

Article

A Multinotch FIR Filter Based on a Stable IIR Filter Prototype with Improved Dynamic Performance via Iterative Determination of Nonzero Initial Conditions Using Vector Projection

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Abstract

In many cases, the removal of individual frequency components from a signal spectrum is achieved using notch and multinotch filters. One of the main disadvantages of such filters is the occurrence of transient states, which depend, among other factors, on the filter order and selectivity, i.e., the bandwidth of the stopband. In this paper, the authors present a method for synthesizing a finite impulse response (FIR) multinotch filter based on a prototype infinite impulse response (IIR) notch filter. The proposed approach is characterized by a significant reduction in the influence of transient effects on the filter response, achieved through the iterative determination of nonzero initial conditions. This allows the dynamic performance of the filter to be improved without compromising its frequency response. Furthermore, the proposed filter structure is characterized by having a lower filter order than conventional filtering structures while maintaining satisfactory filtration quality. To demonstrate the properties of the proposed structure, computer simulations were performed and filtration quality metrics were evaluated. The obtained results indicate that the proposed structure outperforms classical filtering methods by ensuring faster stabilization of the response during the transient state. Moreover, it was demonstrated that the proposed method of reducing the FIR filter order has only a minor effect on filtration quality.

Keywords: digital signal processing; notch filter; short-time signals; nonzero initial conditions

1. Introduction

Digital notch filters, which precisely remove selected frequency components from a signal spectrum, are widely used in modern technology. The primary purpose of digital notch filters is to suppress interference originating from 50 or 60 Hz power lines, particularly in medical devices [1–4], but they are also applied in signal processing for other sources, such as industrial plants [5]. Notch filters are also used for reference signal extraction in shunt active power filters [6], phase and frequency estimation in inverters [7], and resonance compensation circuits in LCL filtering systems [8]. These types of filters are also employed to remove unwanted frequency components in digital lock-in amplifier signals [9], systems for condition monitoring of gearboxes and induction motors [10,11], and resonance compensation systems in servomechanisms [12].

Notch filters are traditionally implemented using FPGA-based solutions [13,14] or microcontrollers [15]; however, attempts have also been made to implement them in hardware using memristors [16]. One of the main challenges associated with the implementation



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of notch filters is the occurrence of a transient state [17]. The transient state prevents full analysis of the denoised signal during the initial period after filter activation or following rapid changes in the input signal. This issue is particularly critical when measuring short-duration signals, such as evoked potentials, partial discharge signals, or seismic signals. Although repeated signal acquisition is possible under constant conditions, this approach is often impractical, especially in medical applications, where repeated measurements may lead to patient fatigue and distortion of test results [18].

Several methods have been proposed to reduce the influence of transient states on filter responses [17]. Among those reported in the literature are filter initialization techniques proposed by Chornoboy [19] and Al-Ahmad and Kamal [20], determination of nonzero initial conditions using vector projection introduced by Pei and Tseng [21] and further developed by Kocoń and Piskorowski [22], gain modification proposed by Amini et al. [23], and the application of filters with time-varying parameters [24]. To reduce the computational cost associated with the classical vector projection method for determining initial conditions, a hybrid approach combining nonzero initial condition determination and time-varying filter parameters was proposed by the authors in [25].

In this paper, the authors present a method for synthesizing a finite impulse response (FIR) mult notch filter based on a stable prototype infinite impulse response (IIR) notch filter. FPGA-based digital FIR and IIR filters are widely used in biomedical ultrasound imaging systems, including intravascular ultrasound (IVUS) applications [26]. This paper proposes a new method that enables reduction of the order of an FIR filter while maintaining filtration quality as measured by a filtration quality index. The method is based on synthesizing an FIR filter from a stable IIR prototype by approximating poles with zeros. A procedure for selecting the parameter ϵ , which determines the accuracy of pole approximation during filter synthesis, is also introduced, with filtration quality assessed in terms of the mean square error. Furthermore, the proposed filter structure is combined with iterative determination of nonzero initial conditions to mitigate transient effects in the filter response. To address the increased computational cost associated with using an FIR filter instead of an IIR filter, the approach includes an order reduction procedure that preserves filtration quality. This combination enables the use of filters with narrower stopbands in short-duration signal processing applications. The proposed approach significantly limits the influence of the transient state on the filter response through the iterative determination of nonzero initial conditions. Additionally, the computational complexity of the synthesized FIR filter is reduced by truncating its impulse response using a method based on mean square error (MSE) calculation. The duration of the transient state is closely related to the locations of the poles of the transfer function in IIR filters and to the order of FIR filters. Transient effects arise from the lack of information about past signal samples, causing some filter coefficients to be multiplied by zero during the initial computation of the filter output.

The transient response can be reduced either by degrading the frequency characteristics of the filter, which decreases its selectivity, or by determining an appropriate vector of nonzero initial conditions. The aim of this paper is to present a method for limiting the amplitude of the transient response in unconditionally stable mult notch FIR filters derived from prototype IIR notch filters. Furthermore, the computational complexity of the proposed filter structure is reduced by truncating the FIR filter impulse response.

To reduce the influence of the transient state on the filter response, the poles of the prototype IIR notch filter are first shifted to zero, resulting in an FIR notch filter structure. This eliminates the influence of previous output samples on the current response, thereby reducing transient effects. Subsequently, to reduce the computational complexity of each FIR notch filter used to construct the mult notch filter, a method for impulse response reduction based on mean square error evaluation is applied. Individual FIR notch filters,

each removing a specific frequency component from the useful signal band, are then cascaded to obtain a mult notch FIR filter. For such a structure, the vector projection method can also be applied to determine the nonzero initial conditions.

The proposed method for reducing transient effects in notch filters consists of the following steps:

- Synthesis of a stable prototype IIR notch filter.
- Synthesis of an FIR notch filter by shifting the poles of the prototype IIR filter to zero.
- Reduction of the number of FIR filter coefficients.
- Cascading FIR notch filters to obtain an FIR mult notch filter.
- Iterative determination of nonzero initial conditions for the FIR mult notch filter.

The paper is organized as follows. Section 2 presents the synthesis of a stable prototype IIR notch filter (Section 2.1.1), the procedure for transferring IIR filter poles to FIR filter zeros (Section 2.1.2), and the synthesis of a mult notch FIR filter based on the prototype IIR filter (Section 2.2). Section 2.3 describes the iterative determination of nonzero initial conditions, while Section 2.1.3 presents the method for FIR filter order reduction aimed at lowering computational complexity. Section 3 presents computer simulation results confirming the effectiveness of the proposed methods and compares them with classical filtering techniques. Section 4 discusses the results in the context of the delay introduced by the filter structure, and Section 5 concludes the paper and outlines directions for future research aimed at further reducing transient effects in digital filters.

2. Material and Methods

This section presents a method for synthesizing a mult notch FIR filter with nonzero initial conditions and reduced order, based on a prototype IIR notch filter. The first part of the section describes the synthesis of a prototype IIR notch filter characterized by stability and maximally flat amplitude response in the passband. The subsequent part presents a method for shifting the poles of the prototype IIR notch filter to zeros in order to obtain an FIR notch filter. Next, a method for reducing the order of the FIR notch filter while maintaining filtration quality is introduced. The following part describes the synthesis of a mult notch filter based on the previously presented FIR notch filter structure derived from the prototype IIR notch filter. Finally, the determination of nonzero initial conditions for the FIR filter is presented in order to reduce the influence of the transient state on the filter response.

