Fault Location Based on Comprehensive Grey Correlation Degree Analysis for Flexible DC Distribution Network

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Abstract: Flexible DC distribution networks have a strong capacity for new energy consumption and have received extensive attention from scholars in recent years, but fault location for DC distribution lines is a difficult task. To solve this problem, we propose using a comprehensive grey correlation degree analysis to analyze the similarity of aerial mode traveling differential current waveforms at the first and last ends of a DC line, thus achieving fault location by obtaining the optimal time shift. Additionally, we built a six-terminal flexible DC distribution network model in Matlab/Simulink for simulation and validation, showing that the method can complete the rapid and accurate location of all types of faults on a DC line, and that it possesses an anti-transition resistance capability, making it suitable and highly reliable for cases of low sampling frequency.

Keywords: DC distribution network; fault location; comprehensive grey correlation degree; traveling differential current

1. Introduction

A flexible DC distribution system based on voltage source converters (VSCs) is an important new form of power distribution system, providing a good way of building a new type of power system with a high percentage of new energy sources [1–4].

The line of a DC distribution network is typically made from a cable, making it difficult to troubleshoot faults using traditional manual patrols, though there are relatively few transient faults [5]. Currently, fault location methods for DC distribution network are insufficient. Accurate fault location is critical for rapid fault recovery and improving reliability. The difficulty of fault location in DC distribution network is due to two major factors: First, faults in the DC system develop quickly, usually in tens of milliseconds, so the requirements for the quick activity of protection devices are high, resulting in insufficient available fault information for fault location. Second, while the more commonly used fault location methods in AC systems are often based on the fundamental frequency, DC distribution networks cannot directly use the fault location methods of AC systems [6].

There have been numerous studies on fault location for AC and HVDC systems that can be applied to DC distribution networks [7]. The main methods currently employed are fault analysis [8,9], the traveling wave method [10], and the use of artificial intelligence algorithms [11–13]. The first of these methods collects voltage and current data from the fault transient process and then identifies the fault line parameters in order to calculate the fault distance. Paper [8] calculates the fault distance using a matrix equation composed of voltage and current based on the capacitor discharge phase. Paper [9] analyzed the change of the current during the fault transient process and derived from this the formula for the fault distance. One of the most commonly used methods in practice is the traveling wave distance measurement method. The traveling wave method can provide rapid calculation and has a high positioning accuracy. However, this method must accurately capture the head of the traveling wave, otherwise the traveling wave method will have reliability
problems. Additionally, the equipment cost is high [10]. In recent years, many scholars have worked on the application of artificial intelligence in fault location. Paper [11] derives a fault distance measuring formula for DC distribution networks from artificial intelligence algorithms. Paper [12] uses transient traveling waves for fault location, which is more effective when the system sampling frequency is high. In order to reduce the requirement for sampling accuracy and obtain accurate fault location information, paper [13] proposed a Hausdorff distance-based HVDC traveling wave differential current similarity location method, and used cubic spline interpolation to compensate for the lack of sampling frequency, but did not take into account the impact of abnormal data on the Hausdorff distance algorithm.

Grey correlation has been widely used in many fields, such as economics and industry, and has been applied in the field of electric power [14–17]. Paper [17] uses grey correlation to compare the similarity of fault signals to achieve AC distribution network fault zone location and fault type identification, and this method can also be applied in DC line protection.

Current differential protection is based on Kirchhoff’s current law, the advantages of which include a clear principle and good selectivity, though it is affected by current amplitude decay. Traveling differential current protection is based on the difference between the traveling currents at both terminals of the fault line, and its theoretical basis is the traveling wave transmission invariance on the distributed parameter line model. Traveling differential current protection naturally takes into account the distributed capacitance current and transmission time delay. In the late 1970s, Japanese scholars first proposed the principle of traveling differential current protection based on the distributed parameter line model of lossless lines for feasibility studies. Subsequently, the world’s first traveling differential current protection device based on low-frequency information was developed by Tokyo Electric Power Company. However, the reliability of the protection device was seriously lacking due to the limitations of the technology level and the lack of knowledge of traveling current at that time, and it failed to achieve popular application. A new method of traveling differential current protection for HVDC lines is proposed in [18]. The relationship between the currents at both terminals of the line is analyzed. However, this method is only applicable to the field of HVDC systems. In paper [19], the authors propose a distributed parameter model-based differential current protection for HVDC. Two different sets of differential criteria are used to ensure the sensitivity. However, the differential criteria composition is complex and computationally intensive.

