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Abstract: A numerical simulation of convective heat transfer coefficient ($h_{\text{conv}}$) was studied with Cu-Water and TiO$_2$-Water nanofluids flowing through a circular tube subjected to uniform wall heat flux boundary conditions under laminar and turbulent regimes. Four different concentrations of nanofluids ($\phi = 0.5, 1, 1.5$ and $2\%$) were used for the analysis and the Reynolds number (Re) was varied from laminar (500 to 2000) to turbulent flow regime (5000 to 20,000). The dependence of $h_{\text{conv}}$ on Re and $\phi$ was investigated using a single-phase Newtonian approach. In comparison to base fluid, average $h_{\text{conv}}$ enhancements of $10.4\%$ and $7.3\%$ were noted, respectively, for the maximum concentration ($\phi = 2\%$) and Re = 2000 for Cu-Water and TiO$_2$—water nanofluids in the laminar regime. For the same $\phi$ under the turbulent regime (Re = 20,000), the enhancements were noted to be $14.6\%$ and $13.2\%$ for both the nanofluids, respectively. The random motion (Brownian motion) and heat diffusion (thermophoresis) by nanosized particles are the two major slip mechanisms that have more influence on the enhancement of $h_{\text{conv}}$. In addition, the Nusselt number (Nu) of the present work was validated for water with the Shah and Dittus Boelter equation and found to have good agreement for both the regimes.

Keywords: maximum copper; titanium oxide; nanoparticle; convection; heat transfer; flow regime

1. Introduction

The substantial need for solving thermal management problems in modern electronic devices is increasing nowadays [1]. As the performance efficiency of these devices decreases when operating at higher temperatures, efficient cooling of these devices to enhance their long term becomes essential [2–6]. Single-phase convective heat transfer which uses conventional liquid coolant has been employed as a common method to extract the generated heat. However, the use of conventional liquid coolants such as water, ethylene glycol and other coolants fails to extract large amounts of heat from these devices. Therefore, a working fluid resulting in a higher heat transfer coefficient value than the conventional liquid coolants, resulting in an efficient cooling of electronic devices, is the pursuit of today’s modern industries and researchers.

Many experimental works have been carried out to enhance the poor thermal properties of existing liquid coolants. In recent years, the suspension of nanosized solid particles...
in the base fluid has attracted great attention as an enhancement of the fluid’s thermal properties \[7–9\]. In particular, adding nanoparticles such as aluminum, copper, silicon and silver to water, ethylene glycol and other conventional liquid coolants resulted in higher heat transfer characteristics. This is due to the higher thermal conductivity of the nanoparticles that subsequently enhance the effective thermal conductivity of the solid liquid mixture. Moreover, the addition of nanoparticles increases the overall heat transport capability of a single-phase and two-phase flow \[10–12\]. Thus, employing nanofluids results in effective heat dissipation which is a promising alternative. Despite there being many experimental studies on convective heat transfer enhancement using nanofluids, very few works have been carried out on computational analysis to study the fluid flow behavior with different nanofluids for both laminar and turbulent regimes. Therefore, analyzing the fluid flow using nanofluids for different flow regimes is still an active research topic in the present context \[13\]. Some of the numerical studies on enhancement in single phase heat transfer with higher thermal conductivity nanofluids are as follows.

Rakhsha et al. \[14\] studied thermal performance using CuO/water nanofluid inside helically coiled tubes. Turbulent forced convection with constant wall temperature condition was solved using OpenFOAM solver. When comparing the results with pure water, enhancements of 6–7% and 9–10%, respectively, were observed for thermal performance and pressure drop. However, for various experimental conditions, the heat transfer coefficient increases by 16–17% and the pressure drop increases by 14–16% for various tube geometries and different Re. Hussein et al. \[15\] numerically analyzed the effect of nanofluid flowing through tubes having different cross-sectional areas. The mesh type used for the tube wall surfaces was rectangular cells, but the internal spaces were meshed with triangular cells. The outputs of the CFD model and experimental data were compared. They show that when the volume concentration was varied from 1–2.5%, a maximum deviation of 4% in friction factor and 6% in \(h_{\text{conv}}\) was observed for the tested nanofluid.

