

Article

Composite Friction Torque Compensation Strategy for PMSM Based on Piecewise Modeling

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Abstract

The performance degradation of Permanent Magnet Synchronous Motors (PMSMs) is caused by nonlinear friction torque during low-speed operation, a phenomenon known as low-speed crawling. To address this issue, a composite compensation strategy based on a piecewise modeling approach is proposed in this paper. To accurately characterize the friction behavior, a dynamic learning rate neural network (DLRNN) is utilized for the ultra-low-speed region, while an optimal 5th-order polynomial model is established for the low-speed region. Building on this piecewise modeling, a composite control strategy with friction compensation is studied. This strategy employs a friction compensator for the feedforward compensation of the deterministic friction torque components. Furthermore, a reduced-order extended state observer (RESO) is designed to estimate and compensate for residual friction errors and random disturbance torques, thereby achieving high-precision motor control. Finally, experimental results on a surface-mounted PMSM validate that the proposed strategy effectively improves tracking accuracy and suppresses torque ripple near zero-speed, all while remaining within the computational constraints of standard industrial control hardware.

Keywords: permanent magnet synchronous motor; friction compensation; piecewise modeling; BP neural network

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1. Introduction

Permanent magnet synchronous motors (PMSMs) are widely applied in servo systems, electric vehicles, and computer numerical control (CNC) machine tools due to their high power density, high efficiency, compact structure, and superior dynamic performance [1–3]. However, in high-precision motion control applications—particularly under low-speed or near-zero-speed operating conditions—the friction generated in bearings, gears, and other mechanical transmission components exhibits strong nonlinear and time-varying behavior [4]. Such nonlinear friction effects can lead to discontinuous transitions between static and dynamic friction states [5], often resulting in the well-known low-speed crawling phenomenon. This phenomenon significantly degrades the tracking accu-

racy and stability of PMSM-based servo systems. Consequently, friction torque compensation has become a critical research topic for improving PMSM control performance.

To address the adverse effects of nonlinear friction torque at low speeds, various compensation strategies have been developed, which fall into two primary categories: model-free and model-based approaches. Model-free methods typically treat friction as an external disturbance and employ robust control algorithms to suppress its impact. Commonly used techniques include high-gain PID control, sliding mode control, disturbance observers, active disturbance rejection control, and fuzzy logic control [6–13]. Reference [12] proposed an enhanced adaptive friction observer combining a bristle deformation predictor and an adaptive extended observer, which improves friction estimation accuracy during the pre-sliding phase and enhances control performance during start-up and velocity reversal. Reference [13] developed an adaptive fuzzy control scheme for stochastic nonlinear systems with full-state constraints; however, its implementation involves a relatively high computational burden. In addition, advanced control strategies, such as composite adaptive super-twisting sliding mode control and super-twisting-like fractional-order control, have been proposed for low-speed PMSM drives. These methods improve robustness and disturbance rejection capability, thereby enhancing low-speed control performance. More importantly, due to the lack of explicit descriptions of friction nonlinearities, their compensation precision, particularly in the ultra-low-speed region, is inherently limited.

Model-based friction compensation methods have been extensively investigated owing to their relatively simple design and high modeling accuracy [14]. An accurate friction model is the foundation of such approaches. Established models range from fundamental static representations, such as the Coulomb-viscous and Stribeck models [15], to dynamic formulations like the LuGre model [16,17] and the generalized Maxwell slip (GMS) model [18]. While the static Stribeck model is often adopted for its simplicity, it is inherently limited to steady-state behavior and fails to capture critical dynamic effects, potentially leading to overcompensation or oscillatory instability. In contrast, dynamic models, particularly the LuGre model, characterize friction more comprehensively by introducing internal state variables that describe the Stribeck effect, pre-sliding displacement, and frictional hysteresis. Nevertheless, the practical application of the LuGre model is often challenged by its multiple parameters, complex identification, and limited robustness under real operating conditions.

To overcome these limitations, recent studies have focused on model structure improvement, intelligent friction modeling techniques, and advanced data-driven control strategies. For example, feedforward compensation schemes combined with filtering methods have been proposed to suppress zero-crossing distortion in PMSM servo systems [19]. In addition, artificial-intelligence-based friction modeling methods have attracted considerable attention due to their powerful nonlinear approximation capability [20–24]. Hybrid approaches that integrate physical mechanisms with data-driven models have demonstrated enhanced generalization ability and robustness under varying operating conditions [21], while physics-informed neural networks have been successfully employed to improve friction compensation performance by incorporating prior physical knowledge into the learning process [24]. More recently, machine-learning-based control algorithms have been utilized to develop parameter-free motor control strategies with improved adaptability [25]. Furthermore, advanced disturbance estimation and feedback correction techniques, such as composite observer structures integrating extended state observers and sliding-mode control, have shown promising performance in disturbance rejection and chattering suppression [26]. In addition, adaptive model predictive control methods incorporating Bayesian inference have been proposed to enhance parameter adaptability and control performance under varying operating conditions [27].

Despite these advances, achieving a satisfactory balance between modeling accuracy, computational complexity, and real-time implementation remains challenging for low-speed PMSM drives. Conventional friction models often exhibit limited accuracy in characterizing highly nonlinear friction behaviors, whereas advanced intelligent control strategies may introduce additional computational burdens. Therefore, a friction compensation method that simultaneously considers modeling accuracy, computational efficiency, and practical applicability is still highly desirable.

To address this issue, this paper proposes a composite friction torque compensation strategy for PMSMs based on piecewise modeling. The friction torque characteristics are first analyzed to determine an appropriate speed threshold, dividing the operating range into an ultra-low-speed region and a low-speed region. Corresponding friction torque models are established for each region to accurately characterize the nonlinear behavior. Furthermore, a composite control structure is developed, integrating feedforward compensation using the piecewise model with feedback correction via a reduced-order extended state observer to suppress friction torque and external disturbances. Experimental results validate the feasibility and effectiveness of the proposed approach.

2. System Mathematical Model

2.1. Mathematical Model of PMSM

In this paper, a surface-mounted permanent magnet synchronous motor (SPMSM) is employed as the research object. By neglecting the eddy current and hysteresis losses, the mathematical model of the PMSM in the d-q rotating reference frame can be expressed as

$$\begin{cases} L_d \frac{di_d}{dt} = u_d - R_s i_d + \omega_e L_q i_q \\ L_q \frac{di_q}{dt} = u_q - R_s i_q - \omega_e (L_d i_d + \psi_f) \\ T_e = 1.5 \psi_f p i_q \\ J \frac{d\omega}{dt} = T_e - T_L - T_f \end{cases} \quad (1)$$

where u_d and u_q denote the d-axis and q-axis stator voltages, respectively; R_s is the stator winding resistance; i_d and i_q represent the d-axis and q-axis stator currents, respectively; $\omega_e = p\omega$, where ω_e is the electrical angular speed, ω is the mechanical angular speed, and p denotes the number of pole pairs; L_d and L_q are the d-axis and q-axis stator inductances, respectively, and for a surface-mounted PMSM, $L_d = L_q$; ψ_f is the permanent magnet flux linkage; J is the rotor moment of inertia; T_e denotes the electromagnetic torque; T_L is the load torque; and T_f represents the friction torque.