In this paper, we propose a fault location method for flexible DC distribution systems based on improved grey integrated correlation and traveling wave differential current, and we built a six-terminal flexible DC distribution network model in Matlab/Simulink to verify the proposed method.

2. Basic Theory of Fault Location
2.1. Traveling Wave Differential Currents

Voltage and current travel waves contain a wide range of fault information. Therefore, for traveling wave protection for power system lines, the line electrical state is described by forward and inverse traveling waves, instead of just using the traditional voltage and current.

Figure 1 shows a schematic diagram of a uniform DC line with a total line length of $L$ and a wave speed of $v$. The propagation delay from the beginning of the line to the end of the line is $\tau$, $t = L/v$. The direction of the arrow in the diagram indicates the positive direction of the positive and negative traveling wave currents. The positive and negative traveling currents at a and b are:
The time-shifted result of the fault current in the line wave transmission constancy. The distance between the a-side and b-side of the DC line, respectively, and the total propagation delay from the a-side to the b-side is \( \tau \), that is:

\[
\tau = \tau_a + \tau_b
\]  

Let the difference in propagation delay from a-side to b-side be \( \Delta \tau \), that is:

\[
\Delta \tau = \tau_a - \tau_b, \quad \Delta \tau \in (-\tau, \tau)
\]  

This gives the fault distance \( x \) as:

\[
x = v \cdot \tau_a = \frac{L + v \cdot \Delta \tau}{2}
\]  

Before the fault occurs, the traveling wave current at terminals a and b of the line satisfies:

\[
\begin{align*}
\begin{cases}
i_{a}^{+}(t) &= \frac{u_{a}(t)}{Z_c} + i_{a}(t) \\
i_{a}^{-}(t) &= \frac{u_{a}(t)}{Z_c} - i_{a}(t) \\
i_{b}^{+}(t) &= \frac{u_{b}(t)}{Z_c} - i_{b}(t) \\
i_{b}^{-}(t) &= \frac{u_{b}(t)}{Z_c} + i_{b}(t)
\end{cases}
\end{align*}
\]  

where \( i_{a}^{+}(t) \) and \( i_{b}^{+}(t) \) denote the positive traveling wave current on the a-side and b-side of the DC line, respectively, \( i_{a}^{-}(t) \) and \( i_{b}^{-}(t) \) denote the reverse traveling current on the a-side and b-side of the DC line, respectively, \( u_{a} \) and \( u_{b} \) denote the voltage on the a-side and b-side of the DC line, respectively, and \( Z_c \) is the wave impedance of the DC line.

\[
\begin{align*}
\begin{cases}
i_{a}^{+}(t) &= \frac{u_{a}(t)}{Z_c} + i_{a}(t) \\
i_{a}^{-}(t) &= \frac{u_{a}(t)}{Z_c} - i_{a}(t) \\
i_{b}^{+}(t) &= \frac{u_{b}(t)}{Z_c} - i_{b}(t) \\
i_{b}^{-}(t) &= \frac{u_{b}(t)}{Z_c} + i_{b}(t)
\end{cases}
\end{align*}
\]  

\[
\begin{align*}
\begin{cases}
i_{a}^{+}(t) &= \frac{u_{a}(t)}{Z_c} + i_{a}(t) \\
i_{a}^{-}(t) &= \frac{u_{a}(t)}{Z_c} - i_{a}(t) \\
i_{b}^{+}(t) &= \frac{u_{b}(t)}{Z_c} - i_{b}(t) \\
i_{b}^{-}(t) &= \frac{u_{b}(t)}{Z_c} + i_{b}(t)
\end{cases}
\end{align*}
\]  

\[
\begin{align*}
\begin{cases}
i_{a}^{+}(t) &= \frac{u_{a}(t)}{Z_c} + i_{a}(t) \\
i_{a}^{-}(t) &= \frac{u_{a}(t)}{Z_c} - i_{a}(t) \\
i_{b}^{+}(t) &= \frac{u_{b}(t)}{Z_c} - i_{b}(t) \\
i_{b}^{-}(t) &= \frac{u_{b}(t)}{Z_c} + i_{b}(t)
\end{cases}
\end{align*}
\]  

This gives the following forward and reverse line wave differential currents under normal conditions:

\[
\begin{align*}
\begin{cases}
di_{a}^{+}(t) &= i_{a}^{+}(t - \tau) - i_{b}^{+}(t) \\
di_{a}^{-}(t) &= i_{a}^{-}(t - \tau) - i_{b}^{-}(t)
\end{cases}
\end{align*}
\]  

