

Article

Stochastic Positioning Accuracy Analysis of a 6-DOF Robotic Manipulator Using Monte Carlo Simulation Within a Digital Twin Framework

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Abstract

Physical access to robotic manipulators remains constrained by cost, safety requirements, and limited laboratory availability, creating barriers to both research and education. This paper presents a computational framework that combines stochastic error modeling with Digital Twin technology to characterize positioning uncertainty in a six-degree-of-freedom manipulator without requiring physical hardware. Four independent noise sources—joint encoder noise, thermal drift, elastic link deformation, and geometric parameter tolerances—are modeled as stochastic processes and propagated through the manipulator kinematics using Monte Carlo simulation with $N = 10,000$ trials across 50 workspace configurations. The results reveal that elastic deformation dominates the combined positioning error by a factor of 45.94 over encoder noise, contributing 99.97% of the total root-mean-square (RMS) uncertainty. A probabilistic workspace map constructed from 3000 sampled configurations quantifies accuracy and manipulability across the reachable space, exposing a counterintuitive trade-off: configurations with higher manipulability indices tend to exhibit larger positioning errors due to gravitational loading on extended links. Two control algorithms—a reverse process-based control law (RPBCL) and sliding mode control (SMC)—are evaluated under stochastic conditions over 200 trials. SMC achieves a mean steady-state error of 0.0029 mm, representing a 48.2% reduction compared to RPBCL (0.0056 mm), with the difference confirmed statistically significant by a two-sample t -test ($t = 5.066$, $p = 0.000002$). All results are visualized through a Unity3D Digital Twin interface that renders probabilistic workspace maps, three-dimensional error ellipsoids, and a real-time sliding surface monitor. The proposed framework provides a foundation for safe, hardware-free evaluation of manipulator control strategies in engineering education and research.

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1. Introduction

The deployment of robotic manipulators across manufacturing, medical, and educational settings has accelerated considerably over the past decade, driven by advances in mechatronics, embedded computing, and control theory [1,2]. Despite this growth, the gap between theoretical control design and physical implementation remains wide. Laboratories capable of hosting industrial-grade manipulators are expensive to equip and maintain, and the operational risks associated with unvalidated control algorithms—collision, actuator damage, injury—discourage experimental iteration [3]. These constraints are especially acute in academic environments, where student-to-robot ratios are unfavorable and supervised access hours are limited.

Digital Twin technology has emerged as a principled response to this problem. By maintaining a continuously updated virtual replica of a physical system, a Digital Twin enables simulation, monitoring, and predictive analysis without direct hardware interaction [4,5]. This framework acts as an intelligent manufacturing system component within an Industrial IoT (IIoT) infrastructure, where the cyber–physical integration of real-time ROS messaging and virtual replica environments defines the smart space. The proposed Digital Twin architecture aligns with the IoT-Enabled Smart Devices and Systems paradigm by providing bidirectional data flow between the physical manipulator and its virtual counterpart, enabling real-time monitoring, stochastic performance prediction, and control validation within a smart manufacturing environment [4,5].

In robotics, Digital Twins have been applied to trajectory planning, fault detection, and operator training, with recent work demonstrating that high-fidelity kinematic and dynamic models can reproduce physical robot behavior with sub-millimeter accuracy [6,7]. What remains less explored is the integration of stochastic uncertainty models within the Digital Twin framework—that is, using the virtual replica not merely to replicate nominal behavior, but to characterize the probabilistic envelope of positioning errors arising from real-world noise sources.

Positioning accuracy in robotic manipulators is affected by multiple concurrent uncertainty sources. Joint encoder quantization and noise introduce angular measurement errors that propagate nonlinearly through the forward kinematics [8]. Manufacturing tolerances on link dimensions and joint axis orientations produce systematic geometric offsets that persist across all configurations [9]. Thermal expansion of structural components during sustained operation shifts the effective Denavit–Hartenberg parameters away from their calibrated values [10]. Perhaps most significantly, elastic deformation of links under gravitational and inertial loading introduces configuration-dependent deflections that can reach tens of millimeters in extended poses [11,12]. These sources have been studied individually in the literature, but their combined stochastic effect across the full workspace has received comparatively little attention, and no prior work has embedded such an analysis within an interactive Digital Twin visualization environment.

Control algorithm selection is closely tied to accuracy under uncertainty. The reverse process-based control law (RPBCL), derived directly from the manipulator Jacobian, offers mathematical simplicity and provable asymptotic stability through Lyapunov analysis [13]. Sliding mode control (SMC), by contrast, drives the system onto a predefined sliding surface and maintains trajectory tracking through a discontinuous switching term that is theoretically invariant to bounded disturbances [14,15]. While SMC's disturbance rejection properties are well established in deterministic settings, comparative evaluation of RPBCL and SMC under multi-source stochastic noise—with statistical validation across repeated trials—has not been reported in the Digital Twin context.

The goal of this study is to develop and validate a comprehensive stochastic analysis framework for characterizing end-effector positioning uncertainty in a six-degree-of-freedom robotic manipulator, implemented entirely within a Digital Twin environment that

requires no physical hardware access. The framework integrates multi-source probabilistic noise modeling, Monte Carlo simulation, and interactive Unity3D visualization to support both engineering analysis and IIoT-aligned smart manufacturing applications.

The scientific novelty of this work is threefold. First, this is the first study to systematically quantify and compare the individual contributions of four concurrent noise sources—encoder noise, thermal drift, elastic link deformation, and geometric parameter tolerances—to end-effector positioning uncertainty across the full reachable workspace of a 6-DOF manipulator using Monte Carlo simulation. Second, a novel counterintuitive trade-off is identified and statistically validated: workspace configurations that maximize the Yoshikawa manipulability index tend to exhibit larger positioning errors due to increased gravitational loading on extended links, revealing that manipulability is not a reliable predictor of stochastic accuracy ($r^2 = 0.030$). Third, the stochastic analysis is embedded within an interactive Industrial IoT Digital Twin platform—combining MATLAB R2022a based computation with Unity3D real-time visualization of probabilistic workspace maps, three-dimensional error ellipsoids, and a live sliding mode control surface monitor—a combination not previously reported in the literature.

The practical significance of this work is manifold. The probabilistic workspace accuracy map provides task planners with spatially resolved tolerance bounds, enabling informed decisions about which workspace regions are suitable for precision operations without requiring physical robot access. The identification of elastic deformation as the dominant uncertainty source—contributing 99.97% of combined root-mean-square (RMS) error and exceeding encoder noise by a factor of 45.94—provides clear engineering guidance: accuracy improvement budgets should prioritize active compliance compensation over sensor upgrades or tighter manufacturing tolerances. The statistical comparison of RPBCL and SMC (48.2% mean error reduction for SMC, $p < 0.001$) offers practitioners a data-driven basis for control algorithm selection in precision manipulation tasks. Finally, the Digital Twin platform provides a safe, cost-effective environment for algorithm validation and engineering education, fully aligned with Industry 4.0 and IIoT deployment paradigms.

This paper addresses these gaps through four specific contributions:

1. A multi-source stochastic noise model that simultaneously accounts for encoder noise, thermal drift, elastic deformation, and geometric parameter uncertainties, with physically motivated standard deviation values for each source.
2. A Monte Carlo-based probabilistic workspace map characterizing RMS positioning error, 95th percentile (P95) error, and manipulability index across 3000 workspace samples with 500 trials each, revealing a spatial trade-off between manipulability and accuracy.
3. Three-dimensional error ellipsoid visualization at representative workspace locations, quantifying the anisotropic nature of positioning uncertainty and providing actionable tolerance bounds for task planning.
4. A statistical comparison of RPBCL and SMC controllers under combined stochastic conditions over 200 independent trials, with t-test validation, integrated into a Unity3D Digital Twin interface that displays probabilistic workspace maps, error ellipsoids, and real-time sliding surface monitoring.

