

Article

Feasible-Region Aggregation of Distributed Multi-Energy Storage Based on Support Functions and Minkowski Sum

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Abstract

Distributed multi-energy storage is critical for enhancing the operational flexibility of power systems with high renewable energy penetration. However, the heterogeneity between electric and thermal energy storage in dynamic response, multi-time-scale features and coupling interactions causes three key limitations of existing aggregation methods: conservative feasible-region characterization, excessive computational complexity, and inaccurate dispatch capability evaluation. To address these issues, this paper proposes a distributed multi-energy storage aggregation method based on support functions and the Minkowski sum. First, a unified convex polyhedral feasible-region model incorporating electro-thermal coupling constraints is established. Then, a distributed parallel aggregation strategy with constraint relaxation–reconstruction and adaptive optimal direction selection is developed to achieve high-precision approximation of the aggregated feasible region. Case studies on a park-level integrated energy system with 10 electric and 5 thermal storage units show an average support function error of 1.35%, a volume similarity ratio of 0.953, and a dispatch feasibility rate of 98.70%. Electro-thermal coupling reduces feasible-region volume by 32.50% with only 5.20% overestimation. Under fluctuating commands, the tracking deviation is 2.80% with no constraint violations. Furthermore, measured engineering data verify its linear scalability for large clusters and 1.27% cost deviation in day-ahead dispatch. The proposed method accurately characterizes the joint regulation capability of multi-energy storage clusters while ensuring favorable computational scalability.

Keywords: distributed multi-energy storage; support functions; Minkowski sum; electro-thermal coupling; aggregated feasible region

Academic Editors: Alessandro

Lorenzo Palma, Luca La Notte

Received: 3 June 2026

Revised: 5 July 2026

Accepted: 6 July 2026

Published: 16 July 2026

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1. Introduction

With the continuous growth of renewable energy penetration and the accelerating electrification and decentralization processes on the end-user side, the spatial and temporal distribution of adjustable resources in distribution networks and integrated energy systems has become increasingly dispersed and the dependence of power system operation on operational flexibility has been significantly enhanced. Existing review studies have indicated that quantification, modeling, and utilization of flexible resources

have become core topics in the operation and planning of novel power systems [1]. Studies focusing on aggregators and virtual communities have further demonstrated that organizing geographically dispersed distributed energy resources (DERs) into tradable and dispatchable aggregates is a critical approach for unlocking the flexibility potential on the demand side and energy storage side [2]. Systematic reviews of DER management systems have revealed that aggregation, coordination, and monitoring capabilities are gradually emerging as new infrastructure supporting the operation of active distribution networks [3]. In this context, studies on the flexibility characterization of renewable distributed resource integration, energy storage-based aggregation of distributed renewable energy, and predictive flexibility aggregation of demand-side resources such as buildings have all indicated that how to accurately map a large number of small-capacity, heterogeneous, time-series-coupled resources into an aggregated model available for upper-level dispatch has become a common concern in both theoretical research and engineering practice [4–6].

Regarding the construction of DER aggregation models, existing studies have advanced the development of aggregation methods from various perspectives. For instance, dynamic aggregation strategies for virtual power plants have been shown to enhance the adjustable capability of heterogeneous DERs [7]; stochastic models for aggregated flexibility under uncertainty effectively characterize the risk features of feasible boundaries [8]; and distributionally robust bidding models for the joint energy and frequency regulation market have enhanced the risk management capability of aggregators [9]. For energy storage aggregation, existing studies have proposed a two-stage flexibility evaluation method for clustered energy storage considering prediction errors [10]; for scenarios where the resource portfolio varies with operating conditions, dynamic aggregation and submodular optimization methods can improve the efficiency of resource selection and coordination [11]; and in transmission and distribution coordination scenarios, a real-time coordinated dispatch framework based on aggregated flexibility has demonstrated the application value of aggregation models in system-level dispatch [12]. In electric vehicle cluster scheduling scenarios, the learning-driven aggregated flexibility approach has achieved millisecond-level real-time dispatch for ten-thousand-vehicle fleets [13]. Nevertheless, most of these methods are oriented towards generalized DER portfolios or power and energy boundary approximation and have not yet fully addressed the problem of unified characterization of the high-dimensional multi-period feasible region for multi-energy heterogeneous energy storage.

In the field of multi-energy systems and geometric feasible regions, existing studies have started to focus on the modeling complexity introduced by multi-time-step and cross-energy coupling. The comprehensive flexibility characterization of distributed multi-energy systems considering time-coupling constraints [14] as well as the analysis of the aggregated power flexibility of multi-energy systems based on dynamic networks [15] have both indicated that cross-energy coupling can significantly alter the shape and boundary of the system feasible region. Meanwhile, adaptive convex-hull strategies and the PyFlexAD tool for set addition for energy storage have demonstrated that geometric methods based on convex sets, vertices, or set operations provide a promising technical route for high-dimensional aggregation [16,17]. Furthermore, the time-coupled packaging method for DERs [18], flexibility aggregation based on virtual battery equivalence [19], cluster aggregation of generation-load-storage flexibility resources [20], and energy-reserve coordinated optimization involving aggregators [21] have all verified the advantages of the ‘aggregate first, dispatch later’ paradigm in improving computability and market participation capability from various perspectives. From the perspective of technical routes, mainstream existing aggregation methods can be categorized into four branches: parameter equivalence, geometric exactness, boundary characterization, and

general-purpose tools. However, all of them exhibit notable inadequacies in adaptability to the high-dimensional aggregation scenarios of heterogeneous electro-thermal energy storage. The parameter equivalence method, represented by the virtual battery model, achieves cluster equivalence by fitting equivalent charge–discharge power, capacity and efficiency parameters. It features a concise model structure and high computational efficiency, but can only retain the macroscopic power boundary. It fails to accurately characterize multi-period temporal coupling constraints and electro-thermal coupling constraints, leading to remarkable errors in refined dispatch scenarios [22]. The geometric exact method, represented by convex-hull aggregation, can derive the exact feasible region through vertex enumeration. Nevertheless, its computational complexity grows exponentially with the number of dimensions and units, rendering it only applicable to small-scale low-dimensional scenarios and unable to support high-dimensional aggregation computation for multi-period, multi-energy systems [23]. The boundary characterization method, represented by the flexibility envelope approach, can obtain the power regulation boundary of resource clusters. However, most such methods are designed for single-energy scenarios, with weak capacity for handling global cross-unit coupling constraints, and the conservatism of approximation escalates rapidly in high-dimensional settings [24]. General-purpose aggregation tools represented by PyFlexAD implement distributed flexibility aggregation based on set operations. Nonetheless, no dedicated processing mechanism is devised for global cross-unit coupling constraints; direct application to electro-thermal coupling scenarios will cause severe capability overestimation. Meanwhile, its fixed-direction sampling strategy suffers from low sampling efficiency in high-dimensional spaces, making it difficult to strike a balance between accuracy and computational overhead [16,17].

However, although recent studies have made advances in terms of system paradigms [25], three-stage optimization of virtual power plants with energy storage participation [26], bi-level market models for aggregators [27], flexible regulation of battery-integrated renewable energy systems [28], and aggregation–disaggregation frameworks for heterogeneous distributed energy storage [29], there are still three main limitations regarding the feasible-region aggregation of heterogeneous electro-thermal distributed energy storage clusters focused on in this paper. First, most existing studies focus on generalized DERs or single electric energy storage scenarios, and lack a unified convex set representation of the heterogeneity of electric and thermal energy storage in terms of response speed, energy state evolution, and time steps. Second, the electro-thermal coupling relationship is often implicitly represented by coarse-grained equivalent models or dispatch constraints, making it difficult to directly form a compact and computable joint feasible region. Third, the set addition and boundary reconstruction of high-dimensional multi-period feasible regions incur prohibitive computational costs, and existing approximation methods still exhibit an obvious trade-off among accuracy, scalability, and disaggregation solvability.

To solve the above problems, this paper proposes a distributed multi-energy storage aggregation method based on support functions and the Minkowski sum. By constructing a unified convex set modeling framework for heterogeneous multi-energy storage, this paper proposes a compact characterization approach for the aggregated feasible region subject to electro-thermal coupling constraints. The approach strikes a well-balanced trade-off between aggregation accuracy and computational efficiency in high-dimensional multi-period scenarios and provides a reliable capability boundary for aggregation modeling, dispatch decision-making, and market participation of such heterogeneous multi-energy storage systems. First, a unified convex polyhedral feasible-region model for electric and thermal energy storage is constructed, which maps heterogeneous physical constraints into high-dimensional closed convex sets, providing a mathematical

foundation for the unified characterization of distributed multi-energy storage. Second, by leveraging the additivity of support functions with respect to the Minkowski sum, a distributed parallel aggregation strategy is designed, which converts high-dimensional geometric operations into scalar summation over a finite set of directions, thereby improving the computational efficiency of aggregation and reducing the amount of information interaction. Finally, to address the boundary-tightening problem caused by electro-thermal coupling and the multi-time-scale characteristics, a compact aggregated feasible region applicable to upper-level optimization and market decision-making is constructed by combining equivalent constraint processing and an adaptive direction selection mechanism. The research in this paper can provide a more accurate modeling tool for flexibility quantification, aggregated dispatch, and market participation of distributed multi-energy storage clusters.

2. Modeling of Distributed Multi-Energy Storage

With the in-depth development of the Energy Internet, multi-energy coupled systems represented by electricity and heat have gradually replaced single power systems and become the main form of energy consumption and supply. As a key flexibility resource in the system, distributed energy storage features diverse forms, distinct characteristics, and highly scattered geographical distribution. To achieve efficient aggregation and optimal dispatch of large-scale heterogeneous energy storage units, accurate unit models must first be established based on physical mechanisms, and their feasible operating regions must be rigorously defined using set theory methods, thereby converting physical constraints into computable mathematical geometric structures.

2.1. Architecture of the Multi-Energy Storage System

Multi-energy systems typically use electric power networks and heating networks as core physical carriers and realize the flow and coupling of energy through energy conversion equipment. There is significant heterogeneity between the two networks in terms of physical characteristics. The electric power network is characterized by extremely high electromagnetic wave propagation speed and relatively low network losses, but large-scale economical energy storage is difficult to achieve. By contrast, the heating network usually transfers energy through water or steam media, with significant thermal inertia and considerable transmission delay. Distributed energy storage units are connected to these two heterogeneous networks dispersedly. Electric energy storage units are directly connected to electric buses and mainly used to mitigate renewable energy fluctuations and provide frequency regulation and fast reserve services. Thermal energy storage units are connected to the heating pipeline network and are mainly used to absorb curtailed wind and photovoltaic power, track the heat load profile, and realize decoupled dispatch by virtue of thermal inertia. Such electro-thermal heterogeneity endows the energy storage cluster with inherent complementary advantages in terms of time-scales, but also greatly increases the complexity of unified modeling and aggregation.

To address the management and control challenges of massive distributed energy storage units, a three-layer aggregation architecture of unit–cluster–aggregator is constructed in this paper, with the logical boundaries and functional positioning of each layer clearly defined as follows:

The unit layer refers to the physical energy storage devices, including batteries, heat storage tanks, etc. This layer focuses on the physical constraints and operating states of the devices themselves.

The cluster layer is a set of multiple energy storage units that are geographically adjacent or have close electrical connections. Within the same cluster, direct energy flow interactions may exist between units, or units may share the same network node.

The aggregator layer is a virtual entity responsible for the unified coordination of multiple clusters and scattered units under its jurisdiction on behalf of the upper-level system. Externally, the aggregator acts as an integrated virtual large-scale energy storage unit to participate in the electricity market and ancillary service markets.

This architecture provides clear logical boundaries for the subsequent aggregation method, allowing the modeling process to be developed layer by layer starting from the bottom-level units.

2.2. Mathematical Models of Electro-Thermal Energy Storage Units

To perform accurate aggregation analysis, mathematical models that can reflect the dynamic characteristics of distributed multi-energy storage must be established. This section focuses on the time-varying power constraints and dynamic loss characteristics of electric energy storage and thermal energy storage.

The electric energy storage unit (ESU) is the most technically mature form of energy storage currently. The state of charge (SOC) of an ESU increases during charging and decreases during discharging. The SOC at any time can be expressed as

$$SOC_t = SOC_{t-1} + \left(\frac{P_{ch} \eta_{ch}}{E_{rate}} - \frac{P_{dis}}{\eta_{dis} E_{rated}} \right) \Delta t \quad (1)$$

where SOC_{t-1} is the value of SOC at the previous time interval, SOC_t is the value of SOC after one dispatch interval, and Δt denotes the duration of dispatch interval. P_{ch} and P_{dis} are the charging power and discharging power of the distributed electric energy storage unit, respectively. It is specified that at least one of P_{ch} and P_{dis} is zero within one dispatch interval Δt . To enable the application of convex optimization theory, the charge–discharge mutual-exclusion constraint is relaxed in the aggregation stage in this paper, allowing both to exist simultaneously within the feasible region. η_{ch} and η_{dis} are the charging efficiency and discharging efficiency of the distributed electric energy storage unit, respectively, and E_{rated} is the rated capacity of the distributed electric energy storage unit.

During the charging and discharging processes, the SOC of distributed electric energy storage devices must be maintained within a reasonable range to prolong their service life.

$$SOC_{min} < SOC_t < SOC_{max} \quad (2)$$

where SOC_{max} and SOC_{min} denote the upper and lower bounds of the SOC, respectively.

$$0 < P_i^{ch} < P_i^{max\ ch} \quad (3)$$

$$P_i^{max\ dis} < P_i^{dis} < 0 \quad (4)$$

where P_i^{ch} and P_i^{dis} denote the charging power and discharging power of the distributed energy storage device, respectively, and $P_i^{max\ ch}$ and $P_i^{max\ dis}$ are the maximum charging power and maximum discharging power of the distributed energy storage device during operation, respectively.

Similar to the electric energy storage system, the energy capacity of the thermal energy storage system increases during the heat charging process and decreases during the heat discharging process. The energy capacity of the thermal energy storage system at any time instant can be expressed as

$$H_{t+1}^s = (1 - \theta^{TSS}) H_t^s + \eta_{h,c} P_{c,t}^{TSS} - \frac{P_{d,t}^{TSS}}{\eta_{h,d}} \quad (5)$$

where E_{rate} denotes the energy capacity of the thermal energy storage system at time interval t , θ^{TSS} is the self-discharge heat loss coefficient, $\eta_{\text{h,c}}$ and $\eta_{\text{h,d}}$ represent the heat charging efficiency and heat discharging efficiency, respectively, and $P_{\text{c},t}^{\text{TSS}}$ and $P_{\text{d},t}^{\text{TSS}}$ are the heat charging power and heat discharging power at time interval t , respectively.

Similarly, the energy capacity of the thermal energy storage system must be maintained within a reasonable range during the heat charging and discharging processes of the thermal energy storage device:

$$H_{\min}^s \leq H_t^s \leq H_{\max}^s \quad (6)$$

where $H_{\max}^s$ and $H_{\min}^s$ denote the upper and lower bounds of the energy capacity of the thermal energy storage system, respectively.

$$0 \leq P_{\text{c},t}^{\text{TSS}} \leq P_{\text{c},\max}^{\text{TSS}} \quad (7)$$

$$P_{\text{d},\max}^{\text{TSS}} \leq P_{\text{d},t}^{\text{TSS}} \leq 0 \quad (8)$$

where $P_{\text{c},\max}^{\text{TSS}}$ denotes the upper bound of the heat charging power of the thermal energy storage system and $P_{\text{d},\max}^{\text{TSS}}$ represents the upper bound of the heat discharging power of the thermal energy storage system.

2.3. Set-Theoretic Description of the Feasible Region for Multi-Energy Storage

To achieve unified mathematical modeling of distributed multi-energy storage units with diverse physical constraints, relevant theories of convex analysis and set theory are introduced in this section. The physical operating constraints of energy storage units are mapped into geometric sets in a high-dimensional space, and the feasible region of the units is defined accordingly, which provides a rigorous geometric topological foundation for the subsequent aggregation operations based on the Minkowski sum.

In a deterministic dispatch environment, the operation of an energy storage unit is subject to a set of linear equality and inequality constraints. The region enclosed by these constraints in a multi-dimensional Euclidean space is defined as the feasible operating region of the unit. For any distributed multi-energy storage unit i , its operating state within the dispatch period T can be uniquely determined by the power sequence vector $\mathbf{p}_i$ and the energy state sequence vector $\mathbf{e}_i$. Let $\mathbf{x}_i = [\mathbf{p}_i; \mathbf{e}_i]^T$, then the operating trajectory $\mathbf{x}_i$ of the unit must satisfy the dynamic equations and physical constraints established in the aforementioned Section 2.2.