After the fault occurs, the left and right parts of the line at point k meet the traveling wave transmission constancy. The time-shifted result of the fault current in the line wave differential current is:

\[
\begin{align*}
\begin{cases}
di_{b}^{+}(t) &= i_{b}(t - t_a) \\
di_{b}^{-}(t) &= i_{b}(t - t_b)
\end{cases}
\end{align*}
\]
The formula (7) shows the traveling differential currents after a short-circuit fault represents the short-circuit currents at different moments, the difference in time between them being exactly $\Delta \tau$. That is to say, $d_i^+(t + \Delta \tau) = i_k(t - \tau_b) = d_i^-(t)$

In other words, $d_i^+(t + \Delta \tau)$ and $d_i^-(t)$ have the highest serial correlation during the translation of the forward-traveling differential current along the time axis when the translation time is exactly $\Delta \tau$. This translation time is recorded and substituted into Equation (4) to obtain the value of $x$ and acquire the fault location target for the DC line.

2.2. Polar-Mode Transformation

As with the coupling between the three-phase voltage and current of a three-phase AC line, the positive and negative voltage and current of a bipolar DC line are coupled and need to be decoupled electrically utilizing a modal analysis.

For the discrete forward and reverse traveling wave data obtained by sampling, we used the Karrenbauer polar-mode transform method to decouple the polar-domain data of the DC distribution line and obtain the individual mode vectors corresponding to the mode domain. The transformed components of the signals are no longer electrically coupled, as when a traveling wave is transmitted on a single line.

The fault characterization in this paper is directed at the fault component network, and therefore the fault components of the DC distribution lines are obtained by the following polar-mode transformations:

$$
\begin{bmatrix}
  x_1 \\
  x_0
\end{bmatrix} = S^{-1} \begin{bmatrix}
  x_p \\
  x_n
\end{bmatrix} = \frac{\sqrt{2}}{2} \begin{bmatrix}
  1 & -1 \\
  1 & 1
\end{bmatrix} \begin{bmatrix}
  x_p \\
  x_n
\end{bmatrix}
$$

where $x_p$ and $x_n$ are the positive and negative electrical quantities, respectively, $x_1$ and $x_0$ are the corresponding aerial mode and ground mode components, respectively, and $S$ is the Karrenbauer transformation matrix:

$$
S = \frac{\sqrt{2}}{2} \begin{bmatrix}
  1 & 1 \\
  -1 & 1
\end{bmatrix}
$$

Equation (4) shows that the wave speed greatly affects the accuracy, while the ground mode and aerial mode traveling wave speeds are directly related to frequency. Since the aerial mode component flows in the opposite direction at the positive and negative poles, the wave speed varies with less frequency and attenuates less on the line compared with the ground mode component. Therefore, the proposed fault location method is implemented using the aerial mode components of the positive and negative traveling wave differential currents.

2.3. Improved Comprehensive Grey Correlation Degree

The similarity between the forward and reverse traveling wave differential currents can be judged and quantified using grey correlation degree. Compared with traditional analysis methods, grey correlation degree analysis has no strict sample size requirements, is efficient, and has anti-interference and anti-current transformer saturation capabilities [20]. Therefore, we used the grey correlation degree index to correlate the forward and reverse differential aerial mode current traveling wave sequence data.

Let the fault characteristics of the system be constituted as a data sequence:

$$
x_i = (x_i(1), x_i(2), \ldots, x_i(n))
$$

Fault electrical quantity correlation analysis can be performed using $k$ to denote the system sampling moment and $x_i$ to denote the monitoring data of the fault characteristics. Then, the monitoring data of $x_i$ at moment $k$ is represented by $x_i(k)$.
Given a reference sequence of \( x_0 = (x_0(1), x_0(2), \ldots, x_0(n)) \) and a comparison sequence of \( x_i = (x_i(1), x_i(2), \ldots, x_i(n)) \), \( i = 1, 2, \ldots, m \), the correlation coefficient of the two sequences would be defined as:

\[
r(x_0(k), x_i(k)) = \frac{\min_{i} |x_0(k) - x_i(k)| + 2\max_{i} |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \max_{i} |x_0(k) - x_i(k)|}
\]  
(12)

where \( \xi \in (0,1) \) is the resolution coefficient of the grey correlation, which can be changed dynamically to enhance the sensitivity and adaptability of the grey correlation analysis if necessary. We take \( \xi = 0.5 \) in this paper. The grey correlation of the two series is:

\[
g(x_0, x_i) = \frac{1}{n} \sum_{k=1}^{n} r(x_0(k), x_i(k))
\]  
(13)

As can be seen from Equation (12), the grey correlation is influenced by the maximum and minimum values of the differences between the sampled data, and once there is an abnormal extreme value at a certain sampling moment, the accuracy of the grey correlation analysis will be affected. At the same time, the value of the discrimination coefficient \( \xi \) is also one of the important factors influencing the grey correlation analysis.

