Article

Pan He * and Jian Xia

College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China; jxia@nuaa.edu.cn
* Correspondence: he_pan@nuaa.edu.cn

Abstract: Accurate prediction of the aerodynamic characteristics of wind rotors subjected to various wind profiles is of considerable importance in the aerodynamics and structural design of wind turbines. As a very complex atmospheric phenomenon, the impact of a low-level jet (LLJ) on the aerodynamic characteristics of wind rotors is becoming more and more significant with the increase in wind turbine height. Additionally, during calculating the aerodynamic characteristics of the wind rotor, the known wind speed, rotor speed, and blade-pitch angle are generally required. However, when the wind profile is in the LLJ condition, it is difficult to determine the blade-pitch angle and rotor speed. Therefore, in this paper, the blade-element-momentum (BEM) method is exploited by considering the coupling with the generator-torque controller and blade-pitch controller. In order to solve the problem of the unknown rotor speed and blade-pitch angle under the LLJ condition, a C++ code is developed. Then, the influence of the LLJ on the aerodynamic characteristics of the wind rotor is exclusively examined. The research results show that the calculation method can precisely evaluate the rotor speed, blade-pitch angle, and aerodynamic characteristics of the wind rotor. The influence of the LLJ on the aerodynamic loads of the wind rotor is greater than that of the wind shear. When the LLJ is placed inside the rotor swept area, the aerodynamic loads of the blade exhibit two local maximums and local minimums with the variation of the azimuth angle in a rotation period. The closer the LLJ height is to the hub height, the greater the average aerodynamic loads of the wind rotor are, and the smaller the amplitude of aerodynamic loads of the blade is relative to the average value. When the LLJ height is positioned outside the rotor swept area, the change law of the aerodynamic loads of the blade would be similar to that of the wind subjected to a very strong wind shear inflow. This study provides a crucial reference for a more rational assessment of the aerodynamic characteristics of wind turbines under the action of complex wind profiles, as well as revealing the influence of the LLJ on the aerodynamic characteristics of wind turbines.

Keywords: wind turbine; wind rotor; aerodynamic characteristics; low-level jet (LLJ); control system

1. Introduction

In recent years, due to the excessive exploitation of non-renewable energy, environmental pollution has become increasingly serious, leading to the great development of the renewable energy technology. As the main device for converting wind energy into electrical energy, wind turbines have turned from onshore to offshore, and their sizes have become larger and larger, and the inflow conditions have become more and more complex [1]. The aerodynamic characteristics of wind rotors can be correctly estimated in the presence of different inflow conditions, which is of great significance to the aerodynamic and structural design of wind turbines.

Wind shear (WS) represents the most representative inflow conditions. Kim et al. [2] examined the effects of atmospheric stability, turbulence intensity, and wind shear index on wind turbine power and annual power generation. Dimitrov et al. [3] proposed a suitable
wind shear model for flat terrain, reducing the inaccuracy of the fatigue load prediction of large wind turbines. Additionally, as a common inflow condition, extreme winds, such as gusts, tropical cyclones, and low-level jets (LLJ), often occur depending on geographic and climatic conditions. Lyu [4] investigated the coupled response of wind rotors due to the extreme shear wind, extreme gust, extreme wind direction, and extreme turbulent wind. By performing an experimental study, Wang [5] examined the wake characteristics of wind turbines under tropical cyclone conditions. Walter et al. [6] studied the structural response of wind turbines due to the LLJ conditions. Na et al. [7] explored the flow field characteristics of wind turbines in the presence of the LLJ conditions by the large-eddy simulation method. Among them, the influence of the LLJ on the aerodynamic characteristics of the wind rotor is the most noticeable with the increase in the hub height [8,9]. When the LLJ occurs, the inflow condition becomes very complicated, characterized by strong winds at the height of the LLJ and strong wind shear. Gutierrez et al. [10] found that the wind speed would significantly grow, and the turbulence intensity would lessen under the LLJ condition, affecting the wind turbine for more than 40 m from the ground. Wilczak et al. [11] found that the wind capacity factor magnifies by more than 60% due to the LLJ condition. Therefore, it is necessary to explore the influence of the LLJ on the aerodynamic characteristics of the wind rotor. Generally, the LLJ can be characterized by the LLJ height, LLJ strength, and wind speed profile shape [12,13]. Few studies have been carried out on the aerodynamic characteristics of wind turbines based on the LLJ structural parameters, such as LLJ intensity, LLJ height, and wind speed profile shape.