2.1. Derivation of a Finite Impulse Response Filter from an Infinite Impulse Response Prototype

The synthesis of a mult notch FIR filter with nonzero initial conditions and reduced order begins with the synthesis of a prototype IIR notch filter. To avoid stability issues in the prototype filter, the method for stable IIR notch filter synthesis presented in [27] was employed.

2.1.1. IIR Notch Filter Prototype

According to [27], to obtain a stable IIR notch filter with a maximally flat amplitude response and unit gain at the DC component and the Nyquist frequency, an all-pass filter should be designed with an order equal to twice the number of frequencies to be removed by the IIR notch filter. The notch filter defined by Equation (1) was obtained by synthesizing the all-pass filter given in Equation (2) and subsequently converting it into a notch filter using Equation (3):

$$H(z) = \frac{1+r^2}{2} \frac{1-2\cos(\omega)z^{-1}+z^{-2}}{1-(1+r^2)\cos(\omega)z^{-1}+r^2z^{-2}}, \quad (1)$$

$$A(z) = \frac{z^{-2} - (1+r^2) \cos(\omega)z^{-1} + r^2}{r^2z^{-2} - (1+r^2) \cos(\omega)z^{-1} + 1}, \quad (2)$$

$$H(z) = \frac{1 + A(z)}{2}, \quad (3)$$

where r denotes the radius of the poles of the designed prototype IIR notch filter, and ω represents the frequency to be removed from the signal spectrum using the prototype IIR notch filter. This method was adopted because the synthesized filter is stable provided that the radius of the poles of the prototype IIR notch filter satisfies the condition $r < 1$.

2.1.2. Transformation of Prototype IIR Filter Poles into FIR Filter Zeros

To limit the impact of the transient state on the response of the notch filter, particularly in the case of a mult notch filter, the poles of the prototype IIR notch filter were shifted to the zeros of the FIR notch filter. Figure 1 shows the output signals of mult notch IIR and FIR filters for input signals consisting of test ECG signals from the MIT-BIH database [28] together with the sum of sinusoidal waveforms with frequencies of 50, 100 and 150 Hz. These filters were designed to remove these frequencies from the signal spectrum. The ECG signal was selected as a test signal to allow for a brief visual assessment of the filtration quality. As illustrated in Figure 1, the transient response of the IIR filter, designed to remove 50, 100, and 150 Hz components from the signal, is significantly longer and exhibits a higher amplitude than that of the FIR filter derived from the prototype IIR filter. This difference arises because the amplitude and duration of the transient response in the IIR filter depend on both previous filter output samples and input signal samples, whereas for the FIR filter, the transient response is calculated solely based on the input signal samples. To convert the prototype IIR filter into an FIR filter, the factoring method described in [29] was employed. Let us denote the transfer function of the IIR notch filter prototype from Equation (1) as $H(z)$:

$$H(z) = \frac{N(z)}{D(z)}, \quad (4)$$

where

$$N(z) = \sum_{m=0}^{P_z} b_m z^{-m} \text{ and } D(z) = \sum_{n=1}^{P_p} a_n z^{-n}, \quad a_0 = 1. \quad (5)$$

The formula for accurately obtaining factored FIR approximations of an IIR filter is based on the expression presented in [29]:

$$\frac{1}{1-z} = \prod_{k=0}^{\infty} (1+z^{2^k}), \quad |z| < 1. \quad (6)$$

The designed IIR prototype filter is stable, so all filter poles are assumed to lie inside the unit circle. Therefore, the denominator of Equation (4) can be expressed as:

$$D(z) = \prod_{n=1}^{P_p} \left(1 - \frac{z_n}{z}\right). \quad (7)$$

Using the relationships in (6) and Equation (7), the transfer function of the prototype IIR filter given in Equation (4) can be expressed as:

$$H(z) = N(z) \prod_{n=1}^{P_p} \prod_{k=0}^{\infty} \left[1 + \left(\frac{z_n}{z}\right)^{2^k}\right]. \quad (8)$$

To truncate the presented expression, for each filter pole z_n , it is necessary to find a value of k_t that satisfies the following condition for the chosen ϵ :

$$|z_n|^{2k_t} < \epsilon. \quad (9)$$

The transfer function of the FIR target filter $\tilde{H}(z)$, which is an approximation of the prototype IIR filter by the given Equation (1), can be written as:

$$\tilde{H}(z) = N(z) \prod_{n=1}^{P_p} \prod_{k=0}^{k_t} \left[1 + \left(\frac{z_n}{z} \right)^{2k} \right]. \quad (10)$$

An FIR notch filter designed in this manner can be used to synthesize a mult notch filter, which allows the removal of not only the fundamental frequency but also its higher harmonics. In the numerical example presented in this paper, the value of ϵ was selected by iteratively determining the Mean Square Error (MSE) over the range $\epsilon \in <10^{-10}, 10>$. The largest value of ϵ that ensures the smallest MSE was selected for the calculations. The Mean Square Error in this paper is calculated using the formula presented in [30]:

$$\text{MSE} = \frac{1}{K} \sum_{i=1}^K |y(n) - c(n)|^2, \quad (11)$$

where $c(n)$ denotes the noise-free signal, K is the window size, and $y(n)$ is the output signal of the filter.

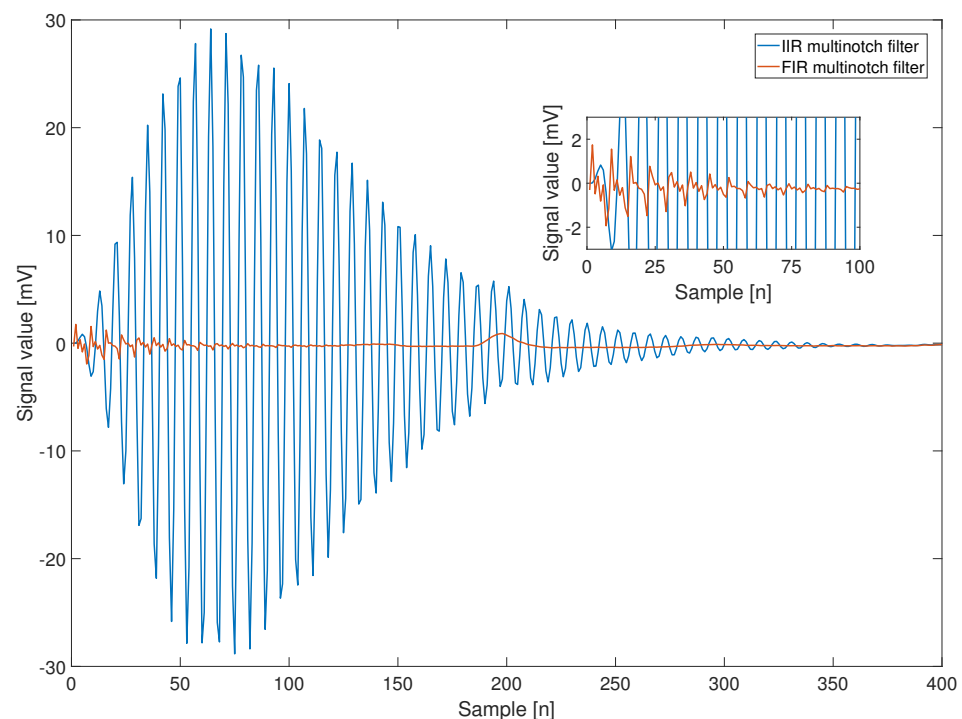


Figure 1. Transient response of the prototype IIR mult notch filter and the FIR mult notch filter when filtering an ECG test signal with interference at frequencies $f = [50, 100, 150]$ Hz.

