

Article

Modeling and Stability Analysis of a PV–Energy Storage AC/DC Integrated Three-Port Grid-Connected Power Electronic Device

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Abstract

Modeling and stability analysis of a PV–energy storage AC/DC integrated three-port grid-connected power electronic device is investigated in this paper for low-voltage single-phase renewable energy applications. The device consists of a PV Boost converter port, a battery-side bidirectional DC-DC converter port, and a single-phase full-bridge grid-connected inverter. To analyze the coupling-induced stability characteristics of this multi-converter system, mathematical models of the PV array, battery, and AC/DC integrated device are established. Considering the periodic time-varying nature introduced by the single-phase grid voltage, a phase-angle-based simplified discrete model is developed to transform the system into a discrete model evaluated at a fixed grid-voltage phase angle. Based on this model, eigenvalue sensitivity, eigenvalue trajectories, and bifurcation diagrams are used to identify the influence of key control parameters on system stability. The results show that excessive proportional gains in the inverter current loop and energy storage control loop reduce the stability margin and may lead to period-doubling bifurcation, Hopf bifurcation, or unstable grid current operation. The period-doubling and Hopf stability boundaries are identified at $k_{p4} \approx 1.40$ and $k_{p2} \approx 1.75$, respectively. In simulation, the grid-current THD increases from 2.07% to 3.60% as k_{p4} rises from 1.3 to 1.6 and from 2.07% to 2.27% as k_{p2} rises from 1.7 to 1.8. MATLAB/Simulink simulations and hardware-in-the-loop experiments further verify that the identified stability boundaries are consistent with the degradation of grid current quality and the increase in total harmonic distortion. The proposed modeling and analysis method provides a reference for parameter tuning and stable operation of single-phase PV–energy storage three-port grid-connected devices.

Keywords: PV–energy storage system; three-port grid-connected device; single-phase inverter; phase-angle-based simplified discrete model; stability analysis; bifurcation; eigenvalue trajectory



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1. Introduction

Against the background of carbon peaking and carbon neutrality goals and the low-carbon transition of the energy structure, the share of PV generation in new power systems continues to increase. Meanwhile, other emerging clean-energy technologies are also advancing. Fuel-cell electric vehicles, particularly fuel-cell buses, have received increasing attention as a sustainable transportation option, with recent studies investigating energy management strategies to improve energy efficiency, fuel-cell durability, and operational

robustness under varying passenger loads [1,2]. However, PV output is strongly affected by solar irradiance, ambient temperature, and weather conditions. Therefore, it exhibits intermittency and fluctuation. When PV generation is integrated on a large scale, it may lead to power fluctuations, DC-bus voltage deviations, and deterioration of grid current quality. Energy storage systems can absorb surplus energy when PV generation is excessive and release energy when PV output is insufficient or the load varies, thereby smoothing PV power fluctuations and improving system operational reliability. Therefore, the coordinated operation of PV generation and energy storage has become an important approach to enhancing renewable energy accommodation and improving grid-connected performance [3,4].

With the simultaneous development of DC loads, AC loads, and distributed energy resources, conventional AC-only or DC-only power supply architectures can hardly meet the access requirements of multiple types of ports. Hybrid AC/DC microgrids connect PV generation, energy storage, and DC loads through a DC bus, while enabling energy exchange with the AC grid or AC loads through an inverter. This architecture combines the advantages of high-efficiency transmission on the DC side and flexible access on the AC side [5]. On this basis, the PV–energy storage AC/DC integrated three-port grid-connected power electronic device combines PV generation, energy storage regulation, and single-phase AC grid connection into one unified power electronic interface. It boasts a compact structure, flexible power flow, and high levels of system integration.

Existing PV grid-connected systems mostly adopt a two-stage architecture consisting of a PV-side DC–DC converter and a downstream DC–AC inverter. Kjaer et al. reviewed single-phase grid-connected inverters for photovoltaic modules and pointed out that this architecture can realize maximum power point tracking and high-quality grid-connected current control, making it a widely used solution for low-power single-phase PV applications [6]. However, when an energy storage unit is introduced, an additional bidirectional DC–DC converter is usually required to enable battery charging and discharging, which increases the number of power conversion stages, circuit components, and control loops. To improve system integration, multi-port converters have attracted increasing attention because they can integrate PV generation, battery energy storage, and load/grid interfaces within a unified power electronic structure [7,8]. Nevertheless, most existing studies mainly focus on topology design, steady-state operation, or DC-side energy management, while the device-level modeling and stability analysis of PV–energy storage AC/DC integrated three-port grid-connected devices, especially under the coupling among the PV port, energy storage port, and single-phase AC grid-connected port, remain insufficiently investigated.

The PV–energy storage AC/DC integrated three-port device is inherently a complex system formed by the coupling of multiple power electronic converters through a common DC bus. Power electronic converters are characterized by high-frequency switching, strong nonlinearity, and multi-time-scale coupling. Different ports interact with each other through the DC bus, capacitive energy storage components, and control loops. Recent studies on converter-dominated systems have shown that the dynamic interactions between PV generation, grid-forming energy storage systems, and multiple grid-connected converters may significantly affect system stability and alter the stability boundary of multi-converter systems [9,10]. When each port operates independently, the corresponding subsystem may exhibit satisfactory dynamic performance. However, after these ports are integrated into a multi-port system, port coupling may lead to DC-bus voltage oscillations, grid current distortion, mutual interference among control loops, and even complex dynamic phenomena such as period-doubling bifurcation, Hopf bifurcation, and chaos [11,12]. Therefore, establishing a mathematical model that can reflect port coupling and switching dynamic characteristics is essential for stability analysis and control parameter design.

At present, several modeling methods have been widely used for the stability analysis of power electronic systems, including state-space averaging, impedance-based modeling, harmonic state-space modeling, and discrete-time mapping. The state-space averaging method can transform a switching converter into an equivalent continuous-time model and is therefore suitable for low-frequency dynamic analysis and controller design. However, the averaging operation filters out the switching-frequency dynamics and makes it difficult to capture fast-scale nonlinear phenomena, such as period-doubling bifurcation. Impedance-based modeling provides an intuitive frequency-domain tool for analyzing source-load interactions and grid-converter coupling. Nevertheless, this method is usually based on small-signal linearization around an operating point and is mainly effective for identifying oscillations caused by impedance mismatch, while its ability to describe switching-induced nonlinear instability is limited. Harmonic state-space modeling can describe periodic time-varying systems by considering harmonic coupling, but it usually leads to high-dimensional models and a relatively high computational burden. In contrast, accurate discrete-time mapping can preserve switching actions and is well suited for bifurcation and nonlinear stability analysis. However, when it is applied to multi-converter AC/DC integrated systems, a large number of switching modes and mode sequences must be enumerated, which significantly increases the modeling complexity and computational cost.

To overcome these limitations, a phase-angle-based simplified discrete modeling method is adopted in this paper. The proposed method inherits the ability of discrete-time mapping to retain key switching-sampling information while reducing the number of operating modes through switching-sequence reconstruction and mode combination. More importantly, for the single-phase grid-connected AC/DC integrated system considered in this study, the AC-side variables are periodic and time-varying within the grid period. By introducing phase-angle analysis, the original time-varying system can be equivalently transformed into a time-invariant discrete model evaluated at a specified grid-voltage phase angle. As a result, the proposed model not only avoids the excessive complexity of accurate discrete mapping but also enables eigenvalue-based stability analysis and bifurcation identification under the combined effects of switching actions, phase-angle variations, and port coupling. Therefore, compared with averaged models, impedance models, harmonic state-space models, and conventional discrete mappings, the proposed method provides a more suitable modeling framework for identifying the stability boundary of single-phase PV-energy storage AC/DC integrated three-port grid-connected devices.

In terms of stability analysis, impedance-based stability criteria, small-signal analysis, Lyapunov-based analysis, robust stability analysis, and nonlinear bifurcation analysis have been widely used for converter-dominated systems. Dragičević et al. pointed out that controlled converters may exhibit negative-impedance characteristics, thereby reducing system damping and causing oscillatory instability [13]. Ding et al. found that low-frequency oscillations and complex instability phenomena such as border-collision events may still occur even when the system satisfies conventional impedance-based stability criteria [14]. Zhao et al. further indicated that the coupling between phase-locked loop parameters and grid impedance may lead to loss of synchronization or even instability of grid-connected converters [15]. These studies show that the stability of PV-energy storage grid-connected devices depends not only on the control parameters of individual converters but also on the dynamic coupling among the DC bus, the energy storage port, the grid-connected inverter, and grid conditions.

To further clarify the differences among these modeling approaches, Table 1 provides a comparative summary of the four representative methods from three perspectives: applicability, accuracy, and simplicity. In Table 1, the symbol “✓” denotes that the method

possesses the corresponding capability or satisfies the specified criterion; the symbol “X” denotes that the method does not possess the corresponding capability or does not satisfy the specified criterion; and the symbol “ Δ ” denotes that the method is applicable under certain conditions but requires additional processing measures or has notable limitations. For example, state-space averaging and impedance-based modeling cannot be directly applied to AC systems and require prior transformation into the dq coordinate frame. This comparison highlights the trade-offs among computational efficiency, switching-dynamic preservation, and applicability to AC time-varying multi-converter systems, thereby further motivating the phase-angle-based simplified discrete modeling method adopted in this paper.

Table 1. Comparison of modeling methods.

Aspect	Specific Criterion	State-Space Averaging	Impedance-Based Modeling	Discrete-Time Mapping	Phase-Angle-Based Simplified Discrete Modeling
Applicability	Same switching frequency	✓	✓	✓	✓
	Different switching frequencies	X	X	X	✓
	AC time-varying systems	Δ	Δ	Δ	✓
Accuracy	Slow-scale instability	✓	✓	✓	✓
	Fast-scale instability	X	X	X	✓
Simplicity	Solution form	Linear continuous	Linear frequency-domain	Nonlinear discrete	Nonlinear discrete
	Computational efficiency	High	High	Low	Medium

As summarized in Table 1, these conventional modeling approaches each have their own limitations: state-space averaging and impedance-based methods sacrifice high-frequency dynamic information for computational efficiency, while exact discrete-time mapping preserves accuracy but suffers from prohibitive modeling complexity. Motivated by the above issues, this study focuses on the modeling and stability analysis of a PV–energy storage AC/DC integrated three-port grid-connected power electronic device for low-voltage single-phase renewable energy applications. The device consists of a PV Boost port, a battery-side bidirectional DC–DC port, and a single-phase full-bridge grid-connected inverter port. To clarify the originality of this work, the main contributions of this paper are summarized as follows:

(1) A PV–energy storage AC/DC integrated three-port grid-connected power electronic device is constructed for low-voltage single-phase applications. Compared with conventional two-stage PV grid-connected systems or DC-side multi-port converters, the proposed device integrates PV generation, battery energy storage regulation, and single-phase AC grid connection into a unified power electronic interface.

(2) A unified mathematical model of the PV array, battery, and AC/DC integrated three-port device is established. The model describes the coupling relationships among the PV port, the energy storage port, and the grid-connected inverter port through the common DC bus, thereby providing a model basis for analyzing coupling-induced stability problems in the multi-converter system.

(3) To overcome the difficulty of simultaneously ensuring modeling accuracy and computational simplicity in existing modeling methods, a phase-angle-based simplified discrete modeling method is innovatively proposed. The proposed method is applicable

to AC time-varying systems with different switching frequencies. By introducing phase-angle analysis, switching-sequence reconstruction, and mode combination, the method transforms the periodic time-varying characteristics caused by the single-phase grid voltage into a discrete model evaluated at a specified grid-voltage phase angle. Compared with averaged and impedance-based models, it retains key switching-sampling information; compared with accurate discrete-time mapping, it significantly reduces the number of operating modes and the modeling complexity.

(4) A systematic stability analysis and validation framework is established by combining eigenvalue sensitivity, eigenvalue trajectories, bifurcation diagrams, MATLAB/Simulink R2024b simulations, and hardware-in-the-loop experiments. The simulation and HIL results verify the correctness of the stability boundaries and instability mechanisms predicted by the proposed model. On this basis, the influence of key control parameters on the stability boundary and grid current quality is further identified, providing guidance for controller parameter tuning and stable operation of single-phase PV–energy storage three-port grid-connected devices.

2. Structure, Operating Principle, and Applications

2.1. Topology

The core objective of the PV–energy storage AC/DC integrated three-port grid-connected power electronic device is to integrate PV generation, energy storage regulation, and AC grid connection within a unified power electronic interface. Compared with conventional PV grid-connected systems, this structure must not only convert the DC power generated on the PV side into AC power delivered to the grid but also provide power buffering and DC-bus voltage support through the energy storage port. Therefore, the three-port grid-connected device constructed in this study uses the common DC bus as the energy coupling node, through which the PV port, the energy storage port, and the AC grid-connected port are interconnected. In this way, an AC/DC integrated topology for coordinated PV–energy storage–grid operation is formed, as shown in Figure 1.

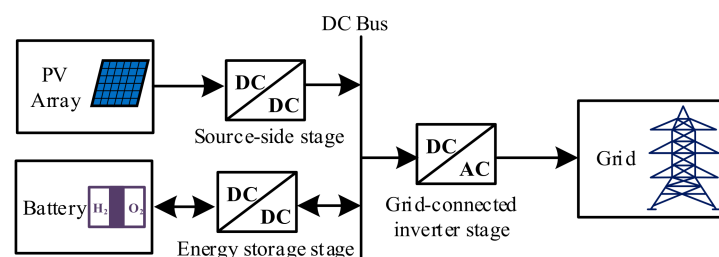


Figure 1. Structure of the PV–battery three-port grid-connected system.

Figure 2 shows the main circuit structure of the PV–energy storage AC/DC integrated three-port grid-connected power electronic device. The device mainly consists of the source-side conversion stage, the energy storage conversion stage, the DC-bus capacitor, and the inverter stage. The source-side conversion stage is connected to the PV array and is used for PV power conversion and maximum power point tracking (MPPT). The energy storage conversion stage is connected to the battery and enables bidirectional energy exchange between the energy storage system and the DC bus. The inverter stage, located on the right side, is connected to the single-phase AC grid through a filter inductor, thereby realizing energy conversion between the DC bus and the AC grid. Accordingly, the PV port, the energy storage port, and the AC grid-connected port together constitute the three-port grid-connected power electronic device.

