

Article

Energy-Saving Applications Based on Train Mass Online Learning Using Time-Varying Train Model

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Abstract: The current train model of the train control system is unable to accurately reflect the influence of nonlinear running resistance, line conditions, the mutative train mass value, and external environment changes on the model in train dynamics, resulting in a defect of poor train model performance. The train basic model and additional resistances are discussed in this paper, a novel neural network online learning method of the time-varying dynamic train model is proposed, combined with the characteristics of rail transit lines, and a neural network learning algorithm is designed by categories and steps. This method can identify the train mass value that changes continuously with passengers during running. The energy savings resulting from using the actual varying train mass in the train control system are calculated. The results show that, when compared to the traditional model's invariant approximate empirical parameters, the time-varying parameter model can follow changes in the train and line environment and obtain quantitative expressions of curve resistance and tunnel resistance with speed. The time-varying train model was used to conduct engineering tests on the Beijing Capital Airport Line; the online learning deviation of train mass was controlled within a margin of 3.08%, and at the same time, energy consumption decreased by 6.13%.

Keywords: rail transit; movement resistances; curve resistances; train modeling; neural network; online learning



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1. Introduction

Rail transit with exclusive right-of-way has the advantages of being fast, efficient, and high capacity, and it is the primary means of resolving urban traffic congestion. Thanks to the government's aggressive promotion, China now has the world's largest urban rail transit and high-speed railway system, generating enormous economic and social value. Furthermore, the rail transit system's low carbon emissions make it an important player in the global goal of achieving carbon neutrality. As a result, the current research is primarily focused on how to further realize more efficient and intelligent rail transit operations on the existing rail transit network [1].

In recent years, there have been more studies using artificial intelligence to promote the intelligent application of train control systems. Wang [2] explored changing the dynamic equation of high-speed trains to a fuzzy model of cascaded vehicles connected by spring-type couplers, and used the optimal control method of fuzzy control to improve the safety, comfort, and energy efficiency of high-speed trains. Huang et al. [3] proposed a train driving strategy method based on an improved genetic algorithm to automatically adapt to the influence of uncertain factors such as the traction weight of heavy haul railway trains. Tang et al. [4] proposed a neural network-based method for generating the running curve of heavy-haul trains to overcome the problems of locomotive traction characteristics. In order to improve the tracking performance of the train nonlinear system, Pu et al. [5] designed a fuzzy controller that adaptively adjusts the parameters. In [6], an adaptive output feedback

trajectory tracking control method is proposed on the basis of neural networks. The multi-particle train dynamics model [7] performed well on heavy-haul railways and high-speed trains [8], but it is still unable to solve the problem of the deterioration of the control performance due to the change of characteristics of the train between different trains. In [9], Wang attempted to investigate parameter online identification, develop an accurate heavy-duty train brake model with accurate model parameters, and provide drivers with more precise control curves. These studies are based on the traditional empirical train dynamics model and employ artificial intelligence to compensate for model inaccuracy, thereby improving control accuracy and effect. However, when the line conditions change and the traction braking characteristics change as a result of the train running for a period of time, the performance of these artificial intelligence methods based on fixed-parameter train models degrades.

With the increase in operating mileage, the energy consumption of urban rail transit has surged, and energy conservation has become one of the key issues for the sustainable development of urban rail transit [10]. Wu et al. developed an integrated optimization model to obtain the speed trajectory with the constraint of on-board energy storage devices properties such as capacity and initial state of energy [11]. Li et al. proposed a multi-objective optimization model for urban railways with timetable optimization to minimize the total energy consumption of trains while maximizing the quality of service [12]. Shang studied the energy-saving train regulation problem by utilizing distributed model predictive control, which is motivated by the breakthrough of vehicle-based train control technology and the pressing real-time control demand [13]. Liao proposed a modified genetic algorithm-gate recurrent unit method, which is a real-time method based on deep learning [14]. All of this literature studied energy-saving methods from the perspective of train control algorithms and scheduling rather than the train dynamics model.

