Maximizing Efficient Power for an Irreversible Porous Medium Cycle with Nonlinear Variation of Working Fluid’s Specific Heat

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Abstract: Considering the specific heat characteristics of working fluid and existence of various losses in a porous medium (PM) cycle, this paper applies finite time thermodynamic theory to study its efficient power performance with nonlinear variable specific heat model. Range of the cycle pre-expansion ratio is obtained by solving the equation, and PM cycle is converted to Otto cycle by choosing appropriate pre-expansion ratio. Influences of pre-expansion ratio, specific heat characteristics, temperature ratio, and various losses on cycle performances are investigated. Thermal efficiencies are compared at operating points of the maximum power output and efficient power. The results show that PM cycle has better performance than Otto cycle. Under certain conditions of parameters, thermal efficiencies at the maximum efficient power and maximum power output operating points are 50.45% and 47.05%, respectively, and the former is 7.22% higher than the latter. The engine designed with the maximum efficient power as the criterion can improve thermal efficiency by losing less power output. The results of this paper can guide parameters selection of actual PM heat engine.

Keywords: porous medium cycle; finite time thermodynamics; efficient power; nonlinear variable specific heat; optimal performance; performance comparison

1. Introduction

Finite time thermodynamics (FTT) [1–11] has attained significant progress in recent years. Many scholars have applied FTT to research performances of thermodynamic cycles, including optimal performance studies [12–29] and optimal path studies [30–42]. Early research work on the internal combustion engine cycles mainly focused on thermal efficiency (η) and power output (P) under the condition of constant specific heat (SH) of working fluid (WF) [43]. When actual heat engines are working, the change in SH of WF is very complicated. In order to study the effects of the SH of WF changing with temperature, which is closer to engineering practice, some scholars have put forward linear [44] and nonlinear [45] models of SH of WFs varying with temperature. In addition to considering the effects of the WF’s SH characteristics on the performances of thermodynamic cycles, it is crucial to take into account the influences of different losses and different objective functions (OFs) on the cycle performance. Common loss models include heat transfer loss (HTL) [43], internal irreversibility loss (IL) [46], and friction loss (FL) [47]. Common OFs include P and η [48], ecological function (E) [49], power density (Pd) [50], efficient power (Ep) [51], etc.
Many scholars have studied the $P$, $\eta$, $P_d$, and $E_P$ optimal performances of the internal combustion engine cycles by applying FTT theory. With the $P_d$ as the OO, Gonca and Genc [52,53] studied the optimal performances of the Gas-Mercury-Steam system and the double-reheat Rankine cycle. Chen et al. [54] derived the $P$, $\eta$, $E$, and $P_d$ of an irreversible modified closed Brayton cycle. Diskin and Tartakovsky [55] introduced the low dissipation model and electrochemistry into the performance optimization of the Otto cycle and investigated the $\eta$ at the maximum $P$ operating point. Wang et al. [56] established irreversible Lenoir cycle with changing temperature heat reservoirs and investigated its $P$ and $\eta$. Gonca et al. [57] combined the Dual cycle and the Diesel cycle and derived the $P$, exergy efficiency, and $P_d$ of a Dual-Diesel cycle. Sahin [58] analyzed the effects of parameters such as compression ratio equivalence ratio cylinder temperature on the $P$, $P_d$, and effective ecological $P_d$ of a modified Dual cycle. Paul et al. [59] investigated the $P$ of Stirling cycle by applying optimal control theory. Bellos et al. [60] analyzed and optimized the $\eta$ of a solar-fed organic Rankine cycle, and compared it with the $\eta$ of organic Rankine cycle.


Ferrenberg [70] put forward the porous medium (PM) engine. Based on PM combustion technology [71], this engine had the characteristics of high efficiency, low emissions, low noise, and stable combustion. Durst et al. [72] simulated how the PM engine operated by modifying the Diesel engine and demonstrated performance of PM engine. Liu et al. [73] studied $P$ and $\eta$ performances of reversible PM engine.

