

Article

A Novel Ambiguity Resolution Method for Array Signals via Wavefront Modulation

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Abstract

Aimed at the elevation ambiguity problem in array synthetic aperture radar (SAR) three-dimensional imaging, this paper proposes a novel ambiguity-resolving method based on wavefront modulation. By introducing measured plasma lens modulation phases and constructing an array SAR signal echo model incorporating wavefront modulation, the method effectively overcomes the physical size limitations of traditional array antennas. Theoretical analysis demonstrates that wavefront modulation significantly reduces the grating lobe level of the array pattern, equivalently increasing the number of array channels and thereby shortening the shortest baseline length, which enhances the system's maximum unambiguous height. At the signal processing level, an observation equation based on compressed sensing is established, and target reconstruction is achieved using the Orthogonal Matching Pursuit (OMP) algorithm. Monte Carlo simulation results indicate that under the same signal-to-noise ratio conditions, when the observation range is extended to twice the theoretical maximum unambiguous height, the proposed method maintains a reconstruction success rate of over 95%, whereas the traditional method's reconstruction success rate drops rapidly below 40% once the maximum unambiguous range is exceeded. This study also investigates the 3D reconstruction of spatial point targets and a rectangular building, with the analysis of their theoretical ambiguous positions confirming the method's effectiveness in suppressing ambiguous targets in the vicinity of spatial point targets as well as in front of and behind the structure. This study provides a new technical approach to overcoming antenna size constraints on airborne platforms, with significant application value in fields such as digital elevation model construction and urban 3D imaging.



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1. Introduction

Traditional array SAR 3D imaging builds upon 2D imaging by arranging multiple transceiver antennas along the cross-track direction to equivalently form a large aperture in the elevation direction, thereby achieving resolution in height [1]. This 3D imaging method only requires registering the images from different channels after 2D imaging and then performing a Fourier transform in the elevation dimension to distinguish targets at different heights. Its implementation and procedure are relatively straightforward, leading

to extensive applications in fields such as topographic mapping, urban 3D modeling, mountainous terrain modeling, resource exploration, and military applications.

In radar 3D imaging [1–5], the longest baseline in the elevation direction determines the elevation resolution, while the shortest baseline determines the maximum unambiguous height resolvable in elevation. Due to the constraint of the shortest baseline, the elevation ambiguity problem has always been a key factor limiting the accuracy of 3D terrain reconstruction. Elevation ambiguity primarily stems from the relationship between the radar wavelength and the baseline length. When the baseline length exceeds half the wavelength, the interferometric phase becomes ambiguous, making it impossible to accurately extract elevation information directly from the interferometric phase. To address this issue, researchers have proposed various methods, each with distinct advantages and limitations.

A common approach is elevation ambiguity resolution based on phase unwrapping techniques [6–8]. This method utilizes multi-baseline interferometric SAR technology, employing baselines of different lengths to increase the elevation ambiguity interval, thereby improving the accuracy of elevation inversion. However, this method can encounter difficulties when dealing with complex terrain, as the difficulty of unwrapping multi-baseline interferometric phases increases significantly with longer baselines, especially in the presence of noise and abrupt terrain changes. Literature [9] proposed an automatic building detection and height estimation method based on phase difference (PD). This method uses the phase difference pattern within building layover areas in SAR interferograms to detect buildings, capable of detection even when building heights exceed the ambiguous height without requiring external data. Paper [10] employs compressed sensing theory to address the elevation ambiguity problem. This method can recover elevation information from limited observation data through sparse signal reconstruction techniques. Theoretically, it can break through the limitations of the Nyquist sampling theorem, but in practical applications, its performance is significantly affected by signal sparsity and noise levels [11]. Paper [12] presents a three-baseline phase unwrapping method based on cluster analysis to resolve elevation ambiguity, but it is still affected by terrain undulations. Paper [13] is based on geometric constraints for elevation ambiguity resolution. This method automatically extracts real point clouds by analyzing the distribution pattern of TomoSAR point clouds and utilizing boundary constraints and elevation continuity constraints. It shows good adaptability in processing complex terrain like mountainous areas but relies on accurate geometric models and prior knowledge, potentially lacking robustness in areas with drastic terrain changes. Therefore, for 3D modeling in complex mountains and cities, there is an urgent need for a novel mechanism that can “equivalently” increase the number of array elements or shorten the baseline, thereby breaking the fundamental performance limit imposed by the physical aperture without significantly increasing system hardware complexity. The wavefront modulation technique studied in this paper is designed to address this challenge at the level of physical principles.

Domestic and international research teams, building upon existing radar systems and referencing optical imaging theory, have proposed a method based on microwave staring correlated imaging [14–17]. This method employs classical pulse compression technology in the range direction to achieve high-precision resolution, while in the azimuth direction, it draws on the concept of optical correlated imaging. It couples target scene scattering information through one-dimensional incoherent wavefront modulation technology and extracts the scattering information using correlated processing, achieving high-resolution imaging in azimuth. The most critical aspect of microwave staring correlated imaging technology is modulating spatiotemporally incoherent wavefronts to increase the number of observation equations. In recent years, with the rapid development of nanotechnology and materials science, various wavefront modulation methods have emerged, including ran-

dom phase modulation [18–22], plasma lens modulation [14–16,23], metasurface antenna modulation [24,25], and Fourier synthesis methods [26], among others. Random phase modulation involves transmitting independent random signals through a multi-channel radar array to form a radiation field with random wavefronts, suitable for high-resolution identification and imaging of ground military targets (especially stationary ones). The plasma lens method modulates the plasma density and dielectric constant by changing the excitation power (e.g., 50–200 W), thereby performing nonlinear spatial modulation of electromagnetic waves, suitable for scenarios requiring flexible adjustment of the wavefront. Metasurface technology achieves wavefront modulation through metasurfaces made of special materials. The radiation field design based on Fourier synthesis modulates the excitation signal amplitude (rather than phase) of the array elements, causing the wavefront phase to vary linearly with the target position while keeping the energy focused.

In 2015, the team from Shanghai Jiao Tong University realized random electromagnetic modulation imaging in the Ku-band and L-band using a plasma lens, verifying the feasibility of plasma lens-based microwave correlated imaging in a microwave anechoic chamber [14,24]. However, current applications of wavefront modulation are primarily limited to microwave staring imaging, where it has demonstrated an azimuth resolution mechanism fundamentally different from SAR. The application of wavefront modulation in array SAR three-dimensional imaging has not yet been explored. Therefore, this paper focuses on introducing wavefront modulation into the elevation direction antenna array to achieve integration with array SAR 3D imaging, constituting the main research objective of this work.

The main contributions of this paper are as follows:

- For the first time, starting from the physical model of the maximum unambiguous height, this paper analyzes the inherent limitations imposed by the array antenna pattern. It innovatively achieves modulation of the array pattern through wavefront phase modulation, derives the equivalent antenna pattern model after wavefront modulation, and explains the principle by which wavefront modulation increases the theoretical maximum unambiguous height.
- This paper derives the echo model after introducing wavefront modulation. By analyzing the relationship between the scattering coefficient matrix and the wavefront phase modulation matrix, it presents the use of the OMP algorithm based on compressed sensing for reconstructing targets in the elevation direction.
- This paper proposes a 3D reconstruction processing flow based on wavefront modulation. Experiment 1 simulates and analyzes the improvement in target reconstruction success rate and maximum unambiguous observation range in the elevation dimension after adding wavefront modulation. Experiment 2 performs 3D reconstruction incorporating wavefront modulation on point targets and volumetric targets. The generated point cloud results verify the enhancement capability of wavefront modulation on the maximum unambiguous height in the elevation direction, i.e., achieving ambiguity-resolving in the height dimension.
- The remainder of the article is organized as follows: Section 2 introduces the modulation model of wavefront modulation on the antenna pattern, used to explain the principle of wavefront modulation ambiguity-resolving it then introduces the echo model after wavefront modulation. Section 3 analyzes the sparsity characteristics of the echo signal after wavefront modulation and provides the solution workflow of the OMP algorithm based on compressed sensing. Section 4 introduces the relevant experimental parameter settings and presents and analyzes the experimental results. Section 5 summarizes the main content of this paper.

