Article

Compensation of the Current Imbalance of an Interleaved DC-DC Buck Converter, Sensorless Online Solution Based on Offline Fuzzy Identification and Post-Linearization

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Abstract: This paper presents a new approach to compensate for the current imbalance of an interleaved DC–DC buck converter (IBC), in which the current sensors are not involved in the operation of the converter when it is connected to an invariable load. The current sensors are only used during the offline identification process that builds the universal fuzzy model of the converter’s steady states. Model building involves an upstream identification phase, followed by further dimensionality reduction of the model and error minimization. The method presented here discusses the mathematical complexity of the analytical modelling of hybrid systems and opposes it with a complexity-reduced identification by learning from data. An offline rendered model of the stable and steady states of the IBC is used as a mapping of the required inverter output current to n-fold asymmetric duty cycles, which are distributed among the IBC phases to allow arbitrarily accurate load sharing. The mapping is carried out in the mathematically normalized space of variables or in the physical sense RMS values, achieving the desired robustness in a noisy environment and stability. The final and canonical feedback control is built from the standard and optimized PI controller, which is compensated by the identified IBC model correction. The only measured feedback of the whole controller is the output voltage. Even when applied to the simulation model (physical MATLAB platform) of a two-phase IBC with the built-in system asymmetry, the presented methodology is also applicable to the n-phase IBC without loss of generality.

Keywords: sensorless interleaved DC–DC buck converter; switched affine systems; hybrid systems; fuzzy identification; fuzzy modeling; fuzzy model-based control

1. Introduction

Interleaved DC–DC buck converters (IBCs) are commonly used in internal combustion engine vehicles (ICE) with 48 V and 12 V voltage systems, battery-powered electric vehicles, and mild-hybrid, plug-in hybrid and fuel cell electric vehicles to supply various loads. Significant power gains can be achieved by splitting the current across multiple parallel phases. The motivation for this article lies in the 48 V IBC power supply for heating catalytic converters in vehicles. Short and slow driving has the disadvantage that the combustion engine and the catalytic converter do not heat up to or maintain the optimum temperature, which leads to increased pollutant emissions. For space and price reasons, the current sensors in each parallel branch of the IBC are usually omitted, leading to natural disadvantages of such systems. This is due to the fuzziness of the load distribution between the converters.

As a representative of pulse energy converters (PEC), the IBC belongs to the same group of hybrid dynamical systems consisting of multiple buck converters connected in parallel, the number of which varies from application to application (Figure 1). The complexity of the dynamics of such a cyber-physical process, which comprises the structure...
of multiple piecewise linear processes, is correspondingly high [1,2]. Therefore, the de-
velopment of an accurate analytical model from a control perspective is tedious, and the
mathematically observed nature of the hybrid process evolves into a complex topology of
hybrid automata [3]. In search of an alternative to derive the complex analytical model
consisting of entangled continuous and discrete states, researchers prefer the widely used
averaged switched model [4–6]. In the state space, it minimizes the matrix ranks and ena-
bles the application of model predictive control (MPC) of the IBCs. In this way, the foun-
dations are laid for comprehensive control, taking into account the constraints and the
possibility of pursuing the optimal control solution. In general, MPC is not widely used
in PEC. Researchers in this field tend to use methods based on established linear control
theory to optimize the investment in terms of the physical merits of the system and its
control problems. In this paper, the way to implement both approaches is described,
and the harmony of both strategic views is established based on heuristics [7].

Figure 1. Two-phase interleaved buck DC–DC converter, standard circuit topology.

PECs are used for a wide range of different applications. As a member of the PECs,
the IBC shares the breadth of applied variants, but its structure is already defined for those
where high currents and energy densities are required. Here, we examine the problem of
current imbalance between IBC phases (Figure 1). For a variety of reasons, it is impossible
to provide physically identical converters, even when “technically identical” DC–DC buck
converters are connected in parallel. All components of the converter have slight differ-
ences that affect the imbalances of the currents differently. A few decades ago, when there
were analogue pulse width modulators (PWM), the problem was mainly the accuracy of
the trigger signals for the transistors or providing the exact duty cycle per phase. Today,
with digital technology, the problem of triggering accuracy has diminished,
and the im-
balance is more a product of the inequality of the physical switching times of the transis-
tors and their inevitable parasitic electrical elements. There is also the problem of the es-
timated series resistance (ESR) of the inductors and their unequal inductance. For this
reason, the IBC traditionally requires the installation of current sensors in all phases of the
conversion, which can provide a comprehensive solution to control the output voltage
and current balance [8–10]. The implementation of current sensors is associated with var-
ious challenges. These range from non-contact current sensors, which are cost-insensitive,
to simpler sensors, which have various anomalies. The latter are susceptible to electro-
magnetic contamination and unavoidable electrical effects on the circuit topology. In ad-
dition, the physical dimensions of the sensors significantly affect the compact form of the
converters in most applications. For these reasons, a sensorless solution is sought for current measurement.