2.2. Friction Torque Model and Characteristic Analysis

In the study of friction compensation methods for PMSM servo systems, dynamic friction models have attracted considerable attention, as they are capable of describing friction characteristics more comprehensively than static friction models. The LuGre model, as a widely applied dynamic friction model, is expressed as

$$\begin{cases} T_f = \sigma_0 z + \sigma_1 \frac{dz}{dt} + B\omega \\ g(\omega) \frac{dz}{dt} = \omega g(\omega) - \sigma_0 |\omega| z \\ g(\omega) = F_c + (F_s - F_c) e^{-(\omega/\omega_s)^\delta} \end{cases} \quad (2)$$

where σ_0 denotes the bristle stiffness coefficient, which characterizes the elastic deformation capability of the microscopic asperities at the contact interface and determines the

pre-sliding friction behavior; σ_1 represents the bristle damping coefficient, accounting for the energy dissipation caused by the relative motion of the contact asperities; σ_2 is the viscous friction coefficient, which reflects the velocity-dependent friction component and becomes dominant at relatively high speeds; z denotes the microscopic bristle deflection between the two contact surfaces; ω is the mechanical angular speed of the motor; $g(\omega)$ is a nonlinear function used to describe the Stribeck effect; F_c and F_s denote the Coulomb friction force and the maximum static friction force, respectively; ω_s is the Stribeck velocity, which determines the velocity range over which the Stribeck effect occurs; and δ is an empirical parameter used to adjust the steepness of the Stribeck curve, typically chosen in the range of 1~2. In this paper, $\delta = 2$.

Where σ_0 denotes the bristle stiffness coefficient (N·m/rad), physically representing the microscopic elastic deformation of the contact surfaces; σ_1 represents the bristle damping coefficient (N·m·s/rad); σ_2 is the viscous friction coefficient (N·m·s/rad), corresponding to the shear resistance of the lubrication layer.

When the system operates at a constant speed, $dz/dt \approx 0$. Taking the case of a positive mechanical angular speed $\omega > 0$ as an example, and assuming that F_c , F_s , ω_s (which is generally very small), and B are all constants, Equation (2) can be rewritten as the static Stribeck friction model:

$$T_f = F_c + (F_s - F_c)e^{-(\omega/\omega_s)^\delta} + B\omega \quad (3)$$

When the system operates at high speed, the mechanical angular speed ω is much larger than ω_s , and the Stribeck effect can be neglected. Accordingly, Equation (3) can be simplified as

$$T_f = F_c + B\omega \quad (4)$$

From Equation (4), it can be seen that the friction torque can be equivalently represented by Coulomb friction and viscous friction at high speeds, with no abrupt changes, thus having a minor impact on the system.

However, during low-speed operation, since the angular velocity ω is not much greater than the Stribeck velocity ω_s , the Stribeck effect can no longer be neglected, and the friction torque exhibits pronounced nonlinear characteristics. Such nonlinear behavior is equally significant when $\omega < 0$. Consequently, under low-speed conditions as well as during startup, shutdown, and direction reversal, the strongly nonlinear friction torque severely degrades the control precision of the system and easily induces the so-called crawling. Therefore, it is necessary to establish an accurate friction torque model to effectively compensate for the adverse effects of friction torque on the control system, thereby improving the control accuracy and stability of the system.

3. Construction of the Piecewise Friction Torque Model

3.1. Data Acquisition and Preprocessing

To establish an accurate piecewise friction torque model, friction-related data were collected from an experimental prototype of a PMSM. During the data acquisition process, the prototype was operated under a cascaded speed-current control structure to ensure precise speed tracking and regulation. Under no-load conditions and assuming an ideal environment where external disturbances are negligible, the electromagnetic torque of the motor can be approximately considered as the friction torque when the motor operates at constant speed. Consequently, the actual friction torque can be indirectly obtained through the measurement of the electromagnetic torque. To acquire high-precision friction characteristic data, a high-resolution incremental optical encoder and a high-accuracy torque sensor were employed to synchronously measure the motor mechanical position

angle θ_m and the friction torque T_f under various speed conditions (speed range: -25 r/min to 25 r/min). The real-time mechanical angular velocity ω was obtained by differentiating the measured position angle.

Firstly, gross error data in the measurement results are eliminated according to the Laida criterion, as illustrated in Figure 1.

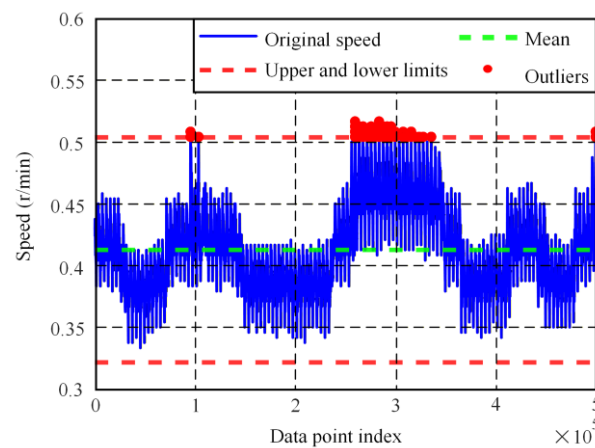


Figure 1. Removal of gross errors using the Laida criterion (taking 0.4 r/min as an example).

Secondly, based on the processed data, the speed–friction torque characteristics of the PMSM servo system investigated in this study are plotted, as shown in Figure 2. It can be seen that, during low-speed operation, the friction torque exhibits obvious nonlinear characteristics. Specifically, within the ultra-low-speed region of ± 2 r/min, the system clearly demonstrates typical Stribeck effects, pre-sliding displacement, and dynamic friction, which lead to speed tracking oscillations. Due to the excessively narrow nonlinear region, the conventional LuGre model struggles to effectively mitigate its impact, thereby limiting the overall performance enhancement of the system. To address this issue, a piecewise modeling approach is adopted to characterize the nonlinear friction torque behavior of the PMSM across different speed regions, thereby enabling effective friction torque compensation.

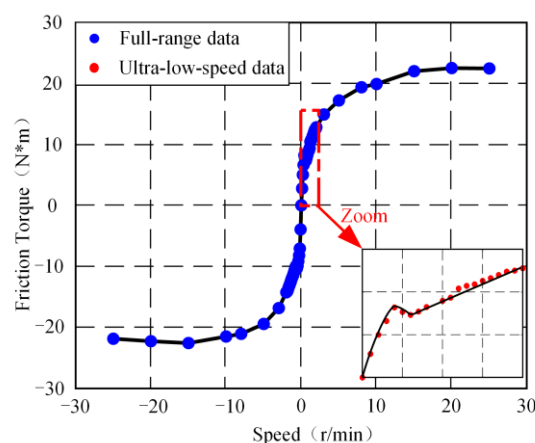


Figure 2. Speed-friction torque characteristic curve.

