

Article

Small-Signal Modeling and Coordinated Optimal Control for an Embedded Heterogeneous MMC-MTDC System Considering AC/DC Bilateral Coupling

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Abstract

Embedded Modular Multilevel Converter-Based Multi-Terminal Direct Current (MMC-MTDC) systems have become an important solution for enhancing transmission capacity and operational flexibility in urban hybrid AC/DC power grids. However, the coexistence of Grid-Following (GFL) and Grid-Forming (GFM) MMC stations introduces complex dynamic interactions through both AC and DC networks. Existing small-signal stability studies often neglect MMC internal dynamics, such as submodule capacitor voltage fluctuations and circulating current-related states, or simplify the AC network as an ideal voltage source, which may lead to inaccurate stability assessment and limited control parameter optimization performance. To address these issues, this paper proposes a coordinated small-signal stability enhancement strategy for an embedded heterogeneous MMC-MTDC system considering AC/DC bilateral coupling. First, a system-level full-order small-signal state-space model is established by incorporating the internal dynamics of both GFL-MMC and GFM-MMC stations, non-ideal AC networks, and DC transmission links. Then, eigenvalue analysis and participation factor-based sensitivity evaluation are performed to identify weakly damped oscillation modes and screen the key variables and control parameters associated with dominant oscillations. Furthermore, a quadratic performance index is constructed by weighting the sensitivities of key control parameters, and particle swarm optimization is employed to obtain coordinated optimized parameters for heterogeneous MMC stations. Comparative case studies and PSCAD/EMTDC time-domain simulations verify the effectiveness of the proposed strategy under different scenarios. The quantitative active-power indices show that, compared with the unoptimized parameters, Strategy 2 reduces the settling time by 49.1% in the power step response, suppresses the power step overshoot from 6.4% to 0%, shortens the settling time by 53.8% under grid-strength variation, and reduces the active-power peak and settling time by 28.0% and 74.8%, respectively, under the fault ride-through scenario.

Keywords: embedded MMC-MTDC system; modular multilevel converter; grid-following control; grid-forming control; small-signal stability; parameters optimization



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1. Introduction

With the escalating severity of global energy shortages and environmental pollution crises, renewable energy sources—represented by wind and photovoltaic power—have achieved rapid development worldwide due to their advantages in flexibility, cost-effectiveness, and environmental friendliness [1,2]. Meanwhile, the increasing penetration of inverter-based resources has also changed the short-circuit and dynamic characteristics of modern power systems [3]. Modular Multilevel Converter-Based High-Voltage Direct Current (MMC-HVDC) transmission technology has become a critical means to support the efficient external transmission of large-scale renewable energy, owing to its superior performance in power flow controllability, transmission capacity, low loss, and power response speed [4–6]. Distinct from traditional long-distance point-to-point MMC-HVDC transmission, embedded Modular Multilevel Converter-Based Multi-Terminal Direct Current (MMC-MTDC) technology constructs a deeply integrated hybrid AC/DC power grid architecture by deploying multiple converter stations within the same AC power system. Related studies have investigated AC/DC interaction, transient control, and operational performance improvement in distribution or transmission systems with embedded DC links [7,8]. This architecture not only achieves a qualitative leap in regional transmission capacity but also significantly enhances system operational economy and power flow regulation flexibility, serving as a core direction for the upgrade and evolution of modern urban power grids. However, facing the multi-scenario operating conditions and strong time-varying characteristics of urban power grids, achieving accurate modeling and robust control design for such embedded heterogeneous hybrid AC/DC systems has become an urgent technical challenge.

Facing complex integration scenarios with varying short-circuit ratios (SCR), the focus of system analysis has shifted toward small-signal oscillatory stability, which characterizes the dynamic damping performance distinct from steady-state frequency or voltage stability. In this context, converter control strategies are mainly categorized into Grid-Following (GFL) and Grid-Forming (GFM) types. GFL relies on a phase-locked loop (PLL) to track the grid voltage phase, whereas GFM possesses the capability to autonomously construct the voltage phase angle [9,10]. Currently, academia has conducted extensive research on the small-signal modeling of MMC under different control strategies. Reference [11] established a detailed GFL-MMC model incorporating internal nonlinear characteristics, such as submodule capacitor voltage fluctuation dynamics and second-harmonic circulating currents, effectively reducing errors introduced by model simplification in system eigenvalue calculation and stability analysis. Reference [12] further revealed that internal variables such as circulating arm currents and submodule capacitor voltage ripples may lead to poorly damped resonant modes and harmonic instability in MMC-HVDC converters. Reference [13] targeted a simplified GFM-MMC model that ignores submodule capacitor dynamic characteristics, deeply analyzing the intrinsic mechanism of stability misjudgment caused by simplification and clearly defining the applicable boundaries of this model in stability analysis. Reference [14] proposed a dual grid-forming control strategy with MMC capacitor-energy regulation capability for integrating offshore wind farms into weak onshore grids. Reference [15] investigated the effects of different MMC energy-control structures and energy-buffering capabilities on the dynamics and AC/DC interactions of a hybrid GFM–GFL system using two eigenproperties-based indices. However, the aforementioned studies are mainly focused on single-terminal or specific MMC-HVDC configurations and still lack characterization of the dynamic coupling effects generated between converters via the DC-side network. Regarding multi-terminal systems, Reference [16] constructed a full-order small-signal model for a multi-terminal hybrid HVDC system and proposed a control parameter optimization method based on stability anal-

ysis. Reference [17] established a small-signal model for multi-terminal VSC-based DC transmission systems and discussed the influence of VSC control parameters on system stability. Reference [18] proposed an impedance-based stability analysis method for a multi-terminal cascaded hybrid HVDC system and investigated its oscillation propagation characteristics. Nevertheless, these studies either focus on specific HVDC configurations or emphasize impedance/network-side stability assessment, while the dynamic interaction between the AC network impedance and the converter port characteristics, i.e., the bilateral AC/DC coupling dynamics, still needs to be further considered in embedded heterogeneous MMC-MTDC systems with coexisting GFL and GFM converters.

In terms of DC protection and stability-oriented control, existing studies have investigated fault protection, GFL/GFM interactions, and stability enhancement from different perspectives. Reference [19] comprehensively reviewed DC fault detection, location, and isolation methods for CSC- and VSC-based HVDC transmission systems. Reference [20] analyzed the small-signal stability characteristics of GFL and GFM converters using state-space models and investigated the effects of their control dynamics on system stability. Reference [21] proposed an adaptive hybrid GFM/GFL control strategy to enhance small-signal stability under varying short-circuit ratios. Reference [22] investigated dual-port GFM control and AC/DC dynamic interactions of MMC-based interconnecting converters and further provided hardware validation. Reference [23] proposed a damping-enhancement control strategy for heterogeneous GFL-PV and GFM-ESS systems under low short-circuit ratio conditions. Although these studies have contributed to DC protection, GFL/GFM coordinated control, and stability enhancement, coordinated parameter optimization for heterogeneous GFL/GFM MMC-MTDC systems has not yet been sufficiently investigated. In particular, the coupled effects of MMC internal dynamics, AC/DC bilateral interactions, and multiple control parameters have not been systematically incorporated into a unified optimization framework.

Therefore, existing studies still lack a unified small-signal analysis framework for embedded heterogeneous MMC-MTDC systems that simultaneously considers detailed MMC internal dynamics, non-ideal AC networks, DC-link coupling, and GFL/GFM converter interaction. To fill this gap, this paper proposes a coordinated small-signal stability enhancement strategy for an embedded heterogeneous MMC-MTDC system considering AC/DC bilateral coupling. The main contributions of this paper are summarized as follows:

- (1) A system-level small-signal model is established for an embedded heterogeneous MMC-MTDC system with coordinated GFL-MMC and GFM-MMC stations. The proposed model explicitly incorporates MMC internal dynamics, including submodule capacitor voltage fluctuations and circulating current-related states, as well as the dynamic interactions with non-ideal AC networks and DC transmission links. Compared with conventional models that either neglect AC/DC bilateral coupling or simplify the AC grid as an ideal voltage source, the developed model provides a more accurate representation of the coupled dynamics in embedded hybrid AC/DC systems.
- (2) The oscillation characteristics and parameter coupling effects of the embedded heterogeneous MMC-MTDC system are analyzed under AC/DC bilateral coupling conditions. Through eigenvalue analysis and participation factor-based sensitivity evaluation, the weakly damped modes are identified, and the key variables and control parameters associated with dominant oscillations are screened. Root locus analysis further illustrates how representative parameters affect eigenvalue migration and damping performance, providing guidance for the subsequent coordinated optimization of GFL-MMC and GFM-MMC control parameters.

- (3) A coordinated parameter optimization method based on a quadratic performance index and particle swarm optimization is proposed. The quadratic index is constructed by weighting the sensitivities of key control parameters, so as to reflect the influence of multi-parameter coupling on system damping performance. The PSO algorithm is then employed to obtain coordinated optimized control parameters for both GFL-MMC and GFM-MMC stations. Comparative studies demonstrate that the proposed strategy improves the damping of dominant modes, suppresses power oscillations, and enhances system robustness under grid-strength variations and fault disturbances.

To more intuitively highlight the above contributions and clarify the research positioning of this paper, Table 1 provides a detailed comparison between this study and the existing representative literature.

Table 1. Comparison of the proposed strategy with the existing literature.

References	System Topology	Converter Model	Control Strategy	MMC Internal Dynamics	AC Network	Optimization Methodology
Ref. [11]	Single-terminal	MMC (Detailed)	GFL	Considered	Ideal	/
Ref. [13]	Single-terminal	MMC (Simplified)	GFM	Neglected	Ideal	Parameter Sweep
Ref. [16]	Multi-terminal	MMC (Detailed)	GFL	Considered	Ideal	PSO
Refs. [20–23]	Hybrid AC/DC	VSC/IBR	GFL + GFM	/	Non-ideal	Parameter Sweep
Proposed	Hybrid AC/DC	MMC (Detailed)	GFL + GFM	Considered	Non-ideal	PSO (Quadratic Index)

The remainder of this paper is organized as follows: Section 2 details the system configuration and small-signal modeling of the hybrid AC/DC power grid. Section 3 investigates system stability using eigenvalue analysis and elaborates on the proposed stability enhancement strategy based on the PSO algorithm. Section 4 presents case studies and validates the effectiveness of the proposed strategy through time-domain simulations. Finally, Section 5 concludes the paper.

2. Small-Signal Modeling of Hybrid AC/DC Power Grids

This study analyzes a hybrid AC/DC power grid system with GFL-MMC and GFM-MMC, with its structure illustrated in Figure 1: AC1 and AC2 represent external 500 kV equivalent AC sources; X_1 and X_2 denote the equivalent reactances of external systems; X_{12} indicates the line reactance between AC substations (marked in red); Region1 and Region2 are two independent AC power supply areas, each connected to the external AC system via 220 kV/500 kV main transformers and containing distributed generation resources of varying types and capacities; depending on the capacity of local generation, MMC1 and MMC2 adopt GFL control and GFM control strategies, respectively; X_{DC} represents the DC line reactance between the converters.

Both MMC stations adopt a pseudo bipolar configuration, with converter valves using hybrid full-bridge and half-bridge structures. The system fully considers the internal dynamic characteristics of converter stations, including submodule capacitor voltage fluctuations, internal circulating currents and their suppression, and circulating current suppression controller parameters.