The calculation methods of aerodynamic characteristics of wind turbine rotors include the blade-element momentum (BEM) method [14,15], vortex wake (VW) method [16,17], and computational fluid dynamics (CFD) method. Generally, the BEM is affected by Reynolds number, three-dimensional rotation effect, tip losses, and root losses, and its predicted results are not very accurate. Therefore, it is necessary to correct the BEM via an appropriate approach, such as three-dimensional rotation correction, and tip and root losses correction. Due to the advantages of the BEM in computational efficiency, this methodology has been extensively employed in the design of wind turbine aerodynamics. The well-known GH-Bladed commercial software [1] and OpenFAST [2] open-source software basically exploited this approach. The CFD is a calculation method developed with the rise of computer technology. This method can directly and accurately analyze the complex flow field with high calculation accuracy, and, in addition, to calculating the aerodynamic force of the wind turbine, the details of the flow field can be readily extracted. However, the CFD method needs to solve a large number of Navier–Stokes equations, indicating huge computational resources and time. Furthermore, its accuracy relies on the selection of the turbulence model, the discrete format of equations in time and space, and the quality of the grid to a certain extent. Regodeseves [18] scrutinized the unsteady flow of a horizontal axis wind turbine with the CFD method, focusing on the influence of the engine compartment, tower, and blade rotation on the induced zone and near wake. Frederik [19] calculated the unsteady aerodynamic force of the wind turbine and carried out numerical calculations on the wind wheel and tower in the presence of the shear inflow condition. The VW method assumes blade and wind turbine wake as a series of vortices, which can be divided into the rigid vortex wake, predetermined vortex wake, and free VW method. The calculation accuracy of this approach is lower than that of the CFD, and its calculation efficiency is lower than that of the BEM. The model also requires to be modified like the BEM method. Therefore, it is not broadly implemented in the examination of the aerodynamic characteristics of the wind turbine. The advantages and disadvantages of different methods have been shown in Table 1. However, irrespective of the calculation method exploited for calculating the aerodynamic characteristics of wind rotors, the wind speed, the rotor speed, and the blade-pitch angle are all known input parameters [20–22]. Hence, when the wind speed profile is non-uniform or very complex, the rotor speed and blade-pitch angle depend on the equivalent uniform wind speed corresponding to the wind speed profile. There are usually two methodologies for evaluating the equivalent wind
speed (EWS) [23,24], one based on the kinetic energy theory and the other based on the equivalent torque. Nevertheless, it is difficult and troublesome to calculate the equivalent wind speed, and the relevant established model is not universal [23]. Irrespective of the used method, only the accuracy of the average value of aerodynamic characteristics can be guaranteed over one rotation cycle. As a general rule, the greater the difference between the equivalent wind speed and the inflow wind speed on the blade element is, the greater the aerodynamic characteristic error calculated using the rotor speed and blade-pitch angle corresponding to the equivalent wind speed is.

Table 1. The calculation methods of aerodynamic characteristics of wind turbine rotors.

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantage</th>
<th>Disadvantage</th>
<th>Limited</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFD</td>
<td>The complex flow field can be solved directly and accurately</td>
<td>Complex pre-processed, very large amount of calculation, the calculation results rely on CFD the method</td>
<td>Calculate cost</td>
</tr>
<tr>
<td>BEM</td>
<td>High efficiency and stability</td>
<td>Low accuracy, Model should be modified</td>
<td>Accuracy of the unsteady solution</td>
</tr>
</tbody>
</table>
| VW     | More efficient than the CFD  
        More accurate than the BEM | Low accuracy, model should be modified | Accuracy of the unsteady solution, calculate cost |

In summary, the previous studies mainly focused on the aerodynamic characteristics of wind turbines under different inflow conditions. In terms of content, the influence of the LLJ on the aerodynamic characteristics of wind turbines is increasing; however, few investigations have been devoted to this issue. In the present research method, the wind speed, the rotor speed, and the blade-pitch angle of blades are all known input parameters, but the present method has particular shortcomings that clearly mentioned above. Therefore, in this paper, firstly, in order to solve that the difficulty of the unknown rotor speed and blade-pitch angle under non-uniform inflow or complex inflow conditions, the BEM theory would be coupled with the generator-torque controller and blade-pitch controller when calculating the aerodynamic characteristics of the rotor, and a C++ program will be developed for implementing the proposed method. The chief reason for employing the BEM is that the iterative solution of the rotor speed and blade-pitch angle requires more calculation steps. Secondly, the influence of the LLJ intensity on the aerodynamic characteristics of the wind turbine can be examined and compared with the WS inflow. Finally, the impact of the LLJ height on the aerodynamic characteristics of the wind turbine is going to be explored as well. The LLJ intensity represents the local maximum of the wind speed on the wind speed profile, and the LLJ height indicates the vertical height of the LLJ intensity.

2. Wind Turbine, Methods, and Scheme

2.1. Wind Turbine

In the present work, the National Renewable Energy Laboratory (NREL) 5 MW wind turbine is taken into account as the calculation object [25]. The wind turbine has complete shape and control parameters. Whether it is implemented as a standard model for verification of the method or as a research object of wind turbine performance, it has been broadly utilized both at home and abroad [21,26,27]. In Table 2, the main parameters of the NREL 5 MW wind turbine have been listed, and other detailed factors can be found in Ref. [25].
Table 2. Main parameters of NREL 5 MW wind turbine.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating/MW</td>
<td>5</td>
<td>Rated Rotor Speed/rpm</td>
<td>12.1</td>
</tr>
<tr>
<td>Rotor Diameter/m</td>
<td>126</td>
<td>Rated wind speed/m·s⁻¹</td>
<td>11.4</td>
</tr>
<tr>
<td>Overhang/°</td>
<td>5</td>
<td>Rotor configuration</td>
<td>3</td>
</tr>
<tr>
<td>Precone/°</td>
<td>2.5</td>
<td>Hub Diameter/m</td>
<td>150</td>
</tr>
</tbody>
</table>

2.2. Methods

2.2.1. The WS and LLJ Models

The WS model includes both logarithmic and exponential law models. Herein, the exponential law model is employed, as displayed by:

\[ v(H) = v_r \left( \frac{H}{H_r} \right)^a = v_r \left( \frac{r \cos \theta + H_s}{H_r} \right)^e \]  

(1)

where \( H \) represents the blade element height, \( v(H) \) denotes the wind speed at \( H \), \( H_r \) is the height of the reference point, \( v_r \) is the wind speed at the reference point, \( e \) is the wind shear index, \( r \) denotes the distance between the blade element and the hub center, and \( \theta \) represents the azimuth angle.