2.1.3. Reduction of FIR Filter Coefficients

As shown, the FIR filtering algorithm can be expressed in the following form:

$$y[n] = \sum_{k=0}^N b_k x[n - k]. \quad (12)$$

Thus, the filter response consists solely of the sum of input signal samples weighted by the filter coefficients. The value N , representing the number of coefficients, is referred to as the filter order. The computational complexity of the filter structure is directly proportional to the filter order N . Moreover, the filter order determines the length of the transient state in FIR notch filters, which arises from the lack of information about past input signal values prior to filter activation. As a result, some filter coefficients are initially multiplied by zeros until full information about the input signal is available, i.e., after N input samples have been recorded. The transient state affects the initial samples of the filter response, which is particularly critical when processing short-duration signals.

On the other hand, increasing the filter order improves the frequency characteristics of the FIR filter. As the order grows, the frequency response of the FIR filter approaches that of an ideal filter. In the case of an FIR notch filter, the width of the stopband depends on the filter order; for an ideal filter, the stopband width is 0 Hz, attenuating only the center frequency. Therefore, the notch width is determined by the FIR filter order and defines the range of frequencies around the center frequency that will be attenuated.

The synthesis of an FIR notch filter requires a compromise among the notch width, transient state duration, and computational complexity. In the mult notch FIR filter structure proposed in this paper, the method of synthesizing FIR filters from prototype IIR filters initially results in a high-order filter to accurately represent the frequency characteristics of the prototype. To reduce the filter order while maintaining filtration quality, it is proposed to set the last b_N filter coefficients to zero, provided their absolute values are below a threshold p . This truncation reduces the filter order by cutting the tail of the impulse response while leaving the initial samples unchanged.

To select a suitable value of p that ensures satisfactory filtration quality while reducing the filter order, the MSE was evaluated according to Equation (11), using as the input signal the sum of the frequencies to be suppressed. In the numerical example presented, this approach allowed the filter order to be reduced without affecting the stopband width. Truncating the final impulse response values decreased the attenuation at the center frequencies to approximately -71 , -85 , and -72 dB, from the original values of -216 , -210 , and -207 dB, while reducing the order from 1536 to 785, corresponding to a 49% reduction. This method also shortened the transient state duration while preserving the filter's stopband width, achieving the primary objective of the study.

2.2. Multinotch Filter

The procedures presented so far pertain to FIR notch filters designed to remove a single frequency component from the signal spectrum. When suppressing power-line interference beyond the fundamental frequency $f_1 = 50$ Hz, higher harmonics of this frequency may also be present in the signal spectrum. Consequently, a single notch filter may not be sufficient to eliminate interference that impedes signal analysis. Multinotch filters are employed to address this issue. To synthesize a multinotch filter capable of suppressing multiple frequency components, individual FIR notch filters targeting the center frequencies $\omega_1, \omega_2, \dots, \omega_m$ were designed and connected in cascade. The multinotch filter structure can be described as follows:

$$H_{multi}(z) = H_{\omega_1}(z) \cdot H_{\omega_2}(z) \cdot \dots \cdot H_{\omega_m}(z). \quad (13)$$

In this paper, for the numerical example, it was assumed that the useful signal is contaminated by noise consisting of the sum of sinusoidal waves with frequencies $f_1 = 50$ Hz, $f_2 = 100$ Hz, and $f_3 = 150$ Hz. The amplitude and phase characteristics of the resulting multinotch filter are shown in Figure 2.

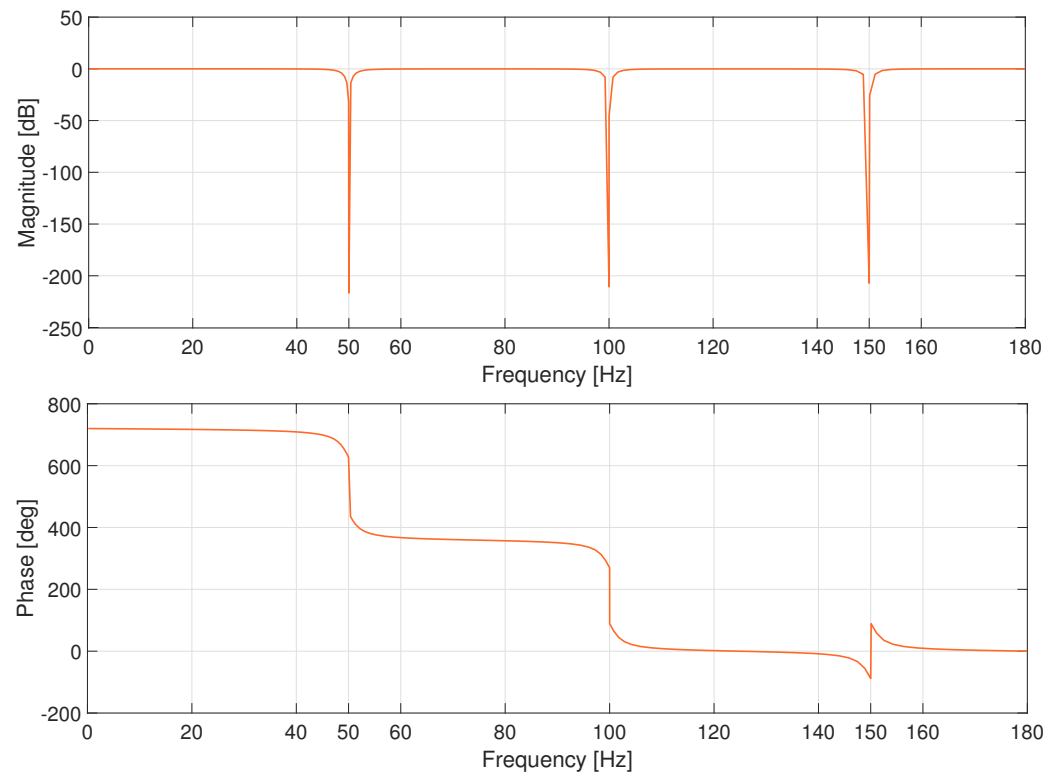


Figure 2. Bode characteristics of a multinotch filter with center frequencies $f_1 = 50$ Hz, $f_2 = 100$ Hz, $f_3 = 150$ Hz.

2.3. Nonzero Initial Conditions

As discussed, the aim of this work is to reduce the impact of transient states on the filter response while preserving its frequency characteristics, particularly the stopband width. In classical filter structures, mitigating the effects of transient states is typically associated with a deterioration of frequency characteristics, most often manifested as an increased stopband width. To maintain the frequency properties while improving the dynamic performance of the filter, it is proposed to use iteratively determined nonzero initial conditions for a reduced-order filter, following the algorithm presented in [22]. This approach enables an enhancement of the filter's dynamic response without compromising its frequency characteristics.

