geometric, and radiation-pattern characterisation. A full VNA calibration moves the reference plane to each feed point, followed by verification of the S-parameters of isolated subarrays and of assembled sectors to confirm fabrication consistency. The actual spatial coordinates and orientations obtained during assembly are integrated into the manifold model for accurate prediction-to-measurement comparison. Radiation-pattern measurements are performed in an anechoic chamber or near-field scanner, obtaining 3D far-field data for multiple steering angles and for any multibeam configurations under test. Key metrics include main-beam pointing accuracy, 3 dB beamwidth, peak gain, sidelobe levels, and degradation at large scan angles. Additional experiments introduce controlled misalignments to assess robustness, validate the sensitivity results obtained through CST simulations, and quantify edge degradation. This combined measurement protocol ensures that the physical demonstrator faithfully reproduces the predicted behaviour of the GP structure and provides a comprehensive assessment of its suitability for scalable, industrially manufacturable wide-angle beamforming systems.

7. Experiments

We conducted several experiments to evaluate the array factor form for a GP antenna array.

1. Computation of the array factor

Expanding upon the foundational theory of spherical arrays presented by Harry L. Van Trees where array elements are assumed to be on a perfect spherical surface, we compute the array factor of our structure. The formula provided by Van Trees for spherical arrays extends naturally to compute the array factor for any arbitrary distribution of isotropic elements in three-dimensional space. One simply requires the active elements and their locations [1].

By exploiting the intrinsic symmetry of Goldberg Polyhedra (GP) structures, we implement an activation function to manage the digital beamforming (DBF) capabilities of the array. For any given steering direction (θ, ϕ) , this function selectively activates only the antenna elements necessary to cover that direction. As mentioned in Section 5, the activation logic under nominal operating conditions is defined as follows: based on the antenna's half-power beamwidth (HPBW)—which depends on the specific antenna type (e.g., patch, horn, Vivaldi, etc.)—we activate only those elements for which the angle between the steering direction and the element's local pointing direction (given by the vector from the centre of the structure to the antenna's position on the sphere, θ_0) is less than the HPBW (θ_{HPBW}). This approach enables beam pattern modelling and synthesis without requiring a strictly spherical geometry, thus allowing for greater flexibility in array design.

Given the known positions p of each element, the array factor can be computed by directly incorporating these coordinates [1], allowing us to model and synthesize the beam pattern without assuming a sphere. The manifold vector and the beam pattern are, respectively:

$$\{v_k(k)\}_{n=1}^N = \exp(-jk^T p_n) \quad (3)$$

$$B(\theta, \phi) = \sum_{n=1}^N a_n [v_k(k)]_n \quad (4)$$

where k stands for:

$$k = -\frac{2\pi}{\lambda} u = -\frac{2\pi}{\lambda} \begin{bmatrix} \sin(\theta)\cos(\phi) \\ \sin(\theta)\sin(\phi) \\ \cos(\theta) \end{bmatrix} \quad (5)$$

and we formulate a_n from Equation (2):

$$a = \prod_{n=1}^N \left(\frac{\cos(\theta_{HPBW})}{\cos(\theta_n)} - 0.5 \right) \quad (6)$$

being θ_n the relative elevation angle for each element.

Furthermore, the beam pattern can be expressed as a function of θ_0 through a rotation matrix M , following the Rodrigues' formula for (θ_0, ϕ_0) . The final equation is:

$$B(\theta, \phi, \theta_0, \phi_0) = \sum_{n=1}^N a_n \cdot \exp \left(-jp_n \begin{bmatrix} k_{1:2} \\ (k_0)_3 \end{bmatrix} M \right)^T \cdot w_n \quad (7)$$

and k_0 stands for:

$$k_0 = -\frac{2\pi}{\lambda} u_0 = -\frac{2\pi}{\lambda} \begin{bmatrix} 0 \\ 0 \\ \cos(\theta_0) \end{bmatrix} \quad (8)$$

w_n are additional weights that may be introduced to this naïve beamforming to match additional constraints (e.g., masks).

This allows us to represent the beam pattern as a function of the distance along the Z' axis of all elements, i.e., $(\cos(\theta_0))$, relative to its maximum value, which corresponds to the radius of the sphere where $\theta_0 = 0$ and the cosine equals 1. This approach enables accurate control of the array factor across various spatial configurations, providing a means to manage pattern coverage, beam direction, and sidelobe levels in complex 3D arrangements.