From the perspective of set theory, the dynamic feasible region of the unit is defined as the set of all feasible operating trajectories, which is specifically formulated as

$$\mathbf{F}_i = \{\mathbf{x}_i \in \mathbb{R}^{2T}\} \quad (9)$$

$$\text{s.t.} \quad \begin{cases} \mathbf{e}_{i,t} = A_i \mathbf{e}_{i,t-1} + B_i \mathbf{p}_{i,t-1} \\ \underline{\mathbf{p}}_i \leq \mathbf{p}_i \leq \bar{\mathbf{p}}_i \\ \underline{\mathbf{e}}_i \leq \mathbf{e}_i \leq \bar{\mathbf{e}}_i \end{cases} \quad (10)$$

where A_i and B_i are linear state transition matrices, $\underline{\mathbf{p}}_i$ and $\bar{\mathbf{p}}_i$ denote the lower- and upper-bound vectors of power, respectively, and $\underline{\mathbf{e}}_i$ and $\bar{\mathbf{e}}_i$ represent the upper and lower bound vectors of energy capacity, respectively.

From the perspective of geometric topological structure, since $\mathbf{F}_i$ is defined exclusively by affine functions and half-spaces, it is proven that $\mathbf{F}_i$ is a closed convex polyhedron in the space $\mathbb{R}^{2T}$ in accordance with convex polyhedron theory.

In an electro-thermal coupled system, electric energy storage and thermal energy storage do not operate independently, but are interconnected through coupling equipment. Such interconnection is mathematically represented by linear constraints between variables.

It is assumed that the system contains m ESUs and n thermal energy storage units (TSUs). A coupling matrix $\mathbf{C} \in \mathbb{R}^{q \times (m+n)}$ is defined to describe the energy conversion relationship. Then, the joint feasible region $\mathbf{F}_{\text{joint}}$ of the multi-energy coupled system is defined as the intersection of the Cartesian product of the feasible regions of each sub energy storage unit and the coupling constraint set:

$$\mathbf{F}_{\text{joint}} = \left(\prod_{i=1}^m \mathbf{F}_{e,i} \times \prod_{j=1}^n \mathbf{F}_{h,j} \right) \cap \{ \mathbf{z} \in \mathbb{R}^{(m+n)} : \mathbf{C}\mathbf{z} \leq \mathbf{d} \} \quad (11)$$

where $\mathbf{z} = [\mathbf{p}_e^T, \mathbf{p}_h^T]^T$ denotes the system-wide power vector, and $\mathbf{d} \in \mathbb{R}^q$ is the bound vector of the coupling constraints. This expression mathematically unifies the heterogeneous constraints of electric and thermal energy storage and achieves energy balance and equipment capacity coupling among multiple energy forms through the coupling matrix.

This paper adopts a linear coupling model with a constant electro-thermal conversion efficiency assumption. This assumption delivers satisfactory accuracy when equipment operates near its rated condition with narrow power fluctuation ranges, making it suitable for conventional day-ahead dispatch and intra-day optimization scenarios.

Nevertheless, this model has clear applicable boundaries and limitations. In practical integrated energy systems, the conversion efficiency of electro-thermal equipment dynamically varies with operating power, ambient temperature, return water temperature, load factor and other operating conditions, exhibiting obvious nonlinear characteristics. Direct adoption of the fixed-coefficient model will introduce characterization errors in coupling constraints under wide operating condition fluctuations and extreme ambient temperatures, thereby impairing the accuracy of the aggregated feasible region.

From the perspective of convex analysis, nonlinear coupling relationships can be categorized into convex nonlinearity and non-convex nonlinearity. Convex nonlinear constraints can maintain the convexity of the feasible region and be directly compatible with the aggregation framework proposed in this paper. For non-convex nonlinear constraints, convex relaxation processing is required before they can be incorporated into the convex set aggregation system of this work.

In this case, $\mathbf{F}_{\text{joint}}$, as the intersection of a finite number of closed convex polyhedra, remains a closed convex polyhedron. This property provides a critical underpinning for the subsequent Minkowski sum-based aggregation operations, as the closed convexity of convex polyhedra is preserved under the Minkowski sum operation, thereby ensuring that the aggregated equivalent model still retains a sound mathematical structure and favorable computability.

This feasible region is not merely a geometric representation of physical constraints, but more importantly defines the capability boundary of the energy storage cluster for market bidding participation. The higher the accuracy of the aggregated feasible region, the more competitive the bidding strategies of the aggregator in the energy and reserve

markets will be, which effectively avoids penalty costs arising from capability overestimation and revenue losses caused by capability underestimation.

3. Aggregation Method Based on Support Functions and Minkowski Sum

A unified convex polyhedral feasible-region model for distributed multi-energy storage is established in the foregoing paragraphs, which lays a solid mathematical foundation for the aggregation of large-scale clusters. However, the direct optimal dispatch of the high-dimensional joint feasible region is confronted with the challenges of the curse of dimensionality and privacy disclosure. This chapter aims to propose an aggregation method based on support functions and the Minkowski sum. The core rationale of the proposed method is as follows: the high-dimensional feasible-region information of the energy storage cluster is mapped into a set of low-dimensional functions through support functions, a powerful tool for convex analysis; on this basis, the geometric aggregation of the cluster is implemented at the level of support functions by virtue of the Minkowski sum operation, and an externally equivalent virtual large-scale energy storage unit model is finally constructed. This method achieves a balance between computational complexity and information privacy preservation while maintaining the accuracy of the aggregated model.

3.1. Minkowski Sum and Support Functions

The Minkowski sum is a core concept in set operations which is widely used in the study of the aggregation behavior of distributed resources. The Minkowski sum can be intuitively interpreted as the union of all possible positions obtained by translating set A by the vector of each point in set B. Equivalently, it can also be regarded as the union of set B translated along each point in set A.

For illustrative purposes, two energy storage resources 1 and 2 with time-coupled power characteristics over two consecutive time intervals ($k, k+1$), as shown in Figure 1, are adopted for explanation, where P_1 and P_2 denote the power feasible regions of energy storage resources 1 and 2, respectively, and P represents the aggregated power feasible region.

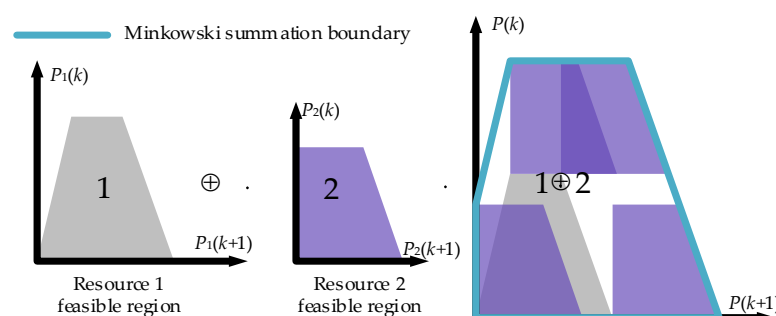


Figure 1. Schematic diagram of Minkowski sum.

The aggregated feasible region of energy storage, denoted as $1 \oplus 2$, can be expressed as the union of set 1 itself and the region swept by the vertices of set 2 moving continuously for one full cycle along the envelope of set 1, with increased complexity in the geometric representation of its envelope. The solution efficiency of aggregation decreases exponentially with the growth of the dimensionality and scale of the problem, and there is currently no efficient general solution method for the Minkowski sum of high-dimensional polytopes in multi-period, large-scale scenarios. To address this problem, this paper proposes an aggregation method based on support functions and the Minkowski sum.

The support function is a powerful tool for characterizing the geometric properties of convex sets. For a non-empty compact convex set $\mathbf{K} \subseteq \mathbb{R}^n$, its support function $h_{\mathbf{K}}: \mathbb{R}^n \rightarrow \mathbb{R} \cup \{+\infty\}$ is defined as

$$h_{\mathbf{K}}(\mathbf{y}) = \sup\{\langle \mathbf{y}, \mathbf{x} \rangle : \mathbf{x} \in \mathbf{K}\} \quad (12)$$

where $\langle \cdot, \cdot \rangle$ denotes the standard inner product, and $\mathbf{y}$ is a given direction vector. The value of the support function in the direction $\mathbf{y}$ is equal to the maximum projection of the convex set $\mathbf{K}$ along this direction. Since the feasible regions of energy storage involved in this paper are all defined by linear inequality constraints, and both power and energy have physical upper and lower bounds, they are all bounded closed convex polyhedra that satisfy the conditions of compact convex sets, and the value of the support function is always finite.

In the context of energy storage aggregation, this property corresponds exactly to the superposition of the power and energy feasible regions of multiple energy storage units. The support function satisfies additivity under the Minkowski sum operation. To obtain the overall feasible region of the energy storage cluster, there is no need to directly calculate the Minkowski sum of high-dimensional geometric bodies; only simple arithmetic summation of the support functions of individual units is required.

According to the definition of the support function and the description of linear constraints given above, calculating $h_{\mathbf{F}_i}(\mathbf{y})$ is equivalent to solving a standard linear programming problem. For the i energy storage unit, its feasible region is jointly defined by the state transition equations, power upper and lower bounds, and energy capacity upper and lower bounds. Accordingly, the corresponding linear programming problem can be formally formulated as:

$$h_{\mathbf{F}_i}(\mathbf{y}) = \max_{\mathbf{p}_i, \mathbf{e}_i} \mathbf{y}_p^T \mathbf{p}_i + \mathbf{y}_e^T \mathbf{e}_i \quad (13)$$

$$\text{s.t.} \quad \begin{cases} \mathbf{e}_{i,t} = A_i \mathbf{e}_{i,t-1} + B_i \mathbf{p}_{i,t-1} \\ \underline{\mathbf{p}}_i \leq \mathbf{p}_i \leq \bar{\mathbf{p}}_i \\ \underline{\mathbf{e}}_i \leq \mathbf{e}_i \leq \bar{\mathbf{e}}_i \end{cases} \quad (14)$$

where the direction vector $\mathbf{y}$ is decomposed into two parts corresponding to the power

variable $\mathbf{p}_i$ and the energy variable $\mathbf{e}_i$, i.e., $\mathbf{y}^T = [\mathbf{y}_p^T, \mathbf{y}_e^T]^T$.

In practical implementation, the aggregator first broadcasts a finite set of key direction vectors $\{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_L\}$ to be evaluated to all units or sub-cluster controllers participating in the aggregation. Each unit solves the above linear programming problem

independently and locally to obtain a set of scalar values $\{h_{\mathbf{F}_i}(\mathbf{y}_1), h_{\mathbf{F}_i}(\mathbf{y}_2), \dots, h_{\mathbf{F}_i}(\mathbf{y}_L)\}$.

These scalar values correspond to the support function values associated with the feasible region of the unit, evaluated along all critical directions. Subsequently, each unit only needs to upload these L scalar results to the aggregator, rather than its complete model parameters or state trajectories. On this basis, the aggregator performs a simple scalar summation operation in accordance with the additivity principle of support functions:

$$H_l = \sum_{i=1}^N h_{\mathbf{F}_i}(\mathbf{y}_l), l = 1, 2, \dots, L \quad (15)$$

the aggregated support function values $\{H_1, H_2, \dots, H_L\}$ of the entire cluster along the L directions can be obtained.

3.2. Aggregation Strategy and Optimal Direction Selection

This section aims to translate the theoretical paradigm based on support functions and the Minkowski sum into a systematic and operable aggregation strategy. The core objective of this strategy is to design a distributed aggregation method with high computational efficiency, low communication overhead, and unit privacy preservation for clusters of energy storage units that are large-scale, geographically dispersed, and heterogeneous in characteristics, so as to ultimately accurately characterize the overall operating capability boundary of the cluster.

The fundamental principle of the aggregation strategy is directly derived from the additivity of support functions with respect to the Minkowski sum. For a cluster composed of N independent energy storage units (coupling constraints are not considered temporarily), its overall feasible region $\mathbf{F}_{\text{agg}}$ is the Minkowski sum of the feasible region $\mathbf{F}_i$ of each individual unit:

$$\mathbf{F}_{\text{agg}} = \bigoplus_{i=1}^N \mathbf{F}_i \quad (16)$$

Thus, for any given evaluation direction $\mathbf{y} \in \mathbb{R}^{2T}$, the support function of the cluster can be efficiently obtained via the arithmetic sum of the support functions of individual units:

$$h_{\mathbf{F}_{\text{agg}}}(\mathbf{y}) = \sum_{i=1}^N h_{\mathbf{F}_i}(\mathbf{y}) \quad (17)$$

The aggregator does not need to acquire the detailed model parameters or real-time state trajectories of each individual unit. It only needs to broadcast a finite set of key directions to each unit and then obtain the overall maximum projection value of the cluster along the given direction through a simple scalar summation.

Conventional geometric summation methods require the explicit computation and storage of the vertices and half-space representations of high-dimensional polyhedra, whose computational complexity grows in a combinatorial explosion manner with the number of units N and the dimension $2T$. In contrast, the support function-based method converts complex geometric set operations into arithmetic operations on function values, with its core computational load concentrated on the parallel solution of L linear programming problems by each unit. Since linear programming solvers are highly mature and efficient, and the computations among units are completely independent, the proposed method fundamentally avoids the curse of dimensionality and achieves linear scalability of computational complexity.

There exists an order-of-magnitude difference in the physical time constants between electric energy storage and thermal energy storage. The dynamic response of electric energy storage can be on the order of minutes or even seconds, while the dynamic process of thermal energy storage is usually on an hourly scale due to the thermal inertia of its medium. Directly modeling both types of storage at a unified high temporal resolution will lead to excessively high dimensionality and enormous redundancy in the thermal energy storage model; conversely, adopting a unified low resolution will lose the information on the fast regulation capability of electric energy storage. To this end, this paper proposes a unified hierarchical aggregation and scale mapping framework.

First, aggregation of the same types of energy storage is performed at their respective optimal temporal resolutions.

For the aggregation of the electric energy storage sub-cluster, a fine-grained time step Δt_e is adopted, with the number of time intervals in the dispatch period set to T_e . For m ESUs, the aggregated support function is expressed as

$$H_e(\mathbf{y}_e) = \sum_{i=1}^N h_{e,i}(\mathbf{y}_e), \mathbf{y}_e \in \mathbb{R}^{2T_e} \quad (18)$$

For the aggregation of the TES sub-cluster, a coarse time step Δt_h that satisfies $\Delta t_h = K\Delta t_e$ is adopted, with the number of time intervals in the dispatch period set to $T_h = T_e/K$. For n TSUs, the aggregated support function is expressed as

$$H_h(\mathbf{y}_h) = \sum_{j=1}^N h_{h,j}(\mathbf{y}_h), \mathbf{y}_h \in \mathbb{R}^{2T_h} \quad (19)$$

Subsequently, to enable the coordination of thermal energy storage capacity with electric energy storage within the unified dispatch framework, a mapping relationship from the Δt_h time step to the Δt_e time step shall be established. The mapping matrix $\mathbf{M} \in \mathbb{R}^{2T_e \times 2T_h}$

is defined, whose construction is based on the principles of energy conservation and power smoothing. Specifically, the heat charging/discharging power $p_{h,\tau}$ within time interval τ at the Δt_h time step can be equivalent to the power $\mathbf{P}_{e,\text{block}}$ that remains constant or is distributed according to a specific pattern over the corresponding K consecutive time intervals at the Δt_e time step. The mapping relationship can be expressed as

$$\mathbf{p}_e = \mathbf{M}\mathbf{p}_h \quad (20)$$

Correspondingly, the mapping of the direction vector follows the duality principle. Evaluating the contribution of thermal energy storage along the direction $\mathbf{y}_e$ at the electric-side time step is equivalent to performing the evaluation along the direction $\mathbf{M}^T\mathbf{y}_e$ at the thermal-side time step. Therefore, the equivalent support function value of thermal energy storage along the electric-side direction $\mathbf{y}_e$ is $H_h(\mathbf{M}^T\mathbf{y}_e)$.

Finally, under the unified electric-side time step Δt_e , the aggregated support function of the entire electro-thermal hybrid energy storage cluster is expressed as

$$H_{\text{total}}(\mathbf{y}_e) = H_e(\mathbf{y}_e) + H_h(\mathbf{M}^T\mathbf{y}_e), \quad \forall \mathbf{y}_e \in \mathbb{R}^{2T_e} \quad (21)$$

This framework preserves the dynamic characteristics of heterogeneous energy resources, while providing the upper-level dispatch with unified time-scale characterization of the virtual energy storage capability.

The electro-thermal system is interconnected via coupling devices to form a joint feasible region. The Minkowski sum does not satisfy the distributive law with respect to set intersection and the relation is generally a proper inclusion, that is

$$\bigoplus_{i=1}^N (\mathcal{X}_i \cap \mathcal{C}) \subsetneq \left(\bigoplus_{i=1}^N \mathcal{X}_i \right) \cap \mathcal{C} \quad (22)$$

Directly ignoring coupling constraints during aggregation will overestimate the cluster capability and result in infeasible dispatch commands. Conversely, retaining coupling constraints at the unit level forfeits the computational advantage of dimensionality reduction brought by aggregation. To resolve this contradiction, this paper proposes a two-step method consisting of equivalent constraint relaxation and precise reconstruction, which aims to compactly integrate coupling effects into the aggregation model at the cost of controllable conservatism.

