

Article

Dynamic Behaviors and Stability Analysis of Closed-Loop Controlled LLC Resonant Converters

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Abstract

The LLC resonant converter constitutes a high-order switching system characterized by multiple operational modes and region-dependent switching sequences. This complexity poses significant challenges to system modeling and dynamic analysis. Furthermore, its inherent high-order nonlinearity tends to induce detrimental nonlinear phenomena, including bifurcation and chaos, which are particularly undesirable in power electronic systems that demand the utmost priority for stability and reliability. To address these concerns, this work focuses on investigating the dynamic behaviors and stability of LLC resonant converter control systems. This study aims to elucidate the origins and evolution of these nonlinear characteristics, thereby facilitating the design of higher-performance power electronic systems. First, a continuous-time model of the closed-loop controlled LLC resonant converter system was established using the sigmoid function modeling method. This model allows direct application of continuous system theory to analyze dynamic behavior, significantly reducing analytical complexity. Second, the system's bifurcation characteristics and stability were comprehensively investigated through Floquet theory, bifurcation diagrams, and Lyapunov exponent spectra. Results reveal that PFM-controlled LLC resonant converters exhibit rich nonlinear dynamics under variations in key parameters. Experiments successfully captured the observed nonlinear phenomena, validating the evolution of system dynamics and stability. This work provides a novel perspective for stability analysis and parameter design in multi-resonant converter systems.

Keywords: LLC converter; dynamic behaviors; stability; sigmoid-function model; Hopf bifurcation



Academic Editor: Fabio Corti

Received: 17 January 2026

Revised: 1 February 2026

Accepted: 3 February 2026

Published: 6 February 2026

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1. Introduction

LLC resonant converters have been widely adopted in high-efficiency power conversion due to their soft-switching capability and high-power density [1–3]. However, the LLC converter is a strongly nonlinear high-order switching system, presenting significant challenges for model formulation, analytical solution derivation, and analysis of dynamic behaviors. First, the LLC resonant converter typically comprises a DC power source, a square-wave inverter bridge, a resonant tank, a rectifier, a filter, and a load. Both the inverter and rectifier bridges incorporate switching devices, while energy storage elements (inductors and capacitors) exist within both the filter stage and resonant tank. Such a structure causes the LLC resonant converter to exhibit a high circuit order and multiple operating modes within one switching cycle. Second, the LLC converter is a classical

multi-resonant converter, exhibiting three operational regions, each containing at least six operational modes with variable switching sequences. Multi-resonant converters exhibit heightened complexity, featuring multiple resonant frequencies that partition the system into distinct operational regions [4]. Each region is likely to encompass multiple operational modes with indeterminate switching sequences during system operation. Consequently, their modeling and dynamic analysis are substantially more intricate than those of non-resonant converters [5].

Switching converters constitute a class of piecewise-smooth dynamical systems exhibiting rich nonlinear dynamics. These systems demonstrate not only conventional bifurcation behaviors such as saddle-node bifurcation [6], period-doubling [7,8], and Hopf bifurcations [9,10], but also non-smooth events, including border collision [11,12] and grazing bifurcations [13,14]. For example, in [15], the dynamical behaviors of the peak current mode memristive buck-boost converter were investigated, and the results indicate that such a converter exhibits complex dynamical behaviors, including chaos, period, and border collision bifurcation. In [16], the study revealed that a second-order DC-AC inverter's state changes not only from period-1 to chaos via period-doubling bifurcation, but also from stability to low-frequency oscillation via Hopf bifurcation. These nonlinear dynamic behaviors can severely degrade converter performance. Therefore, a rigorous analysis of system dynamics is indispensable for predicting stability boundaries under parameter variations and, ultimately, for guiding parameter selection to ensure sustained stable operation. However, most existing studies have primarily focused on relatively simple low-order switching converters, whereas the nonlinear phenomena in LLC converters and other complex high-order resonant converters remain insufficiently explored and warrant further investigation.

Owing to the non-differentiability at switching instants, the primary approach for analyzing piecewise-smooth dynamical systems is to construct a discrete-time map [17,18]. This approach involves deriving local mappings for non-switching phases and discontinuous mappings at switching instants, which collectively form a composite Poincaré map [19]. Stability, chaos, and bifurcation behaviors are then analyzed through this discrete model using Jacobian matrix eigenvalues [20,21], Floquet characteristic multipliers [22], and Lyapunov exponents [23]. For multi-resonant converters, however, the derivation of such discrete maps becomes prohibitively intricate. Consequently, research on nonlinear dynamics in high-order resonant converters remains limited. Establishing sufficiently accurate unified continuous models would circumvent this limitation, enabling the application of continuous system theories to analyze dynamic characteristics while resolving analytical complexities.

A unified continuous-time modeling framework for piecewise-smooth systems, termed the sigmoid function modeling method, was first introduced in [24]. By introducing a continuously differentiable sigmoid function to approximate switching state transitions, this technique enables accurate unified continuous modeling of high-order resonant converters with multiple operational modes, demonstrating broad applicability. Compared to extended describing function (EDF) modeling [25] and generalized state-space averaging (GSSA) modeling [26,27], this method achieves reduced model order and enhanced accuracy. Consequently, this paper adopts the sigmoid-based method to establish a model for the PI-controlled LLC resonant converter with multiple resonant frequencies. Based on this model, we analyzed the nonlinear dynamics of the closed-loop LLC system, investigated mechanisms underlying bifurcation phenomena, and elucidated the parametric influence governing stability in higher-order resonant converters. The main contributions of this work can be summarized as follows. (1) A unified differentiable continuous-time model for closed-loop LLC converters under PFM regulation. A sigmoid-based continuous-time

modeling framework is established for a PI-controlled LLC resonant converter, in which multiple operating modes and resonant-frequency regions are unified into a single continuously differentiable formulation. This allows the Jacobian matrix to be directly evaluated without enumerating mode-dependent switching sequences, thereby providing a mathematical basis for nonlinear stability analysis in high-order resonant converters. (2) Stability boundary identification and bifurcation mechanism interpretation for the closed-loop LLC system. Based on the proposed model, we compute periodic-orbit stability using Floquet multipliers (combined with shooting/Newton iterations) and map the bifurcation diagrams with respect to controller parameters. The resulting bifurcation characterization reveals how PI gains govern closed-loop stability, offering practical guidelines for stability-oriented parameter tuning in LLC resonant converters.

The rest of this article is organized as follows. Section 2 develops the continuous-time model of the closed-loop LLC resonant converter. Section 3 analyzes the nonlinear dynamics and stability of the controlled system. Section 4 provides experimental validation of the theoretical findings. Conclusions are drawn in Section 5.

2. Continuous-Time Modeling of LLC Resonant Converter

The closed-loop LLC resonant converter is depicted in Figure 1. A half-bridge inverter comprises switching devices Q_1 and Q_2 , operating at 50% duty cycle, converting the DC voltage (V_{in}) into a square-wave (v_{ab}) by employing pulse frequency modulation (PFM). The resonant tank comprises a capacitor (C_r), an inductor (L_r), and a magnetizing inductor (L_m). The transformer turn ratio is $n:1:1$, and the secondary is followed by a full-wave rectifier (diodes D_1 and D_2) and an output filter capacitor (C_o). A PI compensator generates the control signal (f_{con}) based on reference voltage (V_{ref}), with proportional gain (k_P) and integral gain (k_I). The parameter ρ denotes the integrator output signal. The load is purely resistive with a resistance value of R_L .