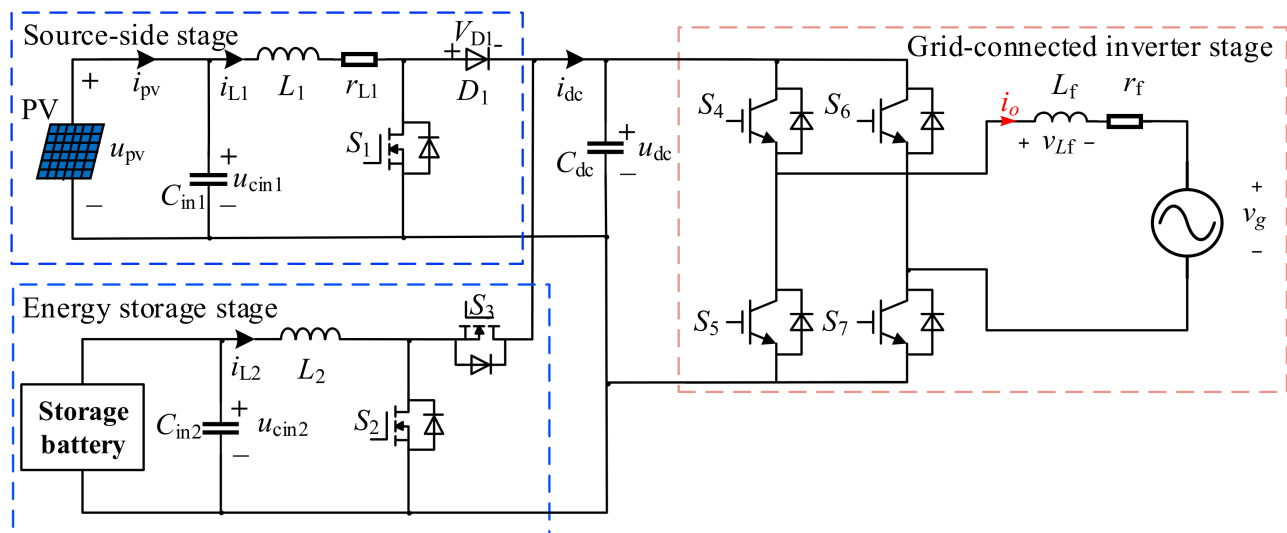


Figure 2. Main circuit of the AC/DC integrated device.

From the perspective of topology, the source-side stage adopts a Boost converter structure. The output voltage and output current of the PV array are denoted as u_{pv} and i_{pv} , respectively. An input-side capacitor, C_{in1} , is connected in parallel with the PV array, and its capacitor voltage is denoted as U_{cin1} , which is used to suppress voltage fluctuations on the PV side. The inductor L_1 , its parasitic resistance r_{L1} , the switch S_1 , and the diode D_1 jointly form the Boost conversion circuit, and the inductor current is denoted as i_{L1} . When the switch S_1 is turned on, the PV array transfers energy to L_1 . When S_1 is turned off, the energy stored in the inductor is released to the DC bus through D_1 , thereby realizing voltage step-up conversion from the low-voltage DC output on the PV side to the higher-voltage DC level on the bus side. The main function of this port is to regulate the duty ratio of S_1 so that the PV array operates near the maximum power point while injecting energy into the DC bus.

The energy storage conversion stage is a battery-side bidirectional DC-DC converter, which consists of the battery, the input filter capacitor C_{in2} , the inductor L_2 , and the switches S_2 and S_3 . The battery terminal voltage is filtered by C_{in2} before being connected to the bidirectional converter. The capacitor voltage and inductor current are denoted as U_{cin2} and i_{L2} , respectively. This port can release energy from the battery to the DC bus when PV output is insufficient or when the grid-connected power demand is high. It can also absorb energy from the DC bus and charge the battery when PV output is excessive or when the DC-bus voltage is relatively high. Therefore, the energy storage conversion stage is an important part for maintaining the power balance of the DC bus and improving the dynamic performance of the system.

The outputs of the source-side conversion stage and the energy storage conversion stage are jointly connected to the DC bus. A DC-bus capacitor, C_{dc} , is installed on the bus side, and its voltage is denoted as U_{dc} . The DC bus serves as the energy collection and distribution center of the three-port device. On the one hand, the PV port and the energy storage port are coupled through the DC bus. On the other hand, the DC bus provides a stable DC input for the downstream inverter stage. Therefore, U_{dc} is a key variable reflecting the power balance of the system. When the PV input power, the battery charging/discharging power, and the grid-connected output power are unbalanced, the DC-bus voltage fluctuates and must be suppressed through coordinated control of the energy storage port and the AC grid-connected port.

The inverter stage adopts a single-phase full-bridge inverter structure, which consists of the switches S_4 , S_5 , S_6 , and S_7 . The DC side of the inverter is connected to the common

DC bus, while the AC side is connected to the single-phase grid through the filter inductor L_f and its equivalent resistance r_f . The grid voltage and grid current are denoted as v_g and i_o , respectively. Through complementary modulation of the four power switches, the single-phase full-bridge inverter converts the DC-bus voltage into an AC current synchronized with the grid, thereby delivering PV and battery energy to the AC grid. The filter inductor is used to suppress the high-frequency current ripple generated by inverter switching actions and to improve the quality of the grid current.

2.2. Operating Principle

As a core carrier for the deep AC/DC integration of new power systems, a typical AC/DC integrated device incorporates a PV access unit, a battery energy storage unit, a Boost conversion unit, a bidirectional DC-DC conversion unit, and a grid-connected inverter unit. It performs multiple functions, including renewable energy access, battery charging/discharging regulation, bidirectional AC/DC power exchange, and grid-interfacing operation. Therefore, it is selected in this study as the representative topology of a typical integrated device, as shown in Figure 2. The corresponding control block diagram is shown in Figure 3.

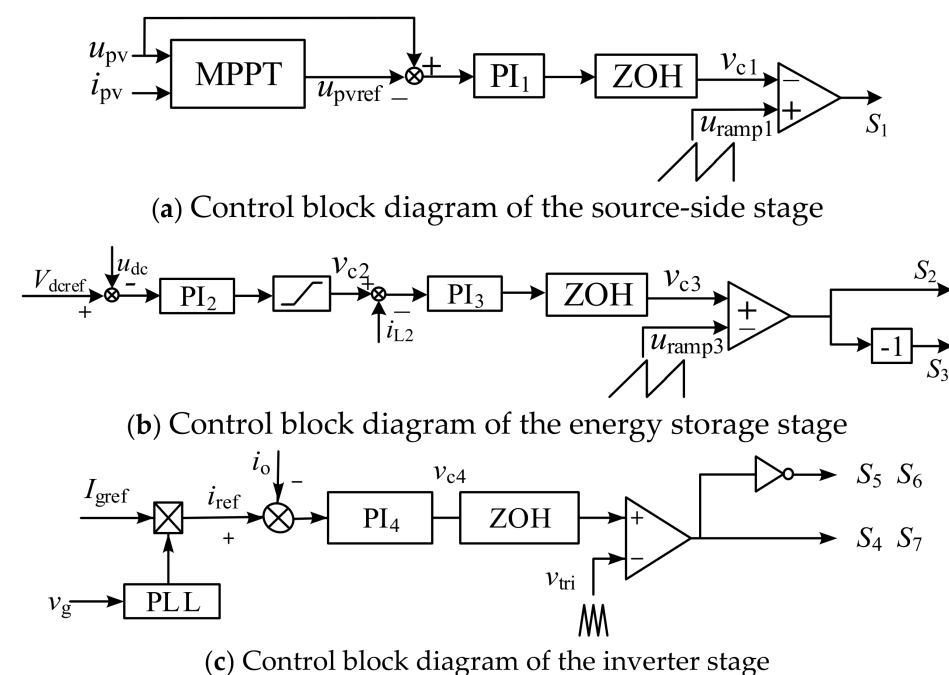


Figure 3. Control block diagram of the AC/DC integrated device.

In Figure 3, the ZOH is a zero-order holder, which maintains the input signal unchanged within one switching period. The switching signal S_1 is generated by the control circuit of the Boost converter. This control circuit adopts a dual-loop control strategy consisting of an outer-loop maximum power point tracking (MPPT) controller and an inner-loop voltage PI controller. Its core objective is to regulate the output voltage of the PV array so that it tracks the reference voltage, thereby ensuring high PV conversion efficiency, as shown in Figure 3a. v_{ci} denotes the output of the i -th PI controller.

Figure 3b shows the control block diagram of the energy storage stage. Figure 3c shows the current closed-loop control block diagram of the inverter stage. Its core objective is to achieve synchronized tracking between the grid current and the grid voltage, thereby ensuring the power quality of grid-connected operation. The phase information is extracted from the grid voltage v_g through the phase-locked loop (PLL). It is then multiplied by the grid current amplitude reference I_{gref} to generate the current reference signal i_{ref} .

which is in phase with the grid voltage. The difference between this reference signal and the actual grid current i_o is sent to a PI controller for proportional-integral regulation, producing the control voltage v_{c4} . After being held constant within one switching period by the zero-order holder, the control voltage is compared with the triangular carrier v_{tri} to generate PWM switching signals. These signals directly drive the switches S_4 and S_7 and drive S_5 and S_6 after inversion. In this way, accurate regulation of the grid current is achieved, enabling it to rapidly track the reference current and satisfy the requirements for grid-connected operation.

2.3. Application Scenarios

The PV–energy storage AC/DC integrated three-port grid-connected device investigated in this study is primarily intended for low-voltage single-phase distributed energy applications. Representative installations include user-side PV–energy storage systems in residential buildings, small commercial facilities, and communication base stations. Although these installations may differ in load profiles and power ratings, they share a common system configuration: a PV source and a battery are connected to a common DC bus through individual DC–DC converters, while a single-phase full-bridge inverter transfers power between the DC bus and the low-voltage AC grid.

In this application scenario, the PV port operates near the maximum power point and injects renewable energy into the DC bus through the Boost converter. The battery port compensates for the instantaneous power mismatch between the PV source and the grid-connected port through bidirectional charging and discharging. When the available PV power exceeds the required grid-connected power, the surplus energy can be absorbed by the battery. Conversely, when the PV output decreases or the grid-connected power demand increases, the battery releases energy to support the DC bus. The grid-connected inverter regulates the single-phase grid current and transfers the combined power supplied by the PV and battery ports to the AC grid.

Although the three ports perform different functions, they are dynamically coupled through the common DC bus and their associated control loops. Variations in PV-side power, battery-side control variables, or inverter control parameters may disturb the DC-bus power balance and subsequently affect the grid current. Meanwhile, because the grid-connected stage adopts a single-phase inverter, the grid voltage, current reference, modulation signal, and switching states vary periodically with the grid-voltage phase angle. The combined effects of converter switching, periodic AC-side variables, and multi-port control interactions may therefore produce DC-bus oscillations, grid-current distortion, and even nonlinear instability.

Consequently, the stability of the proposed device cannot be evaluated solely from the independent dynamic characteristics of each conversion stage. A unified model that incorporates the coupling among the PV port, battery port, DC bus, and single-phase grid-connected inverter is required. Based on this application context, the following sections establish the mathematical model of the low-voltage single-phase three-port system and further develop a phase-angle-based simplified discrete model for analyzing its coupling-induced stability characteristics.

3. Mathematical Description of the System

3.1. Mathematical Model of the PV Array

The output characteristic curves of the PV array shown in Figure 4 illustrate the relationships between the output voltage and current (I–V) and between the output voltage and power (P–V). Here, V_{oc} denotes the open-circuit voltage, i.e., the terminal voltage when the output current is zero; I_{sc} denotes the short-circuit current, i.e., the output current

when the output voltage is zero. The most important operating point is the maximum power point, at which the corresponding voltage, current, and power are denoted as V_M , I_M , and P_M , respectively. These characteristics are not fixed but are significantly affected by external environmental factors such as solar irradiance and ambient temperature. An increase in irradiance mainly increases I_{SC} and the output power, whereas an increase in temperature leads to a significant decrease in V_{OC} , thereby causing a dynamic shift in the maximum power point. Considering that environmental factors vary relatively slowly, and to simplify system modeling, this study does not adopt a complex physical model. Instead, a computationally efficient simplified equivalent model is used to model the PV array [16]:

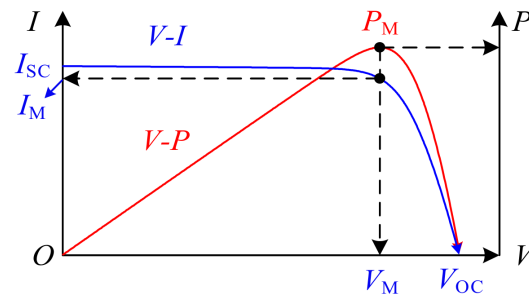


Figure 4. Output characteristic curves of the PV array.

$$i_{pv} = I_{SC} \left[1 - P_1 \left(e^{\frac{u_{pv}}{P_2 V_{OC}}} - 1 \right) \right] = f(u_{pv}) \quad (1)$$

where

$$P_1 = \left(1 - \frac{I_M}{I_{SC}} \right) e^{-\frac{V_M}{A_2 V_{OC}}} \quad (2)$$

$$P_2 = \left(\frac{V_M}{V_{OC}} - 1 \right) \left[\ln \left(1 - \frac{I_M}{I_{SC}} \right) \right]^{-1} \quad (3)$$

The relationship between the output current of the PV array and its terminal voltage exhibits pronounced nonlinearity, as shown in Equation (1). If this nonlinear relationship is modeled directly and accurately in the system model, high-order nonlinear terms will be introduced, making the mathematical representation of the overall model highly complex and causing considerable analytical and computational difficulties for subsequent stability analysis and performance evaluation. However, in practical PV generation systems, the operating point is continuously regulated by the real-time maximum power point tracking (MPPT) strategy and is maintained at the maximum power point or within a small neighborhood around it.

Therefore, the nonlinear function $i_{pv} = f(u_{pv})$ can be small-signal linearized at the maximum power point. It should be noted that this linearization is justified by the fact that the MPPT controller maintains the operating point within a small neighborhood of the MPP. Irradiance variations occur on a much slower time scale (seconds to minutes) than the switching dynamics analyzed in this study (microseconds to milliseconds), so the PV operating point can be treated as quasi-static in the system-level stability analysis. Adopting the full nonlinear PV model would introduce high-order nonlinear terms that substantially complicate the derivation of the discrete mapping and eigenvalue analysis. Thus, the linearization at the MPP represents a reasonable engineering simplification. Specifically, by evaluating its derivative at this operating point, a linear approximate relationship can be obtained as follows [17,18]:

$$i_{pv} = k_0 u_{pv} + b_0 \quad (4)$$

where

$$k_0 = \left. \frac{df}{du_{pv}} \right|_{u_{pv} = V_M} \quad (5)$$

$$b_0 = I_M - k_0 V_M \quad (6)$$

3.2. Mathematical Model of the Battery

In PV–energy storage microgrid systems, energy storage devices play a key role in balancing power fluctuations and improving system reliability, and their technical routes are highly diversified. Among various energy storage technologies, chemical energy storage, typically represented by lithium-ion batteries, has been most widely adopted in engineering applications owing to its combined advantages of flexible deployment, fast response, high energy density, and controllable life-cycle cost. Therefore, the battery is selected in this chapter as the specific research object for the energy storage stage.

The basic idea of an equivalent circuit model for a battery is to abstract the complex internal electrochemical processes into a network topology composed of fundamental electrical elements, such as ideal voltage sources, resistors, and capacitors. In this way, the external terminal-voltage characteristics of the battery can be accurately represented. Among various equivalent circuit models, the Rint model has the simplest topology. It consists only of an ideal voltage source representing the open-circuit voltage in series with an internal resistance. However, its limitation is that it cannot characterize the dynamic response of the battery. The first-order RC model, also known as the Thevenin model, is developed on the basis of the Rint model by adding an RC parallel polarization branch. It is therefore capable of simulating the voltage transient response and relaxation behavior during charging and discharging processes, making it a widely used basic battery model in engineering applications. The second-order RC model further introduces two RC branches with different time constants, corresponding to electrochemical polarization and concentration polarization, respectively. As a result, it can more accurately fit the dynamic voltage characteristics of the battery. Among higher-order models, the PNGV model adds a capacitor associated with current integration to the structure of the first-order RC model, which is used to represent the variation in the open-circuit voltage with the accumulated charge and discharge capacity [19]. In general, the complexity of these models is positively correlated with their modeling accuracy.

