

Article

Entropy Generation Minimization of Two-Phase Flow Irreversibilities in Hydrocarbon Reservoirs

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Abstract: The efficient use of available energy in hydrocarbon extraction processes is essential to reducing overall emissions in the petroleum industry. The inefficient design of an extraction process leads to higher emissions per unit mass of hydrocarbon recovery. Fluid friction and heat transfer are irreversible processes that are vital in decreasing the overall system's operational efficiency. To reduce these irreversible energy losses in the petroleum reservoir production's life, contributing factors such as the characteristic features of a reservoir formation, reservoir fluids, and production rate are investigated in this paper. This study examines irreversible energy loss in porous media and wellbore formations using entropy generation minimization at various stages of production and thermodynamic conditions, eventually achieving higher hydrocarbon recovery factors. Entropy production is used to develop predictive models that calculate reservoir and wellbore energy losses for multiphase flow. The proposed models consider oil and water as the working fluids in a porous medium and a wellbore. This paper also investigates the thermophysical effects around the wellbore by incorporating Hawkin's model. A sensitivity analysis assessed the impact of rock and fluid properties and thermodynamic conditions such as temperature, wettability, and capillary pressure on the total entropy generation. The findings reveal that the capillary pressure significantly impacts the oil and water recovery factor and total entropy production. Additionally, the capillary pressure strongly influences the reservoir production life. The two-phase models show that as the recovery factor increases, the total entropy production decreases at lower production rates. This article helps to address the impact of irreversible processes on multiphase hydrocarbon reservoir operational efficiency. Furthermore, the results obtained from the numerical-simulation model open up a new research area for scholars to maximize the recovery factor using entropy generation minimization in heterogeneous reservoirs.



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Keywords: entropy generation minimization; energy loss; oil–water fluids; porous media

1. Introduction

Thermodynamics laws can examine various hydrocarbon reservoir energy processes [1]. Entropy is a measure of the molecular configuration probability of a system or the ability of a system to perform valuable work [2,3]. Significant efforts to analyze the reservoir energy lost during production processes have been made in recent years [4–6]. This paper focuses on the energy availability that exists in a hydrocarbon reservoir and the irreversible losses that occur during extraction processes.

From a hydrocarbon production point of view, primary reservoir energy plays a vital role in overall petroleum production [7]. The production rate, reservoir pressure, and skin factor influence the reservoir energy and production rate of reservoir fluids [8–11]. This paper will investigate these factors and other reservoir and operating characteristics, such as capillary pressure and relative permeability. The friction that occurs due to turbulence

within the fluid and from the fluid's movement inside a channel is a leading cause of irreversible energy loss. To decrease reservoir energy loss, vital production-related factors, such as reservoir formation characteristics, properties of reservoir fluids, and production rate, should be considered for more effective design and maintenance.

This paper is an extension of the previous work of Elhaj et al. [10]. The previous study focused on the energy losses in three systems—a reservoir, around a wellbore, and a wellbore—due to production for a single-phase flow, while this paper focuses on two systems—a reservoir and a wellbore—for a two-phase flow using the same principles of thermodynamics that were used in the single-phase case. The fundamentals of single-phase fluid flow are relatively well understood; however, a two-phase flow is more complicated due to the existence of moving and deformable interfaces and interactions among the involved phases.

Multiphase flow involves a heterogeneous mixture of multiple fluids or phases, which are not homogeneously mixed at a molecular level, but which can be identified by macroscopic structures such as solid particles, droplets, and/or bubbles in certain regions. One can classify them according to the state of the two phases or components (e.g., gas–liquid, gas–solid, liquid–liquid, and liquid–solid). Various fluid and energy loss aspects of two-phase flow systems are still poorly understood. This paper focuses on investigating the entropy generation in porous media as well as for a wellbore and multiphase flow.

Miller et al. [12,13] developed a transient multiphase flow finite-difference model for a geometrical well. Initially, the model was developed under the condition of the homogenous flow of one component at thermodynamic equilibrium. The extension of the model then included slip and a finite rate of evaporation. The model is only applicable in a flowing well, which limits the use of this model in a reservoir system.

A Representative Elementary Volume (REV) averaging technique was used to develop balance equations for multiphase flow in porous media [14]. Gray [14] used a technique to derive the vertically averaged balance equations of mass, momentum, energy, and entropy. Past research has neglected the entropy equation for flow in porous media; however, Gray [14] included it and showed significant effects. The transport equations were valid for vapor, liquid, and solid phases.

An important thermodynamic property known as the “availability function” can be used for multiphase well cases to calculate the thermodynamic efficiency, including energy losses due to irreversibility [15–17]. The availability function's benefit is that it captures the thermodynamic efficiency of a system even if no work (production or consumption) is performed on the targeted system. In addition, it has the potential to compare different processes, which may lead to good system improvement possibilities.

A thermodynamic model for separated two-phase flow was developed by Kocamustafaogullari [18]. The two-fluid model was formulated by considering each phase separately. The formulation was expressed in two conservation equation sets (one for each phase). In addition to the continuity, momentum, and energy equations, the enthalpy equations were used together with appropriate conditions to formulate the overall model for the two-phase flow. A stability theory for general separated two-phase flow was developed using the model. The study had no applicability in porous media; however, its fundamental concepts were used in porous media [14].

Previous studies conducted experimental investigations on the system's thermal performance and entropy generation [19,20]. Those research studies aimed to examine the effect of fluid concentrations on system efficiency. The experimental work presented by Sheikholeslami et al. [21] investigated the effect of vapor quality and nanofluid concentration on entropy generation and exergy losses of nano-refrigerants. It was found that the relationship between nanofluid concentration and the frictional entropy generation is proportional. Rahbar et al. [22] proposed two new empirical correlations to predict water productivity, which resulted in lower entropy generation in triangular and tubular solar stills. This paper will formulate similar concepts but instead apply the formulation to hydrocarbon reservoirs to reduce irreversible losses and improve overall process efficiency.

2. Pressure Profile Formulation

Past models developed by Elhaj et al. [10,11] for single-phase flow will be extended in this work to two-phase flow (oil–water). The following assumptions are made in the modelling approach: the reservoir is an idealized cylindrical geometry; radial flow exists toward the well; the well is vertical and/or deviated; fluid flow is one-dimensional; the fluid is a two-phase system; an isothermal condition is maintained; the production occurs under pseudo-steady state conditions; and a constant flow rate for each phase is assumed in the well.

A semi-steady state model will be used to describe the fluid flow in porous media. The properties of hydrocarbon mixtures are assumed to be constant as both systems are assumed to be at isothermal conditions. Figure 1 shows a schematic diagram of a production volumetric reservoir. Two parts will be considered for simulation: (i) a wellbore model and (ii) a reservoir model. The primary fluid flow equations in both systems are the continuity (conservation of mass), momentum, energy, and entropy equations (second law of thermodynamics).

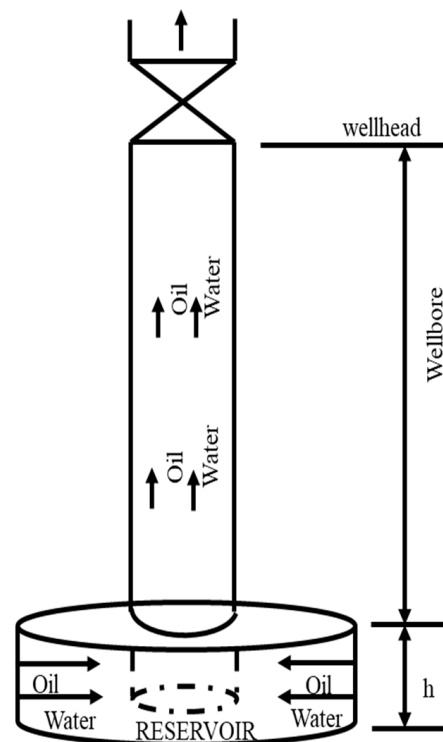


Figure 1. Schematic of a production volumetric reservoir for multiphase flow.

