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Multi-Time-Scale Secure and Economic Dispatch of Active Distribution Networks Integrating Converter Operating Characteristic Constraints

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Abstract

The increasing penetration of renewable energy in active distribution networks introduces severe power fluctuations and uncertainties. This challenges traditional scheduling methods that rely on conservative and static capacity boundaries of converter-interfaced equipment. This paper proposes a multi-time-scale secure and economic dispatch framework that explicitly integrates the dynamic operating characteristic constraints of grid-forming converters. First, a spatial correlation model based on Copula theory is established to handle wind power uncertainties via scenario generation and reduction for the day-ahead and intra-day scheduling phases. This formulation aims to minimize the comprehensive operational costs of the system. For the real-time rolling optimization phase, the short-term overload potential of grid-forming converters, which is unlocked by electro-thermal coupling optimization and discontinuous pulse width modulation phase-shift clamping, is mathematically abstracted into a generalized dynamic elliptical active and reactive power capability envelope. This cross-scale mapping mechanism allows the system to utilize transient thermal margins for enhanced local reactive power support without violating device junction temperature limits. Furthermore, the non-convex scheduling model is transformed into a mixed-integer second-order cone programming problem using convex relaxation techniques to guarantee global optimality and computational efficiency. Comprehensive case studies on the modified IEEE 33-bus and 69-bus systems demonstrate that the proposed strategy reduces tie-line power fluctuations and operational costs under extreme conditions. The results achieve a favorable economic trade-off between brief power quality degradation and global physical stability.



Academic Editor: Jen-Hao Teng

Received: 5 June 2026

Revised: 1 July 2026

Accepted: 3 July 2026

Published: 9 July 2026

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Keywords: active distribution network; grid-forming converter; multi-time-scale dispatch; dynamic capability envelope; electro-thermal coupling; coordinated scheduling

1. Introduction

With the continuous integration of renewable energy at the distribution network level, the scheduling and control of distribution networks have become increasingly complex [1–3]. Traditional distribution networks are gradually evolving into active distribution networks possessing numerous controllable resources. The primary challenge of renewable energy lies in its inherent uncertainty and randomness, particularly regarding wind and

solar power, which significantly impacts the stable operation of distribution networks. Moreover, substantial fluctuations in the output of large-scale renewable energy sources can lead to noticeable power fluctuations at the interface between the distribution and transmission networks, affecting the operation of the transmission network and imposing significant stress on the overall grid [4,5].

To address this issue, extensive research has been conducted on time scales and optimization methods. In terms of time scales, a multi-period day-ahead optimization scheduling method has been proposed [6]. This method controls the operational status of various distributed resources within the distribution network by segmenting the time periods, allowing for a more detailed estimation of renewable energy output. However, the discrepancies between the results obtained from a single offline day-ahead optimization and actual conditions can be substantial [7,8]. Based on the characteristic that the accuracy of renewable energy power forecasts improves as the prediction time scale decreases, literature [9] introduced a multi-time-scale optimization scheduling method. By continuously providing the optimization program with updated forecast data at shorter time scales, the differences between scheduling results and actual conditions are significantly reduced, further addressing the uncertainties and fluctuations associated with photovoltaic output and load demand.

To effectively respond to the uncertainties of renewable energy within active distribution networks, traditional deterministic optimization models are increasingly deemed inadequate, leading researchers to focus on uncertainty optimization models [10–12]. Among these, stochastic optimization has found widespread application, typically employing scenario methods to simulate uncertain variables across multiple scenarios [13–15]. To alleviate the computational burden, scenario reduction techniques are used to obtain representative scenarios for optimized scheduling, ultimately computing expectations for decision-making. Literature [16] utilized scenario methods to characterize the uncertainty of wind power output and to facilitate the scheduling of unit output and the sizing of energy storage devices. In contrast to transmission networks, the resistance and reactance values (R/X ratio) in distribution networks are relatively close, resulting in strong coupling between active and reactive power [17–19]. Therefore, unilateral optimization of active or reactive power is insufficient for a comprehensive approach. Coordinated optimization of active and reactive power has thus become a mainstream method. Coordinated optimization models aimed at minimizing the total generation cost of the power system have been established and have achieved favorable results [20,21]. In these coordinated optimization strategies, the capability of the system to maintain voltage stability and power balance highly depends on the reactive power compensation and active power regulation performance of the underlying power electronic equipment.

However, existing multi-time-scale coordinated scheduling models still face a significant limitation. To reduce the computational complexity in traditional mathematical programming models, they typically treat the operating capacity of controllable distributed generators and converters as rigid fixed boundaries. In actual operation, when sudden changes in wind power output trigger severe voltage fluctuations, the system requires the underlying equipment to respond instantaneously and output a large amount of reactive power. Modern power electronic devices, particularly grid-forming converters, actually possess short-term overload support capabilities that exceed their rated currents. Through advanced control techniques, the active and reactive operating boundaries of these devices can be effectively broadened at the macroscopic level. Traditional scheduling models based on static boundaries ignore this transient regulation potential, which not only leads to low capacity utilization of the underlying equipment and a waste of investment costs but also causes the scheduling model to fail in providing effective power commands when

the system faces extreme conditions because it reaches overly conservative static capacity limits. This eventually leads the system into regulation bottlenecks or even the risk of voltage collapse.

To effectively manage the uncertainty of renewable energy output and bridge the aforementioned research gaps, this paper establishes a correlation model for the output of multiple wind farms within the distribution network based on Copula theory, thoroughly considering the correlations among various wind farms and across different time periods [22–24]. Furthermore, this paper proposes a coordinated scheduling method for active distribution networks for multi-time-scale active and reactive power management, taking into account the dynamic support boundaries of converters. This method combines time scales and optimization approaches, selecting different optimization methods based on varying prediction accuracy across the time scales. Stochastic optimization methods based on scenario techniques are utilized in the day-ahead and intra-day scheduling phases, while deterministic optimization methods are employed in the real-time phase to optimize the distribution network scheduling [25,26]. Additionally, a rolling optimization approach is implemented during real-time scheduling, wherein only the decision values for the first time period are executed in each optimization cycle, necessitating a re-evaluation of the optimal control sequence at the next optimization moment [27]. This paper innovatively abstracts the short-term overload characteristics of grid-forming converters into dynamic boundaries at the real-time scheduling level. By constructing a generalized dynamic active and reactive capability envelope, the macroscopic gains brought by various underlying control strategies are transformed into time-dependent dynamic inequality constraints. This cross-scale mapping mechanism successfully breaks down the barriers between the execution capability of the underlying physical equipment and the high-level mathematical scheduling of the system. Meanwhile, to handle potential conflicts between active and reactive power commands within the expanded operating domain, this paper proposes a method for reasonably setting the optimization objective weights for each time period to further mitigate power fluctuations between the distribution and transmission networks.

In comparison with previous studies, the main contributions are as follows:

The method introduces a coordinated scheduling framework that integrates day-ahead, intra-day, and real-time optimization phases, where each phase employs different optimization techniques based on the accuracy of forecasting data, and incorporates device-level dynamic physical security boundaries, thereby enhancing the adaptability of the distribution network to extreme renewable energy fluctuations.

By constructing a dynamic operating boundary model based on the short-term overload characteristics of grid-forming converters, the limitations of traditional static capacity constraints are broken. This model effectively utilizes the transient support potential under uncertain conditions related to renewable energy, optimizes the operation of distributed resources in active distribution networks, significantly improves device capacity utilization, and minimizes operational costs.

In the real-time scheduling phase, a deterministic optimization based on adaptive weights is employed. By reasonably allocating the weights of power commands under complex conditions, it minimizes power fluctuations between the distribution and transmission networks, ensuring operational stability and maintaining the execution reliability of scheduling commands under extreme power fluctuations.

2. Multi-Time-Scale Scheduling Frameworks and Uncertainty Modelling

2.1. Multi-Time-Scale Coordinated Scheduling Framework for ADN

With the high penetration of wind power, a single day-ahead static scheduling is insufficient to handle the short-term severe fluctuations in wind output. Therefore, this paper

constructs a multi-time-scale coordinated scheduling framework for active distribution networks (ADN) comprising day-ahead (DA), intra-day (ID), and real-time (RT) phases. Based on the variations in forecasting accuracy across different time scales, the framework refines scheduling commands step-by-step to achieve spatial–temporal decoupling and coordination of control variables.

Day-Ahead Scheduling Phase: The time resolution is set to 1 h with a scheduling horizon of 24 h. Based on day-ahead forecast data and typical scenarios of wind power and load, this phase aims to minimize the comprehensive daily operational cost of the ADN. The primary decisions involve slow-dynamic devices with restricted switching frequencies, such as the tap positions of On-Load Tap Changers (OLTC) and the states of Capacitor Banks (CB). The day-ahead results serve as baseline reference trajectories for the intra-day phase.

Intra-Day Scheduling Phase: The time resolution is shortened to 15 min with a scheduling horizon of 4 h. Utilizing updated ultra-short-term forecast data, this phase aims to minimize operational costs by finely adjusting the outputs of fast-response resources, including controllable distributed generators (CDGs), energy storage systems (ESS), and static var compensators (SVC). This adjustment mitigates system power fluctuations caused by day-ahead forecasting errors.

Real-Time Scheduling Phase: The time resolution is refined to 5 min with a scheduling horizon of 1 h. This phase employs a deterministic rolling optimization model focusing on minimizing power fluctuations at the interface between the distribution and transmission networks. Notably, the dynamic operating boundary constraints of grid-forming converters are fully activated during this period. By exploiting the transient support potential of the underlying equipment, the framework ensures reliable global control capabilities even under extreme wind power disturbances, as shown as Figure 1.

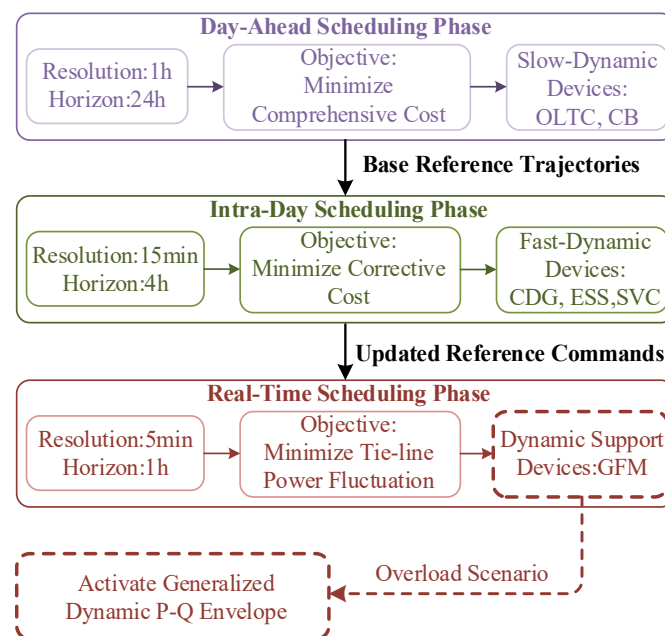


Figure 1. The framework of multi-time-scale coordinated scheduling for active distribution networks considering dynamic support.

2.2. Wind Power Spatial Correlation Modeling Based on Copula Theory

Multiple wind farms within an ADN are typically geographically proximate, exhibiting significant spatial–temporal coupling and tail correlations. Conventional models assuming independent distributions tend to underestimate the probability of extreme conditions. To

address this, Copula theory is introduced to construct a joint probability distribution model for multi-wind-farm outputs.