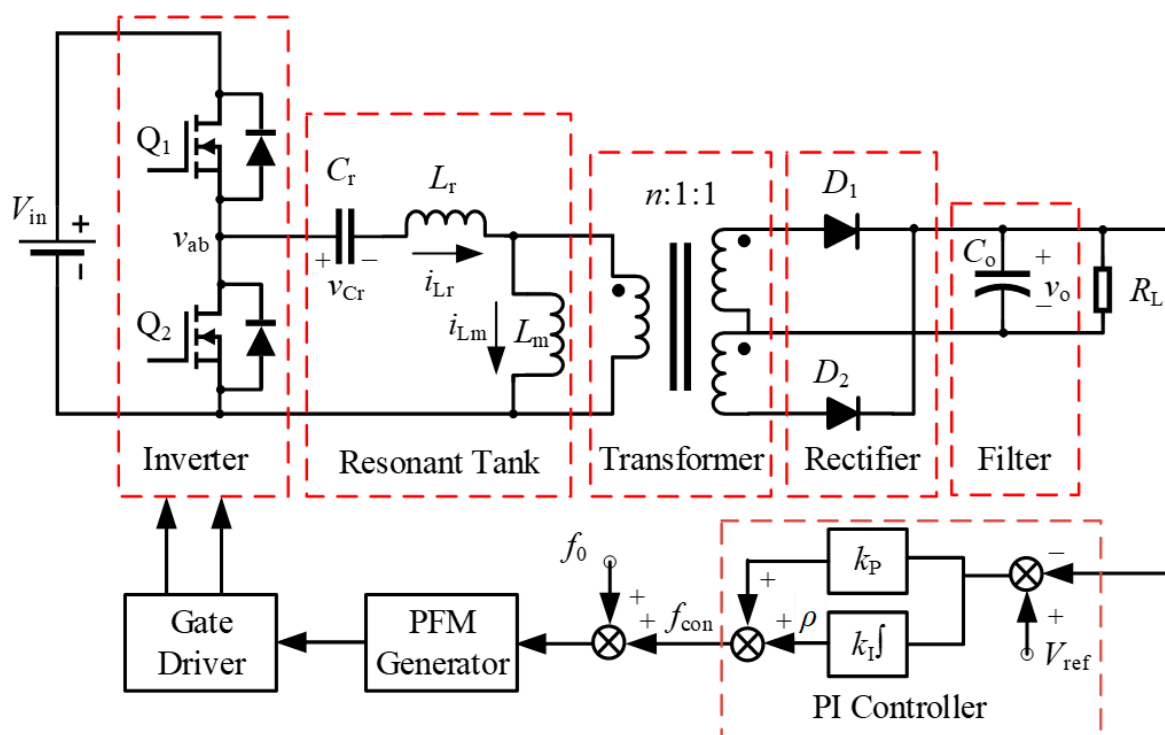


Figure 1. LLC resonant converter with PFM control.

The LLC resonant converter exhibits two resonant frequencies, denoted as f_r and f_m , given by:

$$f_r = \frac{1}{2\pi\sqrt{L_r C_r}}, \quad (1)$$

$$f_m = \frac{1}{2\pi\sqrt{(L_r + L_m)C_r}}, \quad (2)$$

The system operates in three distinct regions, demarcated by the switching frequency (f): $f \leq f_m$, $f_m < f < f_r$, and $f \geq f_r$. Each region encompasses at least six operational modes. Taking the resonant inductor current i_{Lr} , the magnetizing inductor current i_{Lm} , the resonant capacitor voltage v_{Cr} , and the output voltage v_o as the system's state variables, the state-space equations for the main circuit can be derived as follows:

$$\begin{cases} \frac{di_{Lm}}{dt} = \frac{n}{L_m} v_o \operatorname{sgn}(i_{Lr} - i_{Lm}) \\ \frac{di_{Lr}}{dt} = \frac{1}{L_r} [-v_{Cr} - n v_o \operatorname{sgn}(i_{Lr} - i_{Lm}) + v_{ab}] \\ \frac{dv_{Cr}}{dt} = \frac{1}{C_r} i_{Lr} \\ \frac{dv_o}{dt} = \frac{n}{C_o} |i_{Lr} - i_{Lm}| - \frac{1}{R_L C_o} v_o \end{cases}, \quad (3)$$

Equation (3) contains discontinuous functions, such as the signum function, the absolute value function, and the square-wave function. Different combinations of these functions define distinct operating modes of the system. Furthermore, their discontinuities render the derivation of the system's Jacobian matrix intractable. As proven in [24], these discontinuous functions can be approximated by differentiable sigmoid functions. Sigmoid functions, one of the most important activation functions in artificial neural networks [28,29], can effectively describe system mode transitions. Commonly used sigmoid functions include the Tan-Sigmoid (i.e., the hyperbolic tangent function, $\tanh$) and the Log-Sigmoid function (i.e., the logistic function, often simply referred to as the sigmoid function).

In (3), replacing the signum function with the $\tanh$ function yields:

$$\operatorname{sgn}(i_{Lr} - i_{Lm}) \approx \tanh(b(i_{Lr} - i_{Lm})). \quad (4)$$

where b denotes the steepness coefficient.

Similarly, approximating the absolute value function with the $\tanh$ function gives:

$$|i_{Lr} - i_{Lm}| \approx (i_{Lr} - i_{Lm}) \tanh(b(i_{Lr} - i_{Lm})). \quad (5)$$

The voltage v_{ab} , a half-wave square-wave with amplitude V_{in} , can be approximated by the $\tanh$ function as:

$$v_{ab} \approx 0.5V_{in}(1 + \tanh(b \sin(2\pi ft + \theta))). \quad (6)$$

Here, the frequency f is regulated by a PI controller: $f(t) = f_0 + k_P e(t) + \rho$, where f_0 represents the center frequency, and $e(t)$ is the error signal defined by $e(t) = V_{ref} - v_o(t)$.

Figure 2 presents the results of using the hyperbolic tangent function to approximate the sign function and the square-wave function. As shown, a larger value of b produces waveforms that more closely approach the ideal sign and square-wave functions. Specifically, b is a steepness factor that primarily controls the approximation accuracy of the sign function and the square-wave representation. It should be sufficiently large to closely approximate the original switching/piecewise nonlinearity, while still preserving numerical

smoothness for time integration and ensuring solver stability. Accordingly, we set it as $b = 1000$ in this paper.

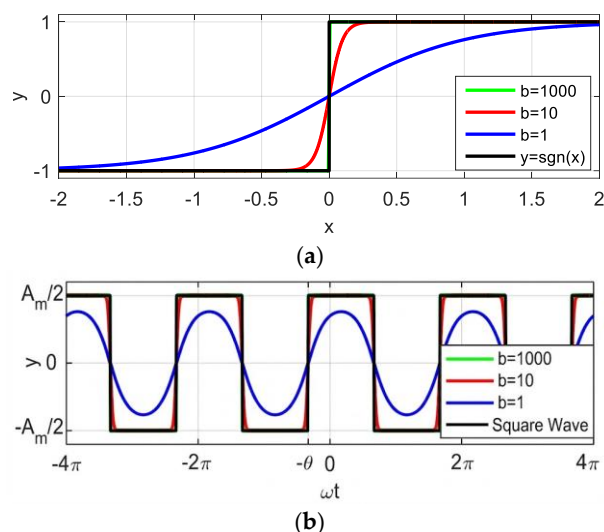


Figure 2. Results of approximating the sign function and the square-wave function using the hyperbolic tangent function. (a) $y = \tanh(bx)$; (b) $y = 0.5A_m \tanh[b \sin(\omega t + \theta)]$.

Substituting the discontinuous functions in (3) with the tanh approximations, the state-space equations for the LLC resonant converter under PI closed-loop control can be expressed as follows:

$$\begin{cases} \frac{di_{Lm}}{dt} = \frac{n}{L_m} v_o \tanh(b(i_{Lr} - i_{Lm})) \\ \frac{di_{Lr}}{dt} = \frac{1}{L_r} [-v_{Cr} - n v_o \tanh(b(i_{Lr} - i_{Lm})) \\ \quad + 0.5 V_{in} (1 + \tanh(b \sin(2\pi(f_0 + k_P(V_{ref} - v_o) + k_I \rho)t + \theta)))] \\ \frac{dv_{Cr}}{dt} = \frac{1}{C_r} i_{Lr} \\ \frac{dv_o}{dt} = \frac{n}{C_o} (i_{Lr} - i_{Lm}) \tanh(b(i_{Lr} - i_{Lm})) - \frac{1}{R_o C_o} v_o \\ \frac{d\rho}{dt} = V_{ref} - v_o \end{cases} \quad (7)$$

Defining the state vector as $\mathbf{x} = [i_{Lm}, i_{Lr}, v_{Cr}, v_o, \rho]^T = [x_1, x_2, x_3, x_4, x_5]^T$, the system (7) yields the following form:

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}, t). \quad (8)$$

The tanh function is infinitely and continuously differentiable. Consequently, model (8) constitutes a continuous-time function whose Jacobian matrix can be derived. Linearizing Equation (8) at the point $\mathbf{x}^*$ yields:

$$\frac{d\mathbf{z}}{dt} = \mathbf{A}(t)\mathbf{z}, \quad (9)$$

where $\mathbf{z} = \mathbf{x} - \mathbf{x}^*$, and $\mathbf{A}(t) = \partial \mathbf{F}(\mathbf{x}, t) / \partial \mathbf{x} |_{\mathbf{x} = \mathbf{x}^*}$ is the Jacobian matrix of the system evaluated at $\mathbf{x}^*$.

Next, the accuracy of the established continuous-time function model (8) is validated. A Simulink circuit model, depicted in Figure 1, is constructed. Simulations of both the circuit model and the mathematical function model are performed under identical conditions. The circuit parameters, selected as specified in Table 1, yield the system's two resonant frequencies: $f_m = 50$ kHz and $f_r = 100$ kHz. For model (8), the steepness coefficient $b = 1000$. The simulation results are presented in Figure 3. In addition, to quantitatively

evaluate the similarity between the waveforms produced by the two models in Figure 3, an amplitude-related error metric is defined as follows:

$$R = \frac{1}{M} \sum_{m=1}^M \left(\frac{1}{\Delta_m} \sqrt{\frac{\sum_{n=1}^{N_m} [x_i(n) - y_i(n)]^2}{N_m}} \right), \quad i = 1, 2, 3 \quad (10)$$

Table 1. Parameters of the LLC converter.