The problem has been approached from many different angles for IBCs with constant switching frequency. The standard control toolbox inevitably provides different observer solutions for the observable systems, such as a solution that integrates the Kalman filter [4,5], or some recent solutions that use a reduced-order observer for the standard DC–DC buck converter [11] and a Luenberger observer for the DC–DC boost converter [12]. These methods use the ripple of the output voltage to reconstruct the phase currents. The biggest problem with these solutions is the correct detection of the phase current, which relies only on a very weak output ripple that decreases with the number of phases used. Moreover, this primarily requires adequate amplification of the measured voltage signal, which, in most cases, leads to problems in synchronizing the measurement and certainly amplifies the omnipresent system noise. Apart from the problems already mentioned in the measurement of voltage ripple and the increase in its complexity due to the introduction of amplifiers and filters, in different control transients, which have traditionally been treated phase-wise, such solutions based on an adapted and uniform voltage measurement additionally obscure the detection of the phase sequence. There are also ways of incorporating additional elements into the circuit that form the basis for the detection of phase currents in the work [13]. This is not the typical solution without a current sensor providing additional circuitry for voltage measurement but is less costly and physically complex for typical current sensors. A more advanced circuit for determining the current values in the respective converter branch is described in [14]. Based on the voltage waveform, conclusions can be drawn about the currents in the individual branches. In contrast to the measurement of the output voltage ripple, the authors of [15,16] provide information about the currents by measuring the input voltage ripple via the additional RC elements. As mentioned earlier, in all these solutions that qualitatively analyze the output or input voltage ripple, the main problem is to distinguish the signal ripple and to identify the correct phase sequence. In [15], signal reconstruction is performed by frequency reconstruction using low-order matrix-vector multiplication and low-order fast Fourier transforms. The authors of [16] use an autotuning method in imbalance control using the differences in the input voltage ripple during a switching cycle. The autotuning control algorithm may well cause stability problems in n-phase applications. Some modern solutions rely more on the natural current balancing of the converters, as briefly presented in [17] and later illustrated in [18], or a solution with more numbered phases with a reduction in current sensors [19]. The self-balancing effect [17,18] can be used in the configuration of the converters to achieve more precise control by replacing the diode with a MOSFET and reducing the controlled time delay together with an external capacitor across the MOSFET to ensure a zero-voltage switching effect. Even with perfect compensation of the elements in the latter examples, where the problem arises of tuning additional MOSFET capacitors that further limit the operating point range, or where there is limitation of a high switching frequency solution, the problem will persist in other elements and cause nonlinear anomalies not only in one phase of the IBC. A cost-related simplification with a reduced number of sensors [19] brings similar problems that have already been mentioned, in addition to the accuracy problem of the chosen modeling method, which requires a regulation of the adjustments of the disequilibrium coefficients.

The measurement of voltage ripple can be enhanced. In [20], a possible solution is presented that uses the measurement of the output voltage ripple per cell to reconstruct the RMS values of the currents. Here the current signals are masked with different switching control frequencies per phase and the final equality of the frequencies guarantees the correct balance. A complex analysis of the output voltage ripple and tracking is presented as a current solution in [21], but is not applicable for IBC examples for more than two phases, which involve more correlations and current dependencies. To anticipate the asymmetry of the phases in the estimated resistance, the authors of [22], and more recently [23], deactivate one of the phases or a combination and, based on this information, balance
the differences. These solutions are efficient and are the most commonly used in practice. Some known disadvantages are the relatively slow current equalization and the imposition of unbalanced triggering, which can lead to increased acoustic and electromagnetic interference. Considering the synchronization problem of the measurements and the complexity of the output voltage ripple, the authors of [24] use an injected perturbation signal per phase to reconstruct the current values from the input and output voltage measurements. The successful results of masking and injecting a previously generated signal into the input side of the converter and measuring it at the output of the converter have certainly inspired some recent and successful work, including the consideration of a more profound and technologically advanced manufacturing process for electronic elements [25,26].

The solutions widely used and available in the market are still burdened with the unresolved problem of IBC current imbalance. In science, the problem of IBC current imbalance is still very present, highlighted by the modern development of power generation systems, but also by examples from energy utilization [27,28].

In order to extend the solution for robust operation of the converter and to avoid an additional electrical element in the already dynamically complex hybrid mathematical modelling structure, we provide an alternative to previous solutions without a current sensor that guarantees an arbitrarily accurate current balance in the steady state of the converter. Unlike other solutions, in our work, we take into account the decomposition of feedback influences on the stability of a complex control system, especially the voltage in the developed modern technical environment. We avoid the additional injection of signals that can lead to electromagnetic pollution of other circuits in the environment and additional heat emission from the converter. Our approach models the steady state of the converter based on its stable open-loop characteristics and avoids dynamic interference with the transients at the converter’s operating point. The modelling is based on fuzzy identification, which covers the entire mathematical solution space for the converter’s steady state and applies heuristics as a universal approximation only when necessary. The output voltage is controlled only to compensate for the steady-state current error through the external feedback loop, which provides an accurate reference current through the fixed resistive load while balancing the currents between the phases of the converter.

Section 2 addresses the modelling paradigm that should be considered when developing accurate modelling. Section 3 considers the reduction of modelling complexity through identification. Section 4 presents the results of the methodology applied to the simulation model and Section 5 concludes the discussion in this article.

2. Mathematical Modeling, Simulation and Current Imbalance Problem

The IBC has a hybrid mathematical model that takes into account the existence of continuous and discrete mathematical expressions. The continuous trajectories of the model are interrupted by discrete system states, resulting in a smooth piecewise change of the system, usually referred to as switching. The hybrid states are generally neither timeless nor invariant and we call them modes. The number of phases of the cyber-physical process presented here, in which we group the IBCs, defines the circuit topology and influences the number of modes of the hybrid model. In the example from Error! Reference source not found., we can define nine modes of operation of the converter, including a continuous conduction mode (CCM) and a discontinuous conduction mode (DCM), where \( i_{\text{CCM}}(t) \) and \( i_{\text{DCM}}(t) \) pass through the 0 state. Even though 16 possible switch positions are theoretically possible, the modes only refer to physically possible process states. Transferred to the mathematical state space, the equations for nine different switch topologies are derived on the basis of Kirchhoff’s laws:

Mode 1 [\( T1 = \text{opened}, D1 = \text{closed}; T2 = \text{closed}, D2 = \text{closed} \]
\[
\frac{d}{dt} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} = \begin{bmatrix}
\frac{(r_{L_1} + r_{L_2})}{L_1} & 0 & -\frac{1}{L_1} \\
0 & 0 & 0 \\
\frac{1}{C} & 0 & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\
0 \\
0
\end{bmatrix} \cdot E
\]

(1)

Mode 2 [T1 = closed, D1 = opened; T2 = closed, D2 = closed]

\[
\frac{d}{dt} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} = \begin{bmatrix}
\frac{(r_{L_1} + r_{L_2})}{L_1} & 0 & -\frac{1}{L_1} \\
0 & 0 & 0 \\
\frac{1}{C} & 0 & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} + \begin{bmatrix} 0 \\
0 \\
0
\end{bmatrix} \cdot E
\]