To synthesize the desired beam pattern in these arrays, techniques similar to those in of Van Trees can be adapted [1]. By adjusting for each element's positional offset and incorporating individual element patterns for non-isotropic elements where necessary, one can effectively mitigate the potential distortions that arise from the non-spherical geometry as depicted in Figure 19 [21]. This approach allows for enhanced control over pattern coverage and grating lobe suppression, making it possible to achieve near-ideal beamforming performance even in arrays with imperfect configurations (Figure 10).

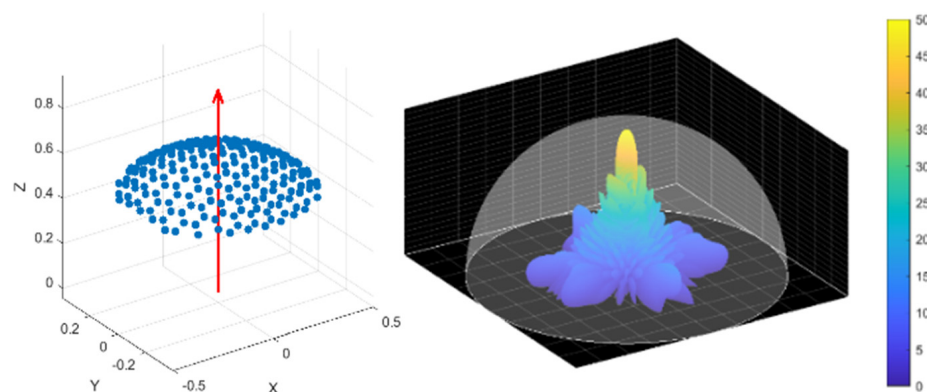


Figure 10. Array factor (right) computed on the GP array attending to the activation function (left) for a desired pointing direction (0,0) (red arrow).

2. Assessment degradation in the borders

The main practical application case is the ability to direct energy in a desired direction with simple activation and without suffering large deformations or losses in the main beam. One can avoid interference from nearby emissions and distinguish between targets or communications close to each other. Our system simplifies steering complexity by using an activation function that accounts for phase shifts between elements just along the pointing dimension. This approach leverages the fact that each virtual set of antennas operates in the “broadside” direction (see the array factor in Figure 11).

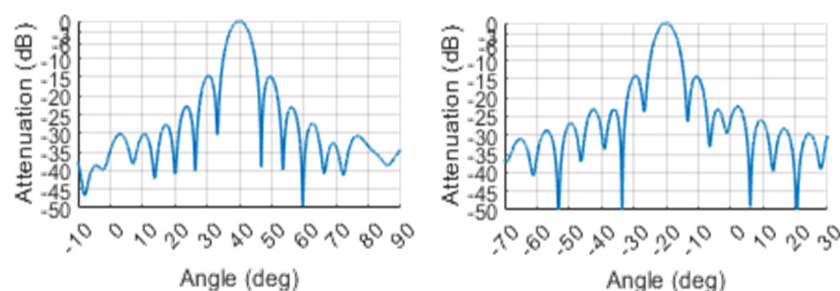


Figure 11. The resulting array factor XZ plane cut for a desired steering direction with the proposed GP section at [40 0] and [−20 0] ([elevation azimuth]).

In essence, during digital processing, we adjust the phase shift similar to the way planar AESA radars operate. Although our system is not a true planar array and includes vertical depth due to its spherical configuration, this method allows us to apply the necessary phase shifts by calculating only the vertical component. By contrast, planar 2D arrays require considering both X and Y position components to achieve steering functionality. As we always work in the “broadside” direction of the working virtual sets of antennas in Figure 10, the position in the orthogonal plane (X,Y) with respect to the pointing direction is dismissed, and we only consider Z.

We must evaluate the limit and undesired case when the number of vertices in the GP, and therefore the sphere is too small. We cannot neglect the fact that our GP is made of planar faces with an orientation, and therefore the antennas contained on each physical subarray (hexagon or pentagon) will be oriented in the same direction as the normal vector of the physical subarray (or grey face of the GP in Figure 1).

In that case, the angle between adjacent subarrays will decrease, and the approximation for large structures where the normal of each element is the same as the normal of its physical subarray no longer holds. For cases such as GP(1,0) (dodecahedron) with angles

between faces of 116° , just a few antennas with a considerable wide enough HPBW will allow the use of more than one face or subarray.

Finally, we analyse the case in which the desired pointing direction approaches a structural edge in reduced GP configurations. Following the previous reasoning, only antennas with at least 116° of HPBW could be employed to cover those regions. This is the rationale behind the bigger the GP structure, the better in order to avoid these limit cases. Similarly, in larger GP structures, employing more directive antennas reduces the number of activated faces compared with configurations that rely on antennas with a higher HPBW.