2. Wavefront Modulation Signal Model

2.1. Principle of Wavefront Modulation for Ambiguity-Resolving

Multi-baseline interferometric altimetry systems suffer from the problem of elevation ambiguity, meaning there exists a maximum unambiguous height. In such systems, the shortest vertical baseline corresponds to the sparsest phase wrapping, which defines the system's maximum unambiguous range. Given a fixed system frequency band, the shortest vertical baseline determines the system's maximum unambiguous range. The relationship between the shortest baseline length and the maximum unambiguous range of the observation system [6] can be expressed as:

$$H_{\max} = \frac{\lambda R}{2b} \quad (1)$$

In Equation (1), λ is the wavelength, R is the target slant range, and b is the shortest baseline length. When the height of the observed scene exceeds the maximum unambiguous height, the excess part causes periodic aliasing. Therefore, phase unwrapping methods are often required in practical observations to resolve the elevation ambiguity problem.

Due to the inverse relationship between the short baseline length and the maximum unambiguous range, and considering practical factors such as manufacturing, assembly, and cost, it is impossible to infinitely reduce the physical length of the short baseline. Practical systems are constrained by Rayleigh resolution and the theoretical unambiguous height. Therefore, this paper proposes introducing wavefront modulation to equivalently increase the number of array elements and reduce the shortest baseline, thereby achieving an improvement in the maximum unambiguous range.

To illustrate the ambiguity-resolving method proposed in this paper, we first derive the relationship between the antenna pattern and the element spacing for a uniform array antenna system without introducing modulation. The uniform array model is shown in Figure 1, with the number of elements being M , and the element positions at $0, d, 2d, \dots, md, \dots, (M-1)d$. The expression for its antenna pattern is derived.

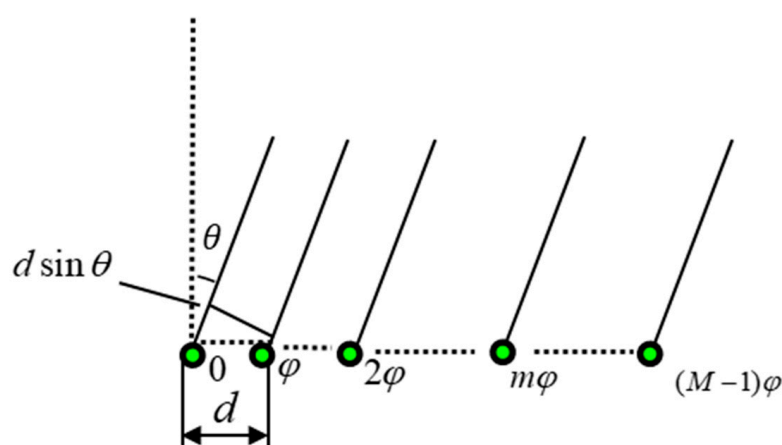


Figure 1. Schematic diagram of a uniform linear array antenna model, showing the structure with equally spaced elements along the elevation direction. The model includes M elements with spacing d , used for subsequent antenna pattern analysis and wavefront modulation modeling.

Assuming a target point in the far-field region at an angle θ deviating from the normal, the field strength at this point can be calculated as Equation (2):

$$E(\theta) = \sum_{k=0}^{M-1} E_k = E \sum_{k=0}^{M-1} e^{jk(\phi - \varphi)} \quad (2)$$

$$\varphi = \frac{2\pi}{\lambda} d \sin \theta \quad (3)$$

Using Euler's formula, this can be simplified to Equation (4):

$$E(\theta) = E \frac{\sin[\frac{M}{2}(\phi - \varphi)]}{\sin[\frac{1}{2}(\phi - \varphi)]} e^{j[\frac{M-1}{2}(\phi - \varphi)]} \quad (4)$$

Therefore, the normalized pattern of the uniform array antenna can be expressed as Equation (5):

$$F(\theta) = \frac{|E(\theta)|}{|E(\theta)|_{\max}} = \left| \frac{1}{M} \frac{\sin[\frac{M}{2}(\frac{2\pi}{\lambda} d \sin \theta - \varphi)]}{\sin[\frac{1}{2}(\frac{2\pi}{\lambda} d \sin \theta - \varphi)]} \right| \quad (5)$$

Based on Equation (5), setting $M = 8$, $\lambda = 0.0284$ m, $d_1 = 0.5\lambda$, and $d_2 = \lambda$, two typical patterns are plotted as shown in Figure 2.

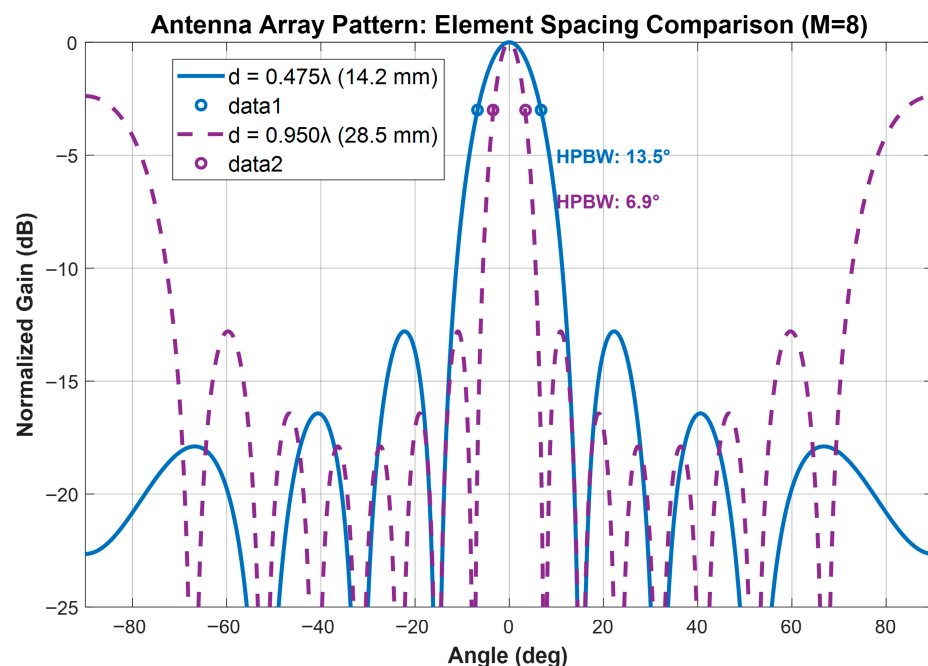


Figure 2. Comparison of antenna patterns under different element spacings. The figure shows normalized array factor patterns for element spacings $d = \lambda/2$ and $d = \lambda$. Increased spacing leads to grating lobes, indicating the significant influence of element spacing on pattern shape and grating lobe occurrence.

Here, M is the number of channels. A larger element spacing results in a longer array length and higher resolution; however, a smaller shortest baseline (i.e., element spacing) leads to a reduced period for the appearance of grating lobes. Theoretical analysis shows that when the element spacing is $d > \lambda/2$, grating lobes appear in the pattern, leading to target ambiguity. The condition for avoiding grating lobes can be derived from Equation (5) as:

$$\frac{d}{\lambda} < \frac{1}{1 + |\sin \theta_0|} \quad (6)$$

where θ_0 is the beam pointing direction. Taking a beam scanning range of $-60^\circ < \theta_0 < 60^\circ$ as an example, $d < 0.53\lambda$ must be satisfied to avoid grating lobes. Therefore, to prevent grating lobes, the shortest baseline length needs to be reduced.

The following section will introduce the concept and implementation of wavefront modulation, analyze how introducing wavefront modulation increases the maximum unambiguous range, and derive the principles and imaging model of microwave staring imaging.

The key to microwave staring imaging is constructing a spatiotemporally uncorrelated radiation modulation matrix. For an antenna array, targets at different elevations have different viewing angles from each channel. By adding different modulation phases to the target echoes from each viewing angle, it is equivalent to introducing new degrees of freedom in observation, increasing the sampling rate in the elevation direction while equivalently reducing the shortest baseline length. Increasing the sampling rate enhances the elevation resolution, and reducing the shortest baseline length increases the maximum unambiguous height. Taking the experiment in this paper as an example, by introducing a wavefront modulation model in the height direction, a modulation phase related to the height and modulation mode is added to targets at different heights, breaking the consistency of the wavefront phase. By constructing an imaging model based on the wavefront modulation matrix, the true target positions can be reconstructed.

In the microwave anechoic chamber, wavefront modulation is achieved through a plasma lens modulation system. A physical image of the modulator is shown in Figure 3. The plasma lens modulation has 14 modulation modes, covering an angular range of 12° around the beam center. Figure 4 shows the variation in the modulation phase with the modulation state at different modulation angles.