Finally, according to the differences in friction characteristics exhibited by the motor under different speed regions, a speed threshold ω_{th} is defined in this paper. Based on this threshold, the motor operating range is divided into two regimes, namely the ultra-low-speed region and the low-speed region. The partitioning rules are defined as

$$\begin{cases} |\omega| \leq \omega_{th}, & \text{Ultra-low speed region} \\ |\omega| > \omega_{th}, & \text{Low speed region} \end{cases} \quad (5)$$

where ω_{th} denotes the speed threshold.

Given that the friction torque exhibits strong nonlinear characteristics in the ultra-low-speed region, approximately 100,000 valid data samples (ω, T_i) were extracted in this region to form the original dataset. To further enhance data reliability, preprocessing operations such as filtering and normalization were applied to the raw data. The processed dataset was subsequently divided into training, validation, and test sets according to a specified ratio for subsequent training and evaluation of the neural network model. In the low-speed region, weighted averaging was applied to the data after eliminating gross errors to suppress random noise interference and extract the dominant trend of the data.

3.2. Determination of the Speed Threshold

In the piecewise modeling of friction torque, the selection of the speed threshold is critical to the effectiveness of friction compensation. An excessively low threshold unduly narrows the ultra-low-speed region, leading to the misclassification of speed points still subject to significant static friction and Stribeck effects into the low-speed region. In such cases, the optimal-order polynomial model fails to adequately capture the friction characteristics, ultimately resulting in reduced compensation accuracy. Conversely, if the selected speed threshold is too large, the ultra-low-speed region is overly expanded, and speed points that have already entered the viscous-friction-dominated region are processed using the friction torque model based on a dynamic learning rate neural network. This leads to increased model complexity and potential overfitting, thereby substantially increasing computational burden. To address this issue, an offline speed threshold selection method based on the analysis of the rate of change in friction torque is proposed in this paper.

The speed-friction torque characteristic curve reflects the transition of the motor from the static-friction-dominated region to the viscous-friction-dominated region and exhibits pronounced nonlinear characteristics. The scattered data corresponding to different steady-state speed points are first processed using weighted averaging to obtain discrete data at each steady-state speed. An interpolated speed–friction torque characteristic curve is generated based on these discrete data. If the first-order derivative $dT_i/d\omega$ and the second-order derivative $d^2T_i/d\omega^2$ of the friction torque with respect to speed are directly approximated using finite differences on the discrete data points, the resulting curves are shown in blue in Figure 3. In contrast, when the first- and second-order derivatives are calculated based on the interpolated characteristic curve, the corresponding results are shown in red in Figure 3.

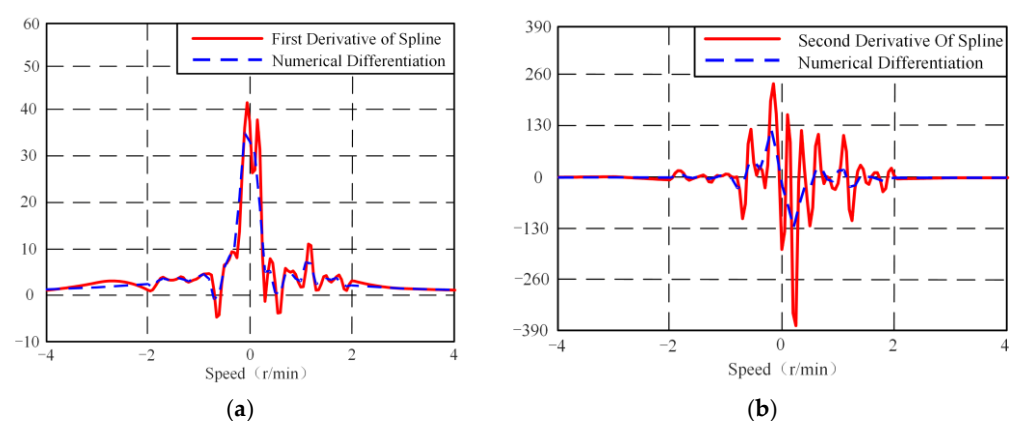


Figure 3. Derivatives of friction torque with respect to speed. (a) First-order derivative of friction torque with respect to speed. (b) Second-order derivative of friction torque with respect to speed.

An analysis of the first-order derivative curve in Figure 3a and the second-order derivative curve in Figure 3b reveals that, within the range of $|\omega| < 2$ r/min, both the first-order and second-order derivatives exhibit pronounced peaks and valleys, indicating that the friction torque is highly sensitive to variations in speed.

By analyzing the first-order derivative curve in Figure 3a and the second-order derivative curve in Figure 3b, it can be observed that in the range of $|\omega| < 2$ r/min, both the first-order and second-order derivative curves exhibit distinct peaks and valleys, indicating that the friction torque is highly sensitive to variations in speed. In this region, the system operates in a strongly nonlinear regime dominated by static friction and the Stribeck effect. At $|\omega| = 2$ r/min, a stable local minimum appears in the second-order derivative, signifying a fundamental transition in the dynamic characteristics of the friction torque.

When $|\omega| > 2$ r/min, the first-order derivative gradually becomes smooth and approaches a constant value, while the second-order derivative rapidly decays and stabilizes, indicating that the rate of change in friction torque is significantly reduced. In this region, the system transitions to a weakly nonlinear regime dominated by viscous friction, and the overall nonlinearity of the system is markedly diminished. Therefore, the speed threshold ω_{th} is selected as 2 r/min in this study.

3.3. Friction Torque Model in the Ultra-Low-Speed Region

In the ultra-low-speed region, the dynamic learning rate neural network (DLRNN) is adopted to model the friction torque. This method utilizes the powerful nonlinear fitting capability of the BP neural network and dynamically adjusts the learning rate, enabling the constructed model to quickly approximate the true friction torque under conditions of strong nonlinearity and weak signal-to-noise ratio.

The structure of the neural network is illustrated in Figure 4. The input layer consists of a single neuron, with the motor speed ω serving as the input signal. Two hidden layers are employed, where the first layer contains 20 neurons and the second layer contains 15. The output layer comprises one neuron, which outputs the estimated friction torque. The hyperbolic tangent sigmoid function (tansig) and a linear function (purelin) are employed as the activation functions for the hidden layers and the output layer, respectively. A fully connected structure is applied between adjacent layers. Accordingly, the proposed neural network model can be formulated

$$\begin{cases} h_i^{(1)} = \tanh(W_i^{(1)}\omega + b_i^{(1)}), & i = 1, 2, \dots, 20 \\ h_j^{(2)} = \tanh(\sum_{i=1}^{20} W_{j,i}^{(2)} h_i^{(1)} + b_j^{(2)}), & j = 1, 2, \dots, 15 \\ \hat{T}_f = \sum_{j=1}^{15} W_j^{(3)} h_j^{(2)} + b^{(3)} \end{cases} \quad (6)$$

where ω denotes the motor mechanical angular speed serving as the input to the neural network; $W_i^{(1)}, W_{j,i}^{(2)}, W_j^{(3)}$ represent the weight matrices of the first hidden layer, the second hidden layer, and the output layer, respectively; $b_i^{(1)}, b_j^{(2)}, b^{(3)}$ denote the bias vectors of the first hidden layer, the second hidden layer, and the output layer, respectively; $h_i^{(1)}$ and $h_j^{(2)}$ are the output vectors of the first and second hidden layers, respectively; and $\hat{T}_f$ is the friction torque estimated by the neural network.