3. Impact of the number of elements in the GP array

It is important to study how the array factor varies when the number of elements in the sphere is modified while the angle of view is preserved, and vice versa.

Figure 12 illustrates the differences observed in array factor as we reduce the angle of view, more elements are unpowered. It reduces power consumption but widens the beam. Conversely, as we increase the angle of view, more elements are activated, causing a narrowing of the beam. For this experiment we vary the HPBW according to three distinct types of antennas.

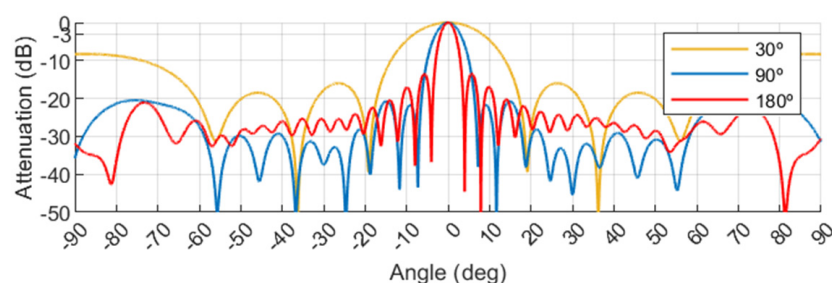


Figure 12. Array factor XZ plane cut with total vision angles of 30° , 90° , and 140° (maximum deviation angles of 15° , 45° , and 70° , respectively).

On the other hand, one can increase the number of vertices under the same solid angle by incrementing the values of m and n for the same distance between elements. A curve of points evenly spaced on the sphere's surface reduces the curvature as the radius increases while preserving the space between points. In other words, preserving the distance between vertices and increasing the number of vertices leads to larger icosahedral and spherical structures and, therefore, to a higher number of vertices under the line of sight of the desired pointing direction that can be activated.

Figure 13 depicts the array factor as we reduce the number of elements in the sphere, the radius decreases and less elements are under the angle of view, causing a widening of the beam. If we increase the number of elements, more elements fall within the angle of view, thereby narrowing the beam. Using various design frequencies and radii to form the array structure, we demonstrate that the minimum beamwidth is achieved when the maximum field of view is reached, corresponding to a total angular span of 180° (90° in each θ_0 direction from the boresight axis). This value is in full agreement across all test cases with the theoretical prediction for planar circular arrays, where the -3 dB beamwidth is given by the expression:

$$BW_{-3dB} = \frac{70\lambda}{2\pi R} \quad (9)$$

Thus, experimental results align with the theoretical expectations for this type of antenna array.

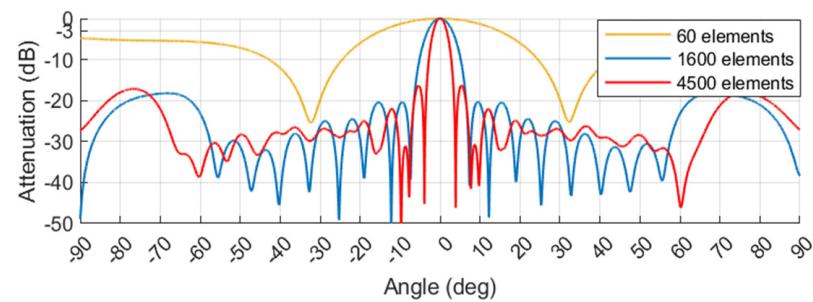


Figure 13. Array factor XZ plane cut with 60, 1600, and 4500 total elements in the sphere. The higher the number of elements, the thinner the main lobe.

4. Angular amplitude weighting

Weighting is a well-known technique. By applying non-uniform amplitude, such as tapering functions (e.g., radial, Taylor, or Chebyshev windows), the power in sidelobes is reduced relative to the main lobe, decreasing interference, and increasing directivity while increasing the beamwidth [22,23].

When studying a 3D semi-spherical array, weights can project points onto a plane orthogonal to the hemisphere's central vector (red arrow in Figure 14). This flattens the surface but distorts distances, making elements at the edges appear closer. As shown on the left of Figure 14, radial weighting reduces the number of highly illuminated elements (blue).

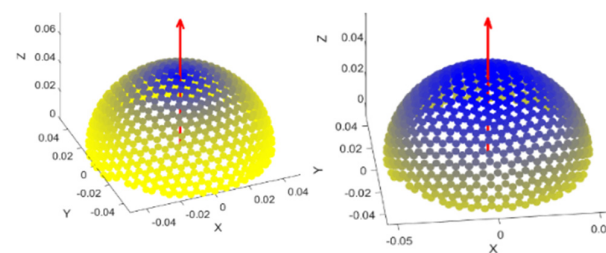


Figure 14. Illumination of the array elements with radial and angular amplitude weighting. The yellow dots represent elements with minimum radiation power, leading to underutilization of the array. In blue the maximum power. The red arrow represents the steering direction of the array.