Assuming an ADN contains n wind farms with marginal cumulative distribution functions $F_1(x_1), F_2(x_2), \dots, F_n(x_n)$ respectively. According to Sklar's theorem, the joint cumulative distribution function $H(x_1, x_2, \dots, x_n)$ of these n random variables can be uniquely determined by a Copula function $C(\cdot)$:

$$H(x_1, x_2, \dots, x_n) = C(F_1(x_1), F_2(x_2), \dots, F_n(x_n)) \quad (1)$$

Given the strong symmetrical tail correlation of wind power output during extremely high or low periods, the t -Copula function is selected to accurately characterize this nonlinear dependence. The probability density function of the t -Copula is expressed as:

$$c(u_1, u_2, \dots, u_n; \rho, \nu) = \frac{f_{\rho, \nu}(t_{\nu}^{-1}(u_1), \dots, t_{\nu}^{-1}(u_n))}{\prod_{i=1}^n f_{\nu}(t_{\nu}^{-1}(u_i))} \quad (2)$$

where ρ denotes the correlation coefficient matrix, ν represents the degrees of freedom, $t_{\nu}^{-1}(\cdot)$ is the inverse of the univariate t -distribution with ν degrees of freedom, and $f_{\rho, \nu}(\cdot)$ is the joint probability density function. By fitting historical output data and solving the model parameters, a high-dimensional joint distribution of wind power output considering spatial correlation is obtained.

Copula theory is suitable for this problem because it can separately model the marginal distribution of each wind farm output and the dependence structure among multiple wind farms. Therefore, it can effectively describe the non-Gaussian, nonlinear, and spatially correlated characteristics commonly observed in renewable power outputs. Existing studies also provide theoretical and practical support for the use of Copula-based methods in this paper. For example, Papaefthymiou and Kurowicka applied Copulas to model stochastic dependence in power system uncertainty analysis, demonstrating the applicability of Copulas in power-system correlation modeling. Tang et al. further employed Copula theory to characterize the spatial and temporal correlations among multiple renewable power plants and applied it to the economic dispatch problem involving multiple renewable power plants. Werho et al. estimated a Copula model based on historical wind farm data for real-time wind power scenario generation, aiming to preserve the spatial and temporal correlations among wind farms. In addition, Pinson et al. pointed out that wind power scenario generation should preserve not only the marginal probabilistic distributions of individual wind farms, but also the dependence structure and joint fluctuation characteristics among wind power outputs [28–31].

To further validate the selection of the Copula function and its parameter estimation, a data-driven fitting procedure is introduced based on historical wind power output data. First, the historical outputs of multiple wind farms are normalized by their installed capacities. Then, the empirical cumulative distribution function is used to estimate the marginal distribution of each wind farm. According to the probability integral transformation, the original wind power samples are transformed into uniformly distributed variables in the interval $[0, 1]$. Based on these transformed samples, several candidate Copula functions, including Gaussian Copula, t -Copula, Clayton Copula, Gumbel Copula, and Frank Copula, are fitted and compared. The parameters of each candidate Copula are estimated using the maximum likelihood estimation method. The fitting performance is evaluated using the log-likelihood value, Akaike information criterion (AIC), Bayesian information criterion (BIC), and goodness-of-fit test.

As shown in Table 1, the t-Copula achieves the highest log-likelihood value, the lowest AIC and BIC values, and the highest goodness-of-fit p -value among the candidate Copula functions. Therefore, the t-Copula is selected to model the spatial dependence among multiple wind farms. This is mainly because the t-Copula can capture not only the general dependence among wind farms, but also the symmetric tail dependence under simultaneous high-output or low-output conditions, which is important for representing extreme wind-power fluctuation scenarios in dispatch studies.

Table 1. Fitting comparison of different Copula functions based on historical wind power data.

Copula Type	Estimated Parameters	Log-Likelihood	AIC	BIC	Goodness-of-Fit p -Value	Evaluation Result
Gaussian Copula	$\rho = 0.64$	412.8	−817.6	−801.2	0.041	Acceptable
t-Copula	$\rho = 0.66$, $\nu = 5.8$	457.6	−905.2	−884.7	0.184	Best
Clayton Copula	$\theta = 1.21$	355.9	−703.8	−687.4	0.018	Poor
Gumbel Copula	$\theta = 1.47$	383.5	−759.0	−742.6	0.033	Poor
Frank Copula	$\theta = 4.36$	398.7	−789.4	−773.0	0.062	Acceptable

Furthermore, to verify whether the selected t-Copula model can reproduce the statistical dependence observed in historical wind power data, the generated wind power scenarios are compared with the historical samples. The validation indicators include the Pearson correlation coefficient, Kendall rank correlation coefficient, joint high-output probability, joint low-output probability, and ramping distribution characteristics.

As shown in Table 2, the generated scenarios closely match the historical wind power samples in terms of Pearson correlation, Kendall rank correlation, joint high-output and low-output probabilities, and ramping characteristics. This indicates that the selected t-Copula model can effectively preserve the spatial dependence and joint fluctuation characteristics of multiple wind farms. Therefore, the Copula function used in this paper is not selected subjectively, but is determined through historical-data-based marginal distribution fitting, maximum-likelihood parameter estimation, model comparison, and statistical validation.

Table 2. Statistical comparison between generated wind power scenarios and historical wind power samples.

Validation Indicator	Historical Samples	Generated Scenarios	Relative Error
Mean pairwise Pearson correlation coefficient	0.642	0.631	1.71%
Mean pairwise Kendall rank correlation coefficient	0.472	0.464	1.69%
Joint high-output probability, $P(P_1 > 0.8, P_2 > 0.8)$	0.116	0.109	6.03%
Joint low-output probability, $P(P_1 < 0.2, P_2 < 0.2)$	0.138	0.132	4.35%
95th percentile of normalized ramping magnitude	0.184	0.191	3.80%

2.3. Wind Power Typical Scenario Generation and Reduction

In the day-ahead and intra-day scheduling phases, a scenario-based approach is adopted to transform the continuous uncertainty of wind power output into a discrete set of deterministic scenarios for optimization.

First, based on the aforementioned t-Copula joint distribution model, the Latin Hypercube Sampling (LHS) method is employed to generate a set of N initial scenarios. However, a massive set of raw scenarios would lead to the “curse of dimensionality” and a surge in computational time if directly integrated into the optimization model.

Consequently, the simultaneous backward reduction method based on the Kantorovich distance is applied to reduce the raw scenarios. The Kantorovich distance quantifies the difference in probability measures between any two scenarios s_i and s_j , defined as:

$$D_K(s_i, s_j) = \sum_{t=1}^T |P_{w,i}^t - P_{w,j}^t| \quad (3)$$

where $P_{w,i}^t$ and $P_{w,j}^t$ denote the forecasted active power output of wind power in scenarios i and j at time t , respectively.

The algorithm iteratively eliminates redundant scenarios with the smallest Kantorovich distances and minimal probabilities, accumulating the probabilities of the deleted scenarios into the nearest remaining scenario. Ultimately, a set of K highly representative typical scenarios is obtained. Each remaining typical scenario s corresponds to a modified probability p_s , satisfying $\sum_{s=1}^K p_s = 1$. These typical scenarios and their probabilities serve as input parameters to drive the stochastic programming models in the day-ahead and intra-day phases.

3. Analysis of the Dynamic Operating Boundary Characteristics of Power Converters

In traditional distribution network scheduling, controllable distributed generators (CDGs) and converter interface devices are typically modeled as ideal power sources with constant capacities. However, this static modeling approach severs the inherent connection between the system-level macroscopic output capability and the underlying physical thermal characteristics of the devices. This section aims to reveal the transient physical support potential of grid-forming (GFM) converters under extreme conditions and rigorously abstract it into dynamic operating boundaries within the real-time scheduling model.

3.1. Physical Limitations of Traditional Static Capacity Constraints

In the original multi-time-scale coordinated optimization models, the active power $P_{\text{CDG},i,t}$ and reactive power $Q_{\text{CDG},i,t}$ injected into the distribution network by the converter are strictly limited by its rated apparent power $S_{\text{CDG},i,t}$. Its operating domain manifests as a fixed static circular boundary on the P-Q plane. This static model implies a highly conservative premise, assuming that the thermal stress on the internal power devices of the converter is uniform and unadjustable under any power factor. When the distribution network faces severe voltage sags due to intense renewable energy fluctuations, the system urgently requires massive reactive currents to maintain stability. If the static circular boundary is strictly followed, the scheduling center will often fail to issue sufficient commands due to reaching the fixed capacity limit, causing the underlying equipment to fall into a regulation bottleneck of “having transient thermal margin but unable to output”.

3.2. Electro-Thermal Coupling and Transient Thermal Limits of Converters

The extreme output capability of modern GFM converters is not a rigid constant; rather, it is deeply coupled with the underlying pulse width modulation (PWM) strategies and the transient thermal impedance of the devices. The core operational bottleneck of a converter is the maximum allowable junction temperature $T_{j,\max}$ of the power semiconductor devices (e.g., IGBT modules). The real-time junction temperature $T_j(t)$ of the device is co-determined by the cooling conditions, the transient thermal impedance, and the total power loss P_{loss} , which mainly consists of conduction losses P_{cond} and switching losses P_{sw} .

Under overcurrent scenarios triggered by extreme system fluctuations, if the underlying control strategy remains unchanged, the surging output current will cause P_{cond} and P_{sw} to increase simultaneously. The junction temperature will rapidly approach $T_{j,\max}$, thereby triggering hardware thermal protection. Therefore, the physical essence of broadening the macroscopic P-Q operating boundary lies in significantly reducing P_{sw} through advanced control techniques without violating the maximum junction temperature con-

straints. This reduction frees up precious transient thermal margins for P_{cond} (i.e., enabling a larger output current).

3.3. Physical Examples of Advanced Control Strategies Broadening Operating Boundaries

To intuitively illustrate how transient thermal margins are transformed into power boundaries, this section provides an in-depth analysis of two typical underlying control techniques employed by converters under extreme conditions:

Switching Frequency Reduction: The switching loss P_{sw} of an IGBT is strictly proportional to the system's switching frequency f_{sw} . During the brief tens of seconds or minutes when the system requires emergency support, if the control system abruptly reduces the switching frequency from the rated 10 kHz to 5 kHz, the P_{sw} will instantly be halved. In the electro-thermal coupling network, this operation manifests as a significant decrease in the rate of junction temperature rise. Utilizing this thermal dissipation space saved by frequency reduction, the converter can safely output a transient large current exceeding the rated value, thereby macroscopically achieving a direct “outward expansion” of the capacity boundary.

DPWM Phase-Shift Clamping Mechanism: In traditional continuous modulation such as SVPWM, devices switch at high frequencies throughout the fundamental period. When the system outputs significant reactive power, the phase difference between the current peak and the voltage peak approaches 90° , leading to massive peak switching losses at the current peak under traditional switching. The Discontinuous Pulse Width Modulation (DPWM) strategy introduced herein does not aim to directly modify or limit the reference current amplitude. Instead, it utilizes specific phase-shift operations to precisely align the peak region of the transient AC current with the clamping region of the converter, which is a 60° —wide interval with no switching actions. As shown in Figure 2, this alignment ensures that the device is in a fully conducting or blocking clamped state ($P_{\text{sw}} \approx 0$) while enduring the maximum transient current. By eliminating high switching losses under worst-case scenarios at their physical root, this precise phase-shift alignment drastically reduces peak junction temperature fluctuations, endowing the equipment with an extremely high short-term overload capability.