For system-level stability analysis, the internal electrochemical dynamics of the battery, whose time constants typically range from seconds to tens of seconds, vary much more slowly than the switching dynamics of power electronic converters, which occur on the microsecond-to-millisecond time scale. In addition, this study focuses on the influence of interactions among power electronic converters on system stability, rather than on the accurate estimation of internal battery states. Therefore, to reduce computational burden while preserving the essential dynamic characteristics and capturing the battery dynamic response to a certain extent, the Rint model is adopted to model the battery in this study. The corresponding equivalent circuit is shown in Figure 5. It is worth noting that the Rint model has been widely employed in system-level stability analysis of power electronic converters. In our previous works [18], the Rint model was also adopted for battery modeling, and its effectiveness in predicting system bifurcation boundaries was successfully validated through both simulations and hardware-in-the-loop experiments. Furthermore, quantitative studies on battery time constants [20] indicate that the polarization time constants of lithium-ion batteries range from seconds to tens of seconds under typical operating conditions, which are significantly larger than the switching period of the converters, further supporting the rationality of adopting the Rint model for system-level stability analysis in this work.

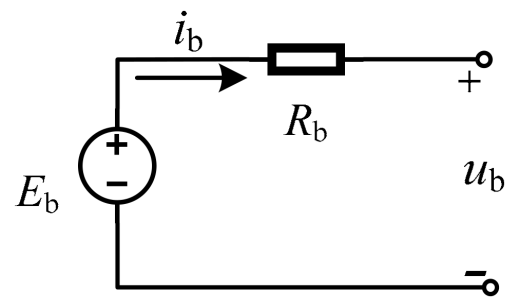


Figure 5. Equivalent circuit of the Rint model.

3.3. Mathematical Model of the AC/DC Integrated Device

The hybrid AC/DC PV–energy storage system mainly has two operating states. In operating state 1, when the grid power demand increases and the battery has sufficient remaining capacity, the PV array is configured as the main power supply unit, while the battery acts as a supplementary support unit. The two units collaboratively deliver electric power to the grid. In operating state 2, when the load demand is low and the PV output power is much higher than the grid power demand, the PV array can simultaneously supply power to the grid and charge the battery. This study takes operating state 1 as an example for analysis, and the model for operating state 2 can be derived by analogy.

Under operating state 1, all inductors are assumed to operate in continuous conduction mode. In addition, diode D_1 and switch S_1 operate complementarily, and switches S_2 and S_3 also operate complementarily. Table 2 shows the operating modes of the system under operating state 1, where s_i ($i = 1, 2, 4$) denotes the switching function of switch S_i . When $s_i = 1$, switch S_i is turned on; when $s_i = 0$, switch S_i is turned off.

Table 2. The operating modes.

Modes	Switch State (s_1, s_2, s_4)
M_1	(0, 0, 0)
M_2	(0, 0, 1)
M_3	(0, 1, 0)
M_4	(0, 1, 1)
M_5	(1, 0, 0)
M_6	(1, 0, 1)
M_7	(1, 1, 0)
M_8	(1, 1, 1)

The six switching-state diagrams of the three conversion stages are shown in Figure 6. Figure 6a,b illustrate the two switching states of the source-side Boost converter; Figure 6c,d illustrate the two switching states of the battery-side bidirectional DC-DC converter; and Figure 6e,f illustrate the two switching states of the single-phase full-bridge inverter. The eight combined operating modes listed in Table 2 are obtained from the possible combinations of these stage-level switching states, as specified by the switching functions s_1 , s_2 , and s_4 .

Based on the switching states shown in Figure 6, each operating mode M_j in Table 2 corresponds to a specific combination of conducting branches in the three conversion stages. After approximating the PV array and the battery using the equivalent models described in Sections 3.1 and 3.2, the state variables of the main circuit are selected as $y = [i_{L1}, i_{L2}, u_{cin1}, u_{cin2}, u_{dc}, i_o]^T$. Applying Kirchhoff's voltage and current laws to the six elementary switching states and representing their combinations by s_1 , s_2 , and s_4 yields the unified piecewise state equations in Equation (7):

$$\left\{ \begin{array}{l} \frac{di_{L1}}{dt} = \frac{u_{cin1}}{L_1} - \frac{i_{L1}r_{L1}}{L_1} - \frac{(1-s_1)u_{dc}}{L_1} \\ \frac{du_{cin1}}{dt} = \frac{k_0 u_{cin1}}{C_{cin1}} + \frac{b_0}{C_{cin1}} - \frac{i_{L1}}{C_{cin1}} \\ \frac{di_{L2}}{dt} = \frac{u_{cin2}}{L_2} - \frac{(1-s_2)u_{dc}}{L_2} \\ \frac{du_{cin2}}{dt} = \frac{E_b - u_{cin2}}{r_b C_{cin2}} - \frac{i_{L2}}{C_{cin2}} \\ \frac{du_{dc}}{dt} = \frac{(1-s_1)i_{L1}}{C_{dc}} + \frac{(1-s_2)i_{L2}}{C_{dc}} - \frac{2s_4-1}{C_{dc}}i_o \\ \frac{di_o}{dt} = \frac{2s_4-1}{L_f}u_{dc} - \frac{1}{L_f}V_g \sin \omega_c t - \frac{i_o r_f}{L_f} \end{array} \right. \quad (7)$$

The state variables of the control circuit are selected as $z = [v_{c1}, v_{c2}, v_{c3}, v_{c4}]^T$. The state equations of the control circuit are expressed as follows:

$$\left\{ \begin{array}{l} \frac{dv_{c1}}{dt} = \frac{k_{p1}}{C_{in}} f(u_{pv}) + k_{i1} u_{pv} - \frac{k_{p1}}{C_{in}} i_{L1} - k_{i1} u_{pvref} \\ \frac{dv_{c2}}{dt} = -\frac{(1-s_1)k_{p2}}{C_{dc}} i_{L1} - \frac{(1-s_2)k_{p2}}{C_{dc}} i_{L2} - \frac{(2s_4-1)k_{p2}}{C_{dc}} i_o - k_{i2} u_{dc} + k_{i2} V_{dcref} \\ \frac{dv_{c3}}{dt} = -\frac{(1-s_1)k_{p3}}{C_{dc}} i_{L1} - \frac{(1-s_2)k_{p3}}{C_{dc}} i_{L2} - \frac{(2s_4-1)k_{p3}}{C_{dc}} i_o - k_{i2} k_{p3} u_{dc} + k_{i2} k_{p3} V_{dcref} \\ \quad - \frac{k_{p3} u_{cin2}}{L_2} + \frac{k_{p3} (1-s_2) u_{dc}}{L_2} + k_{i3} v_{c2} - k_{i3} i_{L2} \\ \frac{dv_{c4}}{dt} = k_{p4} I_{gref} \omega_c \cos \omega_c t - k_{p4} \frac{2s_4-1}{L_f} u_{dc} + \frac{k_{p4}}{L_f} V_g \sin \omega_c t + k_{i4} I_{gref} \sin \omega_c t - k_{i4} i_o \end{array} \right. \quad (8)$$

where k_{pi} and k_{ii} are the proportional and integral gains of the PI controller in Figure 3, respectively.

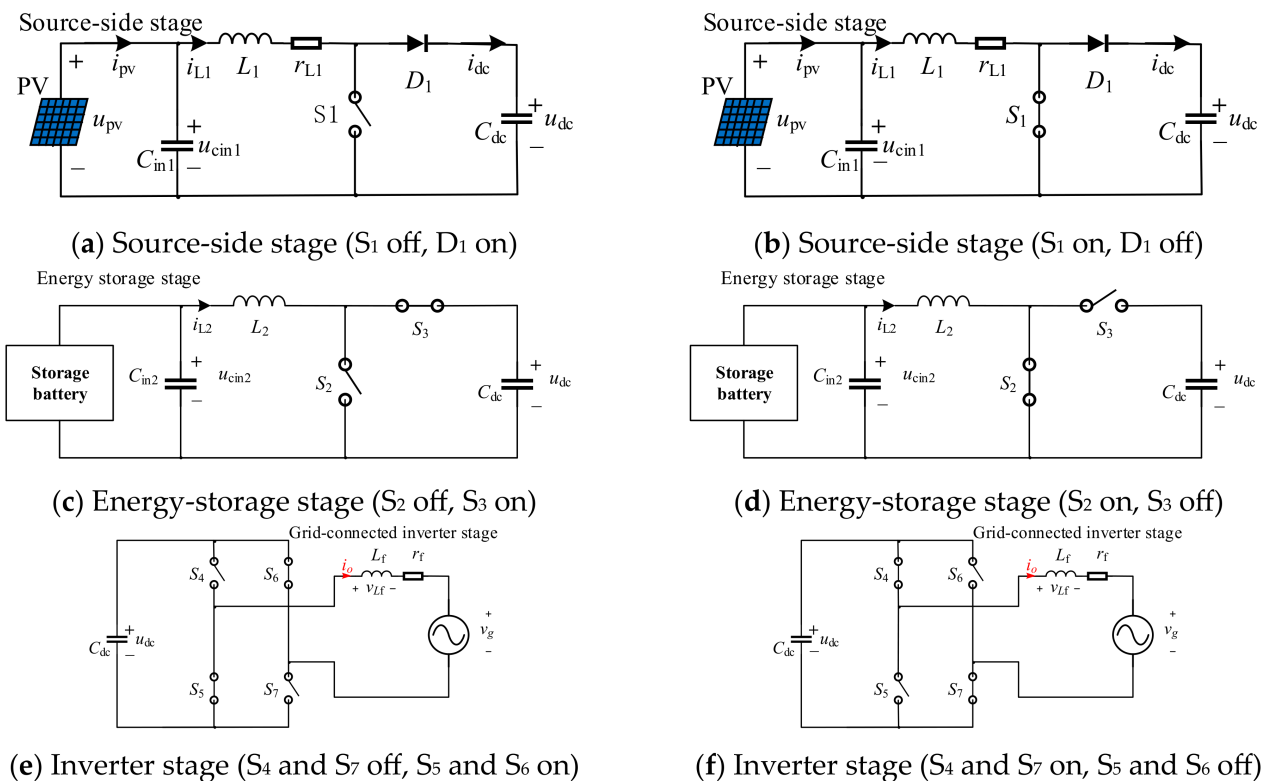


Figure 6. Switching states of the three conversion stages under operating state 1.

The system state variables are selected as $\mathbf{x} = [i_{L1}, i_{L2}, u_{cin1}, u_{cin2}, u_{dc}, v_{c1}, v_{c2}, v_{c3}, i_o, v_{c4}]^T$. By combining Equations (7) and (8), the state equations of the system can be obtained.

$$\dot{\mathbf{x}} = A_j(\sin \omega_c t) \mathbf{x} + B_j(\sin \omega_c t) \quad (j = 1, 2, 3, 4, 5, 6, 7, 8) \quad (9)$$

where A_j and B_j denote the state matrix and input matrix under the j -th operating mode M_j , respectively.

4. System Discrete Modeling Based on Phase-Angle Equivalence

For the hybrid AC/DC PV–energy storage microgrid system, the grid voltage $v_g = V_g \sin \omega_c t$ is included in the main circuit and control circuit. As a result, the mathematical model of the system exhibits the characteristics of time-varying piecewise differential equations. To address this issue, a phase-angle-based simplified discrete modeling method is adopted in this section.

4.1. Phase-Angle Analysis

First, the dynamic characteristics of the grid current i_o are analyzed with emphasis on the period-one state, period-doubling bifurcation, and chaos. This analysis provides a basis for the subsequent phase-angle analysis. The system parameters used in the subsequent analysis are listed in Table 3. These parameters represent a scaled low-voltage single-phase user-side PV–energy storage application, such as a residential or small commercial PV–battery grid-connected system. The 55 V RMS grid voltage, 140 V DC-bus voltage reference, 46 V PV-side voltage reference, and 24 V battery voltage are selected to reproduce the power-conversion relationships and dynamic interactions among the PV port, battery port, and single-phase grid-connected inverter under safe laboratory and hardware-in-the-loop validation conditions.

Table 3. Parameters of the system.

Parameter	Source-Side Stage	Energy Storage Stage	Grid-Connected Inverter Stage
Capacitance/ μF	$C_{in1} = 10$	$C_{in2} = 20$	$C_{dc} = 600$
Inductance/mH	$L_1 = 5$	$L_2 = 3$	$L_f = 8$
Inductor parasitic resistance/ Ω	$r_{L1} = 0.8$		$r_f = 1$
Proportional gain	$k_{p1} = 0.001$	$k_{p2} = 1.6,$ $k_{p3} = 0.5$	$k_{p4} = 1$
Integral gain	$k_{i1} = 40$	$k_{i2} = 10,$ $k_{i3} = 0.02$	$k_{i4} = 3$
Reference current/A		$I_{gref} = 6$	
Reference voltage/V		$V_{pvref} = 46, V_{dcref} = 140$	
Switching frequency/kHz		$f_s = 20$	
RMS grid voltage/V		$V_{grms} = 55 \text{ V}$	
Grid frequency/Hz		$f_c = 50 \text{ Hz}$	
PV module parameters		$V_{OC} = 56 \text{ V}, I_{SC} = 3.2 \text{ A}$	
Battery voltage/V		$V_M = 46 \text{ V}, I_M = 2.85 \text{ A}$ $E_{oc} = 24$	

Based on the accurate model of the system, Equation (7), the time-domain waveforms of the grid current i_o are obtained, as shown in Figure 7. The integral time constant of the PI4 controller in the grid-connected inverter is kept unchanged, and the proportional gain k_{p4} is selected as the bifurcation parameter. As shown in Figure 7a, when $k_{p4} = 1.3$, the system operates stably in the period-one state within the grid period T_c . When $k_{p4} > 1.4$, period-doubling bifurcation occurs over the entire period, as shown in Figure 7b. As k_{p4} further increases, the system evolves from the period-two state to the period-four state and eventually enters chaotic operation over the entire period, as shown in Figure 7c,d respectively.