The appropriate governing equations for pipeline and porous media will address the total entropy generation minimization of both systems. The wellbore and reservoir equations will be solved simultaneously. A one-dimensional, two-phase flow model will be developed for the wellbore, while a radial, pseudo-steady state, two-phase flow (oil and water) is assumed for the reservoir system.

For pseudo-steady state flow, the rate of change of pressure with respect to time is assumed to be constant for a reservoir operating at a total constant terminal rate of q_{tsc} [23]. This type of flow occurs if the outer boundary is completely closed. Therefore, the pressure will decline when fluids are removed from the reservoir, since fluids cannot flow across the reservoir boundaries.

2.1. Reservoir Pressure Profile

Consider the constitutive relationships for fluid and rock properties of two-phase flow in porous media. The auxiliary equations are given below [23–25].

$$s_o + s_w = 1 \quad (1)$$

$$P_{cow} = P_o - P_w \quad (2)$$

$$c_o = -\frac{1}{B_o} \frac{\partial B_o}{\partial P} \quad (3)$$

$$c_w = -\frac{1}{B_w} \frac{\partial B_w}{\partial P} \quad (4)$$

$$c_r = \frac{1}{\phi} \frac{\partial \phi}{\partial P} \quad (5)$$

$$c_T = c_o s_o + c_w s_w + c_r \quad (6)$$

$$k_o(s_o) = k k_{ro} \quad (7)$$

$$k_w(s_w) = k k_{rw} \quad (8)$$

Important parameters characterizing the two-phase flow behavior are auxiliary equations [26] that define the interaction parameter between two phases in a particular system. In this study, the two-phase system is oil and water flowing in the reservoir and wellbore. This study focuses on the relationship between capillary pressure and saturation and the relationship between relative permeability and saturation.

The material balance equations are used for both fluids to find the value of $\partial P / \partial t$ for a two-phase flow. The mass balance for oil is given by Aziz and Settari [25]:

$$\frac{\partial}{\partial t} (V_b \phi \rho_o s_o) = \rho_o q_o \quad (9)$$

where $m_o = V_b \phi \rho_o s_o$ is the oil mass in porous medium; V_b is the bulk volume; ϕ represents the porosity; ρ_o refers to the oil density; s_o introduces the oil saturation; and q_o is the oil flow rate.

Taking the partial derivative of Equation (9) for oil and water and adding these two equations:

$$\frac{\partial P_o}{\partial t} = \frac{q_{tsc} B_o}{\phi c_T h \pi r_e^2} \quad (10)$$

Equation (10) represents the rate of pressure change with respect to time for pseudo-steady state flow.

The continuity equations for the oil and water, respectively, are given by Lee and Wattenbarger [24]:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{k_o}{\mu_o B_o} \rho_{os} \frac{\partial P}{\partial r} \right) = \frac{\partial}{\partial t} \left(\phi \frac{s_o}{B_o} \rho_{os} \right) \quad (11)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{k_w}{\mu_w B_w} \rho_{ws} \frac{\partial P}{\partial r} \right) = \frac{\partial}{\partial t} \left(\phi \frac{s_w}{B_w} \rho_{ws} \right) \quad (12)$$

where r is the radius; ϕ stands is the porosity; P represents the pressure; μ_o and μ_w refer to the viscosities for oil and water; s_o and s_w are the oil and water saturation, respectively; ρ_{os} and ρ_{ws} introduce the densities at standard conditions for oil and water, respectively; and B_o and B_w symbolize the formation volume factors for oil and water, respectively.

Referring to Equations (11) and (12) for slightly compressible fluids, assume that $\left(\frac{\partial P}{\partial r}\right)^2$, $\left[\frac{\partial P}{\partial r} \frac{\partial s_o}{\partial r}\right]$, and $\left[\frac{\partial P}{\partial r} \frac{\partial s_w}{\partial r}\right]$ are negligible. Taking the partial derivative for the formation volume factor for oil and water phases:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P}{\partial r} \right) = \phi \frac{\mu_o}{k_o} \left[\frac{\partial s_o}{\partial P} - \frac{s_o}{B_o} \frac{\partial B_o}{\partial P} \right] \frac{\partial P}{\partial t} \quad (13)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P}{\partial r} \right) = \phi \frac{\mu_w}{k_w} \left[\frac{\partial s_w}{\partial P} - \frac{s_w}{B_w} \frac{\partial B_w}{\partial P} \right] \frac{\partial P}{\partial t} \quad (14)$$

The fluid mobility for each phase is defined as follows:

$$\lambda_o = \frac{k_o}{\mu_o} \quad (15)$$

$$\lambda_w = \frac{k_w}{\mu_w} \quad (16)$$

The total compressibility for multiphase flow in porous media can be determined from the model given below [24]:

$$c_t = \left(\frac{\partial s_w}{\partial P} - \frac{s_w}{B_w} \frac{\partial B_w}{\partial P} \right) \left(\frac{\lambda_t}{\lambda_o} \right) \quad (17)$$

Equation (17) is the total compressibility for oil and water as it has both fluid mobilities, and it was developed initially from a series of equations from the diffusivity equation.

Substituting Equation (17) in Equations (13) and (14) gives:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P}{\partial r} \right) = \frac{\phi c_t}{\lambda_o} \frac{\partial P}{\partial t} \quad (18)$$

Capillary pressure, P_c , is defined as the difference in pressure across the interface between two immiscible fluids.

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P}{\partial r} \right) &= \frac{\phi c_t}{\lambda_w} \frac{\partial P}{\partial t} \\ \text{or } \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P}{\partial r} \right) - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P_c}{\partial r} \right) &= \frac{\phi c_t}{\lambda_w} \frac{\partial P}{\partial t} \end{aligned} \quad (19)$$

Equation (18) describes fluid flow through porous media for slightly compressible fluids. Equation (19) includes the capillary pressure as a variable in the fluid flow.

Substituting Equation (10) in Equation (18) leads to:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P}{\partial r} \right) = \frac{1}{\lambda_o} \frac{q_{sc} B}{h \pi r_e^2} \quad (20)$$

Using the Darcy equation:

$$\frac{d}{dr} \left(\frac{q}{B} \right) = - \left(\frac{2r}{r_e} \right) q_{sc} \quad (21)$$

Integrating Equation (21) and using the volumetric flux, and then substituting the result in the Darcy equation give the following pressure distribution in the reservoir:

$$\frac{\partial P}{\partial r} = B_l \frac{\mu_l}{k_l} \frac{q_{sc}}{2 \pi h} \left(\frac{1}{r} - \frac{r}{r_e^2} \right) \quad (22)$$

Equation (22) represents the pressure distribution in the reservoir for a two-phase flow, which is similar to a single-phase flow except that k_l is the effective liquid (oil or water)

permeability; μ_l is the viscosity of a liquid (oil or water); and B_l introduces the liquid (oil or water) formation volume factor.