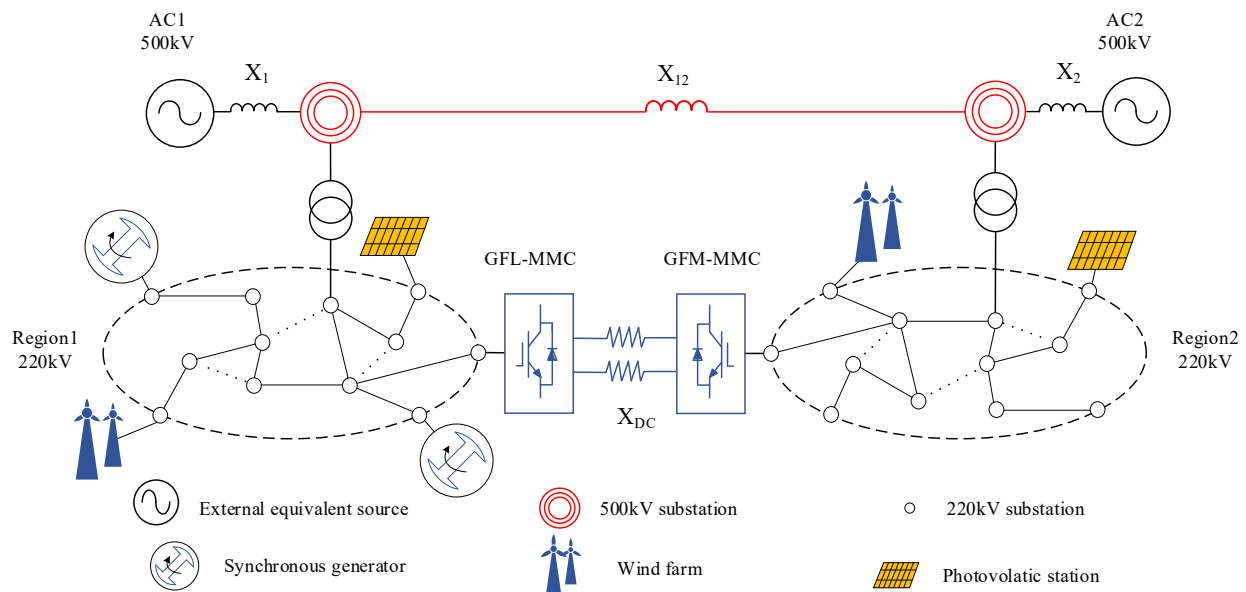


Figure 1. Structure of the hybrid AC/DC power grids.

2.1. Modeling of MMC Valve Control Layer

The topology of a three-phase MMC is shown in Figure 2. The MMC converter station consists of three phases and six arms. Each phase unit contains upper and lower arms. The specific structure of each arm includes one arm inductor L_{arm} , one arm resistor R_{arm} , and N submodules connected in series. The submodule topology is the half-bridge type, and each half-bridge submodule (HBSM) consists of two IGBTs, two anti-parallel diodes, and one capacitor. In Figure 2, i_{sj} ($j = a, b, c$) represents the AC current, I_{dc} represents the MMC DC-side current, U_{dc} represents the MMC DC-side voltage, VT_1 and VT_2 are IGBTs, VD_1 and VD_2 are anti-parallel diodes connected to the IGBTs.

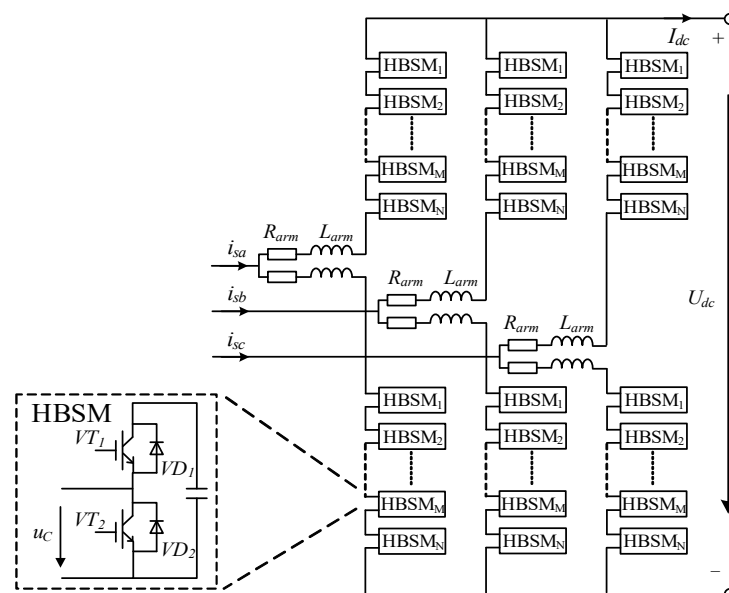


Figure 2. Topology of MMC valve control layer.

During MMC operation, the submodule capacitor voltage fluctuates due to switching actions and dynamic power variations. These capacitor voltage fluctuations are coupled with the AC-side current, DC-side current, and internal circulating current dynamics, and therefore should be included in the MMC internal dynamic model. According to existing

MMC internal dynamic modeling methods [11,12], the submodule capacitor voltage can be expressed as the superposition of a DC component and dominant AC harmonic components. In this paper, the DC component, the fundamental-frequency component, the second-harmonic component, and the third-harmonic component are explicitly retained, while higher-order components are not included because they mainly appear as high-frequency ripples with relatively small amplitudes. The capacitor voltage dynamics can be decomposed as:

$$\frac{du_c}{dt} = \frac{d\bar{u}_c}{dt} + \frac{du_{c_ac1}}{dt} + \frac{du_{c_ac2}}{dt} + \frac{du_{c_ac3}}{dt} \quad (1)$$

Expressing the right-hand side of the DC component's dynamic equation in dq-axis components yields:

$$\frac{d\bar{u}_c}{dt} = \frac{1}{6C} I_{dc} + \frac{U_{cq} I_{sq}}{4CU_{dc}} + \frac{U_{cd} I_{sd}}{4CU_{dc}} + \frac{U_{cirq} I_{cirq}}{2CU_{dc}} + \frac{U_{cird} I_{cird}}{2CU_{dc}} \quad (2)$$

where U_{cd} and U_{cq} are the d-axis and q-axis components of the equivalent arm voltage in the dq reference frame, respectively; I_{sd} and I_{sq} are the d-axis and q-axis components of the AC-side current, respectively; and I_{cird} and I_{cirq} are the d-axis and q-axis components of the circulating current, respectively.

By substituting the arm current and average switching function into the arm capacitor voltage dynamic equation and separating the retained harmonic components, the dynamic equations of the fundamental-frequency, second-harmonic, and third-harmonic capacitor voltage components can be obtained as follows:

$$\begin{cases} \frac{du_{c1d}}{dt} = -\omega u_{c1q} - \frac{1}{4C} I_{sd} - \frac{I_{dc} U_{cd}}{3CU_{dc}} - \frac{U_{cq} I_{cirq}}{2CU_{dc}} - \frac{U_{cd} I_{cird}}{2CU_{dc}} - \frac{U_{cirq} I_{sq}}{4CU_{dc}} - \frac{U_{cird} I_{sd}}{4CU_{dc}} \\ \frac{du_{c1q}}{dt} = \omega u_{c1d} - \frac{1}{4C} I_{sq} - \frac{I_{dc} U_{cq}}{3CU_{dc}} - \frac{U_{cd} I_{cirq}}{2CU_{dc}} + \frac{U_{cq} I_{cird}}{2CU_{dc}} - \frac{U_{cirq} I_{sd}}{4CU_{dc}} + \frac{U_{cird} I_{sq}}{4CU_{dc}} \\ \frac{du_{c2d}}{dt} = -2\omega u_{c2q} + \frac{1}{2C} I_{cird} + \frac{I_{dc} U_{cird}}{3CU_{dc}} + \frac{U_{cq} I_{sq}}{4CU_{dc}} + \frac{U_{cd} I_{sd}}{4CU_{dc}} \\ \frac{du_{c2q}}{dt} = 2\omega u_{c2d} + \frac{1}{2C} I_{cirq} + \frac{I_{dc} U_{cirq}}{3CU_{dc}} + \frac{U_{cd} I_{sq}}{4CU_{dc}} + \frac{U_{cq} I_{sd}}{4CU_{dc}} \\ \frac{du_{c3x}}{dt} = 3\omega u_{c3y} - \frac{I_{cirq} U_{cd}}{2CU_{dc}} - \frac{I_{cird} U_{cq}}{2CU_{dc}} - \frac{U_{cird} I_{sq}}{4CU_{dc}} - \frac{U_{cirq} I_{sd}}{4CU_{dc}} \\ \frac{du_{c3y}}{dt} = -3\omega u_{c3x} + \frac{I_{cirq} U_{cq}}{2CU_{dc}} - \frac{I_{cird} U_{cd}}{2CU_{dc}} + \frac{U_{cirq} I_{sq}}{4CU_{dc}} - \frac{U_{cird} I_{sd}}{4CU_{dc}} \end{cases} \quad (3)$$

where the subscripts 1d and 1q denote the *d*-axis and *q*-axis components of the fundamental-frequency capacitor voltage component, respectively; the subscripts 2d and 2q denote the *d*-axis and *q*-axis components of the second-harmonic capacitor voltage component, respectively; and u_{c3x} and u_{c3y} are the two state variables used to describe the third-harmonic capacitor voltage component. The same subscript convention is used for the related voltage and current variables in the following derivations.

The supplementary derivation of the corresponding AC current, DC current, and circulating current dynamics is provided in Appendix A. Based on the retained internal dynamics, each MMC valve model contains 12 internal dynamic states, including seven capacitor voltage states and five current states.

2.2. Modeling of GFL Controller

This paper considers GFL control with fixed U_{dc}/Q control. The GFL controller consists of an outer loop for fixed U_{dc}/Q control and an inner loop for current control, adopting a typical double closed-loop design. The outer-loop error signals are utilized to calculate the reference values for the inner-loop current. The modulated signals are obtained by tracking the inner-loop current error. Figure 3 illustrates the schematic diagram of the GFL controller structure [11].

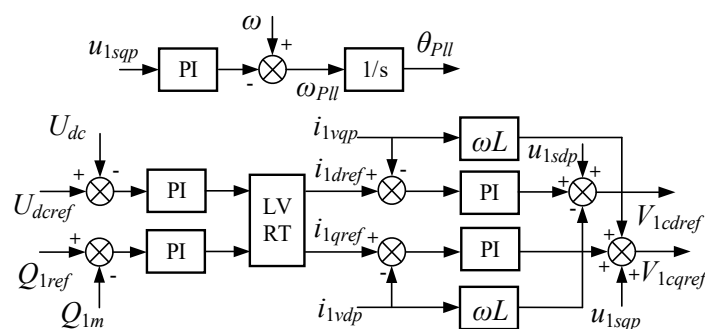


Figure 3. GFL controller structure.

As shown in Figure 3, U_{dcref} and Q_{1qref} represent the DC voltage and reactive power reference values, respectively. i_{1dref} and i_{1qref} are the d-axis and q-axis components of the current loop reference values, while V_{1cdref} and V_{1cqref} denote the d-axis and q-axis components of the modulated voltage signals. U_{dc} is the DC-side voltage, and Q_{1m} is the calculated reactive power value. i_{1vdp} and i_{1vqp} are the d-axis and q-axis components of the converter terminal current, and u_{1sdp} and u_{1sqp} are the d-axis and q-axis components of the voltage at the PCC (Point of Common Coupling). ω_{pll} is the output angular frequency of the phase-locked loop (PLL), θ_{pll} is the output phase angle of the PLL, and ω is the rated angular frequency. L represents the equivalent inductance at the converter terminal, and LVRT stands for the low voltage ride-through module. The LVRT strategy of the GFL-MMC is described in Appendix B.1, where the voltage threshold and current reference limitation are specified. The outer-loop and inner-loop control equations are expressed as follows [11]:

$$\begin{cases} i_{1dref} = k_{p11}(U_{dcref} - U_{dc}) + k_{i11}x_1 \\ i_{1qref} = k_{p12}(Q_{1ref} - Q_{1m}) + k_{i12}x_2 \\ \dot{x}_1 = U_{dcref} - U_{dc} \\ \dot{x}_2 = Q_{1ref} - Q_{1m} \end{cases} \quad (4)$$

$$\begin{cases} V_{1cdref} = u_{1sdp} - \omega L i_{1vqp} - k_{p13}(i_{1dref} - i_{1vdp}) - k_{i13}x_3 \\ V_{1cqref} = u_{1sqp} + \omega L i_{1vdp} - k_{p14}(i_{1qref} - i_{1vqp}) - k_{i14}x_4 \\ \dot{x}_3 = i_{1dref} - i_{1vdp} \\ \dot{x}_4 = i_{1qref} - i_{1vqp} \end{cases} \quad (5)$$

where x_1 , x_2 , x_3 , and x_4 represent the integrals of the error signals in the PI controllers.