The LLJ-based models are usually summarized using observed data, but the data are regional; therefore, an engineering-based model is adopted in the present work [8], and the LLJ is regarded as a superposition of the WS and free jet, which is expressed as follows:

\[ v(H) = \left\{ v_r + v_m \left[ 1 - \tanh^2 \left( C_s \frac{H - H_s}{H_m} \right) \right] \right\} \left( \frac{H}{H_r} \right)^e \]  

(2)

where \( H_s \) denotes the height of maximum velocity of the free jet, \( v_m \) represents the maximum velocity of the free jet, and \( C_s \) is the shape parameter, depending on the shape of the free jet velocity profile. Other parameters have the same physical meaning as those in the WS model.

Figure 1 represents a schematic diagram of the WS and LLJ wind profile. In the demonstrated figure, \( \bar{v}_j \) and \( H_j \) in order are the intensity and height of the LLJ.

![Figure 1. Schematic representation of the WS and LLJ wind profile.](image)

2.2.2. BEM Theory

The BEM belongs to the family of classical methods utilized for solving the aerodynamic loads of the wind rotor. Usually, the Newton iteration method or the fixed-point iteration method is employed to analyze the axial and tangential induction factors to realize
the calculation. However, the singularity associated with the axial induction factor makes it problematic for the traditional methods to converge during the solution process [28]. Therefore, the solution method of Ning [29] is implemented in the present work, the method has been used in FAST software, and it has been proved that this method cannot only be utilized for the frontal wind calculation. When the wind turbine has a yaw angle, it only needs a simple modification to the calculation method [30]. Figure 2 presents a flowchart of two distinct methods. According to the given flowchart in Figure 2b, we give the main calculation steps of the BEM method.

![Flowchart](image)

**Figure 2.** The flowchart of the BEM theory. (a) The traditional method. (b) Ning’s method.

In order to describe the BEM, Figure 3 shows the relationship between the velocity and force on the blade element.

![Diagram](image)

**Figure 3.** Schematic representation of the relationship between the velocity and force on the blade element.
In this figure, $\alpha$ denotes the aerodynamic angle of attack, $\beta$ represents the sum of the installation angle and blade pitch angle, and $\phi$ is the inflow angle. Clearly, we can write:

$$\alpha = \phi - \beta$$  \hspace{1cm} (3)

The factors $C_l$ and $C_d$ in order denote the lift coefficient and drag coefficient, in which their magnitudes are functions of the attack angle and Reynolds number. These factors can be mathematically expressed by:

$$C_l = f_l(\alpha, \text{Re})$$

$$C_d = f_d(\alpha, \text{Re})$$  \hspace{1cm} (4)

Herein, the three-dimensional rotation effect is considered, and the two-dimensional airfoil lift-drag coefficient is corrected by the Du-Selig model. Subsequently, the thrust coefficient $C_n$ and the tangential force coefficient $C_t$ are calculated by the lift–drag coefficient as follows:

$$C_n = C_l \cos \phi + C_d \sin \phi$$

$$C_t = C_l \sin \phi - C_d \cos \phi$$  \hspace{1cm} (5)

Then, the thrust force $F_n$ and the tangential force $F_t$ of the blade-element can be evaluated per the following relations:

$$F_n = \frac{1}{2} \rho W^2 C_n c$$

$$F_t = \frac{1}{2} \rho W^2 C_t c$$  \hspace{1cm} (6)

where $W$ denotes the resultant velocity, $\rho$ represents the air density, and $c$ is the chord of the blade-element. Furthermore, the resultant velocity is expressed by:

$$W = \sqrt{(V(1-a))^2 + (r\omega(1+b))^2}$$  \hspace{1cm} (7)

where $V$ represents the inflow velocity, $a$ is the axial induction factor, and $b$ is the tangential induction factor. In Ning’s method, two dimensionless parameters $\kappa_a$ and $\kappa_b$ are defined based on the momentum theorem in the following form:

$$\kappa_a = \frac{\sigma C_n}{4F \sin^2 \phi}$$

$$\kappa_b = \frac{\sigma C_t}{4F \sin \phi \cos \phi}$$  \hspace{1cm} (8)

where $F$ represents the tip and root correction of the blade, and Prandtl’s correction is employed in the present work. For evaluating the axial induction factor, when the axial induction is less than 0.4, the momentum theory is valid, and the axial induction factor based on the momentum theory is taken as:

$$a = \frac{\kappa_a}{1 + \kappa_a}$$  \hspace{1cm} (9)

