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Article

Linear Modeling and Regulation Quality Analysis for Hydro-Turbine Governing System with an Open Tailrace Channel

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Abstract: On the basis of the state–space method (SSM), a novel linear mathematical model of the unsteady flow for the tailrace system with an open channel is proposed. This novel model is an elastic linearized model of water hammer. The validity of the model has been verified by several examples of numerical simulation, which are based on a finite difference technique. Then, the complete mathematical model for the hydro-turbine governing system of hydropower station with an open tailrace channel, which is used for simulating the transient process of the hydro-turbine governing system under load disturbance, is established by combining the models of hydro-turbine, generator, governor and open tailrace channel. Finally, according to the complete model, the regulation quality for hydro-turbine governing system with an open tailrace channel under load disturbance is studied, and the effects of open tailrace channel and tailrace surge tank on regulation quality are analyzed. The results indicate that: The open tailrace channel has a strong influence on the regulation quality by observing the water level fluctuations in tailrace surge tank. The surge shows a piecewise periodical change along with the variation in the length of an open channel. The open tailrace channel can be used to improve the regulation quality of hydro-turbine governing system.

Keywords: hydropower station; open tailrace channel; hydro-turbine governing system; linear modeling; regulation quality; state–space method (SSM); load disturbance

1. Introduction

In the case of a hydropower station either along a long tailrace tunnel or at a low downstream water level, an open channel often follows the tailrace tunnel [1,2]. The adoption of open tailrace channel can make the layout of pipelines easier on the one hand, and can reduce the project investment on the other [3,4]. However, the superposition of the gravity wave in an open channel, the mass wave (*i.e.*, the mass/volume of the fluid in the pipe and surge tank) due to water level fluctuation in the surge tank, and the water hammer wave in the pressurized tailrace tunnel will impact the stability and regulation quality of the hydro-turbine governing system under the condition of load disturbance, and make the operation and regulation of the hydropower station extremely complicated.

For the regulation quality for hydro-turbine governing system of hydropower station without open tailrace channel, many researchers have carried out correlational studies. For instance, Guo *et al.* [5–9] discussed the effects of a surge tank and penstock on regulation quality. Fu *et al.* [10] studied the dynamic performance of regulation system with a tailrace surge chamber. Wang and Yang [11] studied the hydraulic control simulation and parameters optimization for water diversion systems. In addition, some novel regulation methods, such as chaos [12], computational fluid dynamics and visualization [13] and multivariable generalized predictive theory [14], have been proposed. But for hydropower stations with open tailrace channel, few current achievements on regulation quality for hydro-turbine governing system can be found.

To evaluate the dynamic performance of the regulation of hydro-turbine governing system in a hydropower station under load disturbance, the whole water and power generation system contain several mathematical sub-models, *i.e.*, the pipeline, the generator, the turbine, and the governor system [15–17]. These mathematical models can be combined in the form of a transfer function to develop an integral governing system along with block diagram for transient numerical simulation and/or for system stability analysis. The water system can always be simulated in the form of a linearized/nonlinear model and is based on either rigid or elastic water hammer phenomenon, but it is only applicable to a pressurized conduit system [18–21]. In the case of a partial differential equation for the hydraulic transient in an open channel, it is hard to draw a simple expression of pressure at the cross-section of discharge, in a similar fashion to that of a pressurized conduit system, as the discharge is associated with both the water depth and the flow velocity. Hence, it is difficult to carry out the study on the regulation quality for hydro-turbine governing system of hydropower station without open tailrace channel.

Aiming to overcome the above problem and difficulty, this article: (1) evaluates the sectional linearized discretion of the control equations for the transient flow both for the pressurized conduit and for the open channel, and (2) attempts to develop a response-based relationship between the input and the output quantities at a designated section of the state–space equation. Then, the state equations for the pressurized conduit and the open channel are coupled, and the model of the water system, as either a module of the time-domain simulation or frequency-domain stability analysis of a hydropower station along with the pressurized flow, the open channel system, and the free flow alternative system of surface pressure is improved. This paper is organized as follows. Section 2: on the basis of the state–space method (SSM), a novel linear mathematical model of the unsteady flow for the tailrace system with an open channel is proposed. The validity of the model has been verified by several examples of numerical simulation. Section 3: the complete mathematical model for the hydro-turbine governing system of hydropower station with an open tailrace channel, which is used for simulating the transient process of the hydro-turbine governing system under load disturbance, is established. According to the complete model, the regulation quality for hydro-turbine governing system with an open tailrace channel under load disturbance is studied, and the effects of open tailrace channel and tailrace surge tank on regulation quality are analyzed.

2. Mathematical Models for the Open Channel and the Pressurized Flow System on the Basis of State–Space Method (SSM)

2.1. Discretion of Control Equations

The continuity equation and the momentum equation describing the transient flow in an open channel and the pressurized conduit are given as follows (we assume that open channel flow do not have a jump in water depth or two values depending on the depth Froude number):

In the case of an open channel [1–4]:

$$\begin{cases} B\dfrac{\partial H}{\partial t}+\dfrac{\partial Q}{\partial x}=0 \\ (1-\dfrac{BQ^2}{gA^3})\dfrac{\partial H}{\partial x}+\dfrac{2Q}{gA^2}\dfrac{\partial Q}{\partial x}+\dfrac{1}{gA}\dfrac{\partial Q}{\partial t}+\dfrac{n^2\chi^{4/3}Q|Q|}{A^{10/3}}=0 \end{cases} \tag{1}$$

Whereas, in the case of a pressurized conduit [1–4]:

$$\begin{cases} \dfrac{\partial H}{\partial t}+\dfrac{\alpha^2}{gA}\dfrac{\partial Q}{\partial x}=0 \\ \dfrac{\partial H}{\partial x}+\dfrac{2Q}{gA^2}\dfrac{\partial Q}{\partial x}+\dfrac{1}{gA}\dfrac{\partial Q}{\partial t}+\dfrac{n^2\chi^{4/3}Q|Q|}{A^{10/3}}=0 \end{cases} \tag{2}$$