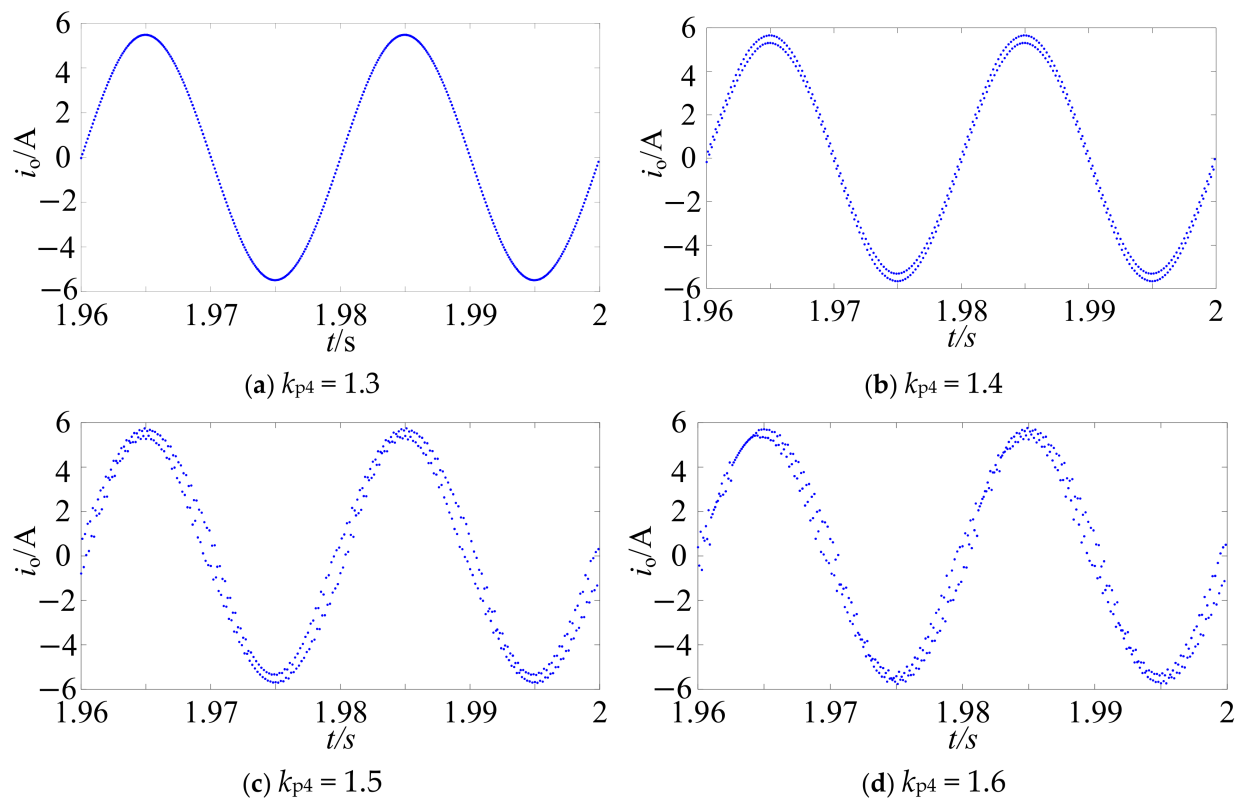
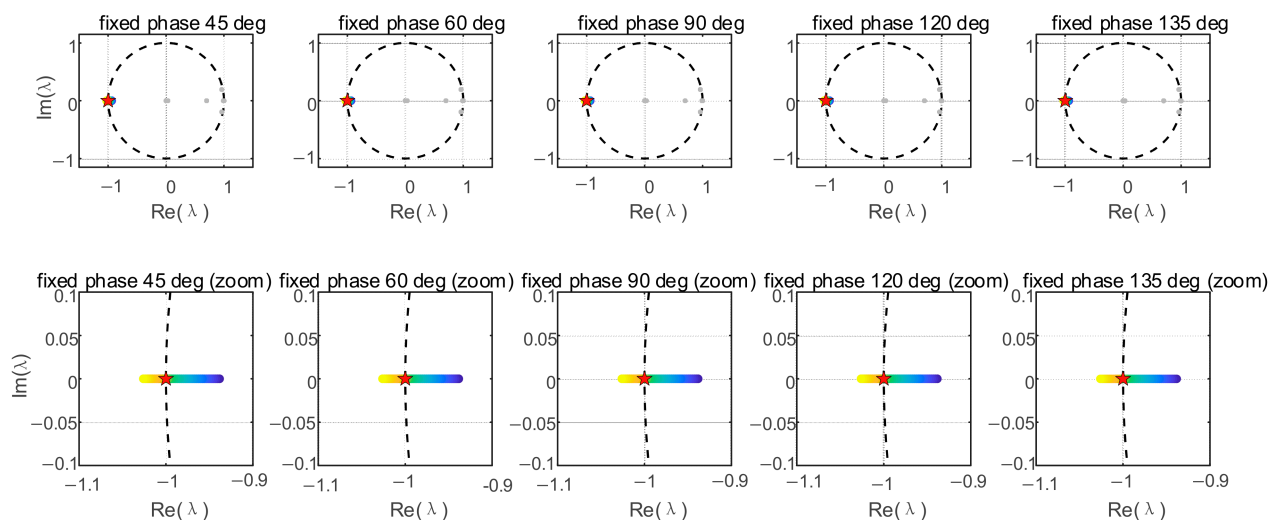


Figure 7. Sampled plot of the grid current i_o .

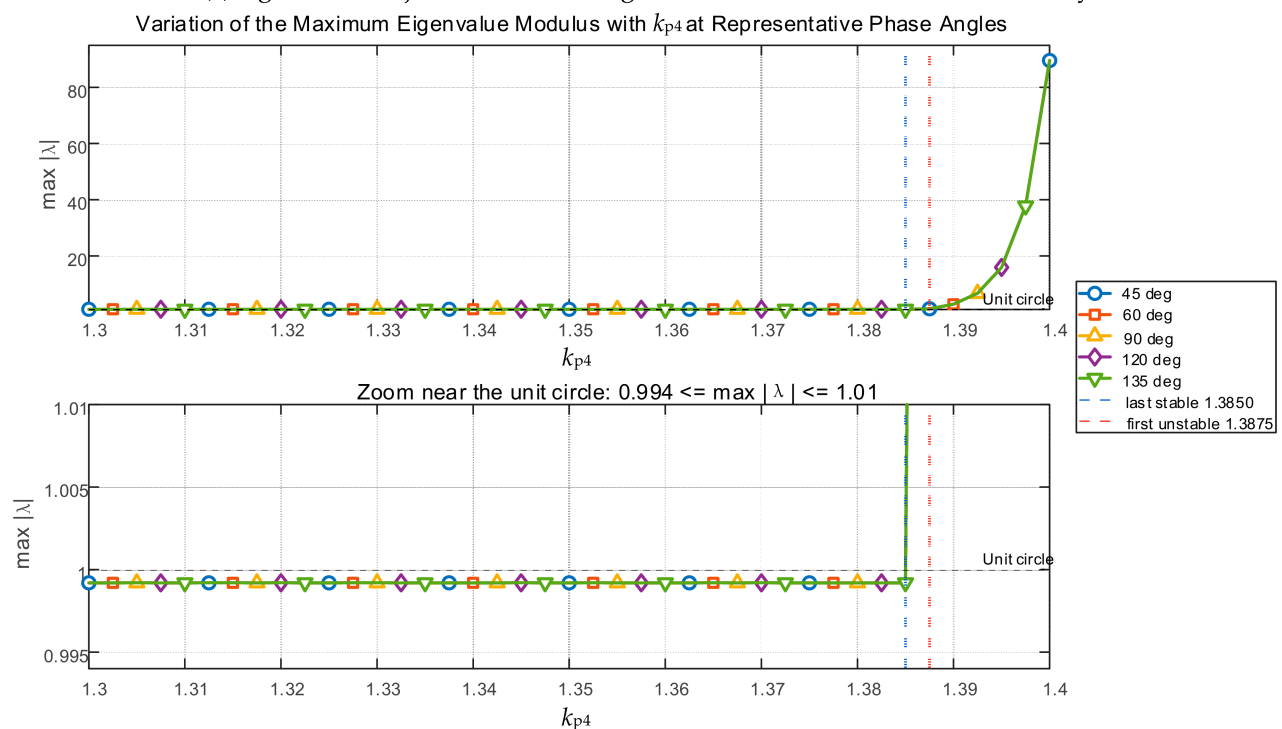
For the single-phase PV–energy storage AC/DC integrated three-port grid-connected device studied in this paper, the grid voltage, current reference, modulation signal, and switching states vary periodically with the grid-voltage phase angle. Therefore, the system exhibits both periodic time-varying characteristics and phase-angle-dependent characteristics. Previous studies have shown that, for systems possessing such phase-angle characteristics, the dynamic behaviors evaluated at different phase angles within one fundamental period generally follow consistent evolution trends. This property is widely observed in H-bridge inverter systems and is not fundamentally dependent on the output-filter structure, such as L, LC, or LCL filters, or on whether the inverter operates in grid-connected or standalone mode [17].

According to the phase-angle-based simplified discrete modeling theory, the original periodic time-varying system can be represented by a family of time-invariant discrete models evaluated at fixed grid-voltage phase angles. If the dominant eigenvalue evolution and critical stability boundaries obtained from these fixed-angle models are consistent, the stability behavior evaluated at one representative phase angle can be used to infer the whole-period stability evolution. This treatment enables eigenvalue and bifurcation analyses to be applied efficiently to the single-phase AC/DC time-varying system without repeatedly evaluating every phase angle over the entire fundamental period.

To verify whether the predicted stability characteristics are materially affected by the selection of the fixed phase angle, the eigenvalue trajectories and maximum eigenvalue modulus are recalculated at $\theta = \pi/4, \pi/3, \pi/2, 2\pi/3$, and $3\pi/4$ under the same operating conditions and k_{p4} sweep settings. The comparative results are shown in Figure 8.



(a) eigenvalue trajectories and enlarged views near the unit-circle boundary



(b) variation in the maximum eigenvalue modulus and enlarged view near the critical stability boundary

Figure 8. Comparison of stability characteristics at representative grid-voltage phase angles under variations in k_{p4} .

As shown in Figure 8a, the eigenvalue distributions obtained at the five representative phase angles are highly similar. In particular, the dominant eigenvalue follows nearly the same trajectory along the negative real axis and approaches the unit-circle boundary near $\lambda = -1$ as k_{p4} increases. The enlarged views further show that the dominant eigenvalue trajectories near the unit circle almost coincide, indicating that the mechanism and direction of the stability transition are essentially independent of the selected phase angle.

Figure 8b compares the corresponding maximum eigenvalue modulus. For all five representative phase angles, the maximum eigenvalue moduli remain below unity before the critical boundary is reached and exhibit nearly identical variation trends. The enlarged view near the unit circle shows that $k_{p4} = 1.3850$ is the last stable parameter value, whereas instability is first detected at $k_{p4} = 1.3875$. Therefore, the critical stability boundary is located within the narrow interval of $1.3850 < k_{p4} < 1.3875$ for all the investigated phase angles.

Beyond this boundary, the maximum eigenvalue modulus increases rapidly, and the five phase-angle cases continue to exhibit consistent instability-evolution trends.

The comparisons in Figure 8 demonstrate that changing the fixed phase angle mainly changes the sampling section within the fundamental period but does not materially alter the dominant eigenvalue trajectory, the stability-transition mechanism, or the predicted critical stability boundary. Therefore, one fixed phase angle can be used to characterize the whole-period stability evolution of the studied system. In the subsequent analysis, $\theta = \pi/2$ is selected as the representative phase angle because it corresponds to the peak grid-voltage region and provides a symmetric and physically intuitive sampling position.

4.2. Phase-Angle-Based Simplified Discrete Model of the System

Step 1: Since the system exhibits periodic time-varying and phase-angle-dependent characteristics, its stability can be inferred by analyzing the dynamic behavior at a specific phase angle. The phase-angle analysis procedure is shown in Figure 9.

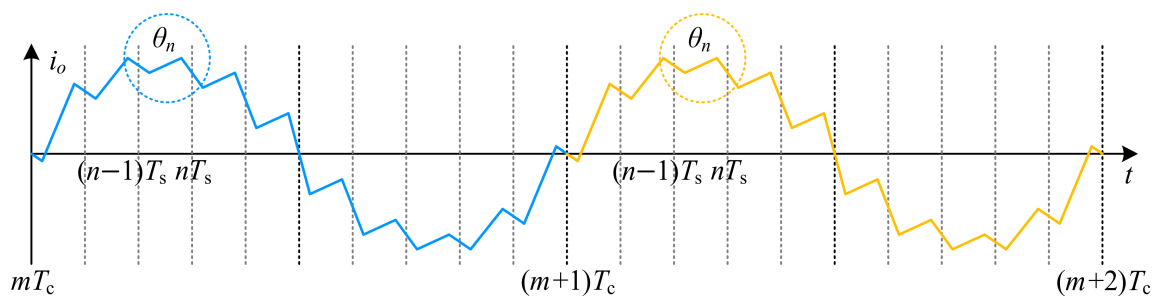


Figure 9. Schematic diagram of phase-angle analysis for the hybrid AC/DC PV-energy storage microgrid system.

Assume that $T_c = NT_s$ ($N \in \mathbb{Z}^+$), T_c is the grid period, and T_s is the switching period. Since $T_s \ll T_c$, the phase angle $\theta = \omega_c t$ can be regarded as a constant and expressed as

$$\theta = \omega_c t = \theta_n \approx \frac{2\pi n}{N} \quad (mT_c + (n-1)T_s < t \leq mT_c + nT_s) \quad (10)$$

Therefore, the time-varying state equation can be transformed into a time-invariant form, as shown in Equation (11). In this way, the Lyapunov indirect method can be applied to evaluate the dynamic behavior of the system.

$$\dot{x} = A_j(\sin \theta_n)x + B_j(\sin \theta_n) \quad (11)$$

Step 2: To avoid deriving separate discrete mappings for different duty-ratio relationships, the switching sequences of the two DC–DC ports are normalized within each switching period. During this procedure, the total conduction interval associated with each switch is maintained, while the original switching sequence of the grid-connected inverter is retained. The reconstruction therefore changes the temporal arrangement of the intermediate operating modes rather than the duty-ratio information used in the discrete model.

The reconstructed model is formulated between two consecutive carrier-synchronized sampling instants. The state evolution inside the switching period is not represented individually for every intermediate mode; instead, its cumulative effect is included in the state update from x_n to x_{n+1} . In this study, x_n is a unified state vector containing the electrical and control states of the PV Boost port, the bidirectional energy-storage port, and the grid-connected inverter. Hence, the resulting discrete update describes the joint evolution of the three ports rather than the isolated response of a single converter stage.

Under high-switching-frequency operation, the state variation over one switching interval is limited. Maintaining the corresponding conduction intervals therefore restricts the influence of sequence normalization mainly to the intra-period trajectories. The sampled state transition used for eigenvalue and bifurcation analyses remains representative of the inter-port dynamic interaction. Accordingly, the reconstructed model is not intended to reproduce the detailed switching waveform within each period but to retain the cycle-to-cycle dynamics required for identifying both fast-scale and slow-scale instability.

Accordingly, the duty-ratio sequences of the Boost converter and the bidirectional DC–DC converter shown in Figure 10a,b are rearranged as shown in Figure 10d,e, respectively. The operating-mode sequence is then expressed in the unified form M_8 – M_4 – M_2 – M_7 – M_3 – M_1 – M_8 – M_4 – M_2 .