The phase angle reference is provided by the PLL:

$$\begin{cases} \omega_{pll} = \omega - k_{ppll}u_{1sqp} + k_{ipll}x_5 \\ \theta_{pll} = \omega t + x_6 \\ \dot{x}_5 = -u_{1sqp} \\ \dot{x}_6 = -k_{ppll}u_{1sqp} + k_{ipll}x_5 \end{cases} \quad (6)$$

where x_5 and x_6 represent the integrals of the error signals.

Figure 4 shows the control block diagram of the circulating current suppression controller, and its internal equations are as follows:

$$\begin{cases} V_{1cird} = -2\omega L i_{1cirq} - k_{p1cird}(i_{1cirdref} - i_{1cird}) - k_{i1cird}x_7 \\ V_{1cirq} = +2\omega L i_{1cird} - k_{p1cirq}(i_{1cirqref} - i_{1cirq}) - k_{i1cirq}x_8 \\ \dot{x}_7 = i_{1cirdref} - i_{1cird} \\ \dot{x}_8 = i_{1cirqref} - i_{1cirq} \end{cases} \quad (7)$$

where x_7 and x_8 represent the integrals of the error signals.

where x_9 , x_{10} and x_{11} represent the integrals of the error signals. The control structures of both the GFM current loop and the circulating current suppression are consistent with that of the GFL controller.

2.4. Small-Signal Modeling Construction

By linearizing Equations (1)–(10), the small-signal model of the hybrid AC/DC power grid system with GFL-MMC and GFM-MMC can be obtained. Its general form is as follows:

$$\begin{cases} \Delta X = [\Delta X_{GFL} & \Delta X_{GFM} & \Delta X_{DC} & \Delta X_{AC}]^T \\ \Delta \dot{X} = A\Delta X + B\Delta U \\ \Delta \dot{Y} = C\Delta X + D\Delta U \end{cases} \quad (11)$$

where ΔX_{GFL} and ΔX_{GFM} represent the state variables of the GFL-MMC and GFM-MMC, respectively, while ΔX_{DC} and ΔX_{AC} denote the state variables of the DC and AC transmission lines. In this model, the DC transmission line is represented by a lumped pi-equivalent line. The AC network is not simplified as an ideal voltage source, but is represented by external equivalent sources behind finite grid impedances together with the inter-area AC line. Therefore, ΔX_{DC} includes the dynamic variables of the DC link, and ΔX_{AC} includes the AC-side network dynamic variables introduced by the non-ideal grid and inter-area coupling. The input matrix $\Delta U = [\Delta U_{dcref}, \Delta Q_{1ref}, \Delta P_{2ref}, \Delta Q_{2ref}, \Delta i_{1circref}, \Delta i_{1cirqref}, \Delta i_{2circref}, \Delta i_{2cirqref}]^T$. The output matrix $\Delta Y = [\Delta P_{1m}, \Delta Q_{1m}, \Delta P_{2m}, \Delta Q_{2m}]^T$. Matrices A, B, C, and D represent the system matrix, control matrix, output matrix, and direct transmission matrix, respectively. The small-disturbance linearization procedure, including the steady-state operating point and the first-order Taylor expansion used to obtain the state-space matrices, is provided in Appendix C.

3. Sensitivity Discrimination and Optimization Design of Control Parameters

3.1. Sensitivity Discrimination and Screening of Main Control Parameters

As indicated in the previous section, the hybrid AC/DC power grids system with GFL-MMC and GFM-MMC comprises 32 control parameters and 58 state variables, which directly leads to complexity in its static stability analysis and control parameter design. Therefore, sensitivity analysis of the control parameters is necessary.

At the steady-state operating point of the hybrid AC/DC power grid system, substitute specific system parameter and control parameter values. Calculate the state matrix A of the small-disturbance model and solve its eigenvalues and corresponding right eigenvectors:

$$\begin{cases} |\lambda E - A| = 0 \\ (\lambda E - A)\xi = 0 \end{cases} \quad (12)$$

where: E is the identity matrix; $\lambda = a + jb$ are eigenvalues of the state matrix (a and b are real and imaginary parts, respectively); ξ is the right eigenvector of the state matrix.

Using Lyapunov's first method, determine if the real parts of all eigenvalues are negative [5]. If all real parts are negative, the system is stable; otherwise unstable. Identify conjugate eigenvalue pairs based on identical real parts and opposite imaginary parts. Each conjugate pair corresponds to an oscillation mode. The oscillation frequency f and damping ratio ζ for each mode are calculated as:

$$\begin{cases} f = b/(2\pi) \\ \zeta = -a/\sqrt{a^2 + b^2} \end{cases} \quad (13)$$

In engineering practice, the damping ratio ζ of a mode is generally required to be greater than or equal to 0.005.

By solving the left eigenvector μ and right eigenvector ζ of the small-disturbance dynamic model, their product yields the participation factor sensitivity P for each oscillation mode:

$$P = \mu \zeta^T \quad (14)$$

3.2. Construction of Quadratic Index

To quantitatively evaluate the stability of the hybrid AC/DC power grid system, this paper adopts the quadratic index as the quantitative evaluation criterion. System stability is determined by observing this objective function value.

Considering the hybrid AC/DC power grids system as linear, the quadratic index describing small-signal stability is defined as:

$$J_x = \frac{1}{2} \int_0^\infty [\Delta x(t) - \Delta x_0]^T Q [\Delta x(t) - \Delta x_0] dt \quad (15)$$

where $\Delta x(t)$ denotes the small disturbance of the state variables, and Δx_0 is the initial disturbance at the steady-state operating point. $Q = \text{diag}(q_1, q_2, \dots, q_n)$ is the state-weighting matrix, whose diagonal coefficients are determined according to the normalized aggregated participation factors of the selected weakly damped dominant modes. State variables with larger overall contributions to the dominant oscillation modes are assigned higher weights, whereas those with weaker modal participation are assigned relatively smaller weights. J_x is a quantitative indicator of the small-signal dynamic performance. The detailed participation factor aggregation, normalization, weight-mapping procedure, and construction of Q are provided in Appendix D.

The output equation of the small-signal model is:

$$\Delta y = C \Delta x \quad (16)$$

Since not all state variables are measurable in engineering practice, an output feedback linear quadratic suboptimal index is more practical. This case constructs control variables from low-dimensional outputs to form a reduced-order output feedback system:

$$\Delta u = -K_x \Delta y = -K_x C \Delta x \quad (17)$$

where K_x is the output feedback coefficient matrix.

$$J_x = \frac{1}{2} \Delta x^T(0) H \Delta x(0) \quad (18)$$

where matrix H satisfies:

$$(A - BKC)^T H + H(A - BKC) = -Q \quad (19)$$

Equation (19) is the quadratic index expression for the hybrid AC/DC power grids system. J_x is solved by first obtaining H from (19). Control parameters affect the state-space matrices A , B , and C and consequently influence H and J_x . Through the participation factor-based weighting matrix Q , the quadratic performance index places greater emphasis on the dynamic states that are more strongly associated with the weakly damped dominant modes. For a stable system, a smaller finite value of J_x indicates improved damping performance and reduced small-signal state deviations.

3.3. Particle Swarm Optimization Algorithm

Due to its characteristics of ease of implementation, high precision, and rapid convergence, the particle swarm optimization algorithm is suitable for optimizing the selection of numerous control parameters in the hybrid AC/DC power grid system. Its expression is as follows:

$$\begin{cases} v_{id}^{k+1} = \omega v_{id}^k + c_1 r_1 (p_{id,pbest}^k - x_{id}^k) \\ \quad + c_2 r_2 (p_{d,pbest}^k - x_{id}^k) \\ x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1} \end{cases} \quad (20)$$

where v_{id}^{k+1} and v_{id}^k represent the particle's velocity at iterations $k + 1$ and k , respectively; ω denotes the inertia weight, reflecting the particle's reliance on its previous motion state; c_1 and c_2 are learning factors; r_1 and r_2 are random numbers within $[0, 1]$ enhancing stochastic exploration; $p_{id,pbest}^k$ indicates the historical optimal position in the d -th dimension for particle i at iteration k ; and $p_{d,pbest}^k$ represents the historical optimal position in the d -th dimension for the swarm at iteration k .

In the PSO-based optimization process, each particle represents a candidate control parameter vector θ . The feasible region of the candidate parameters is mainly determined by the small-signal stability of the closed-loop system. For each candidate θ , the corresponding closed-loop state matrix $A(\theta)$ is reconstructed, and its eigenvalues are recalculated. The basic stability condition is expressed as:

$$\max_i \operatorname{Re}[\lambda_i(A(\theta))] < 0 \quad (21)$$

where $\lambda_i(A(\theta))$ denotes the eigenvalues of the closed-loop state matrix under the current candidate parameter vector. If the stability condition is not satisfied, the particle is penalized in the fitness evaluation. Otherwise, the particle is evaluated by the quadratic performance index. The search range of each control parameter is selected around its initial value by considering practical controller-tuning limits and the feasible small-signal stability region. No additional normalization is applied to the control parameters during the optimization; each parameter is optimized using its original value within the corresponding lower and upper bounds. The electrical variables in the system model are expressed in per-unit values based on the rated quantities.

3.4. Control Strategy Based on Quadratic Performance Index and PSO Algorithm

To address the technical challenges posed by multiple controllers and extensive control parameters in hybrid AC/DC power grid systems, this paper proposes a control parameter analysis method based on classical control theory and an optimization approach utilizing a quadratic index combined with PSO. Leveraging the principles of small-signal stability, the method establishes a mathematical framework for control parameter adjustment, with the detailed process flow illustrated in Figure 6.

The specific control logic diagram is presented in Figure 7. As shown in Figure 7, the system topology consists of an AC/DC hybrid power grid connecting Region 1 and Region 2. On the AC side, the two areas are linked by a high-voltage transmission line with impedance X_{12} . Simultaneously, a parallel DC system connects these two areas via MMC and a DC line with impedance X_{DC} . Each area also contains local generation and loads connected to the 220 kV bus through transformers. Regarding the control architecture, the left-side converter (MMC1) operates as a rectifier using GFL control. Its internal structure includes a PLL, a power control loop, and a current control loop to generate modulation signals. The right-side converter (MMC2) operates as an inverter using GFM control, which consists of a power control loop and cascaded voltage-current control loops. MMC1 and

MMC2 are connected to the local power sources through line reactances X_{L1} and X_{L2} , as well as equivalent reactances X_{Eq1} and X_{Eq2} , respectively. The core of this strategy lies in the PSO algorithm module. This algorithm is responsible for dynamically optimizing the control parameters and sends the optimized GFL parameters (indicated by the red arrow) to MMC1 and the optimized GFM parameters (indicated by the green arrow) to MMC2, thereby ensuring overall system stability and operational performance.