After introducing wavefront modulation into the system, assuming the modulation angular range is $-\varphi < \theta_0 < \varphi$, for a target point A in the far-field region at an angle θ deviating from the normal, the echo phase of this point varies with the modulation cycle through multiple phase modulations, no longer determined solely by its slant range. Using the k -th modulation mode as an example, the modulation phase added to point A relative to all array elements is $\phi_k(\theta)$. Therefore, Equation (7) can be derived from Equation (3):

$$\phi_k(\theta) = \frac{2\pi}{\lambda} \Delta d_k(\theta) \sin \theta \quad (7)$$

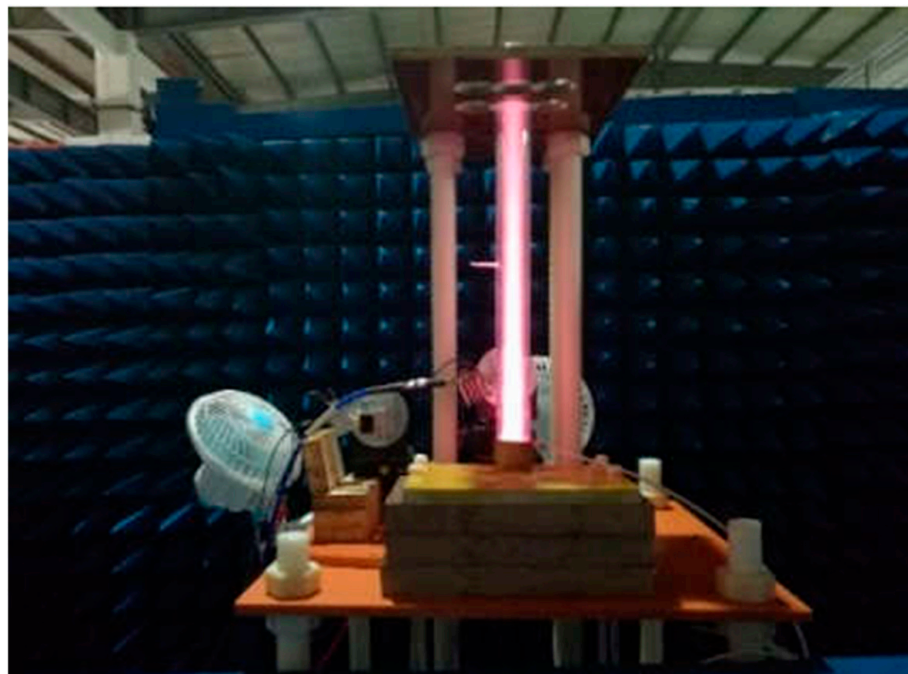


Figure 3. Physical photograph of the wavefront modulation experimental platform. The image shows the plasma lens modulation system used for wavefront modulation in a microwave anechoic chamber, supporting multiple modulation modes for phase modulation at various angles.

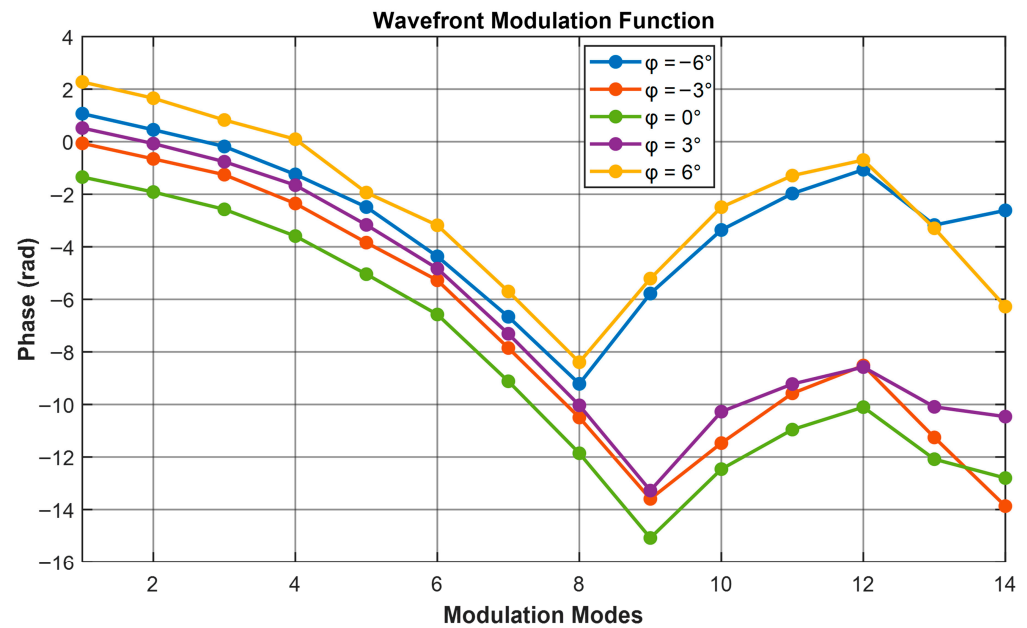


Figure 4. Measured variation in modulation phase with modulation state at different angles in the microwave anechoic chamber. The horizontal axis represents the modulation state index, and the vertical axis shows phase values (in radians), illustrating the phase modulation capability and consistency of the wavefront modulation system across angles.

For angle θ and modulation state k , the equivalent element position is:

$$d_{equiv,m}^{(k)}(\theta) = d_m + \Delta d_k(\theta) \quad (8)$$

Multiple modulation states equivalently increase the number of antenna elements. Here, Δd is the equivalent displacement of the element under a fixed angle and modulation mode. The range of this equivalent displacement can be expressed as:

$$0 \leq \Delta d \leq \frac{\lambda}{2\pi} \cdot \frac{\phi_k}{\sin \theta} \quad (9)$$

Through multiple phase modulations, the number of antenna elements is equivalently increased, and the shortest baseline is reduced, thereby improving the system's maximum unambiguous range. At this point, the array response function becomes:

$$E_{equiv}^{(k)}(\theta) = \sum_{m=1}^M \exp\left(j \frac{2\pi}{\lambda} d_{equiv,m}^{(k)}(\theta) \sin(\theta)\right) \quad (10)$$

Combining Equations (2), (7), (8) and (10), the equivalent array antenna pattern is obtained as:

$$E_{equiv} = \sum_{k=1}^{Nk} \sum_{m=1}^M \exp\left(j \frac{2\pi}{\lambda} d_{equiv,m}^{(k)}(\theta) \sin(\theta)\right) \quad (11)$$

Figure 5 shows the comparison of the array pattern before and after applying the wavefront modulation matrix under the conditions of channel number $M = 8$, element spacing $d = 14.5$ cm, and wavelength $\lambda = 3$ cm. The modulation angle applied in Figure 5 is $(-12^\circ, 12^\circ)$. The specific results are shown below:

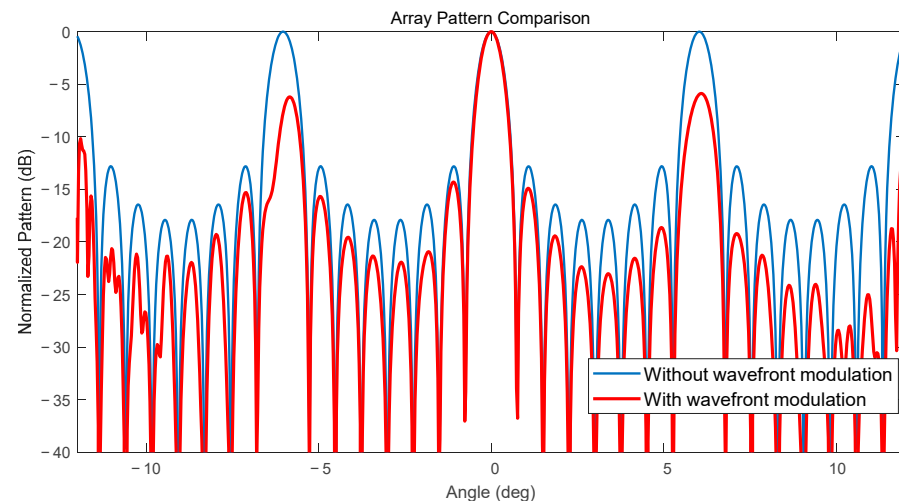


Figure 5. Comparison of array antenna patterns before and after wavefront modulation. Conditions: number of elements $M = 8$, element spacing $d = 0.1427$ m, wavelength $\lambda = 0.03$ m, modulation angle range -12° to 12° .

