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Analysis of Emission Reduction Mechanism of High-Tiered Carbon Tax under Green and Low Carbon Behavior

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Abstract: This article studies the emission reduction mechanism of high-tiered carbon taxes under green and low-carbon behavior in single and two-stage supply chains. First, based on the Cournot game model, it explores the impact of high-tiered carbon tax policies on supply chain carbon reduction decisions in the green exchange market. By analyzing the effects of implementing a high-tiered carbon tax policy, the basic characteristics of its implementation are identified, and the advantages of a high-tiered carbon tax compared to a unified carbon tax are summarized. Second, it establishes a carbon reduction technology investment cost-sharing model and a carbon tax cost-sharing model under the high-tiered carbon tax policy. It analyzes and studies the impact of high-tiered carbon tax policies on balancing the relationship between members of the two-level supply chain through optimal decision-making of the two-level supply chain under two cost-sharing strategies, revealing the emission reduction mechanism of the two-level supply chain under high-tiered carbon tax policies. The results indicate that there are extreme points in the emission reduction rates of producers in the green exchange market under both the high-tiered carbon tax policy and the unified carbon tax policy. It shows that the two cost-sharing strategies can effectively alleviate the cost burden for producers, increase their marginal profits, and promote further improvement in emission reduction. It explores the emission reduction mechanism of high-tiered carbon taxes and future research should delve into the emission reduction mechanism of high-tiered carbon taxes in different carbon emission departments and regions.

Keywords: high-tiered carbon tax; carbon reduction decision-making; emission reduction mechanism; supply chain; cost-sharing

1. Introduction

Due to the potential threat of climate change to global economic and social development, exploring appropriate carbon reduction policy tools has become the only way to achieve sustainable development. As one of the effective policies for controlling carbon emissions, the carbon tax policy aims to guide enterprises to undertake low carbon emission reduction through the externality of endogenous enterprises’ carbon emissions, to drive enterprises to carry out emission reduction innovation and ultimately cultivate enterprises’ green production structure systems [1]. The implementation of carbon tax policies can not only effectively reduce environmental pollution, but also significantly promote economic development, achieving a “dual dividend” of suppressing carbon emissions and promoting economic growth [2]. Existing research has confirmed that carbon taxes are a powerful incentive factor for reducing carbon emissions [3,4]. However, at the same time, such policies may also lead to a series of socio-economic impacts [5,6]. For example, a carbon tax may
have a negative impact on the share and export of most industries in the Chinese market [7]. The collection of carbon tax will reduce the savings and investment of enterprises and narrow the Lebensraum of SMEs [8]. Given the advantages of tiered pricing over unified pricing, there have been studies applying tiered pricing methods to the formulation of carbon taxes. Many studies have proposed that a tiered carbon tax is an optimized form of carbon tax pricing and have shown that a tiered carbon tax can significantly reduce the carbon tax burden on enterprises and promote low-carbon production in the market [9–11].

However, is the tiered carbon tax effective? What are the effects of tiered carbon taxes on market carbon reduction? There is currently little research on these issues, and the current tiered carbon tax is either limited to low-tiered carbon taxes or the impact of high-tiered carbon taxes on market carbon emission reduction has not been systematically studied. Therefore, it is crucial to systematically explore the emission reduction mechanism of high-tiered carbon taxes, which helps to achieve a balance between socio-economic and environmental goals. Guided by the concept of sustainable development and based on the research idea of a tiered carbon tax, this article first establishes a three-stage tiered carbon tax model and analyzes its impact on market participants, revealing the emission reduction mechanism of the three-stage tiered carbon tax; Secondly, this article studies the emission reduction mechanism of a single-level supply chain under a high-tiered carbon tax policy from the perspective of a green exchange market; Finally, it studies the optimal strategies for carbon reduction technology investment cost-sharing and carbon tax cost-sharing in a two-level supply chain under a high-tiered carbon tax policy, and summarizes the emission reduction mechanism of the two-level supply chain under a high-tiered carbon tax policy. This study helps to maintain market stability and improve emission reduction efficiency and has important practical significance and theoretical value in guiding economic structural transformation and promoting green and low-carbon development.

By reviewing the existing literature, it was found that:

1. At present, many studies on the improvement and optimization of an undifferentiated unified carbon tax mainly focus on the collection of differentiated carbon taxes, lacking specific support for carbon tax collection models. The concept of a tiered carbon tax provides specific policy recommendations that can be used for reference; however, many so-called tiered carbon taxes are only converted into two-stage tiered carbon taxes by adding a penalty factor, resulting in low emission reduction efficiency. (2) At present, there is relatively little research on the study of tiered carbon taxes, which includes consumers with green consumption preferences in the tiered carbon tax model to study the impact of tiered carbon taxes on market carbon emission decisions in the context of consumers’ green consumption preferences. Additionally, few studies have included the health impact of environmental pollution on workers in the production costs. (3) At present, most studies on tiered carbon taxes mainly focus on studying the imposition of carbon taxes on producers at the production end, with little consideration given to how producers and retailers in a two-stage supply chain can reasonably share tiered carbon taxes based on cost-sharing strategies.

2. Literature Review

2.1. Optimization and Improvement of Carbon Tax Policy

The carbon tax policy incentivizes the reduction of carbon emissions through price regulation. But at the same time, the collection of carbon taxes also brings a certain economic burden to the production enterprises. Based on this, many scholars have proposed suggestions for improving carbon tax policies. Their research results can be divided into three categories:

The first type of research proposes a dynamic carbon tax policy. Research by Wang Jinnan shows that China’s carbon tax collection scheme should follow the principle of gradually increasing from low to high, in which they set the initial tax rate at 20 yuan/t, increased to 50 yuan/t in 2020, and further increased to 100 yuan/t in 2030, gradually form-
ing a relatively complete carbon tax implementation scheme [12]. De Miguel and Manzano suggest increasing carbon taxes from an environmental perspective and social development efficiency to reduce environmental pollution and improve production efficiency [13]. Zhang et al. investigated the design of a progressive carbon tax mechanism by the government and its impact on the production, pricing, and emission reduction level decisions of producers in a dual-channel supply chain network [14]. The second type of research proposes differentiated carbon tax policies. Wang Kun and Zhao Daozhi studied the optimal production and emission reduction decisions faced by enterprises under differential carbon taxes [15]. Florian Landis et al. explored the economic costs of differential carbon prices and carbon reduction [16]. Yin Weihua studied and analyzed whether China's proactive adoption of differentiated carbon tax policies based on the principle of cost fairness is sufficient to address the threat of carbon tariffs in developed economies [17]. Ma et al. stated that the optimal industrial carbon tax in China is a differentiated tax rate, with energy-intensive sectors taxed at 75 yuan/t and low-carbon sectors taxed at 50 yuan/t [18]. Based on the environmental externality characteristics of carbon emissions, Wang Mingxi and others proposed the basis for the levy of carbon tax policy with differential tax rates [19]. Yu and others showed that regulators can reduce emissions more effectively by adjusting the carbon tax rates of different products [20]. The third type of research proposes low-carbon subsidy policies. For example, to encourage enterprises to reduce carbon emissions, Zhao et al. proposed subsidy policies, which are essentially negative carbon tax policies [21], that will enable low-carbon enterprises to achieve the same goals as carbon taxes. Research by Jafar Hussain and others shows that subsidies can enable enterprises to achieve profit maximization while ensuring the implementation of green technologies [22]. Yu analyzed and studied the choice and impact of enterprises on carbon tax and carbon subsidy policies under the premise of profit maximization [23].

2.2. Research on Stepped Carbon Tax

In terms of formulating a tiered carbon tax policy, Wang Zizhuo’s research combines environmental capacity with a carbon tax system, proposing and demonstrating the correctness of the government’s establishment of a tiered carbon tax system [24]. Zhang Jijian and others have shown that a tiered carbon tax can effectively promote carbon emission reduction, and the setting of carbon tax rates should follow the principle of a low starting point and gradual progress [25]. Che et al. proposed a carbon tax policy for the power industry based on emission intensity that adopts a variable tax rate that gradually increases with the increase in power generation emission intensity [26]. Yang Chunyan and others have shown that compared to a unified carbon tax, a tiered carbon tax can effectively suppress the profits of high energy-consuming industries, and it is concluded that an effective carbon tax price should be maintained within 0–100 yuan/t [11]. An et al.’s research shows that when producers expand their production scale, a tiered carbon tax is beneficial for promoting low-carbon production by producers, thereby achieving a win–win situation of limited social welfare and environmental protection [27].