Solving Equation (1) linearly at the steady condition yields a set of partial differential equations; the integration of the partial differential equations along space can be used to obtain a Saint-Venant system [1–4] of ordinary differential equations in space having a step length of Δx as follows (note that for the partial differential equations, the boundary conditions are the known discharge and head of inlet section, outlet section and branch sections; the initial condition is the known discharge and head of each section at the initial time of calculation):

$$\begin{cases} k_{2j,1}h_j+k_{2j,2}h_{j+1}+k_{2j,3}q_j+k_{2j,4}q_{j+1}+T_{2j,1}(\dfrac{\mathrm{d}q_j}{\mathrm{d}t}+\dfrac{\mathrm{d}q_{j+1}}{\mathrm{d}t})=0 \\ T_{2j+1,2}(\dfrac{\mathrm{d}h_j}{\mathrm{d}t}+\dfrac{\mathrm{d}h_{j+1}}{\mathrm{d}t})+k_{2j+1,5}(q_{j+1}-q_j)=0 \end{cases} \tag{3}$$

where:

$$a_{j,1} = 1 - \frac{B_{j0}Q_{j0}^2}{gA_{j0}^3} \qquad a_{j,2} = \frac{2Q_{j0}}{gA_{j0}^2} \qquad a_{j,3} = \frac{1}{gA_{j0}}$$

$$c_{j,1} = (-\frac{Q_{j0}^2}{gA_{j0}^3}\frac{\partial B_{j0}}{\partial H} + \frac{3B_{j0}^2Q_{j0}^2}{gA_{j0}^4})\frac{\partial H_{j0}}{\partial x} + \frac{4n^2\chi_{j0}^{1/3}|Q_{j0}|Q_{j0}}{3A_{j0}^{10/3}}\frac{\partial \chi_{j0}}{\partial H} - \frac{10n^2\chi_{j0}^{4/3}B_{j0}|Q_{j0}|Q_{j0}}{3A_{j0}^{13/3}}$$

$$c_{j,2} = \frac{2B_{j0}Q_{j0}}{gA_{j0}^3}\frac{\partial H_{j0}}{\partial x} + \frac{2n^2\chi_{j0}^{4/3}|Q_{j0}|Q_{j0}}{A_{j0}^{10/3}}$$

$$k_{2j,1} = (\frac{c_{j,1}\Delta x_j}{2} - a_{j,1})H_r \quad k_{2j,2} = (\frac{c_{j,1}\Delta x_j}{2} + a_{j,1})H_r \quad k_{2j,3} = (\frac{c_{j,2}\Delta x_j}{2} - a_{j,2})Q_r$$

$$k_{2j,4} = (\frac{c_{j,2}\Delta x_j}{2} + a_{j,2})Q_r \quad k_{2j,5} = Q_r \quad T_{2j,1} = \frac{a_3\Delta x_j}{2}Q_r \quad T_{2j,2} = \frac{B_0\Delta x_j}{2}H_r$$

The nomenclatures in Equations (1)–(3) are presented in the Appendix.

In a similar method as described above, the water hammer equation of a pressurized conduit can be used to obtain a system of ordinary differential equations, as has been shown in Equation (3) for the cases of head and discharge. The parameters $a_{j,1}$, $a_{j,2}$, $a_{j,3}$, $c_{j,1}$ and $c_{j,2}$ have been explained as follows:

$$a_{j,1} = 1 \qquad a_{j,2} = \frac{2Q_{j0}}{gA_{j0}^2} \qquad a_{j,3} = \frac{1}{gA_{j0}} \qquad c_{j,1} = 0 \qquad c_{j,2} = \frac{2n^2\chi_{j0}^{4/3}|Q_{j0}|Q_{j0}}{A_{j0}^{10/3}}$$

where all variables have been described earlier in the case of an open channel.

2.2. Development of the State–Space Equation for Open Channel and Pressurized Flow System

Both the open channel Saint-Venant equation and the pressurized conduit water hammer equation have the same form of an ordinary differential equation after discretization and these can be expressed in the form of a state–space equation [1–4]. It has been assumed that there is an open channel followed by a pressurized flow system, as shown in Figure 1, where sections 1–*n* are meant for the pressurized conduit and sections *n*–*m* are meant for the open channel.

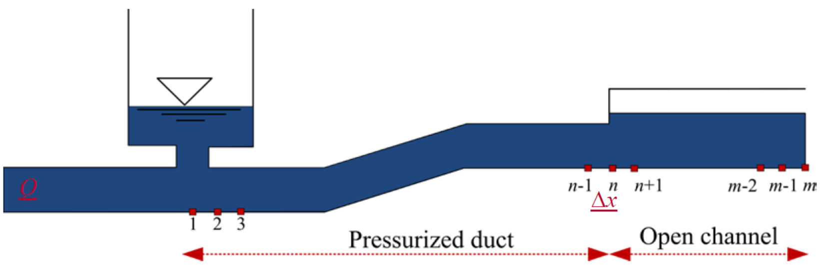

Figure 1. Tailrace system with open channel.

By choosing the head and the discharge of each section as the state variables for the matrix equations, discharges at the inlet of the pressurized conduit are considered as inputs by assuming that the downstream open channel outlet is a reservoir boundary condition. Thereafter, the Equation (3) formed by *m* sections is explained as: $T\dot{X} + KX + LU = 0$, which can be simplified into the following standard form:

$$\dot{X} = AX + B \tag{4}$$

where $A = (T)^{-1}(-K)$ and $B = (T)^{-1}(-L)$.

