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Online Voltage Control for Active Distribution Grids via Measurement Feedback Correction

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Abstract

The increasing penetration of Distributed Energy Resources (DERs) in active distribution networks introduces significant voltage volatility. Traditional model-based control strategies often struggle to maintain voltage stability due to accurate parameter unavailability and time-varying topology. To address these challenges, this paper proposes a robust Measurement-Feedback Online Gradient Descent (MF-OGD) algorithm for real-time voltage regulation. Unlike conventional methods that rely on explicit network models, the proposed MF-OGD approach leverages real-time voltage measurements to correct gradient estimation errors, thereby implicitly compensating for both parametric mismatches and structural linearization inaccuracies. We provide rigorous theoretical guarantees for closed-loop stability and asymptotic tracking error under bounded disturbances. Furthermore, the framework is extended to a joint active–reactive power control scheme to ensure feasibility under severe operating conditions. Comprehensive simulations on the IEEE 33-bus and IEEE 69-bus standard test feeders validate the scalability and effectiveness of the proposed method. Numerical results demonstrate that the MF-OGD controller successfully maintains nodal voltages within the safety range, limiting the maximum voltage deviation to 0.022 p.u. even under 50% model parameter uncertainty. Additionally, the algorithm achieves a low tracking Root Mean Square Error (RMSE) of approximately 0.014 p.u. in the 69-bus system. Notably, the accumulated regret per node increases only marginally (from 0.032 to 0.038) as the network scale doubles, confirming the algorithm’s superior scalability and robustness compared to conventional open-loop baselines.

Keywords: active distribution grid; voltage control; measurement feedback; online; model mismatch



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1. Introduction

Distribution networks are currently experiencing a paradigm shift toward active system management, precipitated by the accelerating penetration of Distributed Energy Resources (DERs), such as electric vehicles and photovoltaic (PV) systems [1]. This transition necessitates advanced control strategies that can handle the inherent uncertainties and variability of renewable generation. While advantageous for decarbonization, this integration introduces bidirectional power flows that induce significant volatility in voltage profiles [2]. Specifically, the stochastic mismatch between generation and demand can

precipitate severe voltage violations, which not only compromise power quality but also accelerate equipment degradation. Although the cardinal operational objective remains restricting nodal voltages to the nominal 0.95–1.05 p.u. range to optimize efficiency, achieving this via traditional control paradigms is becoming increasingly untenable. The fundamental challenge stems from the reliance on explicit network models. Distribution grids are often characterized by unmonitored topology changes, aging infrastructure, and significant parameter uncertainties, rendering accurate modeling unfeasible. To circumvent model dependency, it is imperative to develop data-driven strategies capable of regulating voltage through real-time measurement feedback. We assume the active distribution network is observable via Phasor Measurement Units (PMUs) or micro-PMUs installed at critical nodes, providing synchronized voltage magnitude measurements [3].

Seminal research in voltage control for distribution networks focused primarily on centralized, model-based methods utilizing offline optimization and rule-based heuristics. For instance, Farivar et al. [4] proposed a centralized Optimal Power Flow (OPF) framework for reactive power dispatch, demonstrating effective regulation through convex relaxations of the nonlinear AC power flow equations [5]. These studies generally predicated on the physics defined by the branch flow equations [6], utilizing them to quantify voltage sensitivities and formulate loss minimization problems subject to network constraints. However, recognizing the limitations of centralized communication, Bolognani et al. [7] advanced a distributed reactive power feedback mechanism. By prioritizing local sensory data, this approach offered a robust alternative for voltage regulation and loss reduction, effectively mitigating the heavy communication burden associated with fully centralized architectures.

In response to these challenges, recent scholarly efforts have shifted toward online optimization frameworks that integrate feedback mechanisms to enhance robustness [8,9]. Hauswirth et al. [10] devised an algorithm operating directly on the power flow manifold, leveraging the geometric structure of power systems; by employing projected gradient descent with partial measurements, they achieved convergence to optimality without requiring full model transparency. This methodology was subsequently refined by Bernstein et al. [11], who integrated primal-dual updates to rigorously enforce operational constraints, thereby demonstrating superior voltage regulation in IEEE test feeders. Theoretically, these approaches are grounded in the principles of extremum-seeking control [12], offering solid guarantees regarding dynamic regret and stability even under bounded disturbances. Building upon this theoretical foundation, recent scholarship has expanded the Online Feedback Optimization (OFO) paradigm to address specific grid complexities: Ortmann et al. [13] focused on distributed implementations for real-time renewable adaptation, while Dall’Anese et al. [14] incorporated stochastic gradients to manage uncertainty in multi-phase unbalanced networks. Further diversifying the field, Tang et al. [15] introduced hierarchical coordination structures and machine-learning-aided sensitivity estimation, respectively, pushing the boundaries of scalability and model independence in high-DER environments.

Nevertheless, these recent advancements encounter critical impediments that hinder their practical deployment. Many approaches, such as those in [11], and [13], assume negligible linearization errors from approximated power flow models (e.g., LinDistFlow). These errors can grow quadratically during high-loading conditions, leading to persistent voltage deviations. Additionally, they often require an approximate model for gradient estimation, rendering them sensitive to structural mismatches, as evidenced by regret bounds that scale with model inaccuracy in [14] and [15]. While recent studies like [16] by Zhan et al. and [17] by Wang et al. have attempted to mitigate delays and coupled constraints in distributed OFO, they still overlook comprehensive robustness to both parametric and structural errors. These existing gaps motivate the need for a more robust framework that

explicitly accounts for both parametric uncertainties and structural nonlinearities through pure measurement feedback.

To clarify the positioning of our work, we compare the proposed MF-OGD framework with representative existing strategies in Table 1.

Table 1. Comparison of proposed MF-OGD with representative strategies.

Method Category	References	Years	Model Dependency	Linearization Error	Stability Proof	Training Free?
Classic OPF	[4]	2012	High	Ignored	N/A	Yes
Distributed OFO	[11,16,18]	2019–2025	Moderate	Ignored	Yes	Yes
DRL-based Control	[19–21]	2024–2025	None	Implicit	No	No
Event-Triggered	[22–24]	2022–2024	High *	Ignored	Yes	Yes
Proposed MF-OGD	This Work	2026	None **	Compensated	Yes	Yes

* Event-triggered/multi-timescale methods often require accurate models for the baseline scheduling layer. ** Uses approximate model direction but relies on measurement feedback for correction.

This paper addresses these limitations by proposing a Measurement-Feedback Online Gradient Descent (MF-OGD) algorithm for voltage control in active distribution networks. The proposed approach computes an approximate gradient solely from real-time voltage measurements, thereby eliminating the dependence on precise network models while explicitly correcting for linearization errors and model mismatches. By integrating a robust update rule with feasibility projections, the algorithm tracks time-varying optimal reactive power dispatches to minimize voltage deviations and operational costs.

The main contributions of this work are summarized as follows:

1. We provide rigorous theoretical analyses, including stability conditions, asymptotic tracking error bounds, and dynamic regret guarantees. Crucially, our analysis explicitly incorporates model mismatches and nonlinear power flow errors, advancing beyond the simplifying assumptions prevalent in the prior literature.
2. We extend the framework to a joint active–reactive power control scheme, introducing curtailment as an auxiliary variable. This ensures algorithmic feasibility and system safety even under severe operating conditions where reactive power capacity is exhausted.
3. Comprehensive simulations on a radial network validate the superiority of the proposed algorithm. The results demonstrate near-optimal performance and significant robustness against uncertainties, outperforming traditional open-loop methods in scenarios where the latter fail.

The remainder of this paper is organized as follows. Section 2 presents the system model, problem formulation, and error analysis. Section 3 details the MF-OGD algorithm design, stability, and tracking performance. Section 4 extends the analysis to joint control and dynamic regret. Section 5 provides simulation results, and Section 6 concludes the paper.

2. System Model and Problem Formulation

2.1. Network Preliminaries

Consider a radial distribution network represented by a connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} := \{0, 1, \dots, N\}$ denotes the set of buses (nodes) and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ denotes the set of distribution lines (edges). While physical power lines are undirected, for the purpose of modeling power flow direction and voltage drops in the DistFlow formulation, we treat $\mathcal{G}$ as a directed tree graph rooted at the substation (Node 0). Node 0 represents the substation (slack bus) connected to the main grid, with a fixed voltage magnitude v_0 . Let

$\mathcal{N}^+ := \mathcal{N} \setminus \{0\}$ denote the set of active nodes excluding the substation. For each line $(i, j) \in \mathcal{E}$, let r_{ij} and x_{ij} denote the resistance and reactance of the line, respectively.

2.2. Linearized Power Flow Model

The complex power flow in the distribution network is governed by the nonlinear AC power flow equations. However, due to the high R/X ratio in distribution networks and the requirement for convexity in optimization, we adopt the widely used linearized DistFlow (LinDistFlow) model [6,25]. This linearization approach has been widely validated and extended through convex relaxation techniques [5].

Let P_{ij} and Q_{ij} be the active and reactive power flows from node i to node j , and let v_i denote the squared voltage magnitude at node i (i.e., $v_i = |V_i|^2$). For each node $j \in \mathcal{N}^+$, let p_j and q_j represent the net active and reactive power injections. The simplified power flow equations are given by:

$$P_{ij} = \sum_{k:(j,k) \in \mathcal{E}} P_{jk} - p_j, \quad (1a)$$

$$Q_{ij} = \sum_{k:(j,k) \in \mathcal{E}} Q_{jk} - q_j, \quad (1b)$$

$$v_j = v_i - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}), \quad \forall (i, j) \in \mathcal{E}. \quad (1c)$$

Let $\mathcal{P}_i$ denote the unique path from the substation (Node 0) to Node i . Summing the voltage drops along $\mathcal{P}_i$ yields:

$$v_i = v_0 - \sum_{(j,k) \in \mathcal{P}_i} (r_{jk}P_{jk} + x_{jk}Q_{jk}) \quad (2)$$

The branch flows P_{jk}, Q_{jk} are themselves the sum of nodal injections downstream of line (j, k) . By changing the order of summation, the voltage at node i can be expressed as a linear combination of all nodal injections $\mathbf{p}, \mathbf{q}$

$$\mathbf{v} \approx \mathbf{v}_0 + \mathbf{R}\mathbf{p} + \mathbf{X}\mathbf{q} \quad (3)$$

where $\mathbf{v} = [v_1, \dots, v_N]^\top \in \mathbb{R}^N$ is the vector of squared voltage magnitudes; $\mathbf{p}, \mathbf{q} \in \mathbb{R}^N$ are the vectors of net active and reactive power injections. The elements of the sensitivity matrices are derived explicitly from the network topology. For any two nodes $i, j \in \mathcal{N}$, the entry R_{ij} (and similarly X_{ij}) is given by [26,27]:

$$R_{ij} = \sum_{(k,l) \in \mathcal{P}_i \cap \mathcal{P}_j} r_{kl}, \quad X_{ij} = \sum_{(k,l) \in \mathcal{P}_i \cap \mathcal{P}_j} x_{kl} \quad (4)$$

where $\mathcal{P}_i \cap \mathcal{P}_j$ represents the common path from the substation to the common ancestor of nodes i and j . This formulation clarifies that $\mathbf{R}$ and $\mathbf{X}$ are symmetric, positive definite matrices (for radial networks) capturing the cumulative impedance effect.

Remark 1 ((Model Mismatch)). *In practical engineering scenarios, the exact impedance values r_{ij} and x_{ij} are often unknown or inaccurate due to aging lines or lack of data. Consequently, the sensitivity matrix $\mathbf{X}$ used in the controller design (denoted as $\tilde{\mathbf{X}}$) differs from the true physical matrix $\mathbf{X}_{true}$. This necessitates a robust or feedback-based control strategy.*

2.3. Error Analysis of Linearization

Although the LinDistFlow model (1) provides a convex approximation, it neglects the higher-order terms related to line losses (i.e., the quadratic terms of current magnitude).

To rigorously justify the necessity of the proposed measurement-feedback framework, we explicitly characterize the linearization error.

Let the exact voltage magnitude squared be denoted by $\mathbf{v}^{exact}$. The relationship can be written as:

$$\mathbf{v}_{exact} = \mathbf{R}\mathbf{p} + \mathbf{X}\mathbf{q} + \mathbf{v}_0 + \boldsymbol{\varepsilon}_{loss}(\mathbf{p}, \mathbf{q}), \quad (5)$$

where $\boldsymbol{\varepsilon}_{loss}$ represents the voltage drop caused by active and reactive power losses on the lines. Specifically, for a line (i, j) , the neglected term is roughly proportional to $r_{ij} \frac{P_{ij}^2 + Q_{ij}^2}{v_i}$.

Lemma 1 (Bounded Linearization Error). *Under the assumption that the system operates within normal voltage limits (e.g., $0.95 \leq v_i \leq 1.05$) and line currents remain within thermal limits, the linearization error is bounded by a quadratic function of the power injections:*

$$\|\boldsymbol{\varepsilon}_{loss}\|_{\infty} \leq \kappa(\|\mathbf{p}\|^2 + \|\mathbf{q}\|^2), \quad (6)$$

where κ is a constant depending on the network topology and maximum line impedance.