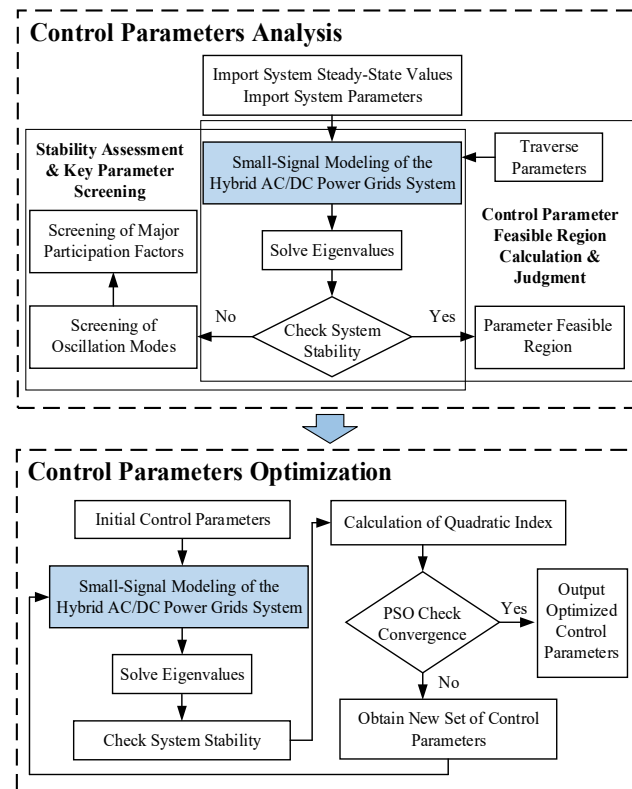


Figure 6. Process flow for control parameters analysis and optimization methodology in the hybrid AC/DC power grids system.

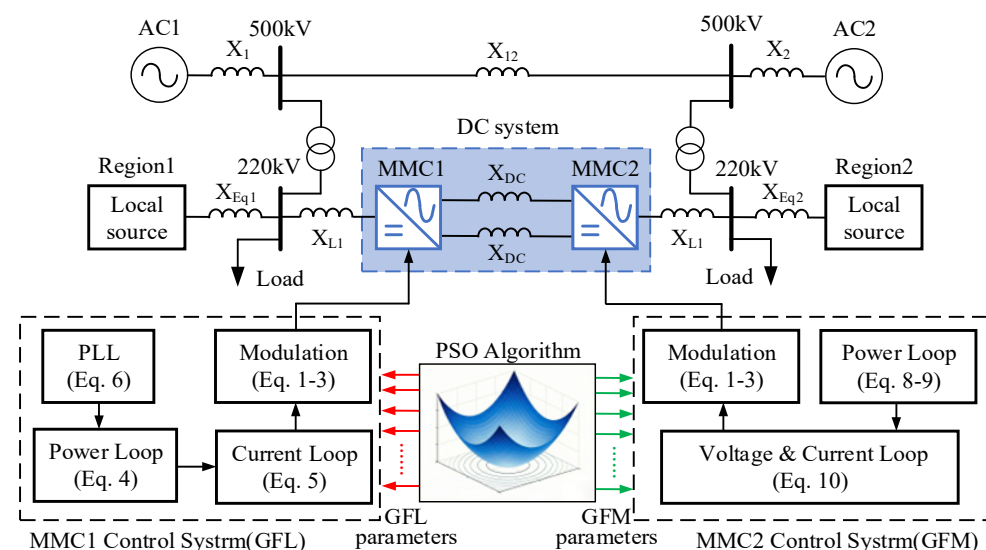


Figure 7. Control logic diagram based on the proposed control strategy.

4. Case Study Analysis

4.1. Small-Signal Model Validation

To further evaluate the accuracy and applicability of the developed small-signal model, comparative validation with the PSCAD/EMTDC electromagnetic transient model was conducted under multiple operating conditions.

The detailed operating conditions and comparison results are presented in Appendix F. The results demonstrate that the small-signal model closely reproduces the dominant transient characteristics of the electromagnetic transient model, including the response trends, oscillation frequencies, damping behavior, and steady-state values. Minor high-frequency fluctuations in the electromagnetic transient results are attributed primarily to the switching dynamics of the MMC, which are not included in the averaged small-signal model. Therefore, the established model provides sufficient accuracy for the subsequent eigenvalue analysis, sensitivity assessment, and control parameter optimization.

4.2. Eigenvalue Calculation and Sensitivity Presentation

Based on Equations (11)–(13), the eigenvalues of the system state-space, oscillation frequencies, and damping ratios can be calculated, as shown in Table 2. Due to space constraints, only the modes with low damping ratios are presented.

Table 2. The dominant mode of the hybrid AC/DC power grid system.

Mode	Eigenvalue	Oscillation Frequency (Hz)	Damping Ratio
7	$-23.3 + 1018.3 i$	162.0	0.003
8	$-13.3 + 932.8 i$	148.4	0.014
9	$-12.7 + 933.1 i$	148.5	0.013
10	$-11.6 + 932.9 i$	148.4	0.012
11	$-44.1 + 586.9 i$	93.4	0.075
12	$-58.5 + 586.1 i$	93.2	0.099
13	$-49.5 + 587.8 i$	93.5	0.084
17	$-29.8 + 380.1 i$	60.5	0.078
19	$-39.3 + 374.2 i$	59.5	0.104
21	$-5.1 + 191.4 i$	30.4	0.026
22	$-5.9 + 188.9 i$	30.0	0.031
23	$-4.2 + 185.9 i$	29.5	0.022

Modes 7, 8, 9, 10, 11, 12, 13, 17, 19, 21, 22, and 23 are weakly damped modes. As they are highly correlated with the small-signal stability of the system, they are typically defined as dominant modes. Notably, Mode 7 has the lowest damping ratio of only 0.003, indicating extremely poor oscillation-decay capability and an unacceptable damping margin for practical control design. Although the corresponding eigenvalue remains in the left half-plane, such a low damping ratio suggests that the original control parameters cannot provide adequate dynamic damping, thereby further justifying the need for coordinated PSO-based parameter optimization. Based on Equation (14), the sensitivity of the control parameters for each dominant mode can be calculated, as shown in Figure 8.

The number below each subplot in Figure 9 indicates the dominant mode number, while the numerical labels within the subplots represent the key participation factor indices. The most frequently observed participation factors in the weakly damped modes (34, 35, 38, 39, 53, 62, 63, 66, 72) are defined in Table A3. Based on the sensitivity results, only the 12 control parameters with relatively strong influence on the weakly damped modes are selected for optimization. The remaining parameters are fixed at their initial values under the operating conditions considered in this study.

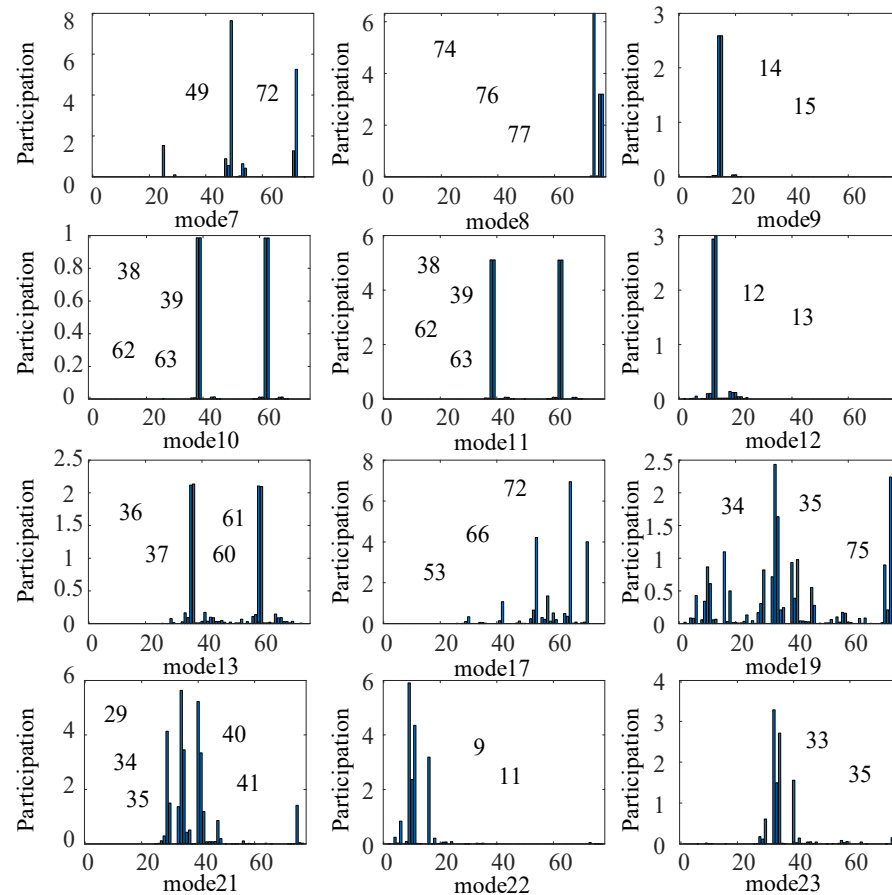


Figure 8. Participation factor sensitivity of the dominant modes.

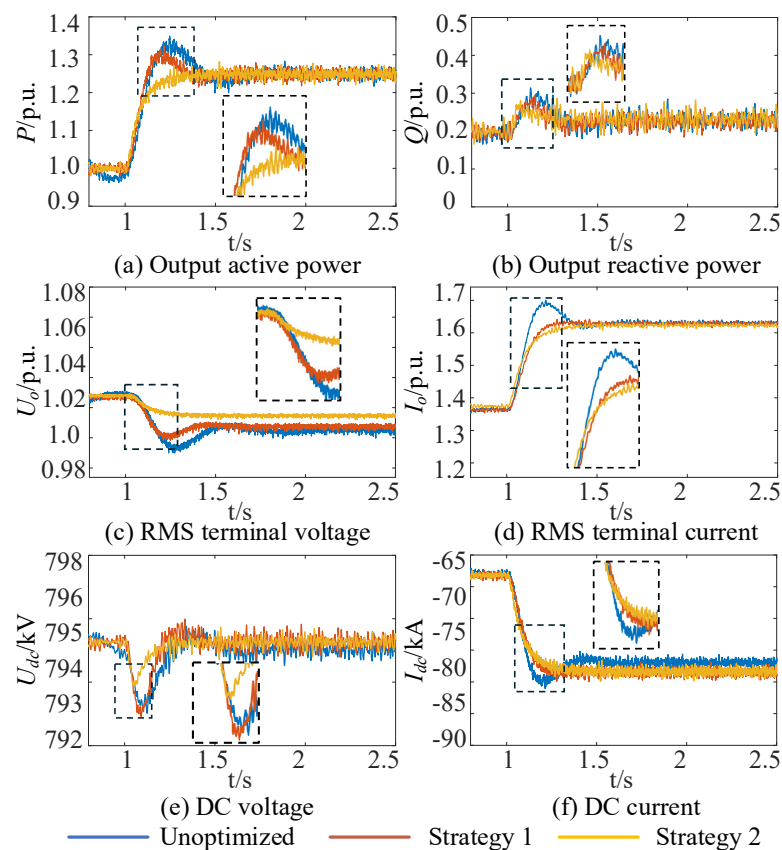


Figure 9. Active power step performance under different control parameters.

4.3. Optimization Based on Quadratic Index and Particle Swarm Algorithm

Following the sensitivity-based screening, the 12 selected parameters are used as the PSO decision variables, while the other control parameters remain fixed. To validate the efficacy of the quadratic index and particle swarm optimization strategy, comparative studies examined small-signal stability under three control schemes:

- (1) Traditional parameter tuning methods (unoptimized).
- (2) Strategy 1: Parameters optimized via Literature [16], employing a weighted sum of damping ratios and real-part eigenvalues as the objective function.
- (3) Strategy 2: Parameters optimized via the proposed strategy.