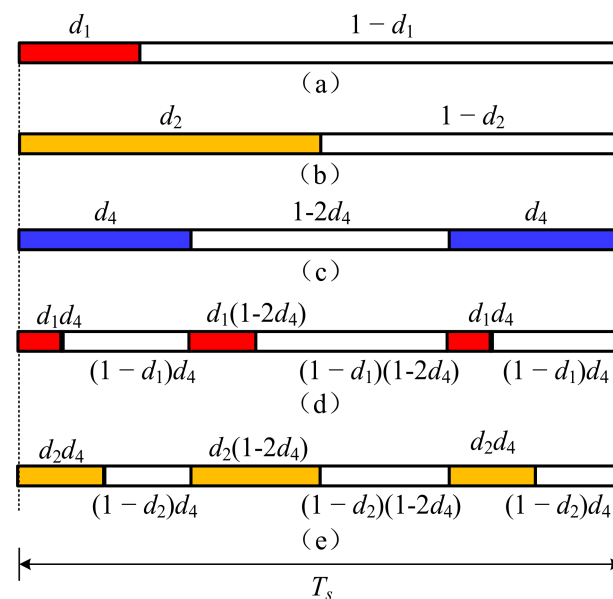


Figure 10. Switching-sequence reconstruction of the hybrid AC/DC PV-energy storage microgrid system. (a) Duty ratio of the Boost converter; (b) duty ratio of the bidirectional DC–DC converter; (c) duty ratio of the inverter; (d) duty ratio of the Boost converter after switching-sequence reconstruction; (e) duty ratio of the bidirectional DC–DC converter after switching-sequence reconstruction.

Step 3: Mode combination. After switching-sequence reconstruction, there are still nine operating modes within one switching period. Based on the state-space averaging method, M_8 , M_4 , and M_2 , as well as M_7 , M_3 , and M_1 , can be merged according to their corresponding duty ratios, as shown in Equations (12)–(15). The schematic diagram is shown in Figure 11. Here, A_{842} and A_{731} are the state matrices after mode combination, while B_{842} and B_{731} are the corresponding input matrices after mode combination.

$$A_{842} = d_1 A_8 + (d_2 - d_1) A_4 + (1 - d_2) A_2 \quad (12)$$

$$B_{842} = d_1 B_8 + (d_2 - d_1) B_4 + (1 - d_2) B_2 \quad (13)$$

$$A_{731} = d_1 A_7 + (d_2 - d_1) A_3 + (1 - d_2) A_1 \quad (14)$$

$$B_{731} = d_1 B_7 + (d_2 - d_1) B_3 + (1 - d_2) B_1 \quad (15)$$

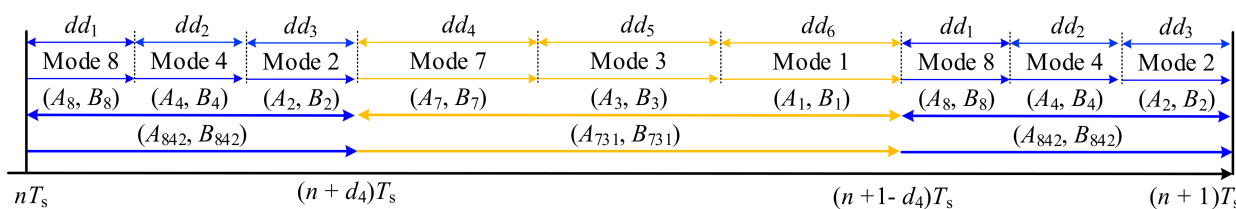


Figure 11. Schematic diagram of mode combination.

The state equation of the system can then be obtained as

$$\dot{x} = F(x) = \begin{cases} A_{842}x + B_{842} & (nT_s \leq t < (n+d_4)T_s) \\ A_{731}x + B_{731} & ((n+d_4)T_s \leq t \leq (n+1-d_4)T_s) \\ A_{842}x + B_{842} & ((n+1-d_4)T_s \leq t < (n+1)T_s) \end{cases} \quad (16)$$

where $x = [v_{pv}, i_{Lb}, v_{dc}, i_o, v_{c1}, v_{c2}, v_{c3}]^T$. By solving the linear differential equation using matrix theory, the phase-angle-based simplified discrete model of the hybrid AC/DC PV-energy storage microgrid system can be obtained as

$$\begin{aligned} x_{n+1} = & e^{A_{842}d_4T_s} e^{A_{731}(1-2d_4)T_s} e^{A_{842}d_4T_s} x_n \\ & + e^{A_{842}d_4T_s} e^{A_{731}(1-2d_4)T_s} \int_{nT_s}^{nT_s+d_4T_s} e^{A_{842}(nT_s+d_4T_s-\tau)} B_{842} d\tau \\ & + e^{A_{842}d_4T_s} \int_{nT_s+d_4T_s}^{nT_s+(1-d_4)T_s} e^{A_{731}(nT_s+(1-d_4)T_s-\tau)} B_{731} d\tau \\ & + \int_{nT_s+(1-d_4)T_s}^{(n+1)T_s} e^{A_{842}((n+1)T_s-\tau)} B_{842} d\tau \end{aligned} \quad (17)$$

Compared with the exact discrete mapping, the proposed phase-angle-based simplified discrete model preserves the state-variable information at the stroboscopic sampling instants while omitting the detailed state evolution during the operating-mode transitions. Therefore, the simplification retains the switching-period mapping from x_n to x_{n+1} , although the intra-period switching trajectories are not reproduced. Since the sampled state vector contains the state variables of all three converter ports, their interactions remain represented in the cycle-to-cycle state evolution. The retained sampling information captures both the switching-cycle variations associated with fast-scale instability and the multi-cycle evolution associated with slow-scale instability. Therefore, the simplified model remains applicable to the analysis of both types of instability.

5. Stability Analysis

5.1. Eigenvalue Sensitivity Analysis of System Control Parameters

In the PV-energy storage AC/DC integrated three-port grid-connected power electronic device, the PV port, the energy storage port, and the grid-connected inverter port are coupled through the common DC bus. The controller parameters of each port not only affect the dynamic response speed of the corresponding port but also influence the overall system stability through the DC bus and the power conversion stages. Since the system contains multiple PI controllers and the established discrete model is of relatively high order, directly plotting the stability boundaries for all control parameters one by one would lead to a complex computational process and low analysis efficiency. Eigenvalue sensitivity is an effective method for determining the variation rate and variation trend of eigenvalues when parameters change. It can be used to identify the parameters that have a significant

influence on the eigenvalues of the model. The sensitivity of the eigenvalue λ_i with respect to the parameter μ can be expressed as [21,22]:

$$S_{\mu}^{\lambda_i} = \frac{\partial \lambda_i}{\partial \mu} = \frac{\omega_i^T (\partial D_x F(x) / \partial \mu) v_i}{\omega_i^T v_i} \quad (18)$$

where ω_i and v_i are the left and right eigenvectors corresponding to the eigenvalue λ_i , respectively, both of which are obtained from the system Jacobian matrix $D_x F(x)$. The real part of the eigenvalue sensitivity determines the horizontal movement trend of λ_i in the complex plane when the parameter varies, whereas the imaginary part governs its vertical movement. When the real part of the sensitivity is positive, an increase in the parameter causes λ_i to move toward the right half of the complex plane, which is unfavorable for system stability. When the real part is negative, an increase in the parameter moves λ_i toward the left half-plane, thereby improving system stability. In addition, the magnitude of the real part of the sensitivity directly reflects the strength of the influence of parameter variations on λ_i . A larger magnitude indicates more pronounced eigenvalue variation.

According to the analysis in Section 4.1, the hybrid AC/DC PV–energy storage microgrid system investigated in this study exhibits both periodic and phase-angle-dependent characteristics. Therefore, the phase angle $\theta = \pi/2$ is selected, and the stability of the grid current at this phase angle is evaluated using eigenvalue trajectories and bifurcation diagrams. The stability of the system over the entire period can then be inferred. The system parameters listed in Table 3 are used to conduct the stability analysis of the hybrid AC/DC PV–energy storage microgrid system.

According to Equation (18), the eigenvalue sensitivities with respect to each controller parameter are calculated in MATLAB, as listed in Table 4. The eigenvalues λ_1 , λ_7 , λ_8 , and λ_9 are most strongly affected by the proportional gain k_{p4} of the current loop in the inverter. As k_{p4} increases, λ_1 , λ_7 , and λ_9 move to the left, whereas λ_8 moves to the right. The eigenvalues λ_2 and λ_3 are most strongly affected by the proportional gain k_{p3} of the inner current-loop PI controller in the bidirectional DC-DC converter. As k_{p3} increases, λ_2 moves to the right, whereas λ_3 moves to the left. The eigenvalues λ_4 , λ_5 , and λ_6 are most strongly affected by the gain k_{p1} of the PI controller in the Boost converter. As k_{p1} increases, $\lambda_{4,5}$ move to the left, whereas λ_6 moves to the right. The eigenvalue λ_{10} is most strongly affected by the integral gain k_{i3} of the inner current-loop PI controller in the bidirectional DC-DC converter. As k_{i3} increases, λ_{10} moves to the left.

Table 4. Eigenvalue sensitivities with respect to the control parameters.

	$S_{\mu}^{\lambda_1}$	$S_{\mu}^{\lambda_2}$	$S_{\mu}^{\lambda_3}$	$S_{\mu}^{\lambda_{4,5}}$	$S_{\mu}^{\lambda_6}$	$S_{\mu}^{\lambda_7}$	$S_{\mu}^{\lambda_8}$	$S_{\mu}^{\lambda_9}$	$S_{\mu}^{\lambda_{10}}$
k_{p1}	0	10	0	$\frac{-90,320}{\pm 427,840 i}$	180,640	0	0	0	0
k_{i1}	0	0	0	$\frac{77.85}{\pm 42.93 i}$	−155.7	−0.01	0	0	0
k_{p2}	0	0	0	0	0	0.0079	−0.0739	0.1691	−0.1031
k_{i2}	0	0	0	0	0	−0.0065	0.0248	−0.0009	−0.0174
k_{p3}	0.0645	16.1639	−33.7562	$\frac{-0.0077}{\pm 0.0044 i}$	0.0262	51.853	−28.2576	−6.1934	0.115
k_{i3}	0.0001	2.0119	−0.0036	0.0001	−0.0001	−0.0527	0.0638	−0.0029	−2.0167
k_{p4}	−27.61	−3.12	0.01	$\frac{-0.03}{\pm 0.01 i}$	0.02	−400.37	534.19	−103.08	0.02
k_{i4}	0.1248	0	0	0	0	−0.0029	0.0096	−0.1314	0

Based on the above sensitivity analysis and the control structure shown in Figure 3, k_{p4} and k_{p2} are selected for further investigation. The parameter k_{p4} is the proportional gain of the grid-connected inverter current controller and is directly associated with grid-current regulation. The parameter k_{p2} is the proportional gain of the DC-bus voltage controller in the energy-storage stage and characterizes the influence of the energy-storage port on grid-connected operation through the common DC bus. Therefore, k_{p4} and k_{p2} are considered as representative parameters for the grid-side control path and the cross-port coupling path, respectively. The effects of k_{p1} , k_{p3} , and k_{i3} on the corresponding eigenvalue modes are presented in Table 4, while the following subsections focus on the detailed stability analysis of k_{p4} and k_{p2} . The combined influence of k_{p2} and k_{p4} over a broader parameter range is further evaluated using the two-parameter stability-region analysis in Section 5.4.

5.2. Influence of the Grid-Connected Inverter on System Stability

The system parameters are selected as listed in Table 3. The time constant of the PI controller in the inverter current inner loop is kept unchanged, and the proportional gain k_{p4} is selected as the bifurcation parameter. Based on the phase-angle-based simplified discrete model in Equation (17), the bifurcation diagram of the grid current i_o is obtained when k_{p4} varies from 1.3 to 1.7, as shown in Figure 12. It can be observed from the bifurcation diagram that when $k_{p4} \in [1.3, 1.4]$, the grid current remains in the period-one state, indicating that the system has satisfactory local stability. When $k_{p4} > 1.4$, the system enters the period-two and period-four states successively through period-doubling bifurcation, indicating that the system has crossed the stability boundary and that the stability margin is significantly reduced. When $k_{p4} > 1.48$, the grid current exhibits pronounced irregular fluctuations, suggesting that the system can no longer maintain stable grid-connected operation.

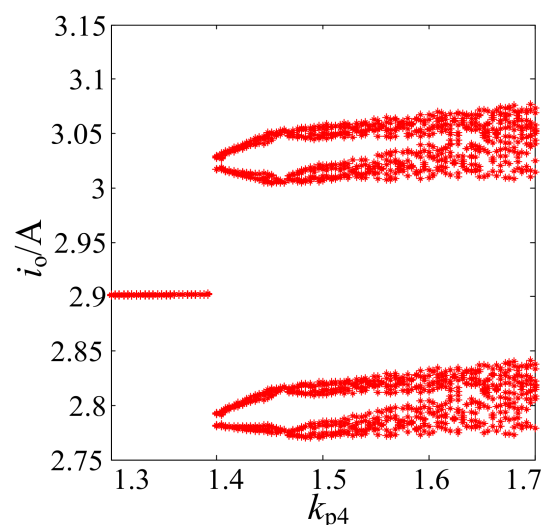


Figure 12. Bifurcation diagram of the hybrid AC/DC PV–energy storage microgrid system with $T_{i4} = 1/3$ and $k_{p4} \in [1.3, 1.7]$.

Subsequently, based on the phase-angle-based simplified discrete model of the system in Equation (17), the eigenvalue trajectories of the system are obtained when k_{p4} varies from 1.3 to 1.7, as shown in Figure 13. When k_{p4} increases from 1.39 to 1.40, the eigenvalue λ_1 crosses the unit circle from inside to outside along the negative real axis, whereas $\lambda_2 \sim \lambda_{10}$ remain inside the unit circle with only slight variations. According to the stability criterion for discrete-time systems, an eigenvalue crossing the unit circle indicates that the system changes from a stable state to an unstable state. Therefore, $k_{p4} \approx 1.4$ can be regarded as the critical stability value of the proportional gain in the inverter current loop. A comparison between the bifurcation diagram in Figure 12 and the eigenvalue trajectories in Figure 13

shows that the two methods yield fully consistent results. This verifies that the phase-angle-based simplified discrete model can effectively identify the system stability boundary and provide a basis for tuning the inverter current-loop parameters.