Figure 5 indicates that the main lobe of the pattern changes little after the addition of phase modulation, but both the side lobes and grating lobes are reduced. In Figure 5, the maximum side lobe reduction is 12.01 dB, and the maximum grating lobe reduction is 9.2 dB. The pattern result for wavefront phase modulation demonstrates that wavefront modulation effectively reduces the minimum element spacing, thereby theoretically increasing the maximum unambiguous range.

2.2. Wavefront Modulation Echo Model

The physical model of wavefront modulation is shown in Figure 6.

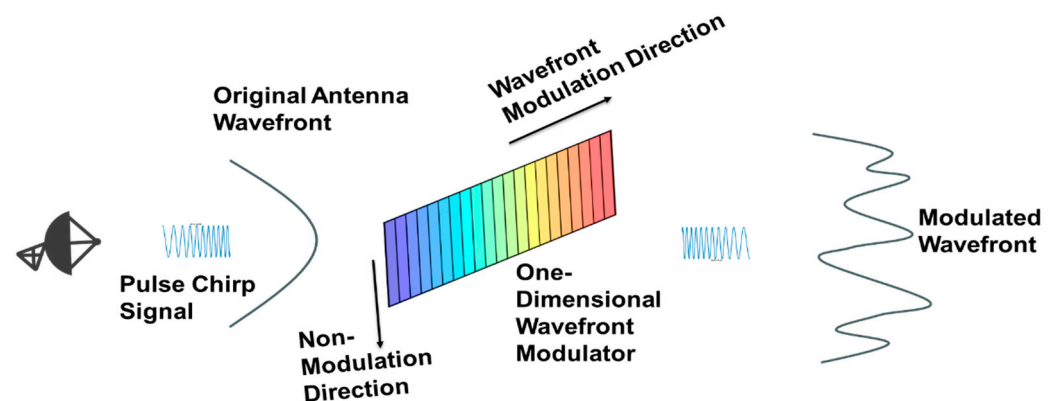


Figure 6. Schematic diagram of the wavefront modulation echo model. After passing through the one-dimensional wavefront modulator, the wavefront of the Chirp signal is modulated, and its phase changes from uniform to non-uniform.

The echo model is derived as follows. Assuming a linear frequency modulated (LFM) signal is transmitted, the transmitted signal can be expressed as:

$$S(t) = \text{rect}\left(\frac{t}{T}\right) \exp\{j\pi k t^2\} \quad (12)$$

The received signal form is:

$$S_r(t) = C(R) \text{rect}\left(\frac{t-t_0}{T}\right) \exp\{j\pi k (t-t_0)^2\} \quad (13)$$

Let σ be the scattering coefficient matrix and W be the wavefront modulation matrix. $C(R)$ represents the combined effect of the wavefront modulation matrix W and the scattering coefficient matrix σ , with dimensions $Nk \times N$, where Nk is the number of modulation modes of the wavefront modulation system, N is the number of range gates, and NL is the number of grids in the height direction. It is expressed as:

$$C(R) = \sum_{n=1}^{NL} w_n \sigma_n(R) \quad (14)$$

Writing C in the form of Equation (15), its columns represent the scattering coefficients combined with wavefront modulation at different range gates. The column at range gate nR can be represented by Equation (16).

$$C = [C(1R) \quad C(2R) \quad \dots \quad C(NR)]_{1 \times N} \quad (15)$$

$$C(nR) = \begin{bmatrix} C_1(nR) \\ C_2(nR) \\ \vdots \\ C_{Nk}(nR) \end{bmatrix}_{Nk \times 1} \quad (16)$$

The scattering coefficient matrix for targets at different ranges and heights across the entire observation scene is in the form:

$$\sigma = \begin{bmatrix} \sigma_1(R) & \sigma_1(2R) & \dots & \sigma_1(NR) \\ \sigma_2(R) & \sigma_2(2R) & \dots & \sigma_2(NR) \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{NL}(R) & \sigma_{NL}(2R) & \dots & \sigma_{NL}(NR) \end{bmatrix} \quad (17)$$

The scattering coefficients of targets at different heights for range gate nR can be expressed as Equation (18):

$$\sigma(nR) = \begin{bmatrix} \sigma_1(nR) \\ \sigma_2(nR) \\ \vdots \\ \sigma_{NL}(nR) \end{bmatrix}_{NL \times 1} \quad (18)$$

The wavefront modulation matrix W is a matrix with Nk rows and NL columns, representing Nk modulation modes applied over NL grids in the modulation dimension. Its form is:

$$W = \begin{bmatrix} w_{1,1} & w_{1,2} & \dots & w_{1,NL} \\ w_{2,1} & w_{2,2} & \dots & w_{2,NL} \\ \vdots & \vdots & \ddots & \vdots \\ w_{Nk,1} & w_{Nk,2} & \dots & w_{Nk,NL} \end{bmatrix} \quad (19)$$

Since W is the wavefront modulation matrix, its elements are phases for different angles under different modulation modes, which can be obtained from the experimentally determined wavefront modulation matrix. The specific form is as follows:

$$W = \begin{bmatrix} \exp(j\varphi_{1,1}) & \exp(j\varphi_{1,2}) & \dots & \exp(j\varphi_{1,NL}) \\ \exp(j\varphi_{2,1}) & \exp(j\varphi_{2,2}) & \dots & \exp(j\varphi_{2,NL}) \\ \vdots & \vdots & \ddots & \vdots \\ \exp(j\varphi_{Nk,1}) & \exp(j\varphi_{Nk,2}) & \dots & \exp(j\varphi_{Nk,NL}) \end{bmatrix} \quad (20)$$

The φ in the exponential terms is the added modulation phase. Combining the above formulas, $C(nR)$ at range gate nR can be calculated as:

$$\begin{aligned}
 C(nR) &= W\sigma(nR) \\
 &= \begin{bmatrix} w_{1,1} & w_{1,2} & \dots & w_{1,NL} \\ w_{2,1} & w_{2,2} & \dots & w_{2,NL} \\ \vdots & \vdots & \ddots & \vdots \\ w_{Nk,1} & w_{Nk,2} & \dots & w_{Nk,NL} \end{bmatrix} \cdot \begin{bmatrix} \sigma_1(nR) \\ \sigma_2(nR) \\ \vdots \\ \sigma_{NL}(nR) \end{bmatrix} \\
 &= \begin{bmatrix} C_1(nR) \\ C_2(nR) \\ \vdots \\ C_{Nk}(nR) \end{bmatrix}
 \end{aligned} \quad (21)$$

Therefore, the comprehensive scattering coefficient C of the target scene can be obtained from Equations (15) and (21):

$$\begin{aligned}
 C &= \begin{bmatrix} w_{1,1} & w_{1,2} & \dots & w_{1,NL} \\ w_{2,1} & w_{2,2} & \dots & w_{2,NL} \\ \vdots & \vdots & \ddots & \vdots \\ w_{Nk,1} & w_{Nk,2} & \dots & w_{Nk,NL} \end{bmatrix} \cdot \begin{bmatrix} \sigma_1(R) & \sigma_1(2R) & \dots & \sigma_1(NR) \\ \sigma_2(R) & \sigma_2(2R) & \dots & \sigma_2(NR) \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{NL}(R) & \sigma_{NL}(2R) & \dots & \sigma_{NL}(NR) \end{bmatrix} \\
 &= \begin{bmatrix} C_1(R) & C_1(2R) & \dots & C_1(NR) \\ C_2(R) & C_2(2R) & \dots & C_2(NR) \\ \vdots & \vdots & \ddots & \vdots \\ C_{Nk}(R) & C_{Nk}(2R) & \dots & C_{Nk}(NR) \end{bmatrix}
 \end{aligned} \quad (22)$$

Equation (22) can be simplified into the following matrix form:

$$W \cdot \sigma = C \quad (23)$$

Therefore, the equation to be solved is Equation (23). Here, W and C can be obtained from the wavefront modulation matrix and the echo signal, respectively. The target scattering coefficient matrix σ needs to be solved. W is the observation equation of the target scene, with dimensions $Nk \times NL$. The rows of the matrix represent the modulation states, and the columns represent different angles or different target heights in the wavefront modulation dimension.