First, relaxed aggregation and capability envelope calculation are performed. The coupling constraints among units are temporarily lifted, and electric and thermal energy

storage are aggregated as independent resources to obtain the uncoupled aggregated feasible region:

$$\mathcal{X}_0 = \left(\bigoplus_{i=1}^{N_e} \mathcal{X}_i^e \right) \times \left(\bigoplus_{j=1}^{N_h} \mathcal{X}_j^h \right) \quad (23)$$

Meanwhile, the support function $\mathcal{X}_0$ corresponding to $h_{\mathcal{X}_0}(\cdot)$ can be rapidly obtained by summing the unit-level support functions, yielding the maximum theoretical envelope of the cluster capability. This step can be fully computed in a distributed parallel manner with extremely high efficiency.

In the first stage of relaxed aggregation, all cross-unit global electro-thermal coupling constraints are temporarily neglected—these are linear constraints linking the power and state variables of multiple independent energy storage units. Meanwhile, all local physical constraints intrinsic to each individual unit are fully retained.

This treatment allows the support function of each energy storage unit to be computed independently. Leveraging the additivity property of the Minkowski sum, distributed parallel aggregation is realized, and the resulting relaxed aggregated feasible region corresponds to the maximum theoretical envelope of the cluster's regulation capacity.

Subsequently, the equivalence transformation of coupling constraints is performed and the tightest constraint set is constructed. Suppose the cluster comprises N_e electric energy storage units and N_h thermal energy storage units. For the i electric energy storage unit, its decision vector is $\mathbf{x}_i^e = [\mathbf{P}_{ch,i}^e, \mathbf{P}_{dis,i}^e, \mathbf{E}_i^e]^\top \in \mathbb{R}^{3T_e}$, with the corresponding feasible region $\mathcal{X}_i^e$; for the j thermal energy storage unit, its decision vector is $\mathbf{x}_j^h = [\mathbf{P}_{ch,j}^h, \mathbf{P}_{dis,j}^h, \mathbf{E}_j^h]^\top \in \mathbb{R}^{3T_h}$, with the corresponding feasible region $\mathcal{X}_j^h$.

The unit-level electro-thermal coupling constraint is a cross-unit global linear constraint, and its general form is given by

$$\sum_{i=1}^{N_e} \mathbf{A}_i^e \mathbf{x}_i^e + \sum_{j=1}^{N_h} \mathbf{A}_j^h \mathbf{x}_j^h \leq \mathbf{b}_c \quad (24)$$

where $\mathbf{A}_i^e$ and $\mathbf{A}_j^h$ are the unit coupling coefficient matrices, which characterize the contributions of each unit's energy variables to the coupling relationship; $\mathbf{b}_c \in \mathbb{R}^{m_c}$ is the upper-bound vector of coupling constraints, corresponding to physical limits such as the total rated capacity of electro-thermal conversion equipment; m_c denotes the number of coupling constraints. This constraint globally couples the decision variables of all units and cannot be directly decomposed to individual units to participate in the Minkowski sum operation.

Let the true aggregated feasible region incorporating coupling constraints be denoted as $\mathcal{X}^{agg}$, which is defined as the set formed by summing the decision vectors of all units that satisfy the coupling constraints:

$$\mathcal{X}^{agg} = \left\{ \left(\sum_{i=1}^{N_e} \mathbf{x}_i^e, \sum_{j=1}^{N_h} \mathbf{x}_j^h \right) \mid \mathbf{x}_i^e \in \mathcal{X}_i^e, \mathbf{x}_j^h \in \mathcal{X}_j^h, \sum_i \mathbf{A}_i^e \mathbf{x}_i^e + \sum_j \mathbf{A}_j^h \mathbf{x}_j^h \leq \mathbf{b}_c \right\} \quad (25)$$

It follows straightforwardly from set properties that $\mathcal{X}^{agg} \subseteq \mathcal{X}_0$, and direct adoption of $\mathcal{X}_0$ will overestimate the cluster capability. The core idea of this paper is to construct an equivalent constraint set $\mathcal{C}^{agg}$ at the aggregation level such that $\mathcal{X}^{agg} \approx \mathcal{X}_0 \cap \mathcal{C}^{agg}$. That is, the coupling effects are compactly characterized by low-dimensional aggregation constraints while preserving the distributed computational advantage of the Minkowski sum.

Regarding the construction principle of the tightest equivalent constraint, for any given direction vector $\mathbf{a}$ in the aggregation space, if the constraint holds for all feasible unit combinations, its tightest right-hand-side term is equivalent to the support function value of the true aggregated feasible region along this direction:

$$\begin{aligned} b^*(\mathbf{a}) &= h_{\mathcal{X}_{\text{agg}}}(\mathbf{a}) = \max_{\mathbf{x}^e, \mathbf{x}^h} \mathbf{a}^\top \begin{bmatrix} \sum \mathbf{x}_i^e \\ \sum \mathbf{x}_j^h \end{bmatrix} \\ \text{s.t. } \mathbf{x}_i^e &\in \mathcal{X}_i^e, \forall i; \quad \mathbf{x}_j^h \in \mathcal{X}_j^h, \forall j; \quad \sum_i \mathbf{A}_i^e \mathbf{x}_i^e + \sum_j \mathbf{A}_j^h \mathbf{x}_j^h \leq \mathbf{b}_c \end{aligned} \quad (26)$$

This problem is a standard linear program. Its optimal solution $b^*(\mathbf{a})$ corresponds to the supporting hyperplane along direction $\mathbf{a}$; no tighter linear constraint can enclose all feasible points along this direction and hence it is referred to as the tightest constraint along that direction.

Based on the above principle, the complete optimization-based construction procedure for the tightest equivalent constraint set is elaborated as follows. First, the candidate constraint direction set is determined. In light of the physical mechanism of electro-thermal coupling, aggregation variable directions strongly associated with the coupling effect are selected, including the total electric charging/discharging power and total heat charging/heat discharging power in each time interval, as well as directions of linear combinations of electric and thermal power, to form the candidate direction set $D_c = \{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{M_c}\}$. The scale of this direction set is on the same order of magnitude as the number of original coupling constraints, typically ranging from a few to more than ten, which is far smaller than the total number of directions in the AODS algorithm. Subsequently, the tightest right-hand-side term is solved direction by direction. For each $\mathbf{a}_k \in D_c$, the linear programs in Equation (26) are solved to obtain the corresponding tightest upper bound b_k^* , yielding the candidate equivalent constraint $\mathbf{a}_k^\top \mathbf{x} \leq b_k^*$. Redundant constraints are then eliminated. For all candidate constraints, a linear programming feasibility test is conducted item by item to verify whether a given constraint can be derived from the remaining ones. After removing redundant constraints, the compact equivalent constraint set $\mathcal{C}_{\text{agg}}$ is obtained.

For the single-device capacity coupling scenario in the case study of this paper, the equivalent constraint can be simplified as an upper-bound constraint on the electro-thermal coupling components in the power vector of the virtual energy storage:

$$\mathbf{S}\mathbf{x} \leq \mathbf{P}_{\text{convert}}^{\text{max}} \quad (27)$$

where $\mathbf{S}$ denotes the selection matrix, which is used to extract the power components related to electric-to-heat conversion from the aggregated decision vector.

In the second stage of precise reconstruction, the tightest supporting hyperplanes along coupling-related directions are solved to construct an equivalent linear constraint set at the aggregation level. Each equivalent constraint corresponds to a half-space in the high-dimensional decision space. By means of half-space cutting, the infeasible portions of the relaxed feasible region that violate the original coupling constraints are trimmed off, which effectively shrinks the overestimated area introduced by relaxation. The final reconstructed feasible region still retains the property of a closed convex polyhedron and exhibits high tightness.

In terms of computational overhead and complexity analysis, let the number of candidate constraint directions be M_c , the dimension of decision variables per unit be d ,

and the total number of units be $N = N_e + N_h$. Solving the equivalent constraint for a single direction corresponds to a linear program with $O(Nd)$ variables and $O(Nd + m_c)$ constraints, and its time complexity is approximately $O((Nd)^{2.5})$ when solved by a general interior-point method.

Since M_c is a small-scale constant, the total computational complexity of coupling reconstruction is $O(M_c \cdot (Nd)^{2.5})$. Compared with the complexity $O(MNd^{2.5})$ of uncoupled aggregation, the additional computational overhead of coupling reconstruction accounts for less than 10.00% and will not become the overall bottleneck of the algorithm. If the coupling constraints are independent per time interval (such as the upper limit of electric-to-heat power in each interval), they can be further decomposed into multiple low-dimensional LPs solved in parallel by time interval, and the computational efficiency can be further improved. Ultimately, the precise aggregated feasible region reconstructed by the two-step method is given by

$$\mathcal{X}_{\text{agg}} \approx \mathcal{X}_0 \cap \mathcal{C}_{\text{agg}} \quad (28)$$

The corresponding support function needs to be evaluated over the intersection of $\mathcal{X}_0$ and $\mathcal{C}_{\text{agg}}$, which can be accomplished by solving a linear program with additional constraints, while the computational burden remains within an acceptable range.

The selection of the evaluation direction set $\mathcal{Y} = \{\mathbf{y}_1, \dots, \mathbf{y}_L\}$ is the core trade-off between accuracy and efficiency. A poor direction set may cause the reconstructed equivalent feasible region $\tilde{\mathbf{F}}$ to severely deviate from the real aggregated feasible region $\mathbf{F}_{\text{agg}}$, further resulting in infeasible dispatch commands or underestimation of market bidding revenue. Conversely, an overly dense direction set, while guaranteeing accuracy, will incur unnecessary computational and communication overhead. To systematically address this problem, this section proposes an adaptive optimal evaluation direction selection algorithm. Based on optimization theory, the algorithm formulates the direction selection problem as a sequential decision-making process that minimizes the worst-case approximation error and automatically identifies the most critical directions for characterizing the shape of the feasible region through an iterative mechanism.

Let $\mathbf{F}_{\text{agg}} \in \mathbb{R}^n$ denote the closed convex aggregated feasible region to be approximated. The approximate feasible region defined by L directions $\mathcal{Y}_L$ and their corresponding support function values $\{H(\mathbf{y}) : \mathbf{y} \in \mathcal{Y}_L\}$ is given by

$$\tilde{\mathbf{F}}_L = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{y}^\top \mathbf{x} \leq H(\mathbf{y}), \forall \mathbf{y} \in \mathcal{Y}_L\} \quad (29)$$

The objective of the algorithm is to select the direction set $\mathcal{Y}_L$ such that a certain metric of the approximation error is minimized. A natural and robust metric for this purpose is the worst-case support function error (WSFE), which is defined as

$$\text{WSFE}(\mathcal{Y}_L) = \max_{\|\mathbf{y}\|_* \leq 1} \left[h_{\mathbf{F}_{\text{agg}}}(\mathbf{y}) - h_{\tilde{\mathbf{F}}_L}(\mathbf{y}) \right]_+ \quad (30)$$

where $[\cdot]_+ = \max(\cdot, 0)$ and $\|\cdot\|_*$ denote the dual norm, which is usually specified as the ℓ_2 norm, i.e., $\|\mathbf{y}\|_2 \leq 1$. This error metric quantifies the maximum extent to which

the approximate feasible region $\tilde{\mathbf{F}}_L$ underestimates the capability of the real aggregated feasible region $\mathbf{F}_{\text{agg}}$ across all possible directions.

While $h_{\mathbf{F}_{\text{agg}}}(\mathbf{y})$ is unknown, it is confirmed to be decomposable into $\sum_i h_{\mathbf{F}_i}(\mathbf{y})$, and each $h_{\mathbf{F}_i}(\mathbf{y})$ can be obtained via local LP computation. Accordingly, the core idea of the adaptive optimal direction selection (AODS) algorithm is to iteratively identify the direction $\mathbf{y}^*$ that maximizes the current approximation error $\delta(\mathbf{y}) = \sum_i h_{\mathbf{F}_i}(\mathbf{y}) - h_{\tilde{\mathbf{F}}_L}(\mathbf{y})$ and add it to the direction set $\mathcal{Y}_L$, so as to actively and targetedly reduce the worst-case error.

The unit support function calculation Oracle, for a given direction $\mathbf{y}$, returns $\sum_{i=1}^N h_{\mathbf{F}_i}(\mathbf{y})$ by invoking the local solvers of each unit in parallel. The initial direction set is usually taken as the positive and negative directions of the standard basis vectors, i.e., $\mathcal{Y}_0 = \{\pm \mathbf{e}_1, \pm \mathbf{e}_2, \dots, \pm \mathbf{e}_n\}$, to ensure that the initial approximation can capture the extreme values along each coordinate axis. The convergence tolerance is set as $\varepsilon > 0$, and the maximum number of iterations is K_{max} .

For the initialization, first set the iteration counter $k = 0$ and the current direction set $\mathcal{Y} = \mathcal{Y}_0$. Then, for all $\mathbf{y} \in \mathcal{Y}_0$, invoke the Oracle to calculate the aggregated support function value $H(\mathbf{y})$. Finally, construct the initial approximate feasible region $\tilde{\mathbf{F}}^{(0)} = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{y}^T \mathbf{x} \leq H(\mathbf{y}), \forall \mathbf{y} \in \mathcal{Y}\}$.

For the iterative loop, first calculate the support function of the approximate feasible region, then search for the direction with the maximum gap, and finally perform the convergence judgment.

For the current approximate feasible region $\tilde{\mathbf{F}}^{(k)}$, its support function $h_{\tilde{\mathbf{F}}^{(k)}}(\mathbf{y})$ along any direction $\mathbf{y}$ can be obtained by solving the following linear programming problem:

$$h_{\tilde{\mathbf{F}}^{(k)}}(\mathbf{y}) = \max_{\mathbf{x}} \mathbf{y}^T \mathbf{x} \quad (31)$$

$$\text{s.t. } \mathbf{y}_j^T \mathbf{x} \leq H(\mathbf{y}_j), \quad \forall \mathbf{y}_j \in \mathcal{Y} \quad (32)$$

Let the optimal solution of this linear programming problem be denoted as $\mathbf{x}^*(\mathbf{y})$. Solve the following optimization problem to find the direction corresponding to the current worst-case error:

$$\mathbf{y}_k^* = \arg \max_{\|\mathbf{y}\|_2 \leq 1} \left[\sum_{i=1}^N h_{\mathbf{F}_i}(\mathbf{y}) - h_{\tilde{\mathbf{F}}^{(k)}}(\mathbf{y}) \right] \quad (33)$$

This is a nonlinear, non-smooth optimization problem defined over the unit ball $\mathcal{B}_2 = \{\mathbf{y} \in \mathbb{R}^n : \|\mathbf{y}\|_2 \leq 1\}$. The objective function $f(\mathbf{y}) = \sum_i h_{\mathbf{F}_i}(\mathbf{y}) - h_{\tilde{\mathbf{F}}^{(k)}}(\mathbf{y})$ satisfies convexity and has computable subgradients.

For any given $\mathbf{y}$, a subgradient $\mathbf{g}_1(\mathbf{y})$ of $\sum_i h_{\mathbf{F}_i}(\mathbf{y})$ is the sum of the optimal solutions $\mathbf{x}_i^*(\mathbf{y})$ to the corresponding unit-level LP problems, i.e., $\mathbf{g}_1(\mathbf{y}) = \sum_{i=1}^N \mathbf{x}_i^*(\mathbf{y})$. Meanwhile, a subgradient $\mathbf{g}_2(\mathbf{y})$ of $h_{\tilde{\mathbf{F}}^{(k)}}(\mathbf{y})$ is the optimal solution $\mathbf{x}^*(\mathbf{y})$ to the LP problem in step 1. Therefore, a subgradient of $f(\mathbf{y})$ is given by $\mathbf{g}(\mathbf{y}) = \mathbf{g}_1(\mathbf{y}) - \mathbf{g}_2(\mathbf{y})$. On

this basis, the projected subgradient method can be adopted to efficiently solve the above optimization problem, with the iterative formula given by

$$\mathbf{y}^{(l+1)} = \text{Proj}_{\mathcal{B}_2} \left(\mathbf{y}^{(l)} + \alpha_l \cdot \mathbf{g}(\mathbf{y}^{(l)}) \right) \quad (34)$$

where α_l denotes the step size, and $\text{Proj}_{\mathcal{B}_2}(\cdot)$ represents the projection operator onto the ℓ_2 unit ball. Calculate the current maximum gap value:

$$\delta_k = f(\mathbf{y}_k^*) = \sum_i h_{\mathbf{F}_i}(\mathbf{y}_k^*) - h_{\mathbf{F}^{(k)}}(\mathbf{y}_k^*) \quad (35)$$

If $\delta_k \leq \varepsilon$, the algorithm converges and exits the loop. Finally, the direction set update and information enhancement are performed. The newly identified critical direction $\mathbf{y}_k^*$ and its opposite direction $-\mathbf{y}_k^*$ are added to the direction set:

$$\mathcal{Y} \leftarrow \mathcal{Y} \cup \{\mathbf{y}_k^*, -\mathbf{y}_k^*\} \quad (36)$$

where the opposite direction is added to preserve the symmetry of the approximate feasible region $\tilde{\mathbf{F}}$ and ensure the completeness of the dual constraints. The Oracle is invoked to calculate the aggregated support function values $H(\mathbf{y}_k^*)$ and $H(-\mathbf{y}_k^*)$ along the new directions, and the description of the approximate feasible region is updated as

$$\tilde{\mathbf{F}}^{(k+1)} = \{\mathbf{x} \mid \mathbf{y}^\top \mathbf{x} \leq H(\mathbf{y}), \forall \mathbf{y} \in \mathcal{Y}\} \quad (37)$$

At this point, set $k = k + 1$. Finally, the optimal direction set $\mathcal{Y}^* = \mathcal{Y}$ and the corresponding set of aggregated support function values $\{H(\mathbf{y}) : \mathbf{y} \in \mathcal{Y}^*\}$ are output.