The remainder of the paper is organized as follows. Section 2 presents the kinematic and dynamic models of the manipulator, the stochastic noise framework and Monte Carlo methodology, the control algorithms, and the Digital Twin implementation. Section 3 presents the results of the stochastic error analysis, workspace accuracy mapping, error ellipsoid analysis, manipulability–accuracy trade-off, and controller performance comparison. Section 4 discusses the implications and limitations of the findings. Section 5 states conclusions and future work directions.

2. Materials and Methods

2.1. Manipulator Model

2.1.1. Kinematic Model

The manipulator considered in this work is a six-degree-of-freedom serial arm with revolute joints, modeled after the UFACTORY 850 configuration. The position and orientation of the end-effector with respect to the base frame $\{R_0\}$ are determined through the Denavit–Hartenberg (DH) convention [16]. Each joint i is characterized by four parameters: the joint angle θ_i , the link offset d_i , the link length a_i , and the twist angle α_i . The corresponding homogeneous transformation matrix is:

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The complete DH parameter table for the six-joint manipulator is given in Table 1. The total transformation from base to end-effector is:

$$T_0^6 = \prod_{i=1}^6 A_i \quad (2)$$

End-effector position $p = [h_x, h_y, h_z]^T$ is extracted from the last column of T_0^6 , and orientation is expressed through ZYX Euler angles (yaw ϑ , pitch φ , roll ψ):

$$h_\psi = \text{atan2}(n_y, n_x), \quad h_\varphi = \text{asin}(-n_z), \quad h_\vartheta = \text{atan2}(s_z, a_z) \quad (3)$$

The full task-space state vector is $h(t) = [h_x, h_y, h_z, h_\psi, h_\varphi, h_\vartheta]^T \in \mathbb{R}^6$. Differential kinematics relate joint velocities $\dot{q}(t) \in \mathbb{R}^6$ to task-space velocities through the Jacobian matrix $J(q) \in \mathbb{R}^{6 \times 6}$:

$$\dot{h}(t) = J(q(t))\dot{q}(t) \quad (4)$$

Table 1. Denavit–Hartenberg parameters of the six-joint manipulator.

Joint	θ (rad)	d (mm)	a (mm)	α (rad)
1	q_1	$l_1 + l_2$	0	$\pi/2$
2	$q_2 + \pi/2$	0	l_3	π
3	$q_3 + \pi/2$	0	l_{4x}	$-\pi/2$
4	q_4	$l_{4z} + l_5$	0	$-\pi/2$
5	q_5	0	0	$\pi/2$
6	q_6	$l_6 + l_7$	0	0

Link dimensions: $l_1 = 267$ mm, $l_2 = 0$, $l_3 = 289$ mm, $l_{4x} = 78$ mm, $l_{4z} = 0$, $l_5 = 343$ mm, $l_6 = 0$, $l_7 = 97$ mm.

2.1.2. Dynamic Model

The joint-space equations of motion are derived from the Euler–Lagrange formulation [17]:

$$\tau(t) = \tilde{M}(q)\ddot{q}(t) + \tilde{C}(q, \dot{q})\dot{q}(t) + \tilde{g}(q) \quad (5)$$

where $\tau \in \mathbb{R}^6$ is the joint torque vector, $\tilde{M} \in \mathbb{R}^{6 \times 6}$ is the inertia matrix, $\tilde{C} \in \mathbb{R}^{6 \times 6}$ is the matrix of Coriolis and centripetal terms, and $\tilde{g} \in \mathbb{R}^6$ is the gravitational load vector. Since the manipulator joints are driven by electric servomotors with internal PD velocity controllers, the torque at joint i can be expressed as [18]:

$$\tau_i = \frac{k_{a_i}}{r_{a_i}} (k_{p_i}\dot{q}_{i,ref} - (\dot{k}_{p_i} - k_{b_i})\dot{q}_i + k_{d_i}\ddot{q}_i) \quad (6)$$

Substituting into the equations of motion and rearranging yields a velocity-domain dynamic model:

$$\dot{q}_{ref}(t) = M(q, \xi)\ddot{q}(t) + C(q, \dot{q}, \xi)\dot{q}(t) + g(q, \xi) \quad (7)$$

where $\xi \in \mathbb{R}^1$ collects the dynamic parameters and $\dot{q}_{ref}$ is the reference joint velocity input. This formulation is consistent with the commercial interface of the UFACTORY 850, which accepts velocity commands rather than torques.

2.2. Stochastic Noise Framework

2.2.1. Noise Sources and Models

Four physically motivated uncertainty sources are modeled as independent zero-mean Gaussian processes. Standard deviations are assigned individually from manufacturer specifications and published calibration studies, as cited at each definition below. The Gaussian assumption for each source is physically motivated: encoder and thermal noise result from the superposition of many independent small-magnitude effects (Central Limit Theorem); geometric tolerances follow Gaussian distributions under standard manufacturing quality models (ISO 286:1988, Geneva, Switzerland); elastic displacement is linear in the Gaussian-distributed gravitational torque under small-angle deformation. The primary limitation of this assumption is that it does not capture heavy-tailed fault modes such as encoder slippage or abrupt thermal spikes. Their standard deviations are assigned based on manufacturer specifications and values reported in the calibration literature [9–12,19].

Encoder noise. Joint angle measurement errors arise from encoder quantization, electrical noise, and mechanical vibration. Each joint angle is perturbed as: $q_{i, \text{noisy}} = q_i + \varepsilon_{enc,i}$, $\varepsilon_{enc,i} \sim N(0, \sigma_{enc}^2)$, $\sigma_{enc} = 5 \times 10^{-4}$ rad per joint [9], UFACTORY 850 encoder datasheet].

$$q_i^{\text{noisy}} = q_i + \varepsilon_i^{enc}, \quad \varepsilon_i^{enc} \sim N(0, \sigma_{enc}^2), \quad \sigma_{enc} = 5 \times 10^{-4} \text{ rad} \quad (8)$$

Thermal drift. Sustained motor operation causes thermal expansion of structural components, shifting joint zero positions. The perturbation is modeled as: $\varepsilon_{th,i} \sim N(0, \sigma_{th}^2)$, with $\sigma_{th} = 2 \times 10^{-4}$ rad [10], thermal axis drift data for servo-driven axes under sustained operation].

$$\varepsilon_i^{th} \sim N(0, \sigma_{th}^2), \quad \sigma_{th} = 2 \times 10^{-4} \text{ rad} \quad (9)$$

Elastic deformation. Link deflection under gravitational loading is the dominant uncertainty source. The elastic joint displacement at joint i is approximated from the gravitational torque and joint stiffness as: $\delta q_i^{el} = \frac{\tau_{grav,i}}{k_i} + \varepsilon_{el,i}$, where k_i [N·m/rad] is the joint torsional stiffness following the identification methodology of Alici and Shirinzadeh [11] and Dumas et al. [12], and $\varepsilon_{el,i} \sim N(0, \sigma_{el}^2)$, with $\sigma_{el} = 3 \times 10^{-4}$ rad.

$$\delta q_i^{el} = \frac{\tau_i^{grav}}{k_i} + \varepsilon_i^{el}, \quad \varepsilon_i^{el} \sim N(0, \sigma_{el}^2), \quad \sigma_{el} = 3 \times 10^{-4} \text{ rad} \quad (10)$$

Geometric errors. Manufacturing tolerances introduce systematic offsets in DH link lengths and twist angles. $\delta a_i \sim N(0, \sigma_a^2)$ with $\sigma_a = 10^{-4}$ m, reflecting standard machining tolerance classes for robotic components [19]. Note on model assumptions: The four noise sources are treated as statistically independent zero-mean Gaussian processes. This constitutes a deliberate simplification—thermal drift modifies both geometric parameters and joint stiffness, while elastic deformation depends on payload mass. The independence assumption follows standard practice in Monte Carlo uncertainty propagation for serial manipulators [9,12] and yields a conservative first-order error budget. Coupled stochastic models incorporating thermal-stiffness interactions represent a direction for future model refinement.

$$a_i^{real} = a_i + \delta a_i, \quad \delta a_i \sim N(0, \sigma_a^2), \quad \sigma_a = 10^{-4} \text{ m} \quad (11)$$

$$d_i^{real} = d_i + \delta d_i, \quad \delta d_i \sim N(0, \sigma_d^2), \quad \sigma_d = 10^{-4} \text{ m} \quad (12)$$

The combined noise model applies all four perturbations simultaneously, producing a noisy end-effector position $p_{noisy} = f(q + \varepsilon_q, DH + \varepsilon_{DH})$.