Li believes that the carbon tax rate should be formulated according to the difference in energy materials, the difference in the industrial structure of enterprises, and the difference in the economic level and industrial structure of different regions to adopt a stepped tax rate policy to enhance the rigor and precision of tax policies [28]. He proposed an innovative “two-tiered + value-added offset” carbon tax collection model. We will implement a tiered carbon tax in the East and the West. Different carbon tax values are set in order based on the technical level, the degree of affluence, the energy structure, the difficulty of afforestation, and the fragility of the ecological environment [29].

2.3. Research on Consumers’ Green Consumption Preference

With the deterioration of the ecological environment, more and more consumers are paying attention to the green and low-carbon nature of consumer goods, which has also sparked scholars to study the impact of consumers’ low-carbon awareness on supply
chain operation and management. Kotchen and Chitra’s research shows a positive correlation between the product prices consumers are willing to pay and their environmental awareness [30,31]. Bemporad and Baranowski conducted a similar survey on American consumers, pointing out that 67% of American consumers believe that purchasing environmentally friendly products is important, and 51% of American consumers are willing to pay higher prices for green and low-carbon products [32]. According to a survey conducted by the European Union in 2009, consumers who tend to pay higher prices for green and low-carbon products increased by 44% from 2005 to 2008 [33]. Sodhi and Tang pointed out through their research that consumers tend to prefer supply chain enterprises that can provide socially responsible and environmentally friendly products or services and encourage these enterprises to sell more products and earn more profit [34]. Secondly, a large number of scholars have investigated and analyzed the impact of consumers’ low-carbon consumption awareness on supply chain operation decisions. Liu investigated the impact of consumer low-carbon environmental awareness and market competition on the main members of the supply chain and pointed out an improvement in consumer environmental awareness [35]. Yang and Chen studied the impact of revenue-sharing and cost-sharing contracts provided by retailers on producers’ production reduction decisions and supply chain members’ profits in the context of consumers’ increasing environmental awareness [36]. The study by He et al. shows that changes in consumer green preference characteristics are important factors that motivate supply chain members to engage in green innovation efforts [37]. Liu et al. stated that under consumer preferences for low-carbon products, carbon reduction cost-sharing contracts can promote and coordinate the development of the supply chain [38]. Yuan Feng stated that over 90% of Chinese consumers have a sense of sustainable consumption, while 70% of consumers have a strong sense of sustainable consumption [39].

2.4. Supply Chain Cost-Sharing Strategy

Supply chain cost sharing refers to the process through which multiple participants in the supply chain jointly bear supply chain costs. Chao et al. pointed out that cost-sharing can effectively reduce uncertainty and improve product quality [40]. Ni et al. studied the investment cost of corporate social responsibility in the Cournot quantity competition model, and their conclusion showed that suppliers can share costs with downstream members of the supply chain by setting reasonable wholesale prices [41]. With the increase in research on sustainable supply chains (Carter and Easton [42] and Sewing [43]), domestic and foreign scholars have begun to study the cost-sharing of carbon emissions. Ghosh and Shah pointed out that cost-sharing strategies can effectively coordinate supply chain relationships when consumers have environmental awareness [44]. Xiao Di et al. pointed out that in a secondary supply chain with producers as the core enterprise and suppliers as followers, long-term responsibility cooperation relationships can be established among supply chain members through relationship contracts, and corresponding cost-sharing strategies can be developed [45]. Lou Gaoxiang and others studied the investment decision for green R&D and Green marketing in the secondary supply chain composed of suppliers and producers under different leadership situations [46]. Yang and Gong’s research shows that cost-sharing contracts can improve the environmental and economic performance of green supply chains [47]. Wu et al. stated that when producers face capital constraints due to increasing investment in low-carbon production technologies, retailers will participate in low-carbon cost sharing [48].

3. Models
3.1. Assumptions and Variables

This section will theoretically analyze the effectiveness of implementing a high-tiered carbon tax policy under green and low-carbon behavior and discuss the emission reduction mechanism of this tax policy. The following section mainly discusses the emission reduction mechanism of high-tiered carbon taxes in the green exchange market under a single-level
supply chain (considering competition between duopoly producers) and a two-level supply chain (considering competition between producers and retailers). The parameters and their definitions in the model are shown in Table 1.

Table 1. Parameters and their definitions.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definitions</th>
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<tbody>
<tr>
<td>( q_i(i = 1, 2) )</td>
<td>Production volume of producer ( i )'s products</td>
</tr>
<tr>
<td>( p_i(i = 1, 2) )</td>
<td>The price per unit product for producer ( i )</td>
</tr>
<tr>
<td>( \kappa(\kappa &gt; 0) )</td>
<td>Market capacity of the product ( (\kappa - \eta &lt; \lambda \epsilon) )</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Production cost per unit product ( (\eta &lt; \kappa) )</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Carbon emissions per unit of product produced by producer</td>
</tr>
<tr>
<td>( \epsilon_i(i = 1, 2)(0 &lt; \epsilon_i &lt; 1) )</td>
<td>The emission reduction rates achieved by the current emission reduction technology level of producer ( i )</td>
</tr>
<tr>
<td>( \lambda(0 &lt; \lambda &lt; 1) )</td>
<td>Emission reduction costs for producer ( i )</td>
</tr>
<tr>
<td>( \lambda_0(0 &lt; \lambda_0 &lt; 1) )</td>
<td>Consumer sensitivity to product carbon emissions (green sensitivity)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Consumer sensitivity to product yield (yield sensitivity)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>The carbon emission reduction input cost coefficient of the producer, which is a sufficiently large number</td>
</tr>
<tr>
<td>( \ell_i(i = 1, 2) )</td>
<td>Grey cost coefficient for the producer</td>
</tr>
<tr>
<td>( \pi_i(i = 1, 2) )</td>
<td>Growth coefficient of a tiered carbon tax</td>
</tr>
<tr>
<td>( \nu )</td>
<td>The wholesale price of a unit product, which is a decision variable for the producer;</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Carbon reduction rate (carbon reduction level), which is a decision variable for the producer</td>
</tr>
<tr>
<td>( \theta )</td>
<td>The order quantity of a product, which is a decision variable for retailers</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>The cost-contribution ratio of carbon emission reduction technology investment, which is a decision variable for the producer</td>
</tr>
</tbody>
</table>

To ensure the effectiveness of the model, the following assumptions were made during model construction:

**Assumption 1.** To meet market demand, it is assumed that the production volume \( i \) of the producers is large.

**Assumption 2.** Producers independently invest in carbon reduction technologies to reduce their carbon emissions and thus reduce the cost of carbon taxes. Drawing on the research by Bai Qingguo et al. [49], this article selects the carbon emission reduction cost function as:

\[
I_i = \frac{1}{2} \delta \epsilon_i^2 (i = 1, 2).
\]

**Assumption 3.** The environmental pollution caused by carbon emissions during the production process of the producer will affect the health of workers, directly affecting the company’s productivity and output, and thereby affecting profit margins. This loss is called the producer’s “gray cost” and is expressed as:

\[
B_i = \mu (1 - \epsilon_i^2) (i = 1, 2).
\]

**Assumption 4.** In the high-tiered carbon tax model, the high-tiered carbon tax is recognized as an extension of the original discrete low-tiered carbon tax. Assuming that the high-order carbon tax can be represented by a continuous function model, and drawing on the research by An et al. [27], this paper defines the carbon tax function as:

\[
t = \ell_i \epsilon_i^\nu = \ell_1 [(1 - \epsilon_i) \epsilon_i]^{\nu} (i = 1, 2),
\]

When \( \nu = 0 \), carbon tax \( t \) is a unified carbon tax \( t = \ell_1 \); when \( \nu = 1 \), the carbon tax \( t \) is a high-tiered carbon tax \( t = \ell_2 [(1 - \epsilon_i) \epsilon_i] \), where \( \ell_2 \) is set to be small enough.