To execute parameter learning on a huge number of train operation data, a time-varying train dynamics model using the neural network method is proposed in this paper. This new method addresses the train model's accuracy decline after a period of operation, as mentioned before. Then, an energy-saving strategy is implemented from the perspective of train mass identification by using this time-varying train dynamics model. In general, the main contributions of this paper are as follows:

1. Fixed parameters are upgraded to variable-parameter train dynamics models.
2. Different from the previous design of high-performance control algorithms to compensate and suppress the impact of model uncertainty, online learning is used to establish an accurate and real-time train model, thereby improving the performance of the control system.
3. The energy-saving method of the urban vehicle onboard control system by utilizing the neural network train mass learning value based on variable-parameter train dynamics models is proposed.

The rest of the paper is organized as follows: Section 1 establishes the parameter-variable train dynamics model. The neural network method is used in Section 2 for online model parameter learning. Section 3 uses updated train mass value learned from neural networks to calculate the energy consumption. Section 4 is the discussion, and Section 5 is the conclusion.

2. Time-Varying Train Model

Because most urban rail transit trains are only 120 m long, the coupling relationship between each carriage and the impact of multiple power distribution on the train model can all be ignored, and the entire train can be regarded as a rigid mass point for analysis [15]. The force analysis of the train is shown in Figure 1.

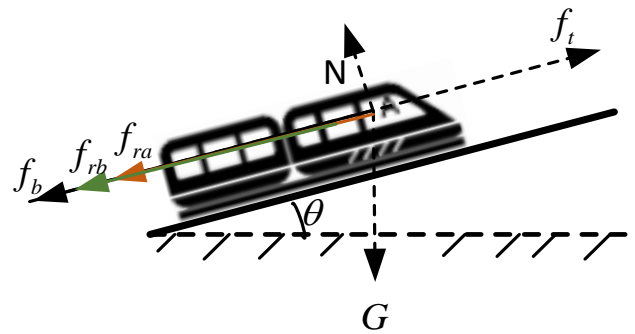


Figure 1. Schematic diagram of the force of the train.

The total force F on the train along the running direction is mainly composed of four parts: traction force f_t , braking force f_b , basic resistance f_{rb} and additional resistance f_{ra} .

$$F = f_t - f_b - f_{rb} - f_{ra} \quad (1)$$

The traction force of Equation (1) is positive, the resistance and braking force are negative, which are related to the running direction of the train.

2.1. Traction Braking Force

As indicated by the Figure 2 traction braking characteristic curve, the train motor system generates traction braking force, and the traction braking force's maximum output value is related to the train's running speed.

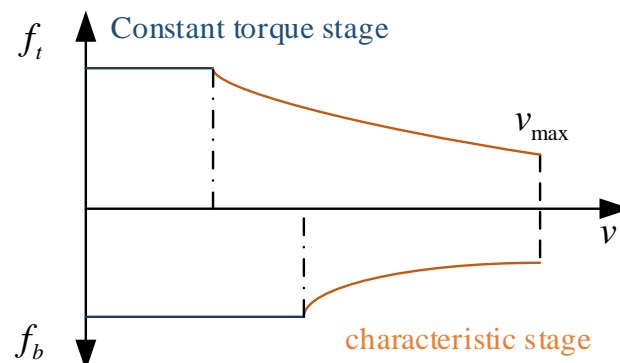


Figure 2. Train traction and braking characteristic curve.

As shown in Figure 2, the maximum tractive force F_t and braking force F_b of the train corresponding to the running speed of the train at a certain time can be obtained from the traction curve table provided by the train manufacturer. It should be noted that the traction braking curves of different trains vary due to equipment aging and individual differences in the fabrication.