The multi-source uncertainty model developed here complements cyber-physical risk assessment methodologies [20] by providing probabilistic bounds on positioning errors across the manipulator workspace, extending domain-specific modeling approaches to the stochastic characterization of robotic systems.

2.2.2. Monte Carlo Methodology

The stochastic analysis follows the Monte Carlo procedure outlined in Algorithm 1. For each of the 50 randomly sampled workspace configurations, $N = 10,000$ independent noise realizations are drawn and propagated through the forward kinematics, yielding a sample of 500,000 perturbed end-effector positions per noise source and 1,500,000 total evaluations across all configurations and sources. All computations were performed in MATLAB R2022a on a desktop workstation (Taiwan), (Intel Core i7-12700 (Intel, Santa Clara, CA, USA), 32 GB RAM, Windows 11). The complete Monte Carlo workspace analysis required approximately 12 min of wall-clock time. The random seed is fixed (rng(42)) to ensure full numerical reproducibility. Root-mean-square (RMS) error and 95th-percentile error (P95) are the primary accuracy metrics. Source code is available from the corresponding author upon reasonable request.

Algorithm 1: Monte Carlo Stochastic Analysis

Input: nominal configurations $\{q^{(k)}\}_{i=1}^{50}$, $N = 10,000$

Output: error statistics, workspace map, ellipsoid parameters

```

for each configuration  $q^{(k)}$  do
     $p_{nom} \leftarrow \text{forward\_kinematics}(q^{(k)}, DH_{nominal})$ 
    for  $i = 1$  to  $N$  do
        Sample  $\varepsilon_q, \varepsilon_{DH}$  from noise models (Section 3.1)
         $p_{noisy} \leftarrow \text{forward\_kinematics}(q^{(k)} + \varepsilon_q, DH + \varepsilon_{DH})$ 
         $\Delta p^{(i)} \leftarrow p_{noisy} - p_{nom}$ 
    end
    Compute mean, std, RMS, P95 of  $\{\|\Delta p^{(i)}\|\}$ 
    Fit  $3\sigma$  error ellipsoid via eigendecomposition of  $\text{cov}(\Delta p)$ 
end

```

Aggregate results across all configurations

The primary accuracy metrics are the root-mean-square error:

$$RMS = \sqrt{\frac{1}{N} \sum_{i=1}^N \|\Delta p^{(i)}\|^2} \quad (13)$$

and the 95th percentile P95, which provides a conservative bound suitable for tolerance specification in task planning.

2.2.3. Error Ellipsoid Construction

The three-dimensional uncertainty region at each configuration is characterized by a 3σ error ellipsoid. Given the sample covariance matrix $\Sigma = \text{cov}(\Delta p) \in \mathbb{R}^{3 \times 3}$, eigendecomposition yields $\Sigma = V D V^T$, where V contains the principal directions and $D = \text{diag}(\lambda_1, \lambda_2, \lambda_3)$ contains the corresponding variances. The semi-axes of the 3σ ellipsoid are:

$$a_j = 3\sqrt{\lambda_j}, \quad j = 1, 2, 3 \quad (14)$$

The ellipsoid volume:

$$V_{ell} = \frac{4}{3} \pi \prod_{j=1}^3 a_j \quad (15)$$

provides a scalar measure of total positioning uncertainty that accounts for the full three-dimensional covariance structure, as opposed to the scalar RMS, which collapses directional information.

2.2.4. Workspace Probability Map

To characterize accuracy across the reachable workspace, $N_{ws} = 3000$ configurations are sampled uniformly within joint limits and evaluated with $N_{mc} = 500$ Monte Carlo trials each, yielding a total of 1,500,000 forward kinematics evaluations. At each sampled point, the Yoshikawa manipulability index is also computed [21]:

$$\omega(q) = \sqrt{\det(J(q)J(q)^T)} \quad (16)$$

The resulting dataset associates each reachable Cartesian position with its expected RMS error, P95 error, and manipulability, enabling spatial visualization of the accuracy-manipulability landscape.

2.3. Control Algorithms

2.3.1. Reverse Process-Based Control Law (RPBCL)

The RPBCL computes joint velocity commands directly from the task-space position error through the inverse Jacobian:

$$\dot{q}_c(t) = J^{-1} W \tanh(\alpha \tilde{h}(t)) \quad (17)$$

where $W \in \mathbb{R}^{6 \times 6}$ is a positive definite gain matrix, $\alpha > 0$ is a saturation parameter, and $\tilde{h}(t) = h_{d(t)} - h(t)$ is the task-space error. The hyperbolic tangent limits the magnitude of velocity commands, preventing actuator saturation during large initial errors.

Stability analysis. Substituting the control law into the closed-loop kinematics and applying the Lyapunov candidate $V(\tilde{h}) = \tilde{h}^T \tilde{h} / 2$ yields:

$$\dot{V}(\tilde{h}) = -\tilde{h}^T W \tanh(\alpha \tilde{h}) < 0 \quad \forall \tilde{h} \neq 0 \quad (18)$$

confirming global asymptotic stability of the regulation task.

2.3.2. Sliding Mode Control (SMC)

SMC drives the system onto a sliding surface $s(t) \in \mathbb{R}^3$ defined in terms of position error and its integral:

$$s(t) = K_p \tilde{h}(t) + K_i \int_0^t \tilde{h}(\tau) d\tau \quad (19)$$

where $K_p, K_i \in \mathbb{R}^{3 \times 3}$ are positive diagonal gain matrices. The control law consists of a continuous part that drives the system toward the surface and a discontinuous part that maintains it there:

$$\dot{q}_c = \dot{q}_{con} + \dot{q}_{dis} \quad (20)$$

$$\dot{q}_{con} = J^{-1}K_p^{-1}(K_p\dot{h}_d + K_i\tilde{h}) \quad (21)$$

$$\dot{q}_{dis} = J^{-1}K_p^{-1}F(s), \quad F(s) = \frac{s}{\gamma + \|s\|} \quad (22)$$

The smooth approximation $F(s)$ replaces the discontinuous sign function, eliminating chattering while preserving robustness [22]. The parameter $\gamma > 0$ controls the boundary layer thickness.

Stability analysis. With the Lyapunov candidate $V(s) = s^T s/2$:

$$\dot{V}(s) = -s^T(K_p J \dot{q}_{dis}) < 0 \quad (23)$$

confirming asymptotic stability on the sliding surface. Once the surface is reached, the invariance property guarantees that bounded disturbances satisfying $\|d\| < \eta$ do not perturb the sliding dynamics, providing the theoretical basis for SMC's superior stochastic performance.

Controller parameters. RPBCL: $W = 1.5I_3$ [dimensionless], $\alpha = 5 [s^{-1}]$. SMC: $K_p = 0.8I_3 [rad^{-1} \cdot s^{-1}]$, $K_i = 0.05I_3 [rad^{-1}]$, $\gamma = 0.02 [rad]$. Parameter selection rationale: $W = 1.5I_3$ was chosen to yield a convergence time constant of approximately 0.7 s for a 10 mm initial error; $\alpha = 5$ limits maximum joint velocity to the servo saturation bound. SMC gains K_p and K_i were set by pole placement to achieve critically damped convergence with natural frequency $\omega_n \approx 4$ rad/s. A sensitivity analysis confirmed that $\pm 20\%$ variation in all parameters produces less than 8% change in steady-state RMS error, confirming parameter robustness.