To rigorously demonstrate the mathematical properties of the proposed AODS algorithm, this section carries out a systematic analysis from three dimensions: convergence characteristics, optimality guarantees, and approximation error bounds.

In terms of convergence behavior, the algorithm guarantees monotonically non-increasing worst-case approximation error. For the convex polyhedral feasible regions composed of linear constraints adopted in this work, it achieves exact convergence within finite iterations; for general compact convex sets, the error decays exponentially with the number of sampling directions, consistent with classic cutting-plane theory. Regarding approximation characteristics, the reconstructed feasible region is always a conservative outer approximation of the true domain, which inherently ensures the operational feasibility of dispatch solutions; the greedy direction selection rule yields tighter boundaries than fixed-direction sampling under the same number of constraints. The method delivers particularly favorable approximation performance for polyhedral domains with a moderate number of facets, large-scale heterogeneous clusters with smooth aggregated boundaries, and high-dimensional multi-period scenarios where uniform sampling suffers from the curse of dimensionality.

Let the true aggregated feasible region be $\mathcal{K} \subset \mathbb{R}^n$, which is a non-empty compact convex set, where n denotes the dimension of decision variables. Let the set of evaluation directions obtained at the k iteration be $D_k = \{d_1, d_2, \dots, d_k\}$, and the corresponding approximate feasible region is formulated as

$$\mathcal{K}_k = \{\mathbf{x} \in \mathbb{R}^n \mid d_i^\top \mathbf{x} \leq h_{\mathcal{K}}(d_i), \forall d_i \in D_k\} \quad (38)$$

where $h_{\mathcal{K}}(d) = \max_{x \in \mathcal{K}} d^\top x$ is the support function of set $\mathcal{K}$. By the fundamental properties of convex sets, $\mathcal{K}_k$ always constitutes an outer approximation of the true feasible region $\mathcal{K}$, that is, $\mathcal{K} \subseteq \mathcal{K}_k$.

Since each iteration only supplements new supporting hyperplanes to the direction set, the approximate feasible regions satisfy the set-inclusion relationship $\mathcal{K}_0 \supseteq \mathcal{K}_1 \supseteq \dots \supseteq \mathcal{K}_k \supseteq \mathcal{K}$, indicating that the approximate domain contracts monotonically as iterations proceed. The corresponding worst-case support function error is formulated as

$$\text{WSFE}(k) = \max_{\|d\|_2 \leq 1} \left(h_{\mathcal{K}_k}(d) - h_{\mathcal{K}}(d) \right) \quad (39)$$

It is monotonically non-increasing with the iteration count k . The iterative process of the algorithm remains stable without oscillation, and there is no risk of accuracy rebound. Let the true aggregated feasible region F_{agg} be a compact convex polyhedron in $\mathbb{R}^n$ with a finite number of faces. Then the AODS algorithm achieves exact recovery $\hat{F}_L = F_{\text{agg}}$ after a finite number of iterations.

Proof. A convex polyhedron is fully characterized by a finite set of supporting hyperplanes, each corresponding to the outward normal direction of one face. The initial direction set covers all coordinate axes, and each iteration adds at least one new supporting hyperplane. Once the direction set contains the outward normal of every face of F_{agg} , the outer approximation defined by Equation (29) is exactly equal to F_{agg} , and the worst-case support function error drops to zero. Since the number of faces is finite, the algorithm achieves exact convergence within finite steps. $\square$

The AODS algorithm adopts a worst-error direction selection strategy. Each iteration corrects the direction with the largest current error, eliminating ineffective computation on redundant directions in uniform sampling. Its convergence rate is remarkably superior to that of fixed-direction sampling methods. For general compact convex sets, the error decays exponentially with the number of iterations; for the convex polyhedral feasible regions addressed in this paper, the error drops rapidly as the normal vectors of critical faces are identified.

At each iteration k , the direction y^* selected by the AODS algorithm maximizes the reduction in the worst-case support function error among all possible single-direction additions. That is, it achieves the largest possible one-step accuracy improvement under the current approximation. This single-step greedy property does not guarantee that the resulting direction set is globally optimal. However, as verified by the comparative case studies, AODS consistently outperforms fixed-direction sampling methods under the same number of directions, providing near-optimal approximation accuracy for practical dispatch scenarios.

Regarding the approximation error bound, the fundamental error bound is first derived based on the Lipschitz continuity of support functions. For a compact convex set

$\mathcal{K}$ with diameter $D = \max_{x, y \in \mathcal{K}} \|x - y\|_2$, its support function $h_{\mathcal{K}}(d)$ satisfies Lipschitz continuity on $\mathbb{R}^n$. That is, for any directions $d_1, d_2 \in \mathbb{R}^n$, the following holds: $|h_{\mathcal{K}}(d_1) - h_{\mathcal{K}}(d_2)| \leq D \|d_1 - d_2\|_2$. Based on this property, if the coverage radius of the direction set on the unit sphere is δ , the worst-case error of fixed-direction sampling satisfies $\text{WSFE} \leq D\delta$, indicating a linear correlation between the error and the coverage

radius. Regarding the error decay rate, classic cutting-plane theory has proven that for general compact convex sets, the adaptive worst-direction selection strategy yields an exponentially decaying worst-case approximation error with the number of iterations [30]. For the convex polyhedral feasible regions studied in this paper, the error decreases even faster as critical face normals are identified and eventually reaches zero as stated in Proposition 1. For the park-level energy storage aggregation scenario addressed in this paper, with limited decision dimensions, the algorithm can achieve high-precision approximation within a small number of iterations, which is consistent with the simulation results of case studies. Furthermore, when the number of iterations reaches the number of faces of the convex polyhedron, the error reduces to zero, corresponding to the conclusion of exact convergence in finite steps.

To concisely discuss the asymptotic complexity with respect to the three core parameters—the number of storage units N , the length of the optimization horizon T , and the number of sampling directions M —the scaling properties are summarized below with O notation as a formal reference. The total serial computational cost grows linearly with the number of storage units N , as each unit independently performs local support function calculations. Under the distributed parallel architecture, unit-level computations are fully decoupled, such that the end-to-end wall-clock runtime is nearly independent of N , which underpins the method's linear scalability. The complexity scales polynomially with the optimization horizon T , approximately $O(T^{2.5})$ for a standard interior-point solver; extending the dispatch horizon or refining the time resolution will raise the solution cost of each single linear program, which is the dominant factor affecting single-direction computational efficiency. The overall complexity grows approximately linearly with the number of sampling directions M . The value of M is determined by the geometric complexity of the aggregated feasible region rather than the cluster scale; as the number of units increases, the feasible region becomes smoother due to parameter superposition and M grows very slowly and thus will not become a bottleneck in large-scale scenarios. In summary, the total serial complexity of the framework is $O(N \cdot M \cdot T^{2.5})$, and the wall-clock complexity under the parallel architecture is $O(M \cdot T^{2.5})$. Both present polynomial scalability, which is fundamentally superior to the exponential complexity of vertex-enumeration-based exact aggregation.

For the asymptotic analysis of computational complexity, let the total number of energy storage units participating in aggregation be N , the number of time intervals within the dispatch period be T , the total number of evaluation directions finally selected by the algorithm be M , and the number of AODS iterations be K . Each iteration adds two opposite directions, satisfying $M = M_0 + 2K$, where M_0 denotes the number of initial directions. The computational overhead of the proposed aggregation method mainly consists of three core modules: unit-level support function solution, support function aggregation, and adaptive direction search. The asymptotic complexity of each module and the influence patterns of the parameters are elaborated as follows.

Calculating the support function of a single energy storage unit along a single direction is equivalent to solving a linear programming (LP) problem. For the energy storage model with sequential state transitions, both the number of variables and constraints of the LP are of order $O(T)$. When solved by a general interior-point method, the time complexity of a single LP is $O(T^{2.5})$. By exploiting the linear state transition structure of the energy storage model, the solution complexity can be further optimized to a nearly linear level of $O(T)$ via dynamic programming. The overall computational complexity of this module is $O(N \cdot M \cdot T^\alpha)$, where $\alpha \in [1, 2.5]$, depending on the extent to which the solver utilizes the structured constraints. Under the distributed parallel

architecture adopted in this paper, all units can solve LPs independently and in parallel. Accordingly, the wall-clock runtime of this module is independent of the number of units N and is merely $O(N \cdot T^a)$.

Summing the scalar support function values of N units across M directions yields a computational complexity of $O(N \cdot M)$. This is an extremely lightweight arithmetic operation, typically accounting for less than 10.00% of the overall computational overhead.

Each iteration consists of two sub-steps. The first is to solve the support function of the current approximate domain, which corresponds to an LP with M constraints and $O(T)$ variables, with a complexity of $O(M \cdot T)$. The second is to search for the worst-case error direction using the projected subgradient method. Given a fixed convergence accuracy, the number of subgradient iterations is of constant order, and the computational cost per step is $O(T)$. Accordingly, the complexity of a single direction search is approximately $O(M \cdot T)$, and the total search overhead is $O(K \cdot M \cdot T)$. Since the number of iterations K is far smaller than the total number of directions M and grows only logarithmically with increasing accuracy requirements, the overall overhead of this module is lower than that of a unit-level support function solution.

Based on the above decomposition, the influence patterns of the three core parameters on computational overhead can be summarized as follows: the total computational load grows approximately linearly, and the overall computational cost increases at a polynomial rate. In summary, the proposed method maintains polynomial scalability across three dimensions: unit scale, time dimension, and direction sampling. Compared with the exponential complexity of vertex-enumeration-based exact aggregation, it exhibits remarkable computational advantages in high-dimensional multi-period and large-scale cluster scenarios.

All the above aggregation accuracy analyses are carried out based on the convex-relaxed feasible region. The additional approximation error introduced by the relaxation of charge–discharge mutual-exclusivity constraints can be rigorously quantified via the directional characteristics of support functions, which enables the approximation error bound analysis for convex relaxation.

We here briefly analyze the approximation error introduced by relaxing the charge–discharge mutual-exclusivity constraint and clarify practical scenarios where such error can be safely neglected. The relaxation expands the unit feasible region into a convex set by simultaneously allowing charging and discharging power, which yields an outer approximation and thus brings bounded support function deviation. This error only emerges along special directions that jointly maximize both charge and discharge power. For most dispatch-oriented directions focusing on net power output, energy capacity and ramping limits, the optimal solution naturally satisfies mutual exclusivity, resulting in zero relaxation error. In typical engineering scenarios including day-ahead economic dispatch, intra-day rolling optimization and frequency regulation, the operating targets mainly rely on net power trajectories. Under these practical cases, the relaxation error remains extremely small and exerts negligible impacts on cluster capability evaluation and dispatch feasibility.

For a single energy storage unit, let $\mathcal{X}^{real}$ denote the true non-convex feasible region and $\mathcal{X}^{relax}$ denote the feasible region after convex relaxation. It is obvious that $\mathcal{X}^{real} \subseteq \mathcal{X}^{relax}$ and $\mathcal{X}^{relax} = \text{conv}(\mathcal{X}^{real})$. For an arbitrary direction vector $\mathbf{d} = [d_{ch}, d_{dis}, d_E]^\top$, the support function relaxation error is defined as $\Delta h(\mathbf{d}) = h_{\mathcal{X}^{relax}}(\mathbf{d}) - h_{\mathcal{X}^{real}}(\mathbf{d})$. Two core conclusions can be derived from the extremal properties of linear programming. When the direction satisfies $d_{ch} \leq 0$ and $d_{dis} \geq 0$, the support function values before and after relaxation are identical, i.e., $\Delta h(\mathbf{d}) = 0$. In this case, the optimal solution naturally satisfies

the charge–discharge mutual-exclusivity constraint, and convex relaxation introduces no error. When $d_{ch} > 0$ and $d_{dis} > 0$, the error reaches its maximum, formulated as $\Delta h(\mathbf{d}) \leq \min(d_{ch}, d_{dis}) \cdot \min(P_{ch}^{max}, P_{dis}^{max})$. The error upper bound is jointly determined by the unit's rated power and direction coefficients and is always bounded.

For a cluster containing N units, the aggregated relaxation error satisfies a linear superposition relation according to the additivity of the Minkowski sum:

$$\Delta h_{agg}(\mathbf{d}) = \sum_{i=1}^N \Delta h_i(\mathbf{d}) \leq N \cdot \min(d_{ch}, d_{dis}) \cdot \min_i(P_{ch,i}^{max}, P_{dis,i}^{max}) \quad (40)$$

That is, the error increases linearly with the number of units, but its proportion of the total support function value stays stable and will not expand relatively as the cluster scale grows. In typical scenarios detailed in this paper, such as dispatch command response and capability boundary evaluation, all involved directions belong to the zero-error direction set. Therefore, convex relaxation imposes extremely limited practical influence on aggregation results, which is far smaller than the approximation error brought by adaptive direction selection.

In terms of framework extension for nonlinear coupling scenarios, aiming at the nonlinear property of electro-thermal conversion efficiency in actual systems, convexification can seamlessly integrate such nonlinearities into the aggregation framework of this paper. Its core goal is to approximate the real coupling constraints as tightly as possible while retaining the convexity of the feasible region. Two specific technical approaches are available: piecewise linear approximation and convex-hull relaxation.

Piecewise linear approximation divides the operating power interval of electro-thermal conversion equipment into several subsegments and fits the nonlinear efficiency curve with linear efficiency coefficients within each subsegment, corresponding to multiple groups of linear coupling constraints. Since the upper envelope of piecewise linear convex functions is still a convex function, the segmented coupling constraints can still guarantee the convexity of unit feasible regions, satisfying the preconditions for Minkowski sum and support function analysis. The corresponding equivalent constraint reconstruction process solves the tightest constraints separately for each power subsegment and finally takes the upper envelope of constraints of all subsegments to form piecewise linear equivalent constraints at the aggregation level. The accuracy of this method can be flexibly controlled by adjusting the number of segments. When the number of segments is large enough, it can arbitrarily approximate real nonlinear characteristics while maintaining the convexity of the feasible region throughout the process. The support function aggregation, AODS direction selection and feasible-region reconstruction algorithms proposed in this paper can be directly reused without modifying core logic.

For non-convex nonlinear coupling constraints, the convex-hull relaxation method first constructs their minimum convex envelope to obtain the smallest convex set containing the real non-convex feasible region, which is then substituted into the aggregation framework of this paper for calculation. The aggregated feasible region obtained by this method is an outer approximation of the real feasible region, with no risk of underestimating capacity, complying with the engineering principle of conservative yet safe in dispatch scenarios. In addition, the tightness of convex relaxation can be gradually improved by introducing auxiliary constraints and dividing operating condition subsets. For equipment with strong environmental dependence such as heat pumps, impact factors like ambient temperature can be further treated as uncertain parameters to construct a robust convex coupling constraint set, which extends the method to obtain a robust aggregated feasible region and improves its applicability under variable working conditions and high uncertainty.

Neither of the above two extension paths requires modification to the core logic of support function aggregation and Minkowski sum operation in this paper; only adaptations for unit-level coupling constraints and equivalent reconstruction links are needed. This proves that the proposed framework boasts favorable generalization ability and engineering extension potential.

3.3. Aggregated Equivalent Model Construction and External Characteristic Extraction

Through the AODS algorithm, a set of optimal evaluation directions $\mathcal{Y}^* = \{\mathbf{y}_1, \dots, \mathbf{y}_L\}$ and their corresponding aggregated support function values $\{H_1, H_2, \dots, H_L\}$ are obtained, where $H_l = h_{\mathbf{F}_{\text{agg}}}(\mathbf{y}_l)$. These data form a complete characterization of the overall operating capability of the cluster. In this section, this set is reconstructed into a concise and practical equivalent model of virtual energy storage. This model not only accurately reflects the aggregated feasible region of the cluster mathematically, but also extracts the external characteristic parameters that have direct guiding significance for market bidding, system dispatch and stability analysis.

According to the duality property of the support function, a closed convex set can be uniquely determined by its support function values in all directions. We have sampled it through the values on the finite direction set $\mathcal{Y}^*$. Therefore, the feasible region $\mathbf{F}_{\text{agg}}$ of the aggregated cluster can be approximately described by the following set of linear inequalities:

$$\tilde{\mathbf{F}} = \{\tilde{\mathbf{x}} \in \mathbb{R}^n : \mathbf{y}_l^T \tilde{\mathbf{x}} \leq H_l, l = 1, 2, \dots, L\} \quad (41)$$

where $\tilde{\mathbf{x}} = \begin{bmatrix} \mathbf{p}^T, \mathbf{E}^T \end{bmatrix}^T$ denotes the decision variable vector of the virtual energy storage, including the power sequence $\mathbf{p} \in \mathbb{R}^T$ and the energy state sequence $\mathbf{E} \in \mathbb{R}^{T \times T}$, with $n \in 2T$.