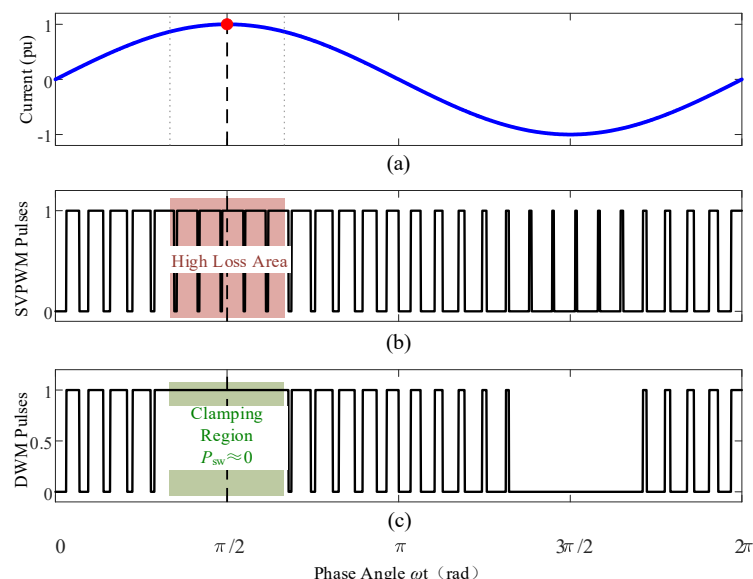


Figure 2. Schematic Diagram of DPWM Phase-Shift Clamping for Switching Loss Reduction: (a) Transient AC Current Waveform. (b) Traditional Continuous Modulation. (c) Proposed Discontinuous Modulation.

3.4. Generalized Dynamic P-Q Capability Envelope Model and Parameter Calibration

To establish a cross-scale mapping bridging underlying physical current constraints and macroscopic system-level power boundaries, this paper proposes a generalized dynamic P-Q capability envelope model. For a three-phase GFM converter, the real-time junction temperature is constrained by the following thermal balance equation:

$$T_{j,t} = T_c + \sum_{\tau=1}^t P_{loss,\tau} Z_{th}(t) \leq T_{j,max} \quad (4)$$

where $T_{j,t}$ denotes the junction temperature of the power semiconductor device, T_c is the case temperature or equivalent cooling reference temperature, and Z_{th} represents the transient thermal impedance of the device. The semiconductor loss consists of conduction loss and switching loss, which can be expressed as follows:

$$P_{cond} = V_{CE0} I_{avg} + r_{CE} I_{rms}^2 \quad (5)$$

$$P_{sw} = f_{sw,t} (E_{on} + E_{off} + E_{rec}) \quad (6)$$

where V_{CE0} and r_{CE} denote the on-state voltage coefficient and equivalent conduction resistance, respectively, while I_{avg} and I_{rms} denote the average current and root-mean-square current, respectively. $f_{sw,t}$ is the real-time switching frequency, and E_{on} , E_{off} , and E_{rec} represent the turn-on, turn-off, and reverse-recovery energies, respectively. Under conventional SVPWM, the power devices perform high-frequency switching throughout the fundamental period. When the converter outputs a large current, especially during intensive reactive power support, the peak-current interval may coincide with high-frequency switching actions, resulting in a rapid increase in instantaneous switching loss and junction temperature. To release the short-term thermal margin, this paper adopts a DPWM phase-shift clamping strategy, which aligns the current peak with the clamping interval without switching actions as much as possible, thereby reducing the switching loss in the high-current region.

$$P_{sw}^{DPWM} = \zeta(\theta_{shift,t}) f_{sw,t} (E_{on} + E_{off} + E_{rec}) \quad (7)$$

Here, $\theta_{shift,t}$ is the equivalent phase-shift angle that characterizes the alignment depth between the current peak and the clamping interval. $\zeta(\theta_{shift,t})$ is the switching-loss correction factor, whose value ranges between 0 and 1. A higher alignment degree between the current peak and the clamping interval leads to a smaller $\zeta(\theta_{shift,t})$. Given the switching frequency, DPWM phase-shift angle, cooling condition, and overload duration, the transient maximum admissible converter current I_{max} can be derived from the junction-temperature constraint. The dynamic current overload coefficient $k_{ov,t}$ is defined as the ratio of the transient maximum admissible current to the rated current:

$$k_{ov,t} = \frac{I_{max}^{transient}(f_{sw,t}, \theta_{shift,t})}{I_{rated}} \quad (8)$$

Considering the apparent power $S_t = \sqrt{3} U_t I_t$, the original static circular operating domain of the converter is expanded into a dynamic inequality constraint that is strongly correlated with the time scale and the underlying control:

$$0 \leq P_{CDG,i,t}^2 + Q_{CDG,i,t}^2 \leq \left(k_{ov,t} \cdot \frac{U_t}{U_{rated}} \cdot S_{CDG,i,t} \right)^2 \quad (9)$$

Based on the derived maximum admissible current I_{max} , the dynamic maximum apparent power under real-time operating conditions can be obtained.

$$S_{\max,t}^{\text{dynamic}} = \sqrt{3}U_t I_{\max} = k_{\text{ov},t} \frac{U_t}{U_{\text{rated}}} S_{\text{CDG},i,t} \quad (10)$$

Considering the asymmetric thermal stress caused by active and reactive currents under different power factors, the dynamic active and reactive power limits can be obtained according to the real-time converter power factor $\cos \varphi_t$:

$$P_{\max}^{\text{dynamic}} = S_{\max}^{\text{dynamic}} \cos \varphi_t \quad (11)$$

$$Q_{\max}^{\text{dynamic}} = S_{\max}^{\text{dynamic}} \sin \varphi_t \quad (12)$$

This boundary can be further refined into an elliptical envelope characterized by dynamic major and minor axes.

$$\left(\frac{P_{\text{CDG},i,t}}{P_{\max,t}^{\text{dynamic}}} \right)^2 + \left(\frac{Q_{\text{CDG},i,t}}{Q_{\max,t}^{\text{dynamic}}} \right)^2 \leq 1 \quad (13)$$

The Electro-Thermal parameters of the converter as shown as Table 3.

Through this rigorous mathematical characterization, the model successfully extracts the short-term overload potential of the converter from the underlying electro-thermal physical black box and maps it into a dynamic spatial-temporal constraint within the macroscopic scheduling layer.

As intuitively illustrated in Figure 3, the traditional operating domain of the converter is strictly confined within the static circular boundary (dashed black line) determined by its rated apparent power. However, by activating the proposed electro-thermal coupling optimization and phase-shift clamping mechanisms, the operational boundary is effectively expanded outward. The shaded blue region enclosed by the dynamic elliptical envelope vividly represents the released transient thermal margin. This morphological evolution on the P-Q plane demonstrates that the converter can legitimately reach the extended dynamic limits, $P_{\max}^{\text{dynamic}}$ and $Q_{\max}^{\text{dynamic}}$, thereby unlocking substantial active and reactive power support capabilities without triggering thermal protection.

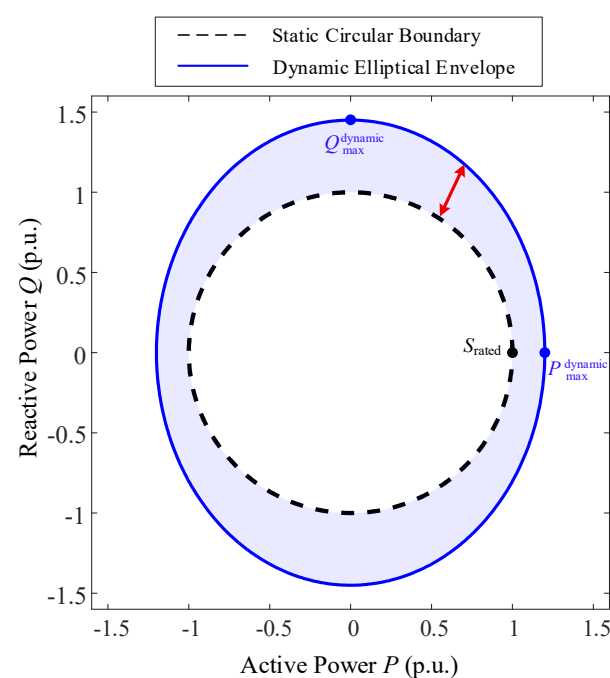


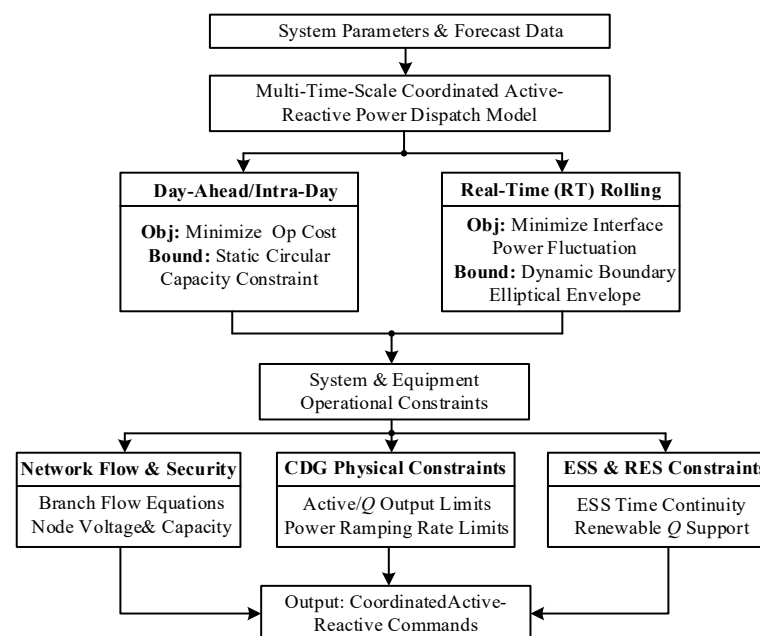
Figure 3. Extended operation domain of the converter.

Table 3. Electro-Thermal Parameters of the Converter.

Parameter	Symbol	Value
Rated apparent power	S_{rated}	1 MVA
Rated AC-side current	I_{rated}	45.6 A
Rated switching frequency	f_{sw}	10 kHz
Maximum allowable junction temperature	$T_{j,\text{max}}$	175 °C
Case temperature	T_c	70 °C

4. Problem Formulation for Secure and Economic Dispatch

This section establishes a coordinated active and reactive power optimization mathematical model for the active distribution network, covering the day-ahead, intra-day, and real-time phases. Considering the differences in control objectives and prediction uncertainties across varying time scales, differentiated objective functions and operational constraints are adopted for each phase. The complete flowchart of the multi-time-scale coordinated optimal dispatch is shown in Figure 4.

**Figure 4.** Flowchart of the multi-time-scale coordinated optimal dispatch.

4.1. Embedding Mechanism of Dynamic Boundaries in the Multi-Time-Scale Model

To achieve seamless integration with the underlying physical characteristics, the dynamic capability envelope derived in Section 3 is embedded into the global multi-time-scale framework as a conditionally activated constraint. To quantitatively determine whether the dynamic boundary should be activated in the real-time scheduling phase, a normalized utilization index of the static converter capacity is introduced as follows:

$$\lambda_{i,t} = \frac{\sqrt{P_{i,t}^2 + Q_{i,t}^2}}{S_i^{\text{rated}}} \quad (14)$$

where $P_{i,t}$ and $Q_{i,t}$ denote the active and reactive power commands of the i -th GFM converter at time t , respectively, and S_i^{rated} is the rated apparent power of the converter. The index $\lambda_{i,t}$ reflects the proximity of the converter operating point to the static circular capacity boundary.