Based on the FTT theory, Liu et al. [74] and Ge et al. [75] studied $P$ and $\eta$ of endoreversible [74] and irreversible [75] PM cycles. When the WF’s SH is constant with temperature, Zang et al. [76] analyzed effects of irreversibility losses, temperature ratio, and pre-expansion ratio on $P_d$, and conducted multi-objective optimization on $P$, $\eta$, $P_d$, and $E$ for irreversible PM cycle. Considering WF’s SH is linear variable with its temperature, Zang et al. [77] derived $E$ and $P_d$, compared $P$ and $\eta$ in the circumstances of maximum $\eta$, maximum $P$, maximum $E$ and maximum $P_d$, and performed multi-objective optimization on $P$, $\eta$, $P_d$, and $E$ for irreversible PM cycle.

In the aforementioned literature, there is no report on the $E_p$ performance optimization for irreversible PM engine. Based on [75–77], this paper will analyze the cycle maximum $E_p$ performance when WF’s SH is nonlinear variable with its temperature [45] and compare $\eta$ at the maximum $E_p$ $(\eta_{\text{max}})$ working point and the maximum $P$ $(P_{\text{max}})$ working point.
2. Model of PM

Irreversible PM cycle model shown in Figure 1 contains two constant volume processes 2–3 and 5–1; isothermal endothermic process 3–4; irreversible adiabatic compression process 1–2; and adiabatic expansion process 4–5. It can be seen from Figure 1 that \( v_1 = v_3 \), \( v_2 = v_5 \), \( T_3 = T_4 \), \( p_1 = p_5 \), and \( p_2 = p_3 \), where \( v \) is specific volume, \( T \) is temperature, and \( p \) is pressure.

![Figure 1](attachment:image.png)

Figure 1. \( p-v \) and \( T-s \) diagrams. (a) \( T-s \) diagram. (b) \( P-v \) diagram.

The cycle pre-expansion ratio \( (\rho) \), temperature ratio \( (\tau) \), and compression ratio \( (\gamma) \) are expressed as:

\[
\rho = \frac{V_4}{V_3} \quad (1)
\]

\[
\tau = \frac{T_3}{T_1} \quad (2)
\]

\[
\gamma = \frac{V_1}{V_2} \quad (3)
\]

In earlier studies [62–64], it was assumed that the WF’s SH was constant when the temperature range was small, but this assumption does not hold for large temperature changes in actual cycles. According to [45], assuming that the SH of the WF is only related to the temperature and has a nonlinear relationship with temperature, the constant pressure SH of the WF can be expressed as:
\[ C_p = a_0 T^2 + a_1 T^{1.5} + a_2 T + a_3 T^{0.5} + a_4 + a_5 T^{-1.5} + a_6 T^{-2} + a_7 T^{-3} \]  

(4)

where \( a_i \) are constants.

Constant volume SH and constant pressure SH have a following relationship:

\[ C_v = C_p - R \]  

(5)

where \( R \) is gas constant.

According to Equations (4) and (5), the constant volume SH can be expressed as:

\[ C_v = a_0 T^2 + a_1 T^{1.5} + a_2 T + a_3 T^{0.5} + a_4 - R + a_5 T^{-1.5} + a_6 T^{-2} + a_7 T^{-3} \]  

(6)

The heat absorption rate is:

\[
Q_{in} = \dot{m} \int_{T_i}^{T_f} C_v dT + \dot{m} \int_{T_i}^{T_f} (7.2674 \times 10^{-10} T^2 + 4.2166 \times 10^{-6} T^{1.5} - 1.23134 \times 10^{-5} T^{1.5} + 9.1968 \times 10^{-4} T^{0.5} + 30.2642 - 4.3848 \times 10^{-10} T^{-1.5} + 8.8827 \times 10^{-8} T^{-2} - 6.4148 \times 10^{-10} T^{-3}) dT \\
= \dot{m} [2.422 \times 10^{-3} + 1.6866 \times 10^{-6} T^{2.5} - 6.1567 \times 10^{-6} T^{2} + 6.1132 \times 10^{-10} T^{1.5} + 30.2642 T + 8.7696 \times 10^{-6} T^{-0.5} - 8.8827 \times 10^{-8} T^{-1.5} + 3.2074 \times 10^{-10} T^{-2} \int_{T_i}^{T_f} + MRT_i \ln \rho \]  