(2)

Mode 3 [T1 = closed, D1 = closed; T2 = closed, D2 = closed]

\[
\frac{d}{dt} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{1}{C} & 0 & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} + \begin{bmatrix} 0 \\
0 \\
0
\end{bmatrix} \cdot E
\]

(3)

Mode 4 [T1 = closed, D1 = closed; T2 = opened, D2 = closed]

\[
\frac{d}{dt} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{1}{C} & 0 & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} + \begin{bmatrix} 0 \\
\frac{1}{L_2} \\
0
\end{bmatrix} \cdot E
\]

(4)

Mode 5 [T1 = closed, D1 = closed; T2 = closed, D2 = opened]

\[
\frac{d}{dt} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{1}{C} & 0 & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} + \begin{bmatrix} 0 \\
0 \\
0
\end{bmatrix} \cdot E
\]

(5)

Mode 6 [T1 = opened, D1 = closed; T2 = closed, D2 = opened]

\[
\frac{d}{dt} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} = \begin{bmatrix}
\frac{(r_{L_1} + r_{L_2})}{L_1} & 0 & -\frac{1}{L_1} \\
0 & 0 & 0 \\
\frac{1}{C} & 0 & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\
0 \\
0
\end{bmatrix} \cdot E
\]

(1)

Mode 7 [T1 = opened, D1 = closed; T2 = opened, D2 = closed]

\[
\frac{d}{dt} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} = \begin{bmatrix}
\frac{(r_{L_1} + r_{L_2})}{L_1} & 0 & -\frac{1}{L_1} \\
0 & 0 & 0 \\
\frac{1}{C} & 0 & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\
0 \\
0
\end{bmatrix} \cdot E
\]

(2)

Mode 8 [T1 = closed, D1 = opened; T2 = opened, D2 = closed]
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space must be evaluated according to the equation:

\[
\frac{d}{dt} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} = \begin{bmatrix} \frac{r_{L_1} + r_{L_2}}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{r_{L_2} + r_{L_2}}{L_2} & -\frac{1}{L_2} \\ \frac{1}{C} & 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_o(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_2} \\ 0 \end{bmatrix} \cdot E
\] (3)

Mode 9 \([T_1 = \text{closed, } D_1 = \text{opened}; T_2 = \text{closed, } D_2 = \text{opened}]

\[
\frac{d}{dt} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \end{bmatrix} = \begin{bmatrix} \frac{r_{L_1} + r_{L_2}}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{r_{L_2} + r_{L_2}}{L_2} & -\frac{1}{L_2} \\ \frac{1}{C} & 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot E
\] (4)

Based on Equations (1)–(9), it is possible to create the hybrid automata in the MATLAB platform [29] by providing the digital PWM that triggers the modes assigned by the physical logic. Thanks to the numerical integration of MATLAB-SIMULINK and the fact that several modes are switched in a natural way, the overall complexity of the simulation model is significantly reduced. The number of modes cannot be neglected when building the hybrid mathematical model, which drastically affects the derived state space modelling. We refer to [1], where the modelling of hybrid dynamical systems is described, which can be applied to the modes of the IBC system. For the IBC in this paper, the authors consider that, among the different and equivalent approaches to hybrid modelling, the mixed logical dynamical (MLD) approach is the most comprehensive mathematical approach that integrates integer and linear/quadratic programming theory simultaneously. Accordingly, the final mathematical IBC model must conform to the unique state space expression for the systems with piecewise linearity:

\[
x_c(k+1) = \begin{cases} 
A_i x_c(k) + B_i u_c(k) & \text{if } \delta_i(k) = 1 \\
+ & \\
A_m x_c(k) + B_m u_c(k) & \text{if } \delta_m(k) = 1
\end{cases}
\] (5)

for \( \delta_1(k) + \cdots + \delta_m(k) = 1 \forall \delta_i \in \{0,1\} \ i = 1, \ldots, m \)

Equation (5) is applicable in cases where the switching period of a converter has a unique state space. Since this is not the case for either the DC–DC buck converter or the IBC, Equation (10) must be reformulated using the MLD method, expanding the new states \( \mathbf{z}(k) = [x_c(k) \ x_e(k)]^T \) for the discrete part of the hybrid formulation. State variables labelled with the subscript \( e \) belong to the continuous space, while those labelled with \( d \) belong to the space of discrete variables from the set \( \{0,1\} \). The discretization time for an \( n \)-phase IBCs MLD model, considered under a sampling time \( k \), must be more than \( 3n \) times faster than the natural switching period of a DC–DC buck converter itself. It must be able to construct the entire state space model and simultaneously evaluate discrete variables that provide the switching logic of the modes. The multiplication by 3 results from the fact that this is the number of modes for a buck DC–DC converter. Let us return to Equation (10). If we consider it with a higher sampling time (processing resolution), we can predict the system response for a switching period \( T_s \) of the IBC. Its common state space must be evaluated according to the equation:
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Model in the steady states of the system space is a tangent space to the dynamical system of duty cycles.

In the equation, unlike (10), the logical variables become real values $\delta_i \in [0,1]$ and it is derived from $x_d(k)$ discrete variables [1,30]. Due to the complexity of modelling and subsequent control, MLD modelling and model predictive control (MPC) are rarely applicable to IBCs, but they certainly represent the most accurate modelling of hybrid dynamical systems.