In this paper, the dynamic elliptical envelope is activated only when the converter approaches the static capacity limit, namely:

$$\lambda_{i,t} \geq \lambda_{\text{on}} \quad (15)$$

where the activation threshold is set as $\lambda_{\text{on}} = 0.98$. This setting ensures that the dynamic boundary is not invoked under normal operating conditions, but is enabled only under near-overload or emergency conditions.

To avoid frequent switching caused by small fluctuations around the static capacity boundary, a hysteresis-based deactivation threshold is further introduced. The dynamic elliptical envelope is deactivated only when:

$$\lambda_{i,t} \leq \lambda_{\text{off}} \quad (16)$$

where $\lambda_{\text{off}} = 0.93$. Since $\lambda_{\text{on}} > \lambda_{\text{off}}$, the interval:

$$\lambda_{\text{off}} < \lambda_{i,t} < \lambda_{\text{on}} \quad (17)$$

forms a hysteresis dead zone. As shown in Figure 5, the interval forms a hysteresis dead zone.

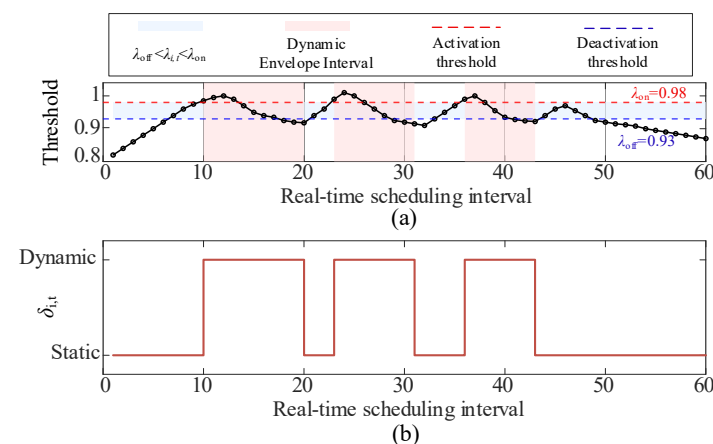


Figure 5. Hysteresis switching of the dynamic envelope. (a) Hysteresis thresholds for dynamic envelope activation (b) Boundary-state switching trajectory.

Within this dead zone, the boundary state remains unchanged from the previous real-time scheduling interval. Therefore, the proposed embedding mechanism enables the converter to release its short-term overload capability only when necessary, while preventing chattering between the static circular boundary and the dynamic elliptical envelope.

In the day-ahead and intra-day scheduling phases, to reserve sufficient physical safety margins for the system and reduce the solving complexity of stochastic optimization scenarios, the capacity of controllable distributed generators (CDGs) still employs the traditional static circular constraint. However, upon entering the real-time rolling optimization phase, when the system faces intense power fluctuations or extreme voltage conditions, the dynamic expansion coefficient is activated. The static capacity constraint in the real-time scheduling model is then replaced by the generalized dynamic elliptical envelope. This nested mechanism preserves the stable operational logic of the original scheduling architecture while endowing the system with robust physical regulation flexibility over extremely short time scales.

4.2. Phased Objective Functions

To ensure the economic operation of the system, the optimization objective for both the day-ahead and intra-day scheduling phases is to minimize the comprehensive operational cost of the distribution network. Based on the scenario method, the objective function is expressed as:

$$\min F^{DA(ID)} = \sum_{s=1}^K p_s \sum_{t=1}^T \left(C_{grid,s,t}^{DA(ID)} + \sum_{i=1}^M C_{GDG,i,s,t}^{DA(ID)} + \sum_{i=1}^N C_{ESS,i,s,t}^{DA(ID)} \right) \quad (18)$$

The superscripts *DA* and *ID* denote the day-ahead and intra-day scheduling phases, respectively. $C_{grid,s,t}^{DA(ID)}$ represents the cost of the distribution network purchasing electrical energy from the transmission network during time period *t* under scenario *s*, applying a time-of-use pricing mechanism. $C_{CDG,i,s,t}^{DA(ID)}$ indicates the generation cost of the *i*-th CDG, and $C_{ESS,i,s,t}^{DA(ID)}$ represents the operating loss cost of the *i*-th energy storage device. Let p_s denote the probability of scenario *s*, *K* represent the total number of scenarios, *M* indicate the number of CDGs, *N* represent the number of energy storage devices, and *T* denote the total number of time periods.

To mitigate the impact of renewable energy uncertainty on grid operation, the optimization objective for the real-time scheduling phase is to minimize power fluctuations at the interface between the transmission and distribution networks:

$$\min F^{RT} = \sum_{t=1}^{T_{RT}} \alpha_t \left(P_{grid,t}^{ID} - P_{grid,t}^{RT} \right)^2 + \sum_{t=1}^{T_{RT}} \alpha_t \left(Q_{grid,t}^{ID} - Q_{grid,t}^{RT} \right)^2 \quad (19)$$

The superscript *RT* denotes the real-time scheduling phase. $P_{grid,t}^{ID}$ and $Q_{grid,t}^{ID}$ represent the committed power exchange references between the transmission and distribution networks determined during the intra-day phase. $P_{grid,t}^{ID}$ and $Q_{grid,t}^{RT}$ are the optimization variables in the real-time phase. The main objective of the real-time dispatch stage is to suppress the power deviation at the transmission-distribution tie line. To quantify the regulation cost caused by this deviation, a tie-line fluctuation penalty cost is introduced:

$$C_{tie} = \sum_t \lambda_{tie} \left[\left(P_{tie,t} - P_{tie,t}^{ref} \right)^2 + \left(Q_{tie,t} - Q_{tie,t}^{ref} \right)^2 \right] \quad (20)$$

where λ_{tie} is the penalty weight for tie-line power fluctuation, and $P_{tie,t}$ and $Q_{tie,t}$ are the active and reactive tie-line powers obtained in the real-time dispatch stage. This term is used to discourage frequent and large power support requests from the main grid under severe renewable-energy fluctuations.

Since the real-time scheduling employs a rolling optimization approach, only the scheduling decision for the first time period is executed during the optimization cycle. Therefore, this paper introduces an adaptive weight α_t to reasonably distribute the optimization objectives across time periods (decaying exponentially with the time step) to ensure tracking accuracy during the rolling optimization process.

4.3. System Operational Constraints

4.3.1. Distribution Network Power Flow and Security Constraints

Based on the radial characteristics of the distribution network, this paper employs the branch flow equations relaxed by second-order cone programming to balance solution accuracy and efficiency:

$$\begin{cases} \sum_{k \in \beta(j)} P_{jk,t} - \sum_{k \in \alpha(j)} (P_{kj,t} - r_{kj} I_{kj,t}) = P_{j,t} \\ \sum_{k \in \beta(j)} Q_{jk,t} - \sum_{k \in \alpha(j)} (Q_{kj,t} - x_{kj} I_{kj,t}) = Q_{j,t} \\ k_{ij,t}^2 U_{j,t} = U_{i,t} - 2(r_{ij} P_{ij,t} + x_{ij} Q_{ij,t}) + (r_{ij}^2 + x_{ij}^2) I_{ij,t} \\ U_{i,t} I_{ij,t} \geq P_{ij,t}^2 + Q_{ij,t}^2 \end{cases} \quad (21)$$

where $U_{i,t}$ and $I_{ij,t}$ represent the square of the voltage magnitude at node i and the square of the current magnitude flowing through branch i - j , respectively; $k_{ij,t}$ represents the tap ratio of the on-load tap changer transformer.

Simultaneously, the system must adhere to branch capacity and node voltage limits:

$$\begin{cases} I_{ij,t} \leq I_{ij}^{max} \\ U_i^{min} \leq U_{i,t} \leq U_i^{max} \end{cases} \quad (22)$$

4.3.2. Cross-Scale Switching Constraints for Converter Capacity

This is the core mechanism of the scheduling model proposed in this paper. The apparent power boundaries of CDG converters switch automatically based on the scheduling phase. For the day-ahead (DA) and intra-day (ID) phases, the conservative static circular boundary is enforced:

$$P_{CDG,i,t}^2 + Q_{CDG,i,t}^2 \leq S_{CDG,i,t}^2 \quad (23)$$

For the real-time (RT) phase, the boundary form is determined by the hysteresis-based boundary-state variable $\delta_{i,t}$, rather than being directly fixed as the dynamic elliptical envelope.

$$\delta_{i,t} = \begin{cases} 1, & \lambda_{i,t} \geq \lambda_{on}, \\ 0, & \lambda_{i,t} \leq \lambda_{off}, \\ \delta_{i,t-1}, & \lambda_{off} < \lambda_{i,t} < \lambda_{on}. \end{cases} \quad (24)$$

where $\delta_{i,t} = 1$ indicates that the dynamic elliptical envelope is activated, while $\delta_{i,t} = 0$ indicates that the static circular capacity boundary is adopted. The interval $\lambda_{off} < \lambda_{i,t} < \lambda_{on}$ serves as a hysteresis dead zone, in which the boundary state remains unchanged from the previous real-time scheduling interval.

When $\delta_{i,t} = 1$, the generalized dynamic elliptical envelope based on transient thermal margins is activated:

$$\left(\frac{P_{CDG,i,t}}{P_{max,i,t}^{dynamic}} \right)^2 + \left(\frac{Q_{CDG,i,t}}{Q_{max,i,t}^{dynamic}} \right)^2 \leq 1 \quad (25)$$

In any phase, the active and reactive outputs, as well as the ramping rates of CDGs, must satisfy the following basic physical limits:

$$\begin{cases} P_{CDG,min} \leq P_{CDG,i,t} \leq P_{CDG,max} \\ Q_{CDG,min} \leq Q_{CDG,i,t} \leq Q_{CDG,max} \\ -R_i^{down} \leq P_{CDG,i,t} - P_{CDG,i,t-1} \leq R_i^{up} \end{cases} \quad (26)$$

4.3.3. Harmonic Distortion Constraints Under DPWM Operation

Since the DPWM-based transient support strategy may introduce additional harmonic components, the harmonic distortion level should not only be evaluated through an economic loss term, but should also be constrained from the perspective of power-quality

security. Therefore, in the real-time scheduling phase, the current total harmonic distortion of the converter output is introduced as a hard constraint:

$$THD_{i,t}^I = \frac{\sqrt{\sum_{h=2}^H I_{i,h,t}^2}}{I_{i,1,t}} \leq THD_{\max}^I \quad (27)$$

where $I_{i,1,t}$ is the fundamental current component of the i -th converter at time t , $I_{i,h,t}$ is the h -th harmonic current component, and $THD_{\max}^I$ is the allowable current harmonic distortion limit.

When the impact of harmonics on sensitive loads and nodal voltage quality is considered, the voltage total harmonic distortion at critical nodes can be further constrained as:

$$THD_{n,t}^V = \frac{\sqrt{\sum_{h=2}^H V_{n,h,t}^2}}{V_{n,1,t}} \leq THD_{\max}^V \quad (28)$$

where $V_{n,1,t}$ and $V_{n,h,t}$ denote the fundamental and h -th harmonic voltage components at node n , respectively. With these constraints, the DPWM-based dynamic support strategy is allowed to operate only when the resulting harmonic distortion remains within the prescribed power-quality limits.