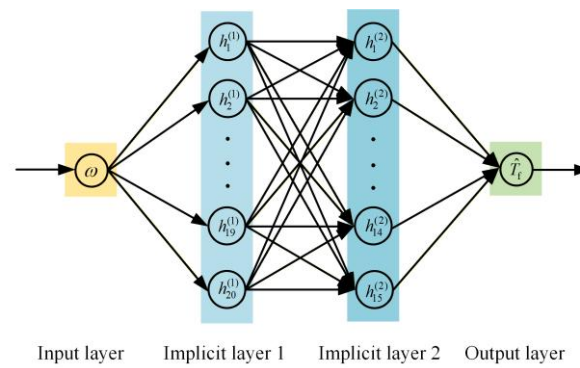


Figure 4. Structure of the neural network.

The training flowchart of the neural network is shown in Figure 5. During the training process, the setting of the learning rate has a crucial impact on the stability and convergence speed of the gradient descent. The mean squared error (MSE) is used as the loss function error for the neural network structure adopted in this paper. During forward propagation, the loss at the current iteration is calculated based on the difference between the network output and the true value, and the loss function is expressed as

$$L^{(n)} = \frac{1}{K} \sum_{k=1}^K (T_{f,k} - \hat{T}_{f,k})^2 \quad (7)$$

where $L^{(n)}$ denotes the loss function error at the n -th iteration; n represents the number of iterations; K is the number of training samples; and $T_{f,k}$ and $\hat{T}_{f,k}$ denote the true friction torque and the neural-network-estimated friction torque of the k -th sample, respectively.

Based on the comparison between the loss function error at the current iteration and a predefined threshold, the learning rate is dynamically adjusted according to the following rules:

$$\begin{cases} \alpha^{(n+1)} = \alpha^{(n)} \times \sigma_{\text{inc}}, & \text{if } L^{(n)} < L_{\min} \\ \alpha^{(n+1)} = \alpha^{(n)} \times \sigma_{\text{dec}}, & \text{if } L^{(n)} > L_{\max} \\ \alpha^{(n+1)} = \alpha^{(n)}, & \text{if } L_{\min} < L^{(n)} < L_{\max} \end{cases} \quad (8)$$

where $\alpha^{(n)}$ denotes the learning rate of the neural network at the n -th iteration; σ_{inc} and σ_{dec} represent the learning rate increasing factor and decreasing factor, respectively; $L_{\min}$ and $L_{\max}$ denote the minimum and maximum thresholds of the loss function error.

To accelerate convergence and suppress oscillations, the stochastic gradient descent with a momentum term is employed to update the neural network parameters. The parameter update rule is expressed as

$$\theta^{(n+1)} = \theta^{(n)} - \alpha^{(n)} \nabla_{\theta} L(\theta^{(n)}) + \beta(\theta^{(n)} - \theta^{(n-1)}) \quad (9)$$

where $\theta^{(n+1)}$ denotes the neural network weights and bias terms after the update at the $(n + 1)$ -th iteration; $\theta^{(n)}$ represents the neural network weights and bias terms before the update at the n -th iteration; $\nabla_{\theta} L(\theta^{(n)})$ denotes the gradient of the loss function with respect to the neural network parameters; β is the momentum coefficient; and $\theta^{(n)} - \theta^{(n-1)}$ represents the weight and bias update increment from the previous iteration.

The convergence behavior, overfitting risk, and generalization capability of the proposed neural network were systematically considered. Dynamic learning-rate adaptation together with momentum-based optimization was adopted to improve convergence characteristics and training stability. In addition, a lightweight network architecture and validation-based training procedure were employed to control model complexity and suppress overfitting. Moreover, the neural network was only applied to the ultra-low-speed region, while the friction characteristics in the low-speed region were represented by a

polynomial model. Such a piecewise modeling strategy not only preserves modeling accuracy in regions with strong nonlinear friction characteristics but also avoids the extrapolation limitations commonly encountered by a single neural network over a wide speed range.

Finally, the weights and bias terms obtained from the trained neural network are used to construct the friction torque model in the ultra-low-speed region, which can be

expressed as $\hat{T}_f = f_1(\omega)$.

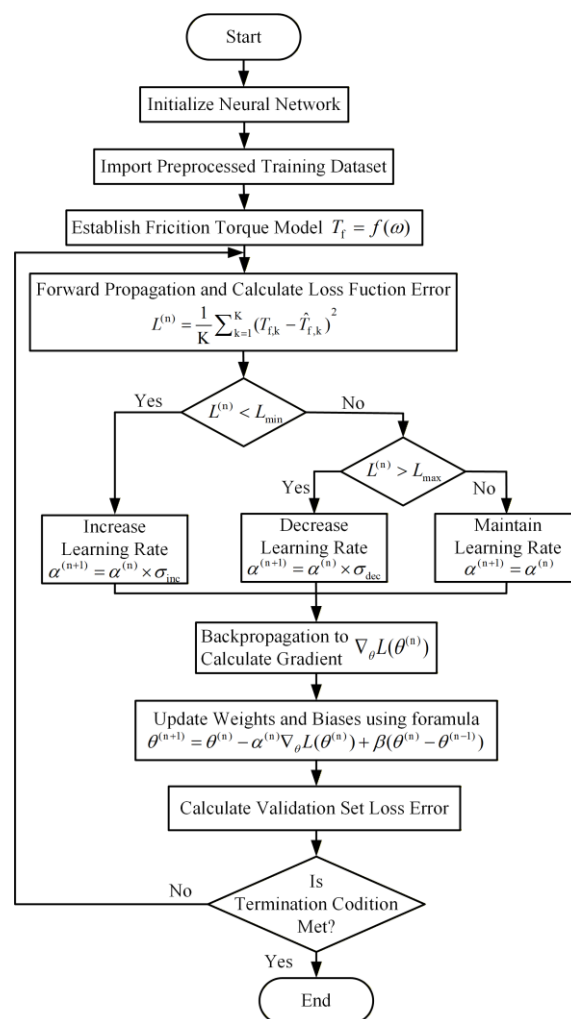


Figure 5. Training flowchart of neural networks.

3.4. Friction Torque Model in the Low-Speed Region

In the low-speed region, the variation in the friction torque tends to be relatively smooth, and the crawling phenomenon is essentially eliminated. The friction characteristics are primarily governed by the combined effects of viscous friction and Coulomb friction. The relationship between friction torque and speed no longer exhibits abrupt strong nonlinear transitions, but instead presents a continuous and approximately smooth variation trend. Therefore, an optimal-order polynomial equation can be employed to describe the nonlinear mapping relationship between the friction torque and the motor speed in the low-speed region.

Based on the preprocessed data, a polynomial model is established with the motor mechanical angular speed ω as the independent variable and the friction torque as the dependent variable. The mathematical expression of the proposed model is defined as

$$\hat{T}_f = \sum_{l=0}^m c_l (\omega)^l \quad (10)$$

where c_l denotes the polynomial coefficient, and l is the order (degree) of the polynomial.