Parameters	Values and Units
voltage of input DC power supply, V_{in}	150 V
capacitance of resonant capacitor, C_r	44 nF
inductance of resonant inductor, L_r	57.6 μ H
inductance of magnetizing inductor, L_m	172.8 μ H
transformer turn ratio, n :1:1	10:1:1
capacitance of filter capacitor, C_o	120 μ F
resistance value of load, R_L	2 Ω

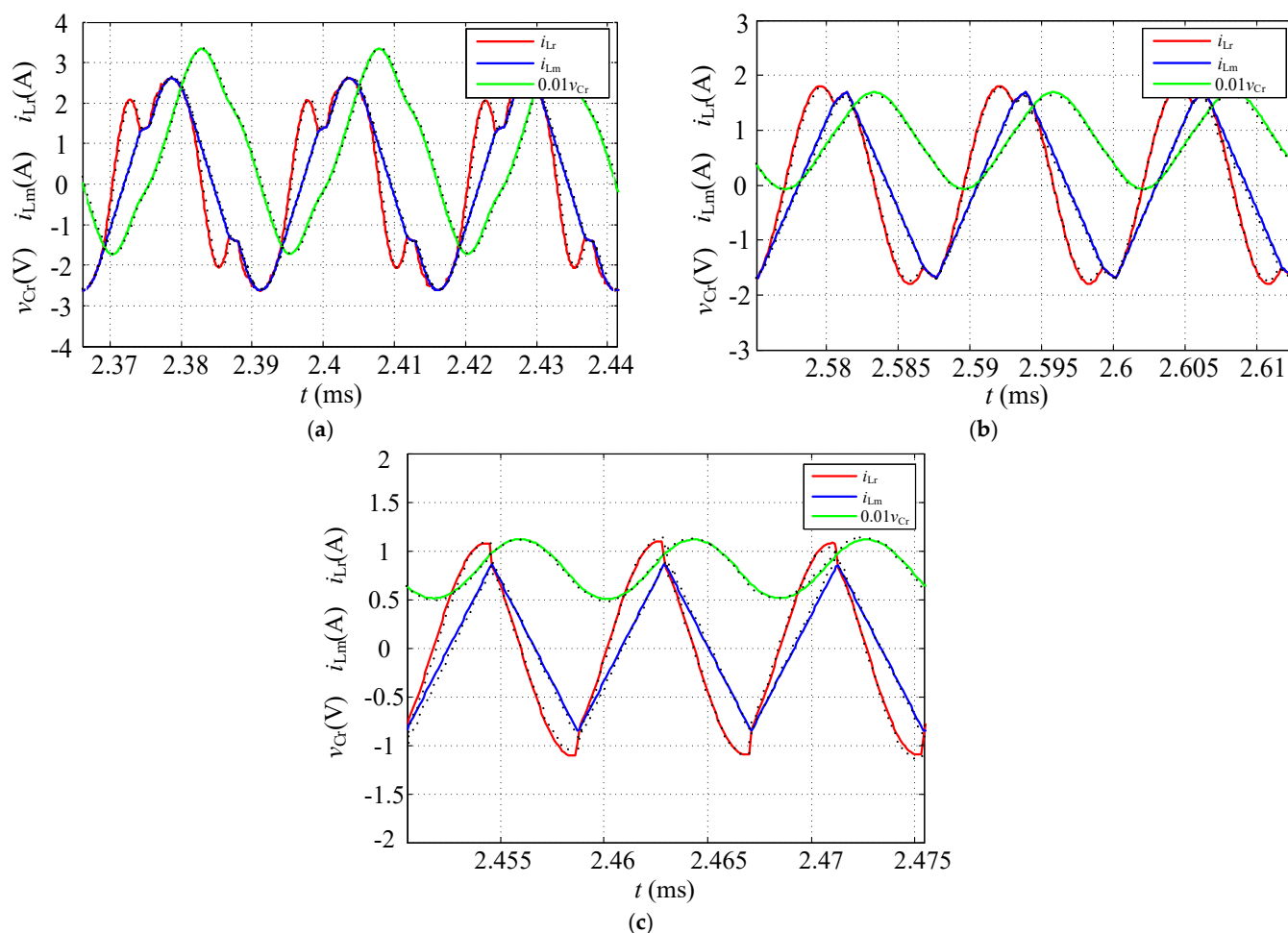


Figure 3. Simulated waveforms of the LLC converter in different operating regions. (a) $f = 40 \text{ kHz} < f_m$; (b) $f_m < f = 80 \text{ kHz} < f_r$; (c) $f = 120 \text{ kHz} > f_r$ (black dotted line: continuous-time function model; solid line: Simulink circuit-simulated model).

In (10), x_1 , x_2 and x_3 are the solutions for i_{Lr} , i_{Lm} and v_{Cr} , respectively, as obtained from the sigmoid continuous model. y_1 , y_2 and y_3 are the solutions for i_{Lr} , i_{Lm} and v_{Cr} , respectively, as obtained from the Simulink circuit simulation model. M is the number of

cycles contained in each solution set, and N_m is the number of solutions contained in the m -th cycle. For AC variables, subtracting the maximum (peak) value from the minimum (trough) value within a cycle can reflect the characteristics of the waveform well. Therefore, Δ_m in the above formula is taken as the difference between the peak and trough values of the m -th cycle of y_i :

$$\Delta_m = y_{\max_m} - y_{\min_m}, \quad m = 1, 2, \dots, M \quad (11)$$

Based on the error metric in (14), the deviations between the output waveforms of the continuous sigmoid-based model and the Simulink circuit-simulated model in Figure 3 were calculated (see Table 2). The results show that, across different operating ranges, the errors are all below 5%, which is within an acceptable tolerance. This, therefore, validates the effectiveness of the sigmoid-based model.

Table 2. Errors (R(%)) of the sigmoid-based model and the Simulink circuit-simulated model.

Operating Range	i_{Lr}/A	i_{Lm}/A	v_{Cr}/V
$f = 40 \text{ kHz} < f_m$	3.36%	3.47%	4.35%
$f_m < f = 80 \text{ kHz} < f_r$	2.45%	2.34%	3.62%
$f = 120 \text{ kHz} > f_r$	3.19%	3.31%	4.29%

Figure 4 compares the output voltage waveforms of the circuit model and the mathematical model (8) under closed-loop control, with the PI controller gains set to $k_P = 0.2$ and $k_I = 300$, and the output voltage reference $V_{\text{ref}} = 12 \text{ V}$. And $f_0 = 70 \text{ kHz}$, and the initial phase angle θ is zero. The results indicate that the two models yield highly similar output waveform characteristics, with nearly identical overshoot and settling time.

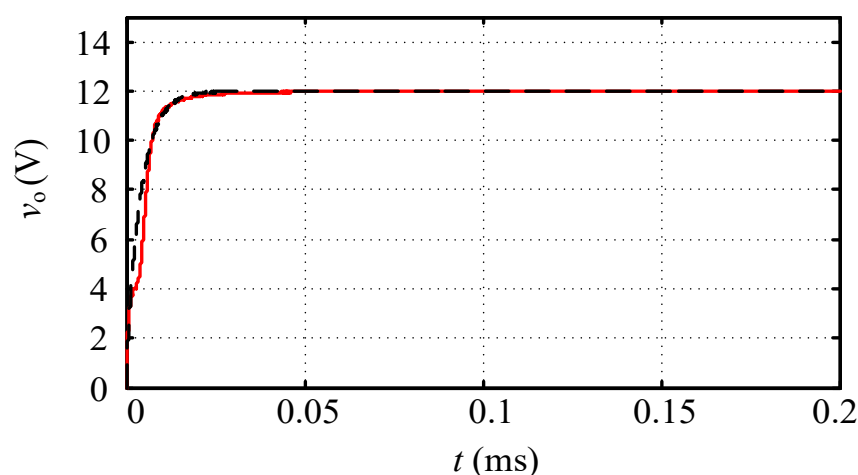


Figure 4. Simulated output voltage waveforms of the closed-loop LLC converter (black dotted line: continuous-time function model; red solid line: the Simulink circuit-simulated model).