4.3.4. Renewable Energy and Energy Storage System Operational Constraints

Utilizing the inverters of renewable energy generation systems to provide reactive power support can effectively improve voltage distribution. The output constraints are:

$$\begin{cases} P_{RES,i,t} \leq P_{RES,i,t}^{pre} \\ -\sqrt{S_i^2 - P_{RES,i,t}^2} \leq Q_{RES,i,t} \leq \sqrt{S_i^2 - P_{RES,i,t}^2} \end{cases} \quad (29)$$

the operational constraints for On-Load Tap Changers (OLTC) are:

$$\begin{cases} K_{ij,t} = k_0 + \Delta k_{ij} \cdot Step_{ij,t} \\ K_{ij}^{min} \leq K_{ij,t} \leq K_{ij}^{max} \end{cases} \quad (30)$$

the energy state and charging/discharging power of the Energy Storage System (ESS) must satisfy time continuity and capacity constraints:

$$\begin{cases} E_{SOC,i,t} = E_{SOC,i,t-1} + P_{ch,i,t} \eta_{ch} \Delta T - \frac{P_{dis,i,t}}{\eta_{dis}} \Delta T \\ E_{SOC,i}^{max} \times 20\% \leq E_{SOC,i,t} \leq E_{SOC,i}^{max} \times 80\% \\ 0 \leq P_{ch,i,t} \leq P_{ch,i}^{max} D_{ch,i,t} \\ 0 \leq P_{dis,i,t} \leq P_{dis,i}^{max} D_{dis,i,t} \\ D_{ch,i,t} + D_{dis,i,t} \leq 1 \end{cases} \quad (31)$$

5. Solution Methodology

The multi-time-scale scheduling model constructed in Section 4 is a typical high-dimensional Mixed-Integer Non-Linear Programming (MINLP) problem. Due to the non-convex nature of the power flow equations and the dynamic capacity constraints, direct solving poses significant computational challenges. Therefore, Second-Order Cone Relaxation (SOCR) and convexification techniques are adopted in this section to reformulate

the original non-convex model into a Mixed-Integer Second-Order Cone Programming (MISOCP) problem. Under the stated relaxation exactness conditions, the relaxed MISOCP problem can be solved to global optimality within the prescribed solver tolerances, while improving computational tractability. The complete flowchart of the solution methodology is shown in Figure 6.

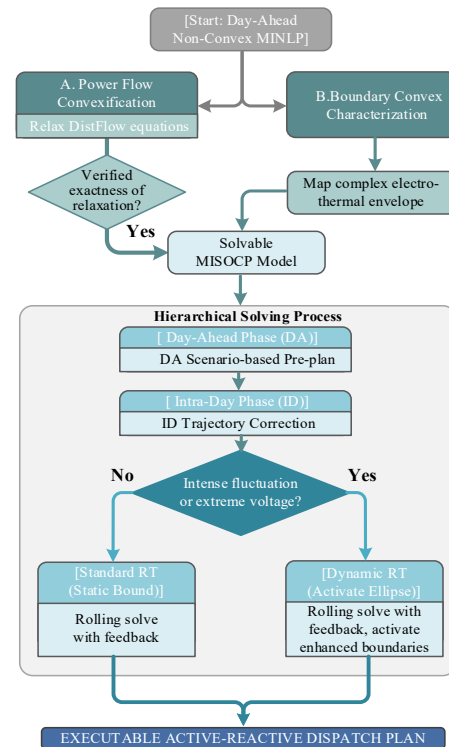


Figure 6. Algorithmic flowchart of the proposed hierarchical solving process.

5.1. SOCR of Power Flow Equations and Exactness Conditions

In the branch flow model in Equation (21), the non-convexity mainly comes from the equality relation among branch power, voltage, and current. It is relaxed as Equation (32). Equation (32) is then transformed into the SOC forms in Equations (33) and (34):

$$P_{ij,t}^2 + Q_{ij,t}^2 \leq U_{i,t} I_{ij,t} \quad (32)$$

To transform Equation (32) into a standard Second-Order Cone (SOC) constraint recognizable by solvers, we employ the algebraic identity $4xy = (x + y)^2 - (x - y)^2$ to derive the following:

$$4P_{ij,t}^2 + 4Q_{ij,t}^2 + (I_{ij,t} - U_{i,t})^2 \leq (I_{ij,t} + U_{i,t})^2 \quad (33)$$

Taking the square root of both sides of Equation (18) yields the standard SOC norm constraint:

$$\left\| \begin{array}{c} 2P_{ij,t} \\ 2Q_{ij,t} \\ I_{ij,t} - U_{i,t} \end{array} \right\|_2 \leq I_{ij,t} + U_{i,t} \quad (34)$$

The exactness of the SOCR is a prerequisite for ensuring the physical validity of the optimized solution. For radial distribution networks, when the objective function f is strictly monotonically increasing with respect to the branch current $I_{ij,t}$ (e.g., incorporating network losses or power purchasing costs in this paper), and the system is not under extreme reverse power flow or strictly binding branch capacity constraints, the optimal solution must be achieved on the boundary of Equation (32). That is,

$$\min f \Rightarrow I_{ij,t} = \frac{P_{ij,t}^2 + Q_{ij,t}^2}{U_{i,t}} \quad (35)$$

At this point, the relaxed inequality constraint (34) is exactly equivalent to the original non-linear equality constraint, ensuring that the calculation results satisfy the original branch-flow equality when the relaxation is exact.

To quantitatively evaluate the exactness of the relaxation, the relaxation error of branch i - j at time t is defined as follows:

$$\varepsilon_{ij,t} = \left| U_{i,t} I_{ij,t} - P_{ij,t}^2 - Q_{ij,t}^2 \right| \quad (36)$$

The maximum and average relaxation errors are further defined as follows:

$$\varepsilon_{\max} = \max_{(i,j),t} \varepsilon_{ij,t} \quad (37)$$

$$\varepsilon_{\text{avg}} = \frac{1}{N_l T} \sum_{(i,j),t} \varepsilon_{ij,t} \quad (38)$$

where N_l is the number of branches and T is the number of optimization periods. When both the maximum and average errors remain at a small order of magnitude, the relaxed optimal solution can be regarded as closely satisfying the original power-flow equality constraints, and the obtained dispatch results have good physical feasibility. In the case studies, the relaxation errors under different time scales and extreme disturbance scenarios are reported to further verify the exactness of the MISOCP model.

It should be emphasized that the exactness of the SOCP relaxation is mainly determined by the network power-flow model rather than by the dynamic converter boundary. According to the branch-flow relaxation theory reported by Farivar and Low [32], Gan et al. [33], and Low [34], SOCP relaxation is generally exact for radial distribution networks under mild and verifiable conditions, such as loss-related or current-related objective terms and the absence of pathological cases caused by severe reverse power flow or strongly binding operational constraints. In the proposed model, the dynamic converter capability boundary is embedded as a nodal injection constraint and can be represented by a convex elliptical set. Therefore, it does not change the structural source of nonconvexity in the branch-flow equations. The relaxation residual is still mainly associated with the relaxed branch-flow equality.

5.2. Convex Characterization of Dynamic Operating Boundaries

The generalized dynamic elliptical envelope introduced in the real-time scheduling phase essentially describes a convex region regarding P and Q . To improve numerical stability, it is rewritten as a normalized rotated second-order cone form:

$$\left\| \begin{array}{c} P_{CDG,i,t} / P_{\max,i,t}^{\text{dynamic}} \\ Q_{CDG,i,t} / Q_{\max,i,t}^{\text{dynamic}} \end{array} \right\|_2 \leq 1 \quad (39)$$

Equation (39) demonstrates that the physical expansion mechanism proposed in this paper is mathematically closed. By mapping complex underlying electro-thermal physical constraints into highly linearized convex set constraints, the rolling optimization algorithm is ensured to complete a single optimization within milliseconds.

5.3. Multi-Time-Scale Solving Process

Following the above processing, the scheduling problems of each phase are transformed into MISOCP models. This paper adopts a hierarchical “predict-roll-correct” strategy:

Day-ahead Phase: Utilize the scenario method to handle stochastic conditions generated by t -Copula and solve the 24-h pre-plan.

Intra-day Phase: Based on the latest ultra-short-term forecasts, correct the output trajectories of CDGs and ESS.

Real-time Phase: Activate the dynamic boundary constraint and perform rolling solving using adaptive decay weights α_t . Only the decision for the initial time step is executed in each sampling period, with feedback corrections continuously suppressing the random fluctuations of renewable energy.

All optimization models are implemented in MATLAB R2019a and solved using the Gurobi optimizer 13.0.1. After second-order cone relaxation and convexification of the dynamic boundary, the day-ahead, intra-day, and real-time dispatch models can all be formulated as MISOCP problems. To ensure solution accuracy, the relative MIP gap of Gurobi is set to 1×10^{-4} , while the feasibility tolerance and optimality tolerance are both set to 1×10^{-6} . In the real-time rolling optimization stage, only the control command corresponding to the current sampling instant is executed, and the optimization problem is solved again at the next sampling instant using updated forecasts and system states. To verify the online applicability of this solution framework, the computation times and relaxation errors at different time scales are further reported in the case studies.

6. Case Studies and Results

6.1. Simulation System and Parameter Settings

To verify the effectiveness of the proposed multi-time-scale coordinated scheduling method considering the dynamic operating characteristics of converters, this paper conducts a simulation analysis based on the IEEE 33-bus active distribution network system. The base voltage of the system is 12.66 kV, and the distribution network is integrated with multiple wind farm nodes, as well as controllable distributed generators equipped with grid-forming, or GFM, converters and energy storage systems. The system topology is shown in Figure 7.

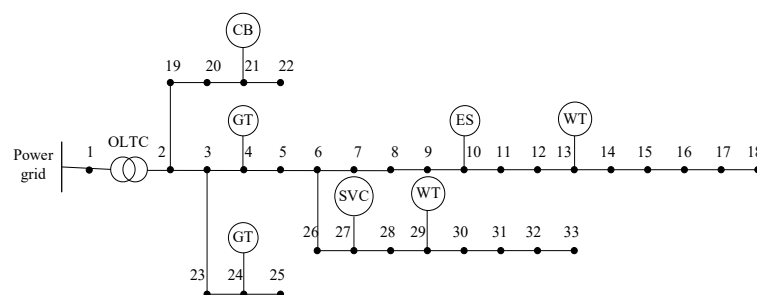


Figure 7. IEEE 33-bus active distribution network topology.

The case simulation is conducted in the MATLAB environment, specifically the R2019a version, utilizing a toolbox for mathematical modeling and calling the Gurobi solver for solutions. The computer configuration for compiling the algorithm features an Intel Core i7-5500 2.40 GHz processor and 8 GB of memory.

To ensure a fair comparison, the four dispatch methods use the same load data, wind-power scenarios, electricity-price parameters, and device capacities. The differences among these methods lie only in the optimization variables, converter capacity boundaries, and real-time support mechanisms, as shown in Table 4. The tie-line fluctuation penalty weight λ_{tie} introduced in Section 4.2 is set to the medium level in the base case, and low, medium, and high levels are further compared in the sensitivity analysis.

Table 4. Model Settings of Different Dispatch Methods.

Method	Optimization Content	Converter Boundary	DPWM Enabled
Single active-power optimization	Active power of DG and ESS	Static boundary	No
Single reactive-power optimization	Reactive power of SVC and converter	Static circular boundary	No
Coordinated optimization with static boundary	Active and reactive power	Static circular boundary	No
Proposed method	Active and reactive power	Real-time dynamic elliptical boundary	Yes

6.2. Effectiveness of Dynamic Boundaries and Suppression of Tie-Line Power Fluctuations

In the real-time scheduling stage, when the system encounters a voltage sag or power imbalance caused by severe fluctuations in wind power output, the traditional static capacity model is often limited by a fixed rated apparent power boundary, resulting in the converter being unable to provide sufficient reactive power support to the grid. The core of the proposed method lies in breaking through the limitation of treating the converter as a constant capacity source in traditional distribution network scheduling, introducing a generalized dynamic active–reactive capability envelope at the system level.