Compared to MLD modelling, the other modelling principles lead to coarser solutions and, accordingly, to all conclusions, including approaches for current sensorless converters. Even with a mathematically perfectly modelled process, open-loop control is never a suitable solution for processes where we can expect physically induced changes in the process parameters. Furthermore, the technology used to manufacture the electrical components, even if very superior in terms of integration, cannot perfectly replicate the components, which would be a necessary prerequisite for our accurate mathematical modelling and subsequent accurate control. For this reason, and especially for open-loop control, we need to identify the process to be controlled by its physical footprint. Apart from this, our IBC cyber-physical process also has its physical limitations. Firstly, the power is limited due to the components used. Secondly, and most importantly in the sensorless solution, is the ultimate purpose or nature of the application. These two limiting parameters can lead to various alternative solutions for sensorless control of IBCs. The heuristics-based identification of the converter’s model presented below provides a suitable basis.

3. Interleaved Buck DC–DC Converter Fuzzy Identification and Fuzzy Inverse Model

Analogous to [31], our task is to study the entire space of steady states of the converter in order to construct the matrix transformation that answers in real time the relationship between the parameters of the converter at each of the operating points. The stability of the converter in the open loop and the possibility to measure it is the sufficient condition for deriving the model approximation of the Banach subspace $(V^{2n+1}, \|\|)$. In Section 2, the example of an entangled topology converter with state vector $x(t) = [i_{L_1}(t)i_{L_2}(t)v_o(t)]$, expressed for $n = 2$ phases, obtains its transformation by $\|\|$ and becomes $\bar{x}(t)$ as a pseudo-norm of its original elements. The input to our process is the vector of duty cycles $d(t) = [d_1(t) \cdots d_n(t)]$, which, together with the norm of the state vector, forms a basis of the Banach subspace $[1_{d_1} \cdots 1_{d_n} 1_{i_{L_1}} \cdots 1_{i_{L_n}} 1_{v_o}] \in (V^{2n+1}, \|\|)$. The simulation of our hybrid model in the three-dimensional state space derived in Section 2 forms as a function $Q: R^3 \to R^5$ without time $(V^5, \|\|)$. The final $(\bar{V}^5, \|\|)$ is obtained by filtering out all transient states from the simulation. Fuzzy identification thus provides the mapping in $(\bar{V}^5, \|\|)$ from our state vector to the duty cycle vector with arbitrary accuracy:

$$d = F(\bar{x})_{t \to \infty} + \varepsilon.$$  

In (7), the argument vector $\bar{x}(t) = \|x(t)\|$ since all the variables in $(\bar{V}^{2n+1}, \|\|)$ are the norm of their origins. Time is omitted from the equation because our $(\bar{V}^{2n+1}, \|\|)$ space is a tangent space to the dynamical $(V^{2n+1}, \|\|)$. The fuzzy model $F(\bar{x})$ is a mapping approximation (for error $\varepsilon \to 0$) of the inverse and unknown hybrid mathematical model in the steady states of the system $G^{-1}$ from equation (8):
\[ \dot{x}_{t \to \infty} = 0 \Rightarrow \dot{\bar{x}}_{t \to \infty} = G(d) \in (V_{t \to \infty}^5, \| \|). \]  

(8)

The fuzzy model is derived for the structure of the MIMO system and its form is expressed in (9).

\[
\begin{bmatrix}
\bar{d}_1 \\
\vdots \\
\bar{d}_i \\
\vdots \\
\bar{d}_n
\end{bmatrix} = F_i \cdot \bar{x} + e
\]

(9)

All fuzzy models \( F_1 \land \ldots \land F_i \land \ldots \land F_n \) are created and based accordingly on their parameter set \( \Theta_1 = \{ c_1, a_1 \} \land \ldots \land \Theta_i = \{ c_i, a_i \} \land \ldots \land \Theta_n = \{ c_n, a_n \} \). In deriving the models, \( n \) processes must be provided for identification. Each process is dedicated to one phase of the IBC and provides the mapping of the common input vector of the states \( \bar{x} \) to a phase duty cycle \( d_i \) in (14). By analogy with [31], the fuzzy rules per phase are as follows:

\[
\text{if } H_i \text{ then } g_{ij}(\bar{x}) .
\]

(10)

In (10), \( H_i \) stands for the phase number. The sets \( X_1 \ldots X_k \) are the discourses of the fuzzy input variables and, in this case, the sets of all possible real values of our state vector variables, \( \bar{x} \in \mathbb{R}^k \). Our outputs are duty cycles in their closed discourse set \( d_i \in Y_i = [0,1] \). The index \( j = 1,2,\ldots,b \) denotes the number of rules from the rule base of dimension \( b \). Once all six steps of the fuzzy identification algorithm from [31] are completed per phase, the fuzzy models for each of the selected state variables provide the grade of membership \( \mu_{H_{ij}}(\bar{x}) \) and yield (11):

\[
F_i(\bar{x}|\Theta_i) = \frac{\sum(a_{ij0} + a_{ij1}\bar{x}_1 + a_{ij2}\bar{x}_2 + \ldots + a_{ijn}\bar{x}_n)\mu_{H_{ij}}(\bar{x})}{\sum_{m=1}^{b} (\bar{x}_m - c_{ijm})^2} .
\]

(11)

The consequence functions in (12) provide the typical linear model of the phase \( i \) of the converter for the respective operating point \( j \).

\[
g_{ij}(\bar{x}) = a_{ij0} + a_{ij1}\bar{x}_1 + a_{ij2}\bar{x}_2 + \ldots + a_{ijn}\bar{x}_n
\]

(12)

The model may bear a resemblance to linear control theory models, but its full expression includes the empirical values of the physical converter tested, which includes natural nonlinearities that would simply be neglected in the analytical type of modelling. The equation in (17) can be recognized in (16), weighted by the grades of membership, to interpolate all possible distances to the previously selected representative models for the respective operating points. The accuracy of the rendered fuzzy model is proportional to the number of rules \( b \) in the fuzzification process or to the characteristic models located in the centers defined by the \( c \)-means clustering method. In the \( c \)-means clustering method, the identification is performed by gradually adjusting the centers of the operating points by minimizing the cost function in (18).

\[
J = \sum_{i=1}^{M} \sum_{j=1}^{b} (\mu_{H_{ij}}(\bar{x}))^p \| \bar{x}_i - c_j \|^2
\]

(13)

In (13), \( M \) denotes the number of selected pairs \( (\bar{x}, d_i) \) from the Banach subspace \( (V_n, \| \|) \), while \( p \) denotes the grade of fuzziness.