In summary, the established continuous-time model shows good agreement with the Simulink circuit simulation results. The model accurately characterizes the steady-state and dynamic performance of the LLC resonant converter under closed-loop conditions. By unifying the piecewise-smooth system model with multiple segments into a single continuous-time representation, this model enables the analysis of converter dynamics using continuous-system techniques. Consequently, it eliminates the need for deriving complex discrete iterative mapping models and significantly reduces analytical complexity. Nevertheless, it should be acknowledged that certain non-ideal effects—such as dead-time, device propagation delays, and saturation—are not yet fully and explicitly accounted for within this continuous-time framework.

3. Dynamic Behavior Analysis

Since the beginning of the 21st century, research on chaos theory and its applications has emerged as a significant field of study. This encompasses two primary domains: theoretical investigations of chaotic systems and the design of chaotic dynamical systems based on theoretical frameworks. Theoretical chaos research involves dynamical analysis of nonlinear systems to reveal their inherent nonlinear phenomena and predict parameter stability domains, thereby guiding systematic parameter optimization design.

Controller parameters critically influence system behavior. Consequently, this paper adopts the proportional gain k_P of the PI controller as the bifurcation parameter. Based on the continuous-time model (8), we conduct a theoretical analysis of periodic solution stability and bifurcation types using Floquet theory combined with the shooting method [30,31]. Subsequently, the nonlinear dynamical characteristics of the system are examined from multiple perspectives through bifurcation diagrams, Lyapunov exponent spectra, phase portraits, and time-domain waveforms.

First, the stability of periodic solutions and bifurcation types is analyzed. Determining periodic solutions for model (8) reduces to solving the two-point boundary value problem of the following differential equation:

$$\begin{cases} \frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}, t) \\ \mathbf{x}(T) - \mathbf{x}(0) = 0 \end{cases}, \quad (12)$$

where $\mathbf{x}(0)$ denotes the initial state. For a given initial value $\mathbf{x}(0) = \mathbf{x}_0^n$, the system is integrated over one period T using the Runge–Kutta method to obtain $\mathbf{x}(T)$. If $\|\mathbf{x}(T) - \mathbf{x}_0^n\| < r$ (where r is the prescribed tolerance), then $\mathbf{x}_0^n$ is regarded as a periodic solution of the system. Otherwise, a Newton iteration is performed to update the initial value for the next computation, and the above procedure is repeated until a periodic solution is found or the maximum number of iterations is reached. Next, let $\Phi(t) = \partial\mathbf{x}(t)/\partial\mathbf{x}_0^n$, where $\mathbf{x}_0^n$ represents the initial value $\mathbf{x}(0)$ at the n -th iteration of the shooting method:

$$\begin{cases} \frac{d\Phi}{dt} = \mathbf{A}(\mathbf{x}, t)\Phi \\ \Phi(0) = \mathbf{I} \end{cases}, \quad (13)$$

Differentiating both sides of Equation (8) with respect to $\mathbf{x}_0^n$ yields:

$$\begin{cases} \frac{d\Phi}{dt} = \mathbf{A}(t)\Phi(t) \\ \Phi(0) = \mathbf{I} \end{cases}, \quad (14)$$

The introduction of the tanh function transforms the LLC resonant converter system model into a continuous-time representation. Consequently, its Jacobian matrix $\mathbf{A}(t)$ can be derived as follows:

$$\mathbf{A}(t) = \begin{bmatrix} -\frac{bn}{L_m}x_4(1-p^2) & \frac{bn}{L_m}x_4(1-p^2) & 0 & \frac{n}{L_m}P & 0 \\ \frac{bn}{L_r}x_4(1-p^2) & -\frac{bn}{L_r}x_4(1-p^2) & -\frac{1}{L_r} & M & k_I Q \\ 0 & \frac{1}{C_r} & 0 & 0 & 0 \\ -N & N & 0 & -\frac{1}{R_o C_o} & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}, \quad (15)$$

where

$$M = -k_P Q - \frac{n}{L_r} P, \quad (16)$$

$$N = \frac{n}{C_o} P + \frac{bn}{C_o} (x_2 - x_1)(1 - P^2), \quad (17)$$

$$P = \tanh(b(x_2 - x_1)), \quad (18)$$

$$Q = \frac{\pi a t V_{in}}{L_r} \left(1 - \tanh^2(a \sin(2\pi f t + \theta)) \right) \cos(2\pi f t + \theta), \quad (19)$$

$$f = f_0 + k_P (V_{ref} - x_4) + k_I x_5, \quad (20)$$

The monodromy matrix $\Phi(T)$ is obtained via numerical integration of the initial-value problem (14); its eigenvalues constitute the Floquet multipliers. The stability of periodic solutions and bifurcation types are determined by the properties of the dominant Floquet multiplier (i.e., the multiplier with the largest magnitude) [32]. When the dominant multiplier resides within the unit circle in the complex plane, the periodic solution is asymptotically stable. Bifurcation occurs and stability is lost when this multiplier crosses outside the unit circle, exhibiting the following bifurcation types: ① Saddle-node bifurcation occurs when the dominant multiplier crosses outward through the unit circle on the positive real axis; ② period-doubling bifurcation occurs when the dominant multiplier crosses outward through the negative real axis; and ③ Hopf bifurcation occurs when a complex-conjugate dominant multiplier crosses the unit circle.

For the numerical computation of the Floquet multipliers, the initial condition is set to $\mathbf{x}(0) = [0, 0, 0, 0, 0]$. A fixed-step Runge–Kutta method (with truncation error $O(h^4)$) is employed to obtain the periodic solution, with the step size set to 1×10^{-7} s, and the maximum number of iterations is limited to 50. In addition, the main circuit parameters selected for the LLC resonant converter are listed in Table 1. With the reference voltage $V_{ref} = 12$ V and the PI controller integral coefficient $k_I = 300$, variations in the proportional coefficient k_P over the range 0~0.9 result in the complex-conjugate dominant multipliers exiting the unit circle on two occasions (as detailed in Tables 3 and 4). These events occur at $k_P = 0.26$ and $k_P = 0.59$, respectively, signifying the onset of Hopf bifurcations in the system. These critical points coincide with the two bifurcation points observed in the bifurcation diagram (Figure 5).

Table 3. Floquet multipliers of the monodromy matrix for different k_P in [0.24~0.25].

k_P	Floquet Multipliers	State
0.24	$0.9997 \pm 0.0254i$	stability
	$0.9999 \pm 0.0021i$	
0.25	$0.9997 \pm 0.0249i$	stability
	$0.9999 \pm 0.0021i$	
0.26	$0.9997 \pm 0.0245i$	Hopf bifurcation
	$1.0000 \pm 0.0021i$	

Table 4. Floquet multipliers of the monodromy matrix for different k_P in [0.57~0.59].

k_P	Floquet Multipliers	State
0.57	$0.9997 \pm 0.0261i$	stability
	$0.9999 \pm 0.0022i$	
0.58	$0.9997 \pm 0.0253i$	stability
	$0.9999 \pm 0.0021i$	
0.59	$0.9997 \pm 0.0246i$	Hopf bifurcation
	$1.0000 \pm 0.0021i$	

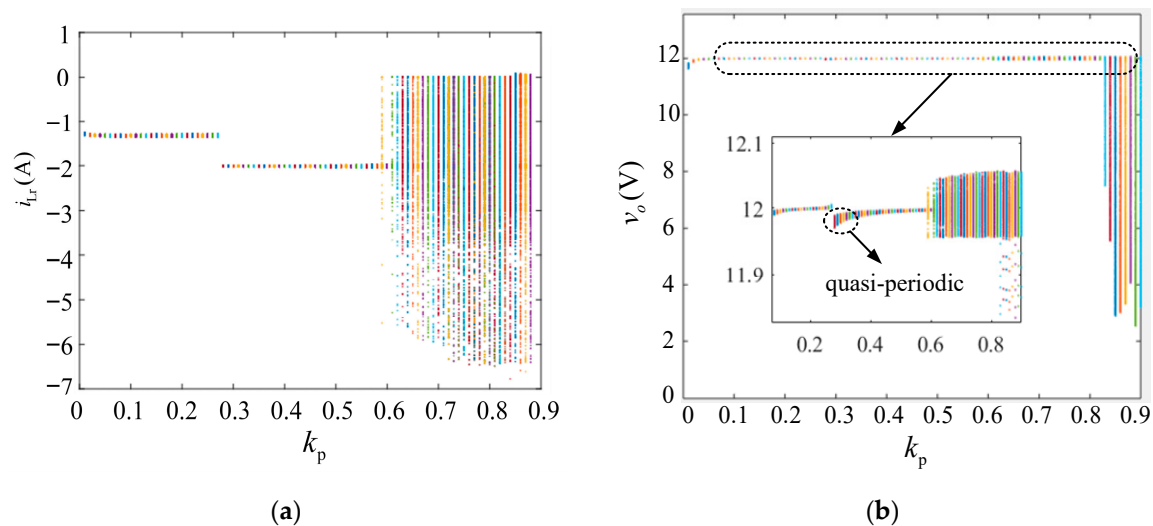


Figure 5. Bifurcation diagram of state variables (a) i_{Lr} and (b) v_o .