The corresponding $\phi$ is greater than 0, and $\kappa_a$ is smaller than $2/3$. When the axial induction factor is greater than 0.4 and less than 1, the momentum theorem is invalid. Using Buhl’s modified Glauert correction model, the axial induction factor can be calculated as:

$$a = \frac{\gamma_1 - \sqrt{\gamma_2}}{\gamma_3}$$  \hspace{1cm} (10)

In Equation (10), the factors $\gamma_1$, $\gamma_2$, and $\gamma_3$ are defined by:

$$\gamma_1 = 2F \kappa_a - (10/9 - F)$$

$$\gamma_2 = 2F \kappa_a - F (4/3 - F)$$

$$\gamma_3 = 2F \kappa_a - F (25/9 - 2F)$$  \hspace{1cm} (11)
When $\gamma_3$ approaches zero, the axial induction factor reduces to:

$$a = 1.0 - \frac{1.0}{2\sqrt{\gamma_2}}$$  \hspace{1cm} (12)

At this time, $\phi$ is greater than 0, and $\kappa_a$ is greater than $2/3$. When the axial induction factor is greater than 1, $\phi$ is less than 0, and $\kappa_a$ is greater than 1.

$$a = \frac{\kappa_a}{\kappa_a - 1}$$  \hspace{1cm} (13)

When the values of $\phi$ and $\kappa_a$ are less than 0 and 1, respectively, it is impossible to seek a solution based on the BEM, and the axial induction factor is set equal to zero. In such a case, the tangential induction factor can be stated by:

$$b = \frac{\kappa_b}{1 - \kappa_b}$$  \hspace{1cm} (14)

Finally, by iterating $\phi$, a solution to residual Equation (15) can be found when it is set equal to zero, and thereby, the aerodynamic force of the blade-element can be evaluated:

$$f(\phi) = \begin{cases} 
\sin \phi - \frac{\cos \phi}{\lambda(1-b)} & \phi > 0 \\
\sin \phi(1 - \kappa_a) - \frac{\cos \phi}{\lambda(1-b)} & \phi <= 0 
\end{cases}$$  \hspace{1cm} (15)

where $\lambda$ is the local tip-speed ratio.

### 2.2.3. Aerodynamics–Controller Interaction Method

Wind turbines are equipped with variable speed-constant frequency generators. In addition, conventional variable-speed and variable blade-pitch configurations are chosen. In order to maximize the captured power in the below-rated wind speed range, a generator-torque controller is designed. To achieve the goal of constant power in the above-rated wind speed range, the blade-pitch controller is designed.

Figure 4 represents the flowchart of the aerodynamics–controller interaction method. In the figure, “step” and “nstep” denote the current calculation step and the total number of calculation steps, respectively. The generator-torque controller is utilized to update the rotor speed of the wind turbine; however, during updating the blade-pitch angle, the blade-pitch controller is implemented. The LLJ wind speed profiles in Section 2.2.1 indicate inflow conditions, and the BEM theory explained in Section 2.2.2 is employed for calculating aerodynamic characteristics of wind rotor. The dotted line indicates that the updated rotor speed and blade-pitch angle have been exploited in the calculation of the next time step.

Figure 5 demonstrates a speed-torque characteristic curve of a variable-speed constant-frequency generator, which is the basis for variable-speed control of the wind turbine. The generator speed can be controlled by following the curves specified by ACEF or ADGF; however, the branches identified by AC and EF have torque jumps at the same speed, which is not easy to achieve in operation, so the branch of ADGF is chosen in the following. The torque control can be divided into five regions, which are specified by $z_1$, $z_2$, $z_3$, $z_4$, and $z_5$. Region $z_1$ represents the region before the generator cut-in torque when the generator is not connected to the grid. Region $z_3$ denotes the region of the tip speed ratio optimal of the wind rotor, and the following relationship between the generator torque and the generator speed can be established:

$$T_g = K\omega^2$$  \hspace{1cm} (16)
in which the parameter $K$ can be written as,

$$ K = \frac{1}{2} \rho \pi R^5 C_p \lambda_{opt} G $$

(17)

where $T_g$ denotes the generator torque, $\omega$ represents the generator speed, $R$ is the swept radius of the wind rotor, $C_p$ is the maximum wind energy utilization coefficient, $\lambda_{opt}$ represents the optimal tip speed ratio, and $G$ denotes the gearbox ratio. The regions $z_2$ and $z_4$ in order represent the linear transition ranges of the regions $z_1$ and $z_3$ and the regions $z_3$ and $z_5$. The rated torque is constant in the region $z_5$, realized by controlling the rotor-collective blade-pitch angle by the blade-pitch controller.