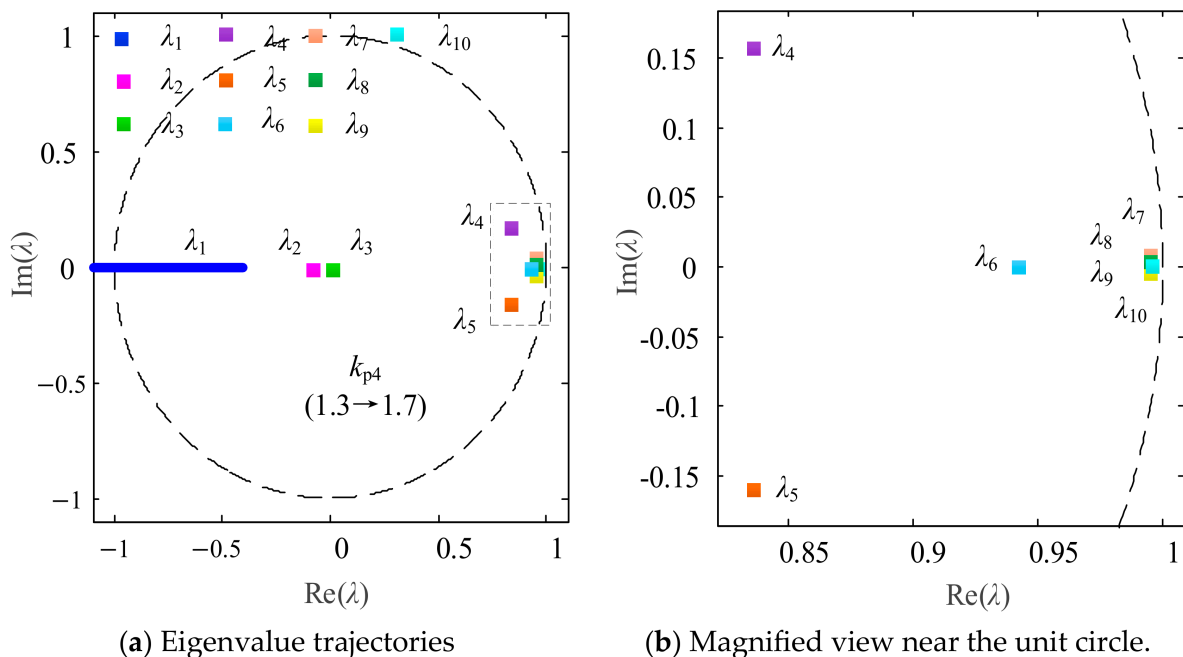


Figure 13. Eigenvalue trajectories of the hybrid AC/DC PV-energy storage microgrid system with $T_{i4} = 1/3$ and $k_{p4} \in [1.3, 1.7]$.

5.3. Influence of Source Converter 2 on System Stability

The system parameters are selected as listed in Table 3. The integral time constant of the PI controller in the energy storage stage is kept unchanged, and the proportional gain k_{p2} is selected as the bifurcation parameter. The corresponding bifurcation diagram of the system is obtained, as shown in Figure 14. The results show that when $k_{p2} \in [1.4, 1.7]$, the grid current remains in the period-one state, indicating that the system can maintain stable grid-connected operation within this parameter range. When $k_{p2} > 1.75$, the system undergoes Hopf bifurcation, and the grid current begins to exhibit pronounced low-frequency oscillations. This indicates that the system has crossed the stability boundary and entered an unstable operating state. As k_{p2} further increases, both the oscillation amplitude and the complexity of the fluctuations continue to increase, and the system operating condition further deteriorates.

To further determine the influence of k_{p2} on system stability, the eigenvalue trajectories of the system Jacobian matrix are calculated based on Equation (17) when k_{p2} varies from 1.6 to 1.8, as shown in Figure 15. The results show that when k_{p2} increases from 1.75 to 1.76, a pair of complex-conjugate eigenvalues, λ_5 and λ_6 , cross the unit circle from inside to outside, whereas the remaining eigenvalues stay inside the unit circle with only slight variations. According to the stability criterion for discrete-time systems, eigenvalues crossing the unit circle indicate that the system changes from stable operation to unstable operation. Therefore, $k_{p2} \approx 1.75$ can be regarded as the critical stability value of the proportional gain of the energy storage stage. The grid current response shown in Figure 14 is consistent with the eigenvalue trajectory analysis shown in Figure 15. This indicates that the control parameters of source converter 2 can affect the grid-connected-side stability through the DC bus, and further verifies that the phase-angle-based simplified discrete model can effectively identify the system stability boundary.

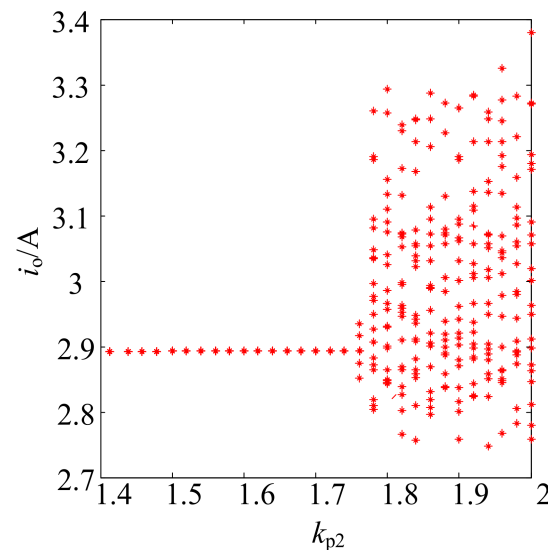


Figure 14. Bifurcation diagram of the hybrid AC/DC PV-energy storage microgrid system under variations in k_{p2} .

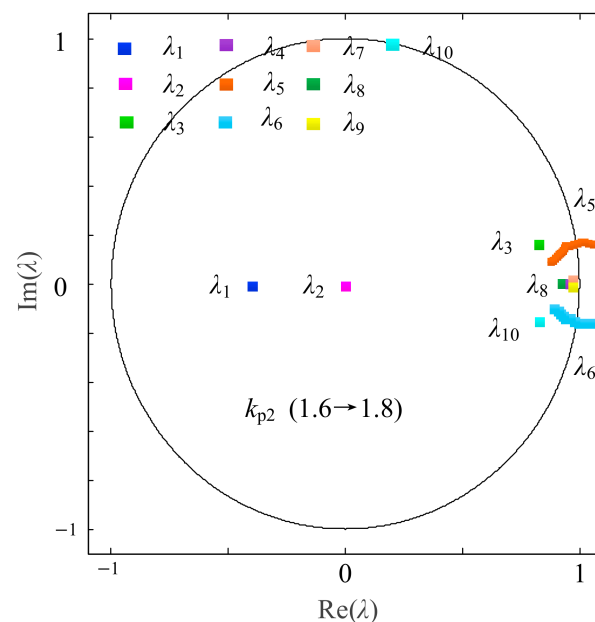


Figure 15. Eigenvalue trajectories of the hybrid AC/DC PV-energy storage microgrid system under variations in k_{p2} .

5.4. Two-Parameter Stability-Region Analysis

The preceding analyses vary k_{p4} and k_{p2} independently and therefore do not directly reveal the interaction between the two control loops. To investigate their combined influence, the proportional gain k_{p2} of the energy-storage-stage voltage loop and the proportional gain k_{p4} of the grid-connected inverter current loop are varied simultaneously.

For each parameter pair, the fixed point of the phase-angle-based simplified discrete model is calculated, and the maximum eigenvalue modulus $\max(|\lambda|)$ of the corresponding Jacobian matrix is evaluated. The condition $\max(|\lambda|) < 1$ denotes a locally stable operating point, whereas $\max(|\lambda|) > 1$ denotes an unstable operating point. The boundary $\max(|\lambda|) = 1$ is therefore used to separate the stable and unstable regions.

The parameter ranges are selected as $k_{p2} = 1.4\text{--}2.0$ and $k_{p4} = 0.8\text{--}1.7$. As shown in Figure 16, the allowable value of k_{p4} remains close to 1.40 when k_{p2} is below approximately

1.60, and then decreases rapidly as k_{p2} approaches 1.70–1.75. The boundary passing near $k_{p2} = 1.75$ and $k_{p4} = 1.0$ is consistent with the Hopf boundary identified from the single-parameter k_{p2} analysis, while the upper-left part of the boundary is consistent with the period-doubling boundary near $k_{p4} = 1.40$.

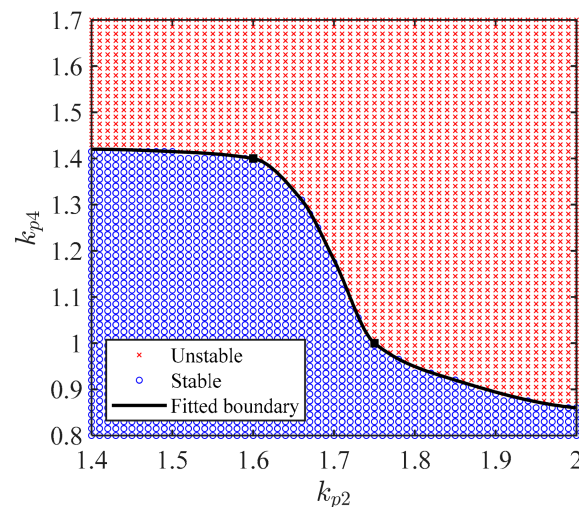


Figure 16. Two-parameter stability region in the k_{p2} – k_{p4} plane.

These results indicate that the stable operating range is jointly determined by the two PI gains. In particular, when either parameter approaches its critical range, the admissible range of the other parameter becomes narrower. The two-dimensional stability region therefore provides a more direct constraint for coordinated PI-controller tuning than separate one-dimensional stability margins.

5.5. Quantitative Comparison with Averaged and Exact Discrete Models

To quantify the accuracy and computational efficiency of the proposed model, an exact switched discrete map and a conventional continuous state-space averaged model are established as benchmarks. All three models use the same ten-dimensional state vector, circuit parameters, controller parameters, sampling frequency, and evaluation phase angle. The exact map retains the chronological order of all PWM events and is therefore used as the numerical reference.

Exact switched discrete map. The switching instants associated with d_1 , d_2 , d_4 , and $1 - d_4$ are sorted within each switching period, and the affine state transition of every constant-topology interval is evaluated in chronological order. At the nominal operating point, these four internal PWM edges are distinct and partition the period into five intervals; hence, five affine transitions are required. If two edges coincide at another operating point, the corresponding intervals are merged and the number of transitions decreases. The one-period map is at the nominal operating point.

$$x_{n+1} = F_{ex}(x_n) = F_q \circ F_{q-1} \circ \dots \circ F_1(x_n), q = 5 \quad (19)$$

Phase-angle-based simplified discrete map. The proposed model reconstructs the inverter switching period into three intervals of duration $d_4 T_s$, $(1 - 2d_4)T_s$, and $d_4 T_s$. The Boost and bidirectional-converter modes are combined according to their dwell fractions. For $d_1 \leq d_2$, the combined matrices are

$$\begin{aligned} A_{842} &= d_1 A_8 + (d_2 - d_1) A_4 + (1 - d_2) A_2 \\ A_{731} &= d_1 A_7 + (d_2 - d_1) A_3 + (1 - d_2) A_1 \end{aligned} \quad (20)$$

The affine input vectors are combined using the same weights. For $d_2 < d_1$, the corresponding mode combinations are formed analogously. The simplified map therefore evaluates three affine transitions per switching period.

Continuous state-space averaged model. The averaged model preserves the modal dwell fractions but removes their chronological order. Its continuous-time eigenvalues are converted into switching-period multipliers for direct comparison with the two discrete models:

$$A_{av} = \sum_j \alpha_j A_j, B_{av} = \sum_j \alpha_j B_j, \lambda_d = \exp(\lambda_c T_s) \quad (21)$$

The common test conditions are $f_s = 20$ kHz, $f_c = 50$ Hz, $\theta = \pi/2$, $L_f = 10$ mH, inverter $P = 2 k_{p4}$, inverter $I = 6 k_{p4}$, and $T_{i4} = 1/3$. During the k_{p2} sweep, $T_{i2} = k_{p2}/k_{i2} = 0.16$ is held constant. Boundary errors are calculated relative to the exact map. The one-step and 250-step NRMSE values are evaluated using the complete state vector, and all runtimes are measured in the same MATLAB session. The structural neutral multiplier associated with the redundant controller state is excluded from the stability assessment.

Table 5 shows that the proposed model reproduces both instability mechanisms with boundary errors below 0.32%. By contrast, the averaged model gives a reasonable estimate of the slow-scale oscillatory boundary but does not detect the fast-scale period-doubling instability because the PWM-event order is eliminated by averaging.

Table 5. Stability-boundary comparison under identical operating conditions.

Model	k_{p4} , Crit	Crossing Type	Error	k_{p2} , Crit	Crossing Type	Error
Exact switched discrete map	1.4293	Period doubling	Reference	1.7060	Neimark-Sacker	Reference
Proposed simplified discrete model	1.4285	Period doubling	0.0492%	1.7005	Neimark-Sacker	0.3194%
Continuous averaged model	Not detected	N/A	N/A	1.7197	Neimark-Sacker	0.8066%

As reported in Table 6, the proposed model reduces the number of affine transitions from five to three per switching period. The resulting 4.14-fold speedup is obtained while maintaining a one-step NRMSE of 0.4723% and a 250-step NRMSE of 4.1003%. Although the averaged model is faster, it cannot represent the switching-sequence information required for fast-scale bifurcation analysis. The proposed model therefore provides the best compromise between stability-boundary accuracy and computational efficiency for the three-port converter considered here.

Table 6. State-trajectory accuracy and measured computational cost.

Model	One-Step NRMSE	250-Step NRMSE	Stages/Step	k_{p4} Sweep	k_{p2} Sweep	Total	Speedup
Exact switched map	0	0	5	1.9343 s	0.5392 s	2.4735 s	1.00×
Proposed simplified model	0.4723%	4.1003%	3	0.3329 s	0.2653 s	0.5981 s	4.14×
Continuous averaged model	0.4855%	3.8064%	1 averaged field	0.1715 s	0.1270 s	0.2985 s	8.29×

6. Simulation and Hardware-in-the-Loop Experimental Verification

6.1. Simulation Verification

To verify the accuracy and reliability of the conclusions obtained in Section 5 regarding the stability of the system, a system simulation model fully consistent with the topology

shown in Figure 1 is established on the MATLAB/Simulink platform. During the simulation, all parameters are set strictly according to the configuration listed in Table 3, so as to ensure consistency between the simulation model and the theoretical analysis model.

Considering that variations in system stability are reflected not only in time-domain waveforms but also in the spectral characteristics and power quality indices of the grid current, the spectrum of the grid current and the total harmonic distortion (THD) are used in this section as auxiliary verification indices. These indices are employed to evaluate the influence of control parameter variations on the stable operating state of the system.

With the integral time constant of the PI controller in the inverter current inner loop kept unchanged, the proportional gain k_{p4} is gradually adjusted, and the spectrum of the simulated grid-current waveform shown in Figure 17 is analyzed. The results show that when $k_{p4} = 1.3$, the system operates in the period-one steady state. The spectrum consists mainly of the fundamental component and the switching-frequency component, with a very small harmonic proportion, and the measured THD is 2.07%. When k_{p4} increases to 1.4, the system approaches the critical stability boundary. A component at one-half of the switching frequency begins to appear in the spectrum, and the THD increases to 2.28%, indicating a reduction in the system stability margin and the initial deterioration of grid current quality. When $k_{p4} = 1.5$, a component at one-quarter of the switching frequency further appears in the spectrum, the harmonic amplitude increases significantly, and the THD rises to 3.47%. This indicates that the system has entered an unstable operating state. When k_{p4} further increases to 1.6, the spectral distribution exhibits continuous and irregular broadband characteristics, and the THD reaches 3.60%, suggesting a further aggravation of system instability.