Figure 5 depicts the bifurcation diagrams of the resonant current (i_{Lr}) and the output voltage (v_o) with k_p as the bifurcation parameter. For fixed system parameters, the observation of a single point, or multiple coincident points corresponding to the system's periodicity in the bifurcation diagram, indicates a periodically stable state. Conversely, the presence of an infinite number of distinct periodic points signifies chaotic behavior. Consequently, bifurcation diagrams clearly characterize the evolution of system performance with parameter variations. As shown in Figure 5, the system exhibits stable period-1 solutions within the range $0 < k_p < 0.26$. At $k_p = 0.26$, an abrupt jump occurs in i_{Lr} and v_o , indicating a switching of periodic orbits (Floquet multiplier analysis further confirms that this jump falls within the Hopf bifurcation class). Despite this jump, the system remains stable, with v_o consistently regulated near its reference value of 12 V. However, the resonant current changes. This distinctive “jump” behavior indicates that the system undergoes a transition between different stable operating regions. Such a jump is worthy of particular attention and must be taken into account when selecting controller parameters to ensure that the system operates in the desired region. Following the transition, the system undergoes a quasi-periodic phase before subsequently entering a new stable period-1 state. When k_p increases to 0.59, the system enters a chaotic state, evidenced by a positive maximum Lyapunov exponent, as shown in Figure 6.

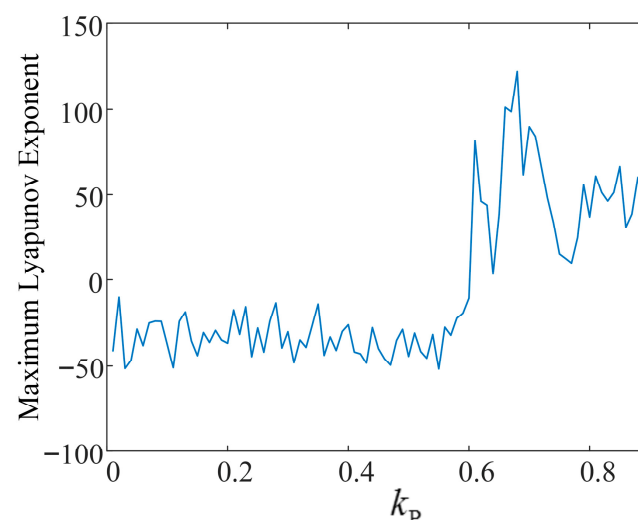


Figure 6. The maximum Lyapunov exponent diagram.

The maximum Lyapunov exponent in Figure 6 is computed using the Wolf method (see Figure 7). The procedure is summarized as follows. Based on the time series $x(t)$ obtained by solving the system with the sigmoid-based model, an m -dimensional phase space is reconstructed using the time-delay embedding technique [32]. Accordingly, an arbitrary point on the system trajectory can be represented as $\{x(t), x(t + \tau), \dots, x(t + [m - 1]\tau)\}$, where τ denotes the selected time delay. After phase-space reconstruction, the phase points of the reference trajectory and its neighboring trajectory at time t_0 are denoted by $Y(t_0)$ and $Y'(t_0)$, respectively, and their separation is denoted by L_0 . As time evolves, the corresponding phase points at time t_1 become $Y(t_1)$ and $Y'(t_1)$, and the separation changes to L'_0 . If L'_0 exceeds a prescribed threshold τ , the trajectories are regarded as diverging; in this case, a suitable replacement point in the neighborhood of $Y'(t_1)$ is sought to substitute for $Y'(t_1)$, denoted by $Y''(t_1)$. The replacement point must satisfy two criteria: (i) the distance L_1 between $Y''(t_1)$ and $Y(t_1)$ is smaller than ε ; and (ii) the angle between the two lines formed by $Y(t_1) - Y'(t_1)$ and $Y'(t_1) - Y''(t_1)$ is minimized. If $L'_0 < \varepsilon$ or no suitable replacement point can be identified, the original point $Y'(t_1)$ is retained. This procedure is repeated M times until the reference trajectory traverses the entire dataset. The maximum Lyapunov exponent is then estimated by averaging the divergence over the M iterations, as given by:

$$\lambda_1 = \frac{1}{t_{M-1} - t_0} \sum_{k=0}^{M-1} \log_2 \left(\frac{L'_k}{L_k} \right), \quad (21)$$

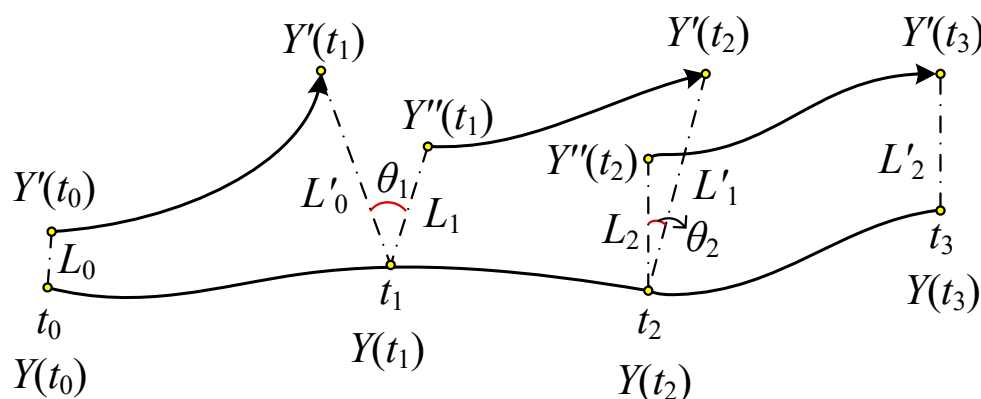


Figure 7. Flowchart of the Wolf method calculation principle.

Phase portraits and time-domain waveforms for the system with k_P set to 0.2, 0.28, 0.5, and 0.62 are shown in Figure 8. Enlarged views highlighting features of Figure 8 are provided in Figure 9. Key observations reveal: when $k_P = 0.2$, the system resides in a stable period-1 orbit with a switching frequency ≈ 73 kHz. Operating within $f_m \leq f < f_r$, the resonant circuit exhibits inductive characteristics. Primary switches achieve zero-voltage switching (ZVS), and the secondary rectifier diode operates in a discontinuous conduction mode (DCM), enabling zero-current switching (ZCS). When $k_P = 0.28$, the system manifests a quasi-periodic solution induced by a Hopf bifurcation. The phase portrait displays a bounded convergence region. When $k_P = 0.5$, the system reverts to a stable period-1 orbit (switching frequency ≈ 35 kHz), operating within $f < f_m$. The resonant circuit demonstrates capacitive characteristics, impeding ZVS achievement. At $k_P = 0.62$, the system operates in a chaotic state, with its phase portrait exhibiting a complex-structured chaotic attractor.