In a fuzzy clustering system, a similarity coefficient is assigned to every two sets of data to determine the relationship between the series. The correlation coefficient method is commonly used to calculate the similarity coefficient \( \eta (x_0, x_i) \), which is given by:

\[
\eta(x_0, x_i) = \frac{\sum_{k=1}^{n} |x_0(k) - \bar{x}_0| |x_i(k) - \bar{x}_i|}{\sqrt{\sum_{k=1}^{n} (x_0(k) - \bar{x}_0)^2} \sum_{k=1}^{n} (x_i(k) - \bar{x}_i)^2}
\]  
(14)

where \( \bar{x}_0 = \frac{1}{n} \sum_{k=1}^{n} x_0(k), \bar{x}_i = \frac{1}{n} \sum_{k=1}^{n} x_i(k) \).

The grey correlation can be understood as quantifying the “similarity” shape variation between the sequences; the more similar the geometry of the corresponding function curve of the two sequences, the greater the grey correlation. The proximity coefficient can be understood as the difference in distance between the quantified sequences; the closer the two sequences are to each other, the greater the proximity coefficient. In the event of a fault in the line, due to the existence of DC line distribution parameters, the actual operation of the process will generate noise interference, and therefore the forward and reverse differential current travel wave will typically exhibit both randomness and ambiguity, simultaneously causing “similarity” and “proximity” differences.

In order to accurately describe the degree of correlation and to find the most accurate translation time \( \Delta \tau \), we combined the standard grey correlation \( \gamma (x_0, x_i) \) with the similarity coefficient \( \eta (x_0, x_i) \) to form the grey integrated correlation \( \omega (x_0, x_i) \).

\[
\omega(x_0, x_i) = \alpha \gamma(x_0, x_i) + \beta \eta(x_0, x_i)
\]  
(15)

where \( \alpha \) and \( \beta \) are weighting factors that satisfy \( \alpha > 0, \beta > 0, \alpha + \beta = 1 \).

2.4. Triangular Hermite Interpolation with Shape Parameters

After a short-circuit fault in a flexible DC distribution system, the current rises extremely quickly and the fault waveform changes in a stepwise manner and contains multiple harmonics. When the sampling frequency is low or the acquisition equipment is faulty and missing, the fault waveform may not be accurately described, and multi-point interpolation is required. Both triangular Hermite interpolation and cubic spline interpolation are common methods of multipoint interpolation, but the former sometimes fails...
to meet interpolation accuracy requirements due to the fixed shape of the spline and the endpoint effect of the latter. The literature [20] proposes an improved cubic triangular Hermite interpolation method that can better compensate for these defects. The specific principle is as follows.

For the given nodes \( a = x_0 < x_1 < \ldots < x_n = b \), the basic functions on the interval \([x_i, x_{i+1}]\) in the cubic triangular Hermite interpolation method with shape parameters are:

\[
\begin{align*}
\ f_i(t) &= \lambda_i S^2 - \lambda_i S^3 + C^3 \\
\ f_{i+1}(t) &= 1 - \lambda_i S^2 + \lambda_i S^3 - C^3 \\
\ g_i(t) &= \frac{2}{h_i}(-\lambda_i + S + \lambda_i S^2 - S^3 + \lambda_i C^3) \\
\ g_{i+1}(t) &= \frac{2}{h_i}(-C + \lambda_i S^2 - \lambda_i S^3 + C^3)
\end{align*}
\]

(16)

where \( S = \sin \frac{\pi t}{h_i} \), \( C = \cos \frac{\pi t}{h_i} \), \( i = 0, 1, 2, \ldots, n \), \( j = 0, 1 \). \( \lambda_i \) is the adjustment parameter.

Equation (16) satisfies the conditions that:

\[
\begin{align*}
\ f_i(x_j) &= \delta_{ij}, \ f_i(x_j) = 0 \\
\ g_i(x_j) &= 0, \ g_i(x_j) = \delta_{ij}
\end{align*}
\]

(17)

The above conclusions show that (16) has similar properties to those of the basic functions of standard cubic Hermite interpolation, but (16) includes an adjustment parameter \( \lambda_i \), which can be adjusted to obtain different basic functions. By finding the partial derivatives separately it can be shown that \( f_i(t) \) and \( g_{i+1}(t) \) are positively correlated with respect to \( \lambda_i \), and that \( f_{i+1}(t) \) and \( g_i(t) \) are negatively correlated with respect to \( \lambda_i \).