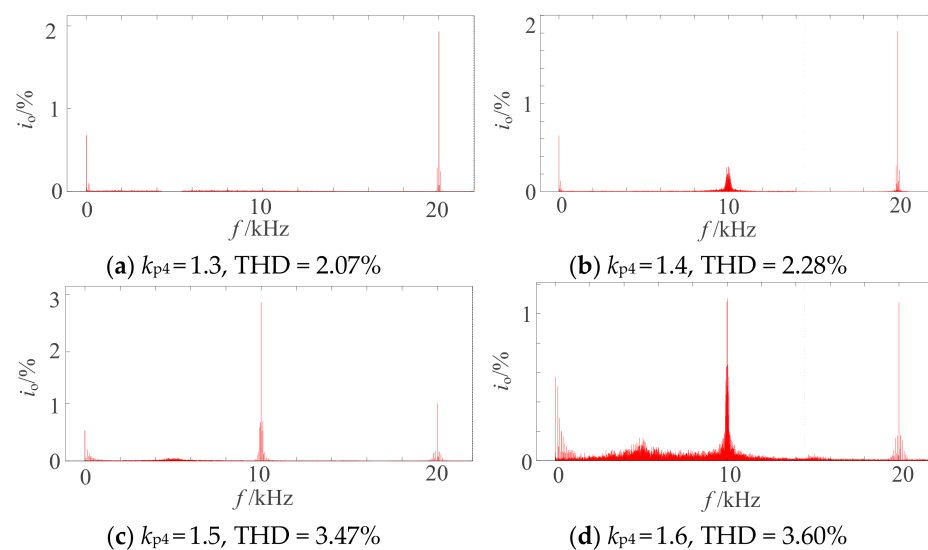


Figure 17. Spectral characteristics of the simulated grid-current waveform with the proportional gain k_{p4} as the bifurcation parameter.

In summary, as k_{p4} increases, the system gradually approaches the stability boundary from stable operation and eventually enters an unstable state. Meanwhile, the harmonic content and THD of the grid current continue to increase. These results are consistent with the stability analysis conclusions obtained from the eigenvalue trajectories in the preceding section. This verifies that an excessively large proportional gain in the inverter current loop reduces the system stability margin and adversely affects the quality of the grid current.

With the integral time constant fixed at $T_{i2} = k_{p2}/k_{i2} = 0.16$, the proportional gain k_{p2} is selected as the bifurcation parameter. The spectral characteristics of the simulated grid-current waveform in Figure 18 are then analyzed. The results show that when $k_{p2} = 1.7$,

the system operates in a stable state. The spectrum mainly contains the fundamental component and the switching-frequency component, with a low harmonic content, and the corresponding THD is 2.07%. This indicates that the energy storage stage can effectively support the DC-bus voltage, and the grid current quality is satisfactory. When k_{p2} increases to 1.75, the system approaches the critical stability boundary. Low-frequency components begin to appear in the spectrum, and the THD increases to 2.17%, indicating that the system stability margin decreases and the coupling oscillation between the DC bus and the grid-connected side becomes stronger. When $k_{p2} = 1.8$, the low-frequency components further increase, and the THD rises to 2.27%. This indicates that the system has entered an unstable operating state, and the grid current quality further deteriorates.

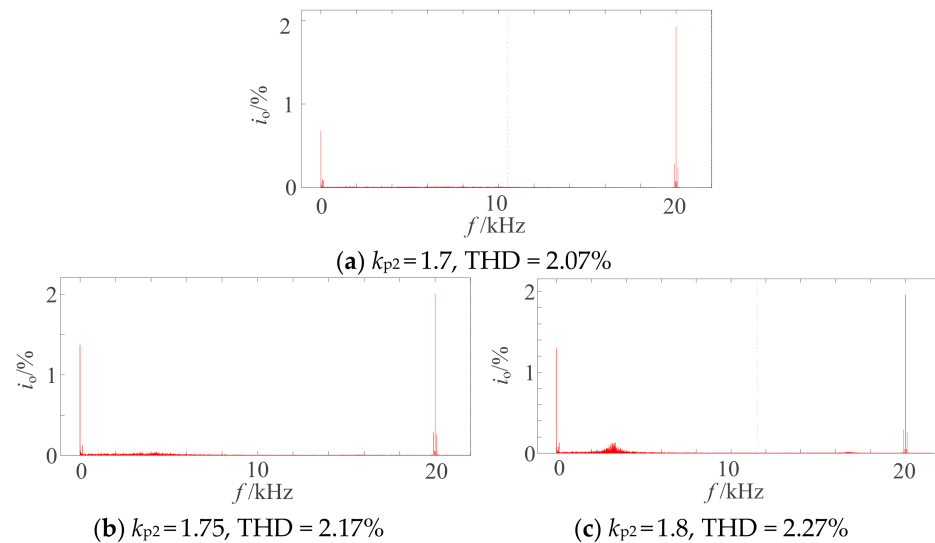


Figure 18. Spectral characteristics of the simulated grid-current waveform under variations in the proportional gain k_{p2} .

6.1.1. Internal Dynamic Responses

To examine whether the grid-connected inverter current-loop gain affects the internal DC-side operating state, the DC-bus voltage v_{dc} is recorded while k_{p4} is varied from 1.5 to 3.0, with all other circuit and control parameters kept unchanged. Figure 19a presents the DC-bus voltage on a full scale, while Figure 19b provides a locally enlarged view of the steady-state voltage ripple.

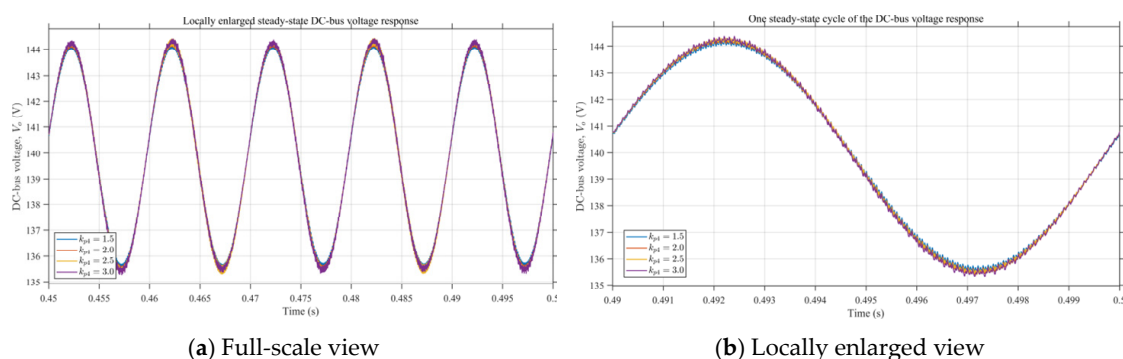


Figure 19. DC-bus voltage responses with k_{p4} varying from 1.5 to 3.0: (a) full-scale view; (b) locally enlarged view.

Within the investigated range of k_{p4} , no appreciable change is observed in the DC-bus voltage waveform. The average voltage remains regulated near its reference value, and

neither the ripple amplitude nor the waveform pattern varies significantly. Therefore, although k_{p4} has a pronounced influence on the bifurcation behavior and spectral characteristics of the grid current, its effect on the DC-bus voltage is limited under the studied operating conditions.

6.1.2. Validation Under Different Grid Conditions

To further evaluate the applicability of the proposed modeling framework, additional grid-side operating conditions are considered by changing the grid-side inductance while keeping the PV-side and battery-side operating points unchanged. This setting isolates the influence of the grid-side impedance and avoids introducing battery state-of-charge dynamics that are outside the scope of the present model.

For each grid condition, the phase-angle-based simplified discrete model is recalculated, and the grid-current waveform and THD are compared. The results show that the selected weaker-grid cases exhibit a smaller predicted stability margin, more pronounced grid-current distortion, and higher THD. These tendencies are consistent with the theoretical prediction of the proposed model.

The comparison demonstrates that the proposed model can capture the variation in the stability boundary and current-quality trend under different grid-side conditions, rather than being valid only for the nominal operating point.

The system parameters are selected as listed in Table 3, while the PV-side operating point, battery-side operating point, and controller parameters are kept unchanged. To further investigate the influence of the grid-side condition on system stability, the grid-side inductance L_f is selected as the bifurcation parameter and is continuously varied from 5 mH to 3 mH. The grid-current responses at four representative inductance values, namely 3, 4, 4.3, and 5 mH, are shown in Figure 20. The corresponding continuous bifurcation diagram of the sampled grid current i_o is shown in Figure 21, in which these four inductance values are marked at their respective positions.

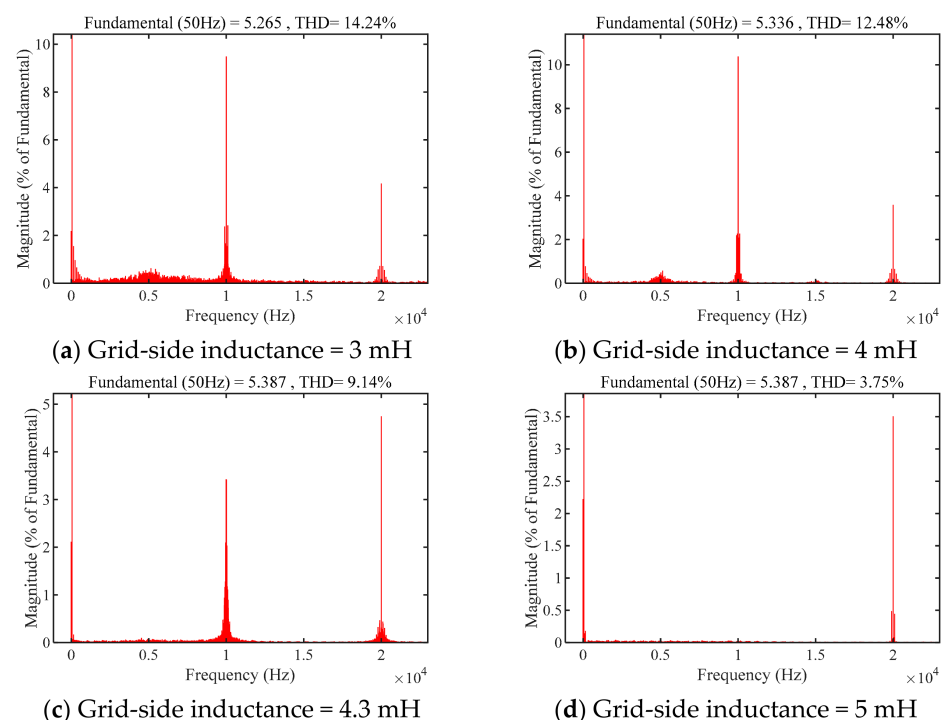


Figure 20. Grid-current responses under different grid-side inductance conditions: (a) 3 mH; (b) 4 mH; (c) 4.3 mH; and (d) 5 mH.

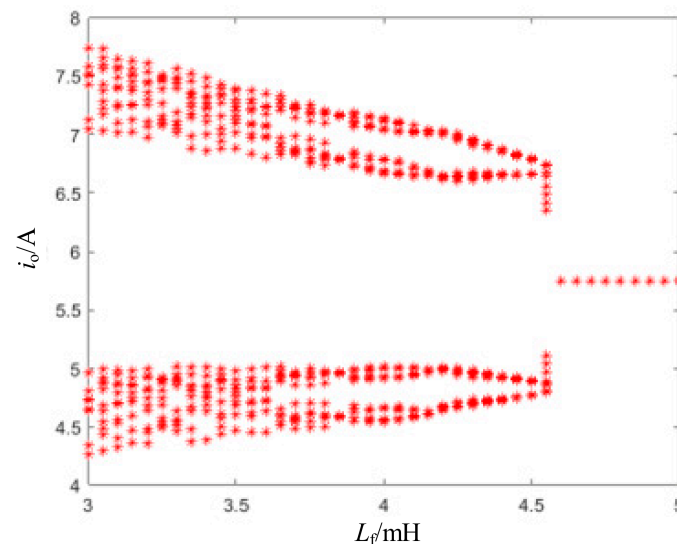


Figure 21. Bifurcation diagram of the sampled grid current i_o with the grid-side inductance L_f as the bifurcation parameter.

The results show that when L_f is approximately within the range of 4.6–5 mH, the sampled grid-current values remain concentrated on a small number of narrow branches, indicating that the system maintains a regular periodic operating state. When L_f decreases below approximately 4.6 mH, the original branches begin to split and expand, indicating that the system crosses the corresponding stability boundary and undergoes bifurcation. As L_f further decreases, the number of bifurcation branches and their distribution range increase continuously, and the grid-current response exhibits increasingly complex fluctuations. In particular, the operating conditions at $L_f = 4.3$, 4, and 3 mH show progressively more dispersed multi-branch characteristics. These results are consistent with the grid-current waveform and spectral comparisons in Figure 20, further demonstrating that variations in the grid-side inductance can significantly affect the stability margin and dynamic behavior of the PV-energy storage AC/DC integrated three-port system.

6.2. Experimental Verification

To verify the correctness of the theoretical analysis and offline simulation results, an RT-BOX-based real-time simulation verification platform, as shown in Figure 22, was established using PLECS 5.0.3 and the RT-BOX real-time simulator developed by Plexim. In this platform, the complete PV-energy storage AC/DC integrated three-port grid-connected device, including the PV Boost converter, the battery-side bidirectional DC-DC converter, the DC bus, the single-phase full-bridge inverter, and the corresponding control loops, was modeled in PLECS and deployed to the RT-BOX for real-time execution. The RT-BOX served as the real-time computational hardware for emulating the switching behavior and dynamic response of the system, while the oscilloscope and host-computer monitoring system were used for waveform observation, data acquisition, and spectral analysis. Therefore, the purpose of this RT-BOX-based validation is to examine whether the stability boundaries predicted by the proposed phase-angle-based simplified discrete model can still be observed under real-time execution conditions.

To improve the reproducibility of the real-time validation, the numerical implementation settings of the RT-BOX-based HIL platform are summarized in Table 7. The complete plant and controller model was executed using a fixed-step discrete state-space solver with a real-time step size of 1 μ s.

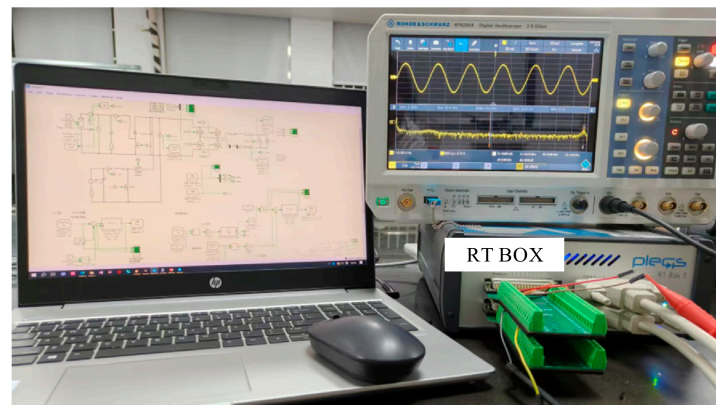


Figure 22. Experimental platform.

Table 7. Real-time implementation settings of the RT-BOX-based HIL validation platform.