In identification in general, the database or data source has a major impact on the accuracy of the model developed later. It remains that accuracy is proportional to \( M \),
(model error $\varepsilon \to 0$ while $M \to \infty$). Although important, authors very often only superficially explain the initialization of the optimization, the considerations in the selection of the model parameters and the way in which the learning database is formed for a given model, when this is not random or arbitrary. In this paper, we discuss the way in which model identification is driven by human knowledge of the system (black/grey/white box identification), as well as analytical methods that can be used to create the database that later serves as the source for the identification that is performed for better accuracy.

Therefore, the identification of the interleaved DC–DC buck converter in this article is considered to be of the grey variety. Through equations (1)–(9), as an example of two-phase converters, we have the analytical knowledge for our identification approach. The basic topologies for the two-phase converter described above represent the analytical complexity without including control of the circuit topologies and their correlations. Nevertheless, even for the n-phase converter and its state space, we know the system basis of our targeted Banach subspace $\{1_{d_1} \cdots 1_{d_n} 1_{i_{t1}} \cdots 1_{i_{tn}}\}$. In the $(2n + 1)$-dimensional normed space of stable points, we have to define our new mapping (9). Despite the analytical state space model, which can be derived but extended by using integer programming equations for discrete states that switch the modes of the converter, our new mapping (9) is a product of inverse modelling thanks to identification, where the new state variables are the duty cycles $d = [d_{i1}, \ldots, d_{in}]^T$. As a reminder, these variables were originally inputs in the direct analytical modelling of the state space, while the rest of the system basis in $(\vec{V}^{2n+1}, \|\|)$ were considered as states. The inverse of modelling in the analytical sense is thus:

$$\dot{d} = F(d, \bar{x}).$$

(14)

The identification of the system (19) must be performed using the database containing sufficient data pairs $H_{i_j}$ for each characteristic operating point $j$ of the converter. Fuzzification of the partially identified models (17) around the operating points helps in complexity reduction of the final and robust fuzzy identified model, generally expressed in (16). The identification process primarily generates a database according to linearization theory [32,33], also known as forward difference approximation (FDA).

Our identification problem from (14), the modelling of a dynamical system, can be represented as a set of differential equations,

$$\dot{x} = f(x_1, \ldots, x_n; u_1, \ldots, u_\sigma; v_1, \ldots v_q; t),$$

(15)

where $n$ differential equations of $n$ state variables $(x_1, \ldots, x_n)$ are equal to the $n$ set of functions of state variables, $\sigma$ inputs $(u_1, \ldots, u_\sigma)$, and $q$ algebraic variables. The indices $n$ are used intentionally to correspond to the previous number of phases. Without loss of generality and valid for our identification of the dynamical system in its steady states $(\vec{V}^{2n+1}, \|\|)$ (applying exclusively to stable open-loop systems), the complexity of the problem is reduced. This is mainly the result of the time exclusion in (15) and the fact that the previously considered input variables in our identification are considered as algebraic variables of the homogeneous inverse problem (14). According to FDA theory, which provides a good system approximation near the stable operating states $d_0$, the forward disturbance is simulated to obtain a suitable learning dataset. This means that the system identification process must simulate the perturbation $\Delta$ expressed in

$$\Delta d = F(\Delta d, \Delta \bar{x}).$$

(16)

For a perturbation, which, in short, means small signal differences of only one of the $n$ state variables at a time, we obtain,

$$d_{i+} \to d_{0} = F(d_{i+} - d_{0}, \bar{x} + \Delta \bar{x}) \to \dot{d}_{i+} = F(d_{i+} - d_{0}, \bar{x} + \Delta \bar{x})$$

(17)

For an $n$ perturbative differential equation (17).
Using the identification terminology in the creation of the database, several different test perturbations in the steady states of the predefined duty cycle $d_{i0}$ or $j$ centers $d_i$ will provide the database for the identification of $F_i$ in (14) for $i = 1, \ldots , n$. For each new equilibrium in the state space where $d_{i+} \rightarrow 0 \forall d_{i+} \in Y_j^X$ for $Y = [0,1]$, the database contains certain $i+$ perturbation sequences or mappings $F_i$ that must be identified. In the identification proposed here, these include the associated changes in the algebraic variables $(\dot{x} + \Delta \dot{x})$, which were the state variables in the analytical approach.

**Identification Algorithm, Applicability Paradigm**

The process of identification already mentioned in the theory has greater practical merits in two respects. First, the theory has a physical and mathematical background consistent with modern and accurate analytical modelling of hybrid systems. Second, the methodology employs machine learning in modelling only where necessary and during offline identification. This allows for a reduction in complexity and model implementation, taking into account the very limited processing and storage capacities of the IBCs studied. In order to achieve as even a load distribution as possible between the IBC phases, the methodology assumes that current measurement for all phases is temporarily and only possible during the identification process. In contrast to the final model evaluation and use, the offline identification itself requires more temporal and spatial processing capacities and algorithms, which are presented below (Algorithms 1–3), starting with the Algorithm 1, which generates the complete Banach subspace $(\bar{V}^{2n+1}, ||||)$.