2.2. Flowing Well Pressure

The governing equation for the well pressure is based on integrating the mechanical energy equation along a streamline within a well at a constant flow rate [27,28], as given below:

$$\frac{1}{\rho} \frac{dP}{d\ell} + v \frac{dv}{d\ell} + g_c \sin \theta + \frac{dW}{d\ell} = 0 \quad (23)$$

where ℓ is the axial coordinate; v refers to the velocity; g_c is the gravitational conversion constant; W represents the work; and θ is the inclination angle.

The following relationship determines the pressure loss due to friction along the well length [27]:

$$\frac{dw}{d\ell} = \frac{f_M v^2}{2D} \quad (24)$$

where f_M is the Moody friction factor and D represents the wellbore diameter.

Taking the derivative of the fluid velocity ($v = \frac{4q_{sc}B}{\pi D^2}$) with respect to pressure leads to:

$$\frac{dv}{dp} = \frac{4q_{sc}}{\pi D^2} \frac{dB}{dp} \quad (25)$$

Using the chain rule and substituting Equations (24) and (25) into Equation (23), the following expression is obtained:

$$\frac{dp_l}{d\ell} = \frac{g \frac{H}{L} + \frac{f_M}{2D} \left(\frac{4q_{sc_l}}{\pi D^2} \right)^2 B_l^2}{\frac{B_l}{\rho_{sc_l}} + \left(\frac{4q_{sc_l}}{\pi D^2} \right)^2 B_l \frac{dB_l}{dp_l}} \quad (26)$$

Equation (26) is applicable for both fluids separately in a wellbore. For both fluids flowing simultaneously, Equation (26) changes to:

$$\frac{dp}{d\ell} = \frac{g \frac{H}{L} + \frac{f_M}{2D} \left(\frac{4q_{tsc}}{\pi D^2} \right)^2 B_t^2}{\frac{B_t}{\rho_{tsc}} + \left(\frac{4q_{tsc}}{\pi D^2} \right)^2 B_t \frac{dB_t}{dp}} \quad (27)$$

The total formation volume factor and density are given as follows:

$$B_t = B_o \quad (28)$$

$$\rho_t = (1 - f_w) \rho_o + f_w \rho_w \quad (29)$$

As the water formation factor is generally equal to 1 and is a fragile function of pressure and temperature, it is assumed that the oil formation volume factor has a primary influence at various pressures. Hence, the total formation factor will only be the equation of the change in the oil formation factor.

Then, f_w is the water volume fraction or quality [16] that is calculated under surface conditions, as defined below:

$$f_w = \frac{q_{\text{water}}}{q_{\text{total}}} \quad (30)$$

3. Entropy Generation Minimization

This section describes Entropy Generation Minimization (EGM) as a useful design tool for achieving a higher recovery factor. The second law of thermodynamics, along with

EGM, is applied in a wellbore and porous system during fluid production. EGM is used as a design tool to optimize the operating parameters and enhance the recovery factor while concurrently consuming the least amount of available energy.

3.1. Reservoir System

It can be shown that entropy generation in porous media can be expressed as [10]:

$$\frac{\partial S_T}{\partial t} = 2\pi h \int_{r_w}^{r_e} \frac{\rho_{sc} f_p \beta u^2}{\phi(1 - s_{wc})TB} r \, dr \quad (31)$$

where S_T represents the total entropy production rate of a radial flow system in a reservoir; h refers to the reservoir thickness; r_w is the wellbore radius; r_e introduces the reservoir radius; ϕ is the formation porosity; ρ_{sc} introduces the fluid mixture density at standard conditions; f_p is the porous medium friction factor; s_{wc} denotes the connate water saturation; T represents the reservoir temperature; β is the non-Darcy coefficient; B is the formation volume factor; and u is the fluid velocity.

3.2. Flowing Well System

A similar equation for single-phase [10] with the modifications of two-phase flow will be used to calculate the total entropy for a flowing well:

$$\left. \frac{\partial S_T}{\partial t} \right|_l = \int_0^L \frac{\rho_{sc} f_M}{2DTB} \left[\frac{4q_{sc} B}{\pi D^2} \right]^3 d\ell \quad (32)$$

3.3. Total Entropy Generation

It is well known in thermodynamics that the total entropy of a system is given by the following equation.

$$S_T = 2\pi h \int_{t_i}^{t_f} \int_{r_w}^{r_e} \langle \dot{S} \rangle r \, dr \, dt \quad (33)$$

$\dot{S}$ where is the rate of entropy production per unit volume.

Using the previous results, the total entropy production rate for two-phase flow in a reservoir and wellbore system over a time interval from an initial time t_i to a final time t_f , where the pressure changes from an initial pressure P_i to a final pressure P_f or spatially averaged pressure in pseudo-steady state, becomes:

$$S_T = \frac{\pi r_e^2 h}{q_{sc}} \int_{\bar{P}_f}^{\bar{P}_i} \left(\left. \frac{\partial S_T}{\partial t} \right|_{\text{well}} + \left. \frac{\partial S_T}{\partial t} \right|_{\text{reservoir}} \right) \frac{\phi(\bar{P}) c_t(\bar{P})}{B_t(\bar{P})} d\bar{P} \quad (34)$$

Here, $B_t(\bar{P})$, $c_t(\bar{P})$, and $\phi(\bar{P})$ are functions of the average pressure.

4. Solution Method

To simulate the fluid flow behavior in the reservoir and wellbore, a computer code was generated in MATLAB. The reservoir and wellbore models involve nonlinear partial differential equations that cannot be solved analytically. The solution of the reservoir models is determined in terms of pressure and entropy. In contrast, the solution of wellbore models is obtained in terms of pressure, mixture entropy, and mixture density.

The computer code was developed to simultaneously simulate and solve the flow of two-phase (oil–water) equations in the reservoir and wellbore systems. Both models have nonlinear equations that can be solved using a numerical solution. As there are only spatial changes but no change in the temperature of the reservoir and wellbore, a finite-difference approximation or grid construction is not used as a solution method. Rather, Euler's method is used for a fast and efficient solution. The outputs of both models are pressure and entropy.

The equations derived in the previous section for two-phase flow in a porous medium and a wellbore are insufficient to describe the fluid flow behavior fully. Additionally, the equations needed are described as follows. The Original Hydrocarbon In Place (OHIP) model by Dake [23] is given by:

$$N = \frac{7758Ah\phi(1 - S_{wi})}{B_{oi}} \quad (35)$$

where N is the OHIP; A introduces the reservoir area; h refers to the reservoir thickness; ϕ is the porosity; S_{wi} represents the initial water saturation; and B_{oi} denotes the initial oil formation volume factor.

The capillary pressure as a function of saturation in field units is defined according to Aziz and Settari [25]:

$$P_c = P_{nw} - P_w \quad (36)$$

where P_c is the capillary pressure; ρ_w introduces the density of water; ρ_o refers to the density of oil; and h is the height of the capillary rise.

The effective water saturation is expressed as follows [29]:

$$S_{ew} = \frac{S_w - S_{wr}}{1 - S_{wr} - S_{or}} \quad (37)$$

In Equation (36), S_{ew} is the effective water saturation; S_{wr} represents the residual water saturation; S_{or} is the residual oil saturation; and S_w introduces the water saturation at a different P_c .

The fractional oil and water flow are given by the following equations [23,24]:

$$f_w = \frac{q_w}{q_t} \text{ and } f_o = \frac{q_o}{q_t} \quad (38)$$

where f_w and f_o are the water and oil fractional flow, respectively; q_w and q_o refer to the water and oil production rates, respectively; and $q_t = q_w + q_o$ denotes the total production rate.