(7)

The heat release rate is:

\[
Q_{out} = \dot{m} \int_{T_i}^{T_f} C_v dT = \dot{m} \int_{T_i}^{T_f} (7.2674 \times 10^{-10} T^2 + 4.2166 \times 10^{-6} T^{1.5} - 1.23134 \times 10^{-5} T^{1.5} + 9.1968 \times 10^{-4} T^{0.5} + 30.2642 - 4.3848 \times 10^{-10} T^{-1.5} + 8.8827 \times 10^{-8} T^{-2} - 6.4148 \times 10^{-10} T^{-3}) dT \\
= \dot{m} [2.422 \times 10^{-3} + 1.6866 \times 10^{-6} T^{2.5} - 6.1567 \times 10^{-6} T^{2} + 6.1132 \times 10^{-10} T^{1.5} + 30.2642 T + 8.7696 \times 10^{-6} T^{-0.5} - 8.8827 \times 10^{-8} T^{-1.5} + 3.2074 \times 10^{-10} T^{-2} \int_{T_i}^{T_f} ] \]  

(8)

where \( \dot{m} \) is the mass flow rate.

Compression and expansion efficiencies are used to represent IIL of PM cycle for the processes \( 1 \rightarrow 2 \) and \( 4 \rightarrow 5 \).

\[
\eta_c = (T_{2s} - T_1) / (T_2 - T_1) \\
\eta_e = (T_5 - T_s) / (T_{5s} - T_4) 
\]

(9)\( (10)\)

where \( \eta_c \) is compression efficiency and \( \eta_e \) is expansion efficiency.

According to [44], it is presumed that the variable SH process of the WF is composed of an infinite number of infinitely small constant SH processes of the WF, and one has:

\[ TV^{k+1} = (T + dT)(V + dV)^{k+1} \]  

(11)

\[ C_v \ln T_i / T_j = R \ln V_i / V_j \]  

(12)

where \( T = (T_i - T_j) / \ln(T_j / T_i) \) is the temperature logarithmically on average between states \( i \) and \( j \).

According to Equations (7) and (8) and the processes \( 1 \rightarrow 2s \) and \( 4 \rightarrow 5s \), one has:

\[ C_v \ln(T_{2s} / T_1) = R \ln \gamma \]  

(13)

\[ C_v \ln(T_{5s} / T_4) = R \ln(\gamma / \rho) \]  

(14)

According to [43], the HTL between the WF and ambient through the cylinder cannot be ignored, and one has:
\[
\dot{Q}_{\text{loss}} = (B/2)(T_2 + T_1 - 2T_0) = (T_2 + T_1 - 2T_0)B_1
\]  
(15)

where \( T_0 \) is ambient temperature and \( B \) is HTL coefficient.

According to [46], the FL due to the movement of the cylinder wall and piston cannot be ignored, and one has:

\[
P = 4\mu(4L\eta)^2 = 64\mu(Ln)^2
\]  
(16)

where \( \eta \) is rotation speed and \( L \) is stroke length.

The \( P, \eta \), and \( E_p \) expressions are shown in Appendix A.