At the same time, heads at the inlet of the pressurized conduit are considered as outputs and are shown as follows:

$$Y = CX + DU \tag{5}$$

where:

$$\dot{X} = \begin{bmatrix} \dot{h}_1 \\ \dot{q}_2 \\ \dot{h}_2 \\ \dot{q}_3 \\ \dot{h}_3 \\ \vdots \\ \vdots \\ \dot{q}_m \\ \dot{h}_m \end{bmatrix} \quad X = \begin{bmatrix} h_1 \\ q_2 \\ h_2 \\ q_3 \\ h_3 \\ \vdots \\ \vdots \\ q_m \\ h_m \end{bmatrix} \quad T = \begin{bmatrix} 0 & T_{2,1} & 0 & 0 & & \vdots & & & \\ T_{3,1} & 0 & T_{3,1} & 0 & & \vdots & & & \\ & T_{4,1} & 0 & T_{4,1} & 0 & \vdots & & & \\ & 0 & T_{5,2} & 0 & T_{5,2} & \vdots & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & & \vdots & T_{2m,1} & 0 & T_{2m,1} & 0 \\ & & & & \vdots & 0 & T_{2m+1,1} & 0 & T_{2m+1,1} \\ & & & & \vdots & 0 & 0 & a & b \end{bmatrix} \quad L = \begin{bmatrix} T_{21} & k_{23} \\ 0 & -k_{35} \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} k_{2,1} & k_{2,4} & k_{2,2} & & & \vdots & & & \\ 0 & k_{3,5} & 0 & & & \vdots & & & \\ & k_{4,3} & k_{4,1} & k_{4,4} & k_{4,2} & \vdots & & & \\ & -k_{5,5} & 0 & k_{5,5} & 0 & \vdots & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & & \vdots & k_{2m,3} & k_{2m,1} & k_{2m,4} & k_{2m,2} \\ & & & & \vdots & 0 & -k_{2m+1,5} & 0 & k_{2m+1,5} \\ & & & & \vdots & 0 & 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} H_r \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad D = \begin{bmatrix} h_{10} \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad U = [\dot{q}_1, q_1]$$

where a and b are defined in accordance with the boundary conditions of the end section of an open channel, and h_{10} is the head at the initial section of the pressurized conduit at the initial time of calculation.

2.3. Method Verification

To verify the accuracy of the space–state equation that describes the unsteady flow in an open channel and a pressurized conduit, comparisons have been made between the results of the three different flow modes (*i.e.*, the open channel, the pressurized conduit, and the combined system of pressurized conduit and open channel) by the SSM and the numerical results by the finite difference method (FDM).

2.3.1. Open Channel Model

It has been assumed that there is a trapezoid channel with the following dimensions: 1000 m long, 10 m wide at bottom, the slope of the side is 0.5, the coefficient of roughness is 0.014, the longitudinal slope of bed is 0.00015, and it is connected with the reservoir at the upstream, where the water level is assumed to be constant. Then, it follows that the reservoir is at its downstream direction. In a steady flow, the stream flows smoothly, where the depth of water is 3.4 m and the discharge at the intake is 60 m^3/s. We assume that the discharge is reduced linearly from 60 m^3/s to 55m^3/s within a time period of 100 s and, thereafter, the unsteady flow process can be computed by the FDM (Preissmann implicit scheme [2,4]) and the SSM, respectively. The change law of the head at the intake section can be obtained (Figure 2).

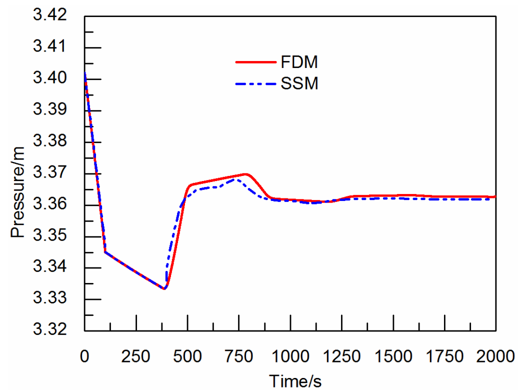

Figure 2. Method verification of the open channel model.

2.3.2. Pressurized Conduit Model

It has been assumed that there is a circular conduit with the following dimensions: the diameter is 5 m; the length is 1000 m; the discharge is 20 m^3/s (*i.e.*, the steady state flow), the pipe roughness is 0.014, and the water hammer wave speed is 1000 m/s, where a reservoir boundary condition at downstream direction exists. The change law of the discharge at the upstream side is given as follows: the discharge value at the outlet is reduced linearly from 20 m^3/s to 15 m^3/s in a time period of 30 s, the upstream head in a steady flow state is 150 m. The transient flow can be computed by the FDM (characteristics method) and the SSM, respectively. Then, the change law of the head at the intake section can be obtained and shown in Figure 3. It should be pointed out that the peak values obtained by SMM are less than the values obtained by FDM. The reason is that Equations (1) and (2) are linearly processed at the steady condition yields, and the results of FDM is more accurate.

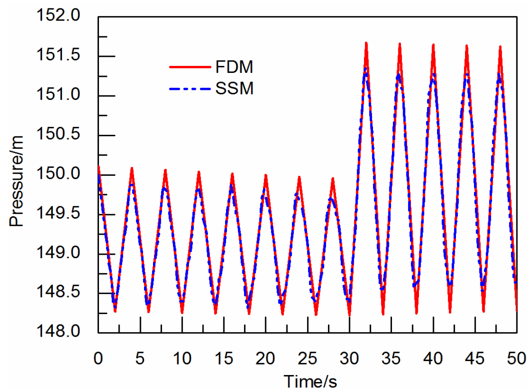

Figure 3. Method verification of pressurized conduit model.

2.3.3. Combined Model of Pressurized Conduit and Open Channel (Pressurized-Open Flow Model)

It has been assumed that a 600 m long pressurized conduit of 5 m diameter is followed by a 300 m long open channel with the following dimensions: 10 m wide at bottom, slope ratio of 1:0.5, longitudinal bottom slope of 0.0002, coefficient of roughness is 0.016, initial discharge is 40 m^3/s, and initial water depth is 2.63 m, which has a reservoir boundary condition at its downstream end. It has been further assumed that the discharge value at the inlet of the pressurized conduit has been reduced linearly from 40 m^3/s to 37 m^3/s in a time period of 50 s. Similarly, the change law of the head at the intake section calculated by the FDM (Preissmann implicit scheme) and the SSM can be obtained, respectively. The results are shown in Figure 4.