In engineering applications, the corresponding proportion of the maximum tractive force and braking force is usually obtained by controller signal.

$$f_t = k_t F_t \quad (2)$$

$$f_b = k_b F_b \quad (3)$$

where k_t and k_b are scale factor of train operating handle, which value belonged to $[0, 1]$. Both f_t and f_b are actual output of the tractive and braking force. Since the train cannot output traction and braking at the same time [16], the traction brake handle switch

$torb \in (0, 1)$ is introduced in Equations (2) and (3), and the traction braking force can be further expressed as follows:

$$f_t - f_b = torb \times k_t \times F_t - (1 - torb) \times k_b \times F_b \quad (4)$$

2.2. Basic Resistance

The mechanical resistance and starting resistance of the wheel-rail relationship between the vehicle and the line are the primary causes of train operation resistance. Due to the complexity of the influencing factors, the basic resistance coefficient per unit mass is difficult to model, so the Davis empirical formula is used for approximate calculation [17].

$$P_{rb} = \alpha_1 + \beta_1 v + \gamma_1 v^2 \quad (5)$$

where P_{rb} is the coefficient of basic resistance per unit, and v is the speed of the train. These α_1 , β_1 and γ_1 are the basic resistance parameters, which usually came from wind tunnel experiments. Unlike in the traditional model, they are all variable parameters in this paper.

2.3. Additional Resistance

When the train runs to the ramp, the component force of gravity in the opposite running direction is called slope resistance [18]. Since the slope of the line is small relative to the design requirements, the slope resistance per unit mass is approximately equal to the slope.

$$P_{ra_slope} = \frac{mg \sin \theta}{mg} \times 1000 = 1000 \times \sin \theta = p \quad (6)$$

where P_{ra_slope} represents the additional slope resistance coefficient of the train unit, θ defines the ramp angle, p is the equivalent thousandth grade slope value, and mg is the gravity of train. When going uphill, the slope resistance hinders the movement of the train and p is a negative value. When going downhill, p is positive.

When all other factors remain the same, the train will encounter greater resistance when traveling to the curved section than when traveling in a straight line. The additional resistance of the curve is related to many factors, including the radius of the curve, the train's running speed, the superelevation of the curve's outer rail, the widening of the gauge, and the train's wheelbase. The curve resistance per unit mass of the train running on the curve is generally expressed by the experience equation by the curve radius R , which a is an empirical parameter in Equation (7).

$$p_{ra_curve} = a/R \quad (7)$$

where P_{ra_curve} is the curve resistance of unit mass. When the train is running through the tunnel, the air in the tunnel and the train rub against each other due to the pressure difference created by the car body and the structure of the train, resulting in additional resistance in the tunnel [19] which is determined by factors such as marshaling parameters, tunnel parameters, and the train's technical condition. The additional tunnel resistance value per unit P_{ra_tunnel} usually adopts the empirical formula obtained from the test like Equation (8) [16].

$$P_{ra_tunnel} = bL_{tunnel} \quad (8)$$

where b is the empirical parameter, and L_{tunnel} is the length of the tunnel.

These three additional resistances may appear alone or at the same time. The model in this paper does not consider the additional resistances that cannot be modeled due to natural factors such as strong wind and severe cold weather. Slope resistance can be obtained as a relatively accurate value according to Equation (6), but other additional resistances caused by curves and tunnels can only be calculated by the approximate empirical value range by Equations (7) and (8) due to the difficulty of modeling.

In previous studies, the additional resistance as an uncertainty term could only be compensated by designing a controller with automatic adjustment capability [18]. However, the goal of this paper is to develop an accurate train dynamics model based on three assumptions listed below in order to revise the expressions of the curve and the tunnel additional resistance.

Assumption 1. The superelevation of the track curve during line construction is to provide the centripetal force mv^2/R of the train when it runs on the curve, so curve resistance of the can be expressed as a polynomial of the train speed v .

Assumption 2. The train tunnel is analogous to a wind tunnel [20], tunnel resistance can also be expressed as a polynomial of the train speed v .

Assumption 3. The primary control variable of the train controller is speed, and the speed value is also the most intuitive and easy-to-measure output value of the train. The curve and tunnel resistance should be expressed by the polynomial of the train speed.

Therefore, Equations (7) and (8) are updated to Equations (9) and (10), respectively.

$$P_{ra_curve} = \alpha_2 + \beta_2 v + \gamma_2 v^2 \quad (9)$$

$$P_{ra_tunnel} = \alpha_3 + \beta_3 v + \gamma_3 v^2 \quad (10)$$

where $\alpha_2, \beta_2, \gamma_2, \alpha_3, \beta_3, \gamma_3$ represent coefficients.