It can be seen from Figure 1 that state point 3 is located between state points 4 and 2, so the value range of \( \rho \) is:

\[
1 \leq \rho \leq V_1/V_2
\]  
(17)

The \( P \) and \( E_p \) after dimensionless processing can be expressed as:

\[
\tilde{P} = P / P_{\max}
\]  
(18)

\[
\tilde{E}_p = E_p / (E_p)_{\max}
\]  
(19)

3. Efficient Power Maximization

According to [45,63,65], the parameters are as follows: \( a_1=7.2674\times10^{-10} \), \( a_2=4.2166\times10^{-6} \), \( a_3=-1.23134\times10^{-4} \), \( a_4=9.1698\times10^{-4} \), \( a_5=38.5787 \), \( a_6=-4.3848\times10^5 \), \( a_7=8.8827\times10^6 \), \( a_8=-6.4148\times10^6 \), \( T_0=300K \), \( T_1=350K \), \( \rho=1-1.6 \), \( \tau=5.78-6.78 \), \( B=2.2W/K \), \( \mu=1.2kg/s \), \( n=30s^{-1} \), \( L=0.07m \), and \( m=1mol/s \).

Figure 2 shows the influence of cycle \( \tau \) on the \( E_p \) characteristics. The figure shows that the curve of the relationship between the \( E_p \) and \( \gamma \) (\( E_p \) vs. \( \gamma \) ) is a parabolic-like one, and it exists an optimal \( \gamma \) (\( \gamma_{E_p} \)) which can make the \( E_p \) reach the maximum (\( (E_p)_{\max} \)). The curve of the relationship between the \( E_p \) and \( \eta \) (\( E_p \) vs. \( \eta \) ) is a loop-shaped one which has the maximum \( \eta \) (\( \eta_{E_p} \)) working point and the \( (E_p)_{\max} \) working point. The corresponding \( \eta \) at the \( (E_p)_{\max} \) working point is \( (\eta_{E_p}) \), and the corresponding \( \eta \) at the \( \tilde{P}_{\max} \) working point is \( \eta_{\tilde{P}} \). As \( \tau \) increases, both \( \gamma_{E_p} \) and \( \eta_{E_p} \) increase. As \( \tau \) increases from 5.78 to 6.78, \( \gamma_{E_p} \) increases from 17.2 to 23.8, and \( \eta_{E_p} \) increases from 0.4851 to 0.5213, by an increase of about 8.20%
Figure 2. Effects of $\tau$ on $\bar{E}_p - \gamma$ and $\bar{E}_p - \eta$. (a) Variation of $\bar{E}_p$ with $\gamma$. (b) Variation of $\bar{E}_p$ with $\eta$.

Figure 3 shows influences of cycle $\rho$ on $\bar{E}_p$ characteristics. $\rho=1$ is performance curve of Otto engine cycle. As $\rho$ increases, both $\gamma_{E_p}$ and $\eta_{E_p}$ increase. As $\rho$ increases from 1.2 to 1.6, $\gamma_{E_p}$ increases from 20.4 to 24.4, and $\eta_{E_p}$ increases from 0.5045 to 0.5285, by an increase of about 4.76%.
Figure 3. Effects of $\rho$ on $\bar{E}_p - \gamma$ and $\bar{E}_p - \eta$. (a) Variation of $\bar{E}_p$ with $\gamma$. (b) Variation of $\bar{E}_p$ with $\eta$.

Figure 4 shows the influences of different losses on the $\bar{E}_p - \gamma$ and $\bar{E}_p - \eta$. Table 1 lists the $\gamma_{\bar{E}_p}$ and $\eta_{\bar{E}_p}$ obtained when different losses are considered. It can be seen from Table 1 that with the increase of loss items considered, both $\gamma_{\bar{E}_p}$ and $\eta_{\bar{E}_p}$ decrease, among which $\eta_\alpha$, $\eta_\epsilon$, and $B$ have bigger effect on $\gamma_{\bar{E}_p}$ and $\eta_{\bar{E}_p}$, and $\mu$ has a small effect on $\gamma_{\bar{E}_p}$ and $\eta_{\bar{E}_p}$. When the three losses are considered at the same time, the $\gamma_{\bar{E}_p}$ is reduced by 32.56% and the $\eta_{\bar{E}_p}$ is reduced by 25.41%.
Figure 4. Effects of $\eta$, $\eta_B$, and $\mu$ on $E_p - \gamma$ and $E_p - \eta$.