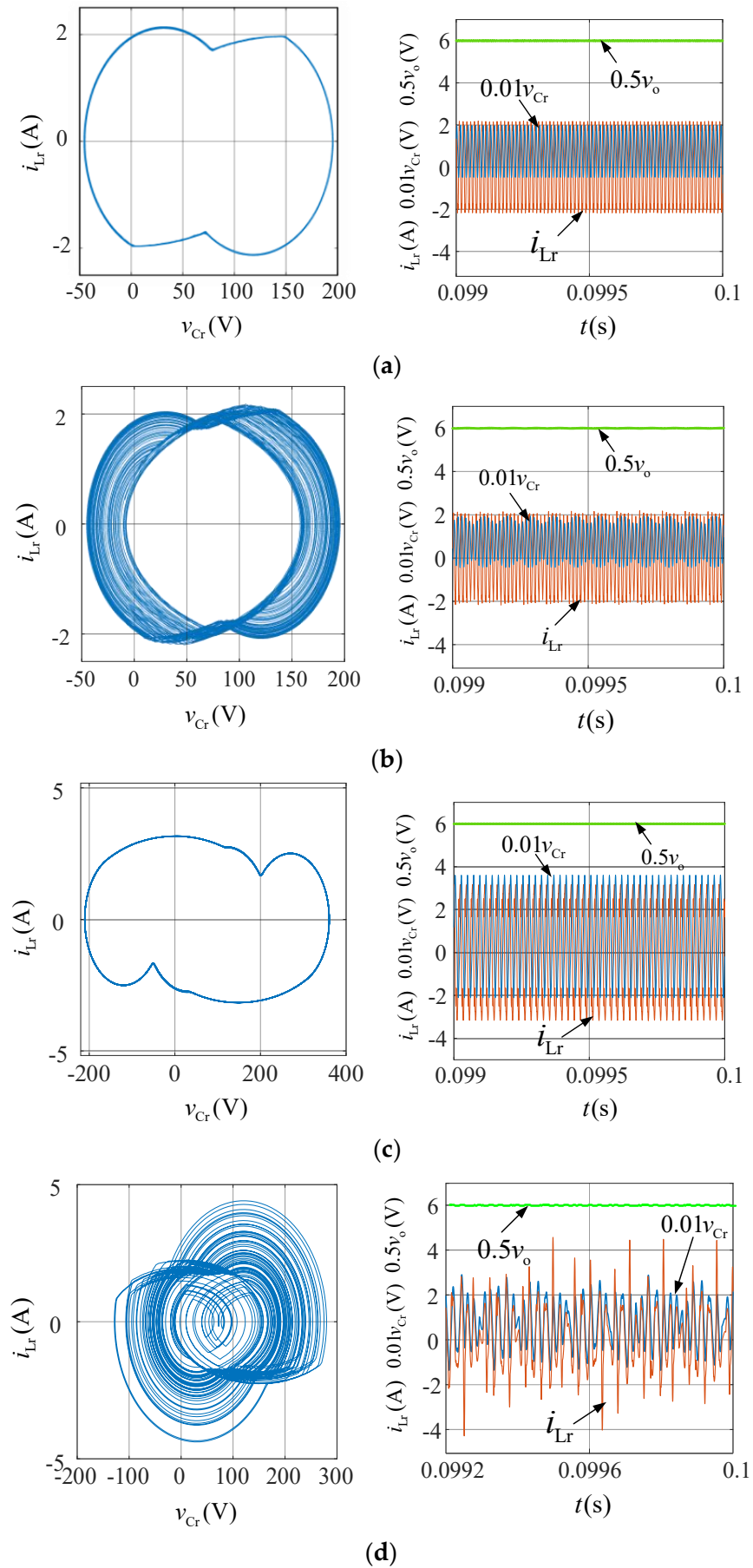


Figure 8. Simulation results: Phase portraits and time-domain waveforms: (a) $k_p = 0.2$; (b) $k_p = 0.28$; (c) $k_p = 0.5$; and (d) $k_p = 0.62$.

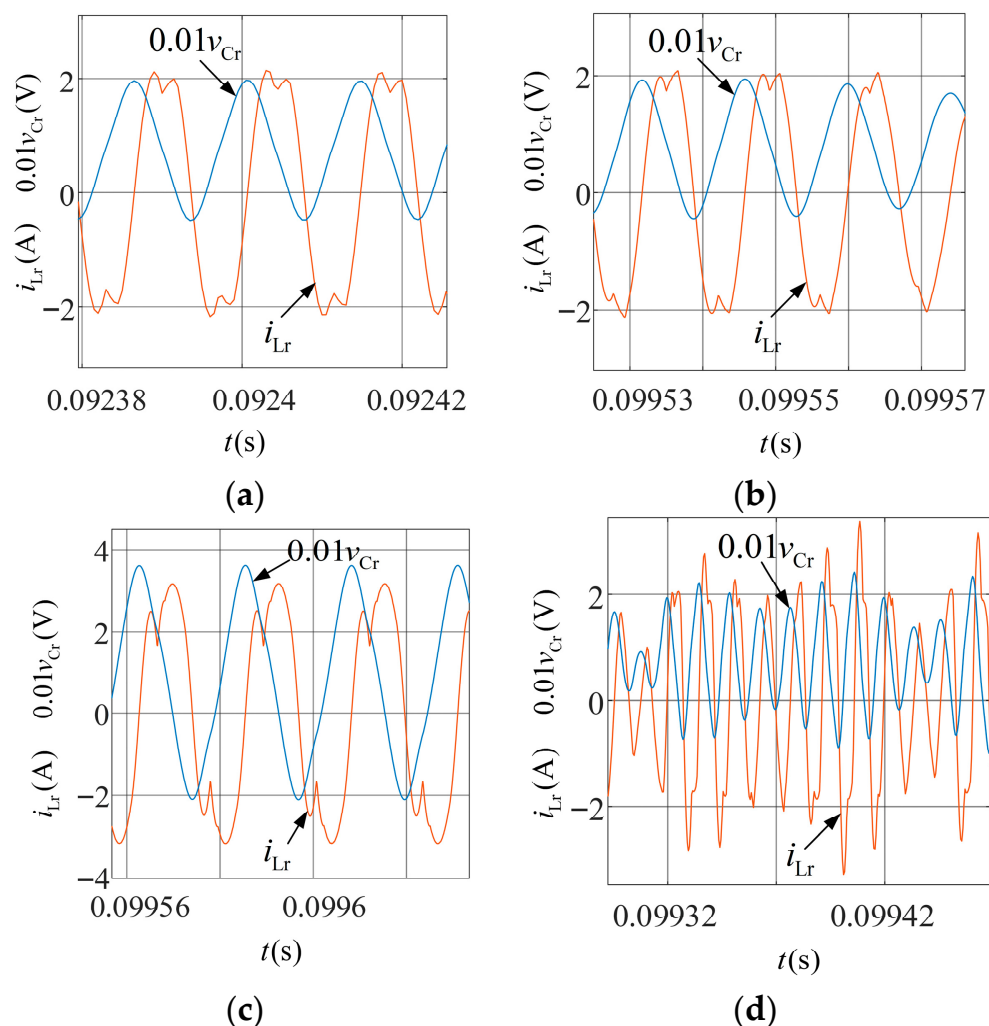


Figure 9. Magnified view of time-domain waveform: (a) $k_P = 0.2$; (b) $k_P = 0.28$; (c) $k_P = 0.5$; and (d) $k_P = 0.62$.

The foregoing analyses demonstrating the system's dynamical behavior characterized through Floquet multiplier analysis, bifurcation diagrams, Lyapunov exponent spectra, phase portraits, and time-domain waveforms exhibit good agreement. These analysis results are not only used to reveal possible nonlinear behaviors, but also to identify instability boundaries, thereby guiding appropriate parameter selection to avoid system oscillations or abnormal operation.

4. Experimental Validation

To validate the preceding dynamical analysis, an experimental platform for the LLC resonant converter control system was implemented, as shown in Figure 10. The platform consisted of the LLC resonant converter main circuit, a DSP28335 (Texas Instruments, Dallas, TX, USA) digital signal processor controller, auxiliary power supplies, and an electronic load. The output voltage was regulated using a PFM control strategy, and the control algorithm was implemented entirely on a DSP28335 controller. The output voltage was sampled by the ADC at a fixed sampling frequency of 20 kHz (once per control period). After being processed by the PI controller, the switching frequency was updated by modifying the ePWM TBPRD register ($f_{TBCLK} = 100$ MHz), while the duty ratio was maintained at 0.5 throughout. The rear-stage rectification circuit employed synchronous rectification technology. The circuit parameters and controller integral coefficients were kept consistent with those in the preceding subsection. The experimental measurements were

performed using an oscilloscope (Tektronix, Inc., Beaverton, OR, USA) with a bandwidth of 20 GHz and a sampling rate of 50 GSa/s. The resonant inductor current was measured with a current probe, which was demagnetized and zeroed prior to testing. The resonant capacitor voltage was measured using a high-voltage differential probe connected directly across the resonant capacitor terminals. The output voltage was measured at the output capacitor using a $10\times$ passive probe. Phase portraits were obtained using the oscilloscope's XY display mode, with the X-axis representing the resonant capacitor voltage and the Y-axis representing the resonant inductor current; persistence was enabled to improve trajectory visualization.

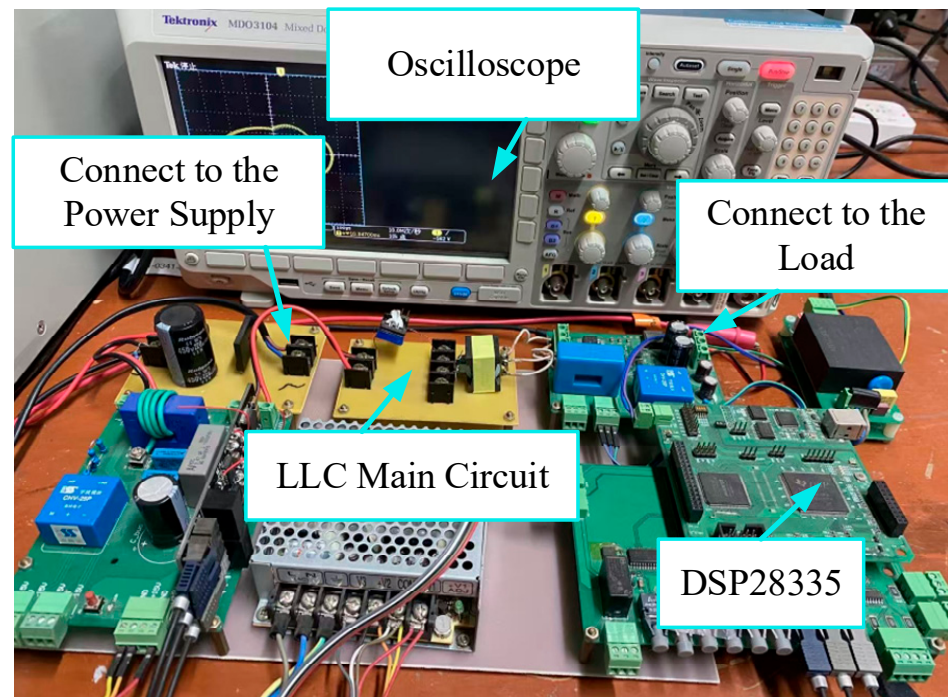


Figure 10. Experimental platform for the LLC resonant converter control system.