**Algorithm 1: Generating the Banach subspace of the IBC steady states**

**Initialization:** Setting up the constraints for the discourses of the variables

1. Definition of the preferred number of equidistant operating points $j \in \mathbb{Z}$ in the IBC parameter space, the characteristic duty cycles for equidistant operating points $d_0 = (d_{10}, d_{20}, \ldots , d_{j0}) \in Y_j^X$, the perturbation step size $\Delta d \in (0,1)$ (infinitesimally small value), the settling time of the system $t_{s0} \approx c \cdot T_s$ for $c \in \mathbb{Z}$ (the time required to reach a stable state of the system), the number of perturbation steps around the operating points $k \in \mathbb{Z}$ and the number of IBC phases $n \in \mathbb{Z}$

   for $i = 1$ to $j$ do
     for $r = 0$ to $k$ do
       1. $d_{ir} = d_{i0}$
       2. Excite the IBC by $d_{ir} = (d_1, d_2, \ldots , d_n)$ where $d_{ir} = (d_i + r \cdot \Delta d) \cup d_{i0} \setminus d_i$
       3. **Pause** Algorithm 1 for $c \cdot T_s$ time
       4. Store the tuple $(d_{ir}, \bar{x})$ in the fuzzy universe $F_i$ (dataset)
       5. $d_{ir} = d_{i0}$
       6. Excite the IBC by $d_{ir} = (d_1, d_2, \ldots , d_n)$ where $d_{ir} = (d_i + r \cdot \Delta d) \cup d_{i0} \setminus d_i$
       7. **Pause** Algorithm 1 for $c \cdot T_s$ time
       8. Store the tuple $(d_{ir}, \bar{x}) \in \{(0,1)^n, R^{n+1}\}$ in the fuzzy universe $F_i$ (dataset)
       9. $r = r + 1$

   $i = i + 1$

Once we have provided the data representing our steady states $(\bar{V}^{2n+1}, ||||)$, the algorithm can proceed to fuzzy modelling, starting from the c-means clustering of our dataset $F = \bigcup F_i \subset (\bar{V}^{2n+1}, ||||)$, see Algorithm 2.
Algorithm 2: Modelling of the IBC steady-state space by fuzzy model (see reference [31])

Initialization: Definition of the universes of the discourses and the iteration parameters

1. Definition of the preferred number of equidistant operating points \( b \in \mathbb{Z} \) in the IBC operating space, the initial parameters, centers of the operating points \( c_0 = (c_{0,1}, \ldots, c_{0,b})^T \in \mathbb{R}^{(n+1) \times b} \), the number of iterations \( k \in \mathbb{Z} \), the minimum model error \( \varepsilon_1 \in \mathbb{R} \), the grade of fuzziness \( p = 2 \) and the number of attempts, ‘attempts’ \( \in \mathbb{Z} \)

\[
\begin{align*}
\text{for } i = 1 \text{ to } n & \text{ do} \\
1. & \text{ load } F_i \\
2. & \text{ watchdog } = 0, \text{ model_error } = 1 \\
3. & \text{ while model_error } > \varepsilon_1 \text{ do} \\
& \quad \text{ watchdog } = \text{watchdog } +1 \\
1. & \quad \text{ for } r = 1 \text{ to } k \text{ do} \\
& \quad \quad \text{ c-means clustering by minimizing the cost function (18), where M is the number of data tuples in } F_i \\
& \quad 2. \quad \text{ Store the new centers } c_{\text{new}} = (c_{1,\text{new}}, \ldots, c_{b,\text{new}})^T \in \mathbb{R}^{1 \times (n+1)} \\
& \quad 3. \quad c_0 = c_{\text{new}} \\
& \quad 4. \quad \text{ Calculate the matrix of parameters } a_i \text{ using the weighted least squares method} \\
& \quad 5. \quad \text{ Store the parameters } \theta_i = (c_i, a_i) \\
& \quad 6. \quad \text{ Calculate the model_error } = \frac{1}{M} \sum_{i=1}^{M} (F_i - d_i)^2 \text{ for } F_i, d_i \in (0,1)^M \\
& \quad \text{ if watchdog } \geq \text{attempts } \text{ do} \\
& \quad \quad \text{ break} \\
4. & \quad i = i + 1 \\
\end{align*}
\]

The data space modelled with fuzzy provides the system knowledge required to predict the parameters of the stable system state based on only one input variable, namely, the new output current IBC in the subsequence \( I_{\text{load}} \). The following Algorithm 3 is a test procedure for the newly developed fuzzy model, which primarily develops a new test data space that already works through the previously created inverse fuzzy system model. In the dataset provided by Algorithm 3, the fuzzy system model asymptotically approaches the ground truth system space of stable states relaxed by \( \varepsilon_2 \), and their intersection provides the opportunity to perform supervised learning, improve accuracy and reduce complexity.
Algorithm 3: Fuzzy model linearization (complexity reduction)

Initialization:
1. Definition of the preferred number of test points \( b \in \mathbb{Z} \) in the IBC operating space, the minimum error \( \varepsilon_2 \leq \varepsilon_1 \in \mathbb{R} \), the number of IBC phases \( n \in \mathbb{Z} \), the settling time of the system (the time required to reach a stable state of the system) \( t_{\text{st}} \approx c \cdot T_s \) for \( c \in \mathbb{Z} \).

1. Generating the \( b \) number of random load current values of the IBC and form
   \[
   I_{\text{Load}} = (I_{\text{Load,1}}, I_{\text{Load,2}}, \ldots, I_{\text{Load,b}}) \in [I_{\text{Load,min}}, I_{\text{Load,max}}]^b \forall I \in \mathbb{R}
   \]
2. for \( r = 1 \) to \( b \) do
   1. Form the input vector for the fuzzy model \( F \)
      \[
      \mathbf{x}^T_m = (1, (\frac{I_{\text{Load,1}}}{n}, \frac{I_{\text{Load,2}}}{n}, \ldots, \frac{I_{\text{Load,b}}}{n}) \in \mathbb{R}^n, I_{\text{Load,1}}, \ldots, I_{\text{Load,b}}) \in \mathbb{R}^{n+2}
      \]
   2. Excite the IBC by \( \mathbf{d}_m^T = (d_{m,1}, d_{m,2}, \ldots, d_{m,i}, \ldots, d_{m,n}) \) which is calculated by (14)
      \[
      \mathbf{d}_m = (F_1, F_2, \ldots, F_n)^T \cdot \mathbf{x}_m, \quad F_i \in \mathbb{R}^{n+2} \quad (F_i \text{ vector of calculated fuzzy coefficients obtained by the explicit form of (16)})
      \]
   3. Pause Algorithm 3 for \( c \cdot T_s \) time
   4. if \( \sum_{i=1}^n |I_{\text{Load,b}}^n - I_{\text{Load,b}}^i| \leq \varepsilon_2 \) do
      Store the tuple \((d_{m,i}, \mathbf{x}_m, \frac{I_{\text{Load,b}}^i}{n})\) in the linearization universe \( L \) (dataset)
   5. \( r = r + 1 \)
3. for \( i = 1 \) to \( n \) do
   1. Fit the function \( d_i = f_i(I_{\text{Load,b}}^i) \) to the dataset
      \[
      \{(d_{m,i}, \frac{I_{\text{Load,b}}^i}{n}) \cdot \forall (d_{m,i}, \mathbf{x}_m, \frac{I_{\text{Load,b}}^i}{n}) \in L\}
      \]
   2. Store the function \( f_i(I_{\text{Load,b}}^i) \) to the matrix \( F \) for \( \mathbf{d}_{m,2} = f_i(I_{\text{Load,b}}^i) \)
   3. \( i = i + 1 \)