Table 3 presents the parameter configuration for the particle swarm optimization algorithm, including the learning factors c_1 and c_2 , inertia weight ω , swarm dimension D (number of control parameters), swarm size N , and maximum iterations M .

Table 3. Parameters configuration for particle swarm optimization algorithm.

Parameters	Value	Parameters	Value
c_1	0.7481	D	12
c_2	0.3842	N	100
ω	0.7298	M	200

The selection basis of the PSO parameters and a detailed comparison of the convergence characteristics are provided in Appendix G. Table 4 presents the specific control parameters before optimization, after optimization using Strategy 1, and after optimization using Strategy 2.

Table 4. Control parameters variation under different strategies.

Parameters	Unoptimized	Strategy 1	Strategy 2
(k_{p11}, k_{i11})	(1.09, 10.11)	(6.74, 25.36)	(5.44, 32.36)
(k_{p13}, k_{i13})	(1.09, 10.11)	(2.26, 33.74)	(5.99, 60.36)
(k_{p14}, k_{i14})	(10.01, 60.33)	(16.38, 44.75)	(40.02, 64.54)
(k_{p21}, k_{i21})	(0.48, 67.03)	(2.53, 45.98)	(9.36, 44.88)
(k_{p22}, k_{i22})	(0.48, 67.03)	(12.96, 84.55)	(13.22, 90.36)
(k_{p24}, k_{i24})	(12.22, 95.88)	(15.32, 89.54)	(14.98, 87.77)

To intuitively verify the impact of control parameters on the system's small-signal stability, the root loci of dominant eigenvalues were analyzed by varying key control parameters, as shown in Appendix H.

4.4. Simulation Verification

To verify the dynamic performance and robustness of the proposed strategy, comprehensive time-domain simulations under active power step, grid impedance variation, and fault scenarios were conducted using the PSCAD/EMTDC 4.6.2 platform.

4.4.1. Verification of Active Power Step Performance

For the hybrid AC/DC power grid system with different control parameters, a step increase of 0.2 p.u. was applied to the receiving-end AC system power reference at $t = 1.0$ s, with simulation waveform variations observed for each control parameter configuration.

As illustrated in Figure 9, the time-domain simulation results reveal distinct differences among the three control parameter configurations. The unoptimized system exhibits a severe underdamped response, characterized by a significant overshoot in active power and RMS current, prolonged low-frequency oscillations, and a substantial voltage sag on

the DC side (Figure 9e) caused by strong AC/DC coupling. While Strategy 1 provides a moderate reduction in overshoot and oscillation amplitude, it fails to achieve a smooth transient transition, indicating limited damping capability. In stark contrast, the proposed Strategy 2 demonstrates superior control performance; it tracks the power reference rapidly with negligible overshoot and a critically damped response, effectively suppressing the potential power oscillations observed in the other cases. Furthermore, the proposed strategy significantly mitigates the fluctuations in both AC terminal voltage (Figure 9c) and DC voltage (Figure 9e), confirming that the PSO-optimized parameters successfully decouple the dynamic interactions and enhance the overall small-signal stability of the hybrid system.

4.4.2. Verification of Grid Adaptability

For the hybrid AC/DC power grid system with different control parameters, grid adaptability validation is conducted under grid-strength variation scenarios. At $t = 1$ s, grid impedance decreases from 0.4 p.u. (short-circuit ratio SCR = 5) to 0.2 p.u. (SCR = 2.5). At $t = 2$ s, grid impedance recovers to 0.4 p.u., with simulation waveform variations observed for each control parameter configuration.

As illustrated in Figure 10, the dynamic responses reveal significant performance disparities among the three strategies. The unoptimized system exhibits severe oscillations and voltage instability, particularly evident in the drastic DC voltage drop to approximately 780 kV at $t = 1.0$ s, indicating a lack of damping against grid-strength variations. Strategy 1, while mitigating high-frequency noise, suffers from a sluggish transient response with excessive overshoot reaching nearly 1.8 p.u. in active power and a prolonged settling time, reflecting poor adaptability to grid-strength changes. In contrast, the proposed Strategy 2 demonstrates superior grid adaptability; it effectively suppresses the power and voltage fluctuations during both the grid stiffening and weakening processes, maintaining the active power overshoot within a narrow margin and restoring steady-state operation within approximately 0.3 s. These results confirm that the PSO-optimized parameters ensure robust operation across different short-circuit ratio scenarios, effectively decoupling the system dynamics from grid impedance variations.

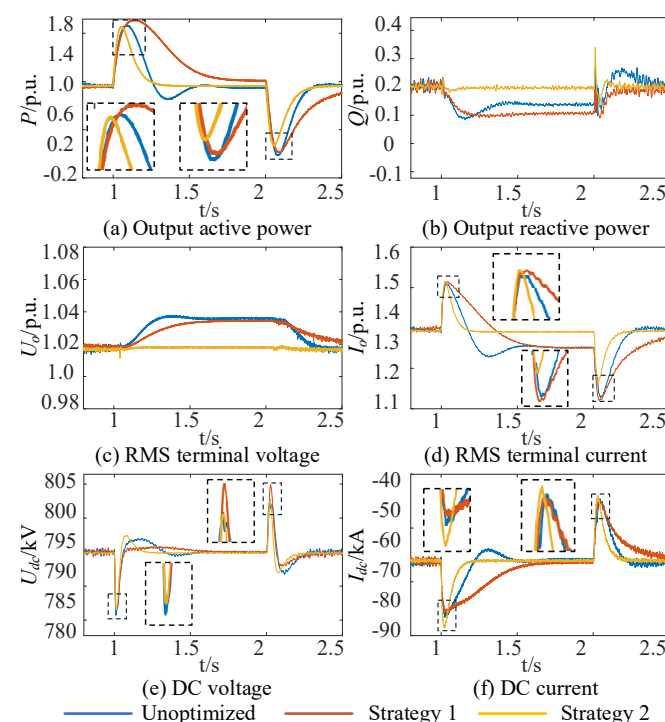


Figure 10. Grid compatibility validation under different control parameters.

4.4.3. Verification of Fault Ride-Through Capability

To further validate the effectiveness of the proposed control parameter optimization method, this section examines the fault ride-through performance of the power grid under different control parameters. Since the study focuses on control parameter design for static stability, a scenario with relatively shallow AC voltage sag is considered: a three-phase short-circuit fault is set to occur on the receiving-end AC grid at $t = 1$ s, causing the RMS AC voltage to drop to 0.8 p.u., and then recover to 1.0 p.u. at $t = 1.1$ s, with simulation waveform variations observed for each control parameters configuration.

As shown in Figure 11, the system dynamic responses vary significantly across the three control configurations. The unoptimized system exhibits severe instability following the fault clearance; the active power surges to approximately 2.2 p.u. with sustained large-amplitude oscillations (Figure 11a), and the DC voltage (Figure 11e) experiences prolonged fluctuations, indicating a high risk of system tripping. Strategy 1 reduces the initial peak to some extent but still suffers from a slow recovery process, taking considerable time to stabilize. In contrast, the proposed Strategy 2 demonstrates superior transient stability. It effectively clamps the post-fault power surge, mitigates the DC voltage fluctuations, and ensures a rapid return to the steady-state immediately after the fault is cleared. These results verify that the parameters optimized by the proposed quadratic index-based PSO algorithm significantly enhance the system's damping and fault recovery capability.

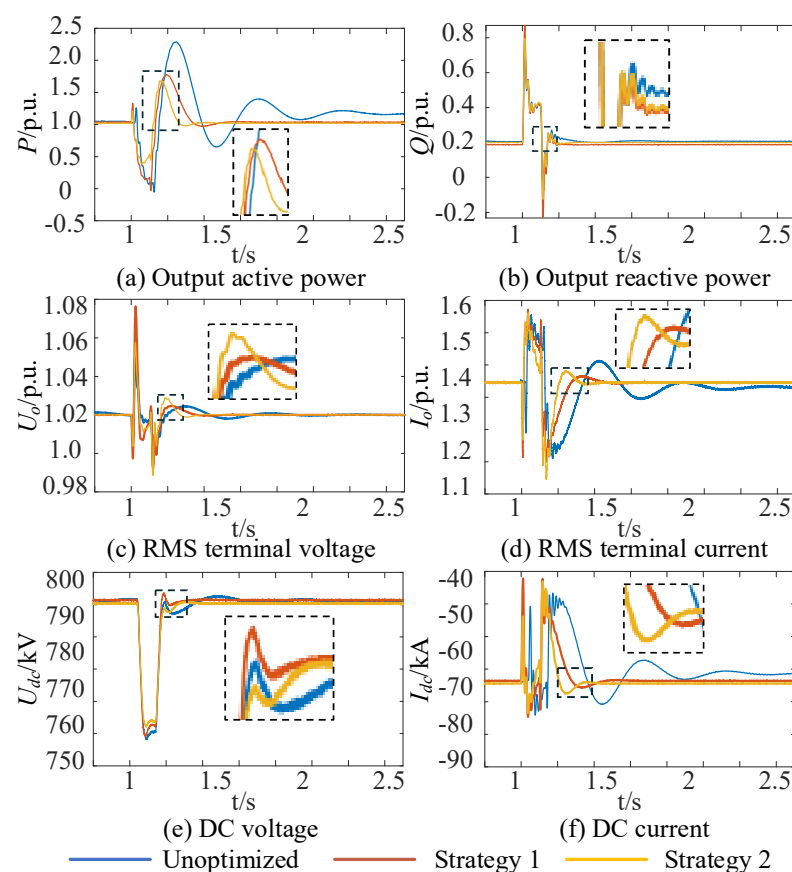


Figure 11. Fault ride-through performance validation under different control parameters.

To further substantiate the time-domain observations, three active power performance indices are extracted from subplot (a) of Figures 9–11, namely P peak, P overshoot, and settling time. These indices are selected because the active power response directly reflects the damping performance of the embedded MMC-MTDC system under the considered disturbances. P overshoot denotes the percentage by which the peak active power exceeds its

final steady-state value, while the settling time is evaluated using a 2% tolerance band around the final steady-state active power. The quantitative results are summarized in Table 5.

Table 5. Quantitative comparison of active power dynamic responses under different disturbance scenarios.

Scenario	Performance Index	Unoptimized	Strategy 1	Strategy 2
Power step response	P_{peak} (p.u.)	1.33	1.30	1.25
Power step response	$P_{\text{overshoot}}$ (%)	6.4	4.0	0
Power step response	Settling time (s)	0.55	0.42	0.28
Grid-strength variation	P_{peak} (p.u.)	1.73	1.80	1.72
Grid-strength variation	$P_{\text{overshoot}}$ (%)	73.0	74.1	72.0
Grid-strength variation	Settling time (s)	0.65	0.90	0.30
AC voltage sag	P_{peak} (p.u.)	2.32	1.74	1.67
AC voltage sag	$P_{\text{overshoot}}$ (%)	114.8	74.0	67.0
AC voltage sag	Settling time (s)	1.43	0.54	0.36

The data in Table 5 indicate that Strategy 2 provides a more damped active power response over the three disturbance scenarios. For the power step response, Strategy 2 reduces the P peak to 1.25 p.u. and suppresses the P overshoot to 0%, with a settling time of 0.28 s. In comparison, the original method presents a P overshoot of 6.4% and requires 0.55 s to settle. Under the grid-strength variation scenario, the P peak values of the three methods are close, whereas the settling time difference is more pronounced. Strategy 2 reaches the steady-state band within 0.30 s, while the original method and Strategy 1 require 0.65 s and 0.90 s, respectively. Under the fault ride-through scenario, the original method exhibits the largest active power excursion, with a P peak of 2.32 p.u., a P overshoot of 114.8%, and a settling time of 1.43 s. Strategy 2 reduces these indices to 1.67 p.u., 67.0%, and 0.36 s, respectively. These results provide quantitative evidence that the proposed coordinated optimization improves the damping of active power oscillations and accelerates post-disturbance recovery.