**Assumption 5.** Consumer green preferences are an important incentive for producers to engage in green innovation to reduce carbon emissions [34]. The market price of a product depends on market
demand and emission reduction level. Normalizing the two parameters, the product market price function follows the following relationship:

\[ p_i = \kappa - \lambda (1 - \varepsilon_i)e - (1 - \lambda)(q_1 + q_2) \quad (i = 1, 2). \] (4)

3.2. Analysis of the Emission Reduction Mechanism of High-Tiered Carbon Tax in a Single-Level Supply Chain

In the Cournot duopoly game, two oligopolies producing homogeneous products make output decisions for the purpose of maximizing their own profits and competing with each other. Whether this competition can maintain stability has a significant impact on the development of enterprises, industries, and society [50,51].

This section studies the high-tiered carbon tax by establishing a Cournot game model for duopoly producers. The model mainly studies the optimal decision-making problem of two oligopoly producers with different emission reduction technologies in the green exchange market and studies the impact of government regulation on the production and emission reduction decisions of the two producers, thus summarizing the emission reduction mechanism of the high-tiered carbon tax policy in a single level supply chain.

If there is a carbon reduction technology exchange or technology spillover between producers in a market, that is, they can share emission reduction technologies, we call it a green exchange market. The green exchange market aims for cooperation and mutual benefit, with participants as partners. In this market, the goal of the two producers is to maximize common profits, which can be expressed as:

\[ q_t = q_1 + q_2, \] (5)

\[ \varepsilon_t = \frac{\varepsilon_1 + \varepsilon_2}{2}, \] (6)

\[ \pi_t = \pi_1 + \pi_2, \] (7)

where \( q_t \) represents the equilibrium total production of two producers and \( \varepsilon_t \) represents the equilibrium emission reduction rates of the market after the two producers exchange emission reduction technologies. The total profits for two producers in the green communication market can be expressed as:

\[
\text{max} \pi_t = \pi_1 + \pi_2 \\
= [\kappa - \lambda (1 - \varepsilon_t)e - (1 - \lambda)(q_1 + q_2)](q_1 + q_2) - \eta(q_1 + q_2) \\
- \frac{1}{2}q_t \varepsilon_t^2 - \mu(1 - \varepsilon_t)^2 - \ell_t[(1 - \varepsilon_t)e]^{v+1}(q_1^{v+1} + q_2^{v+1}).
\] (8)

s.t. \( \pi_1 > 0, \pi_2 > 0 \)

When the market implements a unified carbon tax policy \((v = 0, \ell_i = \ell_1)\), the total profits for two producers can be expressed as:

\[
\pi_t^u = -(\frac{\ell_1}{2} - \mu)\varepsilon_t^2 + [\kappa - (1 - \lambda)q_t - \lambda e - \eta]q_t \\
+(\lambda + \ell_1)e\varepsilon_t - \mu - \ell_1e q_t.
\] (9)

When the market implements a high-tiered carbon tax policy \((v = 1, \ell_i = \ell_2)\), the total profits for two producers can be expressed as:

\[
\pi_t^l = -[\frac{\ell_2}{2} - \mu + \ell_2e^2(q_1^2 + q_2^2)]\varepsilon_t^2 + [\lambda e(q_1 + q_2) - 2\ell_2e^2(q_1^2 + q_2^2)]\varepsilon_t \\
+ [\kappa - (1 - \lambda)(q_1 + q_2) - \lambda e - \eta](q_1 + q_2) - \mu - \ell_2e^2(q_1^2 + q_2^2).
\] (10)
The first-order partial derivatives of $q_1$ and $q_2$ are calculated for formula (8) to obtain:

$$\frac{\partial \tau_i}{\partial q_1} = \kappa - 2(1 - \lambda)(q_1 + q_2) - \lambda(1 - \epsilon_t)e - \eta - (v + 1)\ell_i(1 - \epsilon_t)^{v+1}e^{v+1}q_1^v,$$  \hspace{1cm} (11)

$$\frac{\partial \tau_i}{\partial q_2} = \kappa - 2(1 - \lambda)(q_1 + q_2) - \lambda(1 - \epsilon_t)e - \eta - (v + 1)\ell_i(1 - \epsilon_t)^{v+1}e^{v+1}q_2^v. \hspace{1cm} (12)$$

When the market implements a unified carbon tax policy ($v = 0$, $\ell_i = \ell_1$), it can be concluded that:

$$\frac{\partial \tau_i^u}{\partial q_1} = \kappa - 2(1 - \lambda)(q_1 + q_2) - \lambda(1 - \epsilon_t)e - \eta - \ell_1(1 - \epsilon_t)e,$$  \hspace{1cm} (13)

$$\frac{\partial \tau_i^u}{\partial q_2} = \kappa - 2(1 - \lambda)(q_1 + q_2) - \lambda(1 - \epsilon_t)e - \eta - \ell_1(1 - \epsilon_t)e. \hspace{1cm} (14)$$

When the market implements a high-tiered carbon tax policy ($v = 1$, $\ell_i = \ell_2$), it can be concluded that:

$$\frac{\partial \tau_i^l}{\partial q_1} = \kappa - 2(1 - \lambda)(q_1 + q_2) - \lambda(1 - \epsilon_t)e - \eta - 2\ell_2(1 - \epsilon_t)^2e^2q_1,$$  \hspace{1cm} (15)

$$\frac{\partial \tau_i^l}{\partial q_2} = \kappa - 2(1 - \lambda)(q_1 + q_2) - \lambda(1 - \epsilon_t)e - \eta - 2\ell_2(1 - \epsilon_t)^2e^2q_2. \hspace{1cm} (16)$$

**Proposition 1.** Under the premise of profit maximization, the emission reduction rates of producers in the green exchange market have an extreme point under the unified carbon tax and high carbon tax policies.

**Proof.** When the market implements a unified carbon tax policy ($v = 0$, $\ell_i = \ell_1$), the relationship between the total profit and emission reduction rates of two producers can be obtained by deriving $\epsilon_t$ from Equation (9):

$$\frac{\partial \tau_i^u}{\partial \epsilon_t} = -2(\delta - \mu)\epsilon_t + (\lambda + \ell_1)e_q. \hspace{1cm} (17)$$

When $0 < \epsilon_t < \frac{(\lambda + \ell_1)e_q}{\delta - 2\mu}$, $\tau_i^u$ is proportional to $\epsilon_t$; when $\frac{(\lambda + \ell_1)e_q}{\delta - 2\mu} < \epsilon_t < 1$, $\tau_i^u$ is inversely proportional to $\epsilon_t$.

When the market implements a high-tiered carbon tax policy ($v = 1$, $\ell_i = \ell_2$), the relationship between the total profits of producers and emission reduction rates can be obtained by deriving $\epsilon_t$ from Equation (10):

$$\frac{\partial \tau_i^l}{\partial \epsilon_t} = -2(\delta - \mu + \ell_2e^2(q_1^2 + q_2^2))\epsilon_t + \lambda e(q_1 + q_2) - 2\ell_2e^2(q_1^2 + q_2^2). \hspace{1cm} (18)$$

When $0 < \epsilon_t < \frac{\lambda e(q_1 + q_2) - 2\ell_2e^2(q_1^2 + q_2^2)}{\delta - 2\mu + 2\ell_2e^2(q_1^2 + q_2^2)}$, $\tau_i^l$ is proportional to $\epsilon_t$; when $\frac{\lambda e(q_1 + q_2) - 2\ell_2e^2(q_1^2 + q_2^2)}{\delta - 2\mu + 2\ell_2e^2(q_1^2 + q_2^2)} < \epsilon_t < 1$, $\tau_i^l$ is inversely proportional to $\epsilon_t$.

As in the perfect competition market, the above analysis shows that under the premise of profit maximization, the equilibrium emission reduction rates of two producers in the green exchange market have extreme points under the unified carbon tax and high-carbon tax policies, which proves the proposition.