Item	Setting
Real-time simulation step size	1 μ s
Solver setting	Fixed-step discrete state-space solver
PWM carrier frequency	10 kHz for the PV Boost converter; 20 kHz for the energy-storage converter and the grid-connected inverter
PWM period	100 μ s for the PV Boost converter; 50 μ s for the energy-storage converter and the grid-connected inverter
PWM resolution	A temporal resolution of 1 μ s was adopted. Each Boost PWM period contained 100 calculation steps, corresponding to an equivalent duty-ratio resolution of 1%, whereas each energy-storage-converter and inverter PWM period contained 50 calculation steps, corresponding to an equivalent duty-ratio resolution of 2%
Controller sampling period	50 μ s for the inverter current controller and the energy-storage voltage/current controllers; 1 μ s for the PV Boost voltage controller
Sampling synchronization	The PWM carriers and zero-order holders had zero initial phase delay and were aligned with the 1 μ s fixed-step time grid. The energy-storage and inverter modulation signals were updated once per 50 μ s PWM period, whereas the PV Boost controller was updated at the 1 μ s real-time step

The PV Boost converter operated at a switching frequency of 10 kHz, corresponding to a switching period of 100 μ s, whereas the energy-storage converter and the grid-connected inverter operated at 20 kHz, corresponding to a switching period of 50 μ s. With the real-time simulation step fixed at 1 μ s, each Boost switching period contained 100 calculation steps, providing an equivalent duty-ratio resolution of 1%. Each switching period of the energy-storage converter and the grid-connected inverter contained 50 calculation steps, corresponding to an equivalent duty-ratio resolution of 2%. The PWM carriers and zero-order holders were initialized without phase delay and were aligned with the fixed-step simulation time grid.

With the integral time constant of the PI controller in the inverter current inner loop kept constant, the proportional gain k_{p4} is gradually adjusted. The waveform data of the grid current i_o are collected in real time and then used for spectral analysis. Figure 23 shows the time-domain waveforms and spectral analysis results of the grid current under different values of k_{p4} . The results show that when $k_{p4} = 1.3$, the grid current waveform is relatively stable, and the spectrum mainly contains the fundamental component and the 20 kHz switching-frequency component, indicating that the system operates in a stable state. When k_{p4} increases to 1.4, a 10 kHz component corresponding to one-half of the switching frequency appears in the spectrum. This indicates that the system approaches and crosses the stability boundary, and the stability of the grid current begins to deteriorate.

rate. When $k_{p4} = 1.5$, a 5 kHz component corresponding to one-quarter of the switching frequency further appears in the spectrum, and the harmonic amplitude increases significantly. The spectral distribution tends to become continuous and irregular, indicating that the degree of system instability is further aggravated and that the grid current quality is significantly deteriorated.

For the energy storage stage, the proportional gain k_{p2} is gradually adjusted while the integral time constant of its PI controller is kept unchanged. The waveform data of the grid current i_o are collected in real time and then used for spectral analysis. Figure 24 shows the time-domain waveforms and spectral analysis results of the grid current under different values of k_{p2} . The results show that when $k_{p2} = 1.7$, the grid current remains stable, and the spectrum is mainly composed of the fundamental component and the switching-frequency component, indicating that the system has a satisfactory stability margin. When k_{p2} increases to 1.75, low-frequency components begin to appear in the spectrum, indicating that the dynamic coupling between the energy storage stage and the DC bus is enhanced and that the system stability margin is reduced. When $k_{p2} = 1.8$, the low-frequency harmonic content increases significantly, and the grid current fluctuation becomes more severe, indicating that the system has entered an unstable operating state.

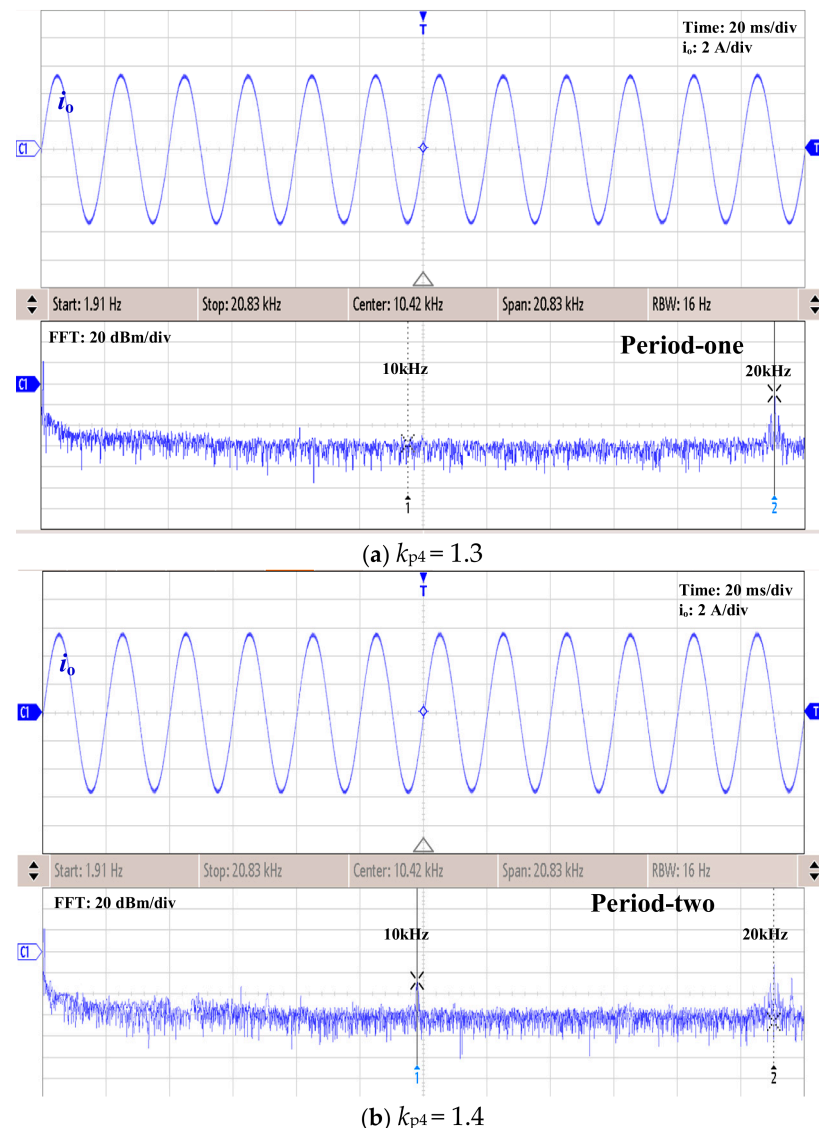


Figure 23. Cont.

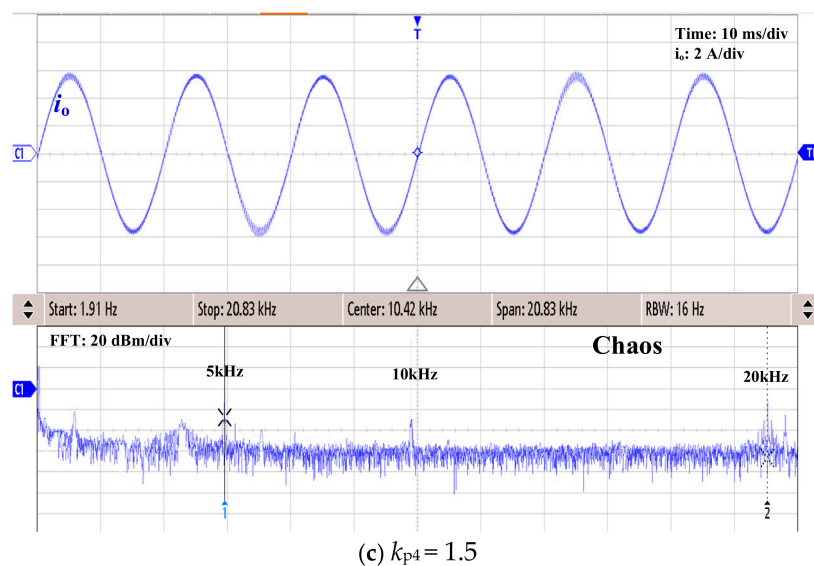


Figure 23. Experimental results with k_{p4} as the bifurcation parameter.

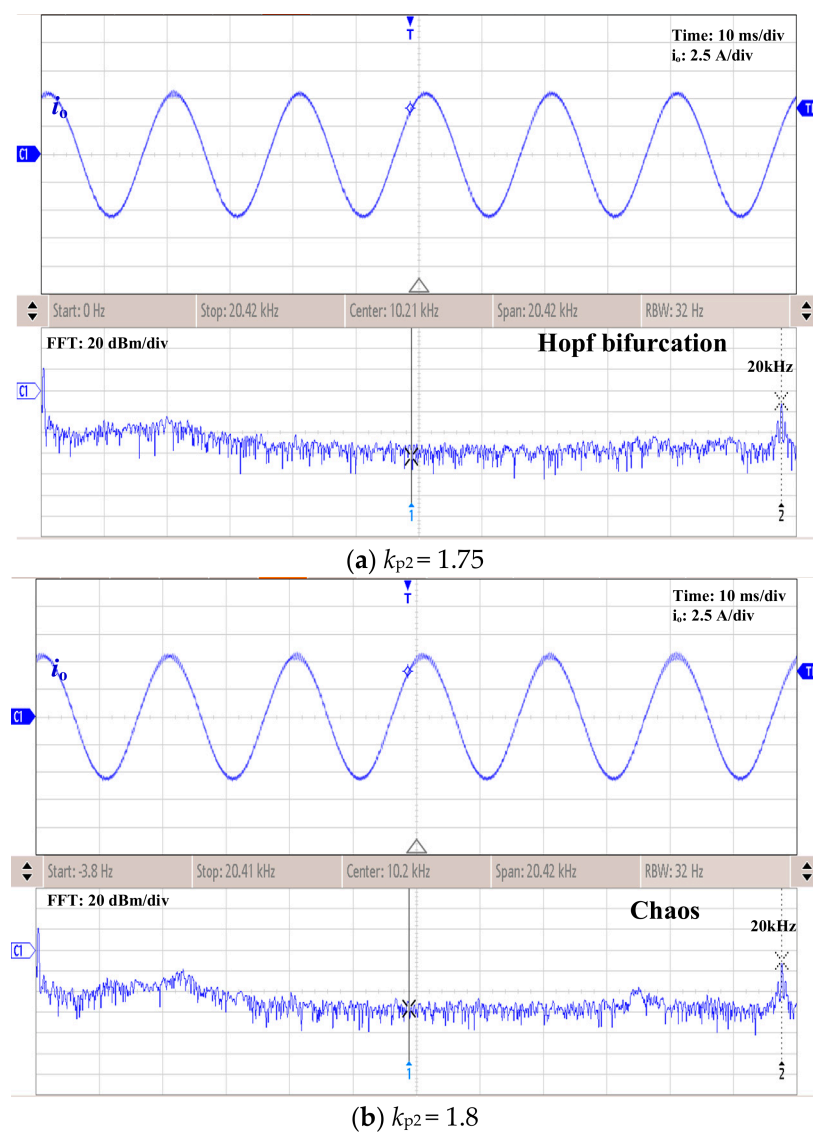


Figure 24. Experimental results with k_{p2} as the bifurcation parameter.

The measured results are highly consistent with the simulated waveforms and theoretical analysis, which verifies the effectiveness of the proposed method. In addition, the experimental results show that when the control parameters exceed their critical stability values, the harmonic components in the grid current increase significantly, leading to a degradation of power quality. Therefore, identifying the bifurcation points provides a theoretical basis for the optimal selection of control parameters in engineering applications. It also helps constrain the total harmonic distortion (THD) to a low level and improve power quality.

To present the consistency among the theoretical prediction, offline simulation, and HIL validation more explicitly, Table 8 compares the critical stability boundaries, dominant eigenvalue behavior, simulated grid-current THD, and the corresponding HIL spectral features for k_{p4} and k_{p2} .

Table 8. Comparison of theoretical predictions, simulation results, and HIL observations near the stability boundaries.

Parameter Case	Theoretical Prediction	Simulation THD	HIL Observation
$k_{p4} = 1.3$	Stable; λ_1 remains inside the unit circle	2.07%	Period-one current; no half-switching-frequency component
$k_{p4} = 1.4$	Boundary near 1.40; λ_1 crosses the unit circle along the negative real axis	2.28%	A 1/2 switching-frequency component appears
$k_{p4} = 1.5$	Unstable region after period-doubling bifurcation	3.47%	Additional subharmonic and broadband components appear
$k_{p4} = 1.6$	Further inside the unstable region	3.60%	Subharmonic/broadband spectral components become more pronounced
$k_{p2} = 1.7$	Stable; λ_5 and λ_6 remain inside the unit circle	2.07%	Stable baseline case
$k_{p2} = 1.75$	Boundary near 1.75; λ_5 and λ_6 approach the unit circle	2.17%	Low-frequency components emerge near the predicted Hopf boundary
$k_{p2} = 1.8$	Unstable; λ_5 and λ_6 have crossed the unit circle	2.27%	Low-frequency components are enhanced beyond the boundary

The comparison shows consistent transition trends among the three validation levels. For k_{p4} , the appearance of the half-switching-frequency component agrees with the period-doubling boundary predicted near $k_{p4} = 1.40$. For k_{p2} , the emergence and enhancement of low-frequency components agree with the Hopf boundary predicted near $k_{p2} = 1.75$.

7. Conclusions

In this study, a PV–energy storage AC/DC integrated three-port grid-connected power electronic device was constructed to realize coordinated energy conversion among PV generation, energy storage, and the single-phase AC grid. A phase-angle-based simplified discrete model was developed to describe the stability evolution of the system under variations in key control parameters. The results obtained from eigenvalue trajectories, bifurcation diagrams, simulation, and HIL experiments show that the stability boundaries identified by the proposed model are consistent with the degradation of grid current quality observed in the frequency-domain analysis. Specifically, the inverter current-loop gain reaches a period-doubling boundary at $k_{p4} \approx 1.40$, whereas the energy-storage-stage voltage-loop gain reaches a Hopf boundary at $k_{p2} \approx 1.75$. The simulated grid-current THD increases from 2.07% to 3.60% when k_{p4} increases from 1.3 to 1.6 and from 2.07% to 2.27% when k_{p2} increases from 1.7 to 1.8. The two-parameter stability map further shows that the allowable value of k_{p4} remains close to 1.40 when k_{p2} is below approximately 1.60 but decreases rapidly as k_{p2} approaches 1.70–1.75, while the grid-side-inductance

sweep identifies a bifurcation boundary near 4.6 mH. These results verify the correctness and effectiveness of the proposed modeling and stability analysis method in identifying stability boundaries and predicting instability phenomena. They also provide a reference for control parameter tuning and stable operation of the proposed three-port device. Future work will further consider the impact of rapid irradiance fluctuations—which may drive the PV operating point away from the MPP—by incorporating a full nonlinear PV model into the proposed discrete modeling framework. In addition, variations in battery state of charge, weak-grid conditions, and the parallel operation of multiple modules will also be investigated. The proposed topology and modeling method can be further extended to higher-power applications or three-phase grid-connected systems.

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