The final system model has the analytical form \( \mathbf{d}_{m,2} = F \left( \frac{I_{\text{Load,b}}^i}{n} \right) \) and was built based on machine learning. When Algorithm 3 does not allow complexity reduction and the system contains complex nonlinearities, the final current compensation can sufficiently rely on the fuzzy model from Algorithm 2 and its duty cycle prediction mapping (12) and (14), \( \mathbf{d}_{m,1} = F \left( \mathbf{x}_m \right) \) with arbitrary accuracy \( \varepsilon_1 \).

4. Results of Identification Applied on Two-Phase Interleaved DC–DC Buck Converter

   For the identification presented in the above section and applied to the converter example in Error! Reference source not found., we built the MATLAB [29] simulation model using two approaches. First, by implementing the state space topologies (1)–(9) switched by discrete functions derived from the duty cycle control logic via PWM. Here, we consider the natural commutation of converters for DCM, resulting in a discrete hybrid automaton of a designed converter. Secondly, the simulation modelling was based on the use of electronic circuits formed by selecting the electronic components available in the system or simply by Simulink blocks [29] from the Simscape/Electronics/Semiconductor_Devices package. Comparison of the approaches produced identical results in the simulation, but, due to the much simpler linking of the identification excitation signals to the model and GUI-based parameterization of semiconductors and electronic components in general, the latter was used for further identification and control, Figure 2. The two-phase interleaved converter is designed for future practical use of high-power converters in electrical heating where we have a constant load and variable power transfer to the load, in the physical sense, a heater. As mentioned in the introduction, it is still tendentious to assume that the interleaved converters on the market are built with identical or physically cloned hardware for each buck converter. Moreover, in the case of parallel connection of
the individual buck converters, even if they have the same technical datasheet, we can expect slight physical differences that have a strong impact on the load sharing in parallel operation. In our simulation, we predicted very common and possible phase element differences, but, in practice, the differences cannot be estimated. In analytical modelling, these differences are hardly measurable and not available in the manufacturer’s technical documentation, especially regarding the dynamic behavior of the converter.

In Figure 1, we see the test results of the currents’ comparison for three different applicable solutions for sensorless open-loop control of IBCs. One of them is based on the most widely used approach of direct arithmetic averaging of the duty cycle per phase, calculated from the standard modelling toolbox (averaged buck DC–DC converter model)
and the linear dependence of the duty cycle and fraction $\frac{D_0}{D}$. The other two are based on fuzzy modelling of the stationary spaces of the duty cycle, with the linearized fuzzy model being the better solution. The simulation shows a high sensitivity to disturbance steps in the duty cycle, which means that all variations in the duty cycle around the operating point must be in close proximity. This IBC phenomenon has already been discussed above and it has been proven here that the error in the resolution of the duty cycle of the controller, including the PWM conversion added to the natural switching point of the semiconductor, must be as small as possible.

**Figure 3.** The currents of the converters as a result of the comparison of the compensation of the converter control, defined by three different methods: the arithmetic calculation for ideal converters, the model mapping based on fuzzy identification, and the model mapping based on fuzzy identification after linear regression. (a) Currents generated by the simulation system for 10 different operating points; (b) Currents generated by the simulation system at the fourth operating point, zoomed section.

In forming the datasets and for the simulation example of this article, the perturbation step of the duty cycle must be in the second or higher decimal place of the scalar value of the duty cycle. We see this example problem clearly in Figure 4, where we show what kind of numerical differences in duty cycles cause the degree of current imbalance at the output of the converter for the simulation test in Figure 3. Following the algorithm from the section above, the identification yields $F_1(\tilde{x}|\theta_1)$ and $F_2(\tilde{x}|\theta_2)$, where $\theta_1 =$
However, for the identified simulation example \( \bar{x} = (i_{\text{t1}}, i_{\text{t2}}, v_{\text{p}}) \), the parameter matrix \( \mathbf{a}_1 \) is extended to include the bias coefficient, as in (16) and (17). The complete parameter matrices can be found in Appendix A. The idea of reducing the granularity of identification by increasing the number of operating points, which increases the complexity of online processing, may not be necessary in the initial identification phase. Therefore, we have promoted post-linearization here, which increases the model accuracy that is obscured by unsupervised process learning. Figure 3 shows the difference in accuracy that post-linearization can achieve in the numerical sense and later in terms of output imbalance. In general, to achieve a better final model result, i.e., optimizing accuracy against online complexity, one needs to reduce the dimensionality of the online model by further optimizing the identified model rather than increasing the number of operating points and, hence, the number of perturbations in the datasets. To do this, the test results of the identified model must be filtered only for the points that agree with the ground truth in order to derive the model towards the minimum of the approximation error \( \varepsilon \) (12), Algorithm 3. In other words, this means that we move from fully unsupervised learning to supervised learning from data. In practice, this means two levels of learning, Algorithm 2 and Algorithm 3. According to the dimensionality of the chosen problem in (14), reducing the dimensionality for the chosen example leads to two linear equations (see Appendix A). In the methodology presented here, the final optimization model mapping is a bijection of the full subsets of the Banach space. Therefore, the final model mapping is in the space of continuous functions, which provide better conditions for control stability. Although this work focuses on the actuator problem, where we generally look for a linear transfer function and control, we present here the solution where our previous identification provides a solid platform for developing a stable model-based control algorithm for actuator current transfer. Figure 5 shows the recommended control approach, where the actuator follows an ideal first-order system response when dynamically exchanging current setpoints. The control design and stability analyses are performed using the standardized MATLAB [29] Control System Designer. Following the ideal parallel operation of converters, the control law can be developed based on an averaged model of a buck DC–DC converter, which is amplified by the number of phases used. The created averaged converter model [34], which includes only two topologies, (1) and (3), provides the necessary basis for deriving the PI model, which is optimized by an optimization algorithm that includes gradient descent and the ideal step response boundaries as design requirements.
Figure 4. Comparison of the duty cycle of the individual converters for the current results in Figure 3: (a) Simulation of the duty cycle of the system for 10 different operating points; (b) Simulation of the duty cycle of the system in 1 operating point, zoomed section.