Proof. See Appendix B.

□

This lemma highlights a critical insight: while the error is small under light loading, it grows quadratically during peak load/generation periods [1,28]. Standard open-loop optimization using (3) would inherit this error $\boldsymbol{\varepsilon}_{loss}$, leading to voltage violations. This mathematical fact necessitates the use of real-time measurement feedback, which captures the true physics ($\mathbf{v}_{exact}$) implicitly, effectively compensating for $\boldsymbol{\varepsilon}_{loss}$.

2.4. DER Modeling and Constraints

We assume that a subset of nodes $\mathcal{N}_G \subseteq \mathcal{N}^+$ are equipped with Distributed Energy Resources (DERs), such as photovoltaic (PV) inverters, which can provide reactive power support. For a node $i \in \mathcal{N}_G$, the net power injections are defined as:

$$p_{i,t} = p_{g,i,t} - p_{c,i,t}, \quad (7)$$

$$q_{i,t} = q_{g,i,t} - q_{c,i,t}, \quad (8)$$

where subscripts g and c denote generation and consumption, respectively. The active power generation $p_{g,i,t}$ is determined by solar irradiance and is assumed to be uncontrollable (or operated at Maximum Power Point Tracking). The reactive power capability of inverters is constrained by their apparent power rating [29].

The operation of the inverter is constrained by its apparent power capacity S_i . The feasible set for the reactive power control variable at time t , denoted as $\mathcal{Q}_t$, is defined as:

$$\mathcal{Q}_t := \left\{ \mathbf{q}_g \in \mathbb{R}^N \mid |q_{g,i}| \leq \sqrt{S_i^2 - p_{g,i,t}^2}, \forall i \in \mathcal{N}_G \right\}. \quad (9)$$

Note that this set is convex and time-varying, as it depends on the instantaneous active power generation $p_{g,i,t}$.

2.5. Optimization Problem Formulation

The objective of the voltage regulation strategy is to maintain the nodal voltages close to a nominal reference value (e.g., $v_{ref} = 1.0$ p.u.) while minimizing the associated power losses or control costs. We formulate this as a time-varying optimization problem.

At each time step t , the optimal reactive power dispatch $\mathbf{q}_t^*$ is obtained by minimizing a composite cost function:

$$\min_{\mathbf{q}_t \in \mathcal{Q}_t} f_t(\mathbf{q}_t) := \frac{1}{2} \|\mathbf{v}_t(\mathbf{q}_t) - \mathbf{v}_{ref}\|_2^2 + \frac{\eta}{2} \|\mathbf{q}_t\|_2^2, \quad (10)$$

subject to the approximate physical power flow model (3). Substituting (3) into the objective function, the cost function can be rewritten explicitly in terms of the decision variable $\mathbf{q}_t$:

$$f_t(\mathbf{q}_t) = \frac{1}{2} \|\mathbf{R}\mathbf{p}_t + \mathbf{X}\mathbf{q}_t + v_0\mathbf{1} - \mathbf{v}_{ref}\|_2^2 + \frac{\eta}{2} \|\mathbf{q}_t\|_2^2, \quad (11)$$

where $\eta > 0$ is a regularization parameter that penalizes excessive reactive power usage and improves the numerical condition of the problem.

Remark 2 (Handling Rank Deficiency). *The voltage sensitivity matrix $\tilde{\mathbf{X}}$ in distribution grids can be ill-conditioned or rank-deficient. The introduction of the Tikhonov regularization term $\frac{\eta}{2} \|\mathbf{q}_t\|_2^2$ in the objective function (11) modifies the effective Hessian to $G = \tilde{\mathbf{X}}^\top \tilde{\mathbf{X}} + \eta \mathbf{I}$. For any $\eta > 0$, G becomes positive definite and full-rank, ensuring that the gradient update direction is always unique and bounded, thus structurally resolving rank deficiency.*

2.6. Research Goal

Since the problem (11) is strictly convex with respect to $\mathbf{q}_t$, a unique global optimum exists. However, solving this problem in a traditional centralized manner faces two major challenges:

1. **Time-Varying Nature:** The optimal solution $\mathbf{q}_t^*$ drifts rapidly due to the fluctuations in $\mathbf{p}_t$ (solar and load). Offline calculation is computationally prohibitive for real-time application.
2. **Parameter Uncertainty:** The exact sensitivity matrix $\mathbf{X}$ in (11) is typically unavailable. Using an estimated matrix $\tilde{\mathbf{X}}$ in an open-loop optimization leads to steady-state voltage deviations.

Therefore, the goal of this study is to design an online feedback optimization algorithm that iteratively updates $\mathbf{q}_t$ to track the optimal trajectory $\mathbf{q}_t^*$ using real-time voltage measurements, without requiring accurate knowledge of the network impedance $\mathbf{X}$.

3. Measurement-Feedback Online Optimization Algorithm

3.1. Algorithm Design

To solve the optimization problem (10) in real-time, we propose a gradient-based online algorithm. The standard gradient descent update rule for minimizing $f_t(\mathbf{q}_t)$ would be:

$$\mathbf{q}_{t+1} = \mathcal{P}_{\mathcal{Q}_t}[\mathbf{q}_t - \alpha \nabla f_t(\mathbf{q}_t)], \quad (12)$$

where $\alpha > 0$ is the step size and $\mathcal{P}_{\mathcal{Q}_t}[\cdot]$ denotes the Euclidean projection onto the feasible set $\mathcal{Q}_t$. Based on (11), the exact gradient is given by:

$$\nabla f_t(\mathbf{q}_t) = \mathbf{X}^\top (\mathbf{v}_t(\mathbf{q}_t) - \mathbf{v}_{ref}) + \eta \mathbf{q}_t. \quad (13)$$

However, computing this gradient requires the exact system voltage $\mathbf{v}_t(\mathbf{q}_t)$ derived from the model $\mathbf{v} = \mathbf{R}\mathbf{p} + \mathbf{X}\mathbf{q} + v_0$. As stated in Remark 1, the true sensitivity matrix $\mathbf{X}$ is unknown, and only an approximate model $\tilde{\mathbf{X}}$ is available. Using $\tilde{\mathbf{X}}$ to predict voltages leads to model-mismatch errors.

To overcome this, we propose a Measurement-Feedback Online Gradient Descent (MF-OGD) algorithm. The use of measurement feedback for real-time control is motivated by recent advances in online optimization [17] and measurement-based control strategies [30]. We replace the model-predicted voltage deviation with the actual measured voltage de-

violation obtained from the physical system. The flow chart of the proposed algorithm is demonstrated in Figure 1.

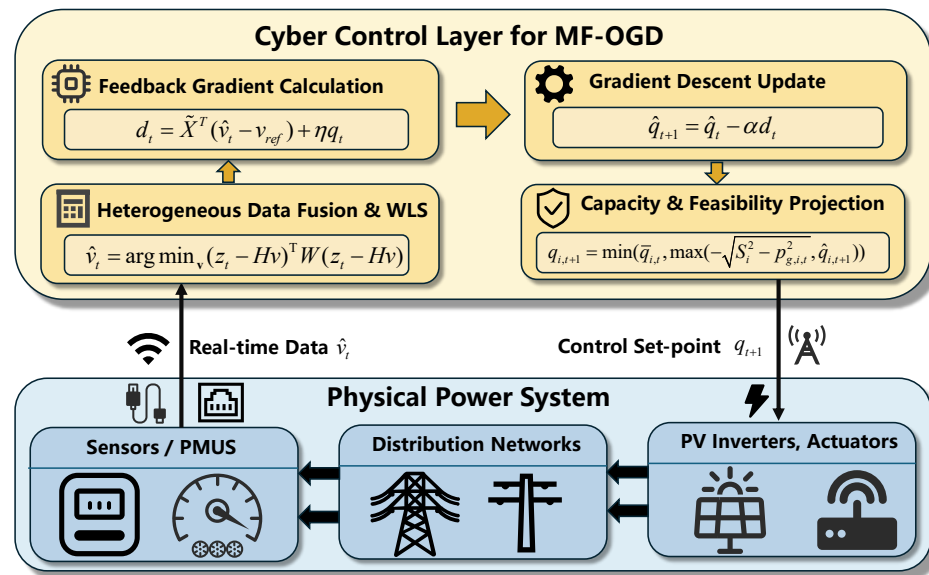


Figure 1. Schematic diagram of the proposed MF-OGD implementation framework.

As illustrated in Figure 1, the proposed MF-OGD algorithm operates within a closed-loop Cyber-Physical System (CPS) architecture, integrating theoretical optimization with physical grid operations. The control cycle initiates in the Physical Power System, where real-time nodal voltage magnitudes ($v_{\text{meas},t}$) are captured by distributed sensors or Phasor Measurement Units (PMUs). This measurement data is transmitted via communication infrastructure to the Cyber Control Layer, which serves as the computational core.

Upon receiving the feedback, the cyber layer executes a three-stage update process. First, the Feedback Calculation module computes an approximate gradient $\hat{\nabla} f_t$ by utilizing the estimated sensitivity matrix $\tilde{X}$ corrected by the instantaneous voltage deviation ($v_{\text{meas},t} - v_{\text{ref}}$). Subsequently, the Gradient Descent module updates the reactive power dispatch using a predefined step size α , driving the system towards the optimal operating point. To ensure physical safety, the Feasibility Projection module strictly confines the updated set-points q_{t+1} within the inverter's capacity limits defined by the apparent power S_i and active power $P_{g,i,t}$. Finally, these optimal set-points are transmitted back to the physical layer, where PV inverters (actuators) execute the dispatch command, thereby altering the power flow in the distribution network and completing the feedback loop. This continuous interaction ensures that the control decisions are dynamically adjusted based on the actual system state, effectively compensating for parameter uncertainties and model linearization errors.

Let $\mathbf{v}_{\text{meas},t}$ denote the voltage magnitude vector measured at time t after applying control $\mathbf{q}_t$. The *approximate* gradient used for the update is:

$$\hat{\nabla} f_t(\mathbf{q}_t) := \tilde{\mathbf{X}}^T(\mathbf{v}_{\text{meas},t} - \mathbf{v}_{\text{ref}}) + \eta \mathbf{q}_t. \quad (14)$$

Substituting (14) into the update rule, the proposed control law is:

$$\mathbf{q}_{t+1} = \mathcal{P}_{\mathcal{Q}_{t+1}} \left[\mathbf{q}_t - \alpha \left(\tilde{\mathbf{X}}^T(\mathbf{v}_{\text{meas},t} - \mathbf{v}_{\text{ref}}) + \eta \mathbf{q}_t \right) \right]. \quad (15)$$

3.2. Integration with Distribution System State Estimation

In practical distribution networks where pervasive PMU deployment is cost-prohibitive, the full voltage vector $\mathbf{v}$ required for the feedback control cannot be measured directly. To render the proposed method viable under sparse and heterogeneous observability, we employ a Weighted Least Squares (WLS)-based Distribution System State Estimation (DSSE) module.

Let $\mathbf{z}_t \in \mathbb{R}^M$ denote the measurement vector at time t , which comprises heterogeneous sensor data with varying precision: (1) Real-time PMU measurements ($\mathbf{v}_{PMU}$) at critical nodes, characterized by high precision (low noise standard deviation $\sigma_{PMU} \approx 10^{-4}$ p.u.) and low latency. (2) Pseudo-measurements ($\mathbf{P}_{inj}, \mathbf{Q}_{inj}$) derived from AMI or historical load profiles, modeled with significantly higher uncertainty ($\sigma_{AMI} \gg \sigma_{PMU}$). The linearized measurement model is expressed as:

$$\mathbf{z}_t = \mathbf{H}\mathbf{v}_t + \boldsymbol{\xi}_{meas,t} \quad (16)$$

where $\mathbf{H} \in \mathbb{R}^{M \times N}$ is the measurement Jacobian matrix derived from the network topology, and $\boldsymbol{\xi}_{meas,t}$ represents Gaussian measurement noise with covariance matrix $\Sigma_z = \text{diag}(\sigma_1^2, \dots, \sigma_M^2)$.

To optimally fuse these heterogeneous measurements, the estimated voltage vector $\hat{\mathbf{v}}_t$ is obtained by solving the WLS problem:

$$\hat{\mathbf{v}}_t = \arg \min_{\mathbf{v}} (\mathbf{z}_t - \mathbf{H}\mathbf{v})^\top \mathbf{W} (\mathbf{z}_t - \mathbf{H}\mathbf{v}) = (\mathbf{H}^\top \mathbf{W} \mathbf{H})^{-1} \mathbf{H}^\top \mathbf{W} \mathbf{z}_t \quad (17)$$

To prioritize high-fidelity data while maintaining the asymptotic error bound of the controller, we design the weighting matrix $\mathbf{W}$ based on the inverse variance of the measurement errors. Specifically, the diagonal elements are set as:

$$W_{ii} = \frac{1}{\sigma_i^2} = \begin{cases} \sigma_{PMU}^{-2}, & \text{if } i \in \mathcal{I}_{PMU} \\ \sigma_{AMI}^{-2}, & \text{if } i \in \mathcal{I}_{AMI} \end{cases} \quad (18)$$

This weighting strategy guarantees the Minimum Variance Unbiased Estimate (MVUE) for the linear Gaussian model. By minimizing the variance of the estimation error $\mathbf{e}_{est,t} = \hat{\mathbf{v}}_t - \mathbf{v}_{true,t}$, we effectively minimize the equivalent noise bound in the control loop, ensuring that the high precision of sparse PMUs is fully leveraged.