A lower order in polynomial fitting leads to increased prediction error of the model, while an excessively high order is prone to overfitting. Therefore, it is necessary to properly constrain the fitting order to avoid the fitted curve deviating from the actual characteristics. In this study, polynomial fitting calculations are performed for orders ranging from 5 to 10, and the optimal order is determined based on the fitting residuals of each order polynomial.

For an arbitrary polynomial order m , the sum of squared residuals is defined as

$$S(m) = \sum_{y=1}^Y (T_f^y - \hat{T}_f^y)^2 \quad (11)$$

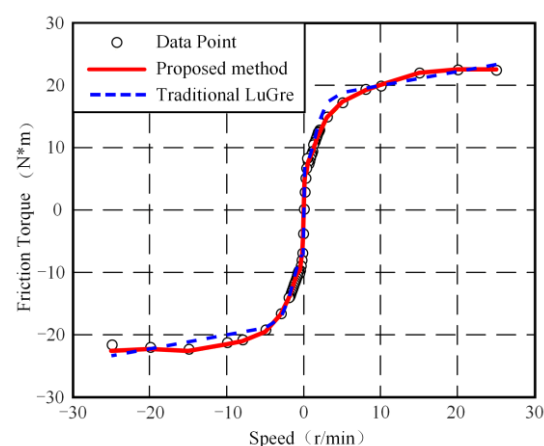
where (ω^y, T_f^y) denotes the mechanical angular speed and the corresponding measured friction torque at the y -th experimental sampling point, respectively; $\hat{T}_f^y$ represents the fitting result of the m -th order polynomial at the same sampling point; and Y is the total number of sampling points. By comparing the sum of squared residuals $S(m)$ corresponding to different polynomial orders, the polynomial order yielding the minimum residual is selected as the optimal order. The calculations show that setting the polynomial order to $m = 5$ achieves the minimum residual sum of squares, striking the best balance between model complexity and fitting precision. Therefore, the 5th-order polynomial is selected as the exact friction torque model for the low-speed region. The resulting model is then adopted as the final friction torque model for the low-speed region, which can be expressed as $\hat{T}_f = f_2(\omega)$.

3.5. Model Validation

Based on the above theoretical derivations, a piecewise friction torque model is established, which can be expressed as

$$T_f = \begin{cases} f_1(\omega), & |\omega| \leq \omega_{th} \\ f_2(\omega), & |\omega| > \omega_{th} \end{cases} \quad (12)$$

To verify the modeling performance of the model, the proposed method is compared with the conventional LuGre friction model. Figure 6 presents the fitting results of the friction torque versus speed obtained using different models, where the black scattered circles represent the experimentally measured friction torque data, the red solid curve corresponds to the proposed piecewise friction torque model, and the blue dashed curve denotes the modeling result of the traditional LuGre model.



4.1. Design of the Friction Feedforward Compensator

In motor servo systems, speed control is a key factor in achieving high-precision speed regulation and fast dynamic response. Under ideal linear conditions, a conventional PI controller can provide satisfactory steady-state accuracy and dynamic performance. However, in the low-speed region, the system is significantly affected by friction torque, which leads to increased speed errors and induces speed oscillations. For this issue, a feedforward compensator based on the piecewise friction torque model is introduced into the speed control loop with a conventional PI controller as the main control structure. By embedding the friction model into the control loop, the proposed feedforward compensation scheme aims to effectively compensate for nonlinear friction effects.

The speed error is defined as

$$e = \omega^* - \omega \quad (13)$$

where ω^* denotes the reference speed.

The control law of the conventional PI controller is given by

$$i_{q\omega}^* = K_p e + K_i \int e dt \quad (14)$$

where K_p and K_i are the proportional and integral gains of the PI controller, respectively.

Based on the established piecewise friction torque model, the corresponding current compensation term $i_{q\text{com}}^*$ can be derived, and it is expressed as

$$i_{q\text{com}}^* = \frac{\hat{T}_f}{K_t} \quad (15)$$

where $\hat{T}_f$ denotes the predicted friction torque, and $K_t = 1.5p\Psi_f$ is the torque constant of the system.

To further enhance the system's anti-interference capability, the current compensation term is superimposed on the output of the speed-loop PI controller, enabling the control system to simultaneously possess error-based feedback regulation and feedforward compensation capabilities against friction disturbances. Accordingly, the equivalent control law of the speed loop can be rewritten as

$$i_q^* = i_{q\omega}^* + i_{q\text{com}}^* = K_p e + K_i \int e dt + \frac{\hat{T}_f}{K_t} \quad (16)$$

4.2. Design of the Reduced-Order State Observer

Although the friction compensator can effectively suppress the influence of nonlinear friction torque, the overall system performance may still deteriorate due to modeling inaccuracies and external disturbances. To solve this problem, a reduced-order extended state observer is adopted to estimate and dynamically compensate for the residual disturbances.

According to the motion equation of the PMSM, the reconstructed state-space equation of the motor under no-load operating conditions can be expressed as

$$\begin{cases} \dot{x}_1 = x_d + \frac{K_t}{J} u_q \\ x_1 = \omega \\ \dot{x}_d = h_d \end{cases} \quad (17)$$

where x_d denotes the lumped disturbance of the system. It characterizes the combined influence of parameter uncertainties, external disturbances, and unmodeled dynamics. In

particular, x_d incorporates the effects of resistance variations due to temperature fluctuations, load torque perturbations, and other higher-order nonlinear factors that are not explicitly included in the nominal system model; $u_q = i_q$ is the q-axis current used as the control input for torque regulation. Subsequently, the following extended state observer is designed as

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_1 + \frac{K_t}{J} u_q + 2\gamma e_1 \\ \dot{\hat{x}}_d = \gamma^2 e_1 \end{cases} \quad (18)$$

where $\hat{x}_1$ and $\hat{x}_d$ denote the estimates of x_1 and x_d , respectively; $e_1 = x_1 - \hat{x}_1$ represents the observation error of x_1 , and γ is the observer gain matrix.

The mechanical position θ_m and the mechanical speed ω of the motor are acquired by a high-resolution encoder. Therefore, the measured value of $\hat{x}_1$ can be regarded as a known quantity and be written as

$$\dot{\hat{x}}_d = \gamma(x_d - \hat{x}_d) \quad (19)$$

By substituting (19) into (17), the designed reduced-order extended state observer can be obtained, which is expressed as

$$\dot{\hat{x}}_d = -\gamma \hat{x}_d - \gamma \frac{K_t}{J} u_q + \gamma \dot{x}_1 \quad (20)$$

To mitigate the noise influence introduced by the differential term, a new intermediate variable is defined as $x_{im} = \hat{x}_d - \gamma x_1$. Accordingly, the final form of the RESO can be expressed as

$$\begin{cases} \dot{x}_{im} = -\gamma x_{im} - \gamma K_t u_q - \gamma^2 x_1 \\ \hat{x}_d = x_{im} + \gamma x_1 \end{cases} \quad (21)$$

To ensure the stability of the RESO described in Equation (20), the Hurwitz stability criterion is employed in this paper to analyze the stability of the observer. Let the observation error of x_d be defined as $e_2 = x_d - \hat{x}_d$. Then, the following relation holds:

$$\dot{e}_2 = \dot{x}_d - \dot{\hat{x}}_d = h - \gamma e_2 \quad (22)$$

That is:

$$\dot{e}_2(t) = h(t) - \gamma e_2(t) \quad (23)$$

Taking the Laplace transform of Equation (23) yields:

$$sE_2(s) = H(s) - \gamma E_2(s) \quad (24)$$

According to the Hurwitz stability criterion, the stability condition of the reduced-order extended state observer is guaranteed when $\gamma > 0$.