Ebrahimnia-Bajestan et al. \[16\] examined the numerical analysis on different nanoparticles with water and ethylene glycol/water mixture. Nanoparticles such as carbon, aluminum oxide (Al\(_2\)O\(_3\)), copper oxide (CuO) and titanate nanotubes were mixed homogeneously in the fluids, and were made to pass horizontally through a circular pipe under a constant heat flux condition. The particle diameter had an inverse effect on the heat transfer coefficient, while an increase in the concentration of nanoparticles and Re resulted in the enhancement of heat transfer coefficients. Akbari et al. \[17\] numerically studied the influence of nanofluid flowing in a horizontal tube with constant heat flux using single- and two-phase models. Bahiraei and Hosseinalipour \[18\] showed that the particle fluxes were in the range of \(10^{-10}\) and \(10^{-8}\) for Brownian diffusion and thermophoresis, respectively. Since thermophoresis affects particle migration much more significantly than Brownian diffusion, it should be taken into account.

Haddad et al. \[19\] investigated the Rayleigh–Bénard natural convection problem with various Rayleigh numbers at different volume fractions of CuO/water nanofluid. The authors suggested that the influence of Brownian motion and thermophoresis should be considered for heat transfer enhancement. Behroyan and Ganesan \[20\] determined the dynamic viscosity of Cu particles suspended in nanofluid on the basis of Newtonian and non-Newtonian models by means of a single-phase approach. Shariat et al. \[21\] studied the flow of nanofluid in elliptic ducts under a laminar mixed convection. The elliptic tube with an AR value of 0.75 had the minimum friction coefficient and maximum Nusselt number (Nu), so the authors suggested it instead of the circular pipe. Akbaridoust et al. \[22\] experimentally and numerically investigated the heat transfer performance of a laminar flow of nanofluid through helically coiled tubes. The authors studied CuO-water nanofluid for the volume concentration (0.1 and 0.2%) with the average particle size of 68 nm and examined the pressure drop, and \(h_{\text{conv}}\) behavior.

Mahian et al. \[23\] reviewed several articles on the modelling and simulation of nanofluids and suggested that most of the numerical analysis did not take into account the effects of particle sedimentation, erosion and corrosion effects, non-uniformity in the shape and size...
of the nanoparticles, and interparticle forces. Rajesh et al. [24] studied numerical analysis on Cu, TiO$_2$, Al and Cu+TiO$_2$ nanofluids by varying the volume concentration (1–3%), nanoparticle diameter (20–80 nm) and Re (200–600) under uniform and non-uniform magnetic fields. The results showed that, at 3% concentration of Cu nanofluid under magnetic field conditions, a maximum heat transfer enhancement of 173% was observed.

Based on the existing literature, it is clear that the heat transfer performance of base fluid can be enhanced by suspending nanosized particles. Most of the authors have suggested metallic nanoparticles as they possess a higher thermal conductivity than metal oxide nano-particles. While there have been many experimental works on single-phase heat transfer enhancement using different nanofluids, very few works on analytical approach have been carried out to study fluid flow characteristics in different flow regimes. Therefore, in this proposed work, a numerical study is performed to analyze heat transfer enhancement using two different nanofluids, namely, Cu-water and TiO$_2$- water nanofluid. Moreover, the effect of Re (500 to 20,000) on wall temperature, heat transfer coefficient and Nu for nanofluids with different concentrations varying from 0.5% to 2% is compared with base fluid. In addition, the slip mechanisms that are responsible for heat transfer enhancement by using nanofluids are discussed clearly and the present numerical data of Nu are validated with the previously published literature.