(a) $P_{E\gamma}$ and $P_{E\eta}$.

(b) $E_p - \gamma$.

Table 1. Comparison of the $P_{E\gamma}$ and $P_{E\eta}$ when considering different losses.

<table>
<thead>
<tr>
<th>Curve</th>
<th>Losses Considered</th>
<th>$\gamma_{E\gamma}$</th>
<th>Percentage of $\gamma_{E\gamma}$ Reduction</th>
<th>$\eta_{E\gamma}$</th>
<th>Percentage of $\eta_{E\gamma}$ Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Without any losses</td>
<td>30.1</td>
<td>0.00%</td>
<td>67.61%</td>
<td>0.00%</td>
</tr>
<tr>
<td>2</td>
<td>Friction</td>
<td>29.6</td>
<td>1.66%</td>
<td>66.32%</td>
<td>1.91%</td>
</tr>
<tr>
<td>3</td>
<td>Heat transfer</td>
<td>27.0</td>
<td>10.30%</td>
<td>60.56%</td>
<td>10.29%</td>
</tr>
<tr>
<td>4</td>
<td>Friction and heat transfer</td>
<td>26.7</td>
<td>11.30%</td>
<td>59.49%</td>
<td>12.01%</td>
</tr>
<tr>
<td>1’</td>
<td>Internal irreversibility</td>
<td>22.3</td>
<td>25.91%</td>
<td>56.85%</td>
<td>15.91%</td>
</tr>
<tr>
<td>2’</td>
<td>Internal irreversibility and friction</td>
<td>22.0</td>
<td>26.91%</td>
<td>55.68%</td>
<td>17.65%</td>
</tr>
<tr>
<td>3’</td>
<td>Internal irreversibility and heat transfer</td>
<td>20.6</td>
<td>31.56%</td>
<td>51.46%</td>
<td>23.89%</td>
</tr>
<tr>
<td>4’</td>
<td>Friction, heat transfer, and internal irreversibility</td>
<td>20.3</td>
<td>32.56%</td>
<td>50.42%</td>
<td>25.41%</td>
</tr>
</tbody>
</table>

Figure 5 shows the influences of $\tau$ and $\rho$ on $\eta_p$ and $\eta_{E\gamma}$. As can be seen from Figure 5a,b, no matter how $\tau$ and $\rho$ change, $\eta_{E\gamma}$ is always greater than $\eta_p$. 
Figure 5. Effects of \( \tau \) and \( \rho \) on \( \eta_{E_P} \) and \( \eta_p \). (a) Effect of \( \tau \). (b) Effect of \( \rho \).

Figure 6 shows the relationships between \( \bar{P} - \eta \) and \( \bar{E}_P - \eta \) when the cycle design parameters are constants. The results show that \( \eta_p \) is 47.05\%, and \( \eta_{E_P} \) is 50.45\%. The latter is about 7.22\% higher than the former. The \( \bar{P} \) is 0.9739 under the condition of \( (\bar{E}_P)_{\max} \), by a decrease of 2.61\%. Compared with the \( P_{\max} \) condition, the PM cycle sacrifices part of the power output under the \( (\bar{E}_P)_{\max} \) condition, but the cycle \( \eta \) can be greatly improved. The \( \bar{E}_P \) objective function reflects compromise between \( \eta \) and \( \bar{P} \).
The relationships of $P - \eta$ and $E_P - \eta$.

4. The Influences of SH Characteristics on Cycle Performance

Figure 7 shows the comparisons of $P$ and $\eta$ of irreversible PM cycle with constant SH ($P_{\text{const}}, \eta_{\text{const}}$), with linear variable SH ($P_{\text{linear}}, \eta_{\text{linear}}$), and with nonlinear variable SH ($P_{\text{nolinear}}, \eta_{\text{nolinear}}$). The performance indicators for the former two cases are shown in Appendix A and Appendix B.