In summary, comprehensive time-domain simulations under step disturbances, grid impedance variations, and fault scenarios consistently validate that the proposed strategy effectively mitigates oscillations and significantly outperforms traditional methods in terms of dynamic damping, system robustness, and transient stability.

5. Conclusions

This paper investigates the small-signal stability enhancement of an embedded heterogeneous MMC-MTDC system with coexisting GFL-MMC and GFM-MMC stations under AC/DC bilateral coupling. A coordinated parameter optimization strategy based on a quadratic performance index and particle swarm optimization is proposed. The main conclusions are summarized as follows:

- (1) A system-level small-signal state-space model of the embedded heterogeneous MMC-MTDC system is established. The model incorporates the internal dynamic characteristics of both MMC stations, including submodule capacitor voltage fluctuations and circulating current-related states, as well as the dynamic interactions between the non-ideal AC network and the DC transmission link. Comparison with the PSCAD/EMTDC electromagnetic transient model verifies that the established small-signal model can effectively represent the dynamic response characteristics of the studied hybrid AC/DC system.
- (2) The dominant oscillation modes and their key influencing factors are identified through eigenvalue analysis and participation factor-based sensitivity evaluation.

The results show that the weakly damped modes are closely associated with MMC internal variables and selected control loop parameters of the heterogeneous converters. Further root locus analysis indicates that representative control parameters have a clear influence on dominant eigenvalue migration and system damping performance, providing a basis for the coordinated optimization of GFL-MMC and GFM-MMC control parameters. In addition, the PSCAD/EMTDC time-domain simulations provide quantitative support for the effectiveness of the proposed strategy. As summarized in Table 5, Strategy 2 reduces the P peak to 1.25 p.u. and suppresses the P overshoot to 0% in the power step response, with a settling time of 0.28 s. Under grid-strength variation, although the P peak values of the three methods are close, Strategy 2 reaches the steady-state band within 0.30 s, compared with 0.65 s for the original method and 0.90 s for Strategy 1. Under the fault ride-through scenario, Strategy 2 limits the P peak, P overshoot, and settling time to 1.67 p.u., 67.0%, and 0.36 s, respectively, whereas the corresponding values of the original method are 2.32 p.u., 114.8%, and 1.43 s. These quantitative results further verify that the proposed optimization strategy improves active power damping and accelerates post-disturbance recovery.

Overall, the proposed modeling and coordinated optimization method can provide a useful reference for small-signal stability analysis and control parameter tuning of embedded hybrid AC/DC power grids with heterogeneous MMC stations. It should be noted that the proposed method is developed within a small-signal stability framework around a specified operating point. Therefore, very severe disturbance scenarios, such as deep voltage sags and complete short-circuit faults, which are mainly associated with large-signal nonlinear transient dynamics and protection actions, are not fully covered in this study. Future work will extend the proposed method to more complex disturbance scenarios and network configurations, while further considering parameter uncertainty, measurement noise, communication delays, and computational efficiency, and validating its applicability through hardware-in-the-loop and large-scale experimental platforms.

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Data Availability Statement: The data used to support the findings of this study can be obtained through the sample construction method described in Sections 3 and 4 and Appendix E.

Conflicts of Interest: Author Jiaqi Wu Bo Zhu, Hao Lu were employed by the company, Guangdong Power Grid Co., Ltd. Author Zhu Guo, Haoli Chen, Yilin Zhong were employed by the company China Southern Power Grid. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

SCR	Short-Circuit Ratio
MMC	Modular Multilevel Converter
HVDC	High-Voltage Direct Current

MTDC	Multi-Terminal Direct Current
GFM	Grid-Forming
GFL	Grid-Following
VSC	Voltage Source Converter
PLL	Phase-Locked Loop
VSG	Virtual Synchronous Generator
PSO	Particle Swarm Optimization
DC	Direct Current
AC	Alternating Current
PI	Proportional-Integral

Appendix A

The MMC valve model used in this paper is established based on the average switching function modeling framework in [12]. For each MMC arm, the relationship among the average switching function, arm current, submodule capacitor voltage, and arm voltage is expressed as:

$$Si_{\text{arm}} = C \frac{du_c}{dt}, u_{\text{arm}} = NSu_c \quad (\text{A1})$$

where S is the average switching function, i_{arm} is the arm current, C is the submodule capacitance, u_c is the submodule capacitor voltage, u_{arm} is the arm voltage, and N is the number of submodules per arm.

After retaining the DC, fundamental-frequency, second-harmonic, and third-harmonic capacitor voltage components, the MMC valve model contains seven capacitor voltage states. In addition, the model includes five current states, namely the DC current, the d - q components of the AC-side current, and the d - q components of the second-harmonic circulating current. The current dynamics are obtained from the corresponding Kirchhoff voltage equations.

The DC current dynamic equation is derived from the DC-side voltage balance:

$$U_{\text{dc}} = 2u_{\text{armdc}} + \frac{2}{3}R_{\text{arm}}I_{\text{dc}} + \frac{2}{3}L_{\text{arm}}\frac{dI_{\text{dc}}}{dt} \quad (\text{A2})$$

where U_{dc} is the DC-side voltage, u_{armdc} is the DC component of the arm voltage, I_{dc} is the DC current, R_{arm} is the arm resistance, and L_{arm} is the arm inductance. The DC current dynamic equation is obtained by substituting the retained DC component of the arm voltage into the DC-side voltage balance. After rearrangement, the state equation of I_{dc} is obtained as:

$$\begin{aligned} \frac{dI_{\text{dc}}}{dt} = & \frac{3U_{\text{dc}}}{2L_{\text{arm}}} - \frac{R_{\text{arm}}}{L_{\text{arm}}}I_{\text{dc}} - \frac{3N}{2L_{\text{arm}}}\bar{u}_c + \frac{3NU_{\text{cq}}u_{\text{c1q}}}{2L_{\text{arm}}U_{\text{dc}}} + \\ & \frac{3NU_{\text{cd}}u_{\text{c1d}}}{2L_{\text{arm}}U_{\text{dc}}} - \frac{3NU_{\text{cirq}}u_{\text{c2q}}}{2L_{\text{arm}}U_{\text{dc}}} - \frac{3NU_{\text{cird}}u_{\text{c2d}}}{2L_{\text{arm}}U_{\text{dc}}} \end{aligned} \quad (\text{A3})$$

The AC-side current dynamic equation is obtained from the AC-side voltage balance:

$$u_s = -u_{\text{armac1}} + L_{\text{eq}}\frac{di_s}{dt} + R_{\text{eq}}i_s \quad (\text{A4})$$

where u_s and i_s are the AC-side voltage and current, u_{armac1} is the fundamental-frequency component of the arm voltage, and R_{eq} and L_{eq} are the equivalent AC-side resistance and inductance. Applying the positive-sequence fundamental-frequency d - q transformation to (A4), the dynamic equations of I_{sd} and I_{sq} are obtained as:

$$\begin{cases} \frac{dI_{sd}}{dt} = -\frac{R_{eq}}{L_{eq}} I_{sd} + \frac{U_{sd}}{L_{eq}} + \frac{Nu_{c1d}}{2L_{eq}} + \frac{N(-2\bar{u}_c U_{cd} - U_{cq} u_{c2q} - U_{cd} u_{c2d})}{2U_{dc} L_{eq}} + \\ \frac{N(U_{c1q} u_{c1q} + U_{c1d} u_{c1d} + u_{c3x} U_{c1q} + u_{c3y} U_{c1d})}{2U_{dc} L_{eq}} - \omega I_{sq} \\ \frac{dI_{sq}}{dt} = -\frac{R_{eq}}{L_{eq}} I_{sq} + \frac{U_{sq}}{L_{eq}} + \frac{Nu_{c1q}}{2L_{eq}} + \frac{N(-2\bar{u}_c U_{cq} - U_{cd} u_{c2q} + U_{cq} u_{c2d})}{2U_{dc} L_{eq}} + \\ \frac{N(U_{c1q} u_{c1d} - U_{c1d} u_{c1q} + u_{c3x} U_{c1d} - u_{c3y} U_{c1q})}{2U_{dc} L_{eq}} + \omega I_{sd} \end{cases} \quad (A5)$$

where I_{sd} and I_{sq} are the d -axis and q -axis AC-side currents, U_{sd} and U_{sq} are the d - q components of the AC-side voltage, and R_{eq} and L_{eq} are the equivalent AC-side resistance and inductance.

The circulating current dynamic equation is obtained from the second-harmonic arm-voltage balance:

$$2u_{armac2} + 2L_{arm} \frac{di_{cir}}{dt} + 2R_{arm} i_{cir} = 0 \quad (A6)$$

where u_{armac2} is the second-harmonic component of the arm voltage, and i_{cir} is the circulating current. Applying the negative-sequence second-harmonic d - q transformation to (A6), the dynamic equations of I_{cird} and I_{cirq} are obtained as:

$$\begin{cases} \frac{dI_{cird}}{dt} = -2\omega I_{cirq} - \frac{Nu_{c2d}}{2L_{arm}} - \frac{\bar{u}_c U_{cird}}{L_{arm} U_{dc}} - \frac{R_{arm}}{L_{arm}} I_{cird} + \\ \frac{NU_{cd} u_{c1d} - NU_{cq} u_{c1q} + Nu_{c3x} U_{cq} + Nu_{c3y} U_{cd}}{2L_{arm} U_{dc}} \\ \frac{dI_{cirq}}{dt} = 2\omega I_{cird} - \frac{Nu_{c2q}}{2L_{arm}} - \frac{\bar{u}_c U_{cirq}}{L_{arm} U_{dc}} - \frac{R_{arm}}{L_{arm}} I_{cirq} + \\ \frac{NU_{cd} u_{c1q} + NU_{cq} u_{c1d} + Nu_{c3x} U_{cd} - Nu_{c3y} U_{cq}}{2L_{arm} U_{dc}} \end{cases} \quad (A7)$$

where I_{cird} and I_{cirq} are the d -axis and q -axis components of the second-harmonic circulating current.

Based on the retained capacitor voltage dynamics and the current dynamics above, the MMC valve model adopted in this paper contains 12 internal dynamic states. The state vector of one MMC valve model is defined as:

$$x_{MMC} = [\bar{u}_c, u_{c1d}, u_{c1q}, u_{c2d}, u_{c2q}, u_{c3x}, u_{c3y}, I_{dc}, I_{sd}, I_{sq}, I_{cird}, I_{cirq}]^T \quad (A8)$$

The corresponding input vector is defined as:

$$u_{MMC} = [U_{cird}, U_{cirq}, U_{cd}, U_{cq}, \omega, U_{sd}, U_{sq}, U_{dc}]^T \quad (A9)$$

Therefore, each GFL-MMC or GFM-MMC valve model contributes 12 internal dynamic states to the system-level small-signal model. This supplementary description clarifies the state composition and external interfaces of the MMC valve model adopted in the main text.

Appendix B

In Appendix B, all voltage, current, and power quantities are expressed in p.u., and the converter current limit is set as $I_{lim} = 1.5$ p.u.

Appendix B.1. LVRT Control of the GFL-MMC

For the GFL-MMC, the LVRT module is activated according to the PCC voltage ratio. The voltage ratio is defined as:

$$\alpha = \frac{U_{PCC}}{U_N} \quad (A10)$$

where U_{PCC} is the PCC voltage magnitude, and U_N is the rated PCC voltage. When $\alpha > 0.9$, the PCC voltage is regarded as normal, and the original outer-loop current references are

directly used. When $\alpha < 0.9$, the LVRT module modifies the active and reactive current references to provide voltage support and avoid converter overcurrent.