To determine the values of the nonzero initial conditions for the filter, it is necessary to construct a matrix $\mathbf{A}$ of dimensions $L \times 2$. The submatrices $\mathbf{A}_i$ of matrix $\mathbf{A}$ are defined as follows:

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ \sin(\omega_1) & \cos(\omega_1) & \sin(\omega_2) & \cos(\omega_2) & \cdots & \sin(\omega_m) & \cos(\omega_m) \end{bmatrix},$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ \sin(\omega_1) & \cos(\omega_1) & \sin(\omega_2) & \cos(\omega_2) & \cdots & \sin(\omega_m) & \cos(\omega_m) \\ \sin(2\omega_1) & \cos(2\omega_1) & \sin(2\omega_2) & \cos(2\omega_2) & \cdots & \sin(2\omega_m) & \cos(2\omega_m) \end{bmatrix},$$

$$\vdots$$

$$\mathbf{A}_L = \begin{bmatrix} 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ \sin(\omega_1) & \cos(\omega_1) & \sin(\omega_2) & \cos(\omega_2) & \cdots & \sin(\omega_m) & \cos(\omega_m) \\ \sin(2\omega_1) & \cos(2\omega_1) & \sin(2\omega_2) & \cos(2\omega_2) & \cdots & \sin(2\omega_m) & \cos(2\omega_m) \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \cos[(L-1)\omega_1] & \sin[(L-1)\omega_1] & \cos[(L-1)\omega_2] & \sin[(L-1)\omega_2] & \cdots & \cos[(L-1)\omega_n] & \sin[(L-1)\omega_n] \end{bmatrix}. \quad (14)$$

The projection operators should be calculated as follows:

$$\mathbf{P}_m = \mathbf{A}_m(\mathbf{A}_m^T \mathbf{A}_m)^{-1} \mathbf{A}_m^T, \quad m = 1, \dots, L$$

The vectors containing the nonzero initial conditions for each time step of the filtering process are then determined using the following equations:

$$\hat{\mathbf{S}}_m = (\mathbf{I} - \mathbf{P}_m)[x(0), \dots, x(m)]^T = [\hat{s}_m(0), \dots, \hat{s}_m(m)], \quad m = 1, \dots, L \quad (15)$$

$$\tilde{H}(z) = \tilde{b}_0 + \tilde{b}_1 z^{-1} + \dots + \tilde{b}_m z^{-m} \quad (16)$$

$$\begin{aligned} y[0] &= 0, \\ y[1] &= 0, \\ y[2] &= \hat{s}_1(0), \\ y[3] &= \hat{s}_2(1), \\ y[4] &= \hat{s}_3(2), \\ &\vdots \\ y[L] &= \hat{s}_L(L-2), \\ y[L+1] &= \hat{s}_L(L-1), \\ y[L+2] &= \hat{s}_L(L), \\ y[L+n] &= \sum_{i=1}^m \tilde{b}_i x[L+n-i] \end{aligned} \quad (17)$$

Applying the procedure described in Equation (17) allows for determining L samples of the filter's initial condition vector. This provides the filter with information about past input values during start-up, which is necessary for calculating individual output samples. As a result, the impact of the transient state on the filter response is reduced. Compared to classical methods for determining initial conditions, the approach proposed by the authors in [22] and applied in this paper reduces the number of samples required to determine the nonzero initial condition vector to just two. The vector of nonzero initial conditions is determined iteratively along with successive recorded input signal samples. This enables the filtering process to start more rapidly than with classical methods while maintaining satisfactory filtration quality, as confirmed by the calculated filtration quality indicators.

3. Results

The use of nonzero initial conditions in a multiband FIR filter derived from a prototype IIR notch filter resulted in a noticeable reduction in the influence of the transient state on the signal waveform. Employing an FIR filter instead of the prototype IIR filter reduced the transient response to the given excitation, and the application of nonzero initial conditions further mitigated the transient effects. However, the use of the vector projection method introduces a delay equal to the length of the nonzero initial condition vector. The adoption of an iterative approach for determining this vector significantly reduces the delay compared to the classical method described in [21].

Since FIR notch filters with narrow stopbands tend to have a high order, a method for reducing the filter order by truncating the final filter coefficients—subject to a minimum coefficient value—is also presented. This section presents the simulation results and compares them with classical filter structures using zero initial conditions and with the prototype IIR notch filter.

Numerical Example

To illustrate the method described in this paper, a numerical example is provided, demonstrating the step-by-step procedure for synthesizing the proposed filter structure, which reduces the influence of the transient state on the filter response. In the numerical

example, a sampling frequency of $f_s = 360$ Hz was assumed. The interference frequencies affecting the useful signal are the mains frequency $f_1 = 50$ Hz and its higher harmonics $f_2 = 100$ Hz and $f_3 = 150$ Hz. The radius of the poles of the IIR prototype filter was set to $r = 0.97$, ensuring a sufficiently narrow stopband width of $BW \approx 3.5$ Hz. The relationship between r and BW is given by Equation (18):

$$BW = \left(\frac{1-r}{\pi} \right) f_s. \quad (18)$$

An ECG signal was used as the input signal due to its characteristic shape, which allows for a quick assessment of filtration quality without calculating explicit filtration quality indicators. This signal is widely used in the literature on notch and mult notch filters. The prototype filter was synthesized following the procedure described in [29], after which the parameter ϵ in Equation (9) was selected. The selection of this parameter is crucial for the quality of the FIR filter's approximation of the IIR filter's frequency response. However, a compromise must be made between the fidelity of this approximation and the filter order, which directly affects computational complexity. A higher filter order ensures a more accurate representation of the filter's frequency characteristics but increases computational cost due to the greater number of required additions and multiplications.

To achieve satisfactory frequency characteristics while maintaining a minimal filter order, the coefficient ϵ was selected by evaluating the MSE according to Equation (11), assuming that the filter input signal is the sum of sinusoids at the interference frequencies f_1 , f_2 , and f_3 . The dependence of the MSE on the coefficient ϵ is shown in Figure 3. As observed, the MSE changes abruptly with ϵ . The largest value of ϵ that ensures the minimum MSE was adopted, as shown on Equation (19):

$$\epsilon^* = \arg \max_{\epsilon} \{ \epsilon \mid \text{MSE}(\epsilon) = \min \}. \quad (19)$$

The parameter ϵ in [29] was selected a priori under the sole assumption that $\epsilon > 0$. As shown in (9) and (10), the value of ϵ directly affects the selection of the parameters k_t , which correspond to the number of zeros used to approximate each filter pole. Consequently, a lower value of ϵ leads to a more accurate match between the frequency response of the prototype filter and the FIR filter; however, it also increases the order of the FIR filter. Therefore, the selection of the parameter ϵ was linked to the filtration quality expressed in terms of the Mean Squared Error (MSE) in order to determine a value of k_t that preserves the frequency properties of the prototype filter. Based on this assumption, preliminary simulations were conducted for values of ϵ in the range $\epsilon \in [10^{-10}, 10^0]$, following the values reported in [29], with the radius of the prototype filter poles set to $r = 0.97$. It should also be noted that the value of r has a significant impact on the FIR filter order N required to match the frequency specification of the prototype IIR filter. As a result, the selection of the parameter ϵ depends on the radius of the prototype filter poles r and the stopband width BW . Specifically, prototype filter poles located closer to the unit circle—resulting in a narrower stopband BW —require a higher FIR filter order N to preserve frequency characteristics comparable to those of the prototype filter. The simulations made it possible to determine the required filter orders, ranging from $N = 256$ for $\epsilon = 10^0$ to $N = 4096$ for $\epsilon = 10^{-10}$, for each frequency removed from the signal spectrum. Within this scope, the MSE was also evaluated for the removal of each target frequency. This enabled the determination of a set of ϵ values with their corresponding MSE values. Subsequently, as shown in (19), the maximum admissible value of ϵ was sought that yields the minimum filter order N while maintaining the required filtration quality in terms of MSE. This approach inherently results in a relatively high FIR filter order due to the filter structure.