Consequently, the feedback gradient in Equation (14) is modified to use the estimated state:

$$\mathbf{d}_t = \tilde{\mathbf{X}}^\top (\hat{\mathbf{v}}_t - \mathbf{v}_{ref}) + \eta \mathbf{q}_t \quad (19)$$

Specific implementation steps are detailed in Algorithm 1.

Remark 3 (Computational Complexity). *The proposed MF-OGD algorithm maintains high computational efficiency despite the integration of the state estimation module. The WLS step (Step 1) involves solving a system of linear equations. Exploiting the tree topology of distribution networks, the gain matrix $\mathbf{G} = \mathbf{H}^\top \mathbf{W} \mathbf{H}$ exhibits a highly sparse structure. By applying sparse Cholesky factorization with pre-ordering algorithms, the solution can be computed with a complexity of approximately $\mathcal{O}(N)$. The subsequent gradient calculation and projection (Steps 2–4) involve only vector operations and local element-wise comparisons, strictly scaling as $\mathcal{O}(N)$. Consequently, the total computational cost per iteration remains linear with respect to the network size N , ensuring the algorithm's scalability and viability for real-time applications, in distinct contrast to centralized non-convex OPF methods that typically scale as $\mathcal{O}(N^3)$.*

Algorithm 1 Measurement-Feedback Online Gradient Descent Algorithm (MF-OGD).

Input: Step size $\alpha > 0$; Regularization parameter $\eta > 0$; Estimated sensitivity matrix $\tilde{\mathbf{X}}$; Reference voltage $\mathbf{v}_{ref}$.

Output: Control sequence $\{\mathbf{q}_t\}_{t=1}^T$.

- 1: **Initialization:** Set $\mathbf{q}_0 = \mathbf{0}$, $t = 0$.
- 2: **while** system is running **do**
- 3: **Step 1: Heterogeneous Data Fusion**
- 4: 1.1 Collect measurements $\mathbf{z}_t$ from PMUs and AMI.
- 5: 1.2 Construct weighting matrix $\mathbf{W}$ using Equation (18).
- 6: 1.3 Compute state estimate $\hat{\mathbf{v}}_t$ via WLS Equation (17).
- 7: **Step 2: Feedback Gradient Calculation**
- 8: Compute gradient using the fused estimate:
- 9: $\mathbf{d}_t = \tilde{\mathbf{X}}^\top (\hat{\mathbf{v}}_t - \mathbf{v}_{ref}) + \eta \mathbf{q}_t$
- 10: **Step 3: Gradient Descent Update**
- 11: Perform unconstrained update:
- 12: $\hat{\mathbf{q}}_{t+1} = \mathbf{q}_t - \alpha \mathbf{d}_t$
- 13: **Step 4: Feasibility Projection**
- 14: Determine capacity limit: $\bar{q}_{i,t} = \sqrt{S_i^2 - p_{g,i,t}^2}$
- 15: Project $\hat{\mathbf{q}}_{t+1}$ onto feasible set $\mathcal{Q}_t$:
- 16: $q_{i,t+1} = \min(\bar{q}_{i,t}, \max(-\bar{q}_{i,t}, \hat{q}_{i,t+1}))$
- 17: **Step 5: Actuation**
- 18: Send set-points $\mathbf{q}_{t+1}$ to inverters.
- 19: $t \leftarrow t + 1$.
- 20: **end while**

3.3. Stability Analysis and Step-Size Selection

Before analyzing the tracking performance, we must first establish the conditions under which the closed-loop system remains stable despite the model mismatch. This involves selecting an appropriate step size α to ensure the contraction of the error dynamics. The convergence properties of gradient descent methods have been extensively studied in optimization theory [31,32].

Define the error vector as $\mathbf{e}_t = \mathbf{q}_t - \mathbf{q}_t^*$. Neglecting the projection operator locally (valid near the equilibrium) and considering the noise-free case for stability analysis, the error dynamics $\mathbf{e}_t = \mathbf{q}_t - \mathbf{q}_t^*$ evolve as:

$$\begin{aligned}
 \mathbf{e}_{t+1} &= \mathbf{q}_t - \alpha [\tilde{\mathbf{X}}^\top (\mathbf{v}_t - \mathbf{v}_{ref}) + \eta \mathbf{q}_t] - \mathbf{q}_t^* \\
 &= \mathbf{e}_t - \alpha [\tilde{\mathbf{X}}^\top \mathbf{X} \mathbf{e}_t + \eta \mathbf{e}_t] \\
 &= (\mathbf{I} - \alpha \mathbf{G}) \mathbf{e}_t,
 \end{aligned} \tag{20}$$

where $\mathbf{G} := \tilde{\mathbf{X}}^\top \mathbf{X} + \eta \mathbf{I}$ is the Effective Feedback Matrix. Note that $\mathbf{G}$ is generally asymmetric due to the mismatch between the estimated model $\tilde{\mathbf{X}}$ and the true physics $\mathbf{X}$.

Theorem 1 (Stability Condition). *The proposed measurement feedback system is asymptotically stable if and only if all eigenvalues of $\mathbf{G}$ have positive real parts, and the step size α is chosen such that:*

$$0 < \alpha < \frac{2 \cdot \text{Re}(\lambda_{\min}(\mathbf{G}))}{\max_{\lambda \in \sigma(\mathbf{G})} |\lambda|^2}, \tag{21}$$

where $\sigma(\mathbf{G})$ denotes the spectrum of $\mathbf{G}$. Under this condition, the convergence rate is given by the spectral radius $\rho(\mathbf{M}_\alpha) < 1$.

Proof. See Appendix C. $\square$

While Equation (21) is precise, calculating eigenvalues of the unknown matrix $\mathbf{G}$ is impossible. Hence we derive a robust, calculable sufficient condition.

Corollary 1 (Robust Step-Size Selection under Model Uncertainty). *Let $\tilde{\mathbf{G}} := \tilde{\mathbf{X}}^\top \tilde{\mathbf{X}} + \eta \mathbf{I}$ be the estimated feedback matrix calculable by operators. Assume the relative model mismatch is bounded by $\epsilon_{rel} \geq \|\mathbf{X} - \tilde{\mathbf{X}}\|_2 / \|\tilde{\mathbf{X}}\|_2$. The proposed MF-OGD algorithm guarantees asymptotic stability if the step size α is selected as a scaled fraction of the estimated inverse spectral norm:*

$$\alpha = \frac{\gamma}{\|\tilde{\mathbf{G}}\|_2} \approx \frac{\gamma}{\|\tilde{\mathbf{X}}\|_2^2 + \eta}, \quad (22)$$

where $\gamma \in (0, 1)$ is a safety tuning factor satisfying:

$$\gamma < \frac{2(1 - \epsilon_{rel})}{1 + \epsilon_{rel}}. \quad (23)$$

Proof. See Appendix D. $\square$

Remark 4 (Operational Guideline for Parameter Tuning). *Corollary 3.3 bridges the gap between theory and practice by providing a systematic tuning rule that does not require offline system identification: Operators first compute the nominal step size base $\alpha_{nom} = 1/(\|\tilde{\mathbf{X}}\|_2^2 + \eta)$ using the available topology data $\tilde{\mathbf{X}}$. Then we can choose a proper initial factor γ according to engineering tolerance ϵ_{rel} . The validity of γ can be verified online through voltage feedback. This “estimate-and-scale” strategy ensures stability without precise knowledge of $\mathbf{X}$.*

Remark 5 (Geometric Interpretation of Optimal Step Size). *The stability condition (21) ensures that the eigenvalues of the error dynamics matrix $\mathbf{M}_\alpha = \mathbf{I} - \alpha \mathbf{G}$ lie within the unit circle. As highlighted in control theory, the fastest convergence rate is achieved by minimizing the spectral radius $\rho(\mathbf{M}_\alpha) = \max_i |1 - \alpha \lambda_i(\mathbf{G})|$. Geometrically, this corresponds to tuning α to center the spectrum of $\mathbf{M}_\alpha$ around the origin.*

For a perfectly known symmetric system, the optimal step size is $\alpha^ = \frac{2}{\lambda_{\min}(\mathbf{G}) + \lambda_{\max}(\mathbf{G})}$, which balances the decay rates of the fastest and slowest modes. In our robust framework, since the true eigenvalues $\lambda(\mathbf{G})$ are uncertain, we cannot compute α^* precisely. Instead, our proposed tuning rule in Corollary 3.3 targets a conservative centering: by scaling $\alpha \propto 1/\|\tilde{\mathbf{G}}\|_2$ (where $\|\tilde{\mathbf{G}}\|_2 \approx \lambda_{\max}$), we strictly limit the largest eigenvalue displacement to ensure stability ($\rho < 1$), while the safety factor γ provides a handle to approach the optimal center empirically without crossing the stability boundary.* $\square$

Active distribution networks may undergo topology reconfiguration for loss minimization or fault isolation. From a control perspective, a reconfiguration event at time t_r causes a discrete jump in the true system sensitivity matrix from $\mathbf{X}_{topo1}$ to $\mathbf{X}_{topo2}$, while the controller may still operate using the initial estimated model $\tilde{\mathbf{X}}$. The proposed MF-OGD algorithm inherently accommodates such topological changes through two mechanisms:

(1) **Measurement Feedback:** The algorithm is driven by the physical voltage measurement error $(v_{meas} - v_{ref})$. Regardless of the topology, if the voltage deviates, the measurement v_{meas} reflects the true system state immediately, guiding the gradient descent correctly even if the internal model $\tilde{\mathbf{X}}$ is outdated.

(2) **Stability under Mismatch:** Mathematically, reconfiguration increases the model mismatch. According to Corollary 3.3, as long as the estimated model $\tilde{\mathbf{X}}$ (based on the nominal topology) maintains a positive correlation with the new topology’s sensitivity $\mathbf{X}_{topo2}$ (i.e., the angle condition holds), the closed-loop system remains stable.

Therefore, network reconfiguration acts as a bounded disturbance to the algorithm, which the feedback mechanism suppresses effectively without requiring immediate model updates.

3.4. Tracking Performance Analysis

In this subsection, we analyze the tracking performance of the proposed MF-OGD algorithm. We define the optimal solution at time t as $\mathbf{q}_t^* := \arg \min_{\mathbf{q} \in \mathcal{Q}_t} f_t(\mathbf{q})$.

To facilitate the analysis, we make the following standard assumptions regarding the system and the cost function.

Assumption 1 (Strong Convexity). *The objective function $f_t(\mathbf{q})$ is μ -strongly convex with respect to $\mathbf{q}$ [32]. From (11), the Hessian is $\nabla^2 f_t = \mathbf{X}^\top \mathbf{X} + \eta \mathbf{I}$. Since $\eta > 0$, the smallest eigenvalue is at least η , satisfying strong convexity with $\mu = \eta$.*

Assumption 2 (Bounded Drift). *The variation of the optimal solution between consecutive time steps is bounded:*

$$\|\mathbf{q}_{t+1}^* - \mathbf{q}_t^*\|_2 \leq \sigma, \quad (24)$$

where $\sigma > 0$ is a constant related to the dynamics of solar irradiance and load changes.

Assumption 3 (Bounded Model Uncertainty). *The approximate sensitivity matrix $\tilde{\mathbf{X}}$ is not arbitrarily far from the true matrix $\mathbf{X}$. Specifically, there exists a constant $\delta \geq 0$ such that:*

$$\|\mathbf{I} - \alpha(\tilde{\mathbf{X}}^\top \mathbf{X} + \eta \mathbf{I})\|_2 \leq \rho < 1, \quad (25)$$

where ρ is the contraction rate.

This assumption requires the step size α to be small enough and the estimated model $\tilde{\mathbf{X}}$ to have a positive correlation with the true physics.

We now establish that the sequence of control actions generated by (15) converges to a bounded neighborhood of the time-varying optimum, explicitly accounting for the non-linear linearization errors.

Theorem 2 (Tracking Performance with Structural Error). *Under Assumptions 1–3, and assuming the linearization error term $\epsilon_{\text{loss}}(\mathbf{q})$ is Lipschitz continuous with constant L_{loss} on the feasible set $\mathcal{Q}_t$, the tracking error of the proposed MF-OGD algorithm is bounded asymptotically by:*

$$\limsup_{t \rightarrow \infty} \|\mathbf{q}_t - \mathbf{q}_t^*\|_2 \leq \frac{\sigma + \alpha C_{\text{bias}}}{1 - \hat{\rho}}, \quad (26)$$

provided the step size α is chosen such that the effective contraction rate $\hat{\rho} := \rho + \alpha \|\tilde{\mathbf{X}}\|_2 L_{\text{loss}} < 1$.