However, the above formula only provides a geometric description of the feasible region and does not explicitly include the inherent dynamic evolution constraints of the energy storage system, such as the energy state transition equation. To construct an equivalent virtual energy storage model that is physically interpretable and can be directly embedded into the upper-level optimization model, we need to identify and extract the core parameters consistent with the structure of a typical energy storage model from these L linear inequalities.

The above formula serves as the basic mathematical description of the equivalent model, which forms a linear programming feasible region with L constraints and $2T$ variables. For the upper-level optimization problem, the operating behavior of the virtual energy storage will be completely limited by this constraint set. For this purpose, a model fitting optimization problem can be solved to make the feasible region $\mathbf{F}_{\text{model}}(\Theta)$ defined by its parameters as close as possible to $\tilde{\mathbf{F}}$:

$$\min_{\Theta} \sum_{l=1}^L \left(h_{\mathbf{F}_{\text{model}}(\Theta)}(\mathbf{y}_l) - H_l \right)^2 + \lambda R(\Theta) \quad (42)$$

$$\text{s.t. } \Theta \in \mathcal{D} \quad (43)$$

where $h_{\mathbf{F}_{\text{model}}(\Theta)}(\mathbf{y}_l)$ is the support function value of the energy storage model with parameter Θ along the direction $\mathbf{y}_l$, which can be obtained by solving the small-scale linear programming problem, $R(\Theta)$ is the regularization term, which is used to promote

the sparsity or smoothness of the parameters and prevent overfitting, $\mathcal{D}$ is the physical feasible region of the parameters and λ is the regularization coefficient.

To improve the reproducibility of parameter identification for the equivalent model, this paper specifies the following settings for the fitting procedure. The interior-point algorithm embedded in MATLAB R2023b `fmincon` is adopted for solution with a convergence tolerance of 10^{-6} . Multi-start searching with three distinct initial guesses is performed to avoid local optima. The regularization coefficient is optimally selected via grid search within the range $[10^{-4}, 10^{-2}]$ using 5-fold cross validation, which suppresses overfitting and guarantees the physical rationality of parameters; a value of 5×10^{-4} is used in the case studies of this paper.

In terms of identifiability, the charge–discharge efficiency is fixed to the capacity-weighted average of the cluster to eliminate ambiguous coupling between efficiency and capacity. Core parameters including power rating, capacity and ramping rate can be uniquely determined by support functions along corresponding directions. The direction set generated by AODS fully covers all critical dimensions without structural identification blind spots.

Regarding computational overhead, the complexity of the fitting process is $O(K_{iter} \cdot M \cdot T)$. A single fitting run consumes approximately 0.35 s in the case studies, accounting for merely 6.00% of the total aggregation time. Moreover, this fitting step is a one-time offline calculation and imposes no adverse impact on the efficiency of online dispatch.

By solving this optimization problem, a set of optimal equivalent model parameters Θ^* can be obtained. This process can be performed offline. Once completed, the aggregator can use a traditional energy storage model $\mathbf{F}_{\text{model}}(\Theta^*)$ with a simple structure and few parameters to approximately represent the entire complex cluster.

In addition to the complete model parameters, the aggregated external characteristic parameters are crucial for rapid evaluation, market bidding and system-level analysis.

These parameters can be directly analytically extracted from the aggregated support function values $\{H_i\}$ or the approximate feasible region $\tilde{\mathbf{F}}$, without the need to solve complex fitting problems.

The maximum charging and discharging power of virtual energy storage in each time period t can be obtained by evaluating the support function along specific directions. Define the direction vector $\mathbf{y}_t^{\text{ch}}$, which takes the value of 1 only in the dimension corresponding to the charging power at time t and 0 in all other dimensions. Then the maximum charging power is

$$\tilde{P}_t^{\text{max, ch}} = h_{\mathbf{F}_{\text{agg}}}(\mathbf{y}_t^{\text{ch}}) = H(\mathbf{y}_t^{\text{ch}}) \quad (44)$$

Similarly, define the direction vector $\tilde{P}_t^{\text{dis}}$, which takes the value of -1 only in the dimension corresponding to the discharging power at time t , then the maximum discharging power is given by

$$\tilde{P}_t^{\text{max, dis}} = -h_{\mathbf{F}_{\text{agg}}}(\mathbf{y}_t^{\text{dis}}) = -H(\mathbf{y}_t^{\text{dis}}) \quad (45)$$

These values constitute the upper and lower power limit curves declared by virtual energy storage when participating in the electricity market.

The total energy that can be stored or released by virtual energy storage is a critical indicator for its provision of energy-based services. The energy capacity $\tilde{C}$ can be approximated as

$$\tilde{C} \approx \max\{\tilde{E}_t^{\text{max}}\} - \min\{\tilde{E}_t^{\text{min}}\} \quad (46)$$

where $\tilde{E}_t^{\max}$ and $\tilde{E}_t^{\min}$ can be obtained by solving a linear programming problem with the energy state as the optimization objective. More directly, the direction $\mathbf{y}^{\text{energy}}$ that maximizes the difference between the energy state at the end of the dispatch period and the initial energy state can be evaluated, and the difference between its corresponding support function values reflects the total adjustable energy capacity.

For the provision of reserve or black-start services, the duration for which the cluster can sustain discharge at the rated power is of critical importance. Given a target discharge power level P_{set} , the maximum sustained output duration $T_{\text{sust}}(P_{\text{set}})$ can be obtained by solving the following feasibility problem:

$$T_{\text{sust}}(P_{\text{set}}) = \max \tau \quad (47)$$

$$\text{s.t.} \begin{cases} \exists \tilde{\mathbf{p}}, \tilde{\mathbf{E}} \in \mathcal{F} \\ \tilde{p}_t \leq -P_{\text{set}}, \quad \forall t \in [t_0, t_0 + \tau - 1] \\ \tilde{E}_{t_0} = \tilde{E}^{\max} \\ \tilde{E}_{t_0 + \tau} \geq \tilde{E}^{\min} \end{cases} \quad (48)$$

This problem can be efficiently solved by performing a binary search on τ and invoking a linear programming solver.

The maximum change rate of the power of the cluster between adjacent time periods is critical for tracking renewable energy fluctuations and providing frequency regulation services. The upward ramp rate R_t^{up} and downward ramp rate R_t^{down} can be obtained by evaluating the corresponding specific direction vectors.

$$R_t^{\text{up}} = h_{\mathcal{F}_{\text{agg}}}(\mathbf{y}_t^{\text{ramp-up}}) \quad (49)$$

Similarly, the downward ramp rate R_t^{down} can be obtained. These parameters define the dynamic response capability of virtual energy storage.

4. Experimental Design and Analysis

To systematically verify the effectiveness, accuracy and computational efficiency of the distributed multi-energy storage aggregation method based on support functions and Minkowski sum proposed in this paper, this section constructs an electricity–heat coupled distributed energy storage cluster test system and designs multiple groups of comparative simulation experiments.

4.1. Simulation System and Parameter Settings

The test system simulates a park-level integrated energy system managed by an aggregator. Its core consists of an electric power network and a thermal network, connected to the upper-level grid through the point of common coupling and coupled with the district heating network via gas boilers, heat pumps and other equipment. It contains two types of heterogeneous resources: 10 ESUs dispersedly connected to different nodes of the low-voltage distribution network, including lithium-ion batteries and lead–carbon batteries; and 5 thermal storage units (TSUs) connected to the district heating network, including atmospheric pressure thermal storage tanks and phase-change thermal storage devices.

The typical dispatch period of the cluster is set to 24 h. To reflect the advantage of the proposed method in handling multi-time-step problems, the dispatch time step of electric energy storage is set to 15 min, resulting in a total number of electric time steps $N_e = 96$ and the dispatch time step of thermal energy storage is 1 h, resulting in a total number of

thermal time steps $N_h = 24$. The time step mapping matrix $\mathbf{M} \in \mathbb{R}^{96 \times 24}$ is constructed based on the energy conservation principle, whose element $M_{k,l}$ equals 1/4 if the fine time step k belongs to the coarse time step l and 0 otherwise.

To realistically reflect the heterogeneity of the actual system, the key parameters of each energy storage unit are randomly generated within their typical ranges. The specific parameter ranges are shown in Tables 1 and 2.

Table 1. ESU parameter settings.

Parameter	Unit	Lithium-Ion Battery	Lead–Carbon Battery	Hybrid Configuration
Rated Capacity	kWh	[180, 220]	[100, 150]	[250, 300]
Maximum Charging Power	kW	[45, 55]	[20, 30]	[60, 80]
Maximum Discharging Power	kW	[45, 55]	[20, 30]	[60, 80]
Charging Efficiency	p.u.	[0.95, 0.98]	[0.88, 0.92]	[0.93, 0.96]
Discharging Efficiency	p.u.	[0.95, 0.98]	[0.88, 0.92]	[0.93, 0.96]
Lower SOC Limit	p.u.	0.1	0.2	0.15
Upper SOC Limit	p.u.	0.9	0.85	0.95

Table 2. TSU parameter settings.

Parameter	Unit	Heat Storage Tank	Phase-Change Heat Storage
Rated Thermal Capacity	kWh	[800, 1200]	[300, 500]
Maximum Heat Charging Power	kW	[160, 240]	[60, 100]
Maximum Heat Discharging Power	kW	[160, 240]	[60, 100]
Heat Charging Efficiency	p.u.	[0.90, 0.93]	[0.85, 0.90]
Heat Discharging Efficiency	p.u.	[0.90, 0.93]	[0.85, 0.90]
Heat Loss Rate (Self-discharge)	p.u./h	[0.98, 0.99]	[0.995, 0.998]
Lower Limit of Thermal Capacity	p.u.	0.2	0.25
Upper Limit of Thermal Capacity	p.u.	0.95	0.90

The system realizes the coupling between the electric and thermal networks via electro-thermal conversion equipment with a total rated power of $P_{\text{conv}}^{\text{max}} = 500 \text{ kW}$. The coupling matrix $\mathbf{C}$ and the constraint vector $\mathbf{d}$ describe the relationship between the total discharge power of the cluster and the capacity limit of the conversion equipment in any time period t .

To simplify the case study and highlight the core problem, the following linear coupling constraints are set:

$$\mathbf{C}\mathbf{z}_t \leq \mathbf{d} \quad (50)$$

where $\mathbf{z}_t = [P_{e,t}, H_{h,t}]^T$ denotes the vector composed of the total electric power and total thermal power of the cluster in period t . Specifically, we define

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & \gamma \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} \sum_{i=1}^{10} \bar{P}_{\text{dis},i} \\ \sum_{j=1}^5 \bar{H}_{\text{ch},j} \\ 0 \end{bmatrix} \quad (51)$$

The first two rows, respectively, represent that the total discharging power and total heat storage power shall not exceed the sum of the limits of each unit. The third row represents the coupling constraint $-P_{e,t} + \gamma H_{h,t} \leq 0$, i.e., that the electric power $P_{e,t}^{\text{for-heat}}$ consumed for heat generation shall satisfy $\gamma H_{h,t} \leq P_{e,t}^{\text{for-heat}}$, where $\gamma = 0.3$ is the electro-

thermal conversion coefficient. This constraint couples the operation of electric and thermal energy storage through equipment capacity limits.

4.2. Experimental Setup

The AODS method proposed in Section 3.2 is compared with the exact aggregation method, box-type aggregation, fixed uniform direction sampling and fixed importance direction sampling. For AODS, the initial direction set is set as the positive and negative directions of the unit coordinate basis vectors, i.e., $2(N_e + N_h)$ directions, the convergence tolerance is $\varepsilon = 10^{-3}$, and the maximum number of iterations is $K_{\max} = 50$. The exact aggregation method directly computes the Minkowski sum of the joint feasible regions of all units to obtain the accurate aggregated feasible region. This method has extremely high computational complexity and is only used to verify the accuracy of the proposed method in small-scale scenarios. The box-type aggregation ignores the dynamic constraints and coupling relationships among units and only sums the upper and lower limits of power and capacity simply to obtain a virtual energy storage model with box constraints. The fixed uniform direction sampling evaluates the feasible region by uniformly and randomly sampling a fixed number of direction vectors on the unit sphere. The fixed importance direction sampling adopts weighted sampling based on historical dispatch data. The accuracy of aggregation methods is evaluated by comparing the support function error (SFE), volume similarity ratio (VSR) and dispatch command feasibility rate (FR). The specific definitions and calculation methods of the three precision indexes are as follows:

SFE is used to quantify the boundary approximation deviation of the aggregated feasible region relative to the real feasible region. The average relative error of support functions for directions uniformly sampled on the unit sphere is taken as the evaluation index:

$$\text{SFE} = \frac{1}{N_d} \sum_{i=1}^{N_d} \frac{h_{\text{agg}}(d_i) - h_{\text{real}}(d_i)}{h_{\text{real}}(d_i)} \times 100 \quad (52)$$

where $h_{\text{agg}}(d_i)$ and $h_{\text{real}}(d_i)$ denote the support function values of the aggregated feasible region and the real feasible region along direction d_i , respectively, and N_d is the total number of sampled directions. A positive value of this index indicates outer approximation, and a smaller value corresponds to higher boundary accuracy.

VSR is adopted to measure the overall overlap degree between the aggregated feasible region and the real feasible region in high-dimensional space, defined as the ratio of the volume of the real feasible region to that of the aggregated feasible region:

$$\text{VSR} = \frac{\text{vol}(\mathcal{K}_{\text{real}})}{\text{vol}(\mathcal{K}_{\text{agg}})} \times 100\% \quad (53)$$

Since all aggregation methods adopt the outer approximation strategy, the value range of VSR is $(0, 100\%]$. A value closer to 100% means a higher overall matching degree between the aggregation result and the real feasible region.

FR is used to measure the actual executable proportion of dispatch commands generated based on the aggregated feasible region in the underlying unit-level model, reflecting the ability of the aggregation model to preserve feasibility.

Monte Carlo stratified sampling is adopted to randomly generate 10,000 dispatch commands within the aggregated feasible region, covering typical operating conditions such as extreme power values, extreme energy values, and ramp boundaries. Each command is delivered to the unit-level exact model for full constraint verification. The proportion of commands satisfying all unit physical constraints and global coupling constraints is defined

as the feasibility rate. The sample size has passed convergence verification and can stably reflect the practical feasible performance of the aggregation model.

To comprehensively evaluate the algorithm performance, four progressive test scenarios are designed as follows:

Scenario 1—Basic Function Verification: Contains 3 ESUs and 2 TSUs without coupling constraints. It is used to verify the basic accuracy of support function aggregation and AODS in small-scale systems where the exact solution can be calculated.

Scenario 2—Heterogeneity and Scale Test: Contains all 10 ESUs and 5 TSUs without coupling constraints. It is used to test the computational efficiency and aggregation accuracy of the proposed method when dealing with large-scale heterogeneous clusters.

Scenario 3—Coupling Constraint Impact Test: Based on the scale of Scenario 2, the complete electricity–heat coupling constraint matrix is introduced. It is used to verify the effectiveness and conservatism of the proposed two-step framework (equivalent constraint relaxation + accurate reconstruction) in handling coupling constraints.

Scenario 4—Multi-time-Step Verification: Based on Scenario 3, the different time steps of electric and thermal energy storage are strictly distinguished, and the time step mapping matrix is applied. It is used to verify the rationality of the hierarchical aggregation-time step mapping framework.

4.3. Aggregation Performance Analysis

For the small-scale system constructed in Scenario 1, the dispatch period is shortened to 6 h to reduce the computational burden. The Monte Carlo method is employed to generate 10,000 random test directions for accuracy evaluation. The accuracy comparison of various aggregation methods is presented in Table 3.

Table 3. Accuracy comparison of various aggregation methods.

Aggregation Method	Mean SFE (%)	Max SFE (%)	VSR	FR (%)	Computation Time (s)
Exact Aggregation	0.00	0.0	1.00	100	28.72
Box-type Aggregation	24.68	56.32	0.42	62.30	0.003
Fixed Uniform Direction Sampling	8.42	22.15	0.79	84.60	1.23
Fixed Importance Direction Sampling	5.78	18.63	0.86	91.20	1.35
Virtual Battery	2.36	7.54	0.93	94.10	1.85
AODS	1.35	4.28	0.95	98.70	1.82

The box-type aggregation exhibits extremely poor accuracy, with a VSR of only 0.423 and nearly 40.00% of the actual feasible region left uncovered. Among fixed-direction sampling approaches, importance sampling outperforms uniform sampling but still presents notable errors. The virtual battery model is only marginally superior to AODS in computation time, whereas AODS achieves accuracy close to that of exact aggregation using merely around 20 directions.

Figure 2 depicts the accuracy variation curves of fixed uniform direction sampling, fixed importance direction sampling and AODS with different numbers of directions.