Therefore, an additional order-reduction technique was proposed, consisting of truncating the negligible tail of the impulse response, whose contribution to the filtration result is insignificant. The obtained results indicate an order of $N_n = 512$ for a single filter cascade element, resulting in a total filter order of $N = 1536$. Based on the simulations, the coefficient value selected for the numerical example presented in this paper was $\epsilon = 10^{-0.4123(23)}$. Considering that FIR digital filtering involves multiplying and summing input signal samples with the corresponding filter coefficients, it follows that as the values of successive coefficients decrease, their influence on the output sample also diminishes. To limit the order of the proposed filter, the impulse response of the FIR filter was truncated to a predetermined length N_t . The value of N_t was determined by introducing a threshold parameter p , defined as the minimum coefficient magnitude that has a noticeable influence on the filter response in terms of filtration quality, for a signal composed of sinusoids at the interference frequencies. Figure 4 presents the impulse response of the full-order FIR filter together with the MSE as a function of the parameter p . The threshold p was selected to minimize the filter order while maintaining high filtration quality, expressed by the minimum achievable MSE, as formalized in (20):

$$p^* = \arg \max_p \{p \mid N(p) = \min, \text{MSE}(p) = \min\}. \quad (20)$$

Since the FIR filter obtained through the procedure of shifting the poles of the prototype filter to zeros is characterized by a relatively high order, an additional order-reduction method based on truncation of the impulse response was proposed. As shown in Equation (12), each output sample $y[n]$ depends directly on the multiplication of the input samples by the FIR filter coefficients, which correspond to the samples of the filter impulse response. Since these coefficients decay exponentially, the coefficients located in the tail of the impulse response have a progressively smaller influence on the output signal. Consequently, truncating the impulse response at an appropriate threshold allows the filtration quality, expressed in terms of MSE, to be preserved while reducing the filter order N . To determine the optimal value of the threshold p , the MSE was evaluated for the removal of a sinusoidal component with a frequency equal to the center frequency of the filter, for different values of p . The threshold was defined as the absolute value of the impulse response coefficient, $p = |h(n)|$, below which the impulse response samples were truncated. In the presented example, the value of p was selected from the range $p \in [10^0, 10^{-10}]$ by determining the MSE for each candidate value and subsequently selecting the maximum value of p that ensured the minimum MSE while yielding the minimum filter order N . As observed in Figure 4, the MSE changes rapidly with variations in the parameter p . Based on the simulations, p was set to $10^{-4.7298(98)}$, which limited the orders of the resulting filters to $N_1 = 263$, $N_2 = 262$, $N_3 = 260$, resulting in a total filter order of $N_t = 785$. Figure 5 compares the impulse response of one of the cascade elements of the proposed filter structure before and after the order reduction procedure. Another parameter that must be selected for synthesizing the proposed filter structure is the length of the vector of nonzero initial conditions. This vector length directly affects the computational complexity of the structure, as it requires matrix multiplication of the input signal samples by the individual matrices $(\mathbf{I} - \mathbf{P}_m)$. Table 1 summarizes all selected and determined parameters required for the synthesis of the filter structure presented in this paper, as well as the orders of the individual component filters in the cascade. The result of the filtering process is illustrated using an ECG signal in Figure 6, allowing for a straightforward assessment of the filtration quality. In addition, several filtration quality metrics were evaluated, quantitatively demonstrating the advantage of the proposed iterative method for determining nonzero initial conditions over filtering systems with zero initial conditions. Since power-line interference includes

not only the fundamental frequency $f_{\text{mains}} = 50$ Hz but also its higher harmonics, the filter structure presented in this paper suppresses three frequencies in the ECG signal spectrum, namely $f_0 = 50$ Hz, $f_1 = 100$ Hz, and $f_2 = 150$ Hz. This evaluation was performed using a signal from the MIT-BIH database sampled at $f_s = 360$ Hz. For the FIR notch filter, the one-percent step response time is 180 samples. Accordingly, the length of the resulting vector of nonzero initial conditions L was set to 180. In order to maintain filtration quality, the value of L should be selected from among the values presented in Equation (21):

$$L = \frac{f_s}{f_n} j, \quad j = 1, 2, 3, \dots \quad (21)$$

To assess filtration quality and compare different filter structures, two metrics were used in this paper: Mean Square Error (MSE) and signal-to-noise ratio improvement (SNR_{imp}). These indicators are commonly used in the literature for evaluating filtration performance [30]. The MSE is defined in Equation (11), while the SNR improvement is given by the following formula:

$$\text{SNR}_{\text{imp}} = 10 \cdot \log_{10} \left(\frac{\sum_{i=1}^N (x(i) - c(i))^2}{\sum_{i=1}^N (y(i) - c(i))^2} \right), \quad (22)$$

This allows for a comparison of the level of the desired signal to the level of background noise, with a higher value indicating a lower noise level. Similarly, a smaller MSE corresponds to a shorter filter transient duration. The results of these calculations are presented in Table 2.

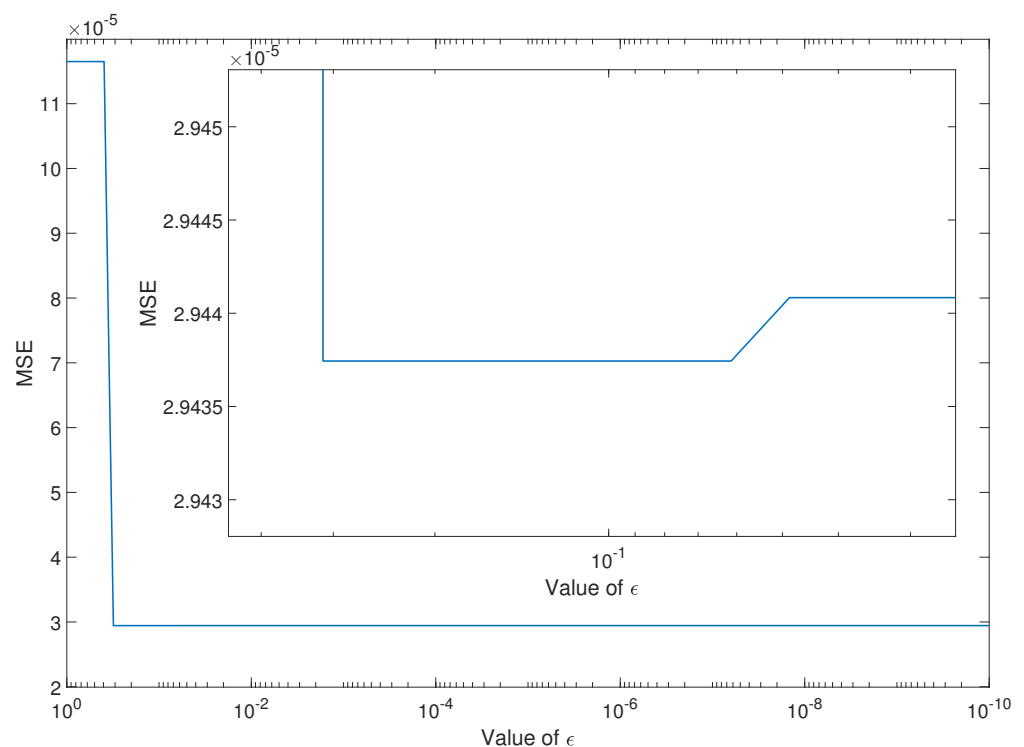


Figure 3. MSE value as a function of the ϵ coefficient.

Table 1. Parameters of Multinotch FIR filter used in computer simulations.

Parameter	Value
Frequencies to be suppressed	$f_1 = 50$ Hz, $f_2 = 100$ Hz, and $f_3 = 150$ Hz
Sample frequency	$f_s = 360$ Hz
Prototype IIR filter pole radius	0.97
Settling time (1%) of IIR filter	1.9917 s
Settling time (1%) of FIR filter based on IIR filter prototype	0.4416 s
Value of coefficient ϵ	$10^{-0.4123(23)}$
Length of nonzero initial conditions vector L	180
Trim value p	$10^{-4.7298(98)}$
Order of FIR filters with 50, 100 and 150 Hz center frequencies	512
Order of FIR filter with 50 Hz center frequency and reduced order	263
Order of FIR filter with 100 Hz center frequency and reduced order	262
Order of FIR filter with 150 Hz center frequency and reduced order	260

Table 2. Filtration quality factor values for the signals shown in Figure 6.