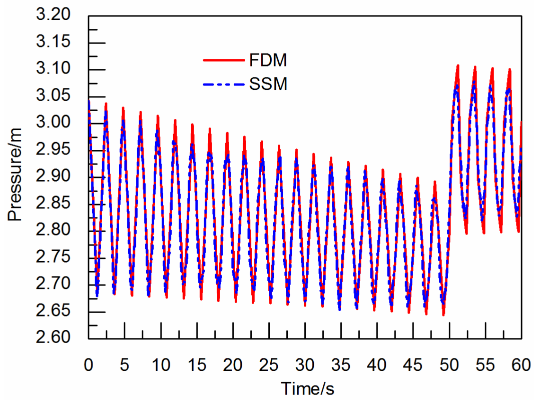

Figure 4. Method verification of pressurized-open flow model.

On the basis of examples and comparisons, as described earlier, it has been inferred that the values obtained from both the methods (*i.e.*, the state–space and finite difference methods) for the cases of waving extreme and the process curves are very close to each other. These observations further conclude that the SSM applied for onward computation of pipe flow in the case of load disturbances by the boundary conditions is accurate; and it can be reliably applied for the computation and analysis for the regulation quality of hydro-turbine governing system with an open tailrace channel under of load fluctuation.

3. Simulation and Regulation Quality Analysis of Hydro-Turbine Governing System

3.1. Simulation Model

The simulation model for the regulation quality analysis of hydro-turbine governing system under load fluctuation in a transient flow mainly includes four sub-models, *i.e.*, the mathematical model of the water system, the turbine model, the generator-grid model and the governor model, which are demonstrated by the following relations:

Turbine model [22,23]:

$$\begin{cases} q = e_{\mathrm{qx}}x + e_{\mathrm{qy}}y + e_{\mathrm{qh}}h \\ m_{\mathrm{t}} = e_{\mathrm{x}}x + e_{\mathrm{y}}y + e_{\mathrm{h}}h \end{cases} \tag{6}$$

Generator rotation model [22,23]:

$$T_{\mathrm{a}}\frac{\mathrm{d}x}{\mathrm{d}t} + e_{\mathrm{n}}x = m_{\mathrm{t}} - m_{\mathrm{g}} \tag{7}$$

Governor model [22,23]:

$$G_{\mathrm{T}}(S) = \frac{Y(s)}{x_{\mathrm{c}}(s)} = \frac{(T_{\mathrm{n}}s+1)(T_{\mathrm{d}}s+1)}{(T_{\mathrm{n}}s+1)[T_{\mathrm{y}}T_{\mathrm{d}}s^2 + (T_{\mathrm{y}} + b_{\mathrm{t}}T_{\mathrm{d}} + b_{\mathrm{p}}T_{\mathrm{d}})s + b_{\mathrm{p}}]} \tag{8}$$

where $Y(s)$ and $x_c(s)$ are the Laplace transformations of the opening and rotation speed of a guide vane. The nomenclatures in Equations (6)–(8) are presented in the Appendix.

The numerical model has been developed by combining and connecting the models of the water system, the turbine model, the generator model and the governor model. The upstream unit of pipeline is simulated as a four-order elastic water hammer linear model [3]. The downstream pressurized-open channel tailrace system is expressed in a state–space equation. The pressure section and the open channel section are separately divided into 10 equal segments. We can solve the 41-order state–space equation in the Simulink of MATLAB as a self-defining function. The effects of b_p and T_y are not considered in the governing model. The T_s is the time constant for the surge tank. When $T_s = 0$, no surge tank is arranged in the tailrace system. The values of the other coefficients are as follows: $e_x = 1$, $e_y = 1$, $e_h = 1.5$, $e_{qy} = 1.0$, $e_{qh} = 0.5$, $e_{qx} = 0.0$, $e_n = 1.0$, $T_a = 10.32$ s, $b_p = 0$, $T_y = 0$ s.

3.2. Simulation without a Tailrace Surge Tank

By considering a hydropower station as an example, its rated head and rated discharge are assumed equal to 120 m and 665 m^3/s, respectively, the upstream pipeline of its headrace and power generation system is 200 m long circular conduit of 12 m in diameter. The downstream section is a tailrace system with an open channel, of which, the pressurized tailrace tunnel is a circular conduit of 14 m in diameter, followed by a flat bottom open channel: 14 m wide at the bottom with a slope ratio of 1:0.5. It is pertinent to mention that the open channel joined the downstream river channel. The initial water level in the open channel is mainly controlled by the downstream river water level, where no surge tank is provided in the tailrace system. By using the above input parameters, the numerical model of power plant regulation system for the tailrace system with an open channel is developed. The step response of the given load disturbance is −0.1. Figure 5 shows the influence of the length of an open channel on the unit rotation speed, when the length of a pressure tailrace tunnel is 500 m, the initial water depth of the open channel is 15 m, $b_t = 0.4$ and $T_d = 6.0$ s. Figure 6 shows the influence of the length on the pressure tailrace tunnel on the basis of unit rotation speed, when the open channel length is 100 m, the initial water depth of the open channel is 15 m, $b_t = 0.4$ and $T_d = 6.0$ s. Figure 7 shows the influence of the governing parameter T_d on the unit rotation speed, when the length of the pressure tailrace tunnel is 500 m, the open channel length is 100 m, the initial water depth of the open channel is 15 m and $b_t = 0.4$.