The active current reference is limited by both the active power command and the remaining current capacity:

$$I_{dref} = \begin{cases} I_{dref0}, & \alpha \geq 0.9 \\ \min\left(\frac{P_{PCCref}}{u_{PCC}}, \sqrt{I_{lim}^2 - I_{qref}^2}\right), & \alpha < 0.9 \end{cases} \quad (A11)$$

where I_{dref0} is the original d -axis current reference generated by the outer loop, I_{dref} is the d -axis current reference after LVRT limitation, P_{PCCref} is the active power reference at the PCC, u_{PCC} is the PCC voltage magnitude, I_{qref} is the q -axis current reference, and I_{lim} is the converter current limit. Under voltage-sag conditions, the d -axis current reference is constrained by the remaining current margin after the reactive current reference has been determined.

The reactive current reference is determined according to the voltage-sag depth. When the PCC voltage remains within the normal range, the original q -axis current reference is retained. When the voltage ratio decreases below 0.9, additional reactive current is injected for voltage support. If the voltage ratio further decreases below 0.4, the reactive current reference reaches the converter current limit:

$$I_{qref} = \begin{cases} I_{qref0}, & \alpha \geq 0.9 \\ -2(1 - \alpha), & 0.4 < \alpha < 0.9 \\ -I_{lim}, & \alpha \leq 0.4 \end{cases} \quad (A12)$$

where I_{qref0} is the original q -axis current reference generated by the outer loop, α is the PCC voltage ratio, and I_{lim} is the converter current limit.

This LVRT strategy gives priority to reactive current injection under voltage sag, while the d -axis current is restricted by the remaining current margin. Therefore, the total current reference satisfies:

$$\sqrt{I_{dref}^2 + I_{qref}^2} \leq I_{lim} \quad (A13)$$

Appendix B.2. Current-Limiting Control of the GFM-MMC

For the GFM-MMC, the current-limiting module is inserted between the voltage control loop and the inner current control loop. The voltage loop first generates the d -axis and q -axis current references. During voltage disturbances, these current references may exceed the allowable converter current. Therefore, a vector-saturation current limiter is adopted to constrain the magnitude of the current reference vector before it is sent to the inner current loop.

The magnitude of the original current reference vector is calculated as:

$$i_{2ref} = \sqrt{i_{2dref}^2 + i_{2qref}^2} \quad (A14)$$

where i_{2dref} and i_{2qref} are the d -axis and q -axis current references generated by the voltage control loop of MMC2, respectively, and i_{2ref} is the magnitude of the original current reference vector. The current-limiting module is activated when:

$$i_{2ref} > I_{lim} \quad (A15)$$

where I_{lim} denotes the maximum allowable converter current. To avoid changing the direction of the current reference vector, the d -axis and q -axis current references are scaled by the same coefficient. The scaling coefficient is defined as:

$$\gamma_2 = \min \left(1, \frac{I_{\text{lim}}}{\max(i_{2\text{ref}}, \varepsilon)} \right) \quad (\text{A16})$$

where γ_2 is the current reference scaling coefficient, and ε is a small positive constant used to avoid division by zero. If $i_{2\text{ref}} \leq I_{\text{lim}}$, then $\gamma_2 = 1$, and the current limiter is inactive. If $i_{2\text{ref}} > I_{\text{lim}}$, then $\gamma_2 < 1$, and the current reference vector is proportionally reduced.

The limited current references are obtained as:

$$i_{2\text{dreflim}} = \gamma_2 i_{2\text{dref}}, \quad i_{2\text{qreflim}} = \gamma_2 i_{2\text{qref}} \quad (\text{A17})$$

where $i_{2\text{dreflim}}$ and $i_{2\text{qreflim}}$ are the limited d -axis and q -axis current references, respectively. Since the same scaling coefficient is applied to both axes, the direction of the original current reference vector is preserved. The resulting limited current reference vector satisfies:

$$\sqrt{i_{2\text{dreflim}}^2 + i_{2\text{qreflim}}^2} \leq I_{\text{lim}} \quad (\text{A18})$$

This current-limiting strategy represents a basic current reference saturation method. It is only activated during voltage disturbances or fault ride-through conditions. Around the normal steady-state operating point, the current reference magnitude is lower than I_{lim} , and therefore the limiter does not affect the small-signal state matrix established in Section 2.4.

Appendix C

The nonlinear dynamic model of the hybrid AC/DC power grid can be written in a compact form as:

$$\dot{x} = f(x, u, p), \quad y = g(x, u, p) \quad (\text{A19})$$

where x is the state vector, u is the input vector, y is the output vector, and p denotes the system and controller parameters. In this paper, x includes the internal dynamic states of the GFL-MMC and GFM-MMC stations, the controller states, the DC-link states, and the AC-network dynamic states.

The small-signal model is established around the initial steady-state operating point of $P_0 = 1.0$ p.u. and $Q_0 = 0.2$ p.u. on the system base. The same operating point is also used as the initial condition for the PSCAD/EMTDC electromagnetic transient validations reported in Appendix F. At this operating point, the steady-state variables satisfy:

$$f(x_0, u_0, p_0) = 0, \quad y_0 = g(x_0, u_0, p_0) \quad (\text{A20})$$

where x_0 and u_0 are the steady-state values of the state and input variables, respectively, and p_0 denotes the parameter vector used for the linearization.

The small disturbances around the steady-state operating point are defined as:

$$x = x_0 + \Delta x, \quad u = u_0 + \Delta u, \quad y = y_0 + \Delta y \quad (\text{A21})$$

Applying the first-order Taylor expansion to (A20) around the steady-state operating point gives:

$$f(x, u, p_0) = f(x_0, u_0, p_0) + \left. \frac{\partial f}{\partial x} \right|_0 \Delta x + \left. \frac{\partial f}{\partial u} \right|_0 \Delta u + O(\|\Delta x, \Delta u\|^2) \quad (\text{A22})$$

Since $f(x_0, u_0, p_0) = 0$ at the steady-state operating point and the higher-order disturbance terms are neglected, the small-disturbance state-space model is obtained as:

$$\Delta \dot{x} = A\Delta x + B\Delta u \quad (\text{A23})$$

where:

$$A = \left. \frac{\partial f}{\partial x} \right|_0, \quad B = \left. \frac{\partial f}{\partial u} \right|_0$$

Similarly, the output equation is obtained as:

$$\Delta y = C\Delta x + D\Delta u \quad (\text{A24})$$

where:

$$C = \left. \frac{\partial g}{\partial x} \right|_0, \quad D = \left. \frac{\partial g}{\partial u} \right|_0$$

The resulting matrices A , B , C , and D are then used for eigenvalue analysis, participation factor calculation, and the subsequent quadratic index-based control parameter optimization.

Appendix D

The detailed construction procedure of the participation factor-based state-weighting matrix Q is illustrated in Figure A1. The weighting strategy is designed to assign greater importance to the state variables that contribute more significantly to the weakly damped dominant modes, thereby enabling the quadratic performance index to focus on the dynamic states most closely associated with the critical oscillatory behavior.

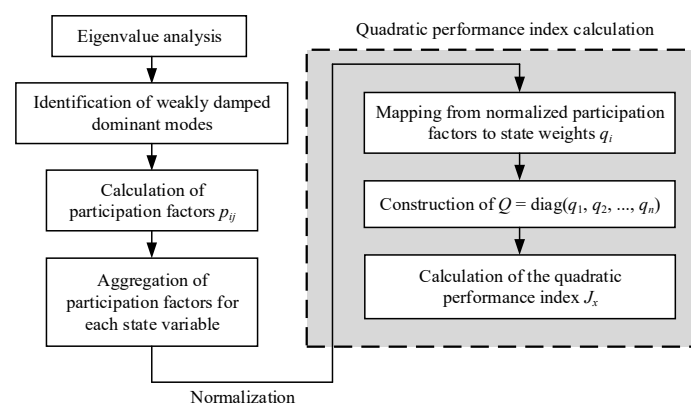


Figure A1. Construction procedure of the participation factor-based state-weighting matrix Q .

First, the weakly damped dominant modes are identified through eigenvalue analysis. For the j -th dominant mode, the participation factor of the i -th state variable is calculated as

$$p_{ij} = |\mu_{ij}\xi_{ij}| \quad (\text{A25})$$

where μ_{ij} and ξ_{ij} are the corresponding elements of the left and right eigenvectors, respectively. The absolute value is adopted to characterize the contribution magnitude of each state variable and to avoid cancellation caused by the signs or phases of the eigenvector components.

The participation factors are then normalized within each dominant mode as

$$\tilde{p}_{ij} = \frac{p_{ij}}{\sum_{r=1}^n p_{rj}} \quad (\text{A26})$$

where n is the total number of state variables. This normalization eliminates the influence of the numerical scale of different eigenvectors and enables a consistent comparison of the state variable contributions among different dominant modes.

For each state variable, the normalized participation factors corresponding to all selected dominant modes are aggregated as

$$s_i = \frac{1}{N_d} \sum_{j \in \Omega_d} \tilde{p}_{ij} \quad (\text{A27})$$

where Ω_d denotes the set of selected weakly damped dominant modes and N_d is the number of modes included in this set. Equal modal weighting is adopted because all selected modes are regarded as critical to the small-signal stability of the system. The aggregated index s_i therefore represents the overall contribution of the i -th state variable to the dominant oscillatory behavior.

The aggregated participation indices are further normalized as

$$\hat{s}_i = \frac{s_i}{\max_{1 \leq r \leq n} (s_r)} \quad (\text{A28})$$

where $0 \leq \hat{s}_i \leq 1$. The normalized aggregated participation index is directly mapped to the corresponding state-weighting coefficient:

$$q_i = \hat{s}_i \quad (\text{A29})$$

Accordingly, the state-weighting matrix is constructed as

$$Q = \text{diag}(q_1, q_2, \dots, q_n) \quad (\text{A30})$$

According to this mapping rule, a state variable with a larger aggregated participation factor is assigned a higher weight because it contributes more strongly to the selected weakly damped modes and has a greater influence on the dominant system oscillations. Conversely, state variables with relatively small modal contributions are assigned lower weights. Therefore, the weighting coefficients are determined quantitatively from the modal participation characteristics rather than through empirical assignment. All state variables remain included in the quadratic performance index, while the weighting matrix adjusts their relative importance according to their contributions to the critical modes.

Appendix E

Table A1. System parameters.

Parameters	Value
Rated DC voltage	± 220 kV
Rated active power	500 MW
Number of bridge arm submodules	200
Rated voltage of AC system	500 kV
Equivalent AC system resistance	0.3333 Ω
Equivalent AC system inductance	0.0005 H
Fundamental frequency	50 Hz
Bridge arm inductance	0.05 H
Submodule capacitance	1500 mF
DC line to ground capacitance	10 mF
DC line resistance	1.35 Ω
DC line inductance	0.125 H

Table A1. *Cont.*

Parameters	Value
Equivalent inductance (220 kV)	0.062 H
Line inductance	0.05 H
Transmission line inductance	0.12 H
Equivalent inductance (500 kV)	0.0005 H

Table A2. Meaning and values of control parameters.