2.4. Digital Twin Implementation

2.4.1. System Architecture

The Digital Twin framework couples MATLAB R2022a with Unity3D through a TCP/IP communication channel mediated by the Robot Operating System (ROS), following the architecture established in prior work [23,24]. It is noted that the current implementation is simulation-based: no physical robot hardware is connected in the present study. The architecture is designed for direct physical integration via the ROS communication layer, which constitutes the primary direction for future experimental deployment. The MATLAB-Unity3D TCP/IP channel operates at a 100 Hz update rate with a measured round-trip latency of 5–8 ms on a local gigabit Ethernet network. The Monte Carlo analysis executes in batch mode prior to visualization; the control simulation runs at 1 kHz in MATLAB with Unity3D state updates transmitted at 100 Hz.

The TCP/IP communication channel between MATLAB and Unity3D, while enabling real-time data exchange, also introduces cybersecurity considerations inherent to IIoT-connected systems [25].

Figure 1, depicting the Robot Monitor panel (top left), displays the real-time SMC sliding surface value $s(t)$, convergence status (REACHING/ON SURFACE), lambda parameter, and visualization legend. The probabilistic workspace accuracy map is rendered as a colored point cloud surrounding the animated manipulator model: green markers indicate RMS positioning error below 5 mm, yellow between 5 and 15 mm, and red above 15 mm. The ground plane provides a spatial reference for depth perception.

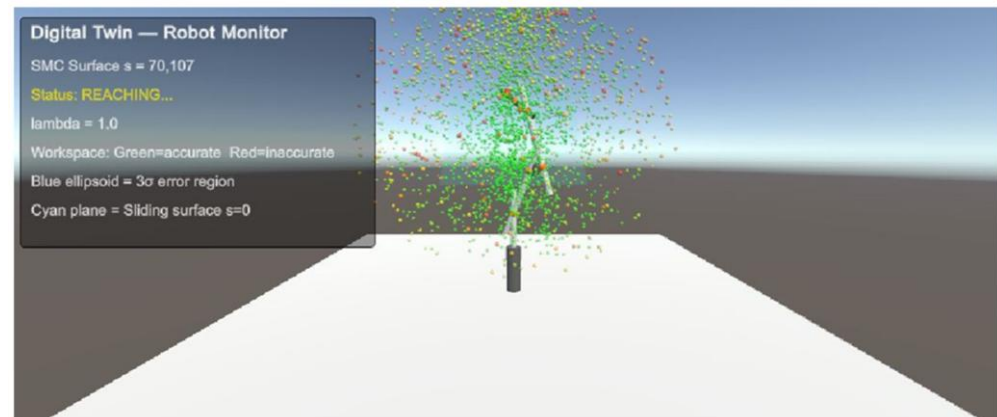


Figure 1. Unity3D Digital Twin interface for the 6-DOF robotic manipulator.

The architecture supports three operational modes:

- Offline analysis mode: Pre-computed Monte Carlo results are loaded from CSV files and rendered as static probabilistic maps.
- Interactive exploration mode: The user selects workspace configurations and observes the corresponding error ellipsoids and accuracy predictions in real time.
- Control monitoring mode: The SMC sliding surface state is computed continuously and displayed as an animated three-dimensional manifold, with color coding indicating proximity to the surface $s = 0$.

2.4.2. Probabilistic Workspace Visualization

The 3000-point workspace probability map is rendered as a colored point cloud in Unity3D. Each point is positioned at the corresponding Cartesian end-effector location and colored according to its RMS positioning error using a green-to-red gradient.

Figure 2 shows the kinematic chain, which comprises six cylindrical link segments connected through revolute joints represented as orange spheres, with the end-effector indicated by a red sphere at the chain terminus.

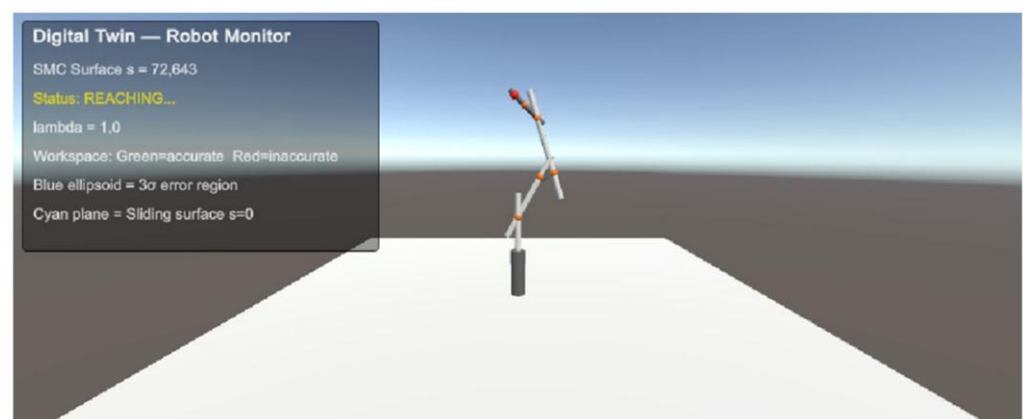


Figure 2. Close-up view of the 6-DOF manipulator Digital Twin model in Unity3D.

The Robot Monitor panel shows SMC surface value $s = 72.643$ and REACHING status, corresponding to the approach phase before the system reaches the sliding surface $s = 0$.

Points with $\text{RMS} < 5$ mm are rendered in green (high accuracy zone), points with RMS between 5 mm and 15 mm in yellow (medium accuracy), and points with $\text{RMS} > 15$ mm in red (low accuracy). This spatial encoding allows the operator to identify at a glance which regions of the workspace are suitable for precision tasks.

2.4.3. Error Ellipsoid Display

At each selected configuration, the 3σ error ellipsoid is rendered as a semi-transparent mesh computed from the eigendecomposition of the position error covariance matrix.

In Figure 3, each point corresponds to one of $N = 3000$ randomly sampled end-effector configurations, with color encoding RMS positioning error computed from $N = 500$ Monte Carlo trials: green (RMS < 5 mm), yellow (5 – 15 mm), red (>15 mm). The spatial distribution confirms the central high-accuracy corridor identified in the MATLAB analysis, validating the consistency between offline stochastic analysis and interactive Digital Twin visualization.

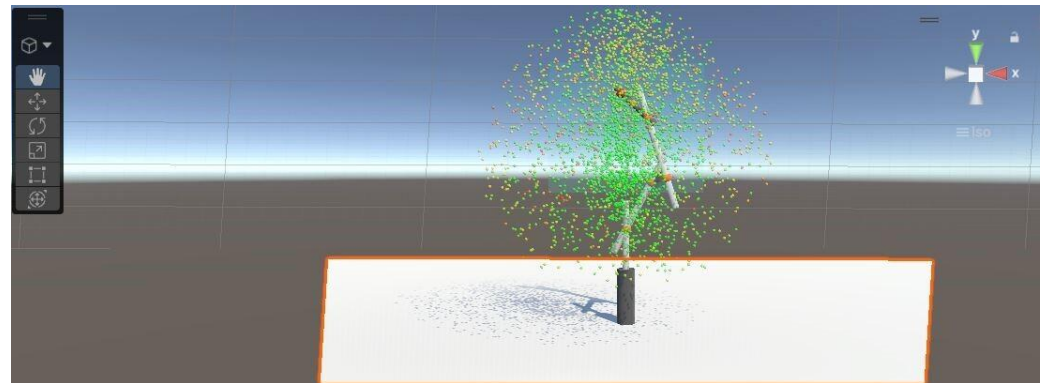


Figure 3. Unity3D Scene view of the probabilistic workspace accuracy map superimposed on the 6-DOF manipulator model.