The wave speed of water hammer in the pressurized conduit may be much faster than that of the wave speed under gravity in the open channel, when there is no surge tank in the tailrace system and the pressurized conduit directly joins the open channel. The fluctuation in the water level in the open channel may not exert substantial influence on the stability of the hydro-turbine governing system. As shown in Figure 5, the influence of the length variation of the open channel and the fluctuation of the unit rotation speed is small. For the influence of the pressure tailrace tunnel length on the unit rotation speed, as indicated in Figure 6, the shorter the pressure tailrace tunnel length is, the smaller the water flow inertia time constant would be; the faster the rotation speed attenuation, the better the regulation quality. Then, as shown in Figure 7, the larger the governor parameters are, the easier the governing system will be operated under stable conditions. Keeping this in view, the regulation quality of the governing system depends on the tailrace tunnel length and the governor parameters, when the tailrace system has an open channel but no surge tank, and, when the influence of the water level fluctuation in an open channel is weak.

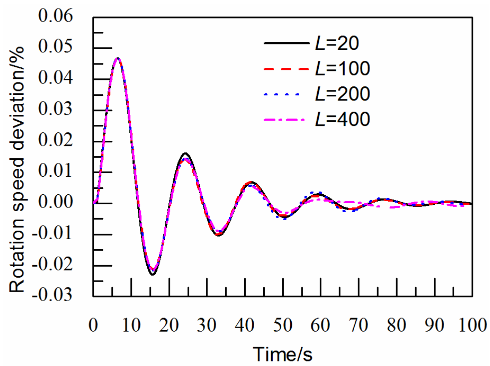

Figure 5. The influence of an open channel length (m) on the unit rotation speed.

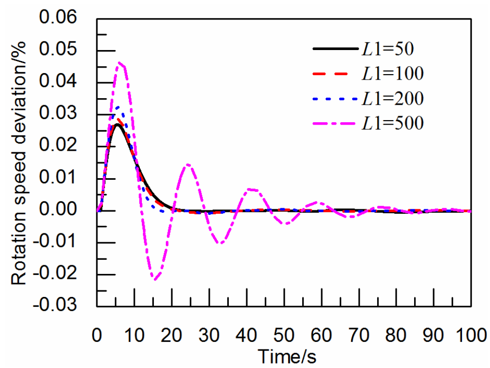

Figure 6. The influence of a pressure flow length (m) on the unit rotation speed.

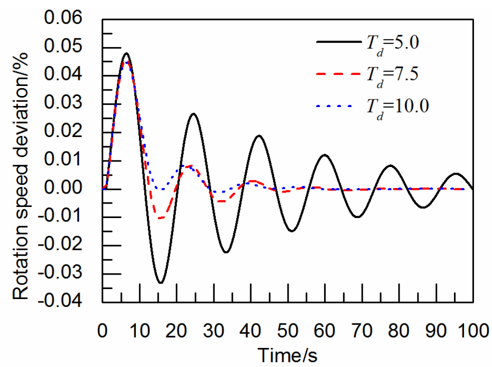

Figure 7. The influence of T_d (s) on the unit rotation speed.

3.3. Simulation with a Tailrace Surge Tank

In the case of the tailrace system having a surge tank, the period of mass waves in the surge tank is the same order of magnitude as the period of gravity waves in the open channel. The fluctuation of water level in the open channel has a noticeable effect on the surge level fluctuations in the surge tank. Next, the fluctuating process of the unit rotation-trailing wave is affected by the level fluctuation in the surge tank, and the period of these two fluctuations are fairly close to each other. Therefore, the effect of the rotation speed on the regulation quality can be analyzed in an indirect way by analyzing the effects of relevant parameters on the wave surge in the surge tank.

Basic parameters of the earlier described example are principally the same, except that a tailrace surge tank has been introduced in front of the pressurized conduit.

3.3.1. The Influence of the Length Variation of an Open Channel on the Regulation Quality

Figures 8 and 9 show the influence of the length variation of an open channel on the fluctuations of surge level in a surge tank. In this regard, the following two aspects are important: (1) the influences of the length variation of the open channel on the amplitude and period of the surge level fluctuation in the surge tank reflect certain stage periodicity. In some specific lengths of the open channel, water level fluctuation in the open channel would noticeably affect the surge level fluctuation in the surge tank, the waveforms of the surge level fluctuation in the surge tank would show a non-sinusoidal feature, and these specific lengths (shown in the following examples) often turn out to be turning points of the level amplitude variation in the surge tank. For instance, when the length of the open channel increases gradually from 50 m to 200 m, the period of the surge level fluctuation in the surge tank get longer while its amplitude reduces. When the length of the open channel increases gradually from 250 m to 450 m, the period of the surge level fluctuation in the surge tank get longer. When the length of the open channel increases gradually from 500 m to 700 m, the period of the surge level fluctuation in the surge tank get longer while its amplitude reduces. It is pertinent to note that with the increase of the length of the open channel, the period of the surge level fluctuation in the surge tank grows in a periodical manner, not in a persistent way. For example, when the open channel length is 200 m, the surge level fluctuation period in the surge tank may be larger than that of the 250 m long open channel. (2) Because of the stage periodicity of the influence of an open channel length on the surge level fluctuation in the surge tank, it may either attenuate or intensify alternately when the open channel length increases. This suggests the complexity of the influence of an open channel length on the regulation quality.

3.3.2. The Influence of the Open Channel Water Depth on the Regulation Quality

The water depth in the open channel is pertinent for the propagation speed of the gravity wave and is influenced by the surge level fluctuation in the surge tank. Figure 10 shows the influence of initial water depth on the surge level fluctuation under the different channel lengths. Within a possible range of variation (*i.e.*, 10–20 m, because the shallow water equations used in the formulations is reasonable for that variation range of the water depth in the open channel, as the velocity is only 1.385 m/s when the water depth is 20 m) for the water depth, the amplitude of surge level fluctuation in the surge tank may increase gradually along with the deepening of the initial water depth in the open channel.

The influence of the water depth in the open channel on the regulation quality is noticeable, which can be represented in the following ways: (1) When the surge tank area and the open channel length are definite, with the increase of water depth in the open channel, the regulation quality gets worse and the surge level fluctuation in the surge tank may take on a gradual transition trend from attenuation to intensification, as shown in Figure 10. (2) Some specific open channel lengths can still guarantee the hydro-turbine governing system has a sound regulation quality. For instance, when the open channel length is 200 m, as shown in Figure 10b, even if the initial water depth in the open channel rises to 20 m, the surge level in the surge tank still shows an attenuating trend.