3. Method

Based on the sparse observation model established in Section 2.2, our goal is to stably recover the sparse scene scattering coefficient vector in the height dimension from limited and noisy observations. This section elaborates on the selection of the echo model solution algorithm and the application workflow of the OMP algorithm in this study.

3.1. Solution Method Selection

When no modulation is applied to the system, the phase of the theoretical target echo depends solely on the distance between the target and the receiving antenna. After adding the wavefront modulation phase, the phase of the target echo also becomes related to the target's elevation angle relative to the antenna. Therefore, compared to the unmodulated case, adding modulation is equivalent to increasing the number of observations for the same target by Nk times. Under traditional sampling, the dimension in the elevation direction is the number of antenna elements, M . After adding phase modulation, the observation

dimension in elevation increases to $M \cdot Nk$. Before wavefront modulation is introduced, the observation matrix is set up based on the number of antenna array elements and is defined as matrix A , with dimensions of $M \times NL$. Matrix A is a variant of the standard discrete Fourier basis. Each row corresponds to a complex sinusoidal plane wave (complex exponential function). Its m -th row takes the form:

$$A(m, :) = \exp(j \cdot 2\pi \cdot \frac{x}{NL} \cdot (m - 1)) \quad (24)$$

where $x = [-NL/2, -NL/2 + 1, \dots, NL/2 - 1]$ and NL is the number of grids in the elevation direction obtained by angle division, and $NL = 1024$ is used in the experiment. Considering the coupling relationship between the number of array elements and the modulation states of the wavefront modulator, the actual observation matrix is obtained by combining matrix A and matrix W , defined as A_W . The combination is performed by element-wise multiplication (dot product) between rows of matrix A and rows of matrix W , extended via broadcasting, resulting in an actual observation matrix with $N \times Nk$ rows and NL columns. Let a_m be the n -th row of matrix A , and W_k be the k -th row of matrix W . The r -th row of A_W (where $r = k + N(m - 1)$) is:

$$A_W_r = a_m \odot W_k \quad (25)$$

In Equation (25), the symbol $\odot$ denotes element-wise multiplication. Therefore, the complete observation matrix A_W can be constructed using Equation (25). This results in a significant increase in the signal dimension. Therefore, in practical processing, W , σ , and C in Equation (23) all undergo a corresponding increase in dimensions. However, the solution method remains the same as in the case of single-element wavefront modulation. Thus, this paper continues to use the notation of Equation (23) for the explanation of the solution method. According to linear algebra theory, the imaging effectiveness of wavefront modulation primarily depends on the correlation of the wavefront modulation matrix. When the correlation between rows of the modulation phase matrix is weak or uncorrelated, an accurate solution for the target position can be obtained. However, in practice, it is difficult to obtain a completely uncorrelated or very weakly correlated modulation matrix. Therefore, the observation matrix constructed from the modulation matrix and the element positions is a low-rank matrix, and Equation (23) becomes an ill-posed equation.

Compressed Sensing (CS) [27–33], as a signal processing technique, can utilize the sparsity of a signal to reconstruct it from far fewer sample points than required by the Nyquist sampling theorem. Its core idea is to transform Equation (26) into solving an optimization problem, which can typically be formulated as:

$$\hat{\sigma} = \arg \min \|\sigma\|_0, \text{ s.t. } \|C - W\sigma\|_2 \leq \varepsilon \quad (26)$$

where limits the noise strength in the data. The most intuitive solution is represented by the l_0 norm. However, considering the presence of noise in the simulation and that the signal is approximately sparse rather than ideally sparse, finding the solution under the l_0 norm constraint is an NP-hard problem. Therefore, this problem can be transformed into an l_1 norm minimization problem with a relaxation constraint:

$$\hat{\sigma} = \arg \min \|\sigma\|_1, \text{ s.t. } \|C - W\sigma\|_2 \leq \varepsilon \quad (27)$$

Under conditions such as the Restricted Isometric Property (RIP) being satisfied by the observation matrix, its solution is equivalent to the norm solution. Although

this convex optimization problem has a global optimum, solving it (e.g., using iterative thresholding algorithms) often involves complex parameter tuning and high computational costs. Particularly in the context of this paper, where the observation matrix dimension increases multiplicatively, the matrix condition number is less than 1, and the matrix rank is much lower than its dimension, this method becomes unsuitable. In contrast, OMP, as a classic greedy iterative algorithm [34,35], directly and efficiently approximates the original norm problem. Its core advantages are:

- High computational efficiency: The iterative process is simple, and for very sparse signals, the computation speed is fast.
- Conceptual intuitiveness: Each iteration identifies one target, making the physical meaning clear.
- Simple parameters: The main parameter is the number of iterations k (sparsity), which is easy to set and interpret.

3.2. OMP Algorithm Workflow

To align with the general sparse reconstruction algorithm framework, we establish the following clear symbolic mapping relationship: let the general observation vector $y \equiv C$, the general observation matrix $\Phi \equiv W$, and the general sparse signal $x \equiv \delta$. Therefore, solving Equation (27) is equivalent to solving the sparse recovery problem in the following standard form:

$$y = \Phi x \quad (28)$$

The OMP algorithm workflow is shown in Algorithm 1:

Algorithm 1: OMP Algorithm Workflow. OMP_Algorithm (Φ, y, k)

Input: $\Phi, \mathbf{y} \in \mathbb{C}^{M \times N}$ its column vector set is $\{\phi_j\}_{j=1}^N, \mathbf{y} \in \mathbb{C}^{M \times 1}$
 k is the iteration count of the algorithm, which is determined by the sparsity of the signal.

Result: x_k

Initialization: $r_0 = y, A_0 = \emptyset$

Normalize all columns of Φ to unit l_2 norm

Remove duplicated columns in A_0

For $k = 1, 2, \dots$ **do**

Step 1. $\lambda_k = \arg \max_{j=1, \dots, N} |\langle \mathbf{r}_{k-1}, \phi_j \rangle|$

Step 2. $\Lambda_k = \Lambda_{k-1} \cup \{\lambda_k\}$

Step 3. $\mathbf{x}_k = \arg \min_{\mathbf{x}} \|\mathbf{y} - \Phi_{\Lambda_k} \mathbf{x}\|_2$

Step 4. $\mathbf{r}_k = \mathbf{y} - \Phi_{\Lambda_k} \mathbf{x}_k$

end

After k iterations, the OMP reconstruction result is obtained. In this paper, the criterion for successful reconstruction is defined as the target amplitude being greater than half of the peak value, and the error between the target position and the actual position being less than half of the theoretical resolution.

4. Experiments and Results

4.1. Elevation Direction Target Reconstruction Realized by Wavefront Modulation

To verify the target reconstruction success rate and ambiguity-resolving effect in the elevation direction before and after introducing wavefront modulation into the array antenna, Experiment 1 investigates the differences in signal reconstruction for height-dimension targets without and with wavefront modulation.

The simulated target scene is shown in Figure 7.

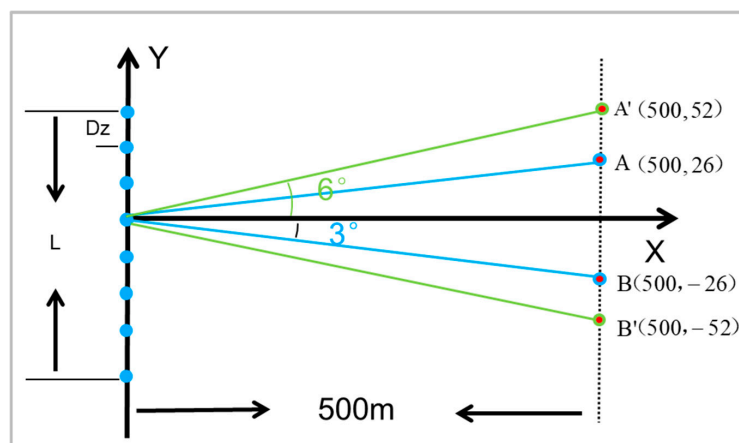


Figure 7. Schematic diagram of the theoretical maximum unambiguous observation height. The figure illustrates the theoretical unambiguous interval in elevation (-26 m to 26 m) and its extended observation range (-52 m to 52 m), used to validate the reconstruction capability of wavefront modulation beyond the unambiguous range. Dz is the element spacing and L is the length of the antenna array.

The system and target parameters are set as shown in Table 1.

Table 1. System Parameters for Elevation Direction Array Wavefront Modulation.