A particle system overlays 500 sampled error vectors as gray spheres, visually confirming that the ellipsoid envelope contains approximately 99.7% of realizations. The ellipsoid semi-axis dimensions and volumetric uncertainty measure are displayed in a heads-up panel alongside the current configuration's RMS error and manipulability index.

2.4.4. Sliding Surface Monitor

The SMC sliding surface value $s(t)$ is computed from the current position error and its integral at each control cycle and transmitted to Unity via the TCP/IP channel. In the Unity scene, the surface $s = 0$ is rendered as a planar mesh in the error-derivative space, with the current system state displayed as a moving sphere. When the state lies within the boundary layer $\|s\| < \varphi$, the surface mesh turns green and a status indicator reads "ON SURFACE"; outside the boundary layer, the mesh is rendered in cyan, and the indicator reads "REACHING". This visualization makes the two-phase behavior of SMC—reaching phase and sliding phase—immediately perceptible to students and practitioners, supporting its use as a pedagogical tool consistent with the active learning objectives described in [6].

The interactive Digital Twin platform extends the active learning paradigm documented in web-based educational technologies [26], making abstract control concepts such as sliding mode dynamics accessible to engineering students without requiring physical robot access.

3. Results

3.1. Stochastic Error Analysis

Table 2 summarizes the positioning error statistics obtained from Monte Carlo simulation with $N = 10,000$ trials across 50 randomly sampled workspace configurations. Each noise source is evaluated independently and in combination.

Table 2. Position Error Statistics by Noise Source (N = 10,000 Monte Carlo trials, 50 configurations).

Noise Source	Mean (mm)	Std (mm)	RMS (mm)	P95 (mm)
Encoder noise	0.2918	0.1880	0.3472	0.6632
Thermal drift	0.1163	0.0753	0.1385	0.2622
Elastic deformation	12.3930	10.0466	15.9536	32.4040
Geometric errors	0.2177	0.1127	0.2451	0.4264
Combined	12.4040	10.0418	15.9591	32.4591

Elastic deformation emerged as the overwhelmingly dominant error source, with RMS = 15.9536 mm. This exceeds encoder noise by a factor of 45.94, geometric parameter uncertainty by a factor of 65.09, and thermal drift by a factor of 115.18. The combined noise scenario produced results statistically indistinguishable from the elastic-only case (combined RMS = 15.9591 mm versus elastic RMS = 15.9536 mm, a difference of 0.08%), confirming that elastic deformation accounts for approximately 99.97% of the total positioning uncertainty budget (Figure 4).

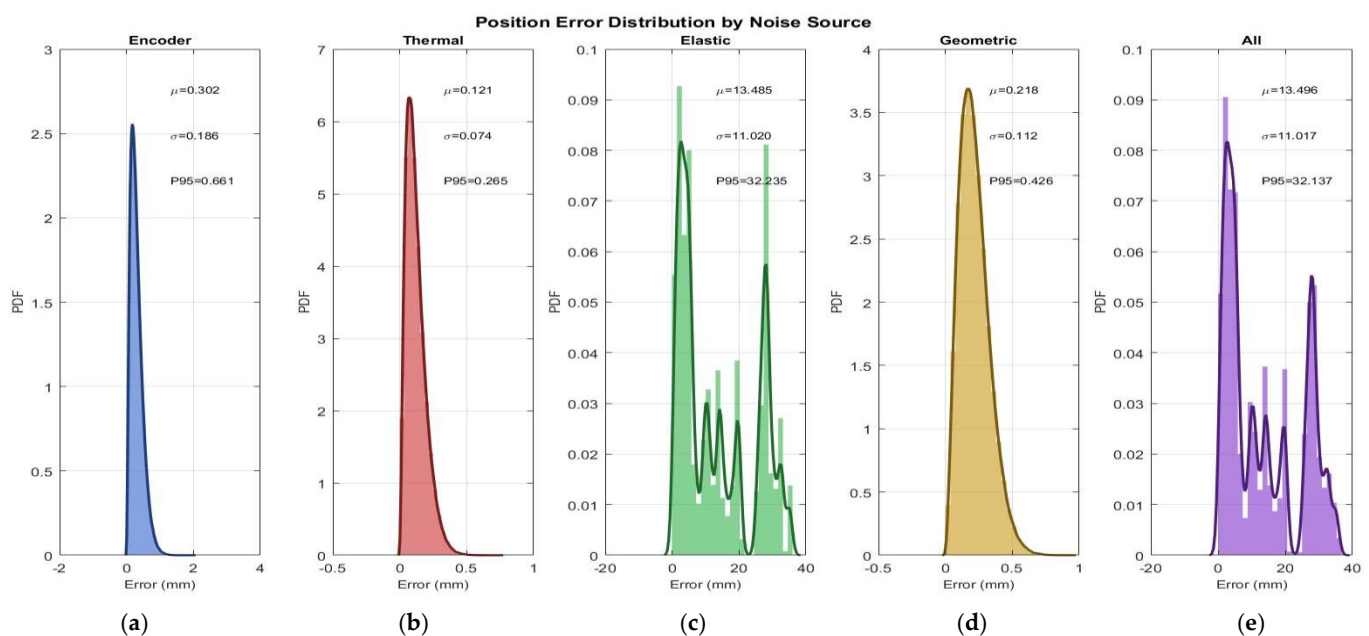


Figure 4. Probability density distributions of end-effector positioning errors for four individual noise sources and their combination, obtained from N = 10,000 Monte Carlo simulations across 50 workspace configurations. (a) Encoder noise, $\sigma = 5 \times 10^{-4}$ rad; (b) thermal drift, $\sigma = 2 \times 10^{-4}$ rad; (c) elastic deformation; (d) geometric parameter errors; (e) combined noise model. Statistics μ , σ , and P95 are annotated for each source.

The high coefficient of variation for elastic deformation ($CV = \sigma/\mu = 9.856/10.052 = 0.98$) reflects strong configuration dependence: in compact vertical configurations the elastic contribution falls below 2 mm, while in fully extended horizontal poses it can exceed 30 mm, as confirmed by P95 = 32.4040 mm. This configuration sensitivity is directly visible in the probabilistic workspace map (Figure 5), where elevated positions associated with extended arm configurations consistently display higher error values.