5. Experimental Results

5.1. Experimental Validation of the Accuracy in IPMSM FVV Acquisition

To verify the effectiveness of the proposed strategy, the experimental validation is conducted on a permanent magnet synchronous motor (PMSM) test platform. The experimental platform is shown in Figure 8, which includes the experimental PMSM prototype, control system, power circuit, and signal acquisition module. The rated power of PMSM is 6.7 kW. The control system adopts a dual-controller architecture based on a DSP (TMS320F28335) and an FPGA (EQ1C6Q240C8). The power circuit is composed of an intelligent power module (IPM) manufactured by Fuji Electric. The friction torque is measured by an HBM T40B torque sensor, while the mechanical angular speed is acquired by

a HEIDENHAIN ERN 1300 series rotary encoder. The sampling frequency and switching frequency of the control system are both set to 10 kHz. The detailed parameters of the experimental motor prototype are listed in Table 2.

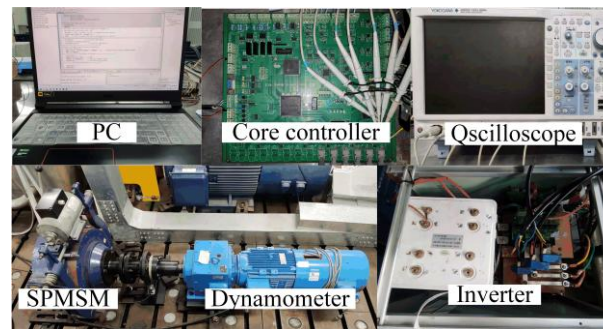


Figure 8. Experimental platform of the control system.

Table 2. Parameters of PMSM.

Parameter	Symbol	Unit	Value
Rated voltage	U_N	V	380
Rated current	i_N	A	15.6
Rated power	P_N	kW	6.7
Number of pole pairs	p	--	10
Stator resistance	R	Ω	1.51
d-axis inductance	L_d	H	0.0215
q-axis inductance	L_q	H	0.0215
Flux linkage	Ψ_f	Wb	2.3237
Moment of Inertia	J	Kg·m ²	8.8295

5.2. Experimental Results and Discussion

To further investigate the influence of friction effects on low-speed servo performance during changes in rotational speed, especially near the zero-crossing point during changes in rotational direction, this paper selects a sinusoidally varying reference rotational speed, expressed as

$$n_{\text{ref}} = N_{\text{ref}} \sin(2\pi f_{\text{ref}} t) \quad (25)$$

where f_{ref} denotes the frequency of the reference speed, which is set to 0.25 Hz and 0.5 Hz in the experiments, and N_{ref} represents the amplitude of the reference speed, which is set to 4 r/min, 6 r/min, and 9 r/min, respectively, in order to evaluate the control performance under different speed conditions. Under the above operating conditions, the speed tracking performance of three control strategies—namely, the method without friction compensation, the friction compensation method based on the conventional LuGre model, and the proposed friction compensation method—is compared by experiments.

When the reference speed frequency is 0.25 Hz, the actual speed tracking responses and corresponding tracking errors under different reference speed amplitudes are illustrated in Figures 9–11. As shown in Figure 9a, when the reference speed amplitude is 4 r/min and no friction compensation is applied, the actual motor speed exhibits a significant deviation from the given speed in the ultra-low-speed region and during direction reversal. The distortion is particularly pronounced around the zero-crossing point, where the speed error reaches a maximum of 1.534 r/min. This is attributed to the intensified nonlinear friction effects during direction reversal, aggravating speed lag and instability, which prevents the motor from tracking the reference speed and leads to the typical low-speed crawling.

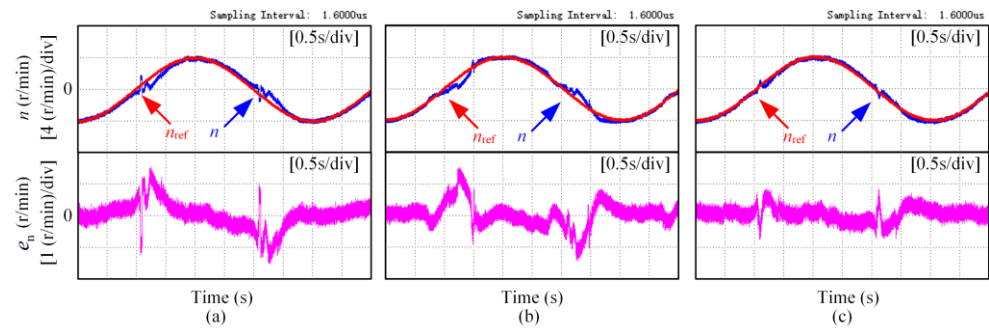


Figure 9. Experimental waveforms at 0.25 Hz and 4 r/min. (a) Without compensation. (b) Traditional LuGre model. (c) Proposed method.

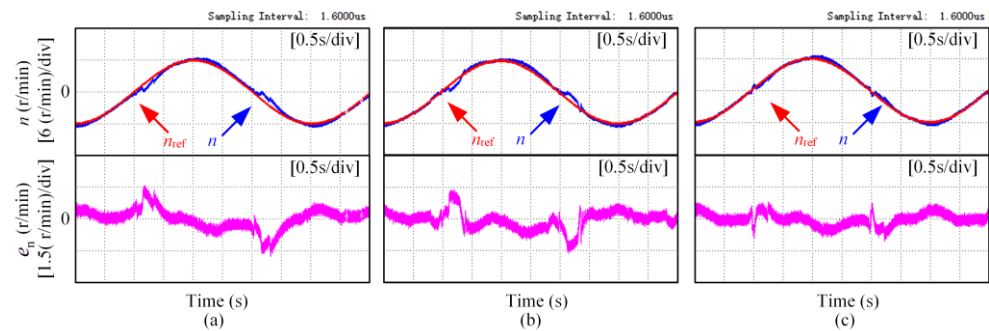


Figure 10. Experimental waveforms at 0.25 Hz and 6 r/min. (a) Without compensation. (b) Traditional LuGre model. (c) Proposed method.