Parameter	Value
Carrier Frequency	10.0 GHz
Element Spacing	0.1427 m
Number of Elements	8
Scene Center Slant Range	500 m
Theoretical Unambiguous Height	52.56 m
Theoretical Height Resolution	7.43 m
Wavefront Modulation Modes	14
Modulation Angle	$[-6^\circ, 6^\circ]$
Height Range of Targets AB in Figure 7	-26 m~ 26 m
Angle of Targets AB Relative to Array Center	$[-3^\circ, 3^\circ]$
Height Range of Targets A'B' in Figure 7	-52 m~ 52 m
Angle of Targets A'B' Relative to Array Center	$[-6^\circ, 6^\circ]$
SNR	10 dB

By focusing on the essence of wavefront modulation ambiguity-resolving, the experiment directly analyzes targets in the elevation dimension, simplifying the azimuth and range dimensions. According to the maximum unambiguous height Formula (1) derived earlier, under the conditions of a carrier frequency of 10 GHz, element spacing of 0.1427 m, and 8 elements, the theoretical maximum unambiguous height is 52.56 m, and the theoretical resolution is 7.43 m. This corresponds to an antenna scanning angle from -3° to 3° . The simulated target scene has a ground range of 500 m. The height falls within the theoretical unambiguous range, and the first unambiguous interval calculated by the formula is -26 m to 26 m. If the target is outside this range, ambiguity occurs, and the signal is folded into the first unambiguous interval periodically.

When the target observation scene is set to AB, both targets are within the maximum unambiguous height. The target spacing ranges from 0 m to 25 m, with a step size of 2.6 m. The specific position of the first target is randomly generated each time, and the position of the second target is determined by the target spacing. 100 Monte Carlo simulations are

performed for each spacing. Successful resolution is defined as the difference between the reconstructed target spacing and the true target spacing being less than half of the theoretical resolution. The target reconstruction success rates before and after introducing wavefront phase modulation are statistically analyzed.

The simulated target reconstruction success rate curves under different spacings, without and with modulation, are shown in Figure 8.

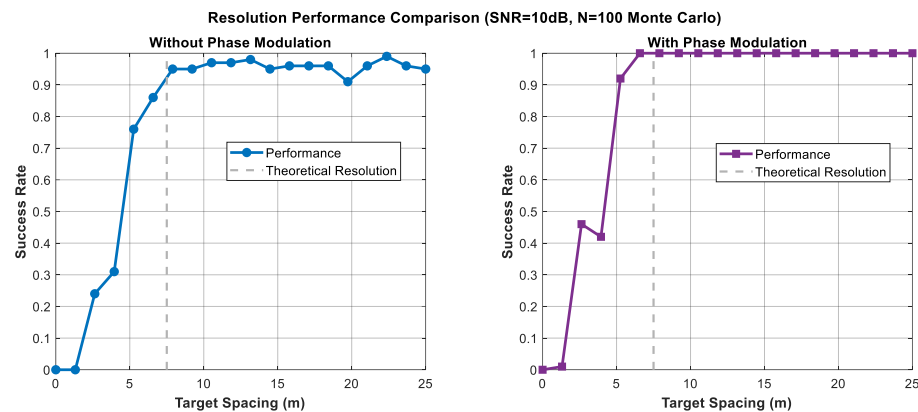


Figure 8. Comparison of Resolution Success Rates Before and After Introducing Phase Modulation under $\text{SNR} = 10 \text{ dB}$, $N = 100$ Conditions. The horizontal axis represents target spacing, and the vertical axis shows reconstruction success rate.

Figure 8 shows that within the theoretical maximum unambiguous height range, the reconstruction success rates are roughly the same without and with wavefront modulation. However, when the spacing is less than or equal to the theoretical resolution, the reconstruction performance with wavefront modulation is better than that of the unmodulated system, indicating that introducing wavefront modulation can improve imaging resolution. When the target spacing is greater than the resolution, the stability of target reconstruction with wavefront phase modulation is also better than that of the unmodulated system.

Figure 9 shows a comparison between the reconstructed results and the true target positions for three typical target spacings after using wavefront modulation. It can be seen that the reconstruction results of both systems are close to the true targets, which validates the reconstruction success rate curves for different spacings before and after modulation shown in Figure 8.

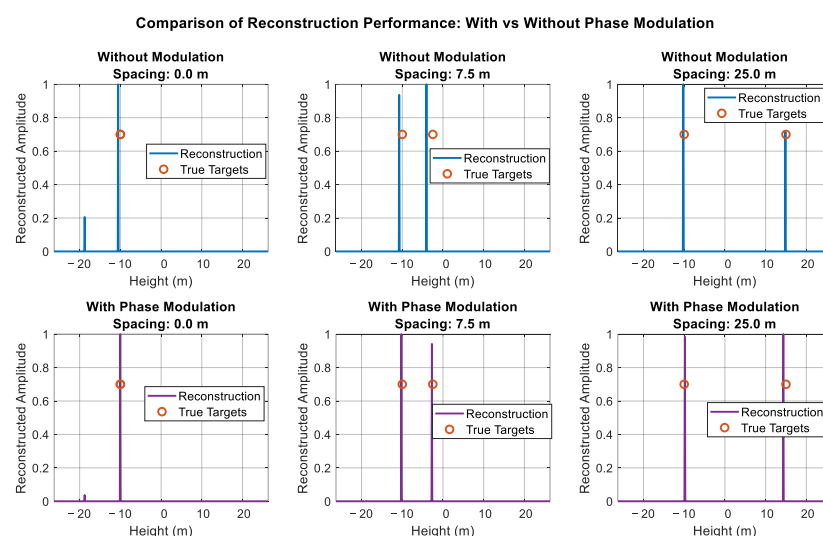


Figure 9. Comparison of reconstruction effects for three typical target spacings before and after introducing wavefront modulation.

When the target observation scene is expanded from AB to A'B' in Figure 7, the observation range extends from the first unambiguous interval to the midpoint of the second unambiguous interval above and below. The target spacing ranges from 0 m to 52 m, with a step size of 2.6 m. The specific position of the first target is randomly generated each time, and the position of the second target is determined by the target spacing. 100 Monte Carlo simulations are performed for each spacing. The target reconstruction success rates before and after introducing wavefront phase modulation are statistically analyzed.

The simulation also plots the target reconstruction success rate curves at different spacings without and with modulation, as shown in Figure 10.

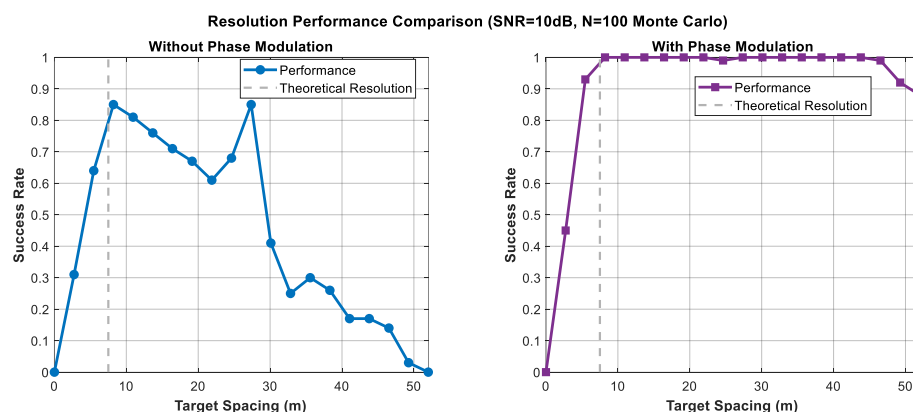


Figure 10. Comparison of Reconstruction Success Rates Before and After Introducing Phase Modulation When the Observation Range is Increased to Twice the Maximum Unambiguous Height. The horizontal axis represents target spacing, and the vertical axis shows reconstruction success rate.

Figure 10 shows that when the target observation range exceeds the maximum unambiguous range, the target reconstruction success rate of the system without wavefront phase modulation decreases significantly. After the target spacing exceeds 26 m, the reconstruction success probability quickly drops below 40%, indicating an inability to reconstruct correctly. The reason is that after doubling the observation range, targets originally outside the theoretical maximum unambiguous range are folded into the first unambiguous interval periodically during reconstruction in the unmodulated system. In contrast, for the system using wavefront phase modulation, when the range exceeds the maximum unambiguous limit, different phases are modulated at different angles. This causes the phases of target echoes that would otherwise be folded periodically to differ, allowing them to be distinguished. Thus, a high reconstruction success rate is maintained within the modulation angle range. The fact that two targets located in different ambiguity intervals can still be correctly reconstructed strongly demonstrates that wavefront modulation can enhance the system's ambiguity-resolving capability.