Table 2 lists the maximum $P$ and maximum $\eta$ under the three kinds of SH models are obtained by deriving the $\gamma$. $P_{\text{linear}}^{\text{max}}$ is 27.14% larger than $P_{\text{const}}^{\text{max}}$, $P_{\text{nolinear}}^{\text{max}}$ is 16.83% larger than $P_{\text{const}}^{\text{max}}$, and $P_{\text{linear}}^{\text{max}}$ is 8.82% larger than $P_{\text{nolinear}}^{\text{max}}$. $\eta_{\text{linear}}^{\text{max}}$ is 2.73% larger than $\eta_{\text{const}}^{\text{max}}$, $\eta_{\text{nolinear}}^{\text{max}}$ is 0.63% larger than $\eta_{\text{const}}^{\text{max}}$, and $\eta_{\text{linear}}^{\text{max}}$ is 2.09% larger than $\eta_{\text{nolinear}}^{\text{max}}$. 

![Diagram showing the relationships of $P - \eta$ and $E_P - \eta$.](image1)

![Diagram showing the comparison of $P$ and $\eta$ under different SH models.](image2)
Figure 7. Effects of variable SH of WF on $P$ and $\eta$. (a) Effects of variable SH of WF on $P$. (b) Effects of variable SH of WF on $\eta$.

It is assumed that the WF’s SH is constant when the temperature range is small, but this assumption does not hold for large temperature changes in actual cycles. It can be seen from Figure 7 and Table 2 that the influences of different models of SH on cycle performance is very obvious. The model with nonlinear variation of WF’s SH is more suitable for the working state of the actual heat engine.

Table 2. Maximum values of $P$ and $\eta$ of the three cycle models.

<table>
<thead>
<tr>
<th>Maximum Power Output</th>
<th>Maximum Value</th>
<th>Maximum Efficiency</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{const}, \text{max}}$</td>
<td>15,858 W</td>
<td>$\eta_{\text{const}, \text{max}}$</td>
<td>0.5240</td>
</tr>
<tr>
<td>$P_{\text{linear}, \text{max}}$</td>
<td>20,162 W</td>
<td>$\eta_{\text{linear}, \text{max}}$</td>
<td>0.5383</td>
</tr>
<tr>
<td>$P_{\text{nolinear}, \text{max}}$</td>
<td>18,527 W</td>
<td>$\eta_{\text{nolinear}, \text{max}}$</td>
<td>0.5273</td>
</tr>
</tbody>
</table>

5. Conclusions

Applying FTT, the $E_P$ objective function is introduced into the study of optimal performances of an irreversible PM engine cycle, and $E_P$ expression is derived. The effects of $\tau$, $\rho$, three kind of losses, and WF’s variable SH characteristics on $E_P$ versus $\gamma$ and $\eta$ are analyzed, and the performance difference of the cycle under the condition of $P_{\text{max}}$ and $(E_P)_{\text{max}}$ are compared. The nonlinear variable SH model of WF with its temperature is adopted for irreversible PM cycle, which is closer to the actual working state of heat engines. The results show that:

1. The $P_d - \eta$ and $P_d - \gamma$ curves of the cycle are a loop-shaped one and parabolic-like one, respectively. As $\rho$ and $\tau$ increase, both $\gamma_{E_P}$ and $\eta_{E_P}$ increase. As $h_i$ and $k_i$ decrease and $B$, $\mu$, $\eta_i$, and $\eta_i$ increase, both $\gamma_{E_P}$ and $\eta_{E_P}$ decrease.
2. By choosing the appropriate pre-expansion ratio, PM engine cycle can be converted to Otto engine cycle.
3. Under specified conditions of different parameters, thermal efficiencies under the conditions of maximum efficient power and maximum power output are 50.45% and 47.05%, respectively. The latter is about 7.22% higher than the former.
4. Efficient power objective function reflects the compromise between thermal efficiency and power output. The results of this paper can guide parameters selection of the actual heat engine.