Filtering Quality Index	Second Order IIR Prototype Filter	FIR Filter with Zero Initial Conditions ¹	FIR Filter with Nonzero Initial Conditions ¹	FIR Filter with Nonzero Initial Conditions and Reduced Filters Orders ²
Mean Square Error	50.660225	0.03506231	0.00072380	0.00072383
Signal-To-Noise Ratio Improvement	−15.28575	16.3125	33.1647	33.1645

¹ Filter orders $N_{f_0} = N_{f_1} = N_{f_2} = 512$. ² Reduced filter orders $N_{f_0} = 263$ $N_{f_1} = 262$ $N_{f_2} = 260$.

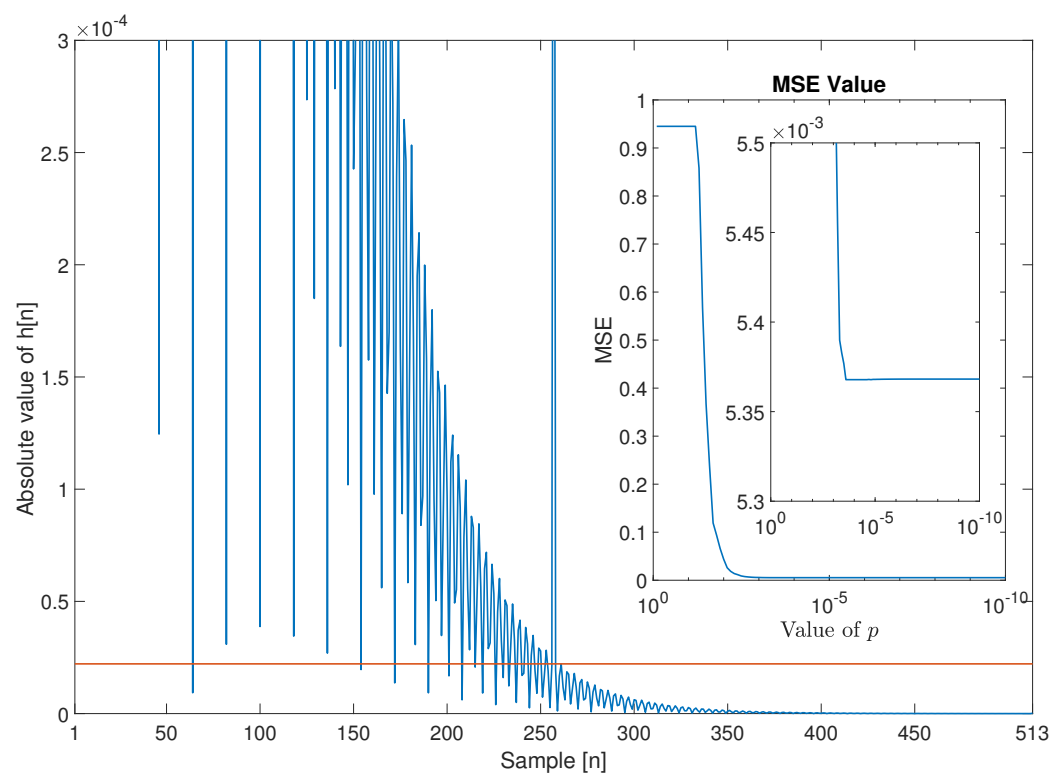


Figure 4. Absolute value of impulse response of the FIR notch filter ($\omega = 2\pi \frac{50}{360} \frac{\text{rad}}{\text{s}}$, $r = 0.97$). The blue line represents the impulse response, the red line indicates the truncation value p and the MSE is shown as a function of p .

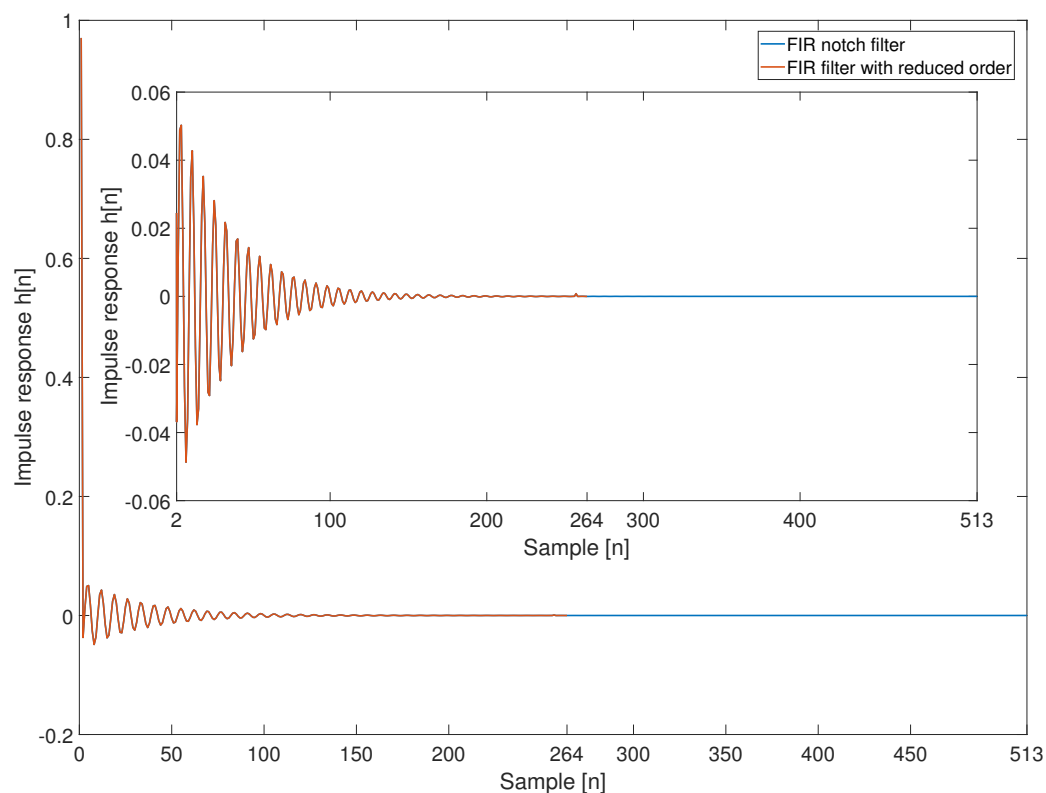


Figure 5. Comparison of the impulse response of a full-order notch filter and a reduced-order notch filter.