Using Euclidean distance in a projection plane is not ideal, as it measures distance over a surface, which in this case is almost spherical. Instead, measuring the angle between the hemisphere's central vector and the normal vectors of the structure preserves power and avoids distortion. Figure 14 (right) illustrates angular amplitude weighting.

Finally, we are able to compare the array factor of the array depicted in Figure 15 with and without the weighting. As it is shown, the windowing generates a widening in the main lobe while reducing the sidelobe levels (SLLs) as expected, by -10 dB. This will not deteriorate the directivity of the beam [1,24].

5. Multibeam

GP arrays allow multiple simultaneous beams. Such flexibility is crucial and could be easily adjusted even for neighbouring directions using virtual sets of antennas. We present a naïve implementation for proof of concept. In Figure 16, two pointing directions have been set: (30,180) and (20,0) (elevation, azimuth). The directions are indicated by black arrows, while the virtual sets are depicted as red and blue sensors, differentiated based on the beam on which they are utilized. To assign each element to a specific virtual set, we calculate the angle between the element's normal and each pointing direction. The element belongs to the virtual set of antennas with the smaller angle to the final direction.

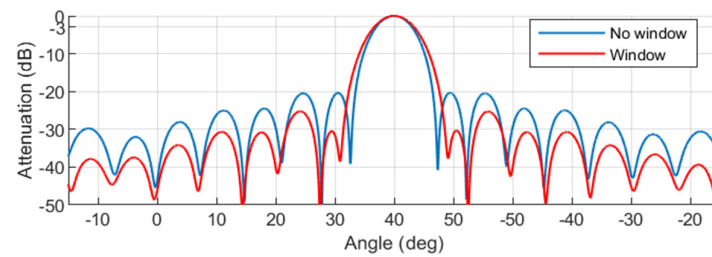


Figure 15. Array factor XZ plane cut with (red) and without (blue) angular Hamming window. With the Hamming window the main lobe is wider, and the SLL lower.

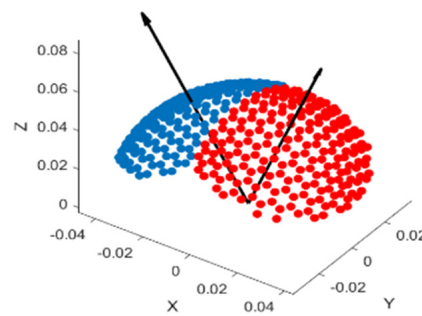


Figure 16. Distribution of antennas to produce two simultaneous beams, one per colour-coded set of antennas.

For each of these virtual sets of antennas the array factor can be computed. Figure 17 depicts that the final array factors point in the desired directions and keep the expected shape as in the former Figure 11 where we had only one beam.

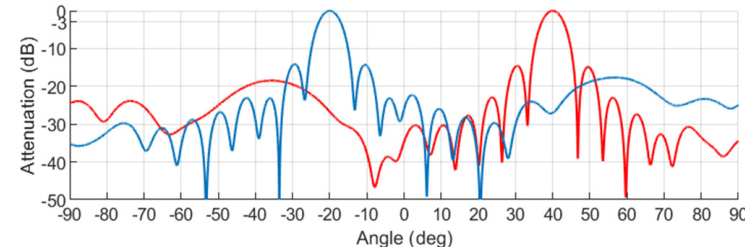


Figure 17. The resulting array factor XZ plane cut for (30,180) and (20,0) steering directions with the corresponding GP virtual colour-coded sets of antennas in Figure 16.

Several considerations must be addressed in this multibeam system. First, when two pointing directions are in close proximity, fewer elements will be available near each direction, as both beams compete for resources in adjacent regions. This competition reduces the number of sensors available, resulting in a distorted broader main beam, or as we will explain, it can be configured for both.

One potential solution for the competition of resources is to alternate the beams by switching them on and off in a manner similar to multifunction radars. Alternatively, increasing the desired field of view could add more elements around the periphery of each virtual set. The optimal yet more complex solution would possibly involve implementing hierarchical virtual sets of antennas [25], allowing elements to be shared concurrently. This approach involves digitally processing information, as if beams operated independently without affecting other beams at a higher computational cost.

6. Computation of the array factor in CST Studio

Regarding the electromagnetic study, we use the CST Studio Suite to focus on the secondary lobe levels, the resulting beam widths, and the coupling between elements.