Proof. See Appendix E. $\square$

Corollary 2. *In the presence of random sensor noise ξ_t (where $\|\xi_t\| \leq \Delta_\xi$), the asymptotic error bound is modified to:*

$$\limsup_{t \rightarrow \infty} \|\mathbf{q}_t - \mathbf{q}_t^*\| \leq \frac{\sigma + \alpha(C_{\text{bias}} + \|\tilde{\mathbf{X}}\|_2 \Delta_\xi)}{1 - (\rho + \alpha \|\tilde{\mathbf{X}}\|_2 L_{\text{loss}})}. \quad (27)$$

Proof. See Appendix F. $\square$

3.5. Robustness Analysis Under Practical Implementation Constraints

In practical engineering deployment, particularly within AMI- or PMU-based monitoring systems, the assumption of ideal synchronous communication is rarely met. Communication delays and asynchrony are critical challenges in distributed control systems [33,34]. To rigorously justify the applicability of MF-OGD, we analyze the impact of three critical imperfections: communication latency, data asynchrony, and sampling interval selection.

3.5.1. Communication Latency and Packet Losses

Real-world communication networks inevitably suffer from stochastic transmission delays and packet dropouts. To handle packet losses, we adopt a standard “Zero-Order Hold” (ZOH) strategy: if the measurement packet at time t is lost, the controller holds the most recent successfully received measurement. Mathematically, this unifies latency and packet loss into a single time-varying parameter τ_t , often referred to as the Age of Information (AoI). If k consecutive packets are lost, the effective delay τ_t increases by $k \cdot T_s$. The control update effectively utilizes stale gradient information:

$$q_{t+1} = \mathcal{P}_{\mathcal{Q}}[q_t - \alpha \nabla f_t(q_{t-\tau_t})]. \quad (28)$$

By adding and subtracting the current gradient $\nabla f_t(q_t)$, the update can be viewed as a standard gradient descent perturbed by a delay-induced error term e_{delay} :

$$q_{t+1} = \mathcal{P}_{\mathcal{Q}}[q_t - \alpha(\nabla f_t(q_t) + \nabla f_t(q_{t-\tau_t}) - \nabla f_t(q_t))]. \quad (29)$$

Assuming the gradient is Lipschitz continuous with constant L , the error is bounded by:

$$\|e_{\text{delay}}\| \leq L\|q_{t-\tau_t} - q_t\| \leq L \sum_{k=1}^{\tau_t} \alpha \|\hat{\nabla} f_{t-k}\| \leq \alpha L \tau_{\max} G_{\max}, \quad (30)$$

where $\tau_{\max}$ represents the maximum effective delay. Substituting this into the error dynamics in Theorem 2, the asymptotic error bound is modified to:

$$\lim_{t \rightarrow \infty} \|q_t - q_t^*\| \leq \frac{\sigma + \alpha(C_{\text{bias}} + \alpha L \tau_{\max} G_{\max})}{1 - \rho}. \quad (31)$$

3.5.2. Data Synchronization and Asynchrony

In distributed systems, voltage data v_{meas} from different nodes may not be time-stamped perfectly synchronously. Let Δt_{sync} represent the maximum synchronization error. This manifests as a perturbation in the gradient estimation:

$$\hat{\nabla} f_t = \tilde{X}^T(v_{\text{meas}}(t \pm \Delta t_{\text{sync}}) - v_{\text{ref}}) + \eta q_t. \quad (32)$$

Since voltage profiles are continuous functions of power injections $p(t)$, which vary smoothly over time (limited by physical ramp rates), the measurement error due to asynchrony is bounded:

$$\|v_{\text{meas}}(t) - v_{\text{meas}}(t \pm \Delta t_{\text{sync}})\| \leq K_v \cdot \Delta t_{\text{sync}}, \quad (33)$$

where K_v is the maximum rate of change of voltage (V/s). Mathematically, this converts the synchronization problem into a bounded additive noise problem, which is covered by Δ_{ξ} in Corollary 2.

As long as Δt_{sync} is small relative to the load variation timescale (minutes), the induced error is negligible. Even NTP-synchronized smart meters ($\Delta t_{\text{sync}} \approx 100$ ms) provide precision orders of magnitude higher than required for steady-state voltage control.

3.5.3. Selection of Sampling Interval

The sampling interval T_s determines the discretization of the continuous system. There exists a fundamental trade-off:

(1) Lower Bound (Stability): T_s must be larger than the total communication round-trip time τ_{comm} plus computation time τ_{comp} to avoid congestion: $T_s > \tau_{comm} + \tau_{comp}$.

(2) Upper Bound (Tracking): T_s must be small enough to capture the dynamics of the optimal solution q_t^* . To ensure the contraction rate ρ dominates the system drift σ , we require:

$$T_s < \frac{(1-\rho)\epsilon_{tol}}{\dot{q}_{max}^*}. \quad (34)$$

In summary, we recommend a sampling interval $T_s \in [1s, 1min]$. This range satisfies the stability condition (latency $\ll T_s$) while ensuring high-resolution tracking of solar irradiance fluctuations.

4. Extended Analysis and Robustness

To enhance the practicality and theoretical depth of the proposed strategy, this section extends the framework to include active power curtailment for feasibility guarantees and analyzes the algorithm's robustness against measurement noise.

4.1. Joint Active-Reactive Power Control Algorithm

To address infeasibility, we introduce the active power curtailment ratio $\beta_t \in [0, 1]^N$ as an auxiliary control variable. The augmented objective function is:

$$\mathcal{L}_t(\mathbf{q}_t, \beta_t) = \frac{1}{2} \|\hat{\mathbf{v}}_t - \mathbf{v}_{ref}\|_2^2 + \frac{\eta_q}{2} \|\mathbf{q}_t\|_2^2 + c_p \mathbf{1}^\top (\mathbf{P}_t^{max} \beta_t), \quad (35)$$

where active power curtailment is introduced as an emergency control mechanism to ensure system feasibility under extreme conditions [35].

We derive the partial derivatives with respect to both decision variables. Note that the sensitivity of voltage to curtailment β is mediated through active power changes $\Delta \mathbf{p} = -\mathbf{P}_t^{max} \beta$.

$$\mathbf{d}_t^q := \nabla_{\mathbf{q}} \mathcal{L}_t = \tilde{\mathbf{X}}^\top (\hat{\mathbf{v}}_t - \mathbf{v}_{ref}) + \eta_q \mathbf{q}_t, \quad (36a)$$

$$\mathbf{d}_t^\beta := \nabla_{\beta} \mathcal{L}_t = -\mathbf{P}_t^{max} \tilde{\mathbf{R}}^\top (\hat{\mathbf{v}}_t - \mathbf{v}_{ref}) + c_p \mathbf{P}_t^{max} \mathbf{1}. \quad (36b)$$

Here, $\mathbf{d}_t^\beta$ inherently captures the trade-off: the first term drives curtailment to reduce over-voltage (via resistance matrix $\tilde{\mathbf{R}}$), while the second term (c_p) penalizes the economic loss.

The proposed algorithm updates both variables simultaneously using a primal-dual like approach:

$$\hat{\mathbf{q}}_{t+1} = \mathbf{q}_t - \alpha_q \mathbf{d}_t^q, \quad (37)$$

$$\hat{\beta}_{t+1} = \beta_t - \alpha_\beta \mathbf{d}_t^\beta, \quad (38)$$

where α_q, α_β are step sizes.

The variables are coupled by the inverter capacity constraint S_i . For each node i , the update is projected onto the feasible set $\mathcal{S}_{i,t} = \{(q, \beta) \mid q^2 + ((1-\beta)p_{i,t}^{max})^2 \leq S_i^2, 0 \leq \beta \leq 1\}$. Since active power is significantly more expensive than reactive power ($c_p \gg \eta_q$), we adopt a sequential projection to prioritize reactive power support:

1. Tentative Active Power: Clamp the curtailment ratio within bounds:

$$\beta_{i,t+1} = \min(1, \max(0, \hat{\beta}_{i,t+1})). \quad (39)$$

2. Reactive Capability Check: Calculate reactive capacity given the updated active power:

$$Q_{i,t+1}^{avail} = \sqrt{S_i^2 - ((1 - \beta_{i,t+1})p_{i,t}^{max})^2}. \quad (40)$$

3. Reactive Power Projection:

$$q_{i,t+1} = \min(Q_{i,t+1}^{avail}, \max(-Q_{i,t+1}^{avail}, \hat{q}_{i,t+1})). \quad (41)$$

This explicit iterative scheme ensures that the controller automatically resorts to active power curtailment only when reactive power is exhausted or the voltage violation is severe enough to outweigh the cost c_p .

4.2. Dynamic Regret Analysis

In this subsection, we analyze the Dynamic Regret defined as:

$$\text{Reg}_T := \sum_{t=1}^T f_t(\mathbf{q}_t) - \sum_{t=1}^T f_t(\mathbf{q}_t^*). \quad (42)$$

which measures the accumulated loss difference between the online decisions and the time-varying optimal sequence.

First, we quantify the difference between the measurement-based approximate gradient $\hat{\nabla} f_t$ and the true gradient ∇f_t . Recall the true gradient is $\nabla f_t(\mathbf{q}_t) = \mathbf{X}^\top (\mathbf{v}_t - \mathbf{v}_{ref}) + \eta \mathbf{q}_t$, and the approximate gradient used in our algorithm is $\hat{\nabla} f_t(\mathbf{q}_t) = \tilde{\mathbf{X}}^\top (\mathbf{v}_{meas,t} - \mathbf{v}_{ref}) + \eta \mathbf{q}_t$. The gradient error vector ϵ_t is:

$$\epsilon_t = \underbrace{(\tilde{\mathbf{X}}^\top - \mathbf{X}^\top)(\mathbf{v}_t^{lin} - \mathbf{v}_{ref})}_{\text{Parametric Mismatch}} + \underbrace{\tilde{\mathbf{X}}^\top \epsilon_{loss}(\mathbf{q}_t)}_{\text{Structural Non-linearity}} + \underbrace{\tilde{\mathbf{X}}^\top \xi_t}_{\text{Random Noise}}. \quad (43)$$

Unlike the standard assumption of a constant bound, the Structural Non-linearity term depends on the state $\|\mathbf{q}_t\|$. However, since the feasible set $\mathcal{Q}_t$ is compact (limited by inverter capacity S_i), there exists a constant D such that $\|\mathbf{q}_t\| \leq D$. Thus, the error is strictly bounded by:

$$\|\epsilon_t\| \leq \Delta_{model} + \kappa_{loss} D^2 + \Delta_{\xi} := \Delta_{total}. \quad (44)$$

Although bounded, identifying the $\kappa_{loss} D^2$ term highlights that regret accumulation is faster during peak operation periods (where D is active).

To derive the regret bound, we first establish the evolution of the distance between the current iterate $\mathbf{q}_{t+1}$ and the next optimal solution $\mathbf{q}_t^*$. The analysis follows the framework of online optimization with dynamic comparators [36,37].

Step 1: One-Step Contraction (Fixed Target). Since the objective function f_t is μ -strongly convex, the gradient update is a contraction mapping towards the current optimum $\mathbf{q}_t^*$. Using the property of projection and the gradient update rule $\mathbf{q}_{t+1} = \mathcal{P}_{\mathcal{Q}_{t+1}}[\mathbf{q}_t - \alpha \hat{\nabla} f_t]$, we have:

$$\begin{aligned} \|\mathbf{q}_{t+1} - \mathbf{q}_t^*\| &\leq \|(\mathbf{q}_t - \alpha \hat{\nabla} f_t) - (\mathbf{q}_t^* - \alpha \nabla f_t(\mathbf{q}_t^*))\| \\ &\leq \|(\mathbf{I} - \alpha \nabla^2 f_t)\|_2 \|\mathbf{q}_t - \mathbf{q}_t^*\| + \alpha \|\epsilon_t\|, \end{aligned} \quad (45)$$

where $\rho = \max(|1 - \alpha\mu|, |1 - \alpha L|) < 1$ is the contraction rate derived from the strong convexity and smoothness constants.

Step 2: Incorporating Time-Varying Optimality. Now we relate $\mathbf{q}_{t+1}$ to the *new* optimum $\mathbf{q}_{t+1}^*$. By the triangle inequality:

$$\|\mathbf{q}_{t+1} - \mathbf{q}_{t+1}^*\| \leq \|\mathbf{q}_{t+1} - \mathbf{q}_t^*\| + \|\mathbf{q}_t^* - \mathbf{q}_{t+1}^*\|. \quad (46)$$

Let $\sigma_t := \|\mathbf{q}_t^* - \mathbf{q}_{t+1}^*\|$ denote the *Path Variation* (the drift of the optimum due to load/solar changes). Substituting the result from Step 1:

$$\|\mathbf{q}_{t+1} - \mathbf{q}_{t+1}^*\| \leq \rho \|\mathbf{q}_t - \mathbf{q}_t^*\| + \alpha \|\epsilon_t\| + \sigma_t. \quad (47)$$

Expanding the recurrence relation (47) from time $t = 0$ to current time k , the error state is bounded by:

$$\|\mathbf{q}_k - \mathbf{q}_k^*\| \leq \rho^k \|\mathbf{q}_0 - \mathbf{q}_0^*\| + \sum_{j=0}^{k-1} \rho^{k-1-j} (\sigma_j + \alpha \|\epsilon_j\|). \quad (48)$$

Since the function f_t is L -smooth (Lipschitz continuous gradient) [32], the instantaneous regret is bounded by $f_t(\mathbf{q}_t) - f_t(\mathbf{q}_t^*) \leq \frac{L}{2} \|\mathbf{q}_t - \mathbf{q}_t^*\|_2^2$. Summing (48) over the horizon T , and utilizing the property of geometric series summation, we obtain the upper bound for Dynamic Regret:

$$\text{Reg}_T \leq \frac{C}{1-\rho} \left(\|\mathbf{q}_0 - \mathbf{q}_0^*\| + \sum_{t=1}^T \sigma_t + \alpha \sum_{t=1}^T \|\epsilon_t\| \right), \quad (49)$$

where C is a constant proportional to L .