2.4. Train Dynamics Model

The total force F can be expressed by traction braking force, basic resistance and additional resistance as follows:

$$F = torb \times k_t \times F_t - (1 - torb) \times k_b \times F_b - f_r \quad (11)$$

$$f_r = G(z_1 \cdot V^T + z_2 \cdot V^T + z_3 \cdot V^T + p) \quad (12)$$

$$z_1 = (\alpha_1, \beta_1, \gamma_1) \quad (13)$$

$$z_2 = (\alpha_2, \beta_2, \gamma_2) \quad (14)$$

$$z_3 = (\alpha_3, \beta_3, \gamma_3) \quad (15)$$

$$V = (1, v, v^2) \quad (16)$$

$$G = mg \quad (17)$$

where z_1, z_2, z_3 are variable parameters vector, V represent vector composed by (0, 1, 2) orders of train speed, and m define the train mass. The relationship between the total force on the train and the mass and speed of the train follows Newton's second law.

$$\dot{v} = F/m \quad (18)$$

In cities with high density of rail transit operations, such as Beijing, Shanghai, and Guangzhou, especially in peak and low periods, since passengers board and exit trains at the station, the train mass will fluctuate greatly, so the train mass m in this paper is also a variable parameter. Equations (11) and (18) constitute the time-varying train dynamics model of this paper.

3. Online Neural Network Parameter Learning

As shown in Figure 3, this paper adopts the neural network method to learn the train dynamics parameters.

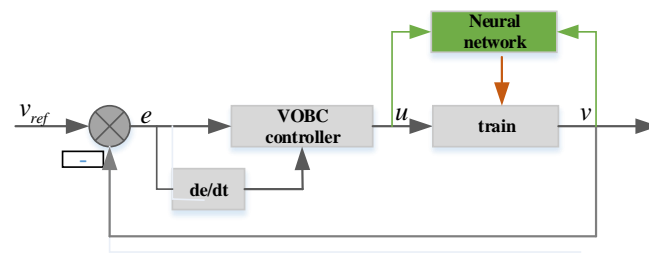


Figure 3. Train model online learning structure diagram.

The neural network learning module is connected in parallel with the train model of the vehicle on-board control (VOBC) system. The controller output k_t , k_b of the VOBC and the vehicle speed v are used as the input of the neural network module, and the output is the parameters of the train dynamic model defined by Equations (11)–(18).

The basic data of the train and the electronic map of the line are also stored in the neural network learner module, including the geographic information, slope, and curve data of the line. The learning module obtains the real-time train speed from the train speed sensor, calculates the train's acceleration by differentiating, and calculates the train's position by integrating. It obtains information such as the slope, curve, and whether or not there is a tunnel at the train's location when combined with the on-board electronic map. The control input f_t and f_b of the train are obtained by checking the traction braking curve table, so as to obtain the sample data of the train dynamics model for online learning of the neural network. The neural network learning module structure is shown in Figure 4.

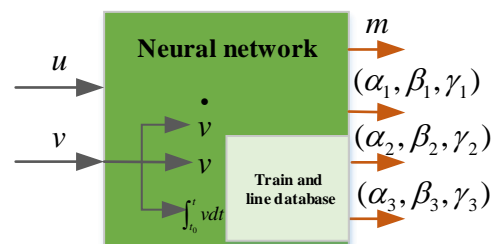


Figure 4. Neural Network learning module structure.

3.1. Neural Network Structure

This paper adopts the neural network method to learn the model parameters. For i input and j hidden layer and k output neural network, the output of the hidden layer is y_j :

$$y_j = f_1(\sum_i w_{ji}x_i - b_j) \quad (19)$$

where w_{ji} is the connection weight between the input layer and the hidden layer, x_i is the input, b_j is the hidden layer threshold, and f_1 is the hidden layer activation function. The output layer of neural network is O_k :

$$O_k = f_2(\sum_j w_{kj}y_j - b_k) \quad (20)$$

where w_{jk} is the connection weight between the hidden layer and the output layer, b_k is the output layer threshold, $f_2(x)$ is the activation function of the output layer.