Parameters	Value
GFL-MMC PLL control (k_{pll} , k_{pli})	(2, 10)
d-axis PI for GFL-MMC power control outer-loop (k_{p11} , k_{i11})	(1.09, 10.11)
q-axis PI for GFL-MMC power control outer-loop (k_{p12} , k_{i12})	(1.09, 10.11)
d-axis PI for GFM-MMC voltage control outer-loop (k_{p21} , k_{i21})	(0.48, 67.03)
q-axis PI for GFM-MMC voltage control outer-loop (k_{p22} , k_{i22})	(0.48, 67.03)
inertia and virtual damping coefficient of the GFM-MMC active power loop (J , D_p)	(3.03, 80)
droop coefficient and integral coefficient of the GFM-MMC reactive power loop (D_q , K)	(500, 0.07)
GFL-MMC circulating current suppression (k_{p1cird} , k_{i1cird})	(0.80, 106.98)
GFL-MMC circulating current suppression (k_{p1cirq} , k_{i1cirq})	(0.80, 106.98)
GFM-MMC circulating current suppression (k_{p2cird} , k_{i2cird})	(0.69, 55.32)
GFM-MMC circulating current suppression (k_{p2cirq} , k_{i2cirq})	(0.69, 55.32)
GFL-MMC d-axis PI for current inner-loop (k_{p13} , k_{i13})	(10.01, 60.33)
GFL-MMC q-axis PI for current inner-loop (k_{p14} , k_{i14})	(10.01, 60.33)
GFM-MMC d-axis PI for current inner-loop (k_{p23} , k_{i23})	(12.22, 95.88)
GFM-MMC q-axis PI for current inner-loop (k_{p24} , k_{i24})	(12.22, 95.88)

Table A3. Explanation of main participation factors.

ID	Meanings
34	GFL-MMC submodule second-harmonic voltage d-axis component
35	GFL-MMC submodule second-harmonic voltage q-axis component
38	GFL-MMC submodule third-harmonic voltage x-axis component
39	GFL-MMC submodule third-harmonic voltage y-axis component
53	GFM-MMC current loop control input signal q-axis component
62	GFM-MMC submodule third-harmonic voltage x-axis component
63	GFM-MMC submodule third-harmonic voltage y-axis component
66	GFM-MMC output current calculated value q-axis component
72	GFM-MMC output current measured value q-axis component

Appendix F

To validate the developed small-signal model under different operating conditions, an electromagnetic transient model of the embedded heterogeneous MMC-MTDC system was established in PSCAD/EMTDC using the system and control parameters listed in Tables A1 and A2.

In Case 1, a 0.5 p.u. step increase was applied to the active power reference of the sending-end AC system. The output active power of MMC1 and the RMS terminal voltage obtained from the small-signal model were compared with those obtained from the electromagnetic transient model.

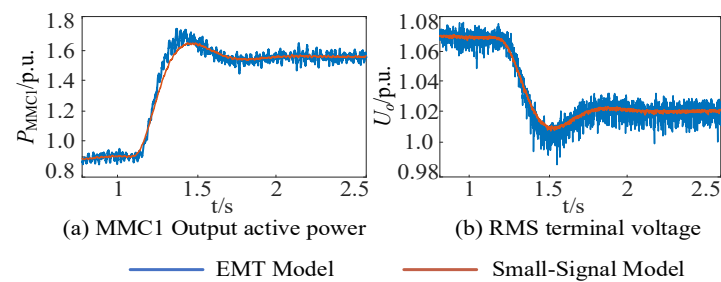


Figure A2. Comparison between the small-signal model and the electromagnetic transient model under Case 1.

As shown in Figure A2, the small-signal model closely matches the electromagnetic transient model in terms of transient trends, oscillation characteristics, damping behavior, and steady-state values. The high-frequency fluctuations in the electromagnetic transient results are mainly caused by converter switching dynamics, which are not represented in the averaged small-signal model.

In Case 2, a 0.2 p.u. step increase was applied to the active power reference of MMC2. The output active powers of MMC1 and MMC2 were selected to evaluate the dynamic power interactions between the GFL-MMC and GFM-MMC stations. The corresponding comparison results are shown in Figure A3.

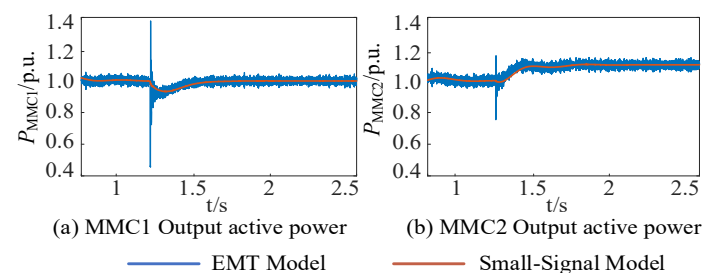


Figure A3. Comparison between the small-signal model and the electromagnetic transient model under Case 2.

As shown in Figure A3, the small-signal model closely matches the electromagnetic transient model and accurately captures the transient power interaction between the two MMC stations. After the active power reference variation, MMC2 reaches the new steady-state, while MMC1 returns to its initial operating point following a brief transient. Together with Figure A2, these results confirm that the developed model effectively represents both local and coupled system dynamics and is suitable for subsequent stability analysis and parameter optimization.

Appendix G

The PSO parameters listed in Table 3 were determined through preliminary tuning by considering the convergence speed, optimization stability, and final objective function value. The adopted parameter combination, $\omega = 0.7298$, $c_1 = 0.7481$, and $c_2 = 0.3842$, provides an appropriate balance between global exploration and local convergence for the considered 12-dimensional control parameter optimization problem.

To further evaluate the convergence behavior of the proposed optimization method, comparative simulations were conducted using the improved PSO, original PSO, and differential evolution (DE) algorithm. The same population size, maximum number of iterations, parameter bounds, and initial population were adopted for all algorithms to ensure a consistent comparison. Here, the improved PSO refers to the proposed PSO-based

optimization scheme incorporating the quadratic performance index, whereas the original PSO represents the conventional PSO benchmark.

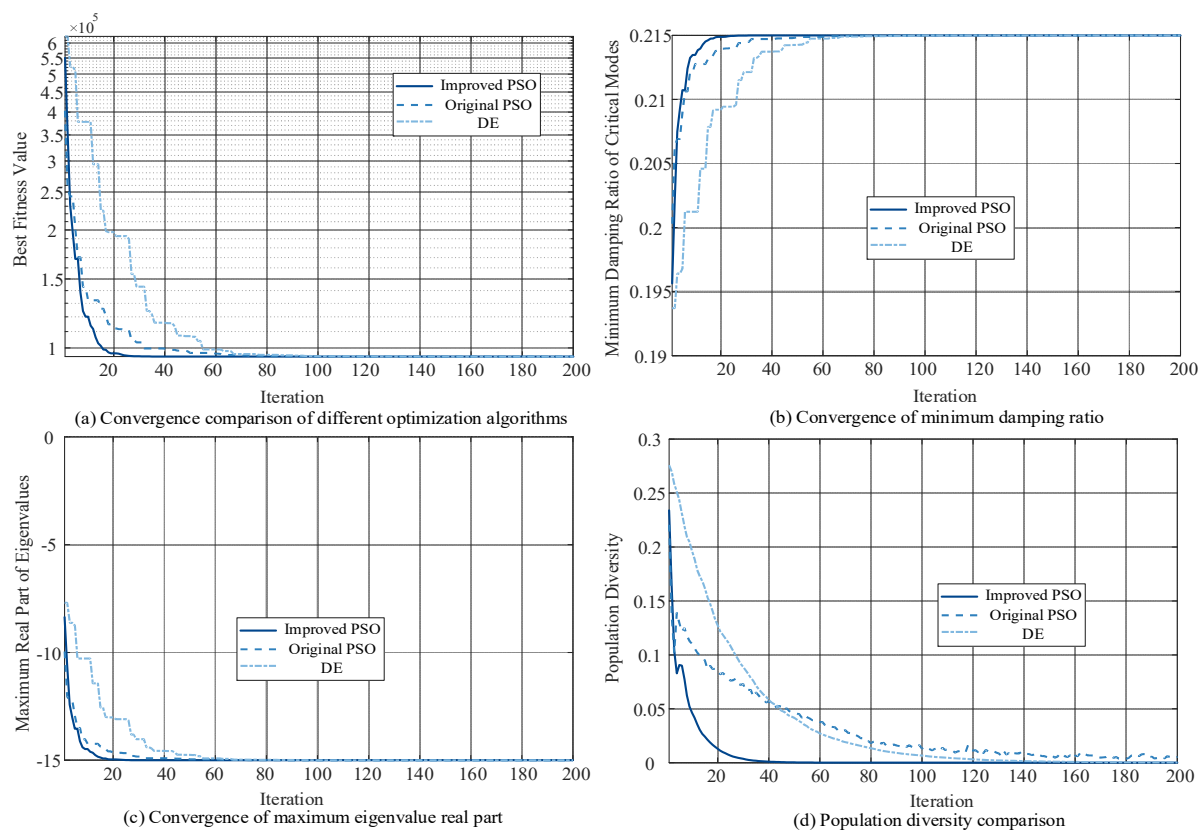


Figure A4. Comparative convergence characteristics of different heuristic optimization algorithms.

As shown in Figure A4a, the improved PSO rapidly reduces the objective function value and reaches a stable solution within approximately 30 iterations, whereas the original PSO and DE require more iterations to approach their final values. The modal indices in Figure A4b,c exhibit consistent convergence behavior. The minimum damping ratio gradually increases to approximately 0.215, while the maximum eigenvalue real part decreases to approximately -15 , indicating improved damping performance and an increased small-signal stability margin. Figure A4d shows that the population diversity of the improved PSO decreases more rapidly than those of the original PSO and DE, indicating faster concentration of the particles around the optimal region. Meanwhile, the objective function and modal indices remain stable after convergence, demonstrating that the reduction in population diversity does not lead to evident premature convergence.

Overall, the proposed optimization method exhibits fast and stable convergence and is suitable for the coordinated optimization of multiple control parameters in the heterogeneous MMC-MTDC system.

Appendix H

To intuitively verify the impact of control parameters on the system's small-signal stability, the root loci of dominant eigenvalues were analyzed by varying key control parameters. This paper selects k_{p21} , k_{i21} , k_{p22} , and k_{i22} for specific analysis.

Figure A5 presents the root loci of dominant eigenvalues with respect to the variations of key control parameters. As illustrated in Figure A5a or H1c, increasing the proportional gains k_{p21} and k_{p22} shifts the dominant oscillation modes $\lambda_{44,45}$ and $\lambda_{26,27}$ significantly towards the left half-plane, thereby enhancing the damping ratio. Similarly, Figure A5b

or Figure A5d demonstrate that increasing the integral gains k_{i21} and k_{i22} drives the poles $\lambda_{14,15}$ and $\lambda_{38,39}$ further away from the imaginary axis, confirming that the optimized parameters contribute to a larger stability margin and faster dynamic response.

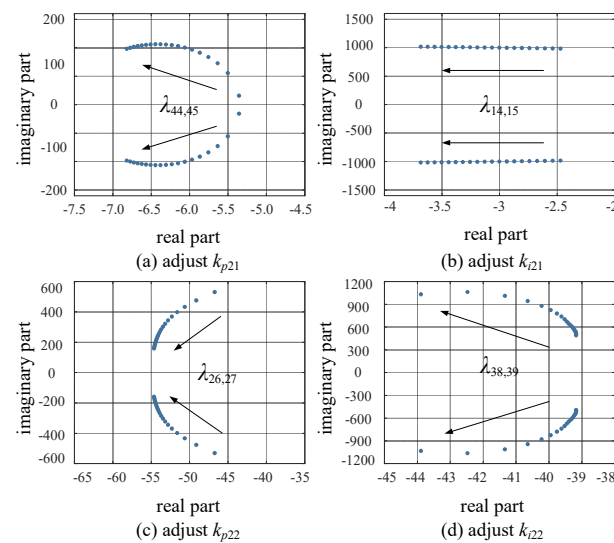


Figure A5. Root locus of the system under varying control parameters.

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