The experimentally measured time-domain waveforms and phase trajectories are presented in Figure 11. When $k_p = 0.2$, the system exhibits a stable period-1 regime with steady waveforms. When $k_p = 0.28$, the system enters a quasi-periodic regime, and the waveforms show small-amplitude oscillations, which is undesirable for stable operation. When $k_p = 0.5$, the waveforms appear stable; however, the corresponding phase trajectory differs from that at $k_p = 0.2$, indicating that the system has transitioned to another stable domain. As k_p is further increased to 0.6, the phase portrait reveals a chaotic regime, and the time-domain waveforms exhibit pronounced large-amplitude oscillations, suggesting loss of stability. Overall, as the proportional gain increases, the system successively undergoes period-1, quasi-periodic, period-1 (in a different stable domain), and chaotic operating states. The experimental observations reproduce the same evolution trend as the simulations, thereby validating the correctness of the dynamic behavior analysis.

In practical engineering applications, controller parameter tuning is commonly performed via a trial-and-error method, which is time-consuming and involves a certain degree of uncertainty. This study investigates the system's dynamical behavior and stability under variations in controller parameters, enabling appropriate controller parameters to be selected directly with reference to the bifurcation diagrams in a simple and effective manner.

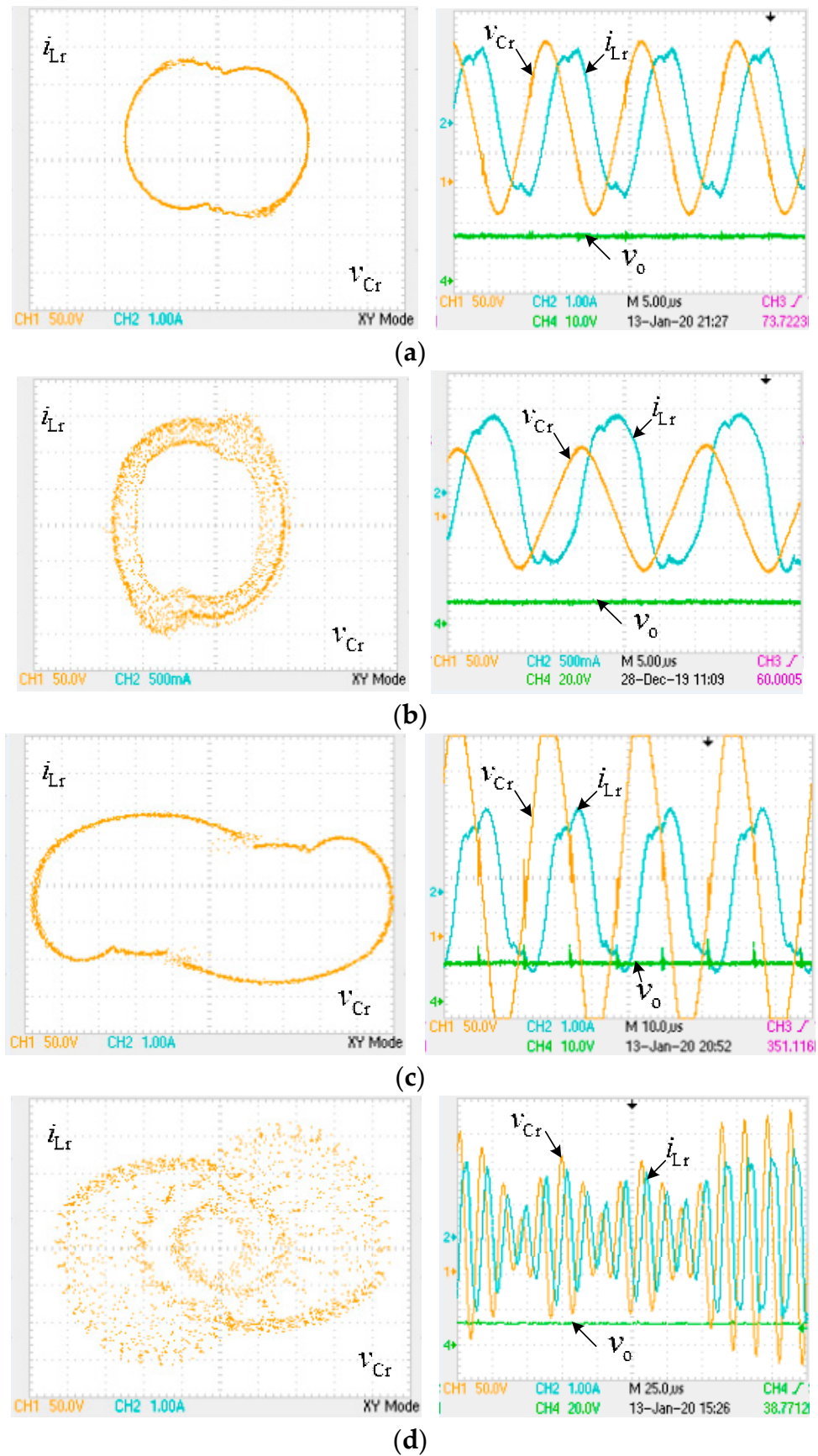


Figure 11. Experimental results: Phase portraits and time-domain waveforms: (a) $k_P = 0.2$; (b) $k_P = 0.28$; (c) $k_P = 0.5$; and (d) $k_P = 0.62$.

5. Conclusions

A continuous-time sigmoid function-based model of the PFM-controlled LLC resonant converter was formulated in this paper, and its nonlinear dynamics and stability were comprehensively analyzed. The principal findings are summarized as follows.

(1) The continuous-time sigmoid function-based model accurately characterizes both steady-state and dynamic behaviors of multi-resonant LLC resonant converters. It explicitly exposes frequency and phase signals, facilitating the implementation of control strategies targeting system frequency and phase. This approach features unified mathematical representation and clear physical interpretability. Moreover, the sigmoid function is smooth, which allows the Jacobian matrix to be explicitly formulated. This facilitates the use of Lyapunov stability theory and Floquet theory for continuous-time systems to analyze the dynamical behaviors of switching converters. Consequently, this methodology circumvents the need for constructing and analyzing complex discrete mapping models, significantly reducing analytical complexity.

(2) Dynamical analysis reveals that, as the proportional coefficient k_p increases, the system exhibits a distinct jump phenomenon. This jump triggers a transition between two distinct stable operating regions. In the first region, the system exhibits inductive characteristics, enabling soft-switching operation. In the second region, it demonstrates capacitive characteristics, where soft switching becomes unattainable. With further increase in k_p , the system experiences transient quasi-periodic behavior before stabilizing into a period-1 solution. As k_p continues to rise, a subsequent Hopf bifurcation occurs, precipitating chaotic operation. Based on the above analysis results, optimal controller coefficients can be selected. This analytical framework can also be applied to determine the optimal range for other system parameters.