This is because in the initial stage of emission reduction, the cost of emission reduction is relatively low, thus emission reduction generates more significant profits for producers;
\[ \varepsilon_1^L = \frac{(\lambda_1 + 2\pi q_1)\varphi_1}{\sqrt{2} + 2\lambda_1 q_1^2} \]

However, as the demand for emission reduction increases, the cost of emission reduction also increases. Once the emission reduction level exceeds the extreme point \( \varepsilon_1^U = \frac{(\lambda_1 + \ell_1)\varphi_1}{\sqrt{2} + \lambda_1 q_1^2} \), the emission reduction behavior of producers leads to negative growth in their profits. □

**Proposition 2.** In the early stage of emission reduction in the green exchange market, a high-tiered carbon tax policy is more effective at encouraging high-yield producers to generate greater profits by increasing the balanced emission reduction rates in the market compared to a unified carbon tax policy; in the later stage of emission reduction, the high-tiered carbon tax policy can effectively reduce the profit losses caused by high-yield producers excessively increasing the balanced emission reduction rates in the market compared to the unified carbon tax policy.

**Proof.** To maximize total profit, two producers will adopt the same production volume, thus \( q_1 = q_2 = \frac{1}{2}q_t \). When two producers continuously improve their balanced emission reduction rates, their profit changes are divided into two stages:

1. **In the early stage of emission reduction**, the total profits of the two producers are in a growth period.

   When \( \frac{\partial \pi_1}{\partial \varepsilon_1} > 0 \), \( \frac{\partial \pi_1}{\partial \varepsilon_2} > 0 \), and \( \left| \frac{\partial \pi_1}{\partial \varepsilon_1} \right| > \left| \frac{\partial \pi_1}{\partial \varepsilon_2} \right| \), that is, \( \frac{\partial \pi_1}{\partial \varepsilon_1} > \frac{\partial \pi_1}{\partial \varepsilon_2} \). This is the stage when the equilibrium emission reduction rates of the two producers change from 0 to the extreme point. In this stage, when the marginal profit of carbon emission reduction under the high carbon tax is greater than the marginal profit of carbon emission reduction under the unified carbon tax, the total output meets:

   \[ q_1 = q_2 = \frac{1}{2}q_t > \frac{\ell_1}{2\ell_2(1 - \varepsilon_1)} \]  \hspace{1cm} (19)

   At this time, \( \ell_2 \) can be considered as a constant and \( \ell_2 \) is infinitely small, thus \( \frac{\ell_1}{2\ell_2(1 - \varepsilon_1)} \) can be considered as a relatively large number, which indicates that for high-yield producers in the cooperative market in the early stage of emission reduction (\( \varepsilon_1 \) is relatively low), the marginal profit generated by increasing the emission reduction rates under the high-tiered carbon tax policy is higher than that generated under the unified carbon tax policy, that is, the high-tiered carbon tax can motivate high-yield producers to increase their emission reduction rates to generate greater profits.

2. **In the later stage of emission reduction**, the total profits of the two producers are in a negative growth period.

   When \( \frac{\partial \pi_1}{\partial \varepsilon_1} < 0 \), \( \frac{\partial \pi_1}{\partial \varepsilon_2} < 0 \), and \( \left| \frac{\partial \pi_1}{\partial \varepsilon_1} \right| < \left| \frac{\partial \pi_1}{\partial \varepsilon_2} \right| \), that is, \( \frac{\partial \pi_1}{\partial \varepsilon_1} < \frac{\partial \pi_1}{\partial \varepsilon_2} \). This is the stage when the equilibrium emission reduction rates of the two producers reach the extreme point and then change towards 1. From the previous conclusion, it can be concluded that at this stage, the emission reduction behavior of producers will generate negative profits for them. When the marginal negative profit of carbon reduction under the high-tiered carbon tax is smaller than the marginal negative profit of carbon reduction under the unified carbon tax, the total production meets:

   \[ q_1 = q_2 = \frac{1}{2}q_t > \frac{\ell_1}{2\ell_2(1 - \varepsilon_1)} \]  \hspace{1cm} (20)

   As above, it can be considered that \( \frac{\ell_1}{2\ell_2(1 - \varepsilon_1)} \) is a relatively large number, indicating that for high-yield producers in the green exchange market who are in the later stage of emission reduction (with higher \( \varepsilon_1 \)), the marginal negative profit generated by increasing emission reduction rates under the high-tiered carbon tax policy is lower than that of the unified carbon tax policy. That is, compared to the unified carbon tax, the high-tiered carbon
tax can more effectively avoid profit losses caused by high-yield producers excessively increasing emission reduction rates in the later stage of emission reduction. The proposition is proven. □

**Proposition 3.** Implement a high-carbon tax policy in the green exchange market and the two oligopoly producers have the best potential to maximize the total profit from the market. When implementing a unified carbon tax policy, there is no such optimal output.

**Proof.** When the market implements a unified carbon tax policy \((\nu = 0, \ell_1 = \ell_2)\), the second-order partial derivatives of Equations (13) and (14) for \(q_1, q_2\) are obtained as:

\[
\frac{\partial^2 \pi_{u}}{\partial \ell^2} = -2(1 - \lambda),
\]

\[
\frac{\partial^2 \pi_{u}}{\partial q^2} = -2(1 - \lambda),
\]

\[
\frac{\partial^2 \pi_{u}}{\partial q_1 \partial q_2} = \frac{\partial^2 \pi_{u}}{\partial q_2 \partial q_1} = -2(1 - \lambda),
\]

The Hessian matrix can be obtained as:

\[
H_i^{u} = \begin{pmatrix}
-2(1 - \lambda) & -2(1 - \lambda) \\ -2(1 - \lambda) & -2(1 - \lambda)
\end{pmatrix}.
\]

And:

\[
|H_i^{u}| = 0.
\]

When the market implements a high-tiered carbon tax policy \((\nu = 1, \ell_1 = \ell_2)\), the second-order partial derivatives of Equations (15) and (16) for \(q_1, q_2\) are obtained as:

\[
\frac{\partial^2 \pi_{l}}{\partial \ell^2} = -2(1 - \lambda) - 2\ell_2(1 - \epsilon_1)^2 \epsilon^2,
\]

\[
\frac{\partial^2 \pi_{l}}{\partial q^2} = -2(1 - \lambda) - 2\ell_2(1 - \epsilon_1)^2 \epsilon^2,
\]

\[
\frac{\partial^2 \pi_{l}}{\partial q_1 \partial q_2} = \frac{\partial^2 \pi_{l}}{\partial q_2 \partial q_1} = -2(1 - \lambda),
\]

The Hessian matrix can be obtained as:

\[
H_i^{l} = \begin{pmatrix}
-2(1 - \lambda) - 2\ell_2(1 - \epsilon_1)^2 \epsilon^2 & -2(1 - \lambda) - 2\ell_2(1 - \epsilon_1)^2 \epsilon^2 \\ -2(1 - \lambda) - 2\ell_2(1 - \epsilon_1)^2 \epsilon^2 & -2(1 - \lambda) - 2\ell_2(1 - \epsilon_1)^2 \epsilon^2
\end{pmatrix}.
\]

From the above analysis, it can be concluded that if \(\frac{\partial^2 \pi_{l}}{\partial q_i^2} < 0\) and \(|H_i^{l}| > 0\), then \((q_1, q_2)\) is the local maximum point, which maximizes the total profits of the two producers. This proposition is supported. The optimal output for two producers can also be obtained as:

\[
q_1^{l} = q_2^{l} = \frac{2\ell_2(1 - \epsilon_1)^2 \epsilon^2[k - \eta - \lambda(1 - \epsilon_1)e]}{3(1 - \epsilon_1)^3 + 4\ell_2^2(1 - \epsilon_1)^3 \epsilon^4 + 8(1 - \lambda)\ell_2(1 - \epsilon_1)^2 \epsilon^2},
\]

Therefore, the total output of the two producers is:

\[
\eta^{l} = \frac{4\ell_2 \epsilon^2[k - \eta - \lambda(1 - \epsilon_1)e]}{3(1 - \epsilon_1) + 4\ell_2(1 - \epsilon_1)^2 \epsilon^4 + 8(1 - \lambda)\ell_2 \epsilon^2}.
\]
According to the previous section’s research and analysis on the emission reduction mechanism of a single-level supply chain under a high-tiered carbon tax policy in the green exchange market, it can be concluded that the emission reduction efficiency of a high-tiered carbon tax policy is better than that of a unified carbon tax. Therefore, the emission reduction mechanism of the single-stage supply chain under the high-tiered carbon tax policy is summarized in two aspects: first, whether in the perfect competition market or the green exchange market, in the early stage of emission reduction, the high-tiered carbon tax policy can encourage high-yield producers to generate higher marginal profits by improving their own emission reduction rates; in the later stage of emission reduction, a high-tiered carbon tax policy can effectively reduce the profit losses caused by high-yield producers increasing emission reduction rates. This indicates that in the early stage of emission reduction, producers expect to achieve greater profit margins through the improvement of emission reduction technologies, and in the later stage of emission reduction when producers expect to avoid reducing their own profits due to the development of emission reduction technologies, the high-tiered carbon tax policy is in line with the actual production emission reduction strategies of producers. Secondly, although the impact of high-tiered carbon tax policies on market participants is roughly the same in different markets, there are slight differences. When implementing the high-class carbon tax policy in a perfect competition market, the emission reduction behavior of producers can increase their marginal production; in the green exchange market, the emission reduction technology exchange between producers can maximize the production space for the market to achieve profit maximization.