2. Thermo-Physical Properties of Nanofluids

The calculation of thermophysical properties such as effective density, thermal conductivity, dynamic viscosity and effective specific heat of nanofluids is relatively straightforward. In the present analysis, nanofluids are called a mixture as they consist of matrixes and particles which are continuous and discontinuous base fluid components.

\[
\rho_{nf} = \phi \rho_{np} + (1 - \phi) \rho_{nf} \\
\mu_{nf} = \mu_{nf} (1 + 2.5\phi) \\
C_{p_{nf}} = \frac{\phi (\rho_{np} C_{p_{np}}) + (1 - \phi) (\rho_{nf} C_{p_{nf}})}{\rho_{nf}} \\
k_{nf} = \left[ k_{np} + 2k_{nf} + 2 \left( k_{np} - k_{nf} \right) (1 + \beta)^3 \phi \right] \frac{k_{nf}}{k_{np}} \\
h_{conv} = \frac{q}{t_{iwx} - t_{fx}}
\]

where

q—Heat flux in W/m$^2$

Inner wall temperature, $t_{iwx} = t_{owx} - \Delta t_x$

where

\[
\Delta t_x = \frac{Q}{2\pi k L_x}
\]

Fluid temperature $t_{fx} = t_{iu} + (t_{out} - t_{iu}) \left( \frac{x}{T} \right)$

For Laminar regime, $Nu = \begin{cases} 1.953 \left( \frac{Re Pr D}{T} \right)^{0.5} & \text{if } \left( \frac{Re Pr D}{T} \right) \geq 33.3 \\ 4.364 + 0.0722 Re Pr D & \text{if } \left( \frac{Re Pr D}{T} \right) < 33.3 \end{cases}$

For turbulent regime, $Nu = 0.023 Re^{0.8} Pr^{0.3}$
Convective heat transfer coefficient ($h$) is calculated using the following equation

$$h_{\text{conv}} = \frac{q}{T_{\text{in}} - T_{\text{f}}x} \quad (10)$$

Nusselt number is calculated as,

$$Nu = \frac{hL}{k} \quad (11)$$

3. Governing Equations

3.1. Single-Phase Model

In this proposed work, a turbulent forced convection condition is considered for the single-phase analysis. The following equations for a steady flow of an incompressible Newtonian fluid are written as follows [25].

1. Continuity Equation

$$\nabla \cdot (\rho_{\text{eff}} \vec{V}) = 0 \quad (12)$$

2. Momentum Equation

$$\nabla \cdot \rho_{\text{eff}} \vec{V} \vec{V} = -\nabla P + \nabla \cdot (\mu_{\text{eff}} \nabla \vec{V} - \rho_{\text{eff}} \vec{V} \nabla) \quad (13)$$

3. Energy Equation

$$\nabla \cdot (\rho_{\text{eff}} C_{\text{p,eff}} \vec{V}T) = \nabla \cdot \left( \left( k_{\text{eff}} + k_t \right) \nabla T \right) \quad (14)$$

3.2. Numerical Solution

In the present study, nanofluids (Cu/Water) and (TiO$_2$/Water) enter a cylindrical horizontal tube made of copper with a 0.004 m diameter and a length of 1 m under a constant wall heat flux condition which is represented in Figure 1a. Meshing is a part of the solver’s second primary stage. The creation of mesh was carried out after creating the geometry. The meshing step chops the domain into cells or elements. On each of the cells, the solver approximates the governing equations and/or boundary condition. The end result consists of a large set of simultaneous algebraic equations and is solved by the simulator in the computer. The boundary conditions such as the left vertical line as velocity inlet, the right vertical line as pressure outlet, the bottom horizontal line as center line and the top vertical line as wall are applied on the geometry.

3.3. Grid Independence Study

The size of the mesh was changed from $40 \times 800$ to $60 \times 1200$. The optimum mesh size of $50 \times 1000$ was chosen based on the accuracy of the results as shown in Table 1. The sectional view of the optimum mesh geometry is shown in Figure 1b. Based on the grid independence study, the optimum quadrilateral mesh was used in the analysis with the edge sizing of $50 \times 1000$. In ANSYS FLUENT the boundary conditions, solution methods, residuals and solution initializations were set and solved. In the post processing, the solutions were established for the optimum mesh. Figure 1c shows the temperature contour plot for the Reynolds of 800 and constant heat flux of 10,000 W/m$^2$. Figure 1d indicates the variation of inner wall temperature along the length for various Re. Figure 2 shows the methodology followed for solving in CFD. The geometry was created in the ANSYS design modeler.
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### Table 1. Grid Independence Study.