The neural network weights w_{ij} are, respectively, set as the parameters of z_1, z_2, z_3 , in this paper the other values are follows:

$$i = 2, j = 3, k = 1 \quad (21)$$

$$b_j = \alpha_j, j \in (1, 2, 3) \quad (22)$$

$$\begin{aligned} w_{j1} &= \beta_i, j \in (1, 2, 3) \\ w_{j2} &= \gamma_i, j \in (1, 2, 3) \end{aligned} \quad (23)$$

$$w_{kj} = 1, j \in (1, 2, 3), k \in (1) \quad (24)$$

$$b_k = 0, k \in (1) \quad (25)$$

The input of the neural network $x_i (i = 1, 2)$ is set as speed v and the square of the speed v^2 , respectively, the output train acceleration as the Equation (26).

$$O_1 = a = \dot{v} \quad (26)$$

The learning factor of neural network is 1×10^{-7} , and the performance index function is chosen as:

$$E = (O_1 - O_{actual})^2 \quad (27)$$

The neural network controller uses the gradient descent method to learn the weight coefficients of the network. This paper uses the Pytorch toolkit to implement the neural network module.

3.2. Learning Algorithm

The learning samples are separated into three categories: non-tunnel curved roads, curved sections, and tunnel sections. The parameters are learned using a systematic gradual step to increase learning accuracy and make full use of the peculiarities of engineering lines. The train is set to the nominal unloaded mass value before the first journey from the depot to the station, taking advantage of the fact that the train is empty before the first trip.

Firstly, in the case of no-load nominal train mass, z_1 is learned from the data of the straight-line segment (excluding curve and tunnel resistance).

Secondly, update the train model using the obtained values after completing the Z_1 parameter learning, and then learn z_2 using data from the train traveling through the curve segment (just the basic resistance and the curve additional resistance). Simultaneously, z_3 is learned using data from the train as it passes through the tunnel segment (only the basic resistance and the additional resistance of the tunnel).

At last, after completing the parameter z_2, z_3 learning, update the train model with the obtained value to conduct online learning of the train mass m .

The sampling period of the learning sample data is consistent with the VOBC period of 200 ms. The specific Algorithm 1 is as follows:

Algorithm 1: Neural network learning of time-varying model.

```

1  Initial basic train and line data, such traction and brake table, train mass  $m$ 
2  Update  $v, u \rightarrow a, s$ , calculate  $F_t$  or  $F_b \rightarrow f_r$ 
3  For cnt = 1, 2, ... Cnt is train running times
4      While (1) prepare samples for NN
5      If (s position is curve)
6          Storage curve samples SC ( $v, a$ )
7      Elseif (s position is tunnel)
8          Storage tunnel samples ST ( $v, a$ )
9      Else
10         Storage normal samples SN ( $v, a$ )
11         Using SN to learn  $z_1$ 
12         Set  $z_1$ , then using SC to learn  $z_2$ 
13         Set  $z_1$ , then using ST to learn  $z_3$ 
14  Update the NN using  $z_1, z_2, z_3$ , then learn the  $m$ 
15  Output all parameter to the operation train model

```

The parameter output of the neural network learning module will not affect the running train model during operation since the vehicle VOBC is a safety integrity level (SIL) 4 safety device, and the learning result output will bypass VOBC.

3.3. Engineering Data

The Beijing Capital Airport Line as shown in Figure 5 includes straight sections, slope sections, large, curved sections, and tunnel sections, encompassing the majority of urban rail transit line characteristics. So, the Beijing Capital Airport Line project data is chosen for online learning in this paper.

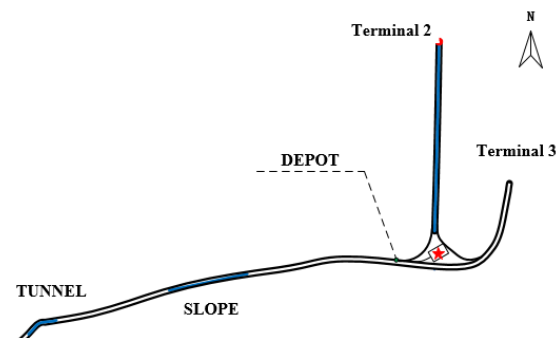


Figure 5. Beijing capital airport line route map.