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**Nomenclature**

- \( B \) Heat transfer loss coefficient (W/K)
- \( C_v \) Specific heat at constant volume (J/(mol·K))
- \( k \) Adiabatic index (-)
- \( m \) Molar flow rate (mol/s)
- \( P \) Power output (W)
- \( Q \) Heat transfer rate (W)
- \( R \) Gas constant (J/mol/K)
- \( T \) Temperature (K)

**Greek symbols**

- \( \gamma \) Compression ratio (-)
- \( \eta \) Thermal efficiency (-)
- \( \eta_c \) Irreversible compression efficiency (-)
- \( \eta_e \) Irreversible expansion efficiency (-)
- \( \mu \) Friction loss coefficient (kg/s)
- \( \rho \) Pre-expansion ratio(-)
- \( \tau \) Temperature ratio (-)

**Subscripts**

- \( \text{in} \) Input
- \( \text{leak} \) Heat leak
- \( \text{out} \) Output
- \( \text{max} \) Maximum value
- \( P \) Max power output condition
- \( \eta \) Max thermal efficiency condition
- \( 1\text{–}5 \) Cycle state points

**Superscripts**

- \( - \) Dimensionless

**Abbreviations**

- FL Friction loss
- FTT Finite time thermodynamics
- HTL Heat transfer loss
Appendix A. Performance Indicators with Nonlinear Model of Variable SH

The cycle $P$ is:

$$P = \dot{Q}_m - \dot{Q}_{out} - P_{\mu}$$

$$= M[2.422 \times 10^{-10}(T_i^3 + T_i^1 - T_i^3 - T_i^0) + 1.6866 \times 10^{-6}(T_i^{2.5} + T_i^{2.5} - T_i^{2.5} - T_i^{2.5})$$

$$- 6.1567 \times 10^{-6}(T_i^1 + T_i^2 - T_i^2 - T_i^1) + 6.1132 \times 10^{-4}(T_i^{1.5} + T_i^{1.5} - T_i^{1.5} - T_i^{1.5}) + 30.2642$$

$$(T_i + T_2 - T_3) + 8.7696 \times 10^3(T_i^{0.5} + T_i^{0.5} - T_i^{0.5} - T_i^{0.5}) + 8.8827 \times 10^4(T_i^3 + T_i^3 - T_i^3 - T_i^3)$$

$$(T_i^3 - T_i^3) + 3.2074 \times 10^6(T_i^2 + T_i^2 - T_i^2 - T_i^2) + RT_i \ln \rho] - 64 \mu (L_i^2)$$

The cycle $\eta$ is:

$$\eta = \frac{P}{Q_m + Q_{out}}$$

$$= M[2.422 \times 10^{-10}(T_i^3 + T_i^1 - T_i^3 - T_i^0) + 1.6866 \times 10^{-6}(T_i^{2.5} + T_i^{2.5} - T_i^{2.5} - T_i^{2.5})$$

$$- 6.1567 \times 10^{-6}(T_i^1 + T_i^2 - T_i^2 - T_i^1) + 6.1132 \times 10^{-4}(T_i^{1.5} + T_i^{1.5} - T_i^{1.5} - T_i^{1.5}) + 30.2642$$

$$(T_i + T_2 - T_3) + 8.7696 \times 10^3(T_i^{0.5} + T_i^{0.5} - T_i^{0.5} - T_i^{0.5}) + 8.8827 \times 10^4(T_i^3 + T_i^3 - T_i^3 - T_i^3)$$

$$(T_i^3 - T_i^3) + 3.2074 \times 10^6(T_i^2 + T_i^2 - T_i^2 - T_i^2) + RT_i \ln \rho] - 64 \mu (L_i^2)$$

$$+ MRT_i \ln \rho + (B/2)(T_i + T_3 - 2T_0)$$

The cycle $E_p$ is:

$$E_p = P \eta$$

$$= [M[2.422 \times 10^{-10}(T_i^3 + T_i^1 - T_i^3 - T_i^0) + 1.6866 \times 10^{-6}(T_i^{2.5} + T_i^{2.5} - T_i^{2.5} - T_i^{2.5})$$