<table>
<thead>
<tr>
<th>Mesh Sizes (Quadrilateral Mesh)</th>
<th>Tw (Wall Temperature)</th>
<th>Tf (Fluid Temperature)</th>
<th>h (Convective Heat Transfer Coefficient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 × 800</td>
<td>299.59</td>
<td>296.59</td>
<td>665.1999</td>
</tr>
<tr>
<td>50 × 1000</td>
<td>300.12</td>
<td>300.08</td>
<td>681.3538</td>
</tr>
<tr>
<td>60 × 1200</td>
<td>300.12</td>
<td>300.09</td>
<td>680.6259</td>
</tr>
</tbody>
</table>

Figure 1. (a) Test Section Geometry (b) Sectional view of the Mesh Geometry (c) Temperature Contour for Re = 800 (d) Inner Wall Temperature along the length for various Reynolds numbers.

**4. Results and Discussion**

4.1. Effect of Re on Wall Temperature with Cu-Water Nanofluid

The effect of Re on mean wall temperature by using Cu-Water nanofluid under laminar flow condition is shown in Figure 3a. The wall temperature decreases with DI water and
nanoparticle concentration. In addition, the wall temperature decreases with increase in Re. The average wall temperatures for laminar flow along the heated wall by using DI water and for the maximum concentration ($\phi = 2.0\%$) are noted as 296.6 K and 294.4 K, respectively. Thus, the maximum reduction in wall temperature is noted with the maximum value of Re = 2000 and nanoparticle concentration. Moreover, an increase in concentration of the nanoparticles enhances the rate of heat absorption and reduces wall temperature. In addition, with an increase in Re value, the velocity of the working fluid that flows through the tube increases. Due to this, the working fluid removes a sufficient amount of heat from the inner walls of the tube which subsequently reduces the outer wall temperature.

![Figure 3](image)

**Figure 3.** Variation of wall temperature with Re for Cu-water nanofluid. (a) Laminar condition. (b) Turbulent condition.

In addition to the laminar condition, the effect of Re on wall temperature along the heated wall under turbulent flow condition is shown in Figure 3b. Similar results are noted in which the wall temperature decreases for Re ranging from 5000 to 20,000 and for the nanoparticle concentration ranging from $\phi = 0.5\%$ to $\phi = 2.0\%$. The highest wall temperature of 294.1 K is noted for DI water, the lowest value of 293.1 K is noted for the maximum nanoparticle concentration. Moreover, the wall temperature decreases as the Re value increases. This is due to the fact that, in case of nanofluids, an increase in nanoparticle concentration enhances the Brownian motion of nanoparticles. Therefore, for all Re values, a decrease in wall temperature is observed when compared to that of DI water. The thermal-fluid properties of nanofluids are tabulated in Table 2.

**Table 2.** Thermal-fluid properties for nanofluids.

<table>
<thead>
<tr>
<th>Nanofluid</th>
<th>Volume Concentration</th>
<th>Dynamic Viscosity (Ns/m$^2$)</th>
<th>Specific Heat (J/kgK)</th>
<th>Thermal Conductivity (W/mK)</th>
<th>Density (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu/Water</td>
<td>0.5</td>
<td>0.000953</td>
<td>4014.45</td>
<td>0.6196</td>
<td>1038.86</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>0.000964</td>
<td>3862.97</td>
<td>0.6363</td>
<td>1078.64</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>0.000976</td>
<td>3722.26</td>
<td>0.6534</td>
<td>1118.41</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>0.000988</td>
<td>3591.21</td>
<td>0.6709</td>
<td>1158.19</td>
</tr>
<tr>
<td>TiO$_2$/Water</td>
<td>0.5</td>
<td>0.000959</td>
<td>4105.48</td>
<td>0.6117</td>
<td>1015.24</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>0.000965</td>
<td>4035.24</td>
<td>0.6259</td>
<td>1031.40</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>0.000981</td>
<td>3967.16</td>
<td>0.6404</td>
<td>1047.55</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>0.000999</td>
<td>3901.44</td>
<td>0.6553</td>
<td>1063.73</td>
</tr>
</tbody>
</table>
4.2. Effect of Re on Wall Temperature with TiO$_2$-Water Nanofluid