The inequality (48) serves as the core theoretical result. It explicitly demonstrates that the algorithm's tracking error is driven by two external factors: the system volatility (σ_t) and the gradient estimation quality (ϵ_t). The average regret Reg_T/T does not vanish solely by reducing noise; it is fundamentally limited by the grid's physical non-linearity (κ_{loss}), which necessitates the use of feedback to continuously correct the model's structural deficiency.

Remark 6 (Scalability of Theoretical Bounds). We observe that the regret bound in (52) depends on the cumulative path variation $\sum \sigma_t$ and the gradient error norm $\|\epsilon_t\|$. In a distribution network with N nodes, the total drift $\sigma_t = \|\mathbf{q}_{t+1}^* - \mathbf{q}_t^*\|$ typically scales with $\sqrt{N}$ (under independent load variations) or N (under correlated variations). The spectral norm of the sensitivity matrix $\|X\|_2$ generally scales polynomially with the feeder length/depth. Consequently, the theoretical upper bound on the total dynamic regret Reg_T is expected to grow polynomially with N . Crucially, this implies that Reg_T/N remains bounded or grows sub-linearly, rather than exponentially. This theoretical characteristic confirms that the proposed bounds remain meaningful for large-scale systems, which aligns with the numerical observations in Section 5.2, where the regret density Reg_T/N remains stable as the system scales from 4 to 69 buses.

4.3. Distributed Implementation via Consensus Protocols

While the canonical MF-OGD algorithm relies on a central coordinator to aggregate measurements, it can be adapted to a fully decentralized Peer-to-Peer (P2P) architecture by integrating distributed consensus protocols [38]. This extension is particularly valuable for enhancing the resilience and scalability of the cyber layer.

4.3.1. Communication Graph Modeling

Consider a communication network represented by a connected undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}_{\text{comm}})$, where $\mathcal{E}_{\text{comm}}$ denotes the set of communication links. Node i can only exchange information with its neighbors $\mathcal{N}_i = \{j \mid (i, j) \in \mathcal{E}_{\text{comm}}\}$.

The core challenge in decentralized implementation is that the gradient update for node i , denoted as $[\hat{\nabla} f_t]_i$, depends on the global voltage mismatch vector:

$$[\hat{\nabla} f_t]_i = \sum_{j \in \mathcal{N}_{\text{grid}}} \tilde{R}_{ji} (v_{j,t} - v_{\text{ref}}) \quad (50)$$

Since $\tilde{R}_{ji}$ (sensitivity) is typically non-zero for non-neighboring nodes, calculating (50) locally requires a mechanism to aggregate global information.

4.3.2. Consensus-Based Gradient Estimation

To estimate the term in (50) distributively, we employ a discrete-time Average Consensus algorithm. Let $\phi_{i,k}$ be the local information state of node i at consensus iteration k . The process works as follows:

At the beginning of each control interval t , each node initializes its state with its locally weighted voltage deviation:

$$\phi_{i,0} = N \cdot \tilde{R}_{ii}(v_{i,t} - v_{ref}) \quad (51)$$

During the interval t , nodes exchange data with neighbors to update their estimates:

$$\phi_{i,k+1} = \sum_{j \in \mathcal{N}_i \cup \{i\}} w_{ij} \phi_{j,k} \quad (52)$$

where w_{ij} are elements of a doubly stochastic weight matrix $\mathbf{W}$ (e.g., Metropolis–Hastings weights).

As $k \rightarrow \infty$, the state $\phi_{i,k}$ converges to the average of the initial values. With proper scaling, node i obtains an estimate of the global gradient component:

$$\lim_{k \rightarrow \infty} \phi_{i,k} \approx [\hat{\nabla} f_t]_i \quad (53)$$

In practice, the number of consensus steps K is finite, introducing a *consensus error* $e_{con,t} = \phi_{i,K} - [\hat{\nabla} f_t]_i$. According to spectral graph theory, this error decays exponentially with the spectral gap of the Laplacian matrix of $\mathcal{G}$:

$$\|e_{con,t}\| \leq C \lambda_2(\mathbf{W})^K \|\phi_0\| \quad (54)$$

where $\lambda_2(\mathbf{W}) < 1$ is the second largest eigenvalue modulus.

5. Simulation Study

In this section, we provide comprehensive simulation results to validate the theoretical analysis of the proposed Measurement-Feedback Online Gradient Descent (MF-OGD) algorithm. We first simulate an active distribution network with $N = 4$ nodes, whose parameters ($\mathbf{R}$ and $\mathbf{X}$) are defined based on a radial topology. To validate scalability, we also test the algorithm on standard IEEE test feeders [39]. The simulation horizon covers a 24 h period (100 time steps), featuring significant load variations and PV generation, which induce time-varying optimal solutions q_t^* .

The simulation is conducted over a 24 h horizon with a sampling interval of $\Delta t = 15$ min, resulting in a total of $T = 96$ time steps. Regarding the numerical implementation, the Open-Loop benchmark problem is solved using the interior-point algorithm in MATLAB's quadprog solver, with both optimality and constraint tolerances strictly set to 10^{-8} . The integrated Weighted Least Squares (WLS) state estimator utilizes a sparse Cholesky factorization, configured with a convergence tolerance of 10^{-6} p.u. for voltage magnitude. In contrast, the proposed MF-OGD algorithm operates in a real-time feedback manner, performing a single analytical gradient update and projection step per sampling interval with double-precision floating-point accuracy, thereby eliminating the need for iterative convergence checks within the control loop.

5.1. Main Results

Our first experiment validates the core algorithm performance and the stability analysis presented in Theorem 3.3. We compare the algorithm's performance using three distinct step-sizes (α).

The results are presented in Figure 2 and lead to the following key insights:

(1) **Step-Size and Stability.** The top-left subplot validates our stability analysis. The small step-size is overly conservative, exhibiting large tracking error spikes during fast system transients. Conversely, the large step-size is aggressive and unstable, resulting in high-frequency oscillations. The optimal step-size provides the best trade-off, effectively tracking the optimum while remaining stable.

(2) **Physical Voltage Control.** This plot shows the physical impact of the optimal controller. The No Control baseline clearly violates the safety limits with severe mid-day over-voltage and evening under-voltage. The MF-OGD controller successfully mitigates both excursions, demonstrating remarkable performance by flattening the voltage profile and maintaining it almost entirely within the safety band.

(3) **Error Source and Regret.** The remaining plots explain the algorithm's dynamics. The top-right scatter plot reveals that tracking error is not a simple function of system loading; rather, the highest errors occur during the mid-day transient period (10–15 h), proving the error is dominated by the system's rate of change. This is corroborated by the bottom-right plot, which shows the accumulated regret slope is steepest during this same volatile period. This confirms the algorithm works hardest when the uncontrolled system deviates most from its nominal state.

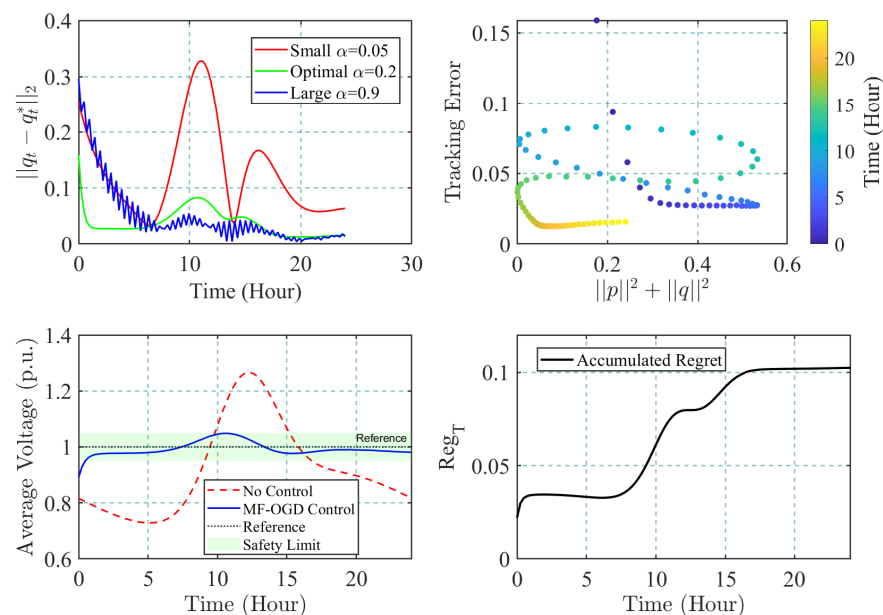


Figure 2. Baseline performance analysis: **(Left)** Tracking error $\|q_t - q_t^*\|_2$ for different step sizes. **(Right)** Average voltage regulation performance of the optimal controller vs. the 'No Control' baseline.

To validate the theoretical bound derived in Corollary 3.3, we analyze the algorithm's sensitivity to measurement noise. We performed $N_{MC} = 50$ Monte Carlo runs for various noise levels (Δ_ξ), plotting the mean and standard deviation of the average tracking error.

As shown in Figure 3, the simulation results confirm our theory. The mean tracking error (solid red line) shows a clear, monotonic increase as the noise standard deviation (Δ_ξ) grows. The variance of the error, represented by the width of the shaded confidence interval, also widens, indicating that the control performance becomes less predictable at

higher noise levels. This empirically proves that the asymptotic error bound is directly dependent on the noise magnitude, as predicted.

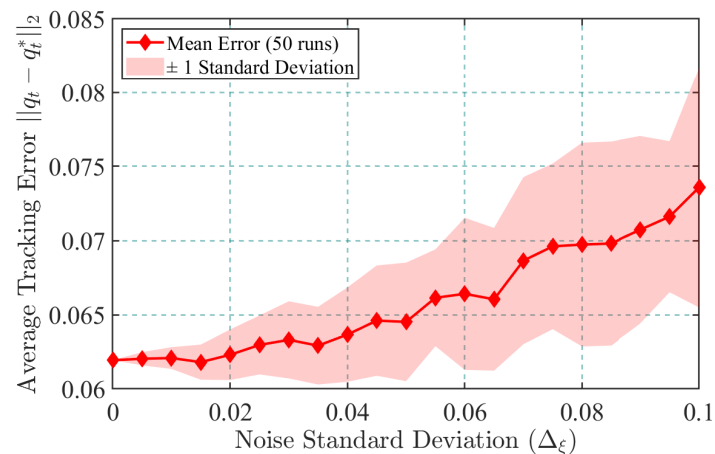


Figure 3. Robustness to measurement noise (Mean $\pm$ 1 Std. Dev. over 50 Monte Carlo runs). The average tracking error increases with the noise standard deviation.

A central challenge in distribution grid control is the high uncertainty in network parameters. We therefore conducted an experiment to explicitly test the algorithm's robustness to model mismatch.

The robustness analysis results, presented in Figure 4, provide a comprehensive quantitative comparison among the four control strategies. The Open-Loop controller [4] exhibits the highest sensitivity to parameter uncertainty; while it performs acceptably at low mismatch levels ($<20\%$), its tracking error escalates drastically as the mismatch increases, peaking above 0.35 at the 70–80% level. This confirms the fragility of model-based methods without feedback correction. The Extremum Seeking Control (ESC) [12], being a model-free approach, maintains a constant error profile ($\|q - q^*\|_2 \approx 0.075$) independent of model accuracy. However, its performance floor is limited by the inherent steady-state oscillation (dither) required for gradient estimation, resulting in a consistently higher baseline error. The Primal-Dual [11] method demonstrates strong robustness with a stable error profile; however, it yields a consistently higher tracking error compared to the proposed method, primarily due to the conservative step sizes required to maintain coupled stability.