Figure 4. Flow chart of aerodynamics–controller interaction method.
The rotor speed is controlled by the equation of motion given by Equation (18), while the blade-pitch angle can be controlled by Equation (19) \[25\]:

\[
T_a - GT_g = (I_r + G^2 l_g)\Delta \Omega
\]

\[
\Delta \theta = (K_p G \Delta \Omega + K_I \int_0^t G \Delta \Omega dt + K_D G \Delta \Omega) G_k
\]

where \(T_a\) represents the torque of the rotor, \(I_r\) denotes the rotor inertia, \(l_g\) is the generator inertia, \(\Delta \Omega\) is the variation of the rotor speed, and the over-dot sign represents the first derivative with respect to time. The factor \(\Delta \theta\) denotes the variation of the blade-pitch angle, and the factors \(K_p, K_I\), and \(K_D\) are the proportional gain, integral gain, and differential gain in the proportion integral differential controller, respectively, \(dt\) is the time step, \(G_k\) denotes the gain correction factor, expressed by:

\[
G_k = \frac{1}{1 + (\theta / \theta_k)}
\]

where \(\theta\) represents the blade-pitch angle of the previous step, and \(\theta_k\) is a constant. The all utilized control parameters of the NREL 5 MW wind turbine are provided in Table 3.

**Table 3.** The control parameters pertinent to the NREL 5 MW wind turbine.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator speed at A/rpm</td>
<td>670</td>
<td>differential gain ((K_D))</td>
<td>0</td>
</tr>
<tr>
<td>Generator speed at D/rpm</td>
<td>871</td>
<td>(\theta_k / ^\circ)</td>
<td>6.30</td>
</tr>
<tr>
<td>Generator speed at G/rpm</td>
<td>1161.96</td>
<td>(K/N \cdot m \cdot rpm^{-2})</td>
<td>2.33</td>
</tr>
<tr>
<td>Generator speed at F/rpm</td>
<td>1173.7</td>
<td>(G)</td>
<td>97</td>
</tr>
<tr>
<td>proportional gain ((K_p))</td>
<td>1.82620057</td>
<td>(l_r/ kg \cdot m^2)</td>
<td>35,444,067</td>
</tr>
<tr>
<td>integral gain ((K_I))</td>
<td>0.78265750</td>
<td>(l_g/ kg \cdot m^2)</td>
<td>534.116</td>
</tr>
</tbody>
</table>
2.3. Scheme

In this part, it is aimed to examine the influence of the LLJ intensity on the aerodynamic characteristics of the wind turbine and compare that with the wind shear. According to the research results of Zhang et al. [8], Shu et al. [24], and Ruchith and Raj [31], the calculation cases presented in Table 4 are determined. Here $v_{hub}$ is wind speed of hub.

Table 4. The parameters of various calculation conditions.

<table>
<thead>
<tr>
<th>Inflow Conditions</th>
<th>$v_{hub}$/m·s$^{-1}$</th>
<th>$e$</th>
<th>$H_{f}$/m</th>
<th>$v_{f}$/m·s$^{-1}$</th>
<th>$H_{j}$/m</th>
<th>$v_{ml}$/m·s$^{-1}$</th>
<th>$C_{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WS</td>
<td>8.0</td>
<td>0.2</td>
<td>150</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.0</td>
<td>0.2</td>
<td>150</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>15.0</td>
<td>0.2</td>
<td>150</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LLJ</td>
<td>8.0</td>
<td>0.2</td>
<td>150</td>
<td>3</td>
<td>150</td>
<td>5</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>10.0</td>
<td>0.2</td>
<td>150</td>
<td>5</td>
<td>150</td>
<td>5</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>15.0</td>
<td>0.2</td>
<td>150</td>
<td>10</td>
<td>150</td>
<td>5</td>
<td>0.6</td>
</tr>
</tbody>
</table>

When the influence of the LLJ height on the aerodynamic characteristics of the wind turbines is of concern, the LLJ height cannot be directly calculated from the wind speed model because the wind speed model is a superposition of the free jet and WS. Therefore, the height of the free jet is adopted as the basic variable. Figure 6 shows the height of the free jet relative to the position of the wind turbine rotor in a schematic manner, where $H_{j}$ denotes the LLJ height associated with the free jet height. The detailed parameters are given in Table 5, in which $\xi$ represents the dimensionless distance and is defined by

$$\xi = \frac{H_{s} - H_{hub}}{R}$$  \hspace{1cm} (21)

where $H_{hub}$ is the height of the wind rotor, and $R$ denotes the radius of the wind rotor.

![Figure 6](image_url)  

Figure 6. Schematic representation of the height of the free jet relative to the position of the wind rotor.
Table 5. Calculation parameters of various LLJ heights.

<table>
<thead>
<tr>
<th>$H_a$/m</th>
<th>$\xi$</th>
<th>$C_s$</th>
<th>$H_r$/m</th>
<th>$v_r$/m·s$^{-1}$</th>
<th>$v_m$/m·s$^{-1}$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>87</td>
<td>−1</td>
<td>0.6</td>
<td>87</td>
<td>5</td>
<td>5</td>
<td>0.2</td>
</tr>
<tr>
<td>119</td>
<td>−0.5</td>
<td>0.6</td>
<td>119</td>
<td>5</td>
<td>5</td>
<td>0.2</td>
</tr>
<tr>
<td>150</td>
<td>0</td>
<td>1</td>
<td>150</td>
<td>5</td>
<td>5</td>
<td>0.2</td>
</tr>
<tr>
<td>181</td>
<td>0.5</td>
<td>1.2</td>
<td>181</td>
<td>5</td>
<td>5</td>
<td>0.2</td>
</tr>
<tr>
<td>213</td>
<td>1</td>
<td>1.2</td>
<td>213</td>
<td>5</td>
<td>5</td>
<td>0.2</td>
</tr>
</tbody>
</table>