Under extreme fluctuation conditions, the proposed scheduling framework can sense and activate the short-term overload potential of GFM converters. Through the underlying electro-thermal coupling optimization, the transient thermal margin is released without violating the safety limits of the device junction temperature. Reflected at the macroscopic power scheduling level, due to the asymmetry of the thermal stress distribution caused by active and reactive currents on underlying devices under different power factors, the safe operation region of the converter adaptively expands from a conservative static circular boundary into a dynamic elliptical envelope with different major and minor axes.

When local nodes experience severe power oscillations, relying on the expanded dynamic operating boundary, the GFM converter can provide high-intensity transient power support locally at the system level. This effectively reduces the phenomenon of the active distribution network frequently and heavily requesting emergency power from the main grid to maintain global voltage stability. Consequently, it greatly weakens the power oscillation amplitude of the tie-line at the source and improves the system anti-disturbance capability against extreme renewable energy fluctuations.

6.3. Transient Harmonic Characteristic Analysis Under Extreme Fluctuation Conditions

Although activating the transient thermal margin can significantly expand the dynamic support boundary of the system, it inevitably leads to an increase in the harmonic distortion rate of the converter output current due to the underlying discontinuous pulse width modulation, also known as DPWM, nature it relies upon. To rigorously evaluate the impact of this transient regulation process on the local power quality and physical network loss of the system, this paper selects key nodes connected to GFM converters for in-depth analysis.

The proposed control strategy adopts a seamless switching mechanism based on threshold triggering: during the steady-state operation of the system, the GFM converter utilizes the traditional high-frequency SVPWM modulation strategy; only when the system encounters extreme disturbances and the converter touches the static capacity boundary will it instantaneously switch to the phase-shifted DPWM strategy for emergency support; it immediately switches back to steady-state modulation after the disturbance subsides.

The waveform in Figure 8 presents the three-phase current waveform under the complete control strategy. In the steady-state interval, the total harmonic distortion, namely THD, of the converter output current is approximately 2.3 percent, which fully meets

grid-connection power quality requirements. When switching to the phase-shifted DPWM strategy for transient limit support, the THD of the output current briefly rises to about 5.6 percent within an extremely short transient window period.

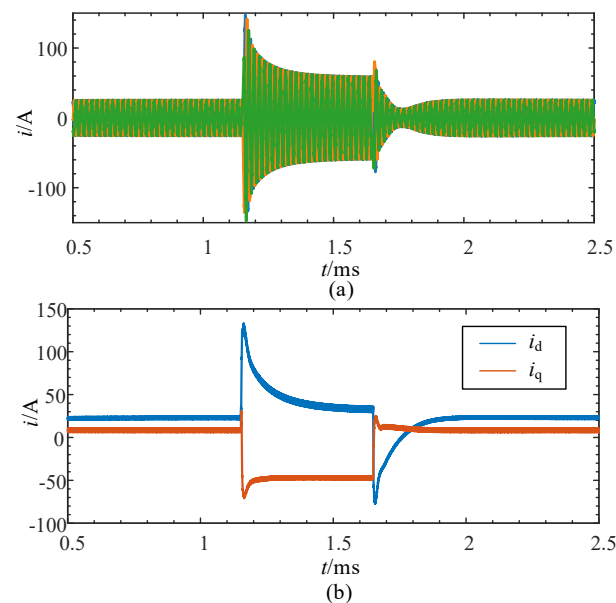


Figure 8. Current waveforms: (a) three-phase current waveform; (b) d-q axis current waveform.

It should be emphasized that the temporary increase in THD is permitted only under the premise that the harmonic distortion constraint is satisfied. In the proposed real-time scheduling model, the DPWM-based transient support mode is not activated if the resulting current THD exceeds the prescribed limit. Therefore, the proposed strategy does not trade excessive harmonic distortion for reduced tie-line power fluctuation. Instead, harmonic compliance is first guaranteed as a hard power-quality constraint, and the harmonic network loss model is then used to quantify the additional economic cost within the feasible operating region.

After the harmonic distortion constraints are satisfied, the additional harmonic network loss is further calculated to evaluate the economic impact of DPWM operation. To quantify the cost associated with this brief decline in power quality, this paper introduces a harmonic network loss calculation model considering the high-frequency skin effect. For any branch i - j in the distribution network, its equivalent resistance $R_{ij,h}$ under the h -th harmonic is approximated as:

$$R_{ij,h} = R_{ij,1} \cdot (1 + \gamma\sqrt{h}) \quad (40)$$

where $R_{ij,1}$ is the fundamental resistance of the branch, and γ is the skin effect coefficient. The total additional system harmonic network loss $P_{loss}^{har}(t)$ caused by high-order harmonics during the transient support period is calculated as follows:

$$P_{loss}^{har}(t) = 3 \sum_{(i,j) \in \Omega} \sum_{h=2}^H I_{ij,h}^2(t) R_{ij,h} \quad (41)$$

Furthermore, the economic cost of the additional harmonic network loss, C_{har} , can be obtained by integrating over the transient action window T_{DPWM} :

$$C_{har} = \sum_{t \in T_{DPWM}} c_{grid,t} \cdot P_{loss}^{har}(t) \cdot \Delta t \quad (42)$$

where $c_{grid,t}$ represents the comprehensive power purchase cost of the grid.

Through the above formulas, it can be clearly observed that although the instantaneous harmonic network loss $P_{loss}^{har}(t)$ increases, the absolute value of the time accumulation is extremely small due to the extremely short duration of the transient action interval T_{DPWM} . In stark contrast, if dynamic support is not provided, the system will incur a huge tie-line fluctuation penalty caused by the power imbalance:

$$C_{fluc} = \sum_{t \in T_{DPWM}} \alpha_t (P_{grid,t}^{HD} - P_{grid,t}^{RT})^2 \quad (43)$$

because the penalty weight is much greater than the basic electricity price $c_{grid,t}$, at the macroeconomic scale, $C_{fluc} \gg C_{har}$. This strictly demonstrates that sacrificing power quality for a very short period in exchange for global physical stability and avoiding exorbitant penalties is a completely reasonable engineering trade-off.

6.4. Junction-Temperature Verification Under Emergency Conditions

To verify the feasibility of the proposed dynamic power boundary under device thermal safety constraints, this section further analyzes the loss components, transient thermal impedance model, overload duration, and junction-temperature response of the GFM converter during grid-voltage sag conditions. The transient thermal impedance of the device is represented by a Foster equivalent thermal network:

$$Z_{th}(t) = \sum_{n=1}^N R_n (1 - e^{-t/\tau_n}) \quad (44)$$

where R_n and τ_n are the thermal resistance and thermal time constant of the n -th order, respectively. This model can capture the rapid rise in junction temperature during the short fault interval and the subsequent temperature recovery after fault clearance. The detailed parameters are listed in Table 5.

Table 5. Parameters of the Foster Equivalent Thermal Network.

Order	$R_n/K/W$	τ_n/s
1	0.0318	0.01
2	0.1749	0.02
3	0.1696	0.05
4	0.1537	0.1

Since the dynamic power boundary relies on the short-term thermal margin of the device, its activation duration cannot be extended indefinitely. Therefore, an overload-duration constraint is imposed in the real-time dispatch stage to limit the continuous operating time of the converter under the dynamic power boundary:

$$0 \leq T_{ov} \leq T_{ov,max} \quad (45)$$

where T_{ov} is the continuous activation time of the dynamic power boundary, and $T_{ov,max}$ is the maximum allowable continuous overload duration determined by the transient thermal impedance model and the junction-temperature safety limit.

Figure 9 shows the junction-temperature responses under three operating strategies. Before the fault occurs, all strategies adopt conventional SVPWM and operate within the static power limit. Therefore, the three junction-temperature curves almost overlap, indicating that the device remains in a normal thermally stable state.

After the grid-voltage sag occurs, the converter is required to provide rapid transient power support. When SVPWM is maintained and the static power limit is enforced, the

device loss increases and the junction temperature rises, but its peak value remains below the safety limit. This result indicates that the conventional static power constraint can ensure device thermal safety, but it also limits the transient support capability of the converter during the fault period and fails to fully utilize the short-term overload margin of the device.

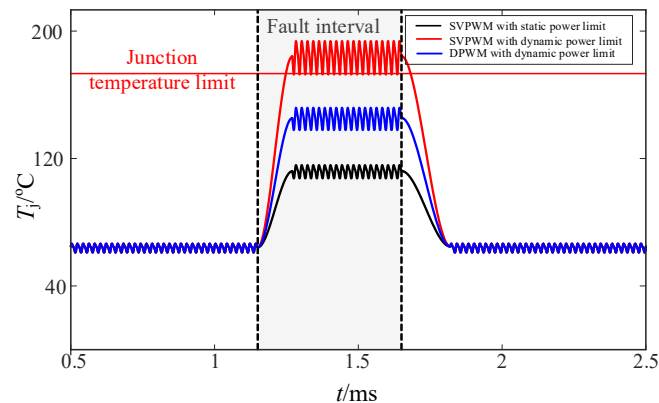


Figure 9. Junction-temperature response comparison under different power-limit and modulation strategies.

When the converter adopts SVPWM and operates under the dynamic power boundary, the dispatch layer allows it to enter the expanded power operating region, significantly improving its output current and power support capability. However, because SVPWM still maintains continuous high-frequency switching during the fault, the switching loss increases significantly with current, causing the junction temperature to rise rapidly and exceed the safety limit. This indicates that if the power boundary is relaxed only at the dispatch layer without a corresponding low-level modulation and loss-suppression mechanism, the dynamic power boundary may lead to excessive device thermal stress and cannot guarantee physical feasibility.

With the proposed DPWM-based dynamic power boundary strategy, the converter can also operate in the dynamically expanded power region. However, DPWM phase-shift clamping significantly reduces switching actions around the current peak, thereby lowering the switching loss during the fault period. As shown in Figure 9, the junction temperature under this strategy rises rapidly after the fault occurs, but its peak value remains below the safety limit. After fault clearance, the junction temperature decreases smoothly and gradually returns to the normal operating range.

In summary, the junction-temperature verification shows that the static power limit is conservative and thermally safe but provides insufficient support capability. The dynamic power boundary under SVPWM enhances fault support capability but may cause junction-temperature violation. In contrast, the proposed DPWM-based dynamic power boundary improves transient support capability while satisfying the device thermal safety constraint, thereby verifying the physical feasibility of the proposed method from an electro-thermal perspective.

To further explain the differences in junction-temperature responses among different strategies, the loss composition of the power devices during fault support is analyzed. The semiconductor loss of the converter mainly consists of conduction loss and switching loss. The conduction loss is mainly related to the on-state voltage drop and output current, and it increases with the support current during the fault. The switching loss is closely related to the switching frequency, DC-link voltage, current amplitude, and modulation strategy. Taking the loss under SVPWM with the static power limit as the reference, the loss distributions under different strategies are listed in Table 6.

Table 6. Comparison of Normalized Losses under Different Operating Strategies.