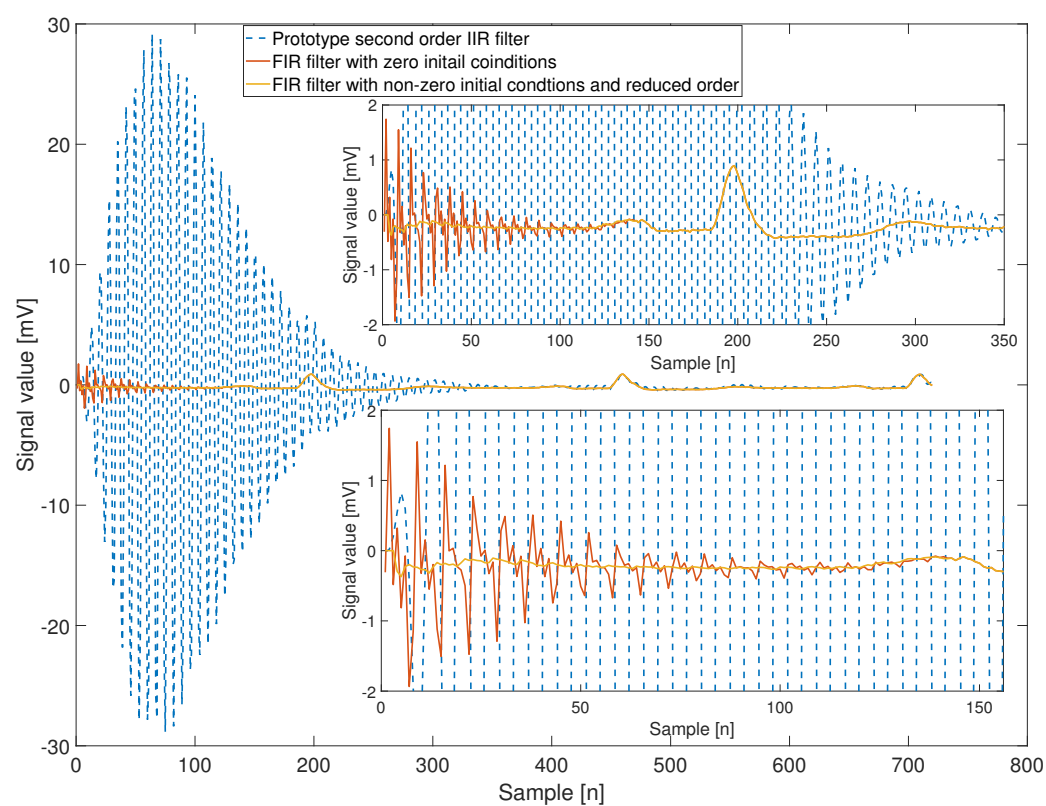


Figure 6. Simulation results of ECG signal denoising with interference at frequencies $f = [50, 100, 150]$ Hz.

4. Discussion

It should be noted that introducing nonzero initial conditions increases the computational requirements for calculating the filter response. However, the benefits of using nonzero initial conditions, particularly in terms of filtration quality, outweigh this drawback. For very short-duration signals, such as seismic signals, evoked potentials, or measurements of transient states or burst signals in electronic systems, the most common approach to interference removal is multiple recordings of the signal followed by averaging. This method is effective for random interference, as described in [18], but it fails for periodic interference, which cannot be removed using averaging. In such cases, the signal must be recorded and processed later, introducing significant delays in signal processing due to the inability to filter in real time.

The iterative determination of nonzero initial conditions presented in this paper enables a substantial acceleration of the filtering process and allows removal of disturbances with known frequencies immediately during signal acquisition. This method allows filtering to begin after recording only the first two signal samples, with delay limited solely by the measurement path. As shown in Figure 6, the transient state is sufficiently reduced to allow the characteristic waveforms of the test signal to be distinguished. The ECG signal was used in this study to facilitate visual assessment of filtration quality. The filtration quality indicators confirm the effectiveness of the proposed method in limiting the influence of the transient state on the signal waveform.

Another technique presented in this paper is the reduction of the FIR filter order by setting the last transformation coefficients with values below a specified threshold to zero. As expected, this slightly reduces filtration quality, but the changes are minor and can be considered negligible upon visual inspection of the test signal waveforms. This approach significantly reduces the computational complexity of the filter structure, lowers the memory required to store the filter coefficients in a microcontroller, and decreases the time needed to compute the filter output.

The results presented in Table 2 show the filtration quality metrics for both the full-order filter and the reduced-order filter obtained by zeroing the coefficients below the threshold. Figure 7 illustrates the difference in responses between the full-order filter ($N = 1536$) and the reduced-order filter ($N_t = 765$). The frequency characteristics of the full-order and reduced-order filters are shown in Figure 8. As one can see, the application of the order reduction method proposed in this paper resulted in a deterioration of attenuation in the stopband, without a significant change in the stopband width. At the expense of poorer interference attenuation, we achieved a reduction in computational cost and a limitation of the filter's transient state. For most applications, the attenuation in the stopband obtained using the proposed method is acceptable and sufficient.

Table 3 compares the computational cost per output signal sample for IIR filters, FIR filters with zero initial conditions, FIR filters with nonzero initial conditions, and the FIR filter structure proposed in this paper. It should be noted that classical IIR and FIR filters are characterized by a computational complexity of $O(n)$. However, the initial samples of the filter output are affected by a transient state, which, in the case of notch filters, depends on the pole radius r for IIR filters or on the filter order for FIR filters. To mitigate the transient response, the use of nonzero initial conditions has been proposed. This approach requires a one-time determination of the matrix $\mathbf{A}$ or a set of matrices $\mathbf{A}_m$, followed by multiplication of the filter input sample vector by these matrices. The determination of the matrices $\mathbf{A}$ and $\mathbf{A}_m$ is performed once during the filter synthesis stage and constitutes a computational problem of complexity $O(L^3)$. Importantly, this computation may be carried out offline on any device and does not need to be performed on the target hardware implementing the filter. The determination of the vector of nonzero initial conditions has a

computational complexity of $O(L^2)$. In return, the one-percent settling time of the filter is reduced to zero samples. It should be noted that the direct introduction of a nonzero initial condition vector introduces a delay equal to its length. Therefore, an iterative determination method is proposed in this paper, in which only two multiplications and one addition are required to compute the first output sample. Although the computational cost increases to 180 multiplications and 179 additions per sample for the L -th output sample, the proposed approach effectively mitigates the impact of transient states on the filter response. At the same time, in order to further reduce the computational cost of the FIR filter, a reduction of the filter order is proposed. This reduction does not affect the selectivity of the filter and only affects the attenuation in the stopband. The presented simulation results confirm the negligible impact of order reduction on the frequency characteristics in terms of selectivity and demonstrate the possibility of reducing the computational cost of the FIR filter. It should also be emphasized that information processing time is a critical issue in real-time systems. With increasing computational capabilities, modern devices integrate an ever-growing number of sensors. In many applications, such as walking robots or autonomous vehicles, delays in signal processing may lead to incorrect control actions and, consequently, to hazardous situations. Furthermore, many events in such systems occur only once, which limits the applicability of averaging techniques. With increasing sampling rates, increasingly shorter signal segments are recorded, often requiring immediate processing. At the same time, the volume of acquired data continues to grow, necessitating the fastest possible signal processing, particularly for interference removal. Classical filtering algorithms do not guarantee correct signal reproduction following abrupt signal changes due to transient effects. Therefore, the development of fast filtering algorithms that reduce or eliminate the impact of transient states is essential from the perspective of technological advancement.