According to the engineering design document, the basic data of the main line and train dynamics model adopted by the neural network online learning module in this paper are shown in Table 1.

Table 1. System Parameters.

NO	Parameters	Value	Note
1	Train mass m	110 t (no load)	Beijing capital line
2	z_1	(2.7551, 0.014, 0.00075)	Empirical value
3	z_2	(1, 1, 1)	Initial value
4	z_3	(1, 1, 1)	Initial value
5	Traction braking characteristics	Data from China management authority	-

3.4. Learning Results

A modified train with neural network module is utilized to accomplish 52 intersections throughout a 7-day operating cycle, each intersection forming roughly 28,000 pieces of learning data, and a total of 52 times online learnings are completed, according to the learning algorithm in Section 3.2. The outcomes are depicted in Figures 6–8.

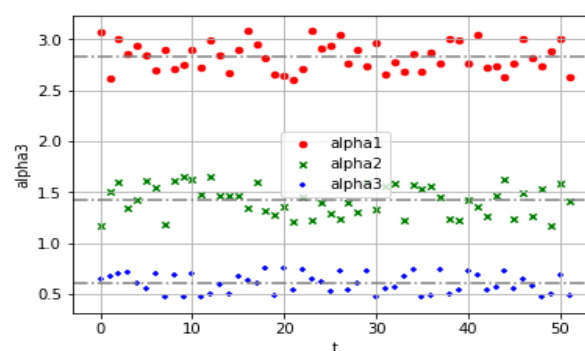


Figure 6. Alpha parameters learning result.

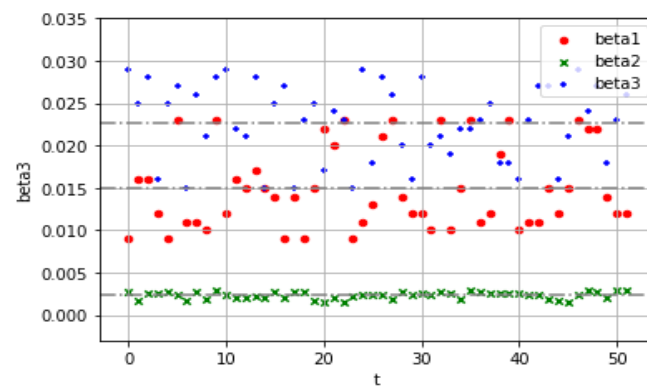


Figure 7. Beta parameters learning result.

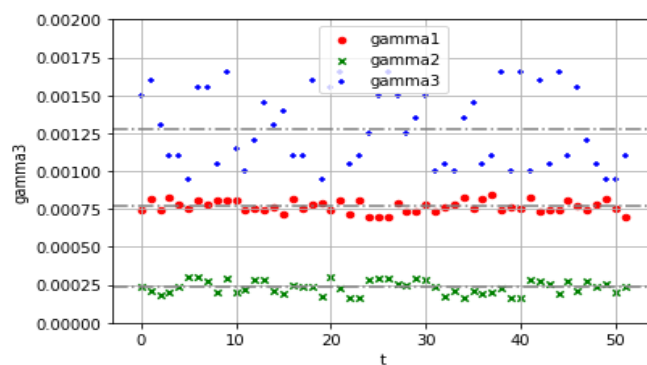


Figure 8. Gamma parameters learning result.

From the 52 times learning results of model parameters, it can be seen intuitively from the graph that they all fluctuate around a certain horizontal axis. Statistical analysis was performed on these results, and the corresponding average values of z_1 , z_2 , z_3 were calculated as shown in Table 2.

Table 2. Parameter learning results.