Operating Strategy	Normalized Conduction Loss	Normalized Switching Loss	Normalized Total Loss
SVPWM with static power limit	1.00	1.00	1.00
SVPWM with dynamic power limit	1.48	1.62	1.55
DPWM with dynamic power limit	1.44	0.87	1.16

Under the static power limit, the converter output current is constrained by the rated capacity, and both conduction loss and switching loss remain at relatively low levels. Therefore, although the junction temperature increases after the fault, it remains below the safety limit. After the dynamic power boundary is adopted, the converter is allowed to output a larger transient support current, and the conduction loss increases significantly with current. If SVPWM is still used, the devices maintain continuous high-frequency switching throughout the fundamental period, and the switching loss also increases. This eventually leads to a significant increase in total loss and causes junction-temperature violation.

In contrast, the proposed DPWM phase-shift clamping strategy reduces switching actions around the current peak, effectively suppressing switching loss in the high-current interval. Although the conduction loss still increases under the dynamic power boundary due to the larger current, the reduction in switching loss offsets part of the thermal stress caused by overload operation, keeping the total loss within the device thermal safety range. Therefore, the DPWM-based dynamic power boundary can provide stronger transient power support while preventing the junction temperature from exceeding the safety limit.

6.5. Verification of Relaxation Exactness and Computational Efficiency

To verify the solution accuracy and computational feasibility of the established MIS-OCF model, the second-order cone relaxation errors, solver tolerances, and computation times are reported for different time scales. All cases are modeled in MATLAB R2019a and solved using the Gurobi optimizer.

To evaluate the exactness of the second-order cone relaxation, the relaxation error index defined in Section 5.1 is calculated for all branches and time periods. Table 7 reports the maximum relaxation error, average relaxation error, and computation time of the IEEE 33-bus and IEEE 69-bus systems at different dispatch stages.

As shown in Table 7, the second-order cone relaxation errors remain at a small order of magnitude in both the IEEE 33-bus and IEEE 69-bus systems, indicating that the relaxed MISOCF solution can closely satisfy the original power-flow equality constraints. As the system scale increases, the solution time of the IEEE 69-bus system increases; however, the single-run solution time in the real-time rolling optimization stage remains shorter than the dispatch sampling interval, satisfying the requirement for online dispatch computation.

Table 7. Relaxation Errors and Computation Times at Different Time Scales.

System	Dispatch Stage	Time Resolution	Maximum Relaxation Error	Average Relaxation Error	Average Solution Time/s	MIPGap
IEEE33	Day-ahead dispatch	1 h	3.6×10^{-6}	8.4×10^{-8}	18.42	1×10^{-4}
IEEE33	Intra-day dispatch	15 min	4.1×10^{-6}	9.7×10^{-8}	6.35	1×10^{-4}
IEEE33	Real-time dispatch	5 min	5.8×10^{-6}	1.2×10^{-7}	0.43	1×10^{-4}
IEEE69	Day-ahead dispatch	1 h	6.9×10^{-6}	1.5×10^{-7}	35.76	1×10^{-4}
IEEE69	Intra-day dispatch	15 min	7.4×10^{-6}	1.8×10^{-7}	12.91	1×10^{-4}
IEEE69	Real-time dispatch	5 min	8.6×10^{-6}	2.1×10^{-7}	0.78	1×10^{-4}

To further examine the validity of the relaxation under extreme wind-power fluctuations and binding dynamic power boundaries, an additional case with a rapid wind-power drop is selected. In this scenario, the GFM converter reaches the dynamic P-Q capability envelope, and some nodal voltage constraints and converter capacity constraints become binding. Table 8 reports the relaxation error statistics under this extreme scenario.

Table 8. Relaxation Error Verification under Extreme Wind-Power Fluctuation Scenarios.

System	Scenario	Dynamic Boundary Status	Maximum Relaxation Error	Average Relaxation Error	Maximum Voltage Deviation/p.u.	Average Solution Time/s
IEEE33	Rapid wind-power drop	Partially binding	6.3×10^{-6}	1.4×10^{-7}	0.047	0.51
IEEE69	Rapid wind-power drop	Partially binding	9.2×10^{-6}	2.5×10^{-7}	0.052	0.86

The results show that even under severe wind-power fluctuations and binding dynamic power boundaries, the second-order cone relaxation errors remain around the order of 1×10^{-6} , and no evident relaxation inexactness is observed. This indicates that the established MISOCP model maintains good numerical stability and physical feasibility under extreme operating conditions. In summary, the effectiveness of the model transformation is verified from three aspects: relaxation error, solver tolerance, and computation time. The results show that after second-order cone relaxation and dynamic-boundary convexification, the original non-convex dispatch model can be transformed into an MISOCP model that can be efficiently solved by commercial solvers. The obtained optimal solutions maintain high power-flow feasibility under different time scales and extreme disturbance scenarios, satisfying the accuracy and real-time requirements of multi-time-scale dispatch.

In addition, the applicability of the proposed relaxation model to more general network structures is discussed as follows. The validation in this paper is conducted on radial or radially operated active distribution networks. For closed-loop ring networks or weakly meshed networks, the standard radial branch-flow SOCP model cannot be directly extended without additional treatment, because cycle angle consistency constraints must also be satisfied. As pointed out by Farivar and Low [32], even if the conic constraints are satisfied in a mesh network, angle recovery should still be checked to ensure that the obtained solution corresponds to a physically feasible AC power-flow solution. Bose et al. [35] also showed that SOCP relaxations are particularly advantageous for radial networks, whereas stronger relaxations or additional cycle constraints may be required for meshed networks. Therefore, for ring or weakly meshed distribution networks, the proposed dynamic converter boundary can still be embedded as a convex nodal injection constraint, but the network power-flow model should be supplemented with cycle angle constraints, phase-shifter-based convexification, or AC power-flow back-checking.

For feeders with high R/X ratios, the branch-flow model adopted in this paper retains the resistance, reactance, and active–reactive power coupling terms. Therefore, a high R/X ratio itself does not invalidate the SOC transformation. However, when severe reverse power flow, tightly binding upper voltage limits, or binding branch ampacity constraints occur, the exactness of the relaxation may be weakened, which is consistent with the sufficient-condition discussions in Low [34]. For active distribution networks with multiple voltage levels, the proposed dynamic boundary model can be extended through per-unit normalization, transformer modeling, and tap-position constraints. If unbalanced three-phase feeders or complex transformer connections are considered, the single-phase balanced branch-flow model should be further extended to a multiphase OPF model, and the relaxation residual or AC power-flow feasibility should be checked case by case.

Therefore, the proposed method is directly applicable to radial or radially operated ADNs. For ring networks, weakly meshed structures, high-renewable reverse-power-flow scenarios, and multi-voltage or multiphase ADNs, the dynamic boundary constraint itself remains convex, but the power-flow relaxation should be supplemented with appropriate topology-specific constraints and feasibility verification.

6.6. Operating Costs and Network Loss Analysis of the IEEE 33-Bus System

To quantitatively evaluate the comprehensive economic performance of the proposed method, this paper selected a 24-h scheduling cycle and compared it with traditional scheduling methods:

Method 1, Single Active Optimization: The optimization variables only consider the active power output of distributed generation and energy storage.

Method 2, Single Reactive Optimization: The optimization variables only consider the reactive power output of local equipment to optimize the voltage profile.

Method 3, Coordinated Optimization with Static Boundaries: Adopts traditional fixed static circular boundaries to perform coordinated active and reactive power scheduling.

Method 4, Proposed Method: Introduces the generalized dynamic elliptical capability envelope and conducts multi-time-scale rolling coordinated scheduling.

As shown in Table 9, the proposed method achieves the global optimum in terms of comprehensive operating cost, network loss, and fluctuation penalty cost. The intrinsic system-level mechanism behind its advantages can be summarized into the following three dimensions:

Table 9. Costs and losses in an IEEE 33-bus system.

Scheduling Method	Operating Costs (10,000 CNY)	Operating Network Loss (kW)	Tie-line Fluctuation Penalty Cost (10,000 CNY)
Single Active Optimization	24.91	2.607×10^2	4.12
Single Reactive Optimization	27.44	2.134×10^2	3.85
Coordinated Optimization	24.46	1.662×10^2	2.86
Proposed Method	21.57	1.317×10^2	1.24

First, breaking the physical limitations of active and reactive power decoupling. Comparing Method 1 and Method 2 reveals that due to the high R/X ratio of the distribution network, active and reactive power are strongly coupled. Although Method 2 improves voltage distribution and reduces network loss to 213.4 kW by optimizing reactive power, it ignores the power generation cost of distributed generation, resulting in its comprehensive operating cost being the highest at 27.44×10^4 Yuan. Method 3 significantly reduces costs and network losses through coordinated optimization, but its support capability remains insufficient when facing extreme disturbances.

Second, the local reactive power compensation substitution effect endowed by dynamic boundaries. When coping with extreme wind power uncertainty, Method 3 is restricted by the rigid constraints of the converter static capacity. When the system urgently requires reactive power support, the limited local equipment forces the distribution network to purchase emergency regulation power from the main grid at high frequencies through the transmission-distribution tie-line, driving the fluctuation penalty cost up to 2.86×10^4 Yuan. In contrast, the proposed method activates the dynamic elliptical envelope of the converter, releasing the underlying transient capacity. This not only avoids the high fundamental network losses caused by long-distance reactive power transmission, further lowering total network loss to 131.7 kW, but also cuts off the demand to purchase highly priced electricity from the main grid at the source. The fluctuation penalty cost drops

sharply to 1.24×10^4 Yuan, achieving an economic game of substituting high main-grid penalties with local transient margins.

Third, the optimal trade-off between multi-time-scales and global economy. Compared to single static scheduling, the rolling optimization mechanism introduced by the proposed method in the intra-day and real-time stages effectively prevents the continuous accumulation of power imbalances within the system. Combined with the quantitative analysis in Section 6.3, the proposed method successfully evades huge power fluctuation penalties at the cost of adding an extremely minimal amount of transient harmonic network loss under extreme operating conditions. This optimal trade-off strategy, combined with the optimized distribution of global power flow, ultimately brings the total daily comprehensive operating cost down to the lowest at 21.57×10^4 Yuan, fully demonstrating the superiority of the proposed method in complex system environments.

6.7. Operating Costs and Network Loss Analysis of the IEEE 69-Bus System

To further verify the applicability and scalability of the proposed multi-time-scale scheduling strategy and dynamic capability envelope model in a larger-scale and more complex topology distribution network, this paper introduces an improved IEEE 69-bus active distribution network system for comparative analysis, as shown as Figure 10. The base voltage of this system is similarly set to 12.66 kV, and the total active and reactive loads of the system are 3.80 MW and 2.69 MVar, respectively. We connected controllable distributed generation and wind farms equipped with GFM converters at the terminal nodes and key branch nodes of this system.

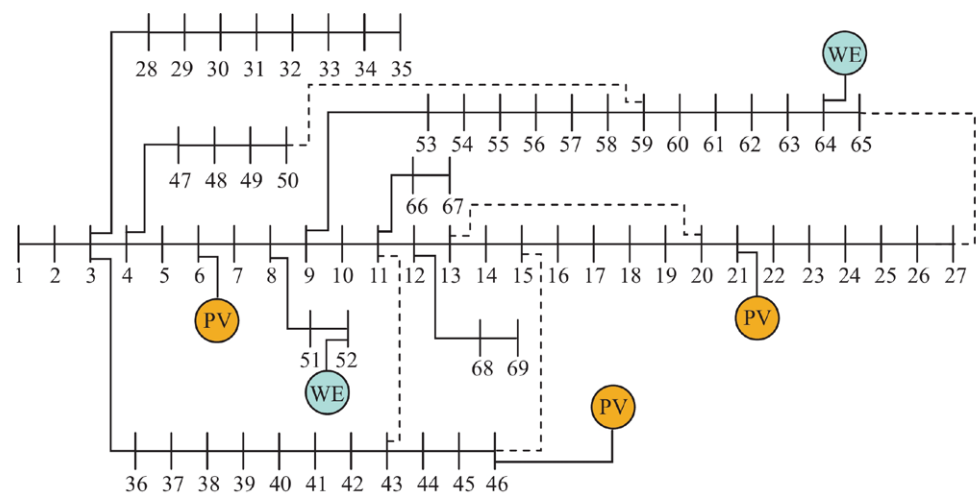


Figure 10. IEEE 69-bus active distribution network topology.