Figure 4a shows the variation of mean wall temperature for TiO$_2$ based nanofluids with a laminar flow regime. A decrease in mean wall temperature is noted when the volume concentration of nanoparticle increases from 0.5% to 2.0%. The highest wall temperature range is noted for DI water (297.7 K to 295.3 K) and the lowest wall temperature range is noted for the maximum concentration of TiO$_2$-water based nanofluid (297 K to 294.6 K). This is due to the fact that when nanosized particles are dispersed in water, the effective thermal conductivity of the nanofluid increases. The increase in thermal conductivity of the mixture is due to the high density of conduction electrons, which move at a faster rate leading to enhancement in heat transfer rate. Thus, an increase in the thermal conductivity of a nanofluid results in a lower wall temperature when compared to that of base fluid for a particular Re value.

![Figure 4a](image1.png)

**Figure 4a:** Variation of wall temperature with Re for TiO$_2$-water nanofluid. (a) Laminar condition. (b) Turbulent condition.

The effect of Re when using TiO$_2$-water based nanofluid for turbulent flow conditions on wall temperature is shown in Figure 4b. Based on the results, a decrease in mean wall temperature is observed when the volume concentrations and Re increase. For the highest Re value of 20,000, the wall temperature of 293.6 K is noted for DI water and 293.3 K is noted for the maximum concentration ($\phi = 2\%$). Thus, an increase in concentration of nanoparticles increases the density of conduction electrons and reduces the wall temperature. Even though the effective thermal conductivity of nanofluid increases with higher concentrations of nanoparticles, the heat conduction mechanism in liquids should also be considered.

4.3. Effect of Re on Heat Transfer Coefficient with Cu-Water Nanofluid

Figure 5a shows the effect of Re on $h_{\text{conv}}$ for a laminar flow. The heat transfer coefficient is higher for all the concentrations of nanofluid than the base fluid and reaches the maximum for $\phi = 2\%$. The highest heat transfer coefficient is noted to be 1100 W/m$^2$K for the maximum concentration of nanoparticles with an enhancement of 10.4% when compared with base fluid. Similarly, the effect of Re on $h_{\text{conv}}$ for a turbulent flow is shown in Figure 5b. As can be observed, the heat transfer coefficient reaches a maximum at $\phi = 2\%$ and is noted to be 17,800 W/m$^2$K. An average enhancement of 14.6% is noted for the maximum concentration of Cu-water nanofluid when compared to that of base fluid.
4.3. Effect of Re on Heat Transfer Coefficient with Cu-Water Nanofluid

The reason for the enhancement in heat transfer coefficient is due to the effect of slip velocity. Generally, in a single-phase model, the velocity variation between water and nanoparticles is assumed to be negligible. However, this assumption cannot be justified when forced convection is involved. As a single-phase model cannot precisely describe the enhancement of a forced convection system, for a clear analysis, many research works have focused on observing the slip velocity (relative velocity that occurs between the suspended nanoparticles and DI water) by considering the nanofluid as a two-phase mixture. This slip velocity occurs due to the temperature difference between the suspended nanoparticles and DI water, nanoparticle concentration gradient, effective viscosity of solid-liquid mixture and gravity. Due to this, migration of nanoparticles occurs to which is attributed the enhancement of the heat transfer coefficient. The slip velocity is higher when the concentration of nanoparticles is increased. This is due to the fact that, for higher concentration nanofluids, the particle density increases which in turn increases the slip velocity. Therefore, drifting of nanoparticles occurs from one place to other and results in a higher heat transfer coefficient value than DI water.

4.4. Effect of Re on Heat Transfer Coefficient with TiO\textsubscript{2}-Water Nanofluid

The effect of Re on heat transfer coefficient for a laminar flow by using TiO\textsubscript{2} -water nanofluid is shown in Figure 6a. The heat transfer coefficient increases when the nanoparticle concentration and Re value is increased. Based on the tested concentrations, the highest $h_{\text{conv}}$ value is noted for the nanoparticle concentration $\phi = 2\%$ and it is noted to be $840 \text{ W/m}^2\text{ K}$. An average enhancement of 7.3% is noted for $\phi = 2\%$ when compared to DI water.