Figure 11 shows three groups of target spacings: 0.0 m, 25.0 m, and 52.0 m. The first target position is fixed at −20.0 m (within the first unambiguous interval). The second target positions are at 5 m and 32 m, respectively. For the first two spacing settings, both targets are within the first unambiguous interval and can be correctly reconstructed. For the third spacing of 52 m, the second target is at 32.0 m, which is not in the same unambiguous interval as the first target at −20.0 m. The traditional array signal OMP algorithm cannot reconstruct this correctly, whereas introducing the wavefront modulation system can increase the maximum unambiguous range of the array antenna.

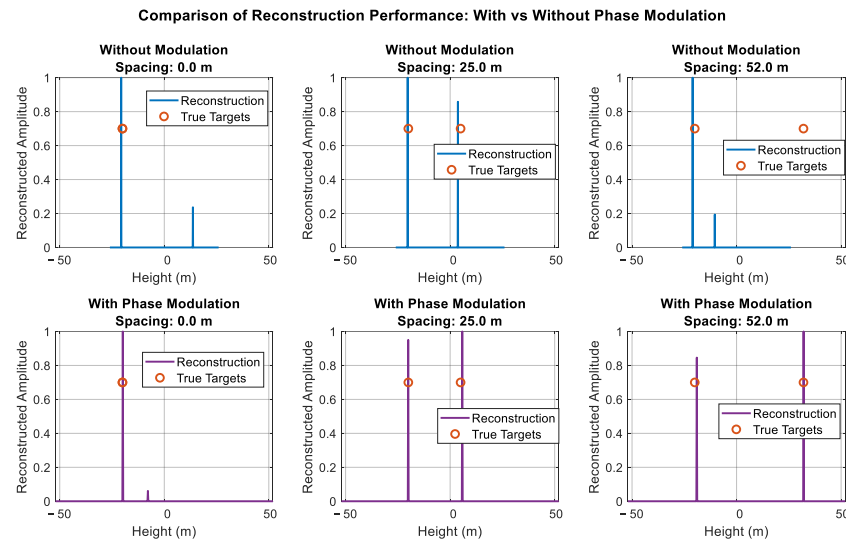


Figure 11. Comparison of reconstruction effects for three target spacings when the observation range is extended to twice the maximum unambiguous height. For the last spacing (52 m), the second target lies in a different unambiguous interval; the traditional method fails to reconstruct correctly, while the wavefront modulation method successfully resolves it.

4.2. 3D Imaging Ambiguity-Resolving with Wavefront Modulation

The implementation of array SAR 3D imaging based on wavefront modulation is as follows: pulse compression in the range direction and synthetic aperture in the azimuth direction are used to achieve 2D imaging. Based on this, the 2D images are registered, and the OMP algorithm is used for elevation direction reconstruction. Finally, the 3D reconstruction result is obtained. The experimental process is shown in Figure 12:

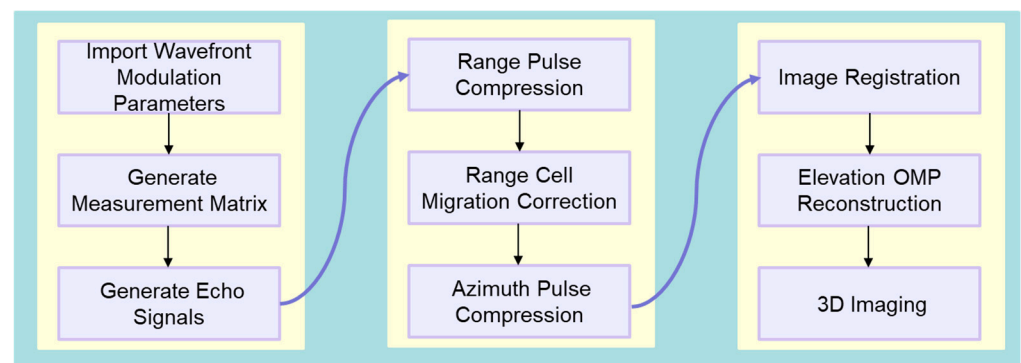
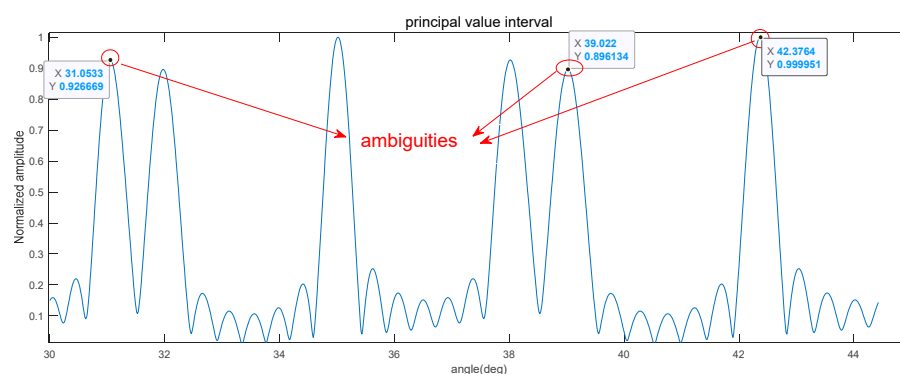
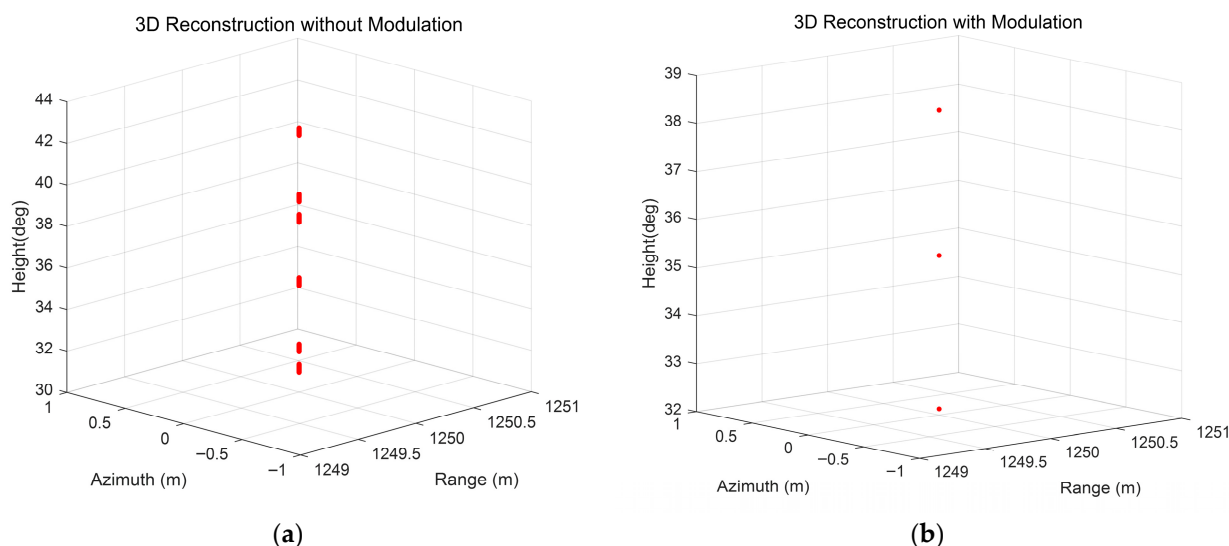


Figure 12. Flowchart of array SAR 3D imaging incorporating wavefront modulation. The process includes range pulse compression, azimuth synthetic aperture imaging, image registration, elevation OMP reconstruction, and finally outputs 3D point cloud results.

The parameter settings for the simulation are shown in Table 2. The simulation results for point targets are shown in Figure 13. For the three targets located at different angles of 32° , 35° , and 38° , corresponding ambiguities exist within the observation interval. The ambiguous positions are at 39.02° , 42.38° , and 31.05° , respectively. As can be more clearly observed in the point cloud diagram of Figure 14, direct three-dimensional imaging without introducing wavefront modulation, where the elevation dimension is solved using the least squares method, results in significant ambiguities in the reconstruction. In contrast, the OMP reconstruction method incorporating wavefront modulation successfully resolves these ambiguities, achieving the goal of ambiguity resolution.