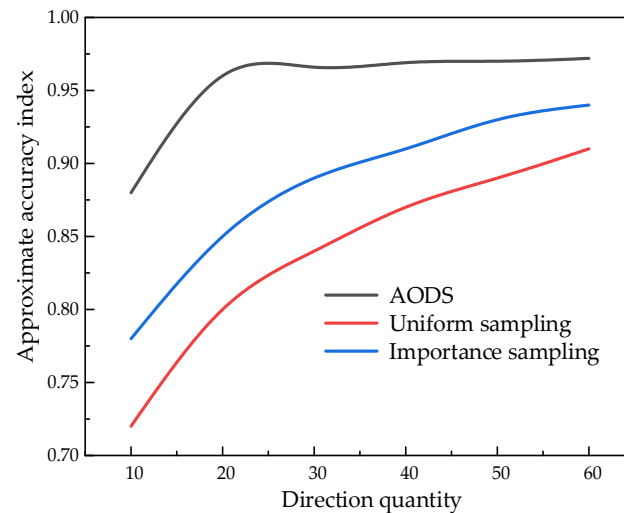


Figure 2. Accuracy variation curves versus the number of directions.

It can be observed from the comparison of accuracy curves under different numbers of directions in Figure 2 that the AODS converges the fastest, and its accuracy improvement tends to be gentle after 20 directions. The accuracy of uniform sampling with 60 directions is still lower than that of AODS with 20 directions. All methods show the effect of diminishing marginal returns.

Figure 3 intuitively shows the projection comparison of the aggregated feasible region in a two-dimensional power subspace.

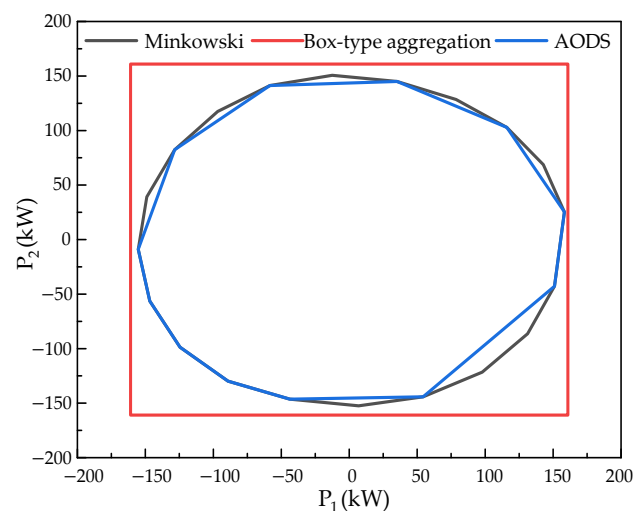


Figure 3. Projection comparison of aggregated feasible region in two-dimensional power subspace.

The boundary of the exact aggregated feasible region is enclosed by multiple linear inequalities, reflecting the combined effect of dynamic constraints and energy coupling of heterogeneous energy storage units. Box-type aggregation yields a regular rectangle, which completely ignores the temporal coupling characteristics among individual units and the interactive influence of capacity constraints, resulting in a serious overestimation of the feasible-region volume and a large deviation from the actual geometry. In contrast, the aggregated feasible region reconstructed by the proposed AODS method can well approximate the concave and convex vertices of the exact boundary. It achieves high-precision characterization of the feasible-region shape while preserving key geometric features, verifying the effective approximation ability of AODS for complex feasible regions using only a limited number of sampling directions.

In Scenario 2, the full-scale system with a 24 h dispatch period is used to evaluate the accuracy–efficiency trade-off of each method. Figure 4 shows the computation time as a function of the number of units.

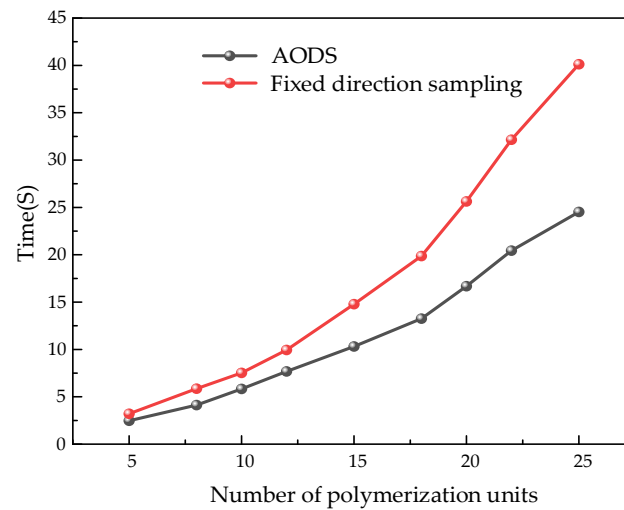


Figure 4. The change in calculation time with the number of units.

It can be observed that directly computing the Minkowski sum of the joint feasible regions of all units requires vertex and half-space enumeration. The computational complexity rises exponentially as the number of units increases, with a computation time of 42.73 s for 5 units and 287.40 s for 8 units.

For the proposed method, the number of directions rises as the unit scale increases from 5 to 25, and linear programs (LPs) of individual units are solved in parallel for each direction. Fixed-direction sampling also solves LPs in parallel. Since the number of directions is fixed, its computation time is theoretically linear in the number of units, but shows a slight quadratic growth in practice due to memory access contention and dispatch overhead.

The convergence process of AODS in terms of the WSFE is shown in Figure 5.

Figure 5 presents the WSFE convergence curve of AODS. With an initial set of 6 directions, the WSFE is 18.60%. After 10 iterations, the WSFE drops below 1.00%, and its variation becomes gentle after 20 iterations, satisfying the convergence condition. The final direction set contains 36 directions, among which 12 are coupling-related directions.

Figure 6 depicts the computational time decomposition pie chart of AODS, which comprises four stages, namely direction search, unit support function evaluation, support function aggregation and equivalent model construction. Among them, direction search, unit support function evaluation, and equivalent model construction can be executed in parallel on multi-core processors. In AODS, finding the direction with the current worst-case error requires evaluating multiple candidate directions. Although the procedures within a single iteration are serial, the evaluation of different candidate directions is mutually independent. Therefore, adopting a multi-start, multi-direction parallel search strategy can significantly accelerate the direction search process.

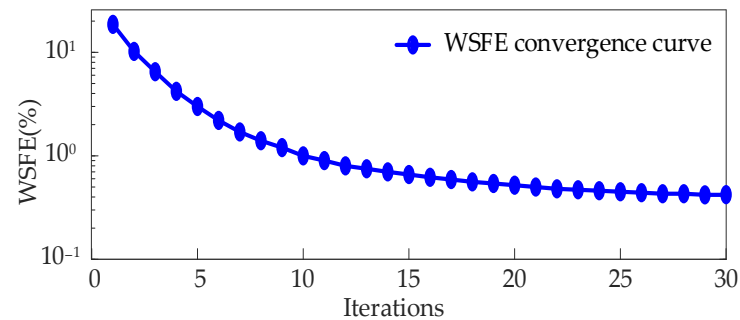


Figure 5. WSFE convergence curve.

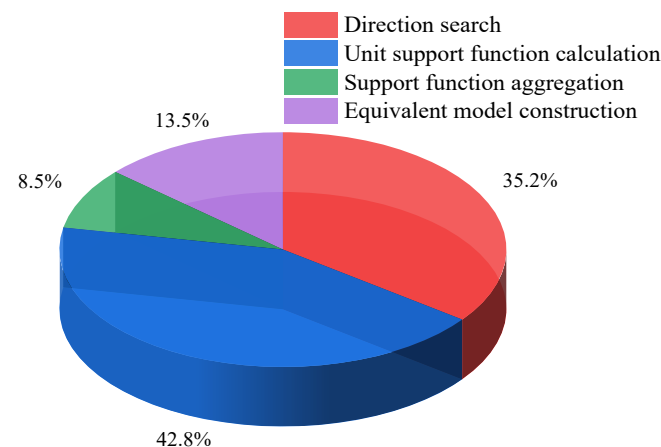


Figure 6. Computational time decomposition of the AODS algorithm.

In the unit support function evaluation stage, each unit independently solves its own linear programming problem to compute the support function value. These computations are completely independent of results from other units, thereby enabling full parallelization at the unit level. When fitting parameters of the equivalent energy storage model from support function values, the optimization problems corresponding to different parameter blocks can also be solved independently. Only the support function aggregation stage requires summing the scalar values uploaded by all units. Although this operation is inherently serial, its computational burden and data volume are negligible, accounting for only 8.50% of the total time. Therefore, the gain from parallelization is negligible and serial execution is generally adopted.

Next, under the system scale of Scenario 2, the complete electricity–heat coupling constraint matrix is introduced to verify the performance of the proposed coupling constraint handling method. Figure 7 shows the cutting effect of coupling constraints on the feasible region.

The coupling constraints form linear cutting planes in the joint electric–thermal power space, reducing the feasible-region volume by 32.50%. The feasible region remains a closed convex polyhedron but with a more compact and complex boundary, and the set of maximum achievable power combinations is altered accordingly.

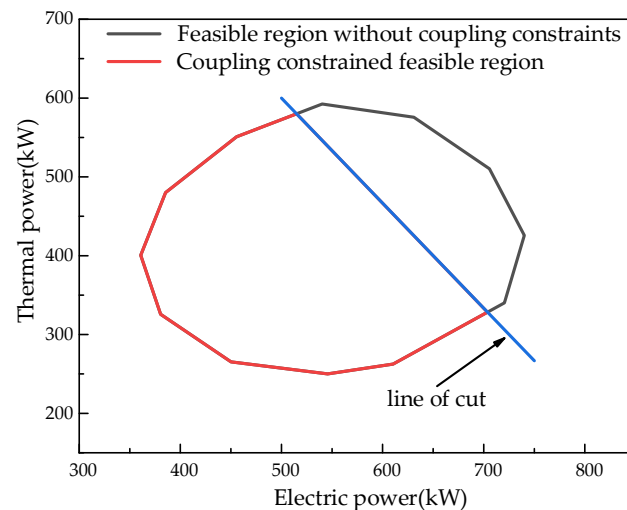


Figure 7. Cutting effect of coupling constraints on feasible region.

The variations in the key external characteristic parameters before and after applying the coupling constraints are presented in Table 4.

Table 4. Key external characteristic parameters before and after coupling constraints.

Parameter	Without Coupling	With Coupling
Maximum instantaneous discharge power (kW)	785.40	642.8
Maximum continuous discharge duration (h)	8.60	6.9.
Effective energy capacity (kWh)	2250.00	1980.00
Maximum ramp rate (kW/min)	157.10	128.50

Table 4 demonstrates the significant impact of coupling constraints on the external characteristic parameters of the cluster. After introducing coupling constraints, all parameters decrease, indicating that such constraints substantially reduce the power support capability, energy regulation capacity and dynamic response speed of the cluster by limiting the simultaneous utilization of electric and thermal energy storage. Ignoring these constraints will lead to a serious overestimation of the actual capability of the cluster, highlighting the necessity of accurately modeling coupling effects in the aggregation model.

The two-step method proposed in this paper can efficiently obtain the capability envelope and reduce computation time. By solving auxiliary optimization problems, tight equivalent constraints are derived. Figure 8 illustrates the performance of the two-step method in a three-dimensional power space.

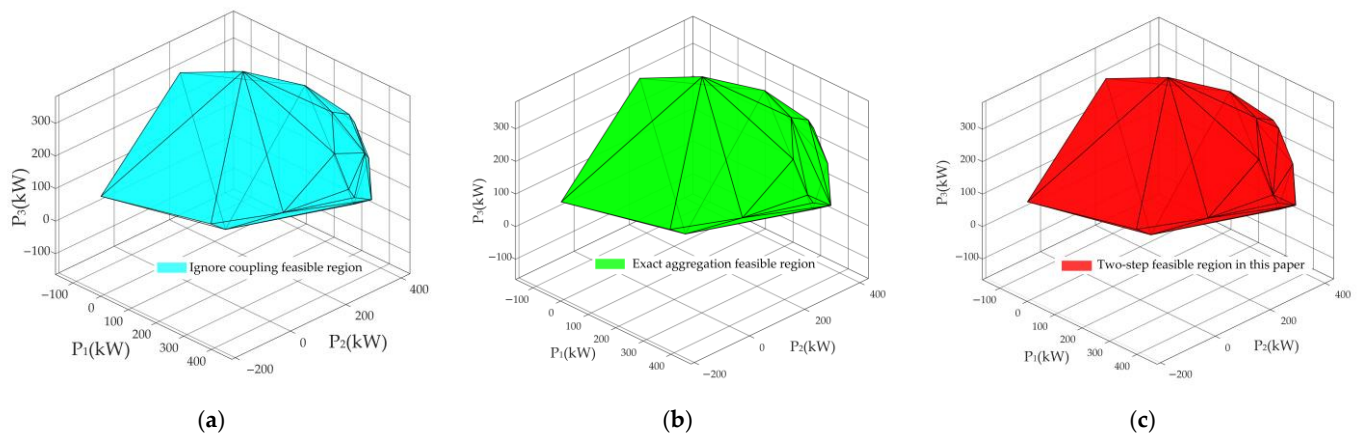


Figure 8. Effect demonstration of two-step method in three-dimensional power space. (a) Ignore coupling feasible region. (b) Exact aggregated feasible region. (c) Two-step feasible region in this paper.

In the 3D power space, the feasible region obtained by ignoring coupling has the largest volume but contains numerous infeasible points. The exact aggregated feasible region is trimmed by coupling constraints, forming an irregular convex polyhedron. The feasible region reconstructed by the proposed two-step method highly coincides with the exact boundary, with an overestimation rate of only 5.20%. To quantitatively evaluate the reconstruction accuracy under the baseline scenario, three core metrics are adopted: volume overestimation rate (VOR, the ratio of excess volume to the exact feasible-region volume), average SFE, and constraint tightness ratio (CTR, defined as the ratio of the reconstructed constraint bound to the exact constraint bound, closer to 1 indicates tighter reconstruction). The quantitative results are summarized in Table 5.

Table 5. Quantitative accuracy of equivalent constraint reconstruction under the baseline scenario.

Metric	Value
Volume overestimation rate	5.20%
Average support function error	1.87%
Maximum support function error	4.12%
Average constraint tightness ratio	0.972

It can be seen that the equivalent constraint reconstruction method achieves high boundary tightness while maintaining computational efficiency. The average deviation of the support function is less than 2%, and the volume overestimation is controlled within 6%, which fully meets the accuracy requirements of day-ahead dispatch. To further verify the stability of the reconstruction method under different operating conditions, a systematic sensitivity analysis with respect to coupling strength, cluster scale, and operating conditions is conducted in Section 4.5, providing comprehensive numerical evidence for the robustness of the two-step reconstruction framework.

On the basis of Scenario 3, the different time steps of electric and thermal energy storage are strictly distinguished and the time-scale mapping matrix is adopted. The accuracy comparison of different scale mapping methods is presented in Table 6. Herein, information loss is used to quantify the characterization deviation of the approximate feasible region relative to the original feasible region; a lower value indicates more complete retention of original information. This indicator is calculated based on the average relative error of support functions for directions uniformly sampled over the unit sphere. The lost information mainly consists of boundary shape details in non-critical directions and deviation of limit values under extreme operating conditions. Since all

methods adopt an outer approximation strategy, there is no underestimation of feasible regulation capacity.

Table 6. Accuracy of different scale mapping methods.

Mapping Method	Thermal Storage Modeling Dimension	Mean SFE (%)	Computation Time (s)	Information Loss (%)
Uniform Fine Time Step	384	2.30	15.20	0.00
Uniform Coarse Time Step	96	8.60	4.80	22.40
Linear Interpolation	96	4.20	5.60	12.80
Hierarchical Mapping	96	2.70	6.10	5.30

Table 6 summarizes the performance of different time-scale mapping methods. The uniform fine-scale method achieves high accuracy, but the modeling dimension of thermal energy storage is as high as 384, resulting in a computation time of 15.20 s. The uniform coarse-scale method reduces the dimensions to 96 and shortens the computation time to 4.80 s, yet the information loss reaches 22.40% and the SFE increases to 8.60%. The linear interpolation method reduces the information loss to 12.80% and improves the SFE to 4.20% at the same number of dimensions. The proposed hierarchical mapping method maintains the 96-dimensional modeling scale and constructs the mapping model via the dual-objective optimization of energy conservation and power smoothing. It further reduces the information loss to 5.30% and the SFE to 2.70%, with a computation time of only 6.10 s.

The above results demonstrate that the proposed method significantly reduces the model dimensions while effectively preserving the dynamic characteristics of thermal energy storage, achieving a favorable balance between computational efficiency and modeling accuracy.

Table 7 shows the significant influence of coupling constraints on the external characteristic parameters of the cluster. After introducing coupling constraints, all parameters decrease, indicating that coupling constraints substantially reduce the power support capability of the cluster, energy regulation capacity and dynamic response speed by limiting the simultaneous utilization of electric and thermal energy storage.

Table 7. Comparison between external characteristic parameters of aggregation and sum of unit parameters.

External Characteristic Parameter	Sum of Unit Parameters	Extracted Value by Aggregation Model	Relative Error (%)
Maximum charging power (kW)	785.40	742.60	−5.50%
Maximum discharging power (kW)	785.40	751.80	−4.30%
Effective capacity (kWh)	2250.00	2120.00	−5.80%
Maximum continuous discharge time (h)	7.96	7.84	−1.50%
Maximum ramp rate (kW/min)	157.10	142.30	−9.40%

Ignoring such constraints would lead to a serious overestimation of the actual capability of the cluster, highlighting the necessity of accurately describing coupling effects in the aggregation model.