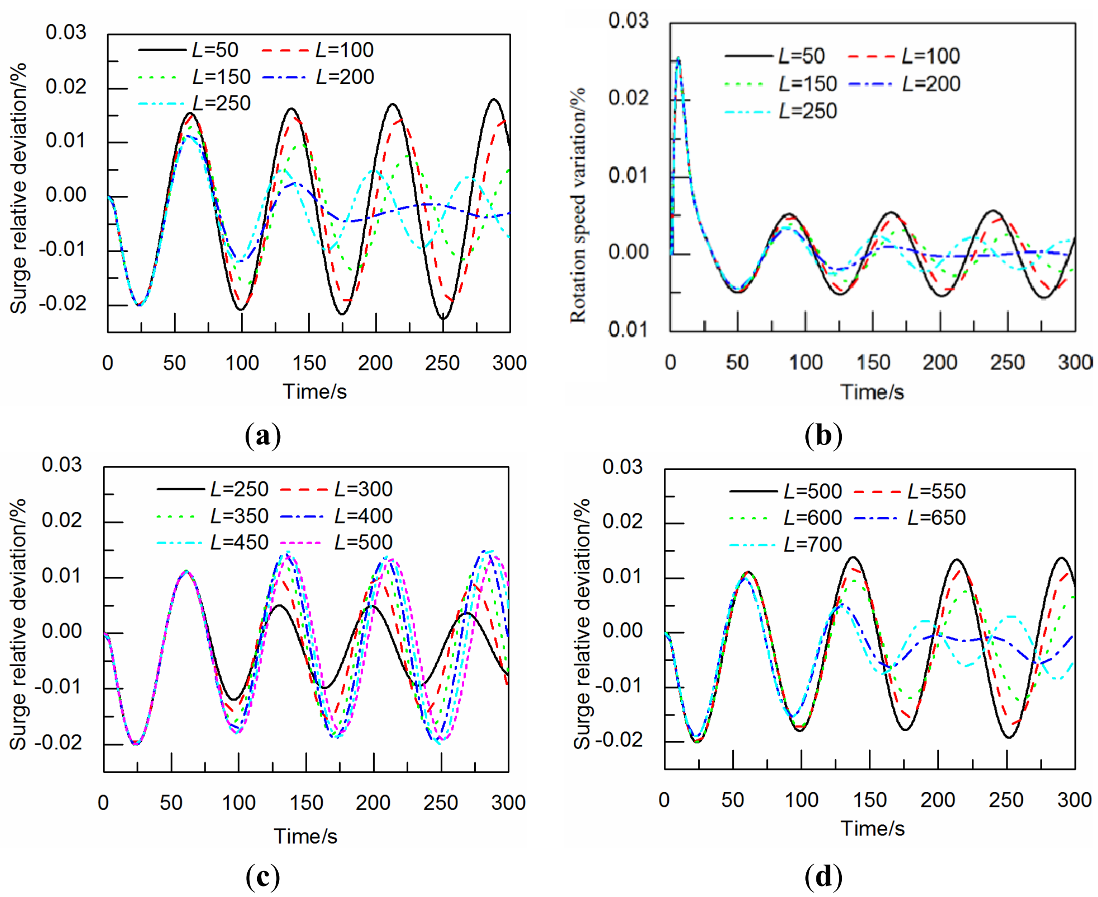

Figure 8. The influence of an open channel length (m) on surge and rotation speed: (**a**) surge variation, when open channel length (*L*) is ranged between 50 m and 250 m; (**b**) rotation speed variation, when open channel (*L*) is ranged between 50 m and 250 m; (**c**) surge variation, when open channel length (*L*) is ranged between 250 m and 500 m; (**d**) surge variation, when open channel (*L*) is ranged between 500 m and 700 m.

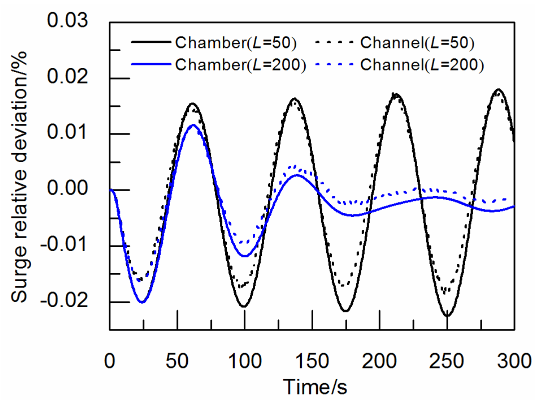

Figure 9. Comparison of surge waves at surge tank and at open channel head.