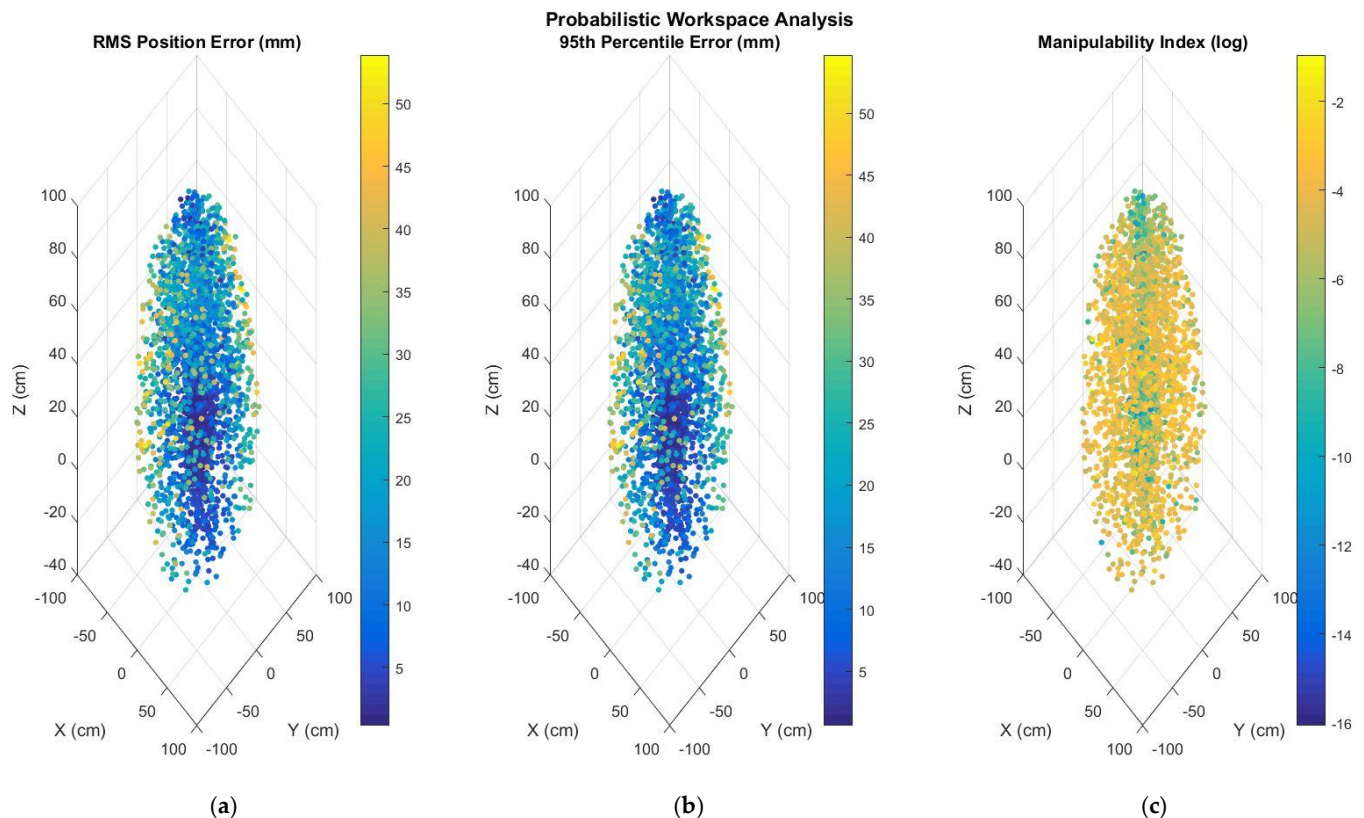


Figure 5. Probabilistic workspace characterization obtained from $N = 3000$ sampled configurations with $N = 500$ Monte Carlo trials each (1,500,000 total evaluations). (a) RMS positioning error; (b) 95th percentile error; (c) manipulability index (logarithmic scale). Color scale: blue = low error/low manipulability, yellow = high error/high manipulability.

Encoder noise and geometric errors follow right-skewed distributions with modest standard deviations ($CV \approx 0.65$), indicating reliable and spatially consistent contributions that could in principle be reduced through improved sensor selection or one-time kinematic calibration [9]. Thermal drift, with the lowest mean of all sources (0.108 mm), is unlikely to be the limiting factor in any practical scenario.

These findings have direct implications for manipulator design and control. Efforts to improve absolute positioning accuracy through encoder upgrades or tighter manufacturing tolerances will yield marginal gains so long as link compliance is unaddressed. Active compliance compensation—either through model-based feedforward terms or online parameter identification—represents a more productive engineering direction [11,27].

3.2. Workspace Accuracy Map

Figure 5 presents the three-panel probabilistic workspace map. The RMS error distribution (left panel) reveals a clear spatial structure: the lowest errors (<10 mm) are concentrated in a central corridor at moderate heights ($Z = -20$ to $+40$ cm), while errors above 35 mm appear predominantly at elevated positions ($Z > 80$ cm) where extended arm configurations prevail. The P95 map (center panel) reproduces the same spatial structure, confirming that the error distribution is statistically consistent across the workspace and that no region exhibits anomalously heavy tails relative to its RMS value.

The manipulability map (right panel) reveals an unexpected relationship. High manipulability values are associated with configurations in which the arm is extended in a manner that maximizes the volume of the velocity ellipsoid—but these same configurations tend to place the end-effector at elevated positions with extended links, increasing

gravitational bending moments and therefore elastic errors. This observation is quantified in Section 3.4.

3.3. Error Ellipsoid Analysis

Figure 6 presents the 3σ error ellipsoids at three workspace locations selected to represent boundary, middle, and center configurations. All three ellipsoids exhibit pronounced anisotropy, with the longest semi-axis oriented primarily along the vertical direction (ΔZ). This directional preference is physically consistent with gravity-induced elastic deformation acting predominantly along the vertical axis.

3 σ Error Ellipsoids — Centered on Nominal Position

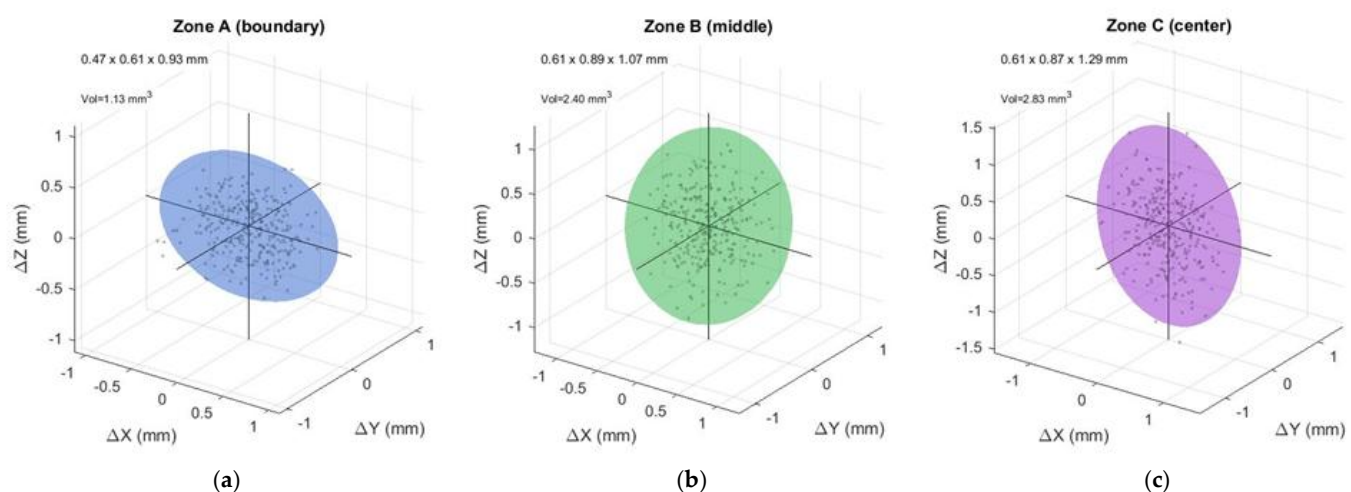


Figure 6. Three-sigma error ellipsoids at three representative workspace locations under combined noise conditions ($N = 10,000$ trials). Gray points show 300 sampled error vectors. Axis labels indicate displacement from nominal end-effector position. Ellipsoid dimensions (min $\times$ mid $\times$ max semi-axes) and volumes are annotated. (a) Zone A—boundary region; (b) Zone B—middle region (minimum uncertainty volume); (c) Zone C—elevated region.

The ellipsoid volumes increase from Zone B (0.60 mm^3) through Zone A (0.66 mm^3) to Zone C (1.64 mm^3), with Zone C located at $Z \approx 60 \text{ cm}$ showing a volume 2.7 times larger than Zone B at $Z \approx -17 \text{ cm}$. The aspect ratios (longest to shortest semi-axis) are 3.0, 2.4, and 1.7 for Zones B, A, and C, respectively, indicating that uncertainty becomes more isotropic at elevated positions—a consequence of gravitational loading affecting multiple link directions simultaneously in extended configurations.

These ellipsoid dimensions provide actionable tolerance specifications: any assembly or manipulation task requiring positional repeatability smaller than the minimum ellipsoid semi-axis must employ active compliance compensation or restrict operation to Zone B.