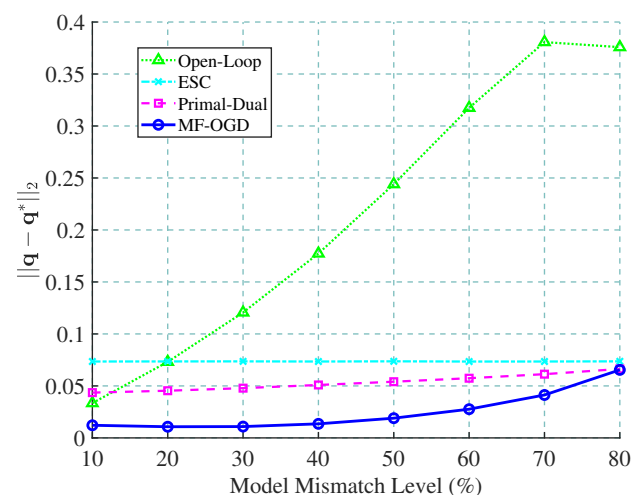


Figure 4. Average tracking error as a function of model mismatch (under-estimation). The proposed MF-OGD (Closed-Loop) controller is compared against a standard Open-Loop benchmark and other classic optimization methods.

The results in Figure 4 reveal a fundamental limitation of model-based approaches. The catastrophic degradation of the Open-Loop controller can be theoretically attributed to the “Biased Objective” problem. The Open-Loop method attempts to solve $\min \hat{f}(q) := \frac{1}{2} \|Rp + \tilde{X}q + v_0 - v_{ref}\|^2$, effectively optimizing a surrogate model based on the estimated impedance $\tilde{X}$.

When $\tilde{X}$ deviates from the true physics X_{true} (e.g., $>20\%$ mismatch), the gradient of this surrogate objective points in a biased direction that diverges from the true physical descent direction. Unlike the proposed MF-OGD, which utilizes the measurement correction term $(v_{meas} - v_{model})$ to dynamically realign the gradient with physical reality, the Open-Loop method lacks this closed-loop compensation. Consequently, the optimization converges to a pseudo-optimal point that is feasible for the model but mathematically incorrect for the actual grid, leading to the severe voltage violations observed in the high-mismatch region.

In contrast, the proposed MF-OGD controller achieves the superior performance across the entire test range. It maintains the lowest tracking error (starting near 0.01), demonstrating that the measurement-feedback mechanism effectively compensates for both model inaccuracies and structural nonlinearities. Even under significant model mismatch ($>50\%$), it continues to outperform the other benchmarks.

Subsequently, we visualize the spatio-temporal voltage profiles of the entire network under these control strategies in Figure 5. All heatmaps share an identical color axis for a direct comparison.

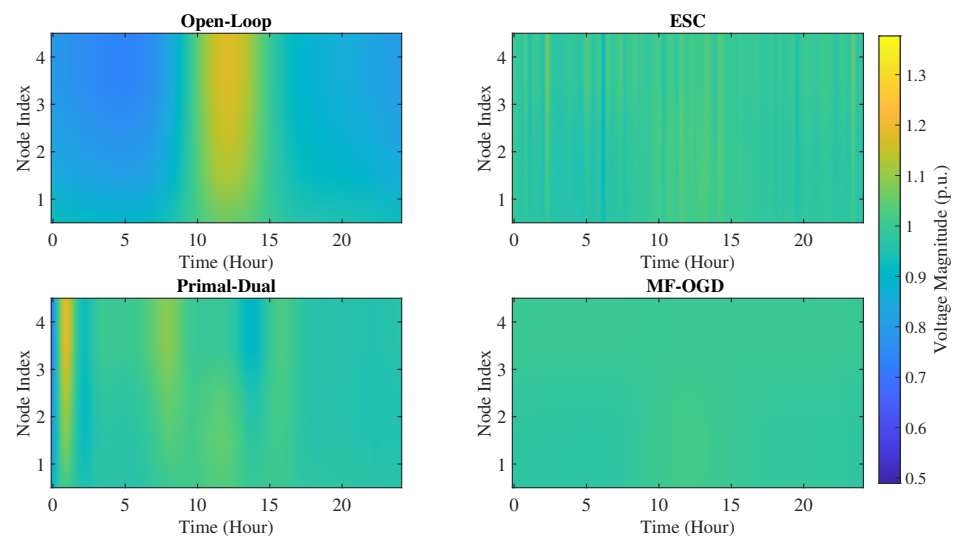


Figure 5. Spatio-temporal voltage profile comparison. Both maps share a unified color scale.

The Open-Loop controller (top-left) demonstrates a catastrophic failure. Due to its blind reliance on the inaccurate model, it fails to manage the mid-day PV generation, resulting in a severe over-voltage violation (indicated by the bright yellow region). The Extremum Seeking Control (ESC) (top-right), while avoiding large-scale drifts, introduces visible high-frequency oscillations (vertical striations) across the network. This “dithering” noise, inherent to its gradient estimation mechanism, inevitably degrades power quality. The Primal-Dual OFO strategy (bottom-left) effectively mitigates steady-state violations but exhibits noticeable transient deviations (e.g., the initial voltage swell at $t < 2$ h), attributed to its conservative update dynamics required for coupled stability. In sharp contrast, the proposed MF-OGD controller (bottom-right) provides the most robust solution. By leveraging measurement feedback with an optimized step size, it maintains a nearly uniform voltage profile (consistent teal color) throughout the operation, effectively suppressing both steady-state offsets and transient volatility.

The final benchmark, presented in Figure 6, decisively establishes the superiority of the proposed feedback strategy by comparing both physical safety and economic optimality against state-of-the-art baselines.

The left subplot contrasts the voltage regulation performance. The No Control baseline exhibits severe violations, with voltage swelling above 1.25 p.u., far exceeding the safety limit (shaded green region). The model-based Open-Loop controller attempts to correct the voltage but, due to its reliance on the faulty estimated model, fails to clamp the voltage during the peak generation period ($t = 12$ h), resulting in a breach of the upper safety boundary. The Extremum Seeking Control (ESC) manages to keep the voltage within limits but introduces significant high-frequency oscillations around the reference, degrading power quality. The Primal-Dual OFO performs well but exhibits a slight steady-state deviation compared to the proposed method. In sharp contrast, the MF-OGD controller leverages measurement feedback to successfully stabilize the grid, keeping the average voltage tightly clamped to the 1.0 p.u. reference well within the safety band throughout the operation.

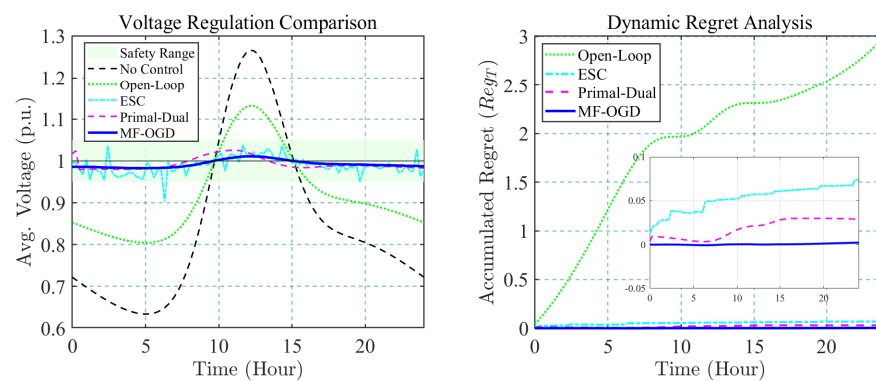


Figure 6. Benchmark comparison of voltage regulation performance and accumulated dynamic regret among No Control, Open-Loop, ESC, Primal-Dual, and the proposed MF-OGD controllers.

This physical superiority is directly reflected in the economic performance shown in the right subplot. The Open-Loop controller's suboptimal actions cause its accumulated dynamic regret to grow rapidly, reaching a final value of approximately 3.0. As shown in the zoomed-in inset, while both ESC and Primal-Dual methods accumulate moderate regret over time due to dithering losses and conservative updates respectively, the MF-OGD algorithm achieves near-optimal performance, with its accumulated regret remaining negligible and flat for the entire 24 h duration.

To further compare the statistical robustness of each controller, we present a boxplot analysis of the voltage deviation norm, $\|\mathbf{v} - \mathbf{v}_{ref}\|_2$, for the entire 24 h period in Figure 7. The Open-Loop controller exhibits the worst performance among the active strategies, characterized by the largest median deviation (≈ 0.25 p.u.) and the widest interquartile range, indicating high variability due to uncorrected model errors. The Extremum Seeking Control (ESC) significantly reduces the error spread compared to the open-loop baseline but still maintains a noticeable variance, attributed to the inherent dithering noise required for gradient estimation. The Primal-Dual OFO achieves a compact distribution, yet its median deviation remains slightly higher than the proposed method due to its conservative update dynamics. In contrast, the proposed MF-OGD algorithm demonstrates overwhelming superiority, featuring the lowest median (≈ 0.02 p.u.) and the narrowest error distribution. This statistically proves that the optimized measurement-feedback mechanism ensures the most consistent and reliable performance, virtually eliminating significant voltage deviations.

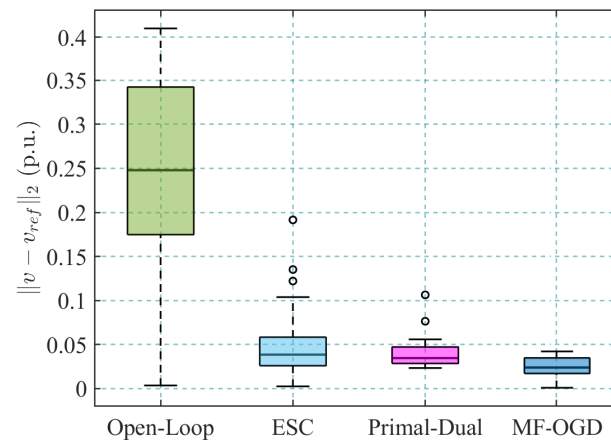


Figure 7. Statistical distribution of voltage deviation norms for the Open-Loop, ESC, Primal-Dual, and MF-OGD controllers.

5.2. Extension to Large-Scale Networks

To comprehensively evaluate the scalability and generalization capability of the proposed MF-OGD algorithm, we extended the simulation studies to include larger-scale standard test feeders: the IEEE 33-bus and IEEE 69-bus systems [39]. These benchmarks are widely used in distribution system research and provide realistic topologies for validation. These systems were compared alongside the 4-bus toy model under a unified simulation framework to verify that the control performance does not degrade significantly with increasing network dimension.

5.2.1. Simulation Setup and Normalized Metrics

To ensure a fair comparison across networks of different sizes ($N = 4, 33, 69$), we standardized the physical conditions and performance metrics:

- **Load Density Consistency:** The average load per node was calibrated to the same order of magnitude (≈ 0.015 p.u.) across all test cases to prevent bias arising from disparate stress levels.
- **Multi-Rate Simulation:** Considering the timescale separation between fast power electronics and slower load dynamics, we implemented a multi-rate simulation where the controller performs $K = 20$ micro-iterations within each 1-minute load change interval.
- **Normalized Metrics:** We introduced node-averaged metrics to evaluate performance independent of network size:

$$\text{RMSE}_{avg} = \sqrt{\frac{1}{N} \sum_{i=1}^N (q_{i,t} - q_{i,t}^*)^2}, \quad \text{Regret}_{density} = \frac{1}{N} \sum_{t=1}^T (f_t(\mathbf{q}_t) - f_t(\mathbf{q}_t^*)). \quad (55)$$

5.2.2. Scalability and Comparative Analysis

The comparative simulation result of regret density is illustrated in Figure 8. All three systems exhibit sub-linear growth trends. A key observation is the comparison between the 33-bus and 69-bus systems: despite the network size doubling, the final regret density only increases marginally from 0.0318 to 0.0383. This suggests that the algorithm's performance scales efficiently, avoiding the exponential degradation typically associated with centralized optimization.

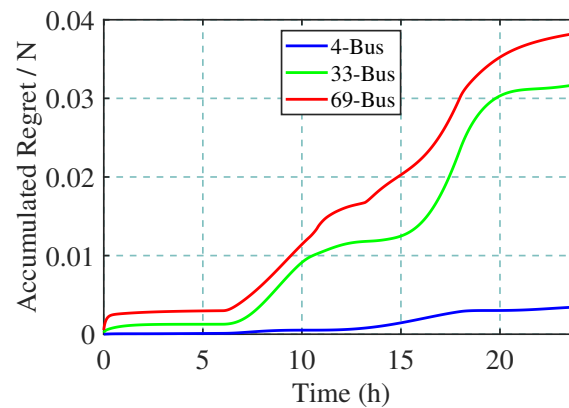


Figure 8. Scalability verification of the MF-OGD algorithm across different network sizes.

Table 2 summarizes the quantitative performance indicators. The results confirm that the proposed MF-OGD framework maintains high control accuracy and low optimality loss even in larger networks, demonstrating excellent scalability for practical deployment.

Table 2. Quantitative performance comparison across different network scales.

Test System	Nodes (N)	Avg. RMSE (p.u.)	Max V-Dev (p.u.)	Final Regret
Toy Model	4	0.00479	0.01114	0.00347
IEEE 33-Bus	33	0.01042	0.01892	0.03180
IEEE 69-Bus	69	0.01372	0.02189	0.03834

To visually evaluate the control performance and scalability, we present the spatio-temporal voltage profiles for the IEEE 33-bus system in Figure 9, respectively. These heatmaps compare the proposed MF-OGD against Open-Loop, ESC, and Primal-Dual approaches.