In addition, the effect of Re for turbulent flow by using TiO\textsubscript{2} -water nanofluid is shown in Figure 6b. Similar results are observed for the turbulent flow regime with a higher heat transfer coefficient value for $\phi = 2\%$. The maximum heat transfer coefficient value of $13,000 \text{ W/m}^2\text{ K}$ is noted for $\phi = 2\%$ with an average enhancement of 13.2%. The improvement in the heat transfer coefficient of TiO\textsubscript{2}—water nanofluid than DI water is due to the following two slip mechanisms. 1. When the temperature of the fluid increases, the Brownian diffusion increases in the nanofluid as the concentration of nanoparticles is higher with lesser particle size. As a temperature gradient occurs, a movement of nanoparticle from a high temperature zone to low temperature zone takes place which is termed thermophoresis. In this condition, the migration of suspended particles towards the center core of the channel takes place at a higher velocity which results in a non-uniform flow. Thus, the nanoparticle that gets dispersed in base fluid rotates in the flow of working fluid thereby generating a lift force called the Magnus effect. Even though diffusiophoresis and settlement of nanoparticles by the influence of gravity contribute to

![Figure 5. Average $h_{\text{conv}}$ with Re for Cu–Water nanofluid. (a) Laminar condition. (b) Turbulent condition.](image-url)
the motion of nanoparticles, Brownian motion and thermophoresis also are considered to be major mechanisms in heat transfer enhancement.

![Figure 6](image_url)  
**Figure 6.** Average $h_{\text{conv}}$ with Re for TiO$_2$–Water nanofluid. (a) Laminar condition. (b) Turbulent condition.

### 4.5. Effect of Re on Nusselt Number for Laminar Flow Conditions

Figure 7 shows the effect of Re on the Nusselt number of DI water and nanofluids of different concentrations in a laminar flow regime. As shown, an increase in Nusselt number of DI water and Cu-water nanofluid is observed as the Re value is increased. Additionally, there is an increase in trend for Nu as Re increases. The highest Nu is noted for the maximum concentration of nanoparticles ($\phi = 2\%$) and the value is noted to be $\text{Nu} = 6.2$. An average enhancement of 7.2% is noted for the nanofluid with the maximum particle concentration ($\phi = 2\%$) when compared to that of base fluid. Enhancements in Nu with increases in Re and nanoparticle concentration are noted. Based on the results, the highest Nu value for laminar flow regime is noted to be $\text{Nu} = 5$ for $\phi = 2\%$. An average enhancement of 14.6% is noted for the maximum particle concentration when compared to the base fluid. It is observed that, the addition of Cu nanoparticles enhances the thermal conductivity of the base fluid. In the case of nanofluids, $h_{\text{conv}}$ increases with higher nanoparticle concentration. In contrast, an increase in nanoparticles enhances viscosity and leads to an increase in thermal boundary layer thickness. This may cause a reduction in Nu and the heat transfer coefficient value. However, based on Figure 7, it can be observed that the addition of nanoparticles increases the Nu which indicates that the thermal conductivity enhancement in the nanofluid overcomes the viscosity effect and leads to higher heat transfer. Moreover, this thermal conductivity enhancement is due to the particle migration hypothesis.

It is noted that for both laminar and turbulent flow regimes, the Nu value for TiO$_2$–water nanofluid is lesser when compared with Cu-water nanofluid. The thermal conductivity of metallic nanoparticles is much higher than the metallic oxide nanoparticles [26]. Due to this, the effective thermal conductivity increases and thereby enhances the value of Nu. In addition, the validation of Nu is carried out with previously published works particularly for laminar flow. Based on the validation, the numerical results of Nu for Cu/water nanofluid obtained from the present study agrees well with Aliabadi et al. [27] and Yang et al. [28] with the average deviation of 11.2 and 13.1%, respectively. However, in case of metallic nanofluids, the stability of nanoparticles must also be focused upon as settlement occurs due to its higher density.
Reduction in wall temperature is observed for both the nanofluids when compared to water and TiO$_2$ were considered and the outputs were compared with those of water. The outcomes were well with Aliabadi et al. [27] and Yang et al. [28] with the average deviation of 11.2 and 13.1%, respectively. However, in case of metallic nanofluids, the stability of nanoparticles must also be considered under constant wall heat flux conditions. The computational results for the Cu-water and TiO$_2$-water nanofluids using single-phase model were verified with the existing literature. The water-based Cu and TiO$_2$ nanofluids with various volume concentrations were considered and the outputs were compared with those of water. The outcomes are discussed below.