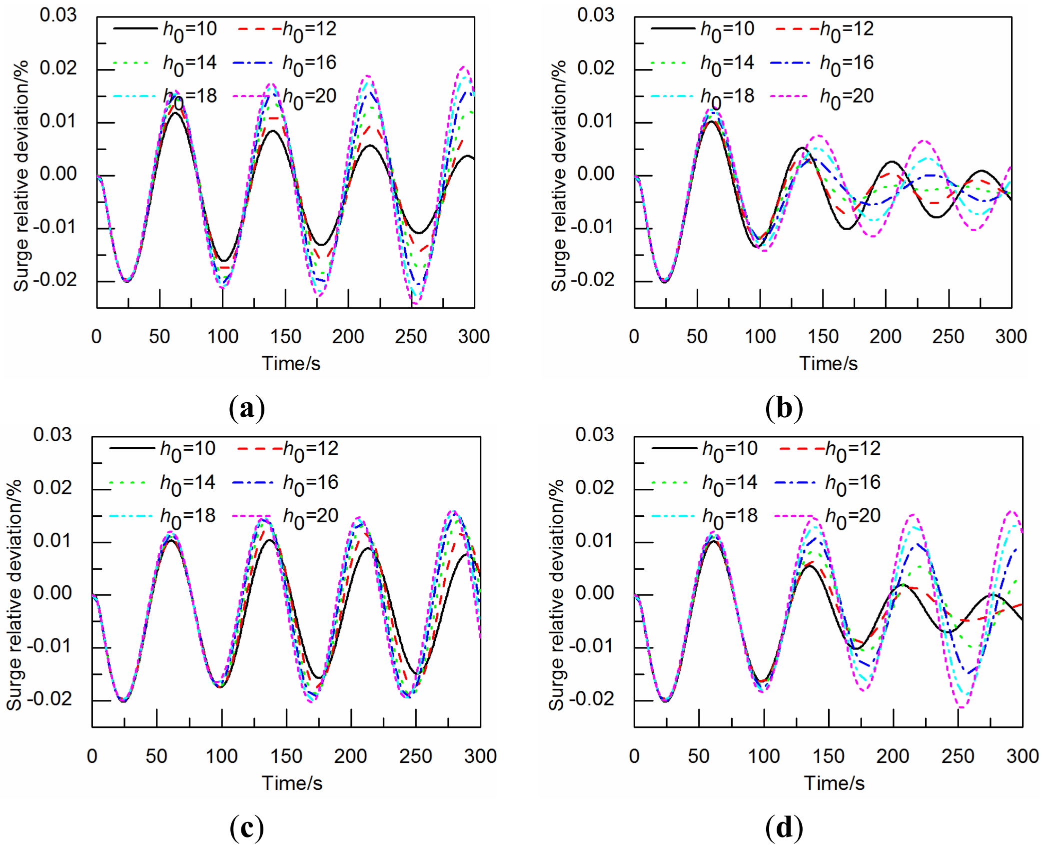

Figure 10. The influence of water depth in the open channel on the regulation quality: (**a**) L = 100 m; (**b**) L = 200 m; (**c**) L = 400 m; (**d**) L = 600 m.

3.3.3. Influence of the Surge Tank Area on the Regulation Quality

The initial water depth in the open channel is maintained at 15 m, and the surge tank area increases from 300 m^2 to 500 m^2. With the increase of the surge tank area, the surge level fluctuation attenuation gradually accelerates, the period gradually becomes longer, and the regulation quality gradually gets better. Besides considering the open channel length variation, the increase of surge tank area will also affect the improvement of regulation quality. As shown in Figure 11, when the open channel length values 100 m and 400 m, respectively, with the increase of surge tank area, the waving shapes (mainly mean the amplitude of the wave) in the surge tank show an attenuation tendency from intensification. When the open channel length values 200 m, the waving shapes in the surge tank attenuate quickly, and the fluctuation overlapping shows favorable effects on the regulation quality.

3.3.4. The influence of Considering an Open Channel or not on the Regulation Quality

When the water depth in the open channel values 15 m, Figure 12 shows the influences of considering an open channel or not on the regulation quality: (1) When the surge tank area is variable, the consideration of the open channel has a favorable influence on the regulation quality; and (2) when the open channel length is variable, the regulation quality becomes better if the open channel effect is considered. However, in some specific channel lengths, such as Figure 12b, when the open channel length is 200 m, the regulation quality of hydro-turbine governing system with an open channel is obviously superior to that without an open channel.

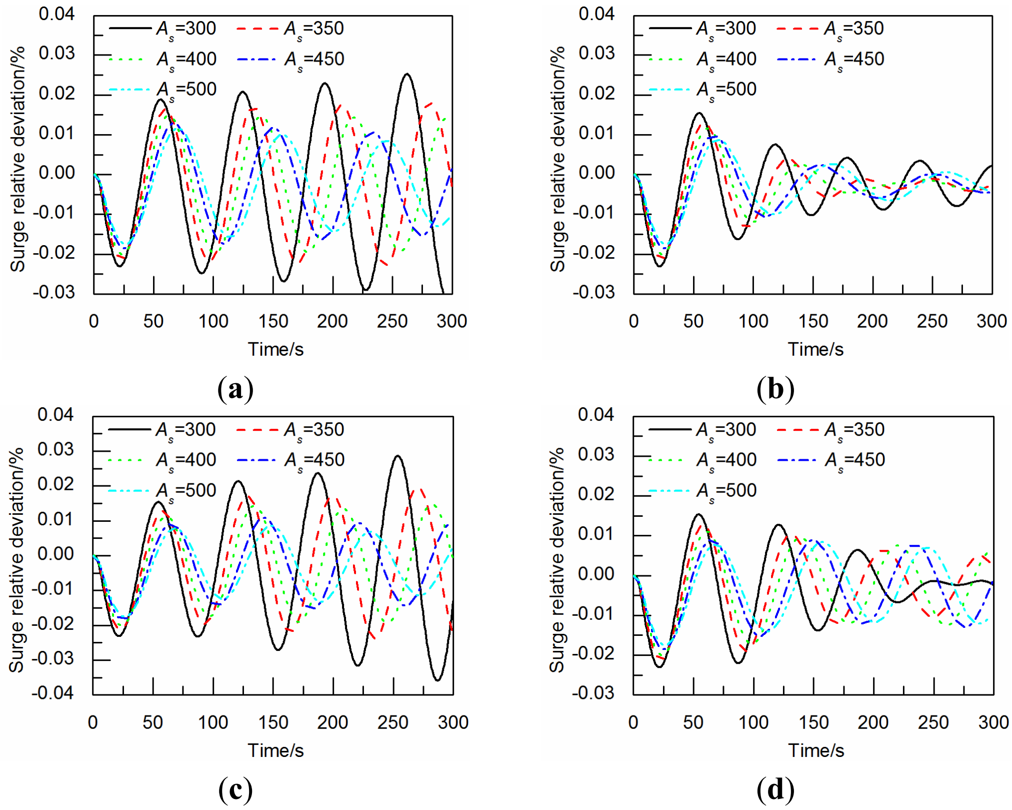

Figure 11. The influence of surge tank area on the regulation quality: (**a**) L = 100 m; (**b**) L = 200 m; (**c**) L = 400 m; (**d**) L = 600 m.