The relative permeabilities as a function of fluid saturation are given below using Brooks and Corey's formula [29]:

$$k_{rw} = (s_{we})^{(2+3\lambda)/\lambda} \quad (39)$$

$$k_{ro} = (1 - s_{we})^2 (1 - s_{we})^{(\frac{2+\lambda}{\lambda})} \quad (40)$$

where k_{rw} is the water relative permeability; k_{ro} introduces the oil relative permeability; and λ characterizes the pore size distribution of the porous medium (ranging from 0.2–5). Higher values of λ are used for highly homogeneous porous media for which the pore size distribution is narrow [30].

The two-phase model's solution differs from the single-phase model in some factors/parameters, such as the relative permeability and pressure profiles. It is not straightforward to control each phase's production at the surface as the pseudo-steady state is required. It is well known that the total production rate (oil and water) that reaches the surface can be controlled. In this paper, an assumption of a constant fractional flow for the entire production time of each phase was made, and the numerical value for each phase was based on a past research by Tiller and Mogollon [31]. In their study, they assumed a fixed percentage of the fluid fraction to maintain the stability of the production for each fluid for simplicity purposes.

The step-by-step algorithm to determine the total entropy production and recovery factor for two-phase flow, as shown in Figure 2, starts by using all the required parameters, such as porosity and relative permeability. Then it calculates the OOIP and the effective water saturation by assuming fractional flow and a degree of reservoir homogeneity. The next step is to calculate the average reservoir pressure using the initial and final relative

permeabilities and the pressure profile equations, and then calculate the total entropy production by solving the reservoir and wellbore entropy equations simultaneously. Finally, the recovery factor is calculated and plotted against the total entropy.

5. Results and Discussion

In this study, past single-phase gas models were extended to two-phase (oil–water) fluid flow models. Significant changes in the models were made to simulate the reservoir pressure profiles for fluids, oil and water. It was found that the multiphase reservoir pressure model has a similar structure to that for single-phase flow except in mathematical formulas for total compressibility, viscosity, and permeability. The same assumptions for single-phase flow were applied in the two-phase flow models.

To maintain the production rate constant for both fluids, the fractional flow of water was assumed to be fixed for the entire period of time. The reservoir pressure and BHP were kept above the bubble point to prevent gas production and to keep only two fluids flowing in the reservoir and wellbore. Here, only natural energy mechanisms influenced the reservoir properties; no secondary or tertiary enhanced oil recovery technique was used.

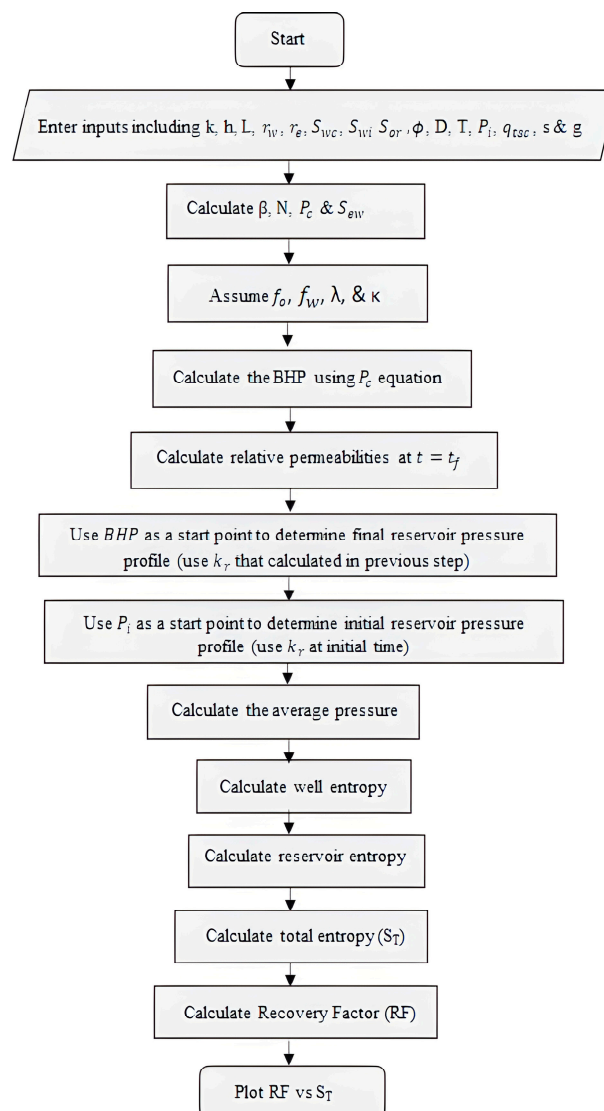


Figure 2. Flowchart for the approach used in two-phase flow.

5.1. Model Validation

Computer Modelling Group (CMG) simulation software was used to compare real field data with the data obtained from CMG to validate the model. Figure 3 shows a good agreement between the actual field data [32] and CMG predictions; they show the same trend, and the CMG results are reasonable. It is worth noting that Figure 3 was developed under semi-steady state conditions as the production rate was kept constant at 1000 bbl/day from day 10 to 50. It was then decreased to 100 bbl/day from day 50 to 70 to maintain the initial reservoir pressure. Another validation was conducted to examine the results obtained from the proposed model and estimated by CMG. An excellent match was achieved between the proposed model data and CMG's data. Figure 4 depicts the recovery factor versus time for the proposed models and CMG. The profiles were obtained at a constant Bottom Hole Pressure (BHP).

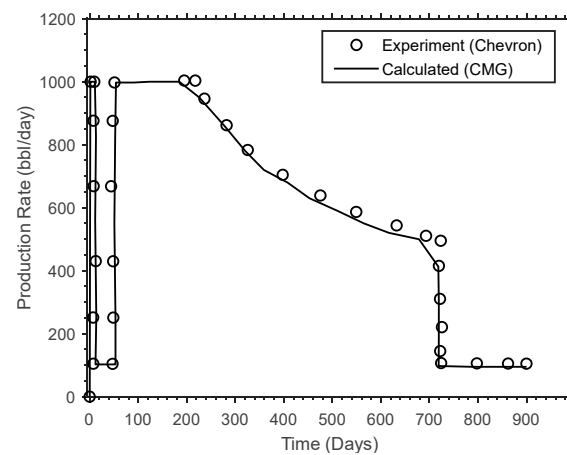


Figure 3. Oil production rate (field data and CMG) in the absence of skin factor.

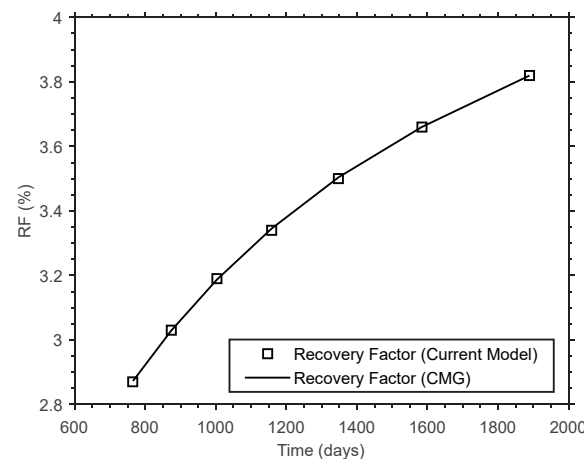


Figure 4. Recovery factor versus time (current model and CMG) at constant BHP.

5.2. Model Results

Based on the two-phase models, the total entropy production decreases at lower production rates as the recovery factor increases. The skin factor effect is significant in recovery, while slight changes occur in total entropy generation. To incorporate the effect of damaged zone around the wellbore, a simple mathematical formula proposed by Hawkins [33] was used in this study and displayed as Equation (41) below:

$$k_d = \frac{k}{s \ln\left(\frac{r_d}{r_w}\right) - 1} \quad (41)$$

where s is the skin factor; r_d is the radius of the damaged region; and r_w is the well radius.