To sum up, in the perfect competition market and the green exchange market, the implementation of high carbon taxes can achieve more efficient carbon regulation, which is an optimized emission reduction system. Due to the establishment of a high-tiered carbon tax through a gradual increase in marginal tax revenue from low to high, it can be achieved at different stages of carbon emissions to limit excessive carbon emissions while reducing the burden on producers, thereby balancing socioeconomic and environmental goals.

3.3. Analysis of Emission Reduction Mechanism of High-Tiered Carbon Tax in a Two-Level Supply Chain

Based on Section 3.2, this section establishes a two-level supply chain composed of producers and retailers, and further studies the emission reduction decisions of the two-level supply chain under the high-tiered carbon tax policy, that is, establishes a joint emission reduction model to study whether the two-level supply chain can achieve more efficient low-carbon development by selecting appropriate cost-sharing strategies. This section will construct two cost-sharing models: carbon emission reduction technology investment cost sharing and high-tiered carbon tax cost sharing, based on which the emission reduction mechanism of the two-level supply chain under the high-tiered carbon tax policy will be analyzed.

This section mainly studies the impact of a high-tiered carbon tax policy on the emission reduction of the two-level supply chain composed of producers and retailers by drawing on the research by Wang et al. [50], and constructs a two-stage Stackelberg model between producers and retailers: in the first stage, producers in the supply chain determine the optimal wholesale price and carbon emission reduction level to achieve profit maximization based on the high-tiered carbon tax policy set by the government. At the same time, they choose and determine the optimal cost-sharing strategy to achieve limited maximization of their own profits. In the second stage, retailers in the two-stage supply chain mainly determine their optimal order quantity based on the producer’s decision. To simplify the model, this section changes the inverse demand function in Section 3.2 to:

$$p = \kappa - (1 - \lambda)q + \lambda \varepsilon.$$  (32)
The market model in this section is based on the premise of the government imposing a high-tiered carbon tax, thus the carbon tax cost sharing in the following text refers to the cost-sharing of high-tiered carbon taxes. Due to the fact that in Section 3.2 of the single-level supply chain both the cost of carbon reduction technology investment and carbon tax are borne by the producer, for the sake of discussion and comparison, the proportion shared by retailers is set to \( \theta \), \( \varphi \), and the proportion shared by producers is set to \( 1 - \theta, 1 - \varphi \).

The models for the two sharing strategies are as follows:

The profit function \( \pi_m(w, \varepsilon) \) for producers under the cost-sharing strategy for carbon emission reduction technology investment is expressed as:

\[
\pi_m(w, \varepsilon) = (w - \eta)q - t - \frac{1}{2}(1 - \theta)\delta e^2 - \mu(1 - \varepsilon^2)
\]

\[
= (w - \eta)q - \ell(1 - \varepsilon)^2q^2 - \frac{1}{2}(1 - \theta)\delta e^2 - \mu(1 - \varepsilon^2)
\]

(33)

The retailer profit function \( \pi_n(q) \) is represented as:

\[
\pi_n(q) = [\kappa - (1 - \lambda)q + \lambda \varepsilon - w]q - \frac{1}{2}\theta \delta e^2.
\]

(34)

The profit function \( \pi_m(w, \varepsilon) \) for producers under the carbon tax cost-sharing strategy is expressed as:

\[
\pi_m(w, \varepsilon) = (w - \eta)q - \frac{1}{2}\delta e^2 - t(1 - \varphi) - \mu(1 - \varepsilon^2)
\]

\[
= (w - \eta)q - \frac{1}{2}\delta e^2 - \ell(1 - \varphi)(1 - \varepsilon)^2q^2 - \mu(1 - \varepsilon^2).
\]

(35)

The retailer profit function \( \pi_n(q) \) is expressed as:

\[
\pi_n(q) = [\kappa - (1 - \lambda)q + \lambda \varepsilon - w]q - \ell \varphi(1 - \varepsilon)^2q^2.
\]

(36)

3.3.1. Analysis of Emission Reduction Mechanism under the Cost-Sharing Strategy for Carbon Emission Reduction Technology Investment

(a) Determine the optimal order quantity for retailer products:

\[
\pi_n(q) \text{ is first and second order conditions for } q:
\]

\[
\frac{\partial \pi_n}{\partial q} = \kappa + \lambda \varepsilon - w - 2(1 - \lambda)q,
\]

(37)

\[
\frac{\partial^2 \pi_n}{\partial q^2} = -2(1 - \lambda) < 0.
\]

(38)

Assume that \( \frac{\partial \pi_n}{\partial q} = 0 \) and we obtain:

\[
q^* = \frac{\kappa + \lambda \varepsilon - w}{2(1 - \lambda)}.
\]

(39)

(b) Determine the optimal emission reduction rates and wholesale price for the producer:

\[
\pi_m^*(w, \varepsilon) = \frac{(\kappa + \lambda \varepsilon - w)(w - \eta)}{2(1 - \lambda)} - \frac{(\kappa + \lambda \varepsilon - w)^2\ell \delta e^2}{4} - \frac{1}{2}(1 - \theta)\delta e^2 - \mu(1 - \varepsilon^2).
\]

(40)

Find the first and second-order derivatives of \( \pi_m^*(w, \varepsilon) \) for \( w \) and \( \varepsilon \):

\[
\frac{\partial \pi_m^*}{\partial w} = \frac{\kappa + \lambda \varepsilon + \eta - 2w}{2(1 - \lambda)} + \frac{(\kappa + \lambda \varepsilon - w)\ell \delta e^2}{2},
\]

(41)

\[
\frac{\partial \pi_m^*}{\partial \varepsilon} = \frac{\lambda(w - \eta)}{2(1 - \lambda)} - \frac{(\kappa + \lambda \varepsilon - w)\lambda \ell \delta e^2}{2} - (1 - \theta)\delta e + 2\mu \varepsilon,
\]

(42)
\[
\frac{\partial^2 \pi_m^*}{\partial \omega^2} = -\frac{1}{(1 - \lambda)} - \frac{\ell \epsilon^2}{2},
\]
\[
\frac{\partial^2 \pi_m^*}{\partial \epsilon^2} = -(1 - \theta)\delta + 2\mu - \frac{\lambda^2 \ell \epsilon^2}{2},
\]
\[
\frac{\partial^2 \pi_m^*}{\partial w \partial \epsilon} = \frac{\lambda}{2(1 - \lambda)} + \frac{\lambda \ell \epsilon^2}{2},
\]
\[
\frac{\partial^2 \pi_m^*}{\partial \epsilon \partial \omega} = \frac{\lambda}{2(1 - \lambda)} + \frac{\lambda \ell \epsilon^2}{2}.
\]

The Hessian matrix obtained from Equations (43)–(46) is:
\[
H = \begin{pmatrix}
-\frac{1}{(1 - \lambda)} - \frac{\ell \epsilon^2}{2} & \frac{\lambda}{2(1 - \lambda)} + \frac{\lambda \ell \epsilon^2}{2} \\
\frac{\lambda}{2(1 - \lambda)} + \frac{\lambda \ell \epsilon^2}{2} & -(1 - \theta)\delta + 2\mu - \frac{\lambda^2 \ell \epsilon^2}{2}
\end{pmatrix}.
\]

From the above analysis, it can be concluded that if \(\frac{\partial^2 \pi_m^*}{\partial \omega^2} < 0, |H| > 0\), then \((w^*, \epsilon^*)\) is the local maximum point, which enables the producer to achieve profit maximization.