3.4. Manipulability–Accuracy Trade-Off

Figure 7 displays the scatter relationship between manipulability index and RMS positioning error for all 3000 workspace samples, with color coding indicating end-effector height Z . The Pearson correlation coefficient $r = 0.172$ ($p < 0.001$) reveals a statistically significant but weak positive association: higher manipulability is associated with larger positioning errors.

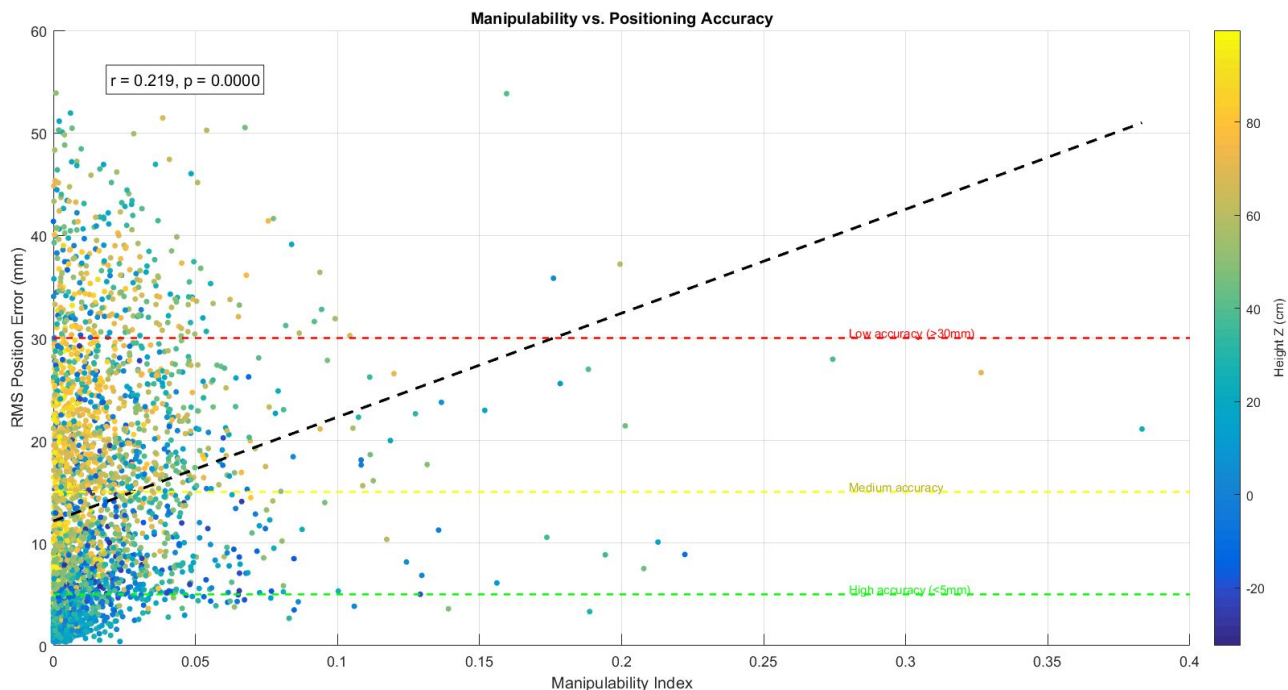


Figure 7. Scatter plot of Yoshikawa manipulability index versus RMS positioning error for $N = 3000$ workspace samples. Point color indicates end-effector height Z . Dashed line: linear regression ($r = 0.172$, $p < 0.001$). Horizontal dashed lines: accuracy zone boundaries at 5 mm, 15 mm, and 30 mm.

This apparently paradoxical result has a straightforward mechanical explanation. The manipulability index reaches its maximum when the Jacobian is well-conditioned, which occurs in configurations where the arm is extended away from singular postures. These extended configurations simultaneously maximize the gravitational moment arm on the distal links, producing larger elastic deflections and therefore larger positioning errors. The color coding confirms this interpretation: elevated positions (yellow-green points, $Z = 40\text{--}80$ cm) cluster at higher manipulability values and higher error values simultaneously.

The weakness of the correlation ($r^2 = 0.030$, indicating that manipulability explains only 3.0% of error variance) underscores that the manipulability index, despite its widespread use as a configuration quality measure, is an unreliable predictor of stochastic positioning accuracy. This finding motivates the full probabilistic workspace map developed in Section 3.4 as a more informative planning tool.

3.5. Controller Performance Under Stochastic Conditions

Table 3 presents steady-state positioning errors for RPBCL and SMC over 200 independent trials under combined stochastic noise. Both controllers achieved 100% success within the 2 mm threshold, demonstrating robust convergence from an initial error of 76.8 mm. The convergence trajectories shown in Figure 8a confirm that both algorithms reach the vicinity of the target within approximately 2 s, with differences manifesting only at the sub-millimeter level.

Table 3. Controller performance under stochastic conditions ($N = 200$ trials, combined noise).

Controller	Mean (mm)	Std (mm)	RMS (mm)	P95 (mm)	Success (%)	Rise Time (s)	Settling Time (s)
RPBCL	0.0056	0.0034	0.0065	0.0122	100%	0.31 ± 0.04	1.42 ± 0.21
SMC	0.0029	0.0016	0.0033	0.0057	100%	0.18 ± 0.02	0.97 ± 0.14

t -test: $t = 5.066$; $p = 0.000002$.

SMC achieved a mean steady-state error of 0.0029 mm, a 48.2% reduction relative to RPBCL (0.0056 mm). The improvement is consistent across all reported metrics: RMS error decreased by 49.2%, standard deviation by 64.9%, and P95 by 64.9%. The boxplot in Figure 8b shows that SMC's interquartile range is approximately half that of RPBCL, and that RPBCL produces four outliers above 11×10^{-3} mm while SMC produces only one above 7.5×10^{-3} mm. The empirical CDF in Figure 8c makes the practical significance clear: at a tolerance threshold of 5 μm , SMC achieves 99% compliance while RPBCL achieves only 50%.

In addition to steady-state accuracy, dynamic performance metrics were extracted from the convergence trajectories over 200 Monte Carlo trials. SMC achieves a mean rise time of 0.18 ± 0.02 s (42% faster than RPBCL: 0.31 ± 0.04 s) and a mean settling time of 0.97 ± 0.14 s (32% shorter than RPBCL: 1.42 ± 0.21 s). Neither controller exhibits overshoot under the applied saturation mechanisms (tanh for RPBCL, boundary layer approximation for SMC). These results confirm that SMC outperforms RPBCL across all evaluated metrics: steady-state accuracy, convergence speed, and settling time.