Table 1 displays the formation and fluid property data, while Table 2 reports the saturation data [32].

Table 1. Data for two-phase flow (oil–water system), adopted from [32].

Parameter	Symbol	Value	Unit
Reservoir radius	r_e	2050	ft
Wellbore radius	r_w	0.25	ft
Wellbore depth	h	1000	ft
Pore compressibility	C_r	0.000004	psi ^{−1}
Water compressibility	C_w	0.000003	psi ^{−1}
Oil compressibility	C_o	0.00001	psi ^{−1}
Oil density	ρ_o	45	lb _m /ft ³
Water density	ρ_w	63.02	lb _m /ft ³
Initial reservoir pressure	P_i	4000	psi
Capillary pressure at water/oil contact	P_c	0	psi
Bubble point pressure	P_b	1000	psi
Absolute permeability	k	265	mD
Porosity	ϕ	0.13	
Reservoir temperature	T	180	°F
Initial water formation factor	B_{wi}	1.003875	bbl/STB
Initial oil formation factor	B_{oi}	1.103875	bbl/STB
Water viscosity	μ_w	0.9975	cp
Oil viscosity	μ_o	0.96	cp

Table 2. Saturation data for the modeling and simulations.

S_w	k_{rw}	k_{ro}	P_{cow}
0.22	0	1	7
0.3	0.07	0.4	4
0.4	0.15	0.125	3
0.5	0.24	0.0649	2.5
0.6	0.33	0.0048	2
0.8	0.65	0	1
0.9	0.83	0	0.5
1	1	0	0

The results obtained from the models are divided into two scenarios: in the absence and presence of capillary pressure. The results from both scenarios are displayed in the figures in this section. To satisfy the assumption of a constant production rate in pseudo-steady state two-phase flow, the fractional flow for oil and water is assumed to be constant for the entire time.

The total entropy production increases as the production rate increases for both oil and water, despite the skin effect, as shown in Figures 5 and 6 for oil and water, respectively. These two figures were obtained at $P_c = 0$. The general trend of total entropy production versus recovery factor for a two-phase flow is similar to that for a single-phase flow.

The skin factor does not significantly affect the total entropy production due to the high oil viscosity and the low values of effective permeabilities for both oil and water phases. However, it affects the recovery factor. This effect is more pronounced in oil, as shown in Figure 5, while in water, the effect is hardly observable in both the entropy production and recovery factor (see Figure 6). As the reservoir is under a natural energy mechanism and no Enhanced Oil Recovery (EOR) technique is used, the recovery factors for both oil and water are relatively low. The BHP and well head pressure were kept equal to or above the bubble point pressure, 1000 psi, to avoid any gas production at the surface.

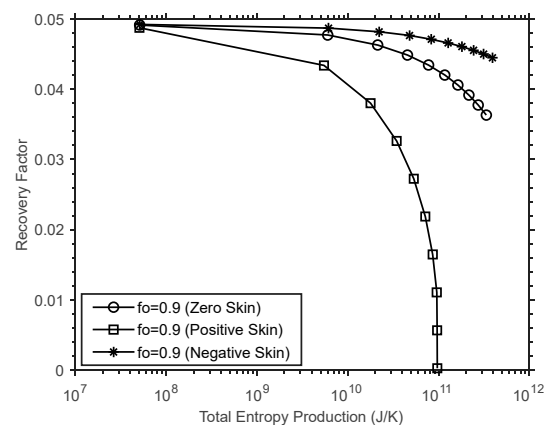


Figure 5. Recovery factor versus cumulative entropy production at different oil fractional flow values and $P_c = 0$.

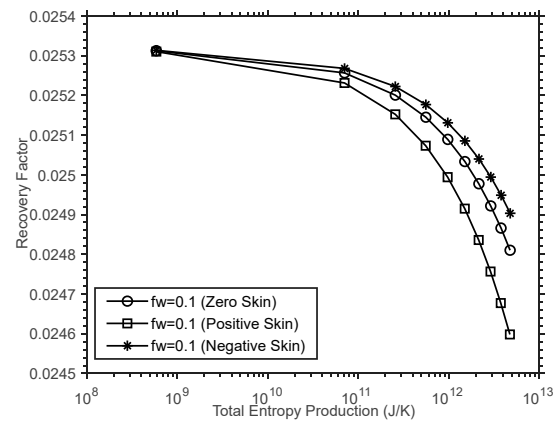


Figure 6. Recovery factor based on different values of water fraction flow at no effect of P_c .

Different fractional flows for oil and water for the same skin effect were obtained and depicted in Figures 7 and 8. As the oil production rate decreases at the same skin effect, both the recovery and total entropy production increase (Figure 7). However, the production time increases as the production rate increases. For the water phase, the contrary is observed. Both recovery factor and total entropy production decrease when the water production rate increases, as illustrated in Figure 8. The production time decreases as water production increases.

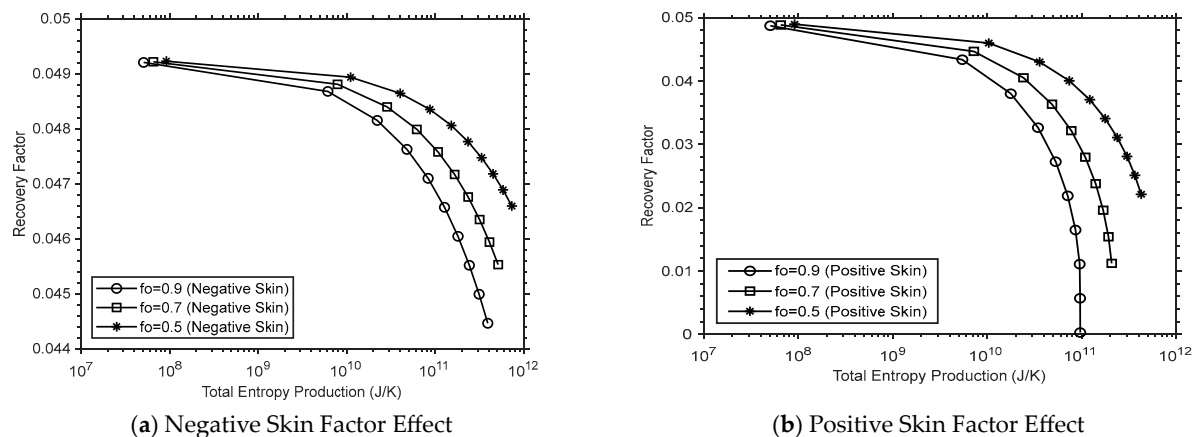


Figure 7. Recovery factor versus cumulative entropy production at different values of oil fractional flow for the same skin factor and no effect of P_c .