Table 2. System Parameters for Elevation Direction Array Wavefront Modulation 3D Imaging.

Parameter	Value
Carrier Frequency	10.0 GHz
Element Spacing	0.1427 m
Number of Elements	8
Platform height	800 m
Scene Center Slant Range	1150 m
Theoretical Unambiguous Angle Range	[30°, 37.5°]
Wavefront Modulation Modes	14
Modulation Angle	[30°, 42°]
Point Target Angles	32°, 35°, 38°
Volumetric Target Angle Range	30.73–44.12°
SNR	10 dB

**Figure 13.** Schematic diagram of ambiguous target positions in point target reconstruction using the conventional least squares method. The figure shows the ambiguous positions (39.02°, 42.38°, 31.05°) corresponding to three real targets (32°, 35°, 38°), indicating elevation ambiguity in the unmodulated system.**Figure 14.** Comparison of 3D reconstruction point clouds for point targets. (a) Without wavefront modulation, ambiguous targets appear in the point cloud; (b) with wavefront modulation, ambiguous targets are effectively suppressed, resulting in clear and unambiguous reconstruction.

The positioning errors are quantified in Table 3.

Table 3. Comparison of 3D reconstruction positioning errors for point targets.

Target Angle	Without Wavefront Modulation	With Wavefront Modulation
Target1: 32°		
Reconstructed position	32.13°	32.12°
Ambiguous position	39.12°	Be suppressed
Positioning error	0.13°	0.12°
Target2: 35°		
Reconstructed position	35.32°	35.32°
Ambiguous position	42.55°	Be suppressed
Positioning error	0.32°	0.32°
Target3: 38°		
Reconstructed position	38.35°	38.35°
Ambiguous position	31.12°	Be suppressed
Positioning error	0.35°	0.35°

The distributed target is a building in the shape of a rectangular prism. The center coordinates of the building's base are (900, 0, 0), with a width of 100 m, a length of 150 m, and a height of 220 m. The simulation results are shown in Figures 15 and 16.

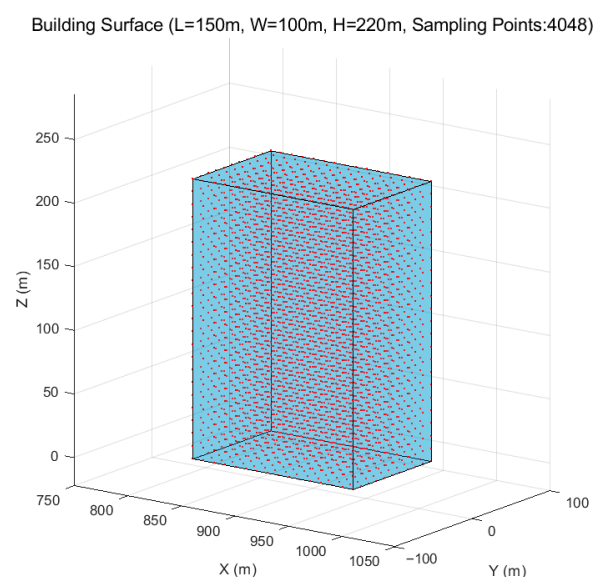


Figure 15. Actual 3D coordinate distribution of the building target. The blue surface represents the building outline, and the red point cloud represents the distributed sampling points with an interval of 10 m in the azimuth, range, and elevation directions.

Figure 15 shows the azimuth, range, and height information of the building in the Cartesian coordinate system, where the internal red points represent the distribution of sampling points. Figure 16a presents the 3D imaging result of the building using the OMP algorithm without wavefront modulation, while Figure 16b displays the corresponding result with wavefront modulation added. It can be clearly observed that the application of wavefront modulation effectively eliminates the ambiguous targets appearing in front of and behind the building. This clearly demonstrates that the system's maximum unambiguous height is effectively enhanced.

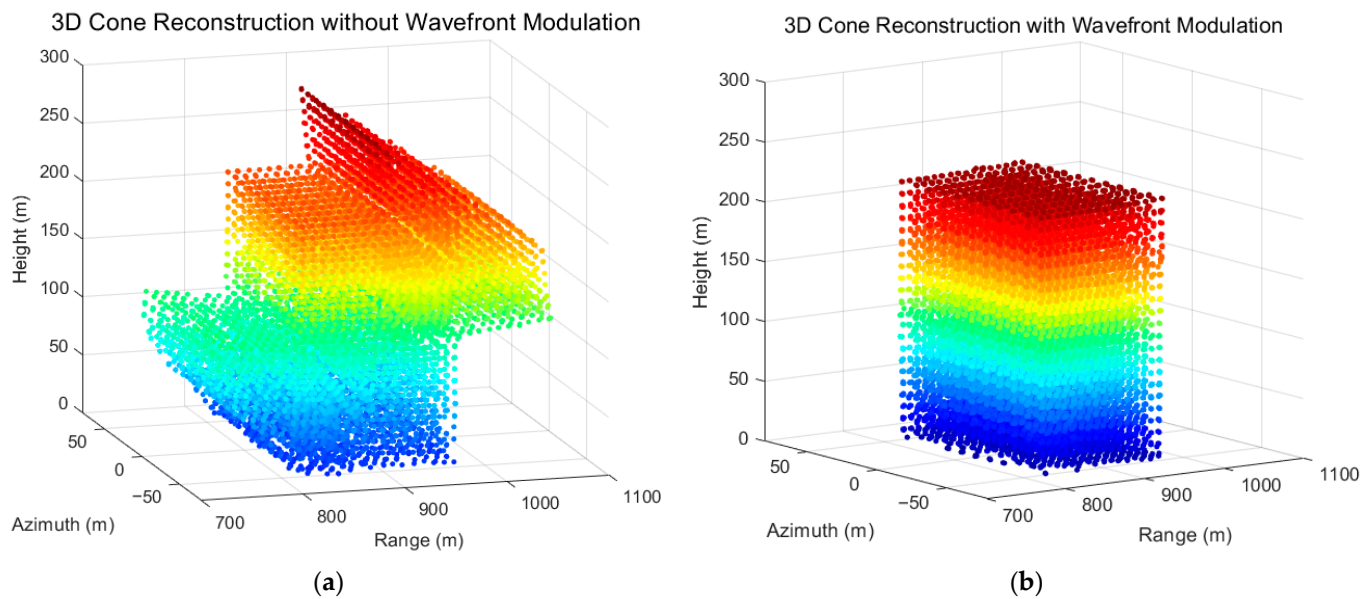


Figure 16. Comparison of 3D reconstruction results for the building target. The colours used in the figure represent the height variation of the reconstruction target. (a) Without wavefront modulation, ambiguous targets appear in front of and behind the reconstructed building. (b) With wavefront modulation, the ambiguous targets are eliminated, and the building structure is clearly reconstructed.

5. Conclusions

This paper has presented a novel ambiguity resolution method for array SAR 3D imaging via wavefront modulation, aiming to break through the physical limit imposed by the shortest baseline on the maximum unambiguous height. The core principle lies in using wavefront phase modulation to equivalently increase the number of array elements and reduce the shortest baseline.

Different modulation states increase the number of observations of the target scene, which is equivalent to increasing the number of antenna elements and shortening the shortest baseline. Consequently, this enhances the maximum unambiguous range of the system in that dimension, achieving the ambiguity-resolving functionality.

Simulation results from two experiments verified the effectiveness of wavefront modulation: Experiment 1 showed that the success rate remained over 95% beyond the theoretical unambiguous range, while Experiment 2 demonstrated effective suppression of ambiguous targets for both point and building targets within 30–45° depression angles, confirming the method's feasibility for elevation direction applications.

The current study validates the principle of ambiguity resolution via wavefront modulation using anechoic chamber data and simulations. Future work will advance along the path from proof-of-concept to practical system implementation: First, a ground-based validation system incorporating the actual modulator will be established to quantitatively analyze the impact of non-ideal factors—such as modulation phase errors and channel inconsistencies—on 3D reconstruction accuracy and to develop corresponding error compensation models. Second, at the algorithmic level, more robust and efficient sparse reconstruction algorithms (e.g., compressive sensing methods based on Bayesian inference) will be explored to enhance performance under challenging conditions with low signal-to-noise ratios or unknown scene sparsity, systematically evaluating the trade-off between computational complexity and reconstruction accuracy. The ultimate goal is to provide theoretical and technical support for parameter design and performance prediction in the engineering application of this technology.

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