These figures are particularly important for the array's directivity, in addition to the array factor that we have been discussing up to this point.

In our study, we analyse the Vivaldi antenna configuration illustrated in Figure 18 as it provides an intermedium beamwidth wider than typical horns and narrower than patch antennas. This allows us to, according to this Vivaldi antenna HPBW, activate a region of the quasi-sphere GP neither too big (patches) nor too small (horns). This selected antenna operates at a central frequency of 45 GHz, which is a common frequency for this type of study, utilizing Perfect Electric Conductor (PEC) material for the front curved structure (depicted in grey) and the microstrip line on the rear side. The substrate layer, positioned between the front and rear components, consists of Taconic RF-60A material [25]. The dimensions of the port to excite the microstrip line were precisely calculated using the CST port simulation tool.

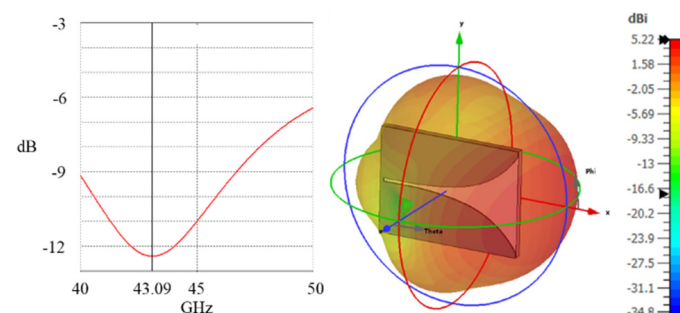


Figure 18. The resulting S-Parameters (S11) for the designed Vivaldi antenna and its beam pattern.

Based on the reflection coefficient $S(1,1)$ presented in Figure 18 (left), the antenna demonstrates a minimum return loss at 43.1 GHz. Consequently, the design of our antenna array was optimized with an element spacing frequency centred at 43.1 GHz. We computed the farfield simulation for a single element at 43.1 GHz (Figure 18, right).

Once the design of the primary unit of the antenna array is finalized, the subsequent step involves building up the complete array model within CST, based on prior MATLAB simulations. To facilitate this process, a TSV (Tab-Separated Values) file is generated in MATLAB. This file encapsulates critical data, including the spatial coordinates of each element in 3D, as well as the corresponding amplitude, phase, and tilt angles of the illumination.

The tilt angles are particularly sensible since each antenna element needs to be oriented perpendicularly to the surface of the sphere at its respective location. The arrangement follows a hexagonal grid pattern with a spacing of $\lambda/2$ between adjacent elements [1], considering that each antenna is surrounded by six neighbouring elements. The total field of view for building the array is determined by the Half-Power Beamwidth (HPBW) of the designed Vivaldi antenna, 78° .

Figure 19 depicts the final configuration of the antenna array within CST, displaying a frontal viewpoint.

The returning losses parameter for the full array finds a minimum value at 42.78 GHz. Slightly deviated from the original 43.1 GHz (Figure 18), this value is acceptable and the designed antennas for 43.1 GHz. The electromagnetic simulation in CST provided the far-field pattern at 42.78 GHz depicted in Figure 20 (left) in 3D. By introducing misalignments in the system, where the Vivaldi antennas deviate from their correct tilt and pitch orientation by angles ranging from 0 to 25 degrees, the overall robustness is significantly affected. As shown in Figure 20 (right), these manufacturing errors lead to notable performance degradations compared to the ideal case previously discussed, highlighting the sensitivity of the system to such misalignments.

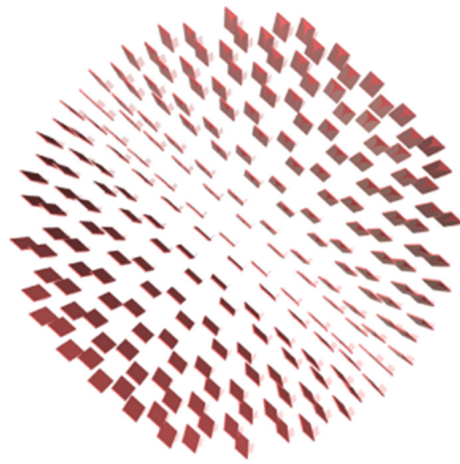


Figure 19. CST array configuration of Vivaldi antennas for a vision angle $\theta_0 \leq 78^\circ$.