Given the data \( (x_i, y_i, d_i), i = 0, 1, \ldots, n \), where \( y_i \) and \( d_i \) are the value of the function and the value of the first order derivative at point \( x_i \), respectively, note that if:

\[
\begin{align*}
\ h_i &= x_{i+1} - x_i \\
\ t &= \frac{x - x_i}{h_i}
\end{align*}
\]

(18)

(19)

where \( \lambda_i \) can be any real number, one obtains a cubic triangular Hermite interpolation spline curve on the interval \([a,b] \):

\[
\begin{align*}
\ TH_i(x) \bigg|_{[x_i, x_{i+1}]} &= f_i(t)y_i + f_{i+1}(t)y_{i+1} + g_i(t)h_id_i + g_{i+1}(t)h_{i+1}d_{i+1}, i = 0, 1, \ldots, n - 1
\end{align*}
\]

(20)

It is easy to verify that \( TH_i(x) \), defined by Equation (20), satisfies:

\[
\begin{align*}
\ TH_i(x_i) &= y_i, \ TH_i(x_{i+1}) = y_i \\
\ TH_i'(x_i) &= d_i, \ TH_i'(x_{i+1}) = d_{i+1}
\end{align*}
\]

(21)

The interpolated spline curve can satisfy up to second-order continuity, and the shape of the interpolated curve can be controlled and modified locally or globally when different values of the parameter \( \lambda_i \) are used. This is much more accurate and universal than the three-time Hermite interpolation method with a fixed shape.

3. Fault Location Method

According to the above analysis process, we propose a fault location scheme for a flexible DC distribution network based on a comprehensive grey correlation degree. The specific steps are as follows:

Step 1: Sample to acquire the voltage and current at each side.

Step 2: Determine whether the sampling frequency complies with the requirements; if the sampling frequency is lower than the threshold \( f_0 \), go to step 3, otherwise, go to step 4.

Step 3: Perform three triangular Hermite interpolations on the data series as required.

Step 4: decouple positive and negative voltages and currents using the Karrenbauer polar-mode transform method.
Step 5: Calculate the line mode traveling wave differential currents in the positive and negative directions at the beginning and end of the line.

Step 6: Use the opposite aerial mode differential line wave current as a reference, gradually shift the forward aerial mode differential line wave current, calculate the similarity of the two waveforms after shifting by using the improved comprehensive grey correlation, and take the corresponding time shift for which the correlation is the largest, this being the best time shift.

Step 7: Bring the optimal time shift into formula (4) to determine the fault location.

The flow chart of the proposed method is shown in Figure 2.

![Flow chart of the fault location method.](image_url)

Within the scope of values of $\Delta \tau$ in Equation (3), it is known that the window length of post-fault data to be extracted by this algorithm is at least $2\tau_1$, and the formula for $\tau_1$ is:

$$\tau_1 = \frac{L}{\nu_1}$$  \hfill (22)

It can be seen that it is determined by the total length of the $\tau_1$ DC line and the line mode traveling current speed. Taking a fault at both ends of a 20 km DC distribution line as an example, the algorithm data window length of 1 ms is chosen, considering a certain redundancy.
Within the scope of values of $\Delta \tau$ in Equation (3), it is known that the window length $\tau$ is: $0.3393$ ms, the grey integrated correlation between $d_i^+(t - \Delta \tau_1)$ and $d_i^-(t)$ reaches its maximum value at the corresponding time shift $\Delta \tau = 0.3380$ ms, that the fault location can be calculated as $x = 4.9630$ km using the fault location formula, and that the error rate of the fault location is $K_{error} = -0.0185\%$. From the positive grounding simulation results, it can be seen that at a time shift of $\Delta \tau = -0.3393$ ms, the grey integrated correlation between $d_i^+(t - \Delta \tau)$ and $d_i^-(t)$ is the maximum, and the fault location can be calculated as $x = 4.9439$ km with $K_{error} = -0.2805\%$ using the fault location formula.

4. Simulation Validation

To verify the effectiveness of the proposed fault location method, a simulation model of a 10 kV six-terminal DC distribution network as shown in Figure 3 was built in Matlab/Simulink.

Figure 3. Topology of six-terminal flexible distribution network.