1. Reduction in wall temperature is observed for both the nanofluids when compared to DI water. However, the lowest temperature is observed for the Cu-water nanofluid rather than the TiO$_2$-water nanofluid and the value is noted to be 293.3 K and 293.1 K for the laminar and turbulent regimes, respectively. The increase in thermal conductivity of conduction electrons results in higher heat absorption leading to reduction in wall temperature.

2. Significant enhancement in $h_{\text{conv}}$ is observed for both the nanofluids and the enhancement percentage increases with increases in $\phi$ and Re. Average enhancements of 10.4% and 14.6% are observed for Cu-water nanofluids under laminar and turbulent regimes, respectively.

3. The Nu value for both Cu-water and TiO$_2$-water nanofluids is found to be higher when compared to that of base fluid and the highest Nu value of 105 is noted for Cu-water nanofluid. Brownian motion of nanoparticles results in the uniform distribution of temperature in the nanofluid which leads to reductions in wall temperature and increases in Nu.

**Figure 7.** Validation of the present results with the existing literature.

5. Conclusions

In this study, forced convection heat transfer of water based nanofluids with Cu and TiO$_2$ nanoparticles in a circular pipe was numerically analyzed for laminar and turbulent flow regimes. Different volume concentrations such as 0.5%, 1%, 1.5% and 2% were examined under constant wall heat flux conditions. The computational results for the Cu-water and TiO$_2$-water nanofluids using single-phase model were verified with the existing literature. The water-based Cu and TiO$_2$ nanofluids with various volume concentrations were considered and the outputs were compared with those of water. The outcomes are discussed below.

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3. The Nu value for both Cu-water and TiO$_2$-water nanofluids is found to be higher when compared to that of base fluid and the highest Nu value of 105 is noted for Cu-water nanofluid. Brownian motion of nanoparticles results in the uniform distribution of temperature in the nanofluid which leads to reductions in wall temperature and increases in Nu.

**Author Contributions:** Methodology, J.R.B.; Software, S.M.; Validation, S.M.; Investigation, J.R.B. and A.A.A.; Resources, A.A.A. and L.G.A.; Data curation, L.G.A.; Writing–original draft, J.R.B. and L.G.A.; Writing–review & editing, S.G. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was not financially supported by any organization.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Acknowledgments:** The authors are thankful to the Karunya Institute of Technology and Sciences, for their guidance and unstinted support for this study.
Conflicts of Interest: This study was not financially supported by any public or private institution. Moreover, the authors declare that they have no competing interests.

Nomenclature

- $H$: Heat transfer coefficient (W/m$^2$K)
- $k$: Thermal conductivity (W/m K)
- $Re$: Reynolds number
- $\phi$: Volume fraction
- $q$: Heat flux (W/m$^2$)
- $Nu$: Nusselt number (h.D/K)
- $Q$: Volumetric flow rate (m$^3$/s)
- $C_p$: Specific heat capacity (J/kg K)
- $Pr$: Prandtl number
- $x$: Distance from tube inlet
- $t_{in}$: Temperature at inlet (K)
- $t_{out}$: Temperature at outlet (K)
- $f$: Darcy’s friction factor
- $L$: Tube length (m)
- $\rho$: Density (kg/m$^3$)
- $\mu$: Dynamic viscosity (Pa)
- $\beta$: Correction factor = 0.1
- $v$: Radial velocity (m/s)
- $t_{iwx}$: Inner wall temperature, (K)
- $t_{fx}$: Fluid temperature (K)
- $t_{owx}$: Outer Wall temperature, (K)

References


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