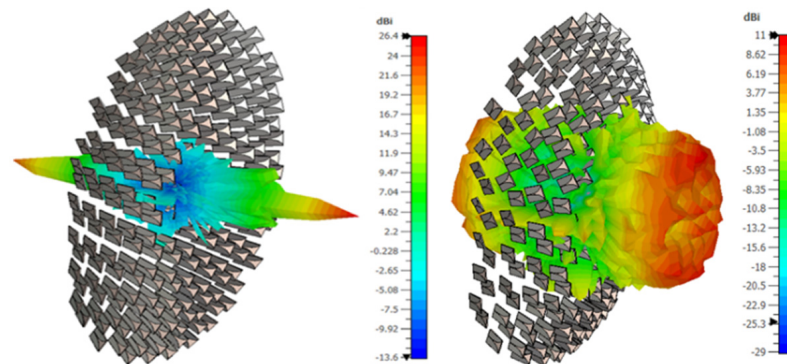


Figure 20. Farfield 3D CST electromagnetic simulation on the test array for 220 elements using Vivaldi antennas and $\theta_0 \leq 80^\circ$ without (left) and with (right) tilt and pitch misalignments.

The peak gain of the Vivaldi array is higher (around 26 dBi) than the expected with isotropic elements, which is a direct consequence of the directive nature of the Vivaldi antennas amplifying the radiated energy in the broadside direction.

7. MATLAB vs. CST Studio simulations

This analysis compares the array factor simulated using isotropic radiators (blue curve in Figure 21) via MATLAB and the radiation pattern of an antenna array composed of Vivaldi antennas (red curve in Figure 21) via CST Studio. Both cases utilize the same array configuration, ensuring a consistent geometric structure for accurate comparison.

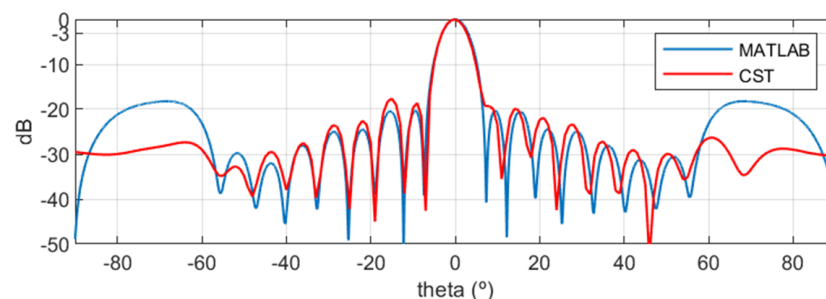


Figure 21. MATLAB array factor XZ plane cut and CST radiation diagram farfield XZ plane cut.

The array factor represents a simple case, where isotropic radiators emit uniformly in all directions. In contrast, the Vivaldi array's pattern results from the convolution of the array factor with the intrinsic angular radiation pattern of the Vivaldi elements.

Side lobe levels display slight differences. The isotropic array factor (AF) produces regularly spaced side lobes with attenuation levels generally below -40 dB, reflecting the predictable nature of an ideal array without directional weighting from the elements. Nulls are sharply defined and evenly spaced, representing complete cancellation due to pure phase interactions. In contrast, the Vivaldi array pattern exhibits more irregular side lobe structures and positions on the right, probably due to constructive interference effects. The resulting pattern displays some less pronounced and shifted nulls, as the directive element patterns contribute non-uniformly, partially filling some of these nulls. The array pattern reflects the practical complexities introduced by directive antenna elements. These findings underscore the importance of individual element patterns in shaping the overall array radiation performance, but also the accuracy and good results under real-environment conditions regarding real elements such as Vivaldi antennas and the EM compatibility.

To strengthen the credibility of the results, experimental measurements will indeed be incorporated; however, these activities form part of our planned future work, as the fabrication and characterisation of a full GP-based prototype fall beyond the scope of the present study.

8. Conclusions

This study introduces a groundbreaking methodology for the design and fabrication of large-scale antenna arrays based on Goldberg Polyhedra configurations, combining hexagonal and pentagonal subarrays to enable an unprecedented balance between structural efficiency and scalability. The proposed near-spherical arrays simplify manufacturing by reducing the system to a small set of regular, flat components, while offering seamless compatibility with both naïve and advanced digital beamforming (DBF) schemes. This allows for highly adaptable and precise beam steering with minimal implementation complexity.