The distance between each adjacent DC bus is 20 km. The resistance per unit length is 0.0139 $\Omega$ and the inductance per unit length is 0.159 mH. The AC/DC and DC/AC converters are two-level VSCs. The A-VSC uses the outer-loop voltage droop control and the inner-loop constant current control. The B-VSC maintains the greatest power control operation except in certain special situations. The energy storage unit, as the slack bus of the system, is in charging or discharging mode to guarantee reliability and stability. The rated power of the energy storage is 20 kW and goes into islanded mode when needed. The PV uses the outer-loop MPPT and inner-loop constant voltage control. The unit power of the load is 20 kW. The wind turbines are permanent magnetic wind turbines. The rated wind speed is 12 m/s and the rated speed of wind power is 75 r/min.

The pole-to-pole short-circuit and positive pole-to-ground short-circuit is introduced within the first 5 km of Line1 at 0.15 s. The sampling frequency is 200 kHz. After the identification of the fault in the DC line, Line1 can be regarded as the DC line as in Figure 1. The aerial mode traveling wave differential current waveforms that can be measured at both terminals are shown in Figures 4 and 5.

As can be seen from Figures 4 and 5, there is a slight difference between $d_i^+(t - \Delta \tau_1)$ and the reference sequence $d_i^-(t)$ after translation, and they do not exactly coincide, which is due to the influence of distribution resistance, soil resistivity, etc. The correlation between the two needs to be judged using the correlation analysis method.

The grey integrated correlation analysis of the inter-pole short-circuit simulation results shows that the grey integrated correlation of $d_i^+(t - \Delta \tau_1)$ and $d_i^-(t)$ reaches its maximum value at the corresponding time shift $\Delta \tau = -0.3380$ ms, that the fault location can be calculated as $x = 4.9630$ km using the fault location formula, and that the error rate of the fault location is $K_{error} = -0.0185\%$. From the positive grounding simulation results, it can be seen that at a time shift of $\Delta \tau = -0.3393$ ms, the grey integrated correlation between $d_i^+(t - \Delta \tau)$ and $d_i^-(t)$ is the maximum, and the fault location can be calculated as $x = 4.9439$ km with $K_{error} = -0.2805\%$ using the fault location formula.
4.1. Effect of Fault Location and Fault Resistance

Simulation verification of pole-to-pole short-circuit and positive pole-to-ground short-circuit at was carried out at different fault locations and at different fault resistances, and the simulation results are shown in Figures 6 and 7, and in Table 1. The absolute value of the error rate is calculated by formula (23):

$$K_{error} = \left| \frac{x_{(measure)} - x_{(actual)}}{L} \right| \times 100\%$$  \hspace{1cm} (23)

As DC distribution lines are mostly cable lines and shorter in length than transmission lines, the fault resistance is small, generally not more than 10 Ω for pole-to-pole short-circuit and not more than 100 Ω for pole-to-ground short-circuit. As can be seen from the results in Table 1, the fault resistance has an impact on fault location accuracy, and the overall error increases when other influencing factors are controlled. However, even with a fault resistance of up to 10 Ω or 100 Ω, the accuracy of the proposed method can still be maintained within 0.6%. The maximum absolute error rate is 0.5415%, indicating that the fault location method proposed in this paper can quickly and accurately achieve fault...
location for the whole DC line under different fault conditions, and is quite resistant to fault resistance.

![Graph](image1)

**Figure 6.** Simulation results for pole-to-pole short-circuit.

![Graph](image2)

**Figure 7.** Simulation results for positive pole-to-ground short-circuit.
Table 1. Simulation results for different fault resistances and fault locations.

<table>
<thead>
<tr>
<th>x (actual)/km</th>
<th>Pole-to-Pole Short-Circuit</th>
<th>Positive Pole-to-Ground Short-Circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rf/Ω</td>
<td>x (measure)/km</td>
</tr>
<tr>
<td>0</td>
<td>0.9882</td>
<td>0.1990</td>
</tr>
<tr>
<td>1</td>
<td>0.9769</td>
<td>0.2655</td>
</tr>
<tr>
<td>10</td>
<td>7.0268</td>
<td>0.1340</td>
</tr>
<tr>
<td>100</td>
<td>6.9982</td>
<td>0.0090</td>
</tr>
<tr>
<td>10</td>
<td>7.0395</td>
<td>0.1975</td>
</tr>
<tr>
<td>100</td>
<td>12.9703</td>
<td>0.1485</td>
</tr>
<tr>
<td>10</td>
<td>13.0273</td>
<td>0.5215</td>
</tr>
<tr>
<td>100</td>
<td>13.0318</td>
<td>0.5330</td>
</tr>
<tr>
<td>100</td>
<td>12.9703</td>
<td>0.1485</td>
</tr>
<tr>
<td>100</td>
<td>13.0273</td>
<td>0.5215</td>
</tr>
<tr>
<td>100</td>
<td>13.0318</td>
<td>0.5330</td>
</tr>
</tbody>
</table>

4.2. Influence of Sampling Frequency

This fault location method based on the traveling wave principle is highly precise. However, the method itself needs a high sampling frequency, which may lead to large location errors at low sampling frequencies, while increasing the sampling frequency will necessarily result in increased equipment investment, which is currently one of the biggest limitations of the application of the traveling wave principle in distribution networks.