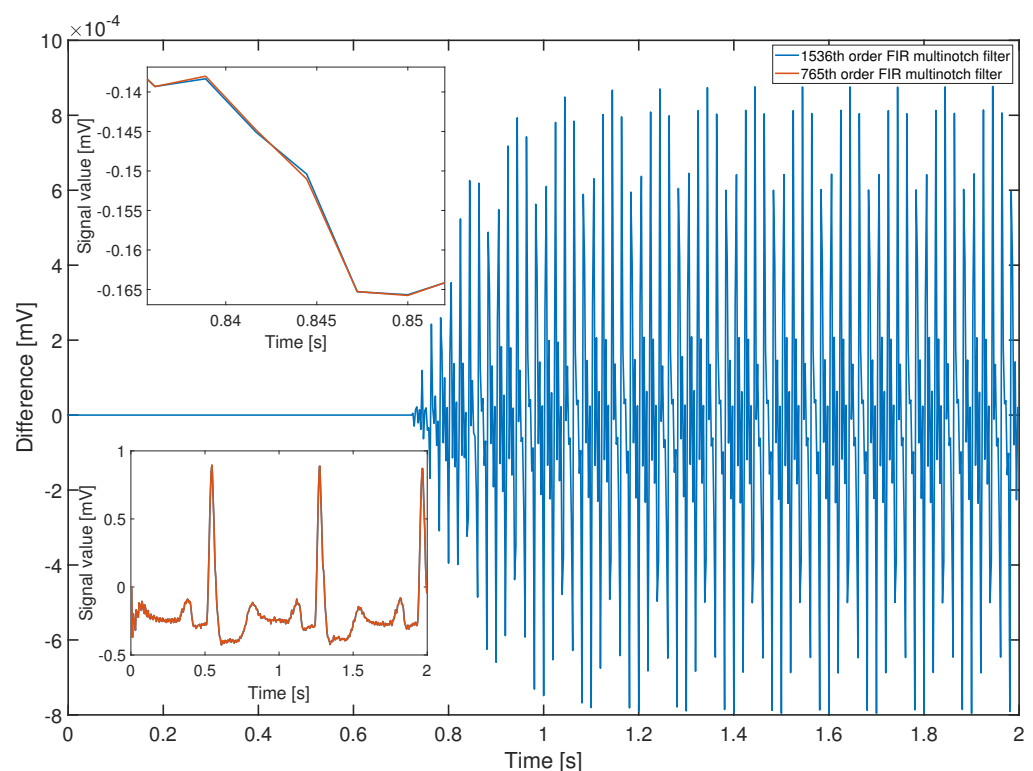


Figure 7. Difference between the responses of the 1536-order and 765-order FIR filters to a test signal, showing the overall filter response and a close-up of the differences.

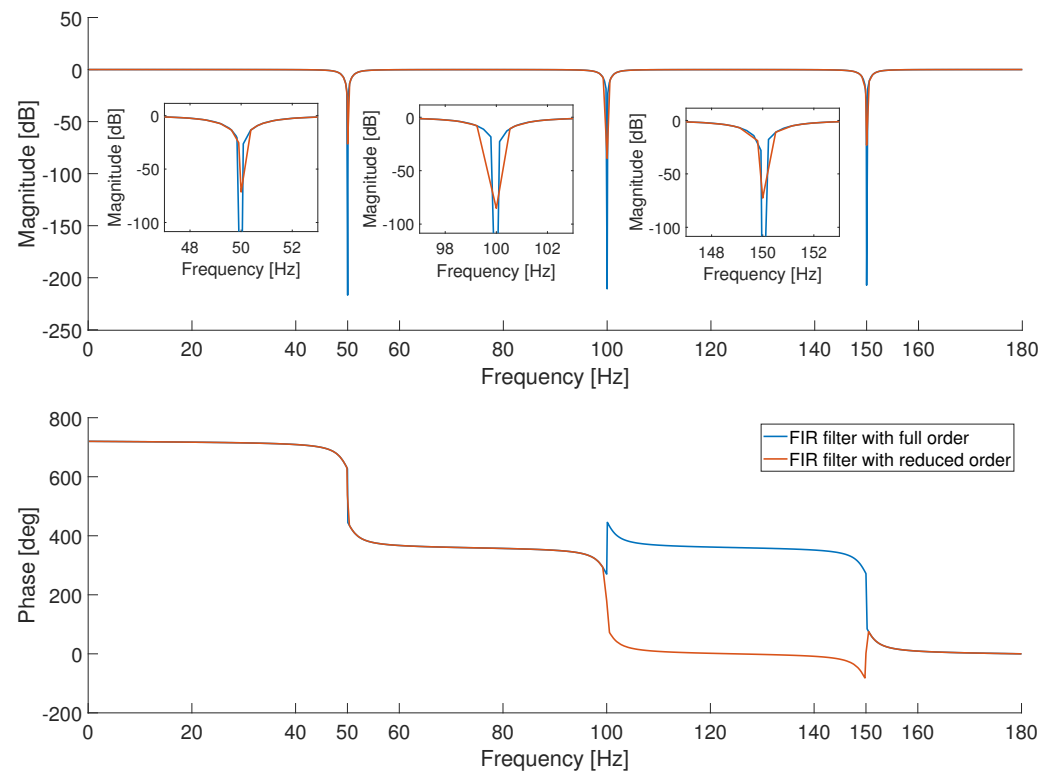


Figure 8. Comparison of Bode characteristics between the full-order filter and the reduced-order filter.

Table 3. Computational cost per sample.

Filter Structure Type	Multiplication	Additions	Transient Duration (1%)	Delay
IIR	$2N_{IIR} + 1$	$2N_{IIR}$	Depends on pole radius r	0 samples (output affected by transient response)
IIR prototype filter	13	12	717 samples	
FIR	$iN_{FIR,i}$	$i(N_{FIR,i} - 1)$	Depends on filter order N	0 samples (output affected by transient response)
Numerical example:	1536	1535	159 samples	
FIR with nonzero initial conditions	First L samples: L From sample $L + 1$: $iN_{FIR,i}$	First L samples: $(L - 1)$ From sample $L + 1$: $i(N_{FIR,i} - 1)$	0 samples	L samples
Numerical example:	First L samples: 180 From sample $L + 1$: 1536	First L samples: 179 From sample $L + 1$: 1535	0 samples	180 samples
FIR proposed in article	First L samples: $k, k = 2, \dots, L$ From $L + 1$ sample: $i(N_{FIR,i}^{reduced})$	First L samples: $(k - 1), k = 2, \dots, L$ From sample $L + 1$: $i(N_{FIR,i}^{reduced} - 1)$	0 samples	2 samples
Numerical example:	First L samples: 2, 3, ..., 180 From sample $L + 1$: 765	First L samples: 1, 2, ..., 179 From sample $L + 1$: 764	0 samples	2 samples

$i = 0, 1, 2, \dots$ —the number of frequencies that the filter is intended to suppress; N_{IIR} —order of IIR filter; $N_{FIR,i}$ —order of i -th FIR filter; $N_{FIR,i}^{reduced}$ —order of i -th FIR filter with reduced order; L —length of nonzero initial condition vector.

5. Conclusions

This paper presents a method for reducing the effect of transients on filter response, which is particularly important when filtering very-short-duration signals contaminated by a prototype IIR filter, where the transient response depends only on the filter coefficients and the initial input signal samples. The use of iterative determination of nonzero initial conditions further reduced the filter transient. Additionally, the filter order was reduced by

zeroing coefficients with minimal impact on the output signal. These methods significantly reduced the transient, which is critical for filtering short-duration signals, while also decreasing computational requirements without compromising filtration quality. Simulations confirmed the effectiveness of the proposed filtering algorithm in removing power-line interference, including both the fundamental frequency and higher harmonics. Comparisons demonstrated that this method outperforms classical filtering algorithms with zero initial conditions, both IIR and FIR. The presented filtration quality metrics validate the approach.

In future work, the authors plan to focus on determining nonzero initial conditions for IIR filter structures and on the hardware implementation of the proposed algorithms.

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Abbreviations

The following abbreviations are used in this manuscript:

ECG	electrocardiogram
IIR	infinite impulse response
FIR	finite impulse response
MSE	mean square error
SNR_{imp}	signal-to-noise improvement

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