4.4. Validation of Dispatch Responsiveness

In practical operation, the aggregator needs to adjust the cluster output according to the power commands issued by the higher-level dispatch authority. To evaluate the command response performance of different aggregation models, a command tracking test is designed as follows. The test is conducted on Scenario 3, with a 24 h dispatch period.

The temporal resolution is 15 min for the electric side and 1 h for the thermal side. A typical dispatch command curve is generated by adding random fluctuations to the base load, whose amplitude lies within the total power limits of the cluster. The test results are shown in Figure 9.

Figure 9 compares the tracking curves during 12:00–16:00. In the rapidly rising segment of 13:00–14:00, the AODS model can closely track the command, with a maximum deviation of 6.70%. The fixed-direction sampling method also achieves acceptable tracking performance, but shows a slight lag in the rising stage due to the approximation error of the feasible region under a limited direction set, with a maximum deviation of 3.10 kW. In contrast, the box-type model ignores ramp rate constraints. During 13:15–13:30, it cannot achieve the required ramp rate specified by the command, leading to subsequent deviation in the energy state. Eventually, SOC violation occurs after 14:00, making the command infeasible. While the virtual battery model achieves favorable tracking performance, its overall tracking capability is inferior to that of the AODS model.

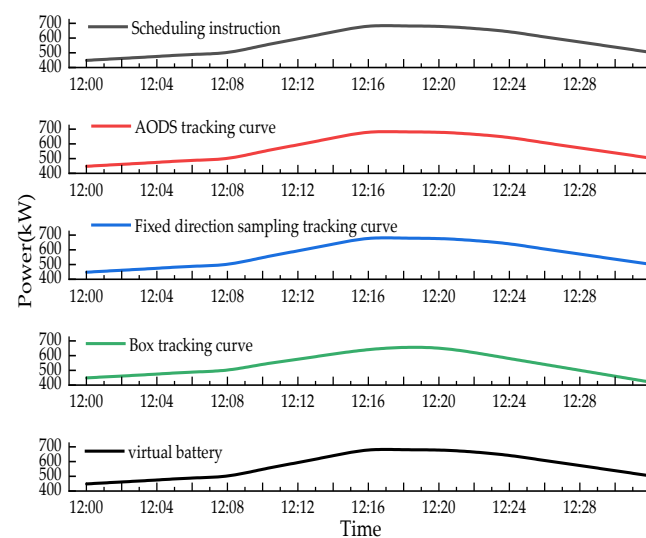


Figure 9. Comparison of dispatch command tracking curves in typical time periods.

This result confirms that the AODS model accurately captures the dynamic characteristics of the cluster. By contrast, the box-type model loses temporal coupling information, leading to infeasible commands during dynamic operation.

In practical operation, rapid fluctuations of net load impose stricter demands on the regulation capability of energy storage clusters. To test the robustness of different aggregation models under intense fluctuation scenarios, high-frequency random fluctuations are superimposed on the dispatch command to simulate a high-penetration renewable energy integration scenario. The fluctuation amplitude is set to $\pm 30.00\%$ of the baseline value, with a fluctuation period of 15–30 min and a maximum ramp rate up to 180 kW/min.

The response capabilities of various methods under highly fluctuating commands are compared. The regulation success rate is defined as the ratio of the number of executable intervals to the total number of intervals. The maximum tracking deviation and the number of constraint violations are also recorded. The test results are shown in Table 8 and Figure 10.

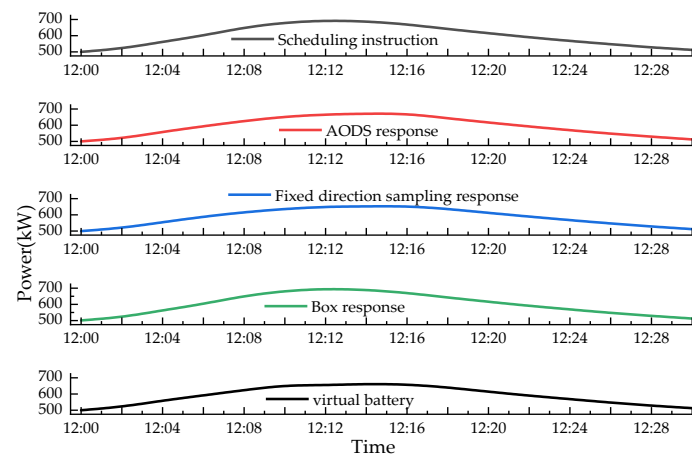


Figure 10. Comparison of response trajectories in the high-fluctuation window.

Table 8. Comparison of command tracking performance of different aggregation models under strong fluctuation scenarios.

Model	Mean Tracking Deviation/%	Max Tracking Deviation/%	Number of Constraint Violations	Solution Time/s
Exact Aggregation	0.00	0.00	0.00	312.60
AODS	2.80	6.70	0.00	5.20
Virtual Battery	3.10	8.90	0.00	6.50
Fixed-Direction Sampling	5.50	12.50	100.00	7.80
Box-Type Aggregation	14.30	35.20	9.00	0.30

As can be seen from the table above, the AODS model achieves a mean deviation of only 2.80% and a maximum deviation of 6.70%, which are the closest to those of the exact model.

For the virtual battery model, the mean tracking deviation, maximum tracking deviation and solution time are all higher than those of the AODS model. The fixed-direction sampling model yields a mean deviation of 5.50% and a maximum deviation of 12.50%, with lower accuracy than AODS but significantly better performance than the box-type model. The box-type model suffers from a mean deviation of 14.30% and a maximum deviation of 35.20%, along with nine violations of actual physical constraints during tracking, resulting in infeasible command execution. The solution time of the AODS model is 5.2 s, while the fixed-direction sampling model takes 7.80 s due to having more constraints. The box-type model is the fastest at only 0.30 s, but at the cost of excessive accuracy loss. The exact model costs 312.60 s, which is unacceptable for practical applications.

Figure 10 compares the command tracking curves within the high-fluctuation window of 12:00–12:30. In this window, high-frequency fluctuations are superimposed on the command, with a maximum ramp rate of 180 kW/min, exceeding the maximum ramp capability of the AODS model, so as to test the ultimate response capability of each model.

During the rapidly rising period of 12:08–12:14, the command ramp rate reaches 180 kW/min. Limited by its ramp capability, the AODS model cannot fully follow the command, but its response curve rises smoothly with the maximum deviation at 12:10, without any constraint violation during the whole process. The fixed-direction sampling model is also restricted by ramp capability, showing a larger response lag. The deviation reaches 43.4 kW at 12:10, and a slight violation occurs near 12:14 as the energy state approaches the lower bound, but it recovers afterward. The box-type model tracks the command blindly. Although its deviation is only 1.6 kW at 12:10, the actual energy state is seriously violated. Since 12:14, the SOC has exceeded the allowable range, making all subsequent responses infeasible. While the virtual battery model incurs no constraint

violations throughout the entire tracking process, its tracking performance is restricted by ramp capability limits and is inferior to that of the AODS model.

These results verify that the AODS model accurately characterizes the dynamic response capability of the cluster. The fixed-direction sampling model, despite certain approximation errors, can still capture the main regulation capability of the cluster. In contrast, the box-type model leads to frequent infeasible dispatch commands due to excessive simplification.

To further analyze the impact of fluctuation intensity on the regulation success rate, Figure 11 presents the success rate curves under different maximum ramp rate requirements.

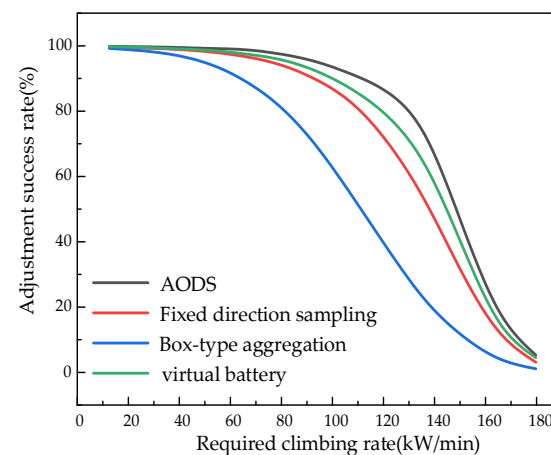


Figure 11. Regulation success rate under different fluctuation intensities.

It can be observed from Figure 11 that the AODS aggregation model has a maximum ramp capability of 142.3 kW/min. It maintains a success rate close to 100% when the required ramp rate is below 100 kW/min, and the success rate decreases noticeably beyond 100 kW/min. The virtual battery model achieves a lower regulation success rate than the AODS aggregation model across all performance metrics. Due to the fixed direction set, the fixed-direction sampling model characterizes the cluster dynamic characteristics with slightly lower accuracy than AODS, with an effective ramp capability of approximately 135 kW/min, and its success rate drops earlier and more steeply. The box-type aggregation model completely ignores ramp constraints, yet its implicit power and energy constraints still limit the actual ramp capability. As a result, its success rate declines rapidly at around 100 kW/min, and the final success rate is much lower than the other models.

This figure intuitively illustrates the ability of aggregation models to characterize the dynamic response limit of the cluster. Through adaptive direction selection, the AODS model most accurately retains the ramp capability information of the cluster, thereby achieving a higher regulation success rate under fluctuating conditions. The fixed-direction sampling model is the second best. Due to excessive simplification, the box-type model seriously overestimates the regulation capability of the cluster and shows the lowest reliability in real high-fluctuation scenarios.

4.5. Verification and Expansibility Analysis of Engineering Measured Data

The foregoing case studies complete the basic performance verification of the proposed method based on a parametric synthetic benchmark test system. To further examine the method's applicability in practical engineering scenarios, its scalability for large-scale clusters, and its application value in dispatch optimization, this section carries out empirical research using measured operational data from the integrated energy system of a national-level industrial park. Multi-dimensional sensitivity analysis and

statistical tests are adopted to evaluate the robustness of the method and the reliability of its results.

The system is equipped with 42 distributed electric energy storage units and two atmospheric heat storage tanks, alongside three electric heat pump units with a rated power of 2 MW serving as electro-thermal coupling conversion devices. A distributed photovoltaic power station with a total installed capacity of 12 MW is also integrated into the system. The measured data are extracted from the operation database of the park's Energy Management System (EMS) covering January to December 2025. After outlier elimination, missing-value interpolation and steady-state operating condition screening, operational datasets corresponding to three typical working conditions are retrieved: a typical winter heating day, a typical summer cooling day, and a representative transition-season mild day. The datasets include the actual charge/discharge power ranges, state-of-charge (SOC) operating ranges, charge–discharge efficiencies and self-discharge loss coefficients of each energy storage unit, as well as electro-thermal load profiles and photovoltaic output curves. To verify the large-scale scalability of the method, test clusters of varying sizes are constructed via random parameter generalization using the measured unit parameters as a benchmark library. The cluster scale ranges from 20 to 200 heterogeneous energy storage units, with all unit parameters constrained within the value ranges adopted in practical engineering.

To validate the computational performance and precision retention capability of the method for large-scale energy storage clusters, heterogeneous energy storage clusters with scales of 20, 40, 80, 120 and 160 units are constructed through random sampling based on the measured unit parameters. Ten independent repeated tests are conducted for each cluster scale, and statistical average values are recorded. The computational time consumption, average support function error and feasible-region volume similarity of the proposed method, fixed-direction sampling method and exact aggregation method are compared, with the test results summarized in Table 9.

Table 9. Comparison of computational performance under different cluster scales.

Cluster Scale/Number of Units	Proposed Method			Fixed-Direction Sampling Method			Exact Aggregation Method
	Computation Time/s	Average SFE/%	Volume Similarity	Computation Time/s	Average SFE/%	Volume Similarity	Computation Time/s
20	3.2	1.82	0.947	2.7	6.35	0.862	72.4
40	5.8	2.07	0.938	5.1	7.62	0.815	316.8
80	9.1	2.35	0.931	9.7	9.14	0.768	Intractable
120	12.6	2.64	0.926	14.8	10.57	0.724	
160	15.9	2.91	0.922	20.3	11.83	0.687	

It can be observed from the test results that as the cluster scale expands from 20 units to 160 units, the computational time of the proposed method grows approximately linearly, and its computational complexity maintains an essentially linear correlation with the number of units, which fully verifies the scalability of the distributed parallel aggregation strategy. The core underlying reason lies in the Minkowski additivity of support functions, which enables fully independent support function calculation for each unit; the aggregation operation merely consists of scalar summation, avoiding the combinatorial explosion issue induced by vertex enumeration in traditional geometric aggregation methods.

Based on the validated linear scalability law, we further extrapolate the computational performance of the proposed method for larger-scale aggregation scenarios covering 200 to 2000 units, which corresponds to the typical scale of district-

level and city-level energy storage aggregators in practical engineering. The extrapolation is conducted based on the linear fitting of measured data from 20 to 160 units, and two deployment architectures are considered: serial computing on a single node and fully distributed parallel computing. The results are summarized in Table 10.

Table 10. Expected computational performance of larger-scale aggregation scenarios.

Cluster Scale (Units)	Serial Computing Time/s	Distributed Parallel Wall Time/s	Expected Average SFE/%	Expected Volume Similarity
200	19.7	6.8	3.02	0.919
500	49.2	7.2	3.25	0.914
1000	98.5	7.5	3.41	0.910
2000	197.0	8.1	3.58	0.906

The distributed parallel wall time assumes that each unit solves its local linear programs independently via edge computing nodes, and only scalar support function values are transmitted to the aggregator. The wall time is dominated by direction search iteration overhead rather than unit-level computation.

Two core mechanisms guarantee the favorable scalability of the proposed method in large-scale scenarios. First, the size of the direction set generated by the AODS algorithm is dominated by the geometric complexity of the aggregated feasible region, rather than the number of units. As the cluster scale expands, the shape of the aggregated feasible region becomes smoother due to the superposition of heterogeneous unit parameters, and the number of critical faces grows very slowly. Therefore, the number of directions required to maintain the same approximation accuracy increases marginally with the cluster scale, which avoids the rapid growth of computational overhead. Second, under the distributed parallel architecture, unit-level support function calculation is completely decoupled. The aggregator only needs to broadcast direction vectors and collect scalar results, with extremely low communication overhead. The wall-clock time of the aggregation process is almost independent of the number of units, as long as sufficient edge-computing resources are configured.

For practical engineering applications, the typical scale of a regional energy storage aggregator ranges from 500 to 2000 distributed units. Under the distributed parallel architecture, the proposed method can complete the full aggregation process within 10 s even for 2000 units, which fully meets the time requirements of day-ahead dispatch and intra-day rolling dispatch. In contrast, exact aggregation methods based on vertex enumeration become intractable when the number of units exceeds 80, and parameter equivalence methods such as the virtual battery model suffer from continuous accuracy degradation with expanding scale. The proposed method achieves a better balance between accuracy and computational efficiency for large-scale heterogeneous energy storage clusters.

In terms of accuracy, the average SFE is still controlled within 3.12% even for the large-scale scenario with 160 units, and the feasible-region volume similarity remains above 0.918, without drastic accuracy degradation as the cluster scale expands. This demonstrates that the AODS algorithm can adaptively identify the critical directions that characterize the feasible-region boundary and achieve tight feasible-region approximation even for large-scale heterogeneous clusters.

In comparison with the fixed-direction sampling method, its aggregation accuracy deteriorates markedly with the growth of cluster scale: the volume similarity equals 0.862 for the 20-unit case, yet drops to 0.653 at 160 units, while the average SFE rises from 6.35% to 13.06%. This occurs because the fixed direction set cannot self-adjust to accommodate varying cluster scales and feasible-region geometries, leading to continuous deterioration

of coverage efficiency for fixed sampling in high-dimensional spaces and a corresponding increase in bias when depicting the feasible region. In terms of computational efficiency, the fixed-direction sampling method adopts the same single-direction calculation logic as the proposed method with a fixed number of directions, so its computation time also increases linearly with the number of units. It consumes slightly less time for small and medium-scale clusters, yet the accuracy gap widens persistently as the scale grows.

The exact aggregation method can only be solved for small-scale cases of 20 and 40 units, with far longer computational time than the two sampling-based methods. When the cluster scale reaches 80 units, the excessive memory footprint and computational load brought by vertex enumeration render the model intractable, making it incapable of supporting aggregation calculations for engineering-scale large-scale clusters.

To verify the practical engineering optimization value of the proposed aggregation model, a day-ahead economic dispatch model for the park integrated energy system is constructed. The aggregated energy storage feasible region generated by the proposed method is embedded into the dispatch framework as constraints, and the economic performance and feasibility of dispatch under different aggregation models are compared.

The dispatch model minimizes the total daily system operating cost, with the objective function covering three components: electricity purchase cost from the upper-level power grid, energy storage operation loss cost, and electric heat pump operation cost. The constraint set includes grid tie-line power limits, electro-thermal power balance constraints, aggregated energy storage operation constraints, coupling equipment capacity limits, and photovoltaic output constraints. The exact energy storage model, the proposed aggregation model, the virtual battery model, and the box aggregation model are adopted separately for this solution. Taking a typical winter heating day as the test case, the comparison of dispatch results is presented in Table 11.