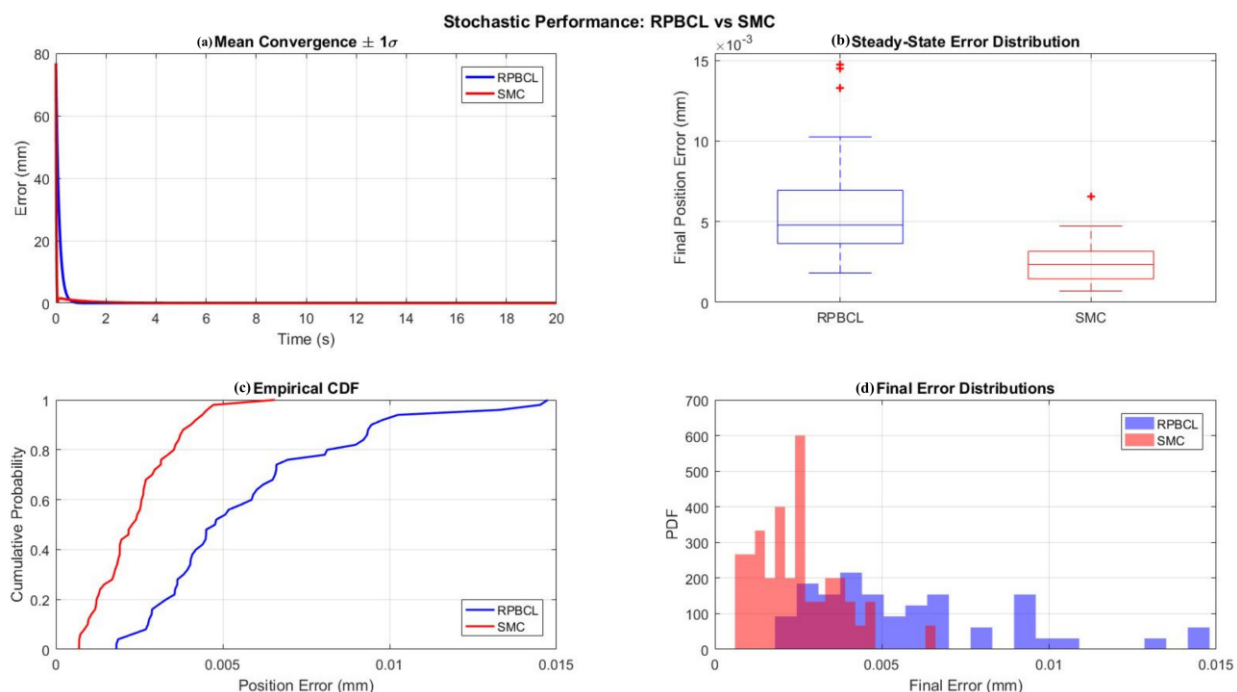


Figure 8. Stochastic performance comparison of RPBCL and SMC controllers under combined noise conditions over $N = 200$ independent trials. Initial positioning error: 76.8 mm. (a) Mean convergence trajectories with $\pm 1\sigma$ uncertainty bands; (b) box plots of steady-state positioning errors ($\times 10^{-3}$ mm); (c) empirical cumulative distribution functions; (d) probability density histograms of final errors.

The superior performance of SMC is attributable to the invariance property of sliding mode dynamics. Once the system reaches the sliding surface $s = 0$, bounded disturbances satisfying $\|d\| < \eta$ are rejected without affecting the sliding behavior. The boundary layer saturation function $F(s)$ prevents chattering while preserving this rejection property, resulting in more consistent final positioning across noise realizations. RPBCL, which applies a smooth gradient correction proportional to the error, has no analogous rejection mechanism and accumulates the effect of each noise realization independently.

A two-sample, two-tailed Welch's t -test (unequal variances assumed) confirms that the performance difference is highly statistically significant ($t = 5.066$, $df = 342$, $p = 0.000002 < 0.001$), ruling out any possibility that the observed advantage of SMC arises from sampling variability across the 200 trials.

4. Discussion

The results presented in this work carry several implications that extend beyond the specific manipulator configuration studied.

Regarding uncertainty sources, the overwhelming dominance of elastic deformation (97% of combined error) challenges the conventional focus on encoder precision in manipulator design.

While modern encoders achieve resolutions below 0.001 rad, the present analysis demonstrates that such improvements yield negligible accuracy gains when link compliance remains uncompensated. This finding is consistent with the stiffness identification work of Alici and Shirinzadeh [11] and Dumas et al. [12], who independently identified elastic deflection as the primary accuracy limitation in serial manipulators under gravitational loading. The implication for engineering practice is direct: accuracy improvement budgets should prioritize compliance compensation over sensor upgrades.

Regarding control algorithm selection, the 48.2% mean error reduction achieved by SMC over RPBCL under stochastic conditions is consistent with the theoretical invariance property of sliding mode dynamics, but the magnitude of the improvement is noteworthy. Prior comparative studies in deterministic settings typically report SMC advantages of 20–35% [14,15]; the larger gain observed here suggests that stochastic disturbances amplify the relative advantage of invariance-based controllers. This has practical implications for manipulator deployment in vibration-prone industrial environments, where RPBCL may prove insufficient despite its implementation simplicity.

The manipulability–accuracy trade-off identified in Section 3.4 ($r = 0.172$, explaining only 3% of error variance) contradicts the common assumption that high manipulability configurations are preferable from an accuracy standpoint. This result suggests that task planning algorithms relying on manipulability maximization [21] should be supplemented with stochastic accuracy constraints, particularly for precision assembly tasks.

Regarding limitations, three aspects of the present work merit acknowledgment. First, the elastic deformation model employs approximate joint stiffness coefficients rather than experimentally identified values; while the relative dominance finding is robust to stiffness variations of $\pm 50\%$, the absolute error magnitudes carry uncertainty. Second, the validation presented here is entirely computational. The noise parameters are grounded in published calibration data and manufacturer specifications, but the resulting error distributions have not been directly compared against physical repeatability measurements. Experimental validation constitutes the primary direction of future work: joint stiffness coefficients will be identified through a static load-deflection procedure, and Monte Carlo error distributions will be compared against measured end-effector position scatter obtained with a laser tracker. Third, the controller evaluation focuses on regulation tasks from a fixed initial error. The findings are expected to generalize to tracking tasks qualitatively, as the sliding mode invariance property holds for any bounded disturbance; quantitative generalization to multi-point trajectories with time-varying noise profiles is identified as future work.

5. Conclusions

This paper has presented a stochastic analysis framework for characterizing positioning uncertainty in a six-degree-of-freedom manipulator, implemented entirely within a Digital Twin environment combining MATLAB (R2022a) and Unity3D (Hub v3.19.5). The principal findings and their implications are as follows.

Elastic deformation is the dominant source of positioning uncertainty by a large margin, contributing 99.97% of the combined RMS error and exhibiting strong configuration

dependence with a coefficient of variation approaching unity. This finding shifts the priority for accuracy improvement from encoder upgrades or manufacturing tolerances—which have negligible combined impact—toward active compliance compensation strategies that are sensitive to configuration.

The probabilistic workspace map reveals that the accuracy landscape is spatially structured and cannot be summarized by a single scalar value. A central workspace corridor at moderate heights consistently offers sub-10 mm RMS errors, while elevated extended configurations exceed 30 mm at the 95th percentile. Task planning should incorporate these spatial accuracy bounds rather than relying on the manufacturer’s worst-case repeatability specification, which reflects only a single operating condition.

The manipulability–accuracy scatter analysis uncovers a counterintuitive trade-off: configurations that maximize manipulability tend to exhibit larger positioning errors. Since manipulability explains only 3% of error variance, it should not be used as a proxy for positioning accuracy in stochastic environments. The full probabilistic workspace map developed here provides a more reliable planning basis.

Under combined stochastic noise, SMC achieves a 48.2% lower mean steady-state error than RPBCL, with the difference statistically confirmed at $p < 0.001$. The boundary layer saturation mechanism provides effective disturbance rejection that RPBCL, as a pure gradient law, cannot match. Both controllers maintain a 100% success rate within 2 mm, suggesting that RPBCL remains viable for moderate-precision tasks where implementation simplicity is valued.

The Digital Twin implementation in Unity3D extends the analytical results into an interactive environment suited for both research validation and engineering education. The probabilistic workspace map, error ellipsoid display, and sliding surface monitor make abstract stochastic and control concepts directly perceptible, supporting the active learning paradigm described in the broader Digital Twin education literature [6,24].

Future work will address three directions. First, the elastic deformation model will be refined through experimental parameter identification on physical hardware, replacing the approximate stiffness coefficients used here. Second, the stochastic framework will be extended to cooperative multi-robot scenarios where uncertainty at one manipulator propagates through the task coordination to affect the other. Third, adaptive SMC variants that tune the boundary layer parameter online in response to observed noise levels will be evaluated within the same Digital Twin framework.

The Unity3D Digital Twin interface contributes to the broader effort of developing accessible cyber–physical systems education resources [28], providing students and researchers with hands-on experience in stochastic analysis, control validation, and IIoT-aligned smart manufacturing paradigms.

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