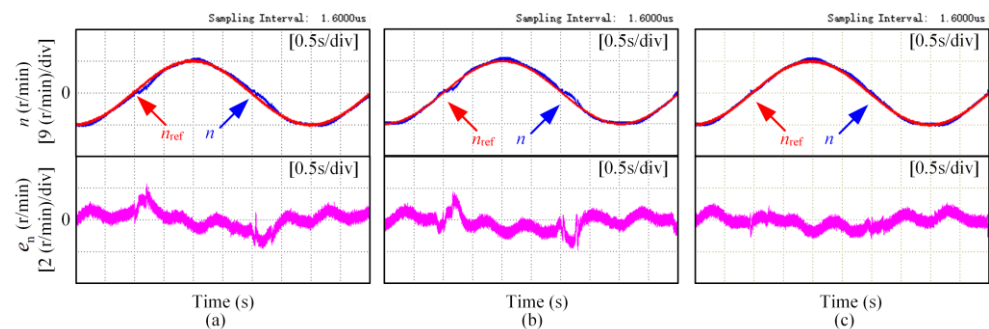


Figure 11. Experimental waveforms at 0.25 Hz and 9 r/min. (a) Without compensation. (b) Traditional LuGre model. (c) Proposed method.

As shown in Figure 9b, after introducing friction compensation based on the conventional LuGre model, the peak speed error at the zero-crossing point increases to 1.540 r/min. Pronounced oscillations persist during the reversal process, and significant high-frequency chattering can be observed in the error curve. This phenomenon can be attributed to the fact that, under practical low-speed operating conditions, the friction characteristics are influenced by factors such as mechanical deformation, temperature-dependent parameter variations, and microscopic contact-condition changes, resulting in deviations from the nominal friction model. Owing to its fixed-parameter structure, the conventional LuGre model is highly sensitive to these variations and cannot accurately characterize the actual friction behavior in the ultra-low-speed region. Consequently, the friction torque estimation becomes inaccurate, leading to friction-model mismatch. The resulting compensation error is introduced into the control loop as an equivalent disturbance, which may cause error accumulation and oscillatory behavior, thereby degrading the control accuracy and stability of the PMSM drive system, particularly in the vicinity of velocity reversal points.

In contrast, as shown in Figure 9c, when the proposed compensation strategy is employed, the actual motor speed is able to closely track the reference speed, and the tracking performance at the zero-crossing points is significantly improved. The peak speed error is reduced to 0.828 r/min, corresponding to a reduction of approximately 46% compared with the case without friction compensation and approximately 46.2% compared with the conventional LuGre-model-based compensation.

By comparing the results in Figures 9–11, it can be observed that as the reference speed amplitude increases from 4 r/min to 6 r/min and 9 r/min, the relative tracking errors under both the no-compensation strategy and the conventional LuGre compensation strategy gradually decrease with increasing speed. However, noticeable phase lag and local waveform distortion can still be observed near the sinusoidal zero-crossing regions. This is attributed to the strong nonlinear characteristics of the friction torque at low speeds, which impede accurate compensation by the control system, particularly near the zero-crossing point where the dynamic behavior of friction effects cannot be effectively addressed. In contrast, the proposed method achieves more accurate friction torque modeling and incorporates an adaptive compensation mechanism, enabling it to effectively handle friction variations at low speeds and near zero-crossing points. As a result, consistent tracking performance is maintained under different reference speed amplitudes, demonstrating superior adaptability to speed variations.

To facilitate quantitative analysis, the RMSE results of speed tracking for different references when the speed frequency is 0.25 Hz are presented in Table 3. It can be observed that, under the conditions of 4 r/min, 6 r/min, and 9 r/min, the RMSEs of all three compensation strategies exhibit an overall increasing trend as the speed amplitude increases, reflecting the growing difficulty of accurate speed tracking in the low-speed region. In comparison, the RMSE of the method proposed in this paper is significantly lower than that of the friction-free compensation and traditional LuGre model compensation strategies, with the most pronounced improvement under the lower speed condition of 4 r/min. This result is consistent with the previously observed suppression of speed distortion near the zero-crossing points, indicating that the proposed method not only improves transient speed waveforms under low-frequency and low-speed operating conditions but also effectively reduces the overall tracking error of the system.

Table 3. Comparison of RMSE among three compensation strategies under the 0.25 Hz operating condition.

Speed	Without Compensation	Traditional LuGre Model	Proposed Method
4 r/min	0.40826	0.37608	0.20838
6 r/min	0.47094	0.44443	0.27806
9 r/min	0.54025	0.44003	0.36594

When the reference speed frequency is set to 0.5 Hz, the actual speed tracking responses and corresponding tracking errors under different reference speed amplitudes for various compensation methods are shown in Figures 12–14, and the RMSE results of speed tracking for different reference when the speed frequency is 0.5 Hz are presented in Table 4. As the reference speed frequency increases from 0.25 Hz to 0.5 Hz, the rate of change in motor speed is doubled, resulting in more frequent speed reversals. Consequently, the requirements on the transient response capability and nonlinear modeling accuracy of the friction compensation algorithm are significantly increased.

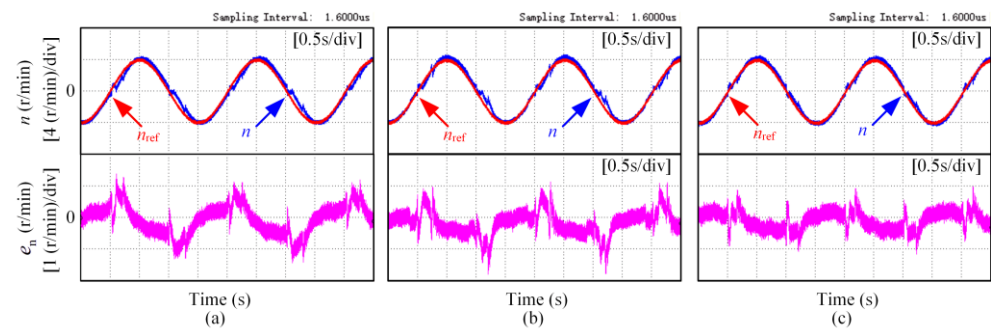


Figure 12. Experimental waveforms at 0.5 Hz and 4 r/min. (a) Without compensation. (b) Traditional LuGre model. (c) Proposed method.

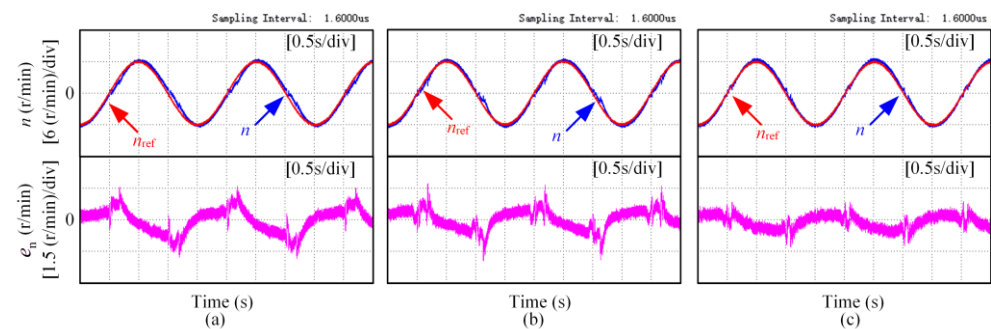


Figure 13. Experimental waveforms at 0.5 Hz and 6 r/min. (a) Without compensation. (b) Traditional LuGre model. (c) Proposed method.