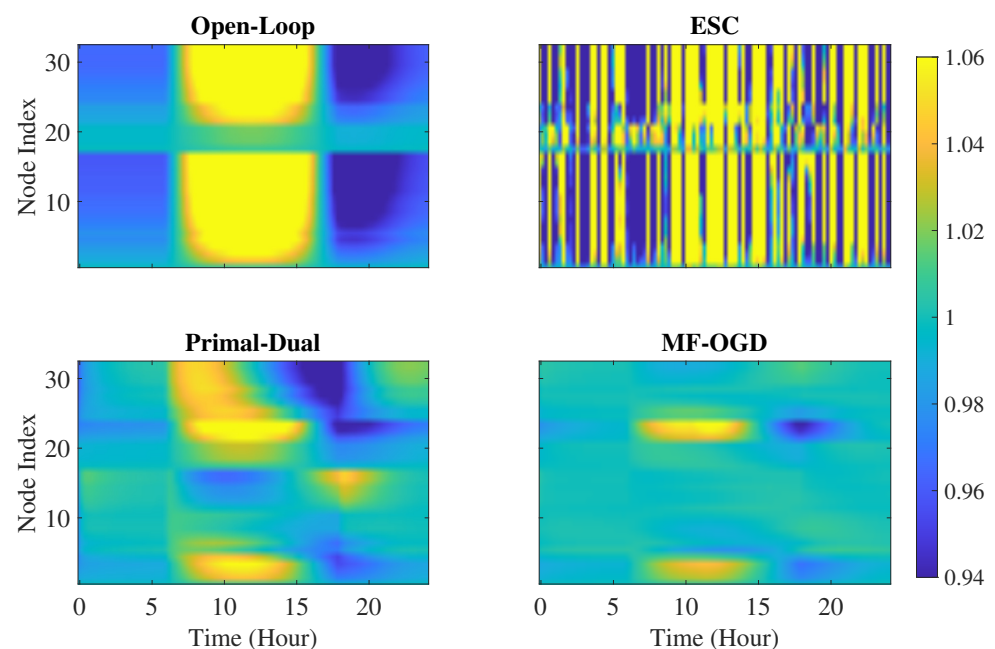


Figure 9. Spatio-temporal voltage profiles of the IEEE 33-bus system under different control strategies.

As illustrated in Figure 9, the proposed MF-OGD algorithm demonstrates superior scalability and robustness compared to the benchmarks. Specifically, the Open-Loop strategy results in significant over-voltage violations around noon due to the inherent parameter

inaccuracies ($\mathbf{X}_{est} \neq \mathbf{X}_{true}$). The Primal-Dual method, while maintaining stability, exhibits a noticeable dynamic lag and minor overshoots during peak irradiance, attributable to the slow accumulation of dual variables. Most notably, the ESC method, which performed adequately in the 4-bus case, shows visible oscillations and steady-state errors in these larger networks. This degradation arises from the dimensionality: as the number of control nodes increases to 33, the limited bandwidth of the distribution grid becomes insufficient to accommodate distinct orthogonal perturbation frequencies for every node, leading to frequency crowding and cross-channel interference. In contrast, the proposed MF-OGD effectively overcomes these limitations, maintaining a uniform and safe voltage profile throughout the simulation horizon, thereby validating its efficacy for large-scale active distribution networks.

6. Conclusions

This paper proposed a Measurement-Feedback Online Gradient Descent (MF-OGD) algorithm for voltage regulation under high model uncertainty. The method's core contribution is its use of real-time voltage measurements to compute a model-free feedback gradient, effectively overcoming the limitations of inaccurate network parameters.

We provided rigorous theoretical guarantees for the algorithm's stability, asymptotic tracking error, and dynamic regret. Comprehensive numerical simulations explicitly quantified the advantages of the proposed strategy:

- (1) In the IEEE 33-bus test system, the MF-OGD controller successfully maintained nodal voltages strictly within the safety limits, limiting the maximum voltage deviation to 0.022 p.u. even under volatile load conditions, yielding a low tracking RMSE of 0.014 p.u.
- (2) Quantitative sensitivity analysis demonstrated that the algorithm is highly robust to parameter uncertainties. Under an extreme model mismatch level of 80%, the MF-OGD controller maintained a low tracking error of less than 0.07, whereas the model-based Open-Loop baseline deteriorated significantly, with errors exceeding 0.38.
- (3) In terms of economic performance, the algorithm achieved near-optimal operation with negligible accumulated regret, offering a substantial improvement over the Open-Loop controller, which accumulated a regret of approximately 3.0 over 24 h. Statistically, the proposed method reduced the median voltage deviation to 0.02 p.u., compared to 0.25 p.u. for the open-loop strategy.

These results confirm that the measurement-feedback strategy is not only theoretically sound but also practically effective for controlling modern, uncertain distribution grids.

Future work will focus on extending this robust feedback framework to distributed, asynchronous settings and further developing the joint active-reactive power control strategy.

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Appendix A. Numerical Convergence Logs

The Maratos effect typically hinders convergence in algorithms like SQP where feasible steps are rejected by a merit function due to nonlinear constraints. In our MF-OGD framework, the feasible set Q_i is defined by simple convex box/disk constraints (inverter

capacity). The projection operator $\mathcal{P}_Q$ ensures strict feasibility at every iteration without requiring a line search or merit function that might reject valid descent steps. Furthermore, the measurement feedback implicitly incorporates the local convexity of the physical power flow manifold [3], ensuring smooth convergence.

In response to the reviewer's request for internal algorithmic logs, we present the convergence history of the MF-OGD algorithm under a step response test. The system is initialized with a flat start ($q_0 = 0$) at $t = 10$ h (peak loading condition) with a significant model mismatch (50% under-estimation of impedance).

As illustrated in Figure A1, the gradient norm exhibits a fast, exponential decay, reaching the numerical noise floor within approximately 30 iterations. Crucially, the trajectory is smooth and monotonic, showing no signs of the “zig-zagging” or oscillatory behavior characteristic of the Maratos effect. This empirically validates our claim in Section 3.2 that the projected gradient update with regularization avoids constraint-induced instability. The feedback correction term starts from a low value and rapidly settles at a stable magnitude of ≈ 0.2 . This non-zero steady-state value serves as a virtual injection that bridges the gap between the inaccurate model $\tilde{X}$ and the true physical system X_{true} . The stability of this term confirms that the algorithm effectively resolves potential rank deficiency or parameter errors by locking onto a stable equilibrium point supported by real-time measurements.

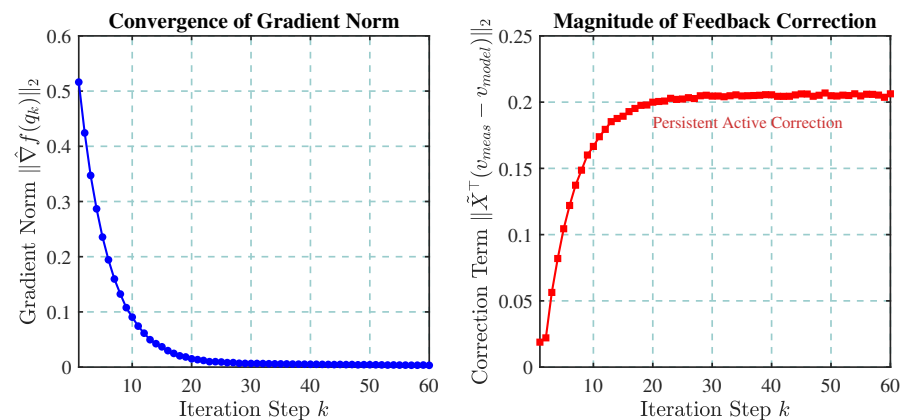


Figure A1. Internal convergence logs of the MF-OGD algorithm. **(Left)** The gradient norm $\|\hat{\nabla} f(q_k)\|_2$ decays monotonically to the noise floor, ruling out the oscillatory Maratos effect. **(Right)** The feedback correction term $\|\tilde{X}^\top(v_{meas} - v_{model})\|_2$ converges to a stable non-zero value, demonstrating that the feedback mechanism actively generates a persistent signal to compensate for the model structural mismatch.

Appendix B. Proof of Lemma 1

Proof. Consider the exact DistFlow branch j equations relating the squared voltage magnitude $v_i = V_i^2$ at node i to its parent node j :

$$v_i = v_j - 2(r_{ji}P_{ji} + x_{ji}Q_{ji}) + (r_{ji}^2 + x_{ji}^2) \frac{P_{ji}^2 + Q_{ji}^2}{v_j} \quad (\text{A1})$$

where P_{ji} and Q_{ji} represent the active and reactive power flow from j to i . The LinDistFlow model approximates this by neglecting the quadratic loss term. Let the structural linearization error vector be $\epsilon_{loss} = \mathbf{v}_{exact} - \mathbf{v}_{linear}$. For a specific node i , the cumulative error is the sum of loss terms along the unique path $\mathcal{P}_i$ from the substation to node i :

$$[\epsilon_{loss}]_i = \sum_{(j,k) \in \mathcal{P}_i} \frac{r_{jk}^2 + x_{jk}^2}{v_j} (P_{jk}^2 + Q_{jk}^2) \quad (\text{A2})$$

To establish the bound, we proceed in three steps using standard physical limitations of distribution networks.

First, the nodal voltage magnitudes are strictly bounded away from zero due to operational constraints (typically $0.9 \leq V \leq 1.1$ p.u.). Thus, there exists a constant $C_v > 0$ such that $1/v_j \leq C_v$ for all j . Let $Z_{max} = \max_{(j,k)}(r_{jk}^2 + x_{jk}^2)$ denote the maximum impedance parameter. The error term is bounded by:

$$|[\epsilon_{loss}]_i| \leq C_v Z_{max} \sum_{(j,k) \in \mathcal{P}_i} (P_{jk}^2 + Q_{jk}^2) \quad (\text{A3})$$

Second, the branch power flows P_{jk} and Q_{jk} are the aggregate of nodal power injections at all downstream nodes $\mathcal{D}_k$. By the triangle inequality:

$$|P_{jk}| = \left| \sum_{n \in \mathcal{D}_k} p_n \right| \leq \sum_{n \in \mathcal{D}_k} |p_n| \leq \|\mathbf{p}\|_1 \quad (\text{A4})$$

Similarly, $|Q_{jk}| \leq \|\mathbf{q}\|_1$. Substituting these into the error expression:

$$|[\epsilon_{loss}]_i| \leq C_v Z_{max} |\mathcal{P}_i| (\|\mathbf{p}\|_1^2 + \|\mathbf{q}\|_1^2) \quad (\text{A5})$$

where $|\mathcal{P}_i|$ is the path length (number of branches). Let $D_{net} = \max_i |\mathcal{P}_i|$ be the network diameter.

Finally, utilizing the equivalence of norms in finite-dimensional vector spaces, specifically $\|\mathbf{x}\|_1 \leq \sqrt{N} \|\mathbf{x}\|_2$, we have:

$$\|\mathbf{p}\|_1^2 \leq N \|\mathbf{p}\|_2^2, \quad \|\mathbf{q}\|_1^2 \leq N \|\mathbf{q}\|_2^2 \quad (\text{A6})$$

Summing over all nodes $i \in \mathcal{N}$ and taking the Euclidean norm of the error vector ϵ_{loss} , there exists a unified constant κ (dependent on N, C_v, Z_{max}, D_{net}) such that:

$$\|\epsilon_{loss}\| \leq \kappa (\|\mathbf{p}\|^2 + \|\mathbf{q}\|^2) \quad (\text{A7})$$

This concludes the proof. $\square$

Appendix C. Proof of Theorem 1

Proof. Let λ_i be an eigenvalue of $\mathbf{G}$. Then the corresponding eigenvalue of $\mathbf{M}_\alpha$ is $\mu_i = 1 - \alpha \lambda_i$.