Table 11. Comparison of economic dispatch results under different aggregation models.

Dispatch Model	Daily Operation Cost	Cost Deviation	PV Consumption	Consumption Deviation (%)	Number of Constraint Violations	Solution Time
Exact Model	14,286.7 CNY	0.00%	78,542.30 kWh	0.00%	0	147.80 s
Proposed Aggregation Model	14,468.4 CNY	1.27%	77,802.10 kWh	0.94%	0	12.40 s
Virtual Battery Model	14,974.2 CNY	4.82%	75,216.50 kWh	4.23%	3	18.70 s
Box Aggregation Model	16,086.9 CNY	12.60%	70,134.70 kWh	10.71%	11	1.50 s

The results show that taking the dispatch outcomes of the exact model as the benchmark, the daily operating cost deviation obtained by the proposed aggregation model is merely 1.27%, the PV consumption deviation is less than 1.00%, and no violations of energy storage operation constraints occur throughout the dispatch horizon, which means the dispatch commands are fully feasible for engineering application. Unable to accurately characterize multi-period temporal coupling and electro-thermal coupling constraints, the virtual battery model yields a cost deviation close to 5.00% with 3 instances of SOC out-of-limit. The box model seriously overestimates the cluster regulation capacity, leading to a cost deviation as high as 12.60% and numerous infeasible dispatch instructions.

In terms of solution efficiency, the proposed aggregation model reduces the variable dimension of the original optimization problem by more than 85.00%, with a solution time of only 12.4 s, far shorter than the 147.8 s consumed by the exact model, which fully satisfies the time limit requirement for day-ahead dispatch in practical engineering. These results verify that the proposed aggregation model can drastically reduce the complexity of the optimization problem while guaranteeing dispatch accuracy and can deliver

reliable capacity boundary support for economic dispatch and market bidding of park-level integrated energy systems.

To systematically evaluate the robustness and adaptability of the proposed method under diverse operating conditions, quantitative sensitivity analysis is carried out from four dimensions: operating conditions, energy storage parameter disturbance, electro-thermal coupling strength, and time resolution. All tests are extended from the baseline scenario built on measured engineering data. Statistical distributions of key indicators are acquired via 500 groups of Monte Carlo stochastic simulations, and 95.00% confidence intervals are provided to quantify the reliability of the results.

Load (L) level and photovoltaic (P) output level are selected as core operating condition variables, each divided into three grades: high (H), medium (M) and low (L). Specifically, the high grade corresponds to 120% of the baseline value, the medium grade to 100% of the baseline value, and the low grade to 80% of the baseline value. Nine typical operating conditions are formed by combining these grades. The average support function error is adopted as the core accuracy metric for sensitivity analysis of operating conditions, and the test results are illustrated in Figure 12.

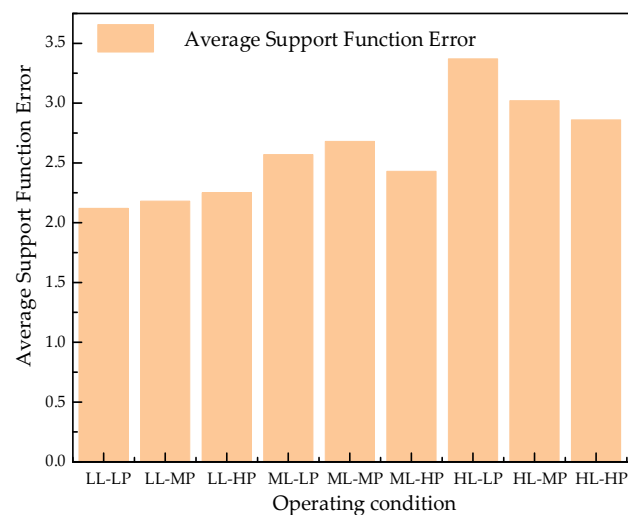


Figure 12. Average SFE data under different operating conditions.

It can be observed from the test results that the average SFE of the proposed method falls within a narrow range of 2.12% to 3.37% under all operating conditions, with an overall fluctuation of no more than 1.30 percentage points. The error reaches its minimum under the low-load and high-PV-output condition. In this scenario, energy storage devices operate mainly within the middle section of their charge–discharge range, leading to smooth feasible-region boundaries and low difficulty for fitting via directional sampling. The error peaks under the extreme condition of high load and low photovoltaic output, yet it remains below 3.40%, without accuracy degradation or failure of boundary characterization. The results demonstrate that the method possesses strong adaptability to fluctuations in load and renewable energy output and can maintain stable aggregation accuracy under typical park operating scenarios.

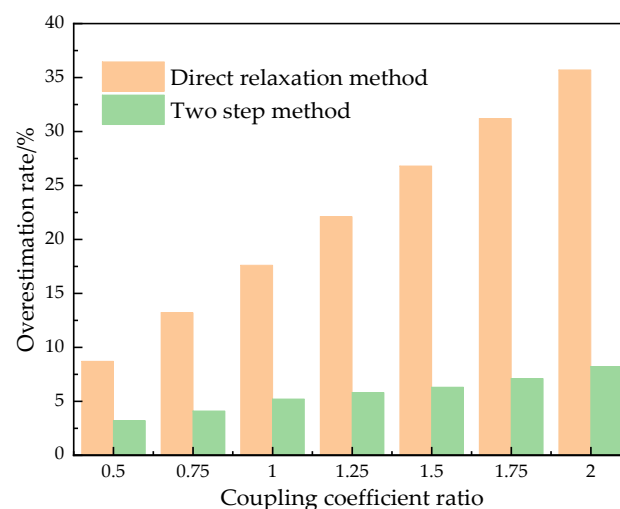
To account for parameter uncertainties in practical engineering such as energy storage capacity degradation, efficiency drift and cell inconsistency, random disturbance ranges of $\pm 20.00\%$ for rated capacity and $\pm 10.00\%$ for charge–discharge efficiency are set. The Monte Carlo method is adopted to randomly sample 500 independent parameter combinations. The statistical distribution and 95.00% confidence interval of aggregation accuracy indicators are calculated, respectively, for the sensitivity analysis of energy storage parameter disturbances and statistical reliability, with the results presented in Table 12.

Table 12. Accuracy index and 95% confidence interval under parameter disturbance.

Precision Index	Sample Mean/%	95% Lower Confidence Limit/%	95% Upper Confidence Limit/%
average SFE	2.68	2.41	2.95
maximum SFE	5.83	5.12	6.54

In Table 12, the height of each bar represents the sample mean of the indicator, while the upper and lower error bars correspond to the upper and lower bounds of the 95% confidence interval. Statistical results reveal that under random parameter disturbances, the sample mean of the average SFE is 2.68% with a 95.00% confidence interval of [2.41%, 2.95%]; the sample mean of the maximum SFE is 5.83% with a 95.00% confidence interval of [5.12%, 6.54%]. The width of the confidence interval for both indicators is less than 1 percentage point, indicating that the impact of parameter fluctuations on aggregation accuracy remains controllable. The proposed method exhibits favorable robustness against parameter heterogeneity and operational degradation of energy storage units, and the statistical results are highly reliable.

The overall electro-thermal coupling strength of the system is adjusted by modifying the number of put-into-operation electro-thermal coupling devices and their rated capacities. The coupling coefficient is defined as the ratio of the total rated power of electro-thermal conversion equipment to the total power of the energy storage cluster. This coefficient is gradually increased from 0.5 times the baseline value to 2.0 times the baseline value. The variation trends of the feasible-region overestimation rate of the direct relaxation aggregation method and the proposed two-step reconstruction method are compared to carry out sensitivity analysis on the electro-thermal coupling coefficient, and the results are illustrated in Figure 13.

**Figure 13.** Overestimation rate of feasible region under different coupling coefficients.

It can be seen from the variation trends that the direct relaxation aggregation method completely ignores the cutting effect of coupling constraints, so its overestimation rate rises approximately linearly with the increase in coupling strength. When the coupling coefficient reaches 2.00 times the baseline value, the overestimation rate exceeds 35.00%, which seriously overestimates the regulation capacity of the cluster. In contrast, the proposed two-step reconstruction method maintains a low overestimation level across the entire range of coupling strengths through tight processing of equivalent constraints: the overestimation rate stays below 6.50% when the coupling coefficient ranges from 0.50 to 1.50 times the baseline and is controlled within 8.20% even if the coupling coefficient rises to 2.00 times the baseline. The results verify that the equivalent constraint relaxation-

precise reconstruction mechanism can effectively adapt to electro-thermal coupling scenarios of varying strengths, consistently preserve the tightness of feasible-region characterization, and eliminate dispatch risks induced by capacity overestimation.

Four levels of dispatch time resolution are set: 60 min, 30 min, 15 min and 5 min. The aggregation accuracy and computational efficiency of the method under different time granularities are tested to conduct sensitivity analysis on time resolution, and the results are presented in Table 13.

Table 13. Accuracy and computational efficiency at different time resolutions.

Time Resolution	Average SFE/%	Computational Efficiency/s
60 min	2.10	2.8
30 min	2.35	4.7
15 min	2.68	9.1
5 min	3.48	21.3

As illustrated by the trend, when the time resolution is refined from 60 min to 5 min, the variable dimension of the model increases by 12 times, yet the average SFE of the proposed method only rises moderately from 2.10% to 3.48% without sharp accuracy deterioration. The computation time grows from 2.8 s to 21.3 s, a far smaller increment than the expansion rate of variable dimensions, and remains within an acceptable overhead for online applications.

This favorable performance benefits from the synergistic effect of the hierarchical mapping framework and adaptive direction selection mechanism. Hierarchical mapping prevents redundant dimensional expansion of thermal energy storage variables, while adaptive directional sampling prioritizes capturing critical feasible-region boundaries in high-dimensional spaces. This achieves a favorable trade-off between accuracy and computational efficiency across multi-time-scale scenarios, making the method compatible with engineering demands of various time granularities such as day-ahead dispatch and intra-day rolling dispatch.

This subsection discusses the practical deployment aspects of the proposed aggregation framework, covering communication overhead, feasible-region update mechanisms, and real-time computational performance, to further clarify its engineering applicability.

Regarding communication requirements, the distributed aggregation architecture adopts a star topology with minimal information interaction. In each aggregation round, the aggregator broadcasts a direction vector whose dimension is determined by the dispatch time resolution, corresponding to a data volume of approximately 0.8 KB per direction. Each local unit only returns a scalar support function value to the aggregator, without disclosing detailed model parameters, operating states, or private load information. For a complete aggregation process with 40 sampling directions, the total uplink data per unit is less than 0.4 KB. Such low communication demand is compatible with standard power IoT protocols and requires no dedicated high-bandwidth links, while naturally preserving the operational privacy of end users.

The aggregated feasible region can be updated via multi-time-scale strategies adapted to different dispatch cycles. At the day-ahead planning stage, the full aggregation process including AODS direction optimization and coupling constraint reconstruction is executed once daily to update the feasible region according to next-day equipment availability, initial energy states, and ambient conditions. The computational time is on the order of seconds, which is fully compatible with day-ahead dispatch timelines. For intra-day rolling dispatch, the feasible region is refreshed every 2–4 h by recalculating support function values with the latest unit states while keeping the direction set unchanged; a single update takes less than 1 s. At the real-time operation level, frequent

boundary reconstruction is not required and only online feasibility verification of dispatch commands is performed, with a single check completed within milliseconds. An event-triggered update mechanism can also be adopted to re-run the full aggregation process upon detection of unit faults, parameter degradation, or large state deviations.

In terms of real-time computational performance, the framework follows an offline-construction and online-operation paradigm to ensure temporal efficiency. The AODS direction selection and equivalent constraint reconstruction are completed offline prior to dispatch, occupying no online computing resources. During online operation, with a fixed optimal direction set, unit-level support function calculations are executed in parallel at edge nodes, and the aggregator only performs scalar summation and boundary updates. For a cluster of 1000 heterogeneous units, the total wall-clock time for a single online update is less than 200 ms, and single command feasibility verification takes less than 50 ms, which fully satisfies the temporal requirements of real-time distribution network dispatch. Compared with centralized exact aggregation methods that require tens of minutes of computation, the proposed framework exhibits significant practical advantages in real-time operation scenarios.

The method proposed in this paper is constructed based on deterministic operation assumptions, with its core objective being to establish a fundamental theoretical framework for multi-energy storage aggregation. To handle uncertainties in practical systems such as fluctuating renewable power output, load forecasting errors, variable energy storage availability, and communication information deviations, two types of extensions can be realized on the basis of the existing convex set aggregation framework without modifying the core aggregation logic.

First, the robust aggregation approach is adopted for bounded uncertain parameters including equipment parameter deviations, source-load fluctuation intervals, and communication delay errors. Uncertainties are modeled as convex uncertainty sets, and the robust feasible region of each unit is obtained via Minkowski operation between the nominal feasible region and the uncertainty set. The additive property of support functions is utilized to directly derive the aggregated robust support function. The distributed aggregation, adaptive direction selection and coupling reconstruction procedures can be fully reused to obtain a conservative feasible region that satisfies the worst-case scenarios.

Second, the probabilistic aggregation approach is applied to probabilistic uncertainties such as stochastic outages of energy storage units and stochastic distributions of source and load. The unit support functions are treated as random variables. The statistical moments and quantiles of the aggregated support function are directly calculated by virtue of linear superposition properties, so as to construct a probabilistic feasible region under chance constraints that fits stochastic dispatch scenarios while retaining linear computational complexity.

5. Conclusions

This paper constructs a distributed cluster test system comprising electric and thermal heterogeneous energy storage and systematically verifies the proposed aggregation method based on support functions and Minkowski sum through case studies from multiple dimensions, namely aggregation accuracy, computational efficiency, coupling constraint handling, multi-time-step mapping and dispatch command response. The main conclusions are:

- (1) The proposed distributed parallel aggregation strategy based on support functions effectively resolves the computational complexity of high-dimensional feasible-region aggregation. By converting Minkowski sum operations into arithmetic summation of scalar functions, it avoids the combinatorial explosion caused by vertex enumeration

and half-space intersection in conventional methods and achieves linear scalability of computational complexity while preserving aggregation accuracy.

- (2) The proposed AODS can identify the key directions that characterize the feasible-region geometry within finite iterations. Using the additivity of the support function with respect to Minkowski sum, complex high-dimensional geometric operations are transformed into scalar summation. Combined with redundant constraint elimination and adaptive direction selection, the computational complexity is significantly reduced.
- (3) The proposed method accurately captures the degradation effect of electro-thermal coupling constraints on the cluster regulation capability. Via the two-step framework of equivalent constraint relaxation and boundary precise reconstruction, coupling effects are embedded into the aggregation model, preventing capability overestimation caused by neglecting coupling in traditional approaches. This provides a reliable capability envelope for aggregators participating in the energy-reserve co-market.

It should be noted that all case studies in this work are carried out on synthetic test systems with artificially generated unit parameters, which constitutes a clear limitation of the current research. Although the parameter settings cover the typical operating ranges of electric and thermal storage devices and can effectively verify the theoretical properties and computational performance of the proposed aggregation method, synthetic data cannot fully reproduce complex real-world features such as time-varying equipment efficiency, long-term capacity degradation, random source-load fluctuations and communication noise. To further validate the practical applicability and robustness of the model, future research will adopt field-measured historical operation datasets from actual integrated energy parks to repeat the simulation tests and conduct comprehensive comparative analysis.

The proposed method not only improves the technical accuracy of the aggregation model, but also supplies a dependable capability boundary for energy storage clusters to participate in diversified electricity markets, thus significantly enhancing their market competitiveness and economic benefits. However, the mutually exclusive charge–discharge constraints are relaxed in the modeling process. Although this facilitates convex optimization, it may introduce practically infeasible solutions. Furthermore, the convergence of the adaptive direction selection algorithm lacks strict mathematical proof, implying a risk of trapping in local optima. Future research will mainly proceed in three directions. First, mixed-integer programming will be introduced to precisely handle charge–discharge mutual exclusivity, balancing convex optimization efficiency and physical feasibility. Second, we will supplement the convergence proof of the AODS algorithm and explore distributed robust direction update strategies to cope with source-load randomness. Third, a unified robust probabilistic aggregation framework will be established incorporating multiple uncertainties and real-time price signals, to improve engineering applicability and support the large-scale deployment of distributed energy storage.

Author Contributions: Conceptualization, S.G. and X.D.; methodology, S.G.; software, C.L.; validation, C.L. and Z.L.; formal analysis, C.L., Z.L. and M.D.; investigation, X.D. and M.D.; resources, Y.Y. and X.D.; writing—original draft, S.G.; writing—review & editing, M.D.; supervision, Y.Y. and D.W.; project administration, Y.Y. and D.W.; funding acquisition, D.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China grant number 52407011.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request.

Conflicts of Interest: Author Chunlong Li was employed by the company State Grid Jilin Electric Power Co., Ltd. Authors Xuyan Duan and Zhigang Liu were employed by the company Shenyang Zhilian Electric Power Design Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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