$$- 6.1567 \times 10^{-6}(T_i^1 + T_i^2 - T_i^2 - T_i^1) + 6.1132 \times 10^{-4}(T_i^{1.5} + T_i^{1.5} - T_i^{1.5} - T_i^{1.5}) + 30.2642$$

$$(T_i + T_2 - T_3) + 8.7696 \times 10^3(T_i^{0.5} + T_i^{0.5} - T_i^{0.5} - T_i^{0.5}) + 8.8827 \times 10^4(T_i^3 + T_i^3 - T_i^3 - T_i^3)$$

$$(T_i^3 - T_i^3) + 3.2074 \times 10^6(T_i^2 + T_i^2 - T_i^2 - T_i^2) + RT_i \ln \rho] - 64 \mu (L_i^2)$$

$$+ MRT_i \ln \rho + (B/2)(T_i + T_3 - 2T_0)$$

+ $MRT_i \ln \rho + (B/2)(T_i + T_3 - 2T_0)$

Appendix B. Performance Indicators with Model of Constant SH

The expressions of $P$, $\eta$, and $E_p$ of the irreversible PM cycle with constant SH [43] can be obtained as:

$$P = Q_m - Q_{out} = M[C_i(T_3 + T_1 - T_2 - T_1) + RT_i \ln \rho] - 64 \mu (L_i^2)$$

$$\eta = \frac{P}{Q_m + Q_{out}} = \frac{M[C_i(T_3 + T_1 - T_2 - T_1) + RT_i \ln \rho] - 64 \mu (L_i^2)}{M[C_i(T_3 - T_2) + RT_i \ln \rho] + B(T_2 + T_1 - 2T_0)}$$
where $C_v = 20.78\, J/(mol\cdot K)$ is constant.

### Appendix C. Performance Indicators with Linear Model of Variable SH

The SH of the WF is only related to the temperature and has a linear relationship with temperature [44]; the constant SH can be expressed as:

$$C_v = b_i + k_i T$$  \hspace{1cm} (A7)

where $b_i$ and $k_i$ are constants.

The expressions of $P$, $\eta$, and $E_p$ of the irreversible PM cycle with linear variable SH can be obtained as:

$$P = \dot{Q}_{in} - \dot{Q}_{out} - P_{\mu}$$

$$= M[b_i(T_1 + T_3 - T_2 - T_3) + 0.5K(T_1^2 + T_3^2 - T_2^2 - T_3^2) + RT_i \ln \rho] - 64\mu(Ln)^2$$  \hspace{1cm} (A8)

$$\eta = \frac{P}{\dot{Q}_{in} + \dot{Q}_{out}}$$

$$= \frac{M[b_i(T_1 + T_2 - T_1 - T_2) + 0.5K(T_1^2 + T_2^2 - T_1^2 - T_2^2) + RT_i \ln \rho] - 64\mu(Ln)^2}{M[b_i(T_1 - T_2) + 0.5K(T_1^2 - T_2^2) + RT_i \ln \rho] + MB[T_2 + T_1 - 2T_i]}$$  \hspace{1cm} (A9)

$$E_p = \frac{M[b_i(T_1 + T_2 - T_1 - T_2) + 0.5K(T_1^2 + T_2^2 - T_1^2 - T_2^2) + RT_i \ln \rho] - 64\mu(Ln)^2}{M[b_i(T_1 - T_2) + 0.5K(T_1^2 - T_2^2) + RT_i \ln \rho] + MB[T_2 + T_1 - 2T_i]}$$  \hspace{1cm} (A10)

### References


64. Chen, L.G.; Tang, C.Q.; Feng, H.J.; Ge, Y.L. Power, efficiency, power density and ecological function optimizations for an irreversible modified closed variable-temperature reservoir regenerative Brayton cycle with one isothermal heating process. *Energies* 2020, 13, 5133.