The sampling frequencies were set at 10 kHz and 100 kHz, and the results of the proposed method to verify the fault location under positive pole-to-ground fault are shown in Figures 8 and 9, and in Table 2.
Fault location results for different sampling frequencies. 

Table 2. Fault location results for different sampling frequencies.

<table>
<thead>
<tr>
<th>x (actual)/km</th>
<th>Rf/Ω</th>
<th>f = 100 kHz</th>
<th>f = 10 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x (measure)/km</td>
<td>Perror/%</td>
<td>x (measure)/km</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.9207</td>
<td>0.3965</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>0.8327</td>
<td>−0.8365</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>0.7903</td>
<td>−1.0485</td>
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<tr>
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<td>0</td>
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<td>−0.7815</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>7.1843</td>
<td>0.9215</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>7.2857</td>
<td>1.4285</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>12.8744</td>
<td>−0.6280</td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td>13.2601</td>
<td>1.3005</td>
</tr>
<tr>
<td>13</td>
<td>100</td>
<td>13.2601</td>
<td>1.3005</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
<td>18.8304</td>
<td>−0.8480</td>
</tr>
<tr>
<td>19</td>
<td>10</td>
<td>18.8197</td>
<td>−0.9015</td>
</tr>
<tr>
<td>19</td>
<td>100</td>
<td>18.6963</td>
<td>−1.5185</td>
</tr>
</tbody>
</table>

A reduction in sampling frequency will directly lead to an increase in the sampling interval, which can lead to distortion of the measured data and waveforms when the interval is too large, thus constituting a limitation on the application of the grey integrated correlation proposed in this paper. As shown in Table 2, the location accuracy is negatively correlated with the sampling frequency. For a 20 km distribution line, the error rates should not exceed 1%, otherwise it will greatly increase the manual patrol workload. With the sampling frequency reduced to 100 kHz, some of the error rates already exceeded 1%, and when the sampling frequency was further reduced to 10 kHz, most of the error rates exceeded 1%, reaching a maximum of 1.9470%, that is, a distance difference of 0.3894 km from the actual fault location. The results show that the accuracy of the proposed fault location method cannot meet the necessary requirements at this low sampling frequency.

The idea of interpolation is important in the approximation of discrete functions. By interpolating the fault location functions, the precision at low sampling frequencies can be improved to a certain extent. Combining the correlation analysis method with the interpolation method can effectively reduce the fault location error and improve the location accuracy. We choose to combine the proposed comprehensive grey correlation.
degree with cubic triangular Hermite interpolation with shape parameters. After simulation verification, this method effectively reduced the error rate, and had was more effective under the influence of different fault resistances. In addition, at lower sampling frequencies, this different method matched well with the grey correlation, which also greatly reduced the errors at low sampling frequencies.

In this paper, the $d_{i1}^+(t)$ and $d_{i1}^-(t)$ of the positive grounding short-circuit at 10 kHz are interpolated using cubic triangular Hermite interpolation with shape parameters, and the interpolated data are used for fault location on the DC distribution lines. The results are shown in Figure 10 and in Table 3.

![Figure 10. Simulation results for proposed fault location method.](image)

Table 3. Location results of proposed fault location method.

<table>
<thead>
<tr>
<th>$x_{\text{actual}}$/km</th>
<th>$R_f$/Ω</th>
<th>$x_{\text{measure}}$/km</th>
<th>$P_{\text{error}}$/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.9047</td>
<td>0.4765</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.8889</td>
<td>0.5555</td>
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<td></td>
<td>100</td>
<td>1.0894</td>
<td>0.4470</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>6.9066</td>
<td>0.4670</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>7.0994</td>
<td>0.4970</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>7.0981</td>
<td>0.4905</td>
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<td>0</td>
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<td>0.4005</td>
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<td>0.3485</td>
</tr>
<tr>
<td>19</td>
<td>10</td>
<td>18.9358</td>
<td>0.3210</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>18.9078</td>
<td>0.4610</td>
</tr>
</tbody>
</table>