Compared with the 33-bus system, the IEEE 69-bus system possesses a longer physical main feeder and more branch structures. The increased electrical distance makes this system more sensitive to reactive power transmission; long-distance reactive power transmission will lead to more severe line network losses and node voltage deviations.

In this 69-bus system, this paper likewise applied the four scheduling methods described previously for a 24-h cycle comparison test. The quantitative results of their comprehensive operating costs, network losses, and tie-line fluctuation penalty costs are shown in Table 10.

As shown in Table 10, in the 69-bus system with a complex topology, the proposed method still maintains globally optimal economics, and its advantages relative to traditional static models are further amplified under the long-feeder characteristic. The deep mechanism analysis is as follows:

Table 10. Costs and losses in an IEEE 69-bus system.

Scheduling Method	Operating Costs (10,000 CNY)	Operating Network Loss (kW)	Tie-line Fluctuation Penalty Cost (10,000 CNY)
Single Active Optimization	28.67	3.254×10^2	5.68
Single Reactive Optimization	31.89	2.685×10^2	4.95
Coordinated Optimization	28.11	2.053×10^2	3.82
Proposed Method	24.35	1.628×10^2	1.63

First, in the topologically extended distribution network, the loss penalty of long-distance reactive power transmission exhibits a nonlinear surge. Because the basic load of the 69-bus system is similar to the 33-bus system, the primary differences in operating costs are concentrated on network loss and support cost. In Method 3, since the GFM converter is restricted by the static circular boundary, it cannot provide adequate reactive power support locally during extreme wind power fluctuations. To maintain the voltage stability of the dispersed terminal nodes, the distribution network is forced to draw a large amount of reactive power from the main grid and transmit it through long-distance feeders. This not only triggers a tie-line fluctuation penalty cost up to 3.82×10^4 Yuan but also generates immense fundamental heating network losses over the long lines.

Second, the proposed method successfully releases the transient capacity of GFM converters deeply dispersed across various nodes by activating the generalized dynamic elliptical envelopes of the underlying converters. This distributed local dynamic support mechanism effectively shortens the physical flow distance of reactive power. Even under conditions where the system is subjected to intense disturbances and accounts for minute transient harmonic additional network losses, this strategy substantially suppresses the total operating network loss to 162.8 kW and slashes the fluctuation penalty cost to 1.63×10^4 Yuan, thereby achieving the lowest comprehensive operating cost.

The simulation results of the aforementioned 69-bus system further indicate that: as the distribution network scales up and line impedance accumulates, the proposed coordinated optimization method, accounting for converter dynamic boundaries and multi-time-scale rolling scheduling, exhibits good scalability and engineering application value in breaking the long-distance reactive power transmission bottleneck, lowering comprehensive system operating costs, and enhancing global power allocation efficiency.

6.8. Sensitivity Analysis

To further verify that the performance improvement of the proposed method does not depend on a single parameter setting, sensitivity analyses are conducted for the dynamic overload coefficient, DPWM activation threshold, and tie-line power fluctuation penalty weight. In the analysis, the load level, wind-power scenarios, device placement, and electricity-price parameters are kept unchanged, while only the target parameter is varied. The total operating cost, network loss, tie-line fluctuation penalty cost, and maximum junction temperature are compared. In this paper, 175 °C is adopted as the maximum allowable junction temperature of the power device.

First, the influence of the dynamic overload coefficient k_{ov} is analyzed. A larger overload coefficient enables the converter to release more short-term support capacity and reduces the system dependence on main-grid power support, but it also increases the thermal stress of the device. Table 11 presents the analysis results for the IEEE 33-bus and IEEE 69-bus systems.

As shown in Table 11, as k_{ov} increases, the total operating cost, network loss, and tie-line penalty cost all show a decreasing trend, indicating that dynamic capacity release enhances local power support capability. However, when k_{ov} increases to 1.20, the maximum

junction temperatures rise to 166 °C and 169 °C, respectively, approaching the safety limit of 175 °C, and the thermal safety margin is significantly reduced. Therefore, k_{ov} is selected as the base value in this paper to balance operating economy and device thermal safety.

Table 11. Sensitivity Analysis of the Dynamic Overload Coefficient.

System	k_{ov}	Total Operating Cost/10,000 CNY	Network Loss/kW	Tie-Line Penalty Cost/10,000 CNY	Maximum Junction Temperature/°C
IEEE33	1.00	24.46	166.2	2.86	118
IEEE33	1.05	23.58	154.7	2.31	132
IEEE33	1.10	22.64	142.9	1.78	140
IEEE33	1.15	21.57	131.7	1.24	146
IEEE33	1.20	21.31	128.5	1.12	166
IEEE69	1.00	28.11	205.3	3.82	121
IEEE69	1.05	27.02	190.8	3.15	134
IEEE69	1.10	25.71	176.4	2.31	141
IEEE69	1.15	24.35	162.8	1.63	148
IEEE69	1.20	24.02	158.6	1.47	169

Second, the influence of the DPWM activation threshold is analyzed. A lower activation threshold triggers DPWM earlier and extends its operating duration, whereas a higher activation threshold may delay dynamic support and increase tie-line power fluctuation. The results are shown in Table 12.

Table 12. Sensitivity Analysis of the DPWM Activation Threshold.

System	DPWM Activation Threshold	Total Operating Cost/10,000 CNY	Tie-Line Penalty Cost/10,000 CNY	DPWM Operating Duration/ms	Maximum Junction Temperature/°C
IEEE33	$S/S_{rated} \geq 0.95$	21.49	1.20	0.68	143
IEEE33	$S/S_{rated} \geq 1.00$	21.57	1.24	0.50	146
IEEE33	$S/S_{rated} \geq 1.05$	22.08	1.56	0.34	149
IEEE69	$S/S_{rated} \geq 0.95$	24.21	1.56	0.71	145
IEEE69	$S/S_{rated} \geq 1.00$	24.35	1.63	0.50	148
IEEE69	$S/S_{rated} \geq 1.05$	24.92	2.04	0.36	152

As shown in Table 12, a lower DPWM activation threshold can slightly reduce the tie-line penalty cost, but it extends the DPWM operating duration and may increase transient harmonic impacts. A higher activation threshold shortens the DPWM operating duration, but insufficient dynamic support leads to higher total operating cost and tie-line penalty cost. The maximum junction temperatures under all three thresholds remain below 175 °C. Considering the dynamic support effect, thermal safety, and power-quality impact, $S/S_{rated} \geq 1.00$ is adopted as the base activation condition.

As shown in Table 13, under different penalty weights, the proposed method maintains relatively low operating cost and network loss while keeping the maximum junction temperature below 175 °C. As the penalty weight increases, the dispatch model tends to use more local dynamic support capability to reduce tie-line power deviation; therefore, the network loss slightly decreases, but the support burden on local converters increases, leading to a higher maximum junction temperature. Overall, the proposed method remains effective under low, medium, and high penalty weights, indicating that the performance improvement does not depend on a single weight setting.

Table 13. Sensitivity Analysis of the Tie-Line Fluctuation Penalty Weight.

System	Penalty Weight Level	Total Operating Cost/10,000 CNY	Network Loss/kW	Tie-Line Penalty Cost/10,000 CNY	Maximum Junction Temperature/°C
IEEE33	Low	22.13	136.8	0.86	139
IEEE33	Medium	21.57	131.7	1.24	146
IEEE33	High	21.48	130.9	1.71	154
IEEE69	Low	25.02	169.4	1.13	141
IEEE69	Medium	24.35	162.8	1.63	148
IEEE69	High	24.18	160.6	2.24	157

In summary, the sensitivity analysis shows that the overload coefficient, DPWM activation threshold, and tie-line fluctuation penalty weight all affect system economy and device thermal margin. Nevertheless, within a reasonable parameter range, the proposed method consistently reduces operating cost, network loss, and tie-line power fluctuation while satisfying the 175 °C junction-temperature safety constraint. Therefore, the performance improvement obtained in this paper is not limited to a narrow parameter range, and the proposed dynamic-boundary dispatch strategy has good parameter adaptability and robustness.

7. Conclusions

This paper presents a novel multi-time-scale coordinated active and reactive power scheduling strategy for active distribution networks (ADNs) facing high wind power penetration. By breaking the physical limitations of traditional static capacity boundaries, the proposed method deeply integrates the underlying electro-thermal characteristics of grid-forming (GFM) converters into macroscopic grid dispatch. The main conclusions are drawn as follows:

Release of Transient Thermal Margins: The construction of a generalized dynamic elliptical capability envelope successfully maps the converter’s short-term overload potential into spatial–temporal constraints at the scheduling layer. By employing DPWM phase-shift clamping to significantly reduce switching losses, the converter frees up thermal margins to output transient large currents safely, enabling high-intensity local reactive power support during severe grid disturbances.

Hierarchical and Robust Dispatch: The multi-time-scale framework optimally balances computational tractability and system security. The day-ahead and intra-day phases leverage a Copula-based scenario method to minimize total operating costs, while the real-time rolling optimization successfully suppresses tie-line power fluctuations utilizing adaptive weights and activated dynamic boundaries.

Superior Economic and Operational Performance: Simulation results on the IEEE 33-bus and 69-bus systems validate that the proposed approach achieves the lowest operating cost among the compared methods. It effectively substitutes high main-grid fluctuation penalties with local transient margins, shortening the physical flow distance of reactive power and reducing associated network losses. Despite an extremely brief and acceptable increase in harmonic distortion, the strategy demonstrates good scalability in the tested systems and engineering application value for resolving long-distance reactive power transmission bottlenecks in complex ADNs.

It should also be noted that the exactness verification in this paper is mainly based on radial or radially operated ADNs. When the proposed dynamic-boundary dispatch model is applied to closed-loop ring networks, weakly meshed networks, or multiphase multi-voltage-level ADNs, additional cycle angle constraints, multiphase power-flow modeling,

or AC feasibility back-checking should be incorporated. This will be further investigated in future work.

Author Contributions: Conceptualization, K.L., J.H., W.Z., R.S., S.W. and N.Y.; Methodology, K.L., J.H., W.Z., R.S., S.W. and N.Y.; Software, K.L., J.H., W.Z., R.S., S.W. and N.Y.; Validation, K.L., J.H., W.Z., R.S., S.W. and N.Y.; Formal Analysis, K.L., J.H., W.Z., R.S., S.W. and N.Y.; Data Curation, K.L., J.H., W.Z., R.S., S.W. and N.Y.; Writing—Original Draft Preparation, K.L., J.H., W.Z., R.S., S.W. and N.Y.; Writing—Review & Editing, K.L., J.H., W.Z., R.S., S.W. and N.Y. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: Authors Ke Liu, Jun Han, Wenqian Zhang, and Rui Song were employed by the Company State Grid Qinghai Electric Power Company Electric Power Science Research Institute, Xining, China. The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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