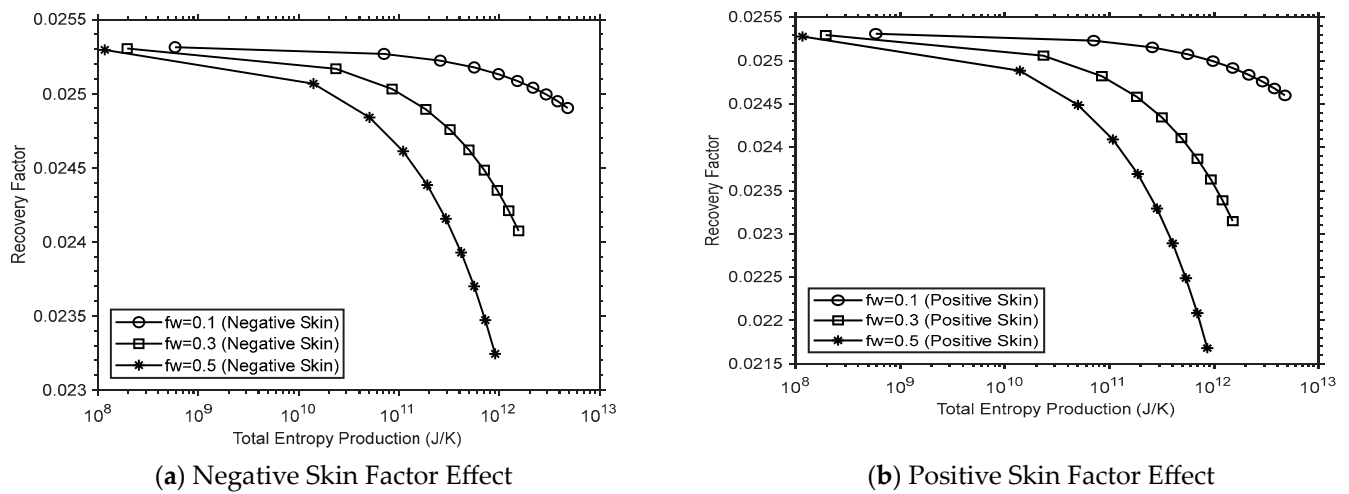


Figure 8. Recovery factor versus cumulative entropy production at different values of water fractional flow with no effect of P_c .

At no skin effect and no capillary pressure ($P_c = 0$) conditions, when a high oil production rate and a low water production rate were investigated, the total entropy production was higher in water than in oil, as depicted in Figure 9. This phenomenon occurs due to the low water production rate. The water production time is significantly higher than the oil production time in this case. On the other hand, when water and oil production rates are the same, the total entropy production is almost similar, as shown in Figure 10.

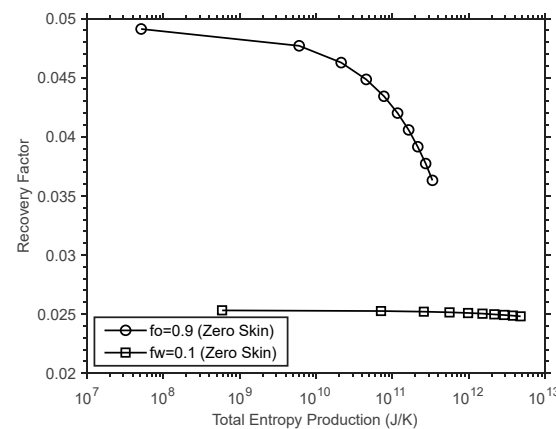


Figure 9. Comparison between $f_o = 0.9$ and $f_w = 0.1$ without skin effect at $P_c = 0$.

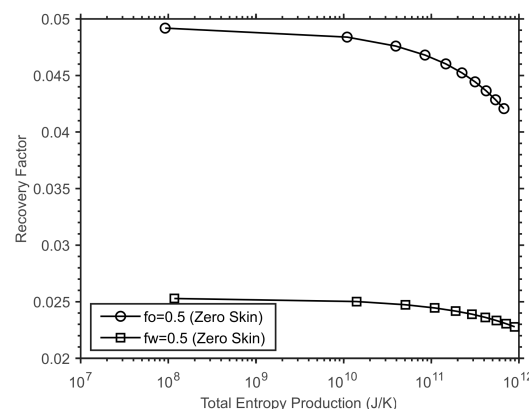


Figure 10. Comparison between $f_o = 0.5$ and $f_w = 0.5$ at no skin effect when $P_c = 0$.

The models' results also reveal that the effect of capillary pressure is significant on both oil and water recovery factors and total entropy production. Figure 11 displays an example of the effect of capillary pressure on the recovery factor and total entropy production when $f_w = 0.1$. Based on Figure 11, it can be concluded that the change in recovery factor (from 2.5% to 5.5%) is more significant compared with the change in the total entropy production (from 4.7×10^{12} J/K to 1.0×10^{13} J/K).

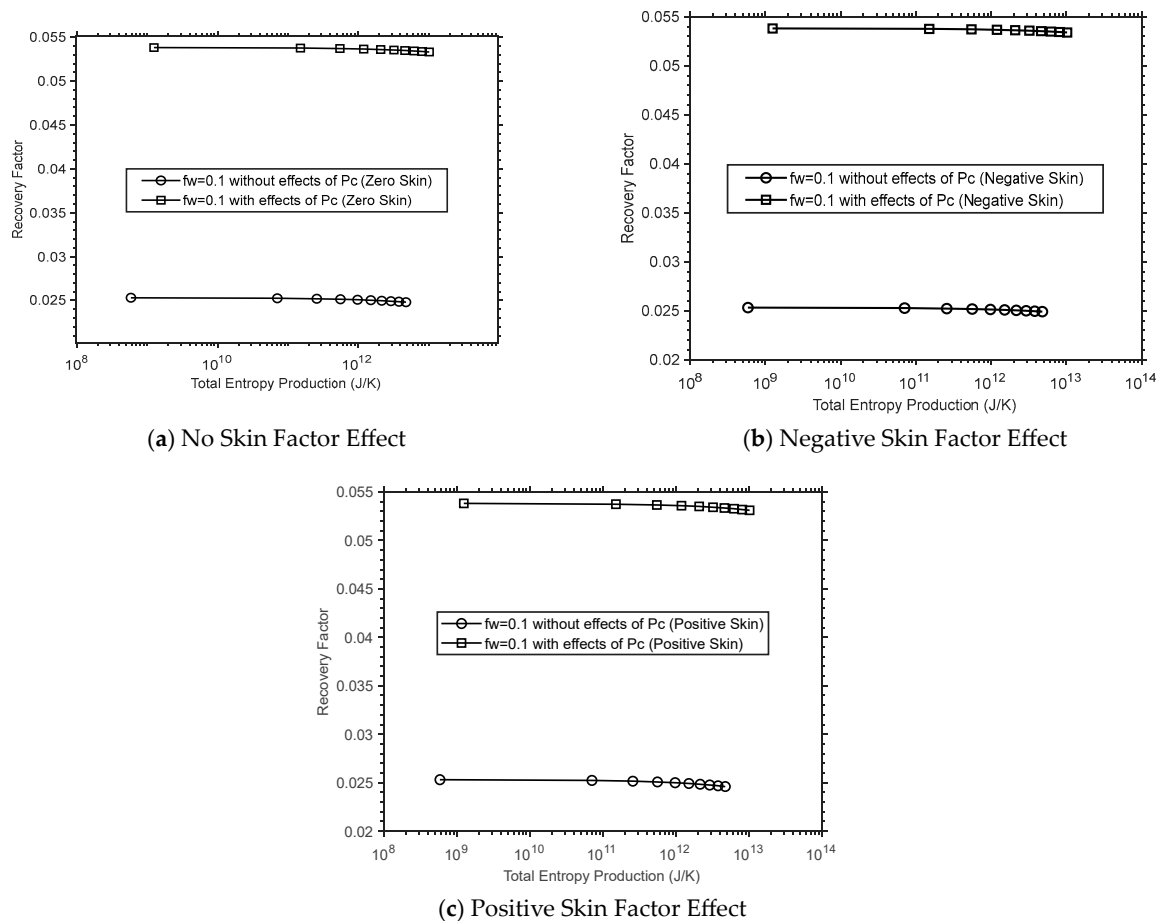


Figure 11. Effect of capillary pressure on the recovery factor of the water phase without skin factor.