No.	Parameters	Value
1	z_1	(2.845737, 0.015846, 0.000765)
2	z_2	(1.362406, 0.002273, 0.000228)
3	z_3	(0.609985, 0.022346, 0.001276)

The learning result of the Davis coefficient z_1 of the basic resistance obtained using online neural network learning is greater than the traditional empirical value, which matches the peculiarities of the line-wheel-rail interaction at the Capital Airport line. It may be that since the Beijing Capital Airport Line has been in operation for 15 years, the performance deterioration of the train's mechanical structure has led to an increase in the basic resistance coefficient. Curve resistance and tunnel resistance learned values are z_2 and z_3 in Table 2, the values α_2 and α_3 from which close to the results calculated in the traditional empirical Equations (7) and (8). This result also shows that the assumptions in Section 1 of expressing train curve resistance and tunnel resistance as polynomials of velocity are reasonable.

The engineering test replaces the corresponding parameters in VOBC with the dynamic model learning results in Table 2 to check the accuracy of the train mass learning outcomes of the online neural network module throughout the night when the Beijing Capital Airport Line operation is stopped.

The test train drove out of the depot without load, and the testers boarded the T3 platform and got off the T2 platform to simulate the change in the number of passengers

during operation. The red dotted line in Figure 9 is the actual trainload, the train mass result was calculated by the model every 200 ms, which is represented by the green scatter in Figure 9.

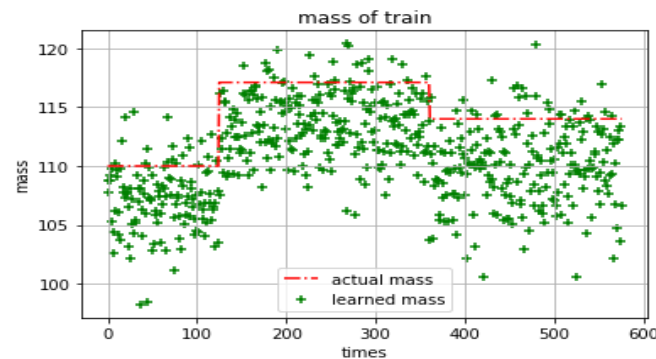


Figure 9. Train mass learning results.

The deviation between the learning mean value and the actual value is shown in Table 3. The three deviation rates are 2.45%, 3.08%, and 2.55%, respectively. It can be seen that the train model in this paper has a high accuracy rate for train mass learning. At the same time, the results also show that the learning value is generally smaller than the actual train value, and there is a systematic error. This error could be due to the disturbance of the wind at the project site on the test day.

Table 3. Train mass learning results.

No.	Actual Value	Learned Value	Deviation
1	110	107.3	2.45%
2	117.1	113.5	3.08%
3	114	111.1	2.55%

4. Energy Consumption Calculation

A portion of the energy in urban rail transit is used to drive the output torque of the train's traction motor, which acts on the moving wheel through the transmission device to generate traction to increase the train's kinetic energy. Another portion is used to ensure the normal operation of the on-board auxiliary equipment such as air conditioning and other equipment. During the operation of the train, the electrical power of the on-board auxiliary equipment remains basically unchanged, and its electrical energy consumption is positively correlated with the train running time and is a linear function, which is not discussed in this paper. The calculation process of this part of the traction energy consumption used to increase the kinetic energy of the train is relatively complex and involves many factors, including the train manipulation strategy, line conditions (slopes, curves, tunnels, etc.), and train mass. The general calculation formula is as follows:

$$E = \int P(u, v) dx \quad (28)$$

$$p(u, v) = \text{torb} \times k_t \times F_t \quad (29)$$

where E is traction energy consumption, x is the train running position. When the train speed is v , and $p(u, v)$ defines the traction force output. The traction energy consumption is the sum of the work by the train applying traction during the running process from Equations (28) and (29). Additionally, the traction energy consumption caused by the change of train mass changes is greater than the rate of change of other factors besides train mass [12]. The following section of this paper uses the above-mentioned online learning

time-varying train mass and fixed experience train mass value to compare and analyze its impact on the traction energy consumption of the train.

Based on the field experiment in Section 2, the result of the neural network module's learning of the train mass is updated to the on-board controller of the train, and the on-board controller outputs the traction braking command to control the train to run from SYQ to DSZ station. The result of traction energy consumption of the train according to formula (28) is shown as the orange line in Figure 10.