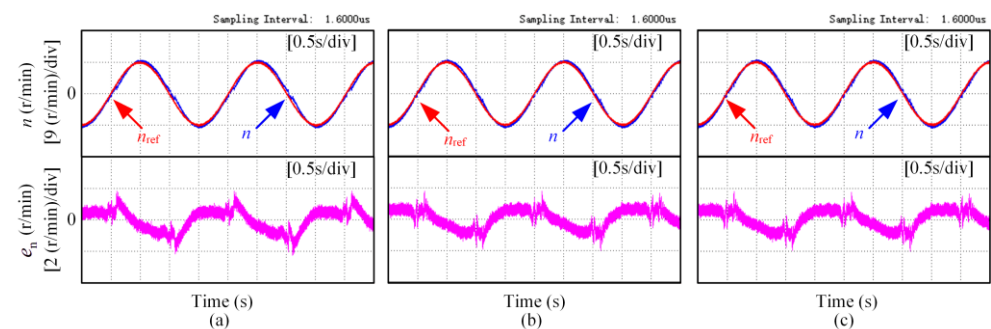


Figure 14. Experimental waveforms at 0.5 Hz and 9 r/min. (a) Without compensation. (b) Traditional LuGre model. (c) Proposed method.

Table 4. Comparison of RMSE among three compensation strategies under the 0.5 Hz operating condition.

Speed	Without Compensation	Traditional LuGre Model	Proposed Method
4 r/min	0.45859	0.42102	0.29175
6 r/min	0.56633	0.45816	0.32891
9 r/min	0.67332	0.59250	0.57010

In the absence of friction compensation, the speed distortion at reversal points becomes increasingly pronounced with higher frequency, due to the negative damping characteristics induced by the Stribeck effect and the static friction constraints near zero-speed. As a result, the amplitude of the tracking error is noticeably larger than that observed under the 0.25 Hz operating condition. For the conventional LuGre-model-based compensation, the model parameters are difficult to adapt to high-frequency dynamic switching, leading to phase lag in the compensation signal and ineffective suppression of reversal-

induced disturbances. In contrast, even when the reference speed frequency is doubled, the proposed method is still capable of achieving relatively smooth speed tracking. The deviation between the actual speed and the reference command is significantly smaller than that of the other two methods, and the error waveform remains smoother. These results demonstrate that the proposed method maintains superior friction compensation capability and dynamic performance under higher-frequency reference speed conditions.

When the reference speed frequency is increased to 0.5 Hz, a comparison of Tables 3 and 4, together with the results shown in Figure 15, indicates that, under the same reference speed amplitude, the RMSE values of all three methods increase to varying degrees compared with the 0.25 Hz operating condition. This observation suggests that more frequent speed reversals caused by the higher reference frequency significantly increase the difficulty of dynamic speed tracking. Under these conditions, the proposed method consistently maintains relatively low RMSE levels across all reference speed amplitudes, demonstrating its effective error suppression capability and a certain degree of dynamic stability even under higher reference frequency operating conditions.

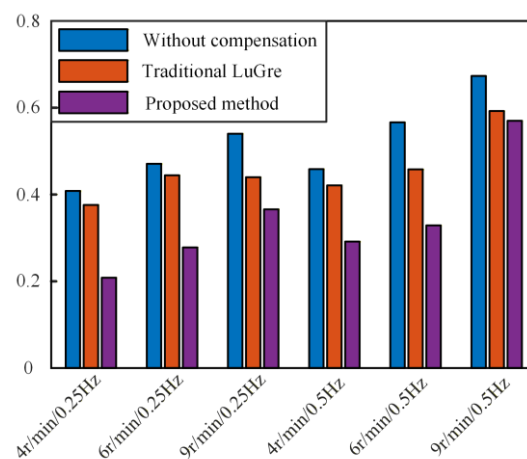


Figure 15. RMSE comparison of three friction compensation strategies under different reference frequencies and speed amplitudes.

6. Conclusions

To effectively suppress the influence of nonlinear friction torque in permanent magnet synchronous motor (PMSM) servo systems under low-speed operating conditions, this paper proposes a composite friction torque compensation strategy for PMSMs based on piecewise modeling. The main contributions of this work can be summarized as follows.

- (1) An offline speed threshold selection method based on the analysis of the rate of change in friction torque is proposed. According to the variation characteristics of the friction torque, the speed threshold is determined, and the motor speed range is divided into an ultra-low-speed region and a low-speed region. Corresponding friction torque models are then established for each region.
- (2) A composite friction torque compensation strategy for PMSMs based on piecewise modeling is developed. On the basis of the established piecewise friction torque model, a friction feedforward compensator is constructed and combined with feedback correction using a reduced-order state observer to form a composite compensation structure. This approach enables effective friction torque compensation in the low-speed region and ensures smooth operation and high-precision control performance of the servo system.

Experimental results demonstrate that the proposed strategy significantly reduces the peak error at zero-crossing points as well as the root-mean-square error in the low-speed operating range, thereby improving the smoothness and control accuracy of the PMSM. Moreover, the proposed method exhibits strong robustness and considerable potential for practical engineering applications.

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Nomenclature

Symbol	Description	Unit
u_d, u_q	stator voltage in d-q frame	V
i_d, i_q	stator current in d-q frame	A
R_s	stator resistance	Ω
L_d, L_q	inductance in d-q frame	H
Ψ_f	permanent magnet flux linkage	Wb
ω_e	electrical angular velocity	rad/s
ω	mechanical angular velocity	rad/s
p	number of pole pairs	--
J	rotor moment of inertia	$\text{Kg}\cdot\text{m}^2$
T_e	electromagnetic torque	$\text{N}\cdot\text{m}$
T_L	load torque	$\text{N}\cdot\text{m}$
T_f	friction torque	$\text{N}\cdot\text{m}$
σ_0	bristle stiffness coefficient	$\text{N}\cdot\text{m}/\text{rad}$
σ_1	bristle damping coefficient	$\text{N}\cdot\text{m}\cdot\text{s}/\text{rad}$
σ_2	viscous friction coefficient	$\text{N}\cdot\text{m}\cdot\text{s}/\text{rad}$
z	microscopic bristle deflection	rad
F_c	coulomb friction force	$\text{N}\cdot\text{m}$
F_s	maximum static friction force	$\text{N}\cdot\text{m}$
ω_s	Stribeck velocity	rad/s
δ	empirical parameter for Stribeck curve steepness	--
ω_{th}	speed threshold for piecewise modeling	rad/s
c_l	polynomial coefficient	--
$\alpha^{(n)}$	learning rate at the n-th iteration	--
$L^{(n)}$	loss function error at the n-th iteration	$(\text{N}\cdot\text{m})^2$
β	momentum coefficient	--
θ	neural network weights and bias terms	--
ω^*	reference speed command	rad/s
e	speed tracking error	rad/s
K_p	proportional gain of the PI controller	--
K_i	integral gain of the PI controller	--

K_t	torque constant of the system	N·m/A
x_d	lumped disturbance of the system	rad/s ²
γ	observer gain matrix	
θ_m	mechanical position angle	rad
f_{ref}	frequency of the sinusoidal reference speed	Hz
N_{ref}	amplitude of the sinusoidal reference speed	r/min

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