Comparing the results in Table 3 with those in Table 2, the accuracy has been improved by the interpolation method, the maximum absolute error value increasing from 1.9470% to 0.5555%, corresponding to a difference of 0.5005 km between the two fault location points in a 20 km DC distribution line. The interpolation method can therefore be applied in practical projects to increase the location precision at low sampling frequencies, thus enabling staff to quickly and accurately locate the fault point, eliminate the fault in time, and ensure reliable and stable operation of the system.
4.3. Comparison with Other Similarity Algorithms

To better reflect the benefits and features of the grey integrated correlation-based fault location method proposed in this paper, some additional anomalously large or small data points were added to the actual sampling sequence and compared with the proposed method, the traditional grey correlation method, and the Person correlation coefficient method. Figures 11 and 12, and Table 4, show the simulation results of three similarity algorithms when a fault occurs at 5 km along the DC cable.

![Figure 11. Simulation results for pole-to-pole short-circuit under three algorithms.](image)

![Figure 12. Simulation results for positive pole-to-ground short-circuit under three algorithms.](image)

<table>
<thead>
<tr>
<th>Type</th>
<th>$R_f/\Omega$</th>
<th>Pearson Correlation Coefficient</th>
<th>Traditional Grey Correlation Degree</th>
<th>Comprehensive Grey Correlation Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$x_{\text{measure}}/\text{km}$</td>
<td>$P_{\text{error}}/%$</td>
<td>$x_{\text{measure}}/\text{km}$</td>
</tr>
<tr>
<td>Pole-to-pole short-circuit</td>
<td>0</td>
<td>4.8862</td>
<td>0.5690</td>
<td>4.9299</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4.8862</td>
<td>0.5690</td>
<td>5.0808</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>4.8326</td>
<td>0.8370</td>
<td>5.0838</td>
</tr>
<tr>
<td>Positive pole-to-ground short-circuit</td>
<td>0</td>
<td>4.8702</td>
<td>0.6490</td>
<td>4.9111</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>5.1683</td>
<td>0.8415</td>
<td>4.9111</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>5.1683</td>
<td>0.8415</td>
<td>5.1105</td>
</tr>
</tbody>
</table>
As can be seen from Table 4, the Pearson correlation coefficient and the traditional grey correlation-based location analysis methods have larger errors than the comprehensive grey correlation degree method proposed in this paper, all other conditions being equal. This is due to the fact that the anomalous extremes have a greater influence on the value of the Pearson correlation coefficient, while the traditional grey correlation can only reflect the “similarity” of the positive and negative differential current traveling wave sequences, while in reality there is often a “similarity” difference between the two sequences. Where the distribution lines themselves are short and the underground cables are not directly identifiable by humans, it is important to minimise the errors inherent in the principle of the algorithm, as this is likely to result in a failure to identify the exact fault location.

Compared with the Pearson correlation coefficient and traditional grey correlation, the fault location error of this method is reduced by 0.4648% and 0.1828%, respectively. Furthermore, the location error of the proposed method is on the whole smaller than that of other similar algorithms. We also found that the comprehensive grey correlation degree works well with the cubic triangular Hermite interpolation method with shape parameters, effectively reducing the required sampling frequency and providing a practical basis for its application to DC distribution networks.

5. Conclusions
In this paper, a fault location method for a flexible DC distribution system based on the comprehensive grey correlation degree is proposed. Based on the correlation of the forward and reverse traveling wave differential currents at both terminals of the DC line, the correlation between the two is quantified and characterized using an improved comprehensive grey correlation degree to extract fault location information, which can accurately locate various fault conditions on the whole line. The use of improved triangular Hermite interpolation at low sampling frequencies overcomes the sampling frequency limitation in traveling wave fault location, improves the fault location accuracy for flexible DC distribution networks to a certain extent, reduces the investment cost of sampling equipment for distribution networks, and provides the possibility of in-depth research on the traveling wave principle in DC distribution networks.

Author Contributions: Conceptualization, T.M. and Z.H.; methodology, T.M.; software, H.D.; validation, Y.X.; formal analysis, Z.H. and H.D.; investigation, Y.X.; resources, Z.H.; data curation, Z.H. and H.D.; writing—original draft preparation, T.M., Z.H. and Y.X.; writing—review and editing, T.M.; visualization, Y.X.; supervision, T.M.; project administration, T.M.; funding acquisition, T.M. All authors have read and agreed to the published version of the manuscript.

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