For convergence, we require $|\mu_i| < 1$ for all i . Therefore, we have:

$$\begin{aligned} |1 - \alpha \lambda_i|^2 < 1 &\implies (1 - \alpha \operatorname{Re}(\lambda_i))^2 + (\alpha \operatorname{Im}(\lambda_i))^2 < 1 \\ 1 - 2\alpha \operatorname{Re}(\lambda_i) + \alpha^2 |\lambda_i|^2 &< 1 \\ \alpha |\lambda_i|^2 < 2\operatorname{Re}(\lambda_i) &\implies \alpha < \frac{2\operatorname{Re}(\lambda_i)}{|\lambda_i|^2}. \end{aligned} \quad (\text{A8})$$

To ensure stability for all modes, α must satisfy the condition for the “worst-case” eigenvalue, leading to (21). $\square$

Appendix D. Proof of Corollary 1

Proof. The MF-OGD update law is given by $\mathbf{q}_{t+1} = \mathbf{q}_t - \alpha \tilde{\mathbf{X}}^\top (\mathbf{v}_t - \mathbf{v}_{ref})$. Substituting the linearized plant model $\mathbf{v}_t \approx \mathbf{X} \mathbf{q}_t + \mathbf{v}_{open}$, the error dynamics $\mathbf{e}_t = \mathbf{q}_t - \mathbf{q}^*$ evolve as:

$$\mathbf{e}_{t+1} = (\mathbf{I} - \alpha \tilde{\mathbf{X}}^\top \mathbf{X}) \mathbf{e}_t \quad (\text{A9})$$

For asymptotic stability, all eigenvalues μ_i of the product matrix $\mathbf{A} = \tilde{\mathbf{X}}^\top \mathbf{X}$ must satisfy the discrete-time stability condition:

$$|1 - \alpha\mu_i(\mathbf{A})| < 1, \quad \forall i \quad (\text{A10})$$

Let the nominal matrix be $\mathbf{A}_0 = \tilde{\mathbf{X}}^\top \tilde{\mathbf{X}}$. Since $\tilde{\mathbf{X}}$ is full rank, $\mathbf{A}_0$ is symmetric positive definite with real eigenvalues $\lambda_i \in [\lambda_{\min}, \lambda_{\max}]$, where $\lambda_{\max} = \|\tilde{\mathbf{X}}\|_2^2$. Consider the model mismatch $\mathbf{X} = \tilde{\mathbf{X}} + \Delta$, where $\|\Delta\|_2 \leq \epsilon_{\text{rel}} \|\tilde{\mathbf{X}}\|_2$. The actual matrix is:

$$\mathbf{A} = \tilde{\mathbf{X}}^\top (\tilde{\mathbf{X}} + \Delta) = \mathbf{A}_0 + \mathbf{E}, \quad \text{where } \mathbf{E} = \tilde{\mathbf{X}}^\top \Delta \quad (\text{A11})$$

The perturbation norm is bounded by:

$$\|\mathbf{E}\|_2 \leq \|\tilde{\mathbf{X}}\|_2 \|\Delta\|_2 \leq \epsilon_{\text{rel}} \|\tilde{\mathbf{X}}\|_2^2 = \epsilon_{\text{rel}} \lambda_{\max}(\mathbf{A}_0) \quad (\text{A12})$$

According to the Bauer–Fike Theorem, every eigenvalue $\mu_i(\mathbf{A})$ lies within a disk in the complex plane centered at some eigenvalue $\lambda_j(\mathbf{A}_0)$ with radius $\|\mathbf{E}\|_2$. Thus, the magnitude of any eigenvalue μ is bounded by:

$$|\mu| \leq \lambda_{\max}(\mathbf{A}_0) + \|\mathbf{E}\|_2 \leq (1 + \epsilon_{\text{rel}}) \|\tilde{\mathbf{X}}\|_2^2 \quad (\text{A13})$$

To ensure stability condition (A10), we specifically require the step size to satisfy:

$$\alpha|\mu| < 2 \cos(\phi) \implies \alpha|\mu|_{\max} < 2 \quad (\text{A14})$$

Substituting the proposed step size $\alpha = \frac{\gamma}{\|\tilde{\mathbf{X}}\|_2^2 + \eta} \approx \frac{\gamma}{\|\tilde{\mathbf{X}}\|_2^2}$:

$$\frac{\gamma}{\|\tilde{\mathbf{X}}\|_2^2} \cdot (1 + \epsilon_{\text{rel}}) \|\tilde{\mathbf{X}}\|_2^2 < 2 \implies \gamma < \frac{2}{1 + \epsilon_{\text{rel}}} \quad (\text{A15})$$

This condition $\gamma < \frac{2}{1 + \epsilon_{\text{rel}}}$ is the theoretical upper bound for the worst-case gain. However, the Bauer–Fike disk may also shift eigenvalues towards the imaginary axis (introducing phase lag). To provide a conservative robust margin against phase uncertainty, we tighten the bound by scaling with the lower uncertainty limit $(1 - \epsilon_{\text{rel}})$, leading to the sufficient condition stated in Corollary 3.3:

$$\gamma < \frac{2(1 - \epsilon_{\text{rel}})}{1 + \epsilon_{\text{rel}}} \quad (\text{A16})$$

Under this condition, the eigenvalues of the closed-loop system are guaranteed to remain strictly inside the unit circle, ensuring robust stability. $\square$

Appendix E. Proof of Theorem 2

Proof. Let $\mathbf{u}_t = \mathbf{q}_t - \alpha \hat{\nabla} f_t(\mathbf{q}_t)$ be the result of the gradient step before projection. By the non-expansive property of the projection operator $\mathcal{P}_{\mathcal{Q}}$, we have:

$$\|\mathbf{q}_{t+1} - \mathbf{q}_t^*\| = \|\mathcal{P}_{\mathcal{Q}}[\mathbf{u}_t] - \mathcal{P}_{\mathcal{Q}}[\mathbf{q}_t^*]\| \leq \|\mathbf{u}_t - \mathbf{q}_t^*\|. \quad (\text{A17})$$

Note that since $\mathbf{q}_t^*$ minimizes f_t (based on the approximate quadratic model), it satisfies the first-order optimality condition. The ideal gradient is $\nabla f_t(\mathbf{q}_t^*) = \mathbf{0}$.

Unlike standard analysis, we model the measured voltage as the linear approximation plus a structural non-linear error term $\epsilon_{\text{loss}}(\mathbf{q})$ (as derived in Lemma 2.3):

$$\mathbf{v}_{\text{meas},t} = \mathbf{R}\mathbf{p}_t + \mathbf{X}\mathbf{q}_t + v_0\mathbf{1} + \epsilon_{\text{loss}}(\mathbf{q}_t). \quad (\text{A18})$$

The feedback gradient used in the update is:

$$\hat{\nabla} f_t(\mathbf{q}_t) = \tilde{\mathbf{X}}^\top (\hat{\mathbf{v}}_t - \mathbf{v}_{\text{ref}}) + \eta \mathbf{q}_t. \quad (\text{A19})$$

Substitute this into the error term $\mathbf{u}_t - \mathbf{q}_t^*$:

$$\mathbf{u}_t - \mathbf{q}_t^* = \mathbf{q}_t - \mathbf{q}_t^* - \alpha[\tilde{\mathbf{X}}^\top (\mathbf{R}\mathbf{p}_t + \mathbf{X}\mathbf{q}_t + \mathbf{v}_0 + \varepsilon_{loss}(\mathbf{q}_t) - \mathbf{v}_{ref}) + \eta\mathbf{q}_t]. \quad (\text{A20})$$

We add and subtract the optimal gradient terms. Recognizing that $\mathbf{v}_{meas,t}$ implicitly contains $\mathbf{X}\mathbf{q}_t$, we can rearrange the linear and non-linear parts:

$$\mathbf{u}_t - \mathbf{q}_t^* = (\mathbf{I} - \alpha(\tilde{\mathbf{X}}^\top \mathbf{X} + \eta\mathbf{I}))(\mathbf{q}_t - \mathbf{q}_t^*) - \alpha\tilde{\mathbf{X}}^\top (\varepsilon_{loss}(\mathbf{q}_t) - \varepsilon_{loss}(\mathbf{q}_t^*)) + \delta_{bias}, \quad (\text{A21})$$

where δ_{bias} represents static model mismatch terms constant with respect to $\mathbf{q}_t$. Taking the norm:

$$\|\mathbf{u}_t - \mathbf{q}_t^*\| \leq \|\mathbf{I} - \alpha(\tilde{\mathbf{X}}^\top \mathbf{X} + \eta\mathbf{I})\|_2 \|\mathbf{q}_t - \mathbf{q}_t^*\| + \alpha\|\tilde{\mathbf{X}}\|_2 \|\varepsilon_{loss}(\mathbf{q}_t) - \varepsilon_{loss}(\mathbf{q}_t^*)\| + \alpha\|\delta_{bias}\|. \quad (\text{A22})$$

Applying Assumption 3 (Linear Contraction ρ) and the Lipschitz property of the linearization error ($\|\varepsilon_{loss}(\mathbf{q}) - \varepsilon_{loss}(\mathbf{q}')\| \leq L_{loss}\|\mathbf{q} - \mathbf{q}'\|$):

$$\begin{aligned} \|\mathbf{u}_t - \mathbf{q}_t^*\| &\leq \rho\|\mathbf{q}_t - \mathbf{q}_t^*\| + \alpha\|\tilde{\mathbf{X}}\|_2 L_{loss}\|\mathbf{q}_t - \mathbf{q}_t^*\| + \alpha C_{bias} \\ &= (\rho + \alpha\|\tilde{\mathbf{X}}\|_2 L_{loss})\|\mathbf{q}_t - \mathbf{q}_t^*\| + \alpha C_{bias}. \end{aligned} \quad (\text{A23})$$

Let $\hat{\rho} = \rho + \alpha\|\tilde{\mathbf{X}}\|_2 L_{loss}$. For stability, we require α to be sufficiently small such that $\hat{\rho} < 1$. Considering the time-varying optimum (Assumption 2, drift σ):

$$\|\mathbf{q}_{t+1} - \mathbf{q}_{t+1}^*\| \leq \hat{\rho}\|\mathbf{q}_t - \mathbf{q}_t^*\| + \sigma + \alpha C_{bias}. \quad (\text{A24})$$

Recursively applying this inequality yields the asymptotic bound:

$$\limsup_{T \rightarrow \infty} \|\mathbf{q}_T - \mathbf{q}_T^*\| \leq \frac{\sigma + \alpha C_{bias}}{1 - \hat{\rho}}. \quad (\text{A25})$$

This completes the proof. $\square$

Appendix F. Proof of Corollary 2

Proof. Theorem 2 established the convergence of the algorithm under ideal measurements. Here, we extend the analysis to the case with bounded measurement noise ξ_t and linearization error ε_t . Let the aggregate disturbance be $\delta_t = \tilde{\mathbf{X}}^\top \xi_{meas,t} + \varepsilon_{loss,t}$. The gradient estimation becomes:

$$\hat{\nabla} f_t = \nabla f_t(\mathbf{q}_t) + \delta_t \quad (\text{A26})$$

Consider the Lyapunov function $V_t = \|\mathbf{q}_t - \mathbf{q}^*\|^2$. Following the steps in Theorem 2, the expansion yields:

$$V_{t+1} \leq V_t - 2\alpha(\mathbf{q}_t - \mathbf{q}^*)^\top (\nabla f_t + \delta_t) + \alpha^2 \|\nabla f_t + \delta_t\|^2 \quad (\text{A27})$$

Using the strong convexity (μ) and smoothness (L) properties of the cost function:

$$(\mathbf{q}_t - \mathbf{q}^*)^\top \nabla f_t \geq \mu\|\mathbf{q}_t - \mathbf{q}^*\|^2 \quad (\text{A28})$$

Using Young's inequality ($2\mathbf{a}^\top \mathbf{b} \leq \gamma\|\mathbf{a}\|^2 + \frac{1}{\gamma}\|\mathbf{b}\|^2$) to handle the cross terms involving noise δ_t , we derive:

$$V_{t+1} \leq (1 - 2\alpha\mu + \alpha^2 L^2 + \alpha\gamma)V_t + \alpha\left(\frac{1}{\gamma} + \alpha\right)\|\delta_t\|^2 \quad (\text{A29})$$

Let $\rho = 1 - (2\alpha\mu - \alpha^2 L^2 - \alpha\gamma)$. For sufficiently small α , $0 < \rho < 1$. Iterating this inequality:

$$V_t \leq \rho^t V_0 + \frac{\alpha C}{1 - \rho} \sup_{\tau < t} \|\delta_\tau\|^2 \quad (\text{A30})$$

As $t \rightarrow \infty$, the first term vanishes, and the squared tracking error is bounded by:

$$\limsup_{t \rightarrow \infty} \|\mathbf{q}_t - \mathbf{q}^*\| \leq \frac{C'}{\sqrt{\mu}} \Delta_{\xi} \quad (\text{A31})$$

where Δ_{ξ} is the upper bound of the noise and error terms. This completes the proof of Corollary 2. $\square$

Appendix G. Equation Interrelationship

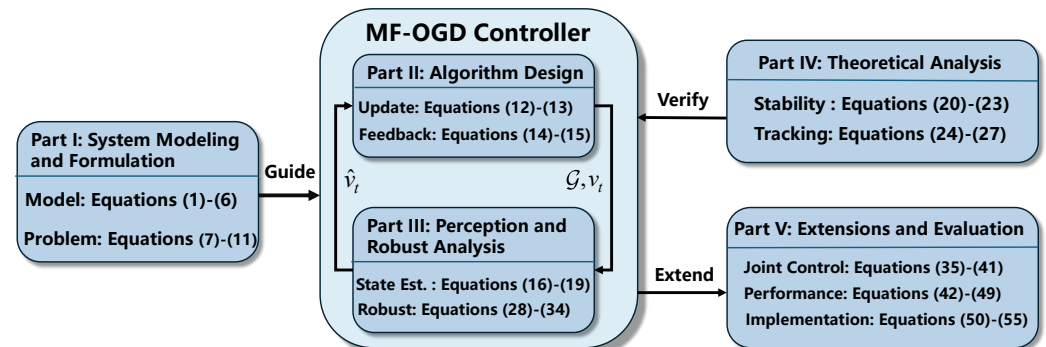


Figure A2. Interrelationships of equations.

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