At the same skin effect, the oil production time required for a higher recovery factor increases as the oil production rate decreases, as depicted in Figure 12. However, at the same oil flow rate and different skin effects, the oil production time for positive skin is less than for negative and zero skin. Figure 12 also reveals that there is not much time difference at the same production rate for negative and zero skin effects.

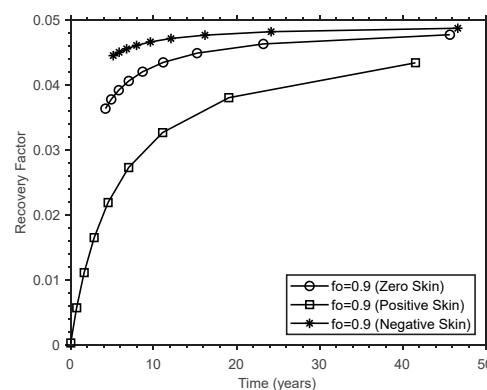


Figure 12. Recovery factor at different values of skin factor at $P_c = 0$ psi..

Capillary pressure has a strong influence on reservoir production life. For example, at a water fractional flow of 0.5, the recovery factor is much higher when capillary pressure is considered than when assuming no capillary pressure effect. However, the time needed to achieve this recovery factor is much higher in the case of capillary pressure, as can be seen in Figure 13.

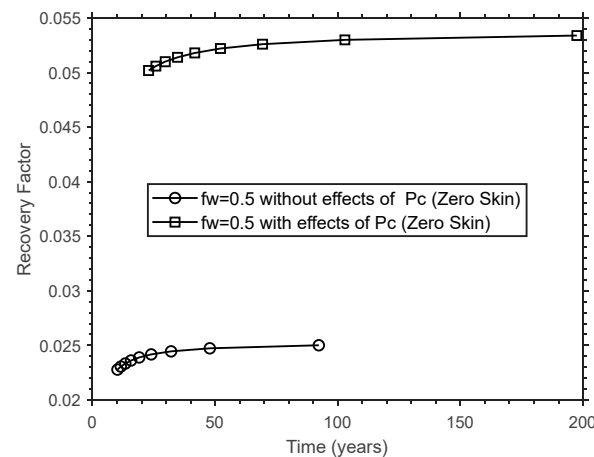


Figure 13. Effect of capillary pressure on the recovery factor at the same value of f_w and skin factor.

The multiphase reservoir pressure profile models are similar to those for single-phase except in total compressibility, viscosity, and permeability models. The results of the two-phase models show the same trend as with single-phase models. The two-phase models show that the total entropy production decreases at lower production rates as the recovery factor increases. The skin factor effect on the recovery factor is appreciable, while slight changes occur in total entropy generation.

5.3. Sensitivity Analysis

Most methods to investigate the effect of temperature on oil viscosity are empirical, such as Beal's correlation, the Beggs–Robinson's model, and Glaso's correlation [34]. Sutton and Farshad [35] concluded that Glaso's correlation yields higher accuracy than Beal's and Beggs–Robinson's correlations. Thus, Glaso's correlation will be used in this work to consider the temperature effect on viscosity. Glaso's correlation is given below:

$$\mu_{od} = \left[3.141 \left(10^{10} \right) \right] (T - 460)^{-3.444} [\log(\text{API})]^A \quad (42)$$

The temperature is in R (Rankine), and the coefficient A is defined as follows:

$$A = 10.313 [\log(T - 460)] - 36.447 \quad (43)$$

The effect of reservoir temperature was investigated over a wide range from 120 °F to 200 °F. Viscosity is a function of mainly temperature and density. Therefore, any temperature change will dramatically affect viscosity. The recovery factor increases as the temperature increases, while the entropy production also increases. Figure 14 displays the recovery factor versus entropy production on a semi-log scale at different temperatures.

Figure 15 shows a standard scale for entropy production versus viscosity. When viscosity changes, there is no considerable change in entropy production at low production rates. However, there is a noticeable decrease in entropy production at higher production rates as viscosity increases. The influence of viscosity on the recovery factor at a constant production rate was also investigated. Based on Figure 16, as viscosity increases, the recovery factor decreases at a constant production rate. This decrease is more pronounced at higher and lower production rates.

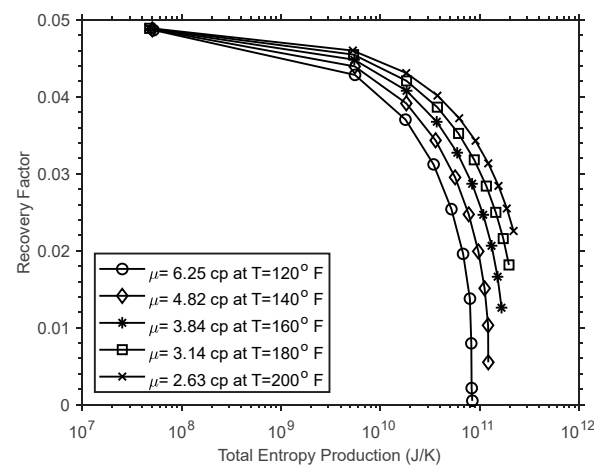


Figure 14. Effect of viscosity on entropy production on the recovery factor.

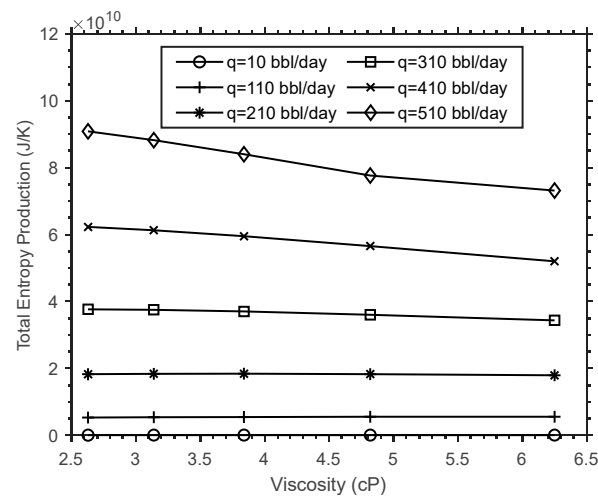


Figure 15. Entropy production versus viscosity at a constant flow rate.