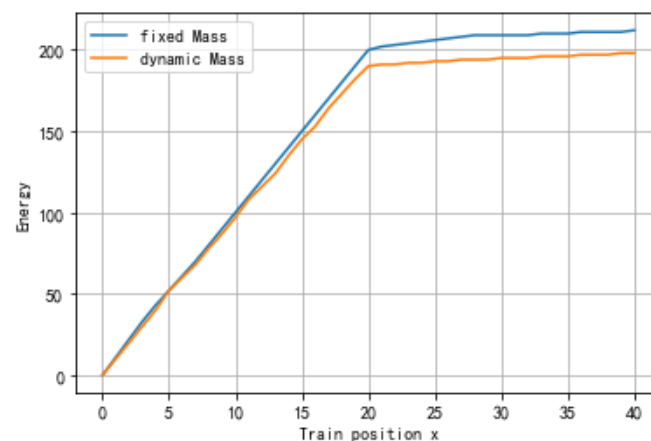


Figure 10. Energy consumption of fixed and online learning train mass value.

The blue line in Figure 10 represents the energy consumption of the tractive force of the train under the same operating line using a fixed empirical value of train mass by the on-board controller. Compared with the energy consumption of train traction under the condition of a fixed value of train mass, the method of online learning of train mass through the neural network method in this paper can reduce the energy consumption of train traction by 6.13%. The reason is that the time-varying train dynamics model is used to obtain the real-time train mass, which can more accurately reflect the boarding and alighting of passengers in the actual operation of the train. The controller can give precise traction and braking commands according to the exact train mass. Compared with the method of using the fixed train quality value, the method of online updating the train's quality can avoid the controller outputting excessive tractive force, thereby avoiding the waste of energy.

5. Discussion

Even though the coefficients in the basic resistance Equation (5) have the lowest variance in the learning results, there are still fluctuations, indicating that the influence of the line environment and external factors will result in differences in the sample data of the 52 times dynamics characteristics of the test train. Furthermore, different learning results will be obtained if sample data were collected from different trains or if the sample data were expanded to a year or more. When the dynamic model of the train changes with the line conditions and traction braking characteristics after running for a period of time, the time-varying train model can be corrected in time to adapt to this change, preventing the dynamic model from becoming inaccurate and causing VOBC performance degraded. Compared with the currently fixed-parameter empirical train model, the results of the neural network learning method in Section 3 show that the proposed train model in this paper can better express the real dynamic characteristics of the train. Similarly, the energy-saving research based on this model is also more accurate than the fixed parameter empirical train model.

It can also be seen from Figures 6–9 that this method still has some limitations that need to be further studied. As can be seen from Figures 6–8, although the results of α_2 and α_3 can illustrate the validity of the assumption in this paper, but the variance of the

learning results γ_2 and γ_3 is larger than that of γ_1 . This paper uses a data-driven approach to verify this hypothesis, but the learning results such as variance still need to be further explained from the perspective of the formation principle of the curve resistance.

The results also show that the learning train mass value is generally smaller than the actual train value in Figure 9. This systematic error may be due to the disturbance of the wind at the project site on the test day. However, due to the difficulty of obtaining weather data, this paper does not consider it in the model. Weather influence will be another important future research topic for dynamic models.

6. Conclusions

The current train model uses empirical formulas to approximate the train's basic resistance and additional resistance, resulting in low accuracy and an inability to reflect the time-varying characteristics of train dynamics. A new variable-parameter train dynamics model is developed, and an online parameter learning neural network is used. The results of the test train's online learning show that:

1. The train model with variable parameters solves the current train control system's inaccurate model problem.
2. The neural network method can accurately complete online parameter learning. The model can accurately track the dynamics of the train as well as changes in the external environment, and it can provide real-time feedback on train mass.
3. Energy can be saved by the actual train mass of online learning using a time-varying train model.

Further research on the impact of wind speed on the model and formation principles of the additional resistances are required in the model's follow-up study. Simultaneously, in order to adapt to the online update of safety equipment such as on-board controllers and promote the method into engineering practice, the time-varying train model neural network online learning method must be combined with European standard safety certification in engineering applications.

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