3. Validation of the Calculation Methods

3.1. Validation of BEM Theory

From the results illustrated in Figure 7, it is observed that when the BEM method is not coupled with the control system, the below-rated wind speed, the error of the calculated value relative to the reference value is less than 3%. For the case of the above-rated wind speeds, the relative error of the power is lower than that of the uncoupled control system. For the calculations associated with the reference power equal to the calculated power. At this time, an error in the calculated power, the calculated rotor speed is the value of the reference rotor speed associated with the reference power equal to the calculated power. Therefore, the relative error of the power is lower than that of the uncoupled control system. For the

![Figure 7. The power of the wind rotor in the lack of coupling of the control system.](image-url)

3.2. Validation of the BEM-Controller Interaction Method

3.2.1. Validation of the Control System

Based on the presented results in Figure 8, when the BEM method is coupled to the control system, for below-rated wind speeds, the blade-pitch action does not occur, and the power and rotor speed correspond to each other at various wind speeds. When there exists an error in the calculated power, the calculated rotor speed is the value of the reference rotor speed associated with the reference power equal to the calculated power. At this time, the power error is reflected in the power error itself and the rotor speed error. Therefore, the relative error of the power is lower than that of the uncoupled control system. For the
above-rated wind speeds, the calculated rotor speed and the calculated power are equal to the reference value, and the power error is reflected in the blade-pitch angle error. The above results reveal that all of the resulted relative errors are less than 5%.

**Figure 8.** The power of the wind rotor, rotor speed, and blade-pitch angle accounting for the coupling of the control system.

### 3.2.2. Convergence Verification of the Rotor Speed and Blade-Pitch Angle Subjected to Uniform Inflows

For further verification of the crucial role of the control system in aerodynamic calculations, the convergence curves of the rotor speed and blade-pitch angle with time step are provided for different wind speeds in Figures 9 and 10. In the calculation process, the initial rotor speed and initial blade-pitch angle in order are 10 rpm and 0°. For the case of the wind speed equal to 8 and 10 m·s⁻¹, the pitch angle is always 0°, and the speed is adjusted by the generator-torque controller. In the case of the wind speed equal to 15 and 20 m·s⁻¹, the speed converges to the rated speed, and the pitch angle is adjusted by the blade-pitch controller, and the final results are consistent with those demonstrated in Figure 8.

**Figure 9.** Variation of the rotor speed in terms of the time step for various wind speeds.
profile would be generally equivalent to the wind speed at the hub. Therefore, we compare the rotor speed, blade-pitch angle, and aerodynamic loads under different methods.

3.2.3. Validation of the Thrust and Torque under the WS Inflow Condition

As mentioned above, for calculating the aerodynamic characteristics of the wind turbine when the wind speed is non-uniform, the wind turbine speed and pitch angle can be set as the rotor speed and pitch angle corresponding to the uniform wind speed equivalent to the non-uniform wind speed. When the inflow condition is represented by the wind shear, this calculation method is commonly utilized, and the WS wind speed profile would be generally equivalent to the wind speed at the hub. Therefore, we compare the predicted results by the EWS method and those obtained by the aerodynamic control coupling method subjected to the wind shear condition, as presented in Table 6. It is seen from the obtained results that the predicted results by the above-mentioned two methods are very close, such that the maximum relative error is only 3.76%.

Table 6. The rotor speed, blade-pitch angle, and aerodynamic loads under different methods.

<table>
<thead>
<tr>
<th>Inflow</th>
<th>$H_r$/m</th>
<th>$v_r$/m·s$^{-1}$</th>
<th>The EWS Method</th>
<th>Control System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thrust/kN</td>
<td>Torque/kN·m</td>
</tr>
<tr>
<td>WS</td>
<td>150</td>
<td>8</td>
<td>375.50</td>
<td>1932.99</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>10</td>
<td>586.52</td>
<td>3020.99</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>15</td>
<td>418.31</td>
<td>4150.26</td>
</tr>
</tbody>
</table>