Author Contributions: Conceptualization, X.-F.W., B.Z., and M.J.; methodology, X.-F.W., B.Z., and M.J.; software, X.-F.W. and M.J.; validation, X.-F.W. and C.-G.H.; formal analysis, X.-F.W.; investigation, X.-F.W.; resources, X.-F.W.; data curation, C.-G.H. and X.-F.W.; writing—original draft preparation, X.-F.W.; writing—review and editing, X.-F.W., B.Z., and M.J.; visualization, C.-G.H. and X.-F.W.; supervision, B.Z. and M.J.; project administration, X.-F.W., B.Z., and M.J.; funding acquisition, B.Z. and M.J. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Guang Dong Basic and Applied Basic Research Foundation, 2024B1515250003, and the Foshan City Self-Funded Science and Technology Innovation Projects (Public Service Capability Enhancement), 2520001002535.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this research are available on request from the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Jo, C.H.; Kim, D.H. Reconfigurable LLC Resonant Converter for Bidirectional Electric-Vehicle Chargers. *IEEE Trans. Power Electron.* **2023**, *38*, 15168–15172. [\[CrossRef\]](#)
2. Shahsevani, J.; Beiranvand, R. Application-Oriented Review of the LLC-Based Resonant Converters. *IEEE Access* **2024**, *12*, 52687–52726. [\[CrossRef\]](#)
3. He, Q.; Tang, S.; Ren, D.; Tang, Z.; Zhu, Q.; Tang, C.; Zhou, K. A Novel Wide-Gain-Range Variable-Structure DC/DC Converter Based on an LLC Resonant Converter. *Energies* **2025**, *18*, 3664. [\[CrossRef\]](#)
4. Zhou, Y.; Qiu, M.; Wang, Q.; Ma, Z.; Sun, H.; Zhang, X. Research on Double LCC Compensation Network for Multi-Resonant Point Switching in Underwater Wireless Power Transfer System. *Electronics* **2023**, *12*, 2798. [\[CrossRef\]](#)

5. Ye, Z.; Abramson, R.A.; Ge, T.; Pilawa-Podgurski, R.C.N. Multi-Resonant Switched-Capacitor Converter: Achieving High Conversion Ratio with Reduced Component Number. *IEEE Open J. Power Electron.* **2022**, *3*, 492–507. [\[CrossRef\]](#)
6. Sun, Y.; Yan, S.; Xu, G.; Deng, G.; Su, M. Stability Analysis of Dual Active Bridge Converter with Input Filter and Constant Power Load. *IEEE J. Emerg. Sel. Top. Ind. Electron.* **2022**, *3*, 658–669. [\[CrossRef\]](#)
7. Li, X.; Tan, Z.; Wang, X.; Liu, Y. Evolution of the Discharge Mode from Chaos to an Inverse Period-Doubling Bifurcation in an Atmospheric-Pressure He/N₂ Dielectric Barrier Discharge in Increasing Nitrogen Content. *IEEE Trans. Plasma Sci.* **2022**, *50*, 619–634. [\[CrossRef\]](#)
8. Song, Y.; Liang, Z.; Zhang, H.; Zhang, Q.; Zhao, L.; Shen, D.; Tang, D. Period-Doubling and Quadrupling Bifurcation of Vector Soliton Bunches in a Graphene Mode Locked Fiber Laser. *IEEE Photonics J.* **2017**, *9*, 4502308. [\[CrossRef\]](#)
9. Sakellaridis, N.G.; Karystianos, M.E.; Vournas, C.D. Homoclinic Loop and Degenerate Hopf Bifurcations Introduced by Load Dynamics. *IEEE Trans. Power Syst.* **2009**, *24*, 1892–1893. [\[CrossRef\]](#)
10. Yang, J.; Tse, C.K.; Huang, M.; Liu, D. Bifurcations of Grid-Following Rectifiers and Routes to Voltage Instability in Weak AC Grids. *IEEE Trans. Power Syst.* **2023**, *38*, 1702–1713. [\[CrossRef\]](#)
11. Ma, Y.; Tse, C.K.; Kousaka, T.; Kawakami, H. Connecting Border Collision with Saddle-Node Bifurcation in Switched Dynamical Systems. *IEEE Trans. Circuits Syst. II Express Briefs* **2005**, *52*, 581–585. [\[CrossRef\]](#)
12. Liu, W.; Zhang, H. Cycle Skipping-Induced Bifurcation Analysis of Single-Inductor Bipolar-Output DC-DC Converters with One-Cycle Control. *IEEE Trans. Circuits Syst. II Express Briefs* **2023**, *70*, 3134–3138.
13. Fan, J.; Gao, M.; Chen, S. Discontinuous Dynamic Analysis of a Class of 2-DOF Oscillators with Strong Nonlinearity under a Periodic Excitation. *IEEE Access* **2021**, *9*, 77997–78016. [\[CrossRef\]](#)
14. Glendinning, P.; Kowalczyk, P.; Nordmark, A.B. Multiple Attractors in Grazing–Sliding Bifurcations in Filippov-Type Flows. *IMA J. Appl. Math.* **2016**, *81*, 711–722.
15. Bao, B.; Zhang, X.; Bao, H.; Wu, P.; Wu, Z.; Chen, M. Dynamical Effects of Memristive Load on Peak Current Mode Buck-Boost Switching Converter. *Chaos Solitons Fractals* **2019**, *122*, 69–79. [\[CrossRef\]](#)
16. Wu, R.; Zhang, X.; Jiang, W.; Zhong, S. Bifurcation Analysis and Control in a Second-Order DC–AC Inverter with Quasi-PIR Controller. *Commun. Nonlinear Sci. Numer. Simul.* **2025**, *145*, 108702. [\[CrossRef\]](#)
17. Zhou, G.; Mao, S.; Zhou, S.; Wang, Y.; Yan, T. Discrete-Time Modeling and Symmetrical Dynamics of V²-Controlled Buck Converters with Trailing-Edge and Leading-Edge Modulations. *IEEE J. Emerg. Sel. Top. Ind. Electron.* **2020**, *8*, 3995–4008.
18. Geng, X.; Zhang, B.; Qiu, D.; Chen, Y.; Xiao, W.; Xie, F. Modeling and Nonlinear Dynamic Analysis of a Photovoltaic System with Multiple Parallel Branches Based on Simplified Discrete Time Model. *IEEE Trans. Power Electron.* **2024**, *39*, 10226–10238. [\[CrossRef\]](#)
19. Huang, L.; Lu, Y. Discrete Modeling and Period-Adding Bifurcation of DC-DC Converter Feeding Constant Power Load. *IEEE Access* **2021**, *9*, 52773–52783.
20. Liu, H.; Guo, W.; Cheng, D.; Wang, Y.; Wang, M. Stability and Bifurcation Analysis of DC Microgrid with Multiple Droop Control Sources and Loads. *IEEE Trans. Power Electron.* **2021**, *36*, 2361–2372. [\[CrossRef\]](#)
21. Ning, J.; Sun, Y.; Lin, J.; Xu, G.; Su, M. Filippov-Based Stability Analysis Method for Digitally Controlled DC–DC Converters. *IEEE Trans. Transp. Electrification* **2024**, *10*, 2511–2520. [\[CrossRef\]](#)
22. Wang, F.; Zhang, H.; Ma, X. Analysis of Slow-Scale Instability in Boost PFC Converter Using the Method of Harmonic Balance and Floquet Theory. *IEEE Trans. Circuits Syst. I Regul. Pap.* **2010**, *57*, 405–414. [\[CrossRef\]](#)
23. Li, Q.D.; Guo, J.L. Algorithm for Calculating the Lyapunov Exponents of Switching System and Its Application. *Acta Phys. Sin.* **2014**, *63*, 100501–100510. [\[CrossRef\]](#)
24. Lu, Y.; Huang, X.; Huang, Y.; Liu, D. Sigmoid Function Model for a PFM Power Electronic Converter. *IEEE Trans. Power Electron.* **2020**, *35*, 4233–4241. [\[CrossRef\]](#)
25. Foster, M.P.; Gould, C.R.; Gilbert, A.J.; Stone, D.A.; Bingham, C.M. Analysis of CLL Voltage-Output Resonant Converters Using Describing Functions. *IEEE Trans. Power Electron.* **2008**, *23*, 1772–1781. [\[CrossRef\]](#)
26. Qin, H.; Kimball, J.W. Generalized Average Modeling of Dual Active Bridge DC–DC Converter. *IEEE Trans. Power Electron.* **2012**, *27*, 2078–2084.
27. Yakala, R.K.; Nayak, D.; Kumar, M.; Pramanick, S.K. The Effect of Cross-Coupling on the Dynamic Performance of a Dual Phase Shift Controlled BiWPT System. *IEEE Trans. Transp. Electrification* **2023**, *9*, 4583–4593. [\[CrossRef\]](#)
28. Rumelhart, D.E.; Hinton, G.E.; Williams, R.J. Learning Internal Representations by Error Propagation. In *Parallel Distributed Processing: Explorations in the Microstructure of Cognition: Foundations*; Rumelhart, D.E., McClelland, J.L., Eds.; MIT Press: Cambridge, MA, USA, 1986; pp. 318–362.
29. Mitchell, T.M. *Machine Learning*; McGraw-Hill: New York, NY, USA, 1997.
30. Rüdiger, S. *Practical Bifurcation and Stability Analysis*; Springer: New York, NY, USA, 2010.

31. Linaro, D.; Giudice, D.D.; Brambilla, A.; Bizzarri, F. Application of Envelope-Following Techniques to the Shooting Method. *IEEE Open J. Circuits Syst.* **2020**, *1*, 22–33. [[CrossRef](#)]
32. Kantz, H.; Schreiber, T. *Nonlinear Time Series Analysis*, 2nd. ed.; Cambridge University Press: Cambridge, UK, 2004.

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