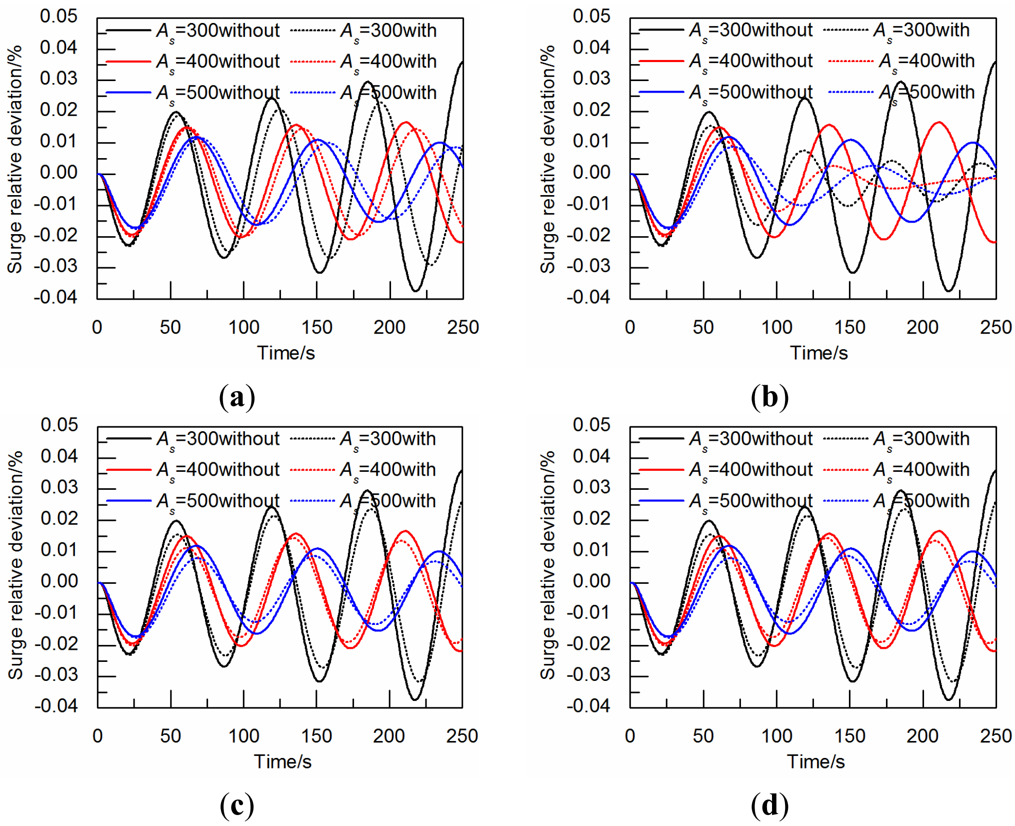

Figure 12. The influence of considering an open channel or not on the regulation quality: (**a**) L = 100 m; (**b**) L = 200 m; (**c**) L = 400 m; (**d**) L = 600 m.

4. Conclusions

By using SSM, a novel elastic water hammer model is developed along with other models of hydro-turbine governing system for the case of the tailrace system with an open channel. Then, a numerical model for load disturbance transient process is established. Finally, according to the complete model, the regulation quality for hydro-turbine governing system with an open tailrace channel under load disturbance is studied, and the effects of open tailrace channel and tailrace surge tank on regulation quality are analyzed. The major conclusions are summarized as follows:

(1) The SSM can reliably be applied to the computation and analysis for the regulation quality of hydro-turbine governing system with an open tailrace channel under load fluctuation.

(2) For the case of a hydropower station without tailrace surge tank, the influence of the open tailrace channel on the regulating quality of hydro-turbine governing system under disturbance can be neglected.

(3) Under load disturbance, the open tailrace channel influences the regulating quality of hydro-turbine governing system by affecting the surge level fluctuation in the surge tank, and its length is a dominant factor. The regulation quality of hydro-turbine governing system can be improved by considering the influence of the water level fluctuation in the open channel.

In future work, this model and method should be improved to be applied for a greater range of the operating conditions, such as load rejection, unit starting up, load adjustment, and frequency adjustment.

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Author Contributions

Jiandong Yang and Mingjiang Wang performed programming works, simulations and discussions, and wrote the manuscript; Chao Wang conducted part of case studies and discussions; and Wencheng Guo engaged in the discussion, coordinated the main theme of this paper and revised the manuscript. All of the authors supervised and approved the final version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

Nomenclature

x	Position along the axis of the pipeline
Q	Discharge
A	Cross-sectional area of the water passage
n	Coefficient of roughness
g	Acceleration of gravity
Δx	Step length

B_{j0}, Q_{j0}, A_{j0}, χ_{j0}	Water surface breadth, the discharge, the wetted area and the wetted perimeter of section J at the initial timestep
Δx_j	Space step length between the sections j and $j + 1$
x, y, h	Relative values of the rotation speed, the aperture and the head
T_a	Hydro-turbine unit inertia time constant
m_g	Relative difference of generator and the drag torque
t	Time
H	Head
B	Breadth of the water surface
χ	Wetted perimeter
α	wave speed of the water hammer
H_r, Q_r	Initial head and the initial discharge
h_j, Q_j	Relative deviation values of the head and the discharge at section j
e_x, e_y, e_h, e_{qx}, e_{qy},e_{qh}	Transfer coefficients for the different turbine characteristics
q m_t	Relative discharge and relative moment of the turbine
e_n	Comprehensive self-regulation coefficient of the turbine-generator
T_n, T_d, T_y, b_p, b_t	Parameters of the governor

Appendix

The relative deviation values are defined as follows (taking h_j as example):

$$h_j = (H - H_r)/H_r$$

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