4. Results and Discussion

4.1. Influence of the LLJ Strength on the Aerodynamic Characteristics of Wind Turbines

Table 7 shows the rotor speed, blade-pitch angle, and aerodynamic loads of the wind rotor under different working conditions. The reference values in the table are values in the presence of the uniform inflow condition when the control system is coupled to the BEM. When the wind speeds are 8, 10, and 15 m·s$^{-1}$, respectively, the thrust of the wind rotor decreases by 5.06%, 4.41%, and $-1.09\%$, and the torque decreases by 5.87%, 4.50%, and 0.69%, respectively, relative to the reference value. At this time, the decrease in the thrust and torque is considerably greater than that of the wind shear since the shear gradient of the LLJ is larger than that of the WS. At the speed level of 15 m·s$^{-1}$, the variation of the thrust and torque of the wind rotor is moderately small; however, the variation of the blade-pitch angle is large, presenting a reduction of about 3.59%.
The aerodynamic characteristics of the wind rotor are the result of the superposition of the aerodynamic characteristics of the blades. In order to further scrutinize the influence of different working conditions on the aerodynamic characteristics of the wind turbine, it is, therefore, necessary to examine the change laws of the aerodynamic characteristics of the blades. Considering that the thrust and flapping torque, power, and rotational torque have the same change law, this paper only aims to analyze the change law of the flapping torque and rotational torque. Figure 11 demonstrates the variation of the torque in terms of the azimuth angle. The dimensionless torque in this figure is set as the torque divided by the average torque. The azimuth angle of 0° is specified as the 12 o’clock direction, and the wind turbine rotates clockwise. The plotted results show that under the condition of the WS inflow, the torque reaches its largest value when the blade arrives at the azimuth angle of 0°. For the case of the azimuth angle equal to 180°, the smallest torque is observed. For the wind speeds of 8, 10, and 15 m·s⁻¹, the amplitudes of the rotational torque are approximately equal to 17.41%, 17.44%, and 20.80% of the average value, and the amplitudes of the flapping torque are 8.51%, 8.52%, and 17.44% of the average value, respectively. Under the LLJ flow condition, the plots associated with the rotating blade exhibit two local maximums, as well as two local minimums in one rotation cycle. The local maximums appear in the first and second quadrants of the Cartesian coordinate system, while the local minimums occur at 0° and 180° azimuth, respectively. Such a scenario is basically related to the wind speed profile of the LLJ. The wind speed does not increase with the height, but first increases to a maximum value and then decreases with the growth of the height. In addition, because the vertical gradient of wind speed of the LLJ is larger than that of the WS, the torque amplitude is also larger. For the wind speeds of 8, 10, and 15 m·s⁻¹, the amplitudes of the rotational torque are about 23.63%, 22.33%, and 24.67% of the average value, respectively, and the flapping torque amplitudes in order are 11.98%, 11.26%, and 21.09% of the average value.

Table 7. The rotor speed, blade-pitch angle, and aerodynamic loads under different working conditions.

<table>
<thead>
<tr>
<th>Inflow</th>
<th>$v_{hub}$/m·s⁻¹</th>
<th>Reference Value</th>
<th>Calculated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thrust/kN</td>
<td>Torque/kN·m</td>
<td>Speed/rpm</td>
</tr>
<tr>
<td>WS</td>
<td>8</td>
<td>377.30</td>
<td>1948.30</td>
</tr>
<tr>
<td>LLJ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WS</td>
<td>10</td>
<td>589.52</td>
<td>3044.23</td>
</tr>
<tr>
<td>LLJ</td>
<td>15</td>
<td>421.56</td>
<td>4180.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inflow</th>
<th>$v_{hub}$/m·s⁻¹</th>
<th>Reference Value</th>
<th>Calculated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thrust/kN</td>
<td>Torque/kN·m</td>
<td>Speed/rpm</td>
</tr>
<tr>
<td>WS</td>
<td>8</td>
<td>377.30</td>
<td>1948.30</td>
</tr>
<tr>
<td>LLJ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WS</td>
<td>10</td>
<td>589.52</td>
<td>3044.23</td>
</tr>
<tr>
<td>LLJ</td>
<td>15</td>
<td>421.56</td>
<td>4180.07</td>
</tr>
</tbody>
</table>

Table 7. The rotor speed, blade-pitch angle, and aerodynamic loads under different working conditions.

<table>
<thead>
<tr>
<th>Inflow</th>
<th>$v_{hub}$/m·s⁻¹</th>
<th>Reference Value</th>
<th>Calculated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thrust/kN</td>
<td>Torque/kN·m</td>
<td>Speed/rpm</td>
</tr>
<tr>
<td>WS</td>
<td>8</td>
<td>377.30</td>
<td>1948.30</td>
</tr>
<tr>
<td>LLJ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WS</td>
<td>10</td>
<td>589.52</td>
<td>3044.23</td>
</tr>
<tr>
<td>LLJ</td>
<td>15</td>
<td>421.56</td>
<td>4180.07</td>
</tr>
</tbody>
</table>

Figure 11. Variation of the blade torque as a function of the azimuth under different working conditions.
4.2. Influence of LLJ Height on Aerodynamic Characteristics of Wind Rotor

The LLJ height, one of the key parameters to describe the characteristics of the LLJ, has a great impact on the aerodynamic characteristics of wind turbines. Table 8 presents the thrust, torque, power, and rotor speed at different LLJ heights.

Table 8. The aerodynamic characteristics of the wind turbines at various LLJ heights.

<table>
<thead>
<tr>
<th>$H_s$/m</th>
<th>$H_J$/m</th>
<th>$\xi$</th>
<th>Thrust/kN</th>
<th>Torque/kN·m</th>
<th>Power/MW</th>
<th>Rotor Speed/rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>87</td>
<td>123</td>
<td>−1</td>
<td>596.10</td>
<td>3075.77</td>
<td>3.70</td>
<td>11.48</td>
</tr>
<tr>
<td>119</td>
<td>168</td>
<td>−0.5</td>
<td>613.15</td>
<td>3162.65</td>
<td>3.85</td>
<td>11.64</td>
</tr>
<tr>
<td>150</td>
<td>176</td>
<td>0</td>
<td>563.54</td>
<td>2907.27</td>
<td>3.40</td>
<td>11.16</td>
</tr>
<tr>
<td>181</td>
<td>212</td>
<td>0.5</td>
<td>505.78</td>
<td>2621.19</td>
<td>2.91</td>
<td>10.60</td>
</tr>
<tr>
<td>213</td>
<td>250</td>
<td>1</td>
<td>445.64</td>
<td>2316.59</td>
<td>2.42</td>
<td>9.97</td>
</tr>
</tbody>
</table>