Conventional conformal and quasi-spherical array implementations such as GEODA-type semi-spherical platforms, geoid-based structures, and isolated large-scale truncated-pyramid systems have demonstrated valuable proof-of-concept performance but remain hindered by intrinsic limitations. These include high manufacturing costs, the absence of scalable and repeatable fabrication processes, complex calibration requirements, and limited transferability to other antenna configurations or industrial deployments. In contrast, the proposed Goldberg Polyhedra methodology introduces a structurally regular, fully replicable, and inherently scalable framework. Its reliance on a small set of identical hexagonal and pentagonal subarrays enables true industrial manufacturability [26], reduces calibration overhead, and supports wide-angle, low-distortion digital beamforming. Table 1 summarises the principal advantages of the proposed design relative to representative state-of-the-art methods.

Future work focuses on optimizing antenna set configurations for specific operational scenarios, exploring hierarchical activation of virtual antennas to facilitate control and management, and developing robust calibration strategies to minimize edge degradation. Further electromagnetic simulations will help refine the practical deployment of these arrays in real-world environments, paving the way for scalable, high-performance, and industry-ready beamforming systems.

Table 1. Comparison of the proposed GP-based method with existing conformal array approaches.

Feature	Conventional Conformal/Semi-Spherical Arrays	Large Bespoke Truncated-Pyramid Systems	Proposed Goldberg Polyhedra (GP) Methodology
Manufacturing cost	High: non-replicable geometries and bespoke fabrication	Extremely high: single-purpose structures	Low-to-moderate: relies on few repeated planar subarrays
Scalability/industrial reproducibility	Limited: structures are not easily scalable and lack identical modules	None: purpose-built, not reusable	Excellent: modular and fully repeatable GP(m,n) structures
Calibration complexity	High: 3D arrays require continuous recalibration	Very high: calibration strongly environment-dependent	Reduced: structural regularity simplifies calibration
Beam steering performance	Moderate; strong deformation at large scan angles	Good within restricted operational window	High: near-spherical coverage with minimal distortion
Flexibility for multibeam operation	Limited: constrained by geometry	Very limited: designed for single mission	High: virtual antenna sets easily support multibeam steering
Adaptability to different applications	Poor: designs are non-transferable	None: single-use systems	High: structure configurable for radar, SATCOM, EW, etc.
Industrial viability	Low	None	High: simple subarray repetition supports mass production

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Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

LAA	Large Antenna Array
AF	Array Factor
TTD	True Time Delay
MIMO	Multiple-Input and Multiple-Output
DFRC	Dual-Function Radar-Communication
MUSIC	MUltiple SIgnal Classification
DBF	Digital Beam Forming
AESA	Active Electronically Scanned Array