By setting \(\frac{\partial \pi_m^*}{\partial w} = 0\) and \(\frac{\partial \pi_m^*}{\partial \epsilon} = 0\), we can obtain:
\[
\epsilon^* = \frac{\lambda(w - \eta) - \lambda \ell \epsilon^2(\kappa - \omega)(1 - \lambda)}{(1 - \lambda)[2(1 - \theta)\delta - 4\mu + \lambda^2 \ell \epsilon^2]},
\]
\[
w^* = \kappa + \lambda \epsilon - \frac{\kappa + \lambda \epsilon - \eta}{2 + \ell \epsilon^2(1 - \lambda)}.
\]

(c) Determine the optimal cost-sharing rates for producers:
\[
\frac{d \pi_m^*}{d \theta} = \frac{1}{2} \delta \epsilon^2 > 0.
\]

**Proposition 4.** In the Stackelberg game of sharing the cost of carbon reduction technology investment between producers and retailers, increasing emission reduction rates can increase the demand for their products in the market, and the higher the green preference of consumers, the greater the increase in product demand.

**Proof.** In the Stackelberg game of cost sharing for carbon emission reduction technology investment, the optimal order quantity for retailers, i.e., the market demand for producer products, is:
\[
q^* = \frac{\kappa + \lambda \epsilon - \omega}{2(1 - \lambda)},
\]
At this point:
\[
\frac{\partial q^*}{\partial \epsilon} = \frac{\lambda}{2(1 - \lambda)} = f(\lambda) > 0,
\]
This indicates that increasing producers’ own emission reduction rates will increase retailers’ order volume for products, and there is:
\[
\frac{df(\lambda)}{d\lambda} = \frac{1}{2(1 - \lambda)^2} > 0,
\]
That is to say, the extent to which retailers increase their order volume for products increases with the increase in consumers’ green preferences, which proves the proposition.

**Proposition 5.** After the retailer makes the order quantity decision to maximize their profit maximization, there is an optimal product wholesale price and carbon emission reduction level $e$ ($w^*, e^*$) in the market, and the two promote each other, that is, the producer can determine the optimal emission reduction rates and product wholesale price to achieve their own profit maximization.

**Proof.** By using Equations (48) and (49), the optimal wholesale price and carbon determined by the producer can be obtained. Taking the derivative of $w^*$ with respect to $e^*$, we obtain:

$$
\frac{dw^*}{de^*} = \frac{\lambda [1 + \ell e^2 (1 - \lambda)]}{2 + \ell^2 e^2 (1 - \lambda)} > 0,
$$

Taking the derivative of $e^*$ with respect to $w$, we obtain:

$$
\frac{de^*}{dw} = \frac{\lambda [1 + \ell e^2 (1 - \lambda)]}{(1 - \lambda) [2(1 - \theta)\delta - 4\mu + \lambda^2 \ell^2]} > 0,
$$

At this point, there is a mutually reinforcing relationship between the optimal wholesale price of products and the optimal emission reduction rates, that is to say, when producers increase their own emission reduction rates, they also promote the increase in wholesale prices. This indicates that producers not only achieve the goal of reducing carbon emissions but also promote further increases in their own profits. This proposition is supported.

**Proposition 6.** In the cost-sharing strategy for carbon emission reduction technology input between producers and retailers, the producers’ profits are in direct proportion to the cost-sharing rates of retailers. As long as retailers are willing to share costs, producers will continue to improve their emission reduction rates to achieve profit maximization.

**Proof.** Formula (50) shows that in the cost-sharing strategy for carbon emission reduction technology input, the producer’s profit increases with the increase in the cost-sharing rates and the extent of the increase depends on their own emission reduction rates, that is, the producer’s marginal profit increases with the increase in their own emission reduction rates. Therefore, when retailers are willing to share the cost of emission reduction technology, producers will inevitably continue to improve their emission reduction rates to achieve profit maximization, which proves the proposition.

### 3.3.2. Analysis of Emission Reduction Mechanism under a Carbon Tax Cost-Sharing Strategy

(a) Determine the optimal order quantity for retailers

$\pi_n(q)$’s first- and second-order conditions for $q$:

$$
\frac{\partial \pi_n}{\partial q} = \kappa + \lambda e - w - 2(1 - \lambda)q - 2\ell \varphi (1 - \varepsilon)^2 e^2 q,
$$

$$
\frac{\partial^2 \pi_n}{\partial q^2} = -2(1 - \lambda) - 2\ell \varphi (1 - \varepsilon)^2 e^2 < 0.
$$
Make $\frac{\partial \tau_{m}}{\partial \phi} = 0$, and we obtain:

$$q^* = \frac{\alpha + \lambda c - w}{2(1 - \lambda) + 2k\phi(1 - \epsilon)^2 \epsilon'^2}. \quad (58)$$

(b) Determine the optimal emission reduction rates and wholesale price for the producer:

$$\pi_{m}^* = \frac{(w - \eta)(\kappa + \lambda c - w)(1 - \epsilon)}{2(1 - \lambda) + 2k\phi(1 - \epsilon)^2 \epsilon'^2} + R_{m}, \quad (59)$$

Here, $R_m$ is an infinitesimal term, which will be ignored in the following operation.

We find the first and second-order derivatives of $\pi_m^*(w, \epsilon)$ for $w$ and $\epsilon$:

$$\frac{\partial \pi_m^*}{\partial \epsilon} = 2\ell \phi e^2 (w - \eta) (\kappa + \lambda c - w) (1 - \epsilon) + 2(w - \eta)(1 - \lambda) \lambda 2(1 - \lambda) + 2k\phi(1 - \epsilon)^2 \epsilon'^2, \quad (60)$$

$$\frac{\partial \pi_m^*}{\partial w} = \frac{\alpha + \lambda c - \epsilon - 2w}{2(1 - \lambda) + 2k\phi(1 - \epsilon)^2 \epsilon'^2}, \quad (61)$$

$$\frac{\partial^2 \pi_m^*}{\partial w^2} = \frac{1}{2(1 - \lambda) + 2k\phi(1 - \epsilon)^2 \epsilon'^2}, \quad (62)$$

$$\frac{\partial^2 \pi_m^*}{\partial \epsilon^2} = -\frac{12\ell^2 \phi e^4 (w - \eta)(\kappa + \lambda c - w)(1 - \epsilon)^2}{[2(1 - \lambda) + 2k\phi(1 - \epsilon)^2 \epsilon'^2]^3}$$

$$-\frac{4k\ell^2 (w - \eta)(1 - \lambda)(\kappa + \lambda c - w) + 2\lambda(1 - \epsilon)]}{[2(1 - \lambda) + 2k\phi(1 - \epsilon)^2 \epsilon'^2]^3}, \quad (63)$$

$$\frac{\partial^2 \pi_m^*}{\partial \epsilon \partial w} = \frac{2\ell \phi \lambda e^2 (1 - \epsilon)^2 + 4\ell \phi e^2 (\kappa + \lambda c + \eta - 2w)(1 - \epsilon) + 2\lambda(1 - \lambda)}{2(1 - \lambda) + 2k\phi(1 - \epsilon)^2 \epsilon'^2}, \quad (64)$$

$$\frac{\partial^2 \pi_m^*}{\partial w \partial \epsilon} = \frac{2\ell \phi \lambda e^2 (1 - \epsilon)^2 + 4\ell \phi e^2 (\kappa + \lambda c + \eta - 2w)(1 - \epsilon) + 2\lambda(1 - \lambda)}{2(1 - \lambda) + 2k\phi(1 - \epsilon)^2 \epsilon'^2}. \quad (65)$$

Then the Hessian matrix is:

$$H = \begin{pmatrix} \frac{\partial^2 \pi_m^*}{\partial w^2} & \frac{\partial^2 \pi_m^*}{\partial w \partial \epsilon} \\ \frac{\partial^2 \pi_m^*}{\partial \epsilon \partial w} & \frac{\partial^2 \pi_m^*}{\partial \epsilon^2} \end{pmatrix}, \quad (66)$$

From the above analysis, it can be concluded that there is no local optimal $(w^*, \epsilon^*)$ to enable producers to achieve profit maximization.