For the case of the wind speed at the center of the hub equal to 10 m·s$^{-1}$, the rotor speed takes different values under different working conditions. When $\xi$ is equal to $−0.5$, the largest rotor speed is detectable, which is increased by 4.29% compared with the case of $\xi = 0$. By growing $\xi$ from $−0.5$ to $0$, the thrust, torque, and power of the wind rotor magnify by 8.80%, 8.78%, and 13.45%, respectively. In the case of $\xi = 1$, the smallest rotor speed is observable, whose magnitude is about 20.32% lower than the rotor speed associated with $\xi = 0$. Further, the corresponding thrust, torque, and rotor power lessen by 20.32%, 28.84%, and 10.70%, respectively. The main reason for this fact is that the LLJ height generally occurs at a height above the free jet height, and the value is related to the LLJ strength and the wind shear index. In the case of $\xi = −0.5$, the height of the LLJ is closest to that of the hub, and the rotor speed is adjusted by the control system to make the tip speed ratio optimal to capture more wind energy. In addition, the aerodynamic loads and the rotor speed exhibit the largest values. For the case of $\xi = 1$, the wind rotor places below the LLJ height, the inflow condition is the same as the very strong wind shear, and the wind energy captured by the wind rotor is the least. In such a case, the rotor speed reaches its lowest level, and the same story holds true for the aerodynamic loads and torque.

Further scrutiny regarding the effect of the LLJ height on the aerodynamic characteristics of the wind turbine blades is also of considerable concern. In Figure 12, the variations of the rotation torque and flapping torque of the blade in terms of the azimuth for different LLJ heights are given. In the cases of $\xi = −1, −0.5, 0$, the demonstrated results exhibit that the LLJ height would be close to that of the hub, while the wind speeds in the swept area are 16.30%, 11.82%, and 22.39% larger than the average wind speed, respectively. Further, the amplitudes of rotational torque and flapping torque are approximately equal to 7.95%, 5.83%, and 11.29% of the average value, respectively. The plotted results associated with the blade exhibit two local minimums at the azimuths $0^\circ$ and $180^\circ$. When the LLJ height is above the hub height, two local maximums appear in the first and second quadrants of the Cartesian coordinate system. For the case of the LLJ height below the hub center height, two local maximums occur in the third and fourth quadrants. In the case of $\xi = 0.5$, the LLJ height appears above the center of the hub and close to the blade tip, and the wind speed gradient becomes larger. The amplitudes of the rotational torque and flapping torque are 34.24% and 17.49% of the average value, respectively, and their change laws are similar to those of the WS inflow conditions. In the case of $\xi = 1$, the LLJ height is outside the swept area of the wind rotor, and the rotational torque and flapping torque take their largest levels, which are 37.75% and 19.12% of their average values, respectively. In such a case, the change law of torque of the blade is identical to a very strong WS inflow.
Figure 12. Variation of the rotation torque and flapping torque of the blade as a function of the azimuth for different LLJ heights. (a) rotation torque. (b) flapping torque.

5. Conclusions

In the present paper, the BEM theory was coupled with the generator-torque controller and blade-pitch controller, and a C++ code was developed to implement the method. The influence of the LLJ strength on the aerodynamic characteristics of the wind rotor and the influence of different LLJ heights on the aerodynamic characteristics of the wind rotor were examined in some detail. The following crucial results are achieved.

Firstly, the difficulty of the unknown rotor speed and blade-pitch angle under non-uniform inflow or complex inflow conditions can be solved by coupling the control system with the aerodynamic characteristic calculation method. Secondly, when the wind speed is the same at the hub, the effect of the LLJ on the aerodynamic characteristics of the wind rotor is greater than that of the WS. The average aerodynamic loads of the wind rotor are less than those of the WS, and the amplitudes of the aerodynamic loads of the blade are greater than those of the WS. Finally, the influence of the LLJ should be considered in the aerodynamic and structural design of wind turbines. When the LLJ height is placed within the swept area of the wind rotor, the change curve of the aerodynamic loads of the blade in terms of the azimuth angle exhibits two local maximums and two local minimums. When the LLJ height is above the hub, local maximums are detectable in the first and second quadrants. For the case of the LLJ height lower than the hub height, maximums appear in the third and fourth quadrants. Further, two local minimums are observed at 0° and 180° azimuth angles. In addition, the closer the LLJ height is to the hub height, the greater the average aerodynamic loads of the wind rotor and the smaller the amplitude of the blade relative to the average value. The closer the LLJ height is to the tip, the smaller the average aerodynamic loads of the wind rotor are and the greater amplitudes of the blade are relative to the average value. When the LLJ height is placed outside the swept area of the rotor, the change law of aerodynamic loads of the blade is similar to a very strong WS inflow.

Author Contributions: Methodology, P.H.; codes, P.H.; validation, J.X.; data curation, P.H.; writing—original draft preparation, P.H.; writing—review and editing, J.X. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.
References