Figure 5. Control of an interleaved DC–DC buck converter compensated by identification-based modelling of duty cycle mappings.

The simulation results, shown in Figures 3 and 4, confirm the expected stable operation of the actuators in the robust range of operating points.

With the expected change in current setpoint, the control system is able to compensate and maintain the ideal load distribution of the parallel converters (Figure 6). The results in Figure 6 are self-explanatory, starting from the corner of the normalized current output (Figure 6a), where the RMS value of the output current is displayed, to the detailed view of the current balance per phase (Figure 6b,c) without RMS filter normalization. The settling time in the system dynamics of the reference change is smoothly delivered by the PI controller in the manner of the reference model in MPC algorithms and is not burdened by the dynamic voltage feedback pattern, which is normalized. Thus, the smooth and scheduled reference is provided by the function $d_{m2} = f \left( \frac{I_{	ext{Load}}}{n} \right)$ as a fixed mapping generated previously and offline by Algorithm 3.
Figure 6. Control results with gradual change in the reference values for the load current: (a) the output load current of the converter compared to the reference, (b) the inductance currents of the same simulation test, (c) the inductance currents in detail for the specific transient time window.
5. Conclusions

The article presents a new method to compensate for the IBC asymmetry of output currents that load the constructive converters differently. The method envisages the use of current sensors only during offline system identification, but not during subsequent use. The physical limitations of the system in the context of the application example can lead to a reduction in system complexity. As shown in this article, the example of the IBC designed as a function of energy transfer to the invariant load opens this possibility. The methodology thus consists of a two-stage identification algorithm. The first stage of identification is based on unsupervised learning from data and creates a system map for the truths of the physical built-in asymmetry. In the second stage of identification, the algorithm filters the data and creates only the ground truth database for model reduction and supervised learning optimization. The resulting model enables the model-based compensation of the asymmetry of the output currents in the IBC and the integration into the load current control of the IBC. The methodology is generalized and developed for n-phase IBCs. In addition to the identification algorithm and fuzzy modelling, a unique approach to the formation of the database is presented, which is mathematically based on small signal approximation theory. The identified fuzzy model or dimensionality reduced time invariant mapping after linear regression preserves the nonlinearity of the system and provides model-based asymmetry compensation for the classical PI control signal. The final control is configured in the canonical connection of the optimized PI controller with the ID model-based compensation and distribution of the control signal. Subsequent research will focus on physical implementation and application of the IBC configuration for variable load systems.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

a) Fuzzy Model Coefficients

\[
\begin{align*}
\mathcal{a}_1 &= \\
0.0164368 & 0.016407 & 0.0164237 & 0.016404 & 0.0164839 & 0.0163233 & 0.0164113 & 0.0164895 & 0.0168527 \\
-2.35 \times 10^{-5} & 0.0002101 & 0.0002428 & 0.0003372 & 0.00023 & 0.0004977 & 0.0003358 & 0.0003804 & 9.46 \times 10^{-6} \\
-0.000358 & -0.00012 & -8.57 \times 10^{-3} & 9.52 \times 10^{-6} & -9.72 \times 10^{-8} & 0.0001713 & 9.09 \times 10^{-6} & 5.49 \times 10^{-6} & -0.000315 \\
0.0240283 & 0.0216619 & 0.0213271 & 0.0203759 & 0.0214449 & 0.0187643 & 0.0203853 & 0.019928 & 0.0236219
\end{align*}
\]

\[
\begin{align*}
\mathcal{c}_1 &= \\
4.1063468 & 21.550839 & 50.040896 & 32.680635 & 70.375985 & 86.682426 & 105.05255 & 122.04902 & 150.6788 \\
\end{align*}
\]

\[
\begin{align*}
\mathcal{a}_2 &= \\
0.0165152 & 0.0173812 & 0.0171376 & 0.017914 & 0.0170732 & 0.016745 & 0.0164965 & 0.0161993 & 0.0160417 \\
-0.000741 & -0.002574 & -0.001507 & -0.001596 & -0.000904 & -0.000341 & 7.00 \times 10^{-8} & 0.0004543 & 0.000612 \\
-0.000515 & -0.00236 & -0.001279 & -0.001372 & -0.000678 & -0.000115 & 0.000293 & 0.0006758 & 0.0008327
\end{align*}
\]
b) Post Linear Regression Model of Duty Cycle

\[ d = \begin{bmatrix} 0.01618974524 \frac{u_i}{u_{ij}} \\ 0.016324370811 \end{bmatrix} + \begin{bmatrix} 0.02211991399 \\ 0.02160889206 \end{bmatrix} \]

\[ g_{pi} = 1.393,364 \cdot 10^{-6} + 142.18 \cdot d \]

References


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