By setting $\frac{\partial \pi_m^*}{\partial w} = 0$ and $\frac{\partial \pi_m^*}{\partial \epsilon} = 0$, we can obtain:

$$w^* = \frac{\kappa + \lambda c + \eta}{2}, \quad (67)$$

$$\epsilon^* = 1 + \frac{(1 - \lambda)\lambda}{\ell \phi e^2 (\kappa + \lambda c - w)}. \quad (68)$$

(c) Determine the optimal cost-sharing rates for producers:

$$\frac{\partial \pi_m^*}{\partial \phi} = \frac{2(1 - \lambda) + 2k\phi(1 - \epsilon)^2 \epsilon'^2 - 2\ell(1 - \epsilon)^2 \epsilon^2 (w - \eta)(\kappa + \lambda c - w)}{[2(1 - \lambda) + 2k\phi(1 - \epsilon)^2 \epsilon'^2]^2} < 0. \quad (69)$$
Proposition 7. In the Stackelberg game between producers and retailers under carbon tax cost-sharing, there is no clear relationship between the optimal order quantity of retailers and the emission reduction rates of producers, and it is negatively correlated with the wholesale prices of producers’ products.

Proof. Taking the derivative of Equation (58) with respect to $\epsilon$ and $w$, we obtain:

$$
\frac{dq^*}{d\epsilon} = \frac{\ell \varphi(1 - \epsilon)e^2[2x + \lambda(1 + \epsilon) - 2w] + \lambda(1 - \lambda)}{2[(1 - \lambda) + \ell \varphi(1 - \epsilon)^2e^2]^2} > 0,
$$

(70)

$$
\frac{dq^*}{dw} = -\frac{1}{2(1 - \lambda) + 2\ell \varphi(1 - \epsilon)^2e^2} < 0,
$$

(71)

From the above, it can be concluded that the increase or decrease in the emission reduction rates of producers cannot directly affect the ordering decisions of retailers. However, as experience has shown, the wholesale price for producers and the ordering quantity for retailers show a trade-off relationship. □

Proposition 8. In the carbon tax cost-sharing strategy between producers and retailers, retailers’ increasing share rates will increase the marginal profits for producers, and the increasing range will decrease first and then increase.

Proof. In carbon tax cost-sharing between producers and retailers, we analyze Equations (59) and (69):

$$
A_1 = 2\ell (1 - \epsilon)^2 e^2 > 0,
$$

(72)

$$
A_2 = (w - \eta)(\kappa + \lambda e - \omega) > 0,
$$

(73)

Then there is:

$$
\frac{\partial \pi_m^*}{\partial \varphi} = \frac{2(1 - \lambda) + A_1 \varphi - A_1 A_2}{(2(1 - \lambda) + A_1 \varphi)^2} < 0,
$$

(74)

Taking the second-order condition of $\varphi$ for $\pi_m^*$ and making it equal to 0, we obtain:

$$
\frac{\partial^2 \pi_m^*}{\partial \varphi^2} = \frac{A_1}{2(1 - \lambda) + A_1 \varphi^2} - \frac{2A_1[2(1 - \lambda) + A_1 \varphi - A_1 A_2]}{(2(1 - \lambda) + A_1 \varphi)^3} = 0,
$$

(75)

Then there is:

$$
\varphi^* = 2A_2 - \frac{2(1 - \lambda)}{A_1}.
$$

(76)

When $\varphi^* = 2A_2 - \frac{2(1 - \lambda)}{A_1}$ and $\frac{\partial^2 \pi_m^*}{\partial \varphi^2} \neq 0$, therefore, when $\varphi \in [0, 2A_2 - \frac{2(1 - \lambda)}{A_1}]$, the increase in $\pi_m^*(\varphi)$ decreases continuously, and when $\varphi \in [2A_2 - \frac{2(1 - \lambda)}{A_1}, 1]$, the increase in $\pi_m^*(\varphi)$ increases continuously. The sharing rate $\varphi$ between producers and retailers has a turning point in the process of changing from 0 to 1, and the corresponding sharing rate at the turning point is:

$$
\varphi^* = 2(w - \eta)(\kappa - \lambda e - \omega) - \frac{(1 - \lambda)}{\ell (1 - \epsilon)^2 e^2}.
$$

(77)

As $\varphi$ increases, $\pi_m(\varphi)$ also increases, and as $\varphi$ increases from 0 to $\varphi^*$, the growth rates of $\pi_m(\varphi)$ decreases. As $\varphi$ changes from $\varphi^*$ to 1, the growth rates of $\pi_m(\varphi)$ gradually increase, and the proposition is proven. □

Proposition 9. Only by continuously developing disruptive carbon reduction technologies can producers achieve the maximum profit margin in sharing carbon tax costs with retailers.
Proof. After the retailer makes the order quantity decision to maximize their profits, there is no optimal combination strategy of the wholesale price and carbon emission reduction rates in the market to enable the producer to achieve their own profit maximization. $\epsilon^* = 1 + \frac{(1-\lambda)\lambda}{\lambda e^2(k+\lambda+w)} \geq 1$. $\epsilon^* = 1$ indicates the emergence of zero carbon technology in the market. $\epsilon^* > 1$ indicates the emergence of negative carbon technologies in the market, also known as disruptive carbon reduction technologies. This indicates that the emission reduction technologies at this time not only reduce carbon emissions to zero, but also achieve carbon sequestration. When producers and retailers share the cost of carbon taxes, simply increasing the level of carbon emissions reduction is not enough to achieve maximum profit. Only by continuously innovating carbon emission reduction technologies and developing disruptive carbon emission reduction technologies can they achieve maximum profit margins, which is proven. □

In the trend for developing green and low-carbon industries, producers’ investment in carbon reduction technology is a major cost of production. By sharing the cost of carbon reduction technology investment between producers and retailers in a two-level supply chain, we found that cost sharing can effectively alleviate the burden on producers and promote further improvement in emission reduction levels. Three main conclusions can be drawn from the analysis of the emission reduction mechanism of the two-level supply chain under the cost-sharing strategy for carbon reduction technology investment under the high-tiered carbon tax policy. Firstly, increasing the emission reduction rates of producers can increase market demand for their products, and the higher the green sensitivity of consumers ($\lambda$), the larger the range of product demand increases. This indicates that the market is sufficiently sensitive to green and low-carbon products at this time; Second, the producer can determine the optimal emission reduction rates and wholesale price of products to maximize the profit of the two-level supply chain; Third, retailers’ cost-sharing will encourage producers to improve their emission reduction rates to achieve profit maximization. Conversely, the increase in producers’ emission reduction rates will also reduce the pressure on retailers’ cost-sharing.

When the government begins to implement a high-tiered carbon tax policy, the cost of carbon tax for producers will continue to increase with the increase in carbon emissions during their production process. Through the analysis of the cost-sharing of carbon tax between producers and retailers in the two-level supply chain, we found that cost-sharing can increase the marginal profits of producers and encourage them to continue to carry out emission reduction technology innovation. Through the analysis of the emission reduction mechanism of the two-level supply chain under the carbon tax cost-sharing strategy under the high-tiered carbon tax policy, three main conclusions can be drawn: firstly, the optimal order quantity of retailers will not increase with the increase in the emission reduction rates of producers, but will increase with the decrease of wholesale prices of products, indicating that the market is not sensitive enough to green and low-carbon products at this time; Second, the retailer’s increasing share will increase the producer’s marginal profits, and the extent of increase will decrease first and then increase; Thirdly, in order for producers to achieve the maximum profit margin in sharing carbon tax costs with retailers, they must conduct research and development of disruptive carbon reduction technologies.