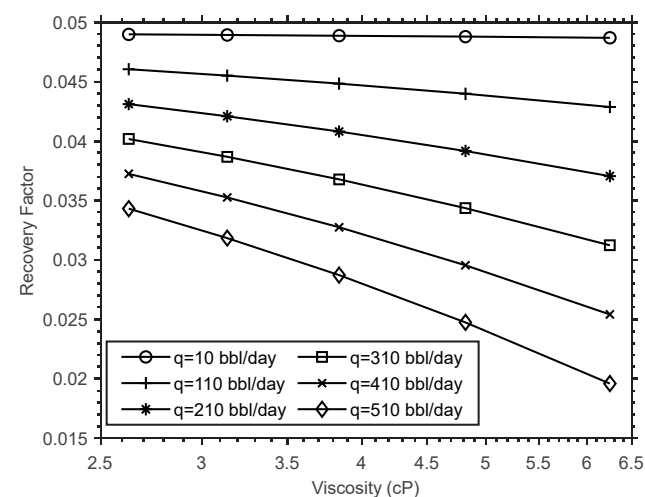


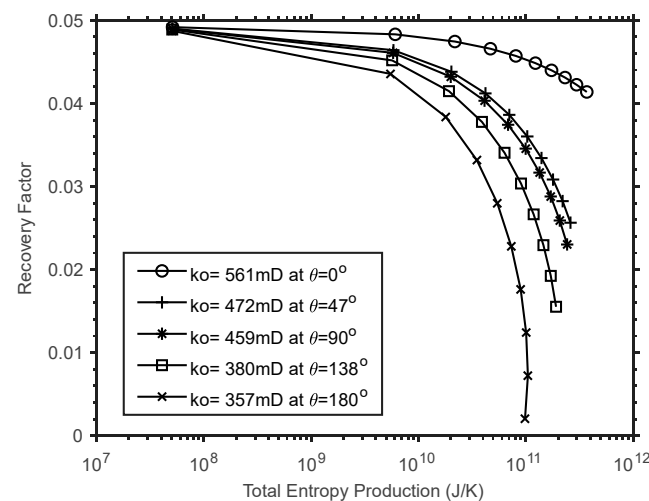
Figure 16. Recovery factor versus viscosity at a constant flow rate.

The impact of wettability on the fluid location and distribution in porous media is essential, as the wettability considerably affects relative permeability. The data used in this paper to investigate the effect of wettability on relative permeability is adopted from [36], as listed in Table 3.

Table 3. Effective oil permeability as a function of contact angle.

Effective Oil Permeability (mD)	Contact Angle (Degrees)
561	0 (water-wet)
472	47
459	90
380	138
357	180 (oil-wet)

Rock wettability is one of the key factors that affects entropy production. As the relative permeability and capillary pressure are affected by rock wettability, both variables (i.e., relative permeability and capillary pressure) were studied. Based on the literature, the effective oil permeability decreases as wettability increases [37]. The effect of wettability on relative permeability is apparent, as shown in Figure 17. At 100% water-wet, both recovery factor and entropy production exist at their maximum, which agrees with the literature.

**Figure 17.** Effect of wettability as a function of relative permeability on entropy.

On the other hand, when the rock is 100% oil-wet, the recovery factor and entropy are at their minimum levels. Both cases (fully water-wet and fully oil-wet) were tested under different flow rates. The wettability effect on entropy production was investigated at a constant flow rate. As the contact angle increases, the entropy production decreases only for higher production rates. Figure 18 shows entropy production versus contact angles on a standard scale. The same phenomenon was observed when the recovery factor versus contact angle at a constant flow rate was studied (see Figure 19). However, as the production rate increases with wettability changes to become fully water wet, the recovery factor decreases due to the low effective oil permeability. On the contrary, entropy production increases with an increase in production rate.

In conclusion, the effect of capillary pressure is critical in two-phase flow models. However, the change of capillary pressure does not significantly affect the results of two-phase flow models. The high impact of capillary pressure on the model is due to a change in BHP, because when the capillary pressure is assumed to be zero, the BHP for oil and water is equal.

The effects of temperature and rock wettability were considered in the models. As the temperature increases, the total entropy production increases at high production rates. This occurs because the viscosity decreases when the temperature increases, leading to a lower recovery factor and an increase in total entropy production. As the water contact angle increases, the effective permeability of the oil increases. Both the recovery factor and entropy production are at their maximum at complete water saturation condition.

However, the recovery factor and entropy are at their minimum levels when the rock is fully oil-wet.

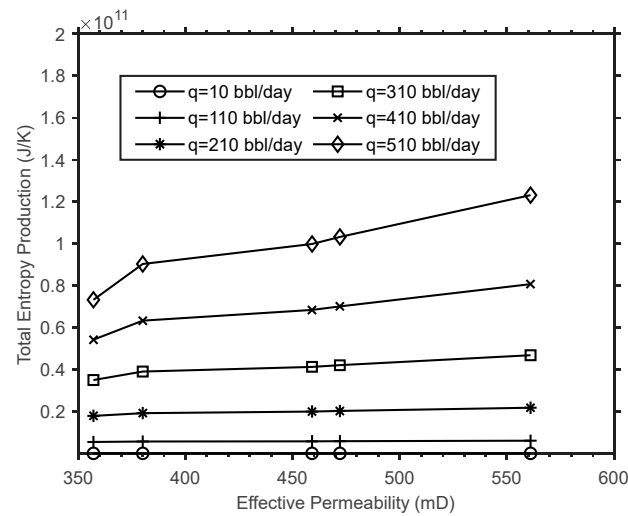


Figure 18. Entropy production versus permeability at a constant flow rate.

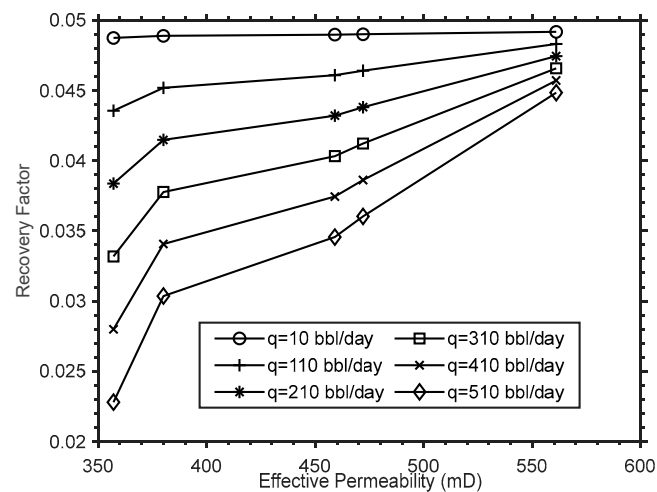


Figure 19. Recovery factor versus permeability at a constant production rate.

6. Conclusions

This paper has demonstrated the use of entropy generation minimization as a helpful design tool for improving the overall process efficiency of hydrocarbon extraction. The second law of thermodynamics was used to predict the entropy generation in a wellbore and porous system during fluid production. The paper analyzed multiphase flow in both porous medium and wellbore systems. Saturation, capillary pressure, and relative permeability factors were considered. The primary equations included the continuity, momentum, energy and entropy balance equations (second law of thermodynamics). Further supplementary equations, such as capillary pressure equation, effective water saturation equation, fractional flow equation, and relative permeability equations, were also used. Computer Modeling Group (CMG) was used to verify the predicted results. The results of the models showed that the recovery factor and total entropy production change slightly for water upon an increase in water fractional flow. Compared to the oil phase, this might be due to the low water production rate (low water fraction) and low value of water's relative permeability. As the oil production rate decreases, the recovery factor and total entropy production increase at the same skin factor effect. In contrast, the recovery factor and total entropy production decrease as the water production rate increases. The

results also revealed that the capillary pressure significantly impacts both the oil and water recovery factors and total entropy production. Additionally, the capillary pressure strongly influences the reservoir production life. For example, at a water fractional flow of 0.5, the recovery factor is much higher when the capillary pressure is considered compared with the case with no capillary pressure effect. From all the above results and interpretations, we conclude that it is wise and recommended for production engineers and designers to avoid having high values of entropy production in a reservoir, which eventually leads to low recovery factors with high reservoir energy loss.

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Disclosure

This manuscript is a part of the PhD thesis of the first author at Memorial University, Canada. Graduate students can submit their thesis either in a traditional way or on the basis of manuscripts. In the latter case, each chapter (manuscript) may be published or submitted or in the preparation process at the time of thesis submission. Thus, the authors confirm that they have followed the Memorial University thesis regulations/guidelines with regard to the manuscript style.

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