References

1. Van Trees, H.L. *Optimum Array Processing, Part IV of Detection, Estimation, and Modulation Theory*; John Wiley & Sons: Hoboken, NJ, USA, 2002.
2. Delos, P.; Broughton, B.; Kraft, J. Phased Array Antenna Patterns—Part 3: Sidelobes & Tapering. *Analog Dialogue* **2020**, *54*, 1–7.
3. Wang, C.; Wang, Y.; Zhou, J.; Wang, M.; Zhong, J.; Duan, B. Compensation method for distorted planar array antennas based on structural–electromagnetic coupling and fast Fourier transform. *IET Microw. Antennas Propag.* **2018**, *12*, 954–962. [\[CrossRef\]](#)
4. Jay Schein, S. Convex Equilateral Polyhedra with Polyhedral Symmetry. U.S. Patent No. US 2015/0037766A1, 5 February 2015.
5. Sumalatha, K.; Phani Kumar, C.R. Advances in Conformal Antennas. *Int. J. Eng. Adv. Technol.* **2019**, *8*, 1883–1887. [\[CrossRef\]](#)
6. Sierra, M.; Torres, A.; Masa, J.L.; Gomez, M.; Montesinos, I. GEODA: Adaptive Antenna Array for Satellite Signal Reception. In Proceedings of the 2nd European Conference on Antennas and Propagation, EU CAP 2007, Edinburgh, UK, 11–16 November 2007; pp. 1–4. [\[CrossRef\]](#)
7. Natera, M.A.S.; Rodríguez-Orsorio, R.M.; de Haro Ariet, L.; Pérez, M.S. Calibration Proposal for New Antenna Array Architectures and Technologies for Space Communications. *IEEE Antennas Wirel. Propag.* **2012**, *11*, 1129–1132. [\[CrossRef\]](#)
8. Mohamadzade, B.; Simorangkir, R.B.; Maric, S.; Lalbakhsh, A.; Esselle, K.P.; Hashmi, R.M. Recent developments and state of the art in flexible and conformal reconfigurable antennas. *Electronics* **2020**, *9*, 1375. [\[CrossRef\]](#)
9. Kirtania, S.G.; Handique, M.U.; Hosain, M.S.; Hossain, M.M.; Nguyen, N.T.; Yoo, H. Flexible Antennas: A Review. *IEEE Access* **2020**, *8*, 164108–164139. [\[CrossRef\]](#) [\[PubMed\]](#)
10. Marks, R.W.; Fuller, R.B. *Dymaxion World of Buckminster Fuller*; Anchor Books: Manhattan, NY, USA, 1973.
11. Geerarts, J.M.; Theis, G.; Smolders, A.B.; Caratelli, D. Constrained deterministic synthesis of conformal supershaped antenna arrays. *IEEE Trans. Antennas Propag.* **2023**, *71*, 8726–8738. [\[CrossRef\]](#)
12. Zeng, Y.; Liu, Q.; Li, W. A Fast Compensation Method for Deformed Conformal Arrays Considering Mutual Coupling Effect. *IEEE Trans. Antennas Propag.* **2023**, *71*, 9581–9592. [\[CrossRef\]](#)
13. Geerarts, J.M.E.; Theis, G.; Smolders, A.B.; Caratelli, D. Synthesis of Elliptically Shaped Aperiodic Antenna Arrays Subject to Complex Design Constraints. *Radio Sci.* **2024**, *59*, e2023RS007914. [\[CrossRef\]](#)
14. Goldberg, M. A class of multi-symmetric polyhedra. *Tohoku Math. J. First Ser.* **1937**, *43*, 104–108.
15. Caspar, D.L.; Klug, A. Physical principles in the construction of regular viruses. *Cold Spring Harb. Symp. Quant. Biol.* **1962**, *27*, 1–24. [\[CrossRef\]](#) [\[PubMed\]](#)
16. Johnson, N.W. Convex polyhedra with regular faces. *Can. J. Math.* **1966**, *18*, 169–200. [\[CrossRef\]](#)
17. Rezaee Javan, A.; Liu, Y.; Xie, Y.M. New families of cage-like structures based on Goldberg polyhedra with non-isolated pentagons. *J. Comput. Des. Eng.* **2023**, *10*, 527–538. [\[CrossRef\]](#)
18. Schein, S.; Gayed, J.M. Fourth class of convex equilateral polyhedra with polyhedral symmetry related to fullerenes and viruses. *Proc. Natl. Acad. Sci. USA* **2014**, *111*, 2920–2925. [\[CrossRef\]](#) [\[PubMed\]](#)
19. Liu, Y.; Lee, T.U.; Rezaee Javan, A.; Xie, Y.M. Extending Goldberg’s method to parametrize and control the geometry of Goldberg polyhedra. *R. Soc. Open Sci.* **2022**, *9*, 220675. [\[CrossRef\]](#) [\[PubMed\]](#)
20. Vásquez-Peralvo, J.A.; Martínez-Espina, M.H.; López-Orozco, H.; Bautista-Quintero, P.; Hernández-Sánchez, J.C. Genetic Algorithm-based Beamforming in Subarray Antenna Systems. *arXiv* **2023**, arXiv:2311.01366.
21. Loza-Morcillo, S.; Blanco-Murillo, J.L. Advanced beam steering and topological optimization in large-scale antenna arrays through hexagonal subarray implementation over Goldberg polyhedral configurations. In Proceedings of the 2025 IEEE Radar Conference (RadarConf25), Krakow, Poland, 4–10 October 2025.
22. Oliveri, G.; Rocca, P.; Massa, A. Differential evolution as applied to electromagnetics: Advances, comparisons, and applications. In Proceedings of the 6th European Conference on Antennas and Propagation (EUCAP), Prague, Czech Republic, 26–30 March 2012.
23. Bai, Y.-Y.; Xiao, S.-L.; Liu, C.; Wang, B. A Hybrid IWO/PSO Algorithm for Pattern Synthesis of Conformal Phased Arrays. *IEEE Trans. Antennas Propag.* **2013**, *61*, 2328–2332. [\[CrossRef\]](#)
24. Yang, K.; Zhao, Z.; Ouyang, J.; Nie, Z.; Liu, Q. Optimisation method on conformal array element positions for low sidelobe pattern synthesis. *IET Microw. Antennas Propag.* **2012**, *6*, 646. [\[CrossRef\]](#)
25. Expósito Domínguez, G. Contribution to Active Array Antennas at Microwave Bands. Ph.D. Thesis, Universidad Politécnica de Madrid, Escuela Técnica Superior de Ingenieros de Telecomunicación, Madrid, Spain, 2013.
26. Meng, F.; Wang, J.; Xia, H.; Long, Y.; Liu, W. Fabrication of electrical devices on conformal surfaces using array nozzles. *Sens. Actuators A Phys.* **2024**, *364*, 115563.

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