4. Results and Discussion

A high-tiered carbon tax restricts the large-scale production of high carbon emissions, but in order to ensure that market supply and demand are not greatly affected, this study sets smaller-tiered carbon tax growth parameters, larger market capacity, and larger carbon emission reduction input cost coefficients. Drawing on the research by Luo et al. [52], a corresponding numerical experiment was conducted ($\kappa = 800; w = 20; \eta = 5; \delta = 2000; e = 2; \lambda = 0.5; \mu = 0.5; \ell_1 = 8; \ell_2 = 0.005$). As shown in Figure 1, we analyzed the impact of producers’ emission reduction rates and production on profit under the high-tiered carbon tax and unified carbon tax policies in the green exchange market and
found that: (1) In most cases, in terms of producers in the green exchange market, a high-tiered carbon tax policy is more cost-effective than a unified carbon tax policy. This is mainly because, in the high-carbon tax policy, there are fewer carbon taxes levied on low carbon emission levels, which can save producers more carbon tax costs; (2) If producers continue to expand their production space while increasing investment in carbon reduction technologies, their profits under the high-tiered carbon tax policy will be much higher than the unified carbon tax. This is mainly because although producers have increased carbon emissions by expanding production space, the increase in investment in emission reduction technology has led to a significant reduction in emissions, thereby increasing their own profits.

Figure 1. The impact of producer emission reduction rates and production on profits in the green exchange market.

As shown in Figures 2 and 3, we compared and analyzed the impact of high-tiered carbon tax and unified carbon tax policies on producer profits before and after emission reduction. We found that in the early and late stages of emission reduction in the green exchange market, the producers’ profits under high-tiered carbon taxes are greater than those under unified carbon taxes. The main reason is that in the early stage of emission reduction, the marginal benefits of increasing emission reduction rates under the high-tiered carbon tax policy are higher than those under the unified carbon tax policy; while producers invest heavily in carbon reduction technology in the later stage of emission reduction, they also increase their own emission reduction rates, reducing their carbon tax costs under the high-tiered carbon tax policy. Therefore, compared to a unified carbon tax, a high-tiered carbon tax can effectively incentivize large producers to increase their own emission reduction rates and achieve greater production space. The blue, green, and yellow colors on the thermal diagram layer represent the value of the value, and from blue to yellow, the carbon tax value is getting larger and larger.

As shown in Figures 4 and 5, we analyzed the impact of the contribution ratio \( \theta \) and carbon tax \( t \), as well as the emission reduction rate \( \epsilon \), on the producers’ profits under the carbon emission reduction technology sharing strategy. We found that: (1) When the intensity of external government regulation (carbon tax \( t \)) decreases or the sharing force within the supply chain (contribution ratio \( \theta \)) increases the producers’ profits increase; A decrease in carbon tax \( t \) indicates a decrease in the carbon tax costs that producers need to bear, while an increase in the contribution ratio \( \theta \) further alleviates the cost pressure on producers in carbon reduction technology, thereby increasing their profits. (2) When the internal sharing force of the supply chain is large enough, with the increase in the producer’s emission reduction level (emission reduction rates \( \epsilon \)), although the producer’s improvement of carbon emission reduction level increases the cost of technology investment, the cost-sharing within the supply chain alleviates some of the pressure, thus the producer will achieve profit maximization. This indicates that cost-sharing within the supply chain is an important factor driving producers to innovate in carbon reduction technologies.
Figure 1. The impact of producers’ emission reduction rates and production on profits in the later stage of emission reduction.

Figure 2. The impact of producers’ emission reduction rates and production on profits in the early stage of emission reduction.

Figure 3. The impact of producers’ emission reduction rates and production on profits in the later stage of emission reduction.

Figure 4. The impact of contribution ratio $\theta$ and carbon tax $t$ on producer’s profit under the carbon reduction technology sharing strategy.

As shown in Figures 6 and 7, we analyzed the impact of the contribution ratio $\varphi$ and carbon tax $t$, as well as the emission reduction rate $\epsilon$ on producer’s profit under the carbon tax cost-sharing strategy. We found that: (1) When the internal sharing force (contribution ratio $\varphi$) in the supply chain is large enough, the external regulatory intensity (carbon tax $t$) of the government increases. At this time, the producers’ carbon tax burden increases,
but because the supply chain shares part of the pressure, the producers will achieve profit maximization. This indicates that cost-sharing within the supply chain contributes to government carbon regulation; (2) When the emission reduction level (emission reduction rate $\epsilon$) of a producer decreases or the internal sharing force (contribution ratio $\phi$) within the supply chain increases, the producer’s profit increases. A decrease in the emission reduction rate $\epsilon$ indicates a decrease in the investment cost of emission reduction technology by the producer, and an increase in the contribution ratio $\phi$ further reduces the carbon tax cost pressure on the producer, thereby increasing the producer’s profit.

**Figure 5.** The impact of contribution ratio $\theta$ and emission reduction rate $\epsilon$ on producer’s profit under the carbon reduction technology sharing strategy.

**Figure 6.** The impact of contribution ratio $\phi$ and carbon tax $t$ on producers’ profits under the high-tiered carbon tax cost-sharing strategy.

**Figure 7.** The impact of contribution ratio $\phi$ and emission reduction rate $\epsilon$ on producers’ profits under the high-tiered carbon tax cost-sharing strategy.
5. Summary and Outlook

5.1. Summary

Due to the potential threat of climate change to global economic and social development, exploring appropriate carbon reduction policy tools has become the only way for China to achieve sustainable development. Carbon tax is a powerful incentive measure to reduce carbon emissions and promote the energy revolution. The tiered carbon tax is considered a further optimization of the carbon tax system. Therefore, it is necessary to systematically analyze the emission reduction mechanism of the tiered carbon tax to achieve an ideal balance between socio-economic and environmental goals. This article mainly studied the impact of high-tiered carbon tax policies on supply chain production and emission reduction decisions to summarize the emission reduction mechanisms of high-tiered carbon taxes.

Firstly, based on the Cournot game model, this article explores the emission reduction mechanism of high-tiered carbon tax policies in the green exchange market from the perspective of the production side, namely the single-level supply chain. It summarizes the effects of implementing high-tiered carbon tax policies and analyzes their impact on the single-level supply chain. The research results indicate that there are extreme points in the emission reduction rates of producers in the green exchange market under both the high-tiered carbon tax policy and the unified carbon tax policy. However, in the early and late stages of emission reduction, compared to the unified carbon tax, the high-tiered carbon tax policy can create greater profit space for producers actively engaged in emission reduction; at the same time, in the green exchange market, the high-tiered carbon tax policy can generate the best output for producers engaged in green emission reduction technology exchange to maximize the limited total market profit.

Then, based on the perspective of a two-level supply chain consisting of producers and retailers, this article explores the emission reduction mechanism of high-tiered carbon taxes. It mainly focuses on the optimal decision-making of the two-level supply chain through two sharing strategies of carbon reduction technology investment cost sharing and carbon tax cost sharing and analyzes the impact of the two cost-sharing strategies on balancing the relationship between members of the two-level supply chain under the high-tiered carbon tax policy. The results show that the two cost-sharing strategies can effectively alleviate the cost burden of producers, increase their marginal profits, and promote further improvement of emission reduction; at the same time, under the cost-sharing strategy for carbon emission reduction technology investment, a high-tiered carbon tax will enable producers to expand their production space by increasing their own emission reduction rates, and the higher the green sensitivity of consumers, the greater the production space of producers. Under the carbon tax cost-sharing strategy, the high-tiered carbon tax policy will incentivize producers to research and develop disruptive emission reduction technologies to generate greater profit space in the carbon tax cost-sharing with retailers; in addition, the cost-sharing strategy for carbon reduction technology in the early stage of emission reduction is superior to the carbon tax cost-sharing strategy, and the implementation of the carbon tax cost-sharing strategy in the later stage of emission reduction may lead to breakthrough changes in emission reduction technology.

5.2. Outlook

This article aimed to study the emission reduction mechanism of high-tiered carbon taxes and analyze the emission reduction mechanism of high-tiered carbon taxes based on single-level supply chain and two-level supply chain models. However, due to various factors, the results obtained have certain limitations and further improvements are needed: (1). The operability of the conclusion still needs to be improved. For research convenience, this article has made several assumptions during model construction, which to some extent weaken the complexity of real-world research. Further research should consider more complex conditions to expand the applicability of research conclusions.
Further detailed research on the emission reduction mechanism of high-tiered carbon taxes is needed. This article mainly systematically explored the emission reduction mechanism of high-tiered carbon taxes and future research should delve into the emission reduction mechanism of high-tiered carbon taxes in different carbon emission departments and regions.

More efficient standards for dividing high-tiered carbon taxes still need to be explored. This article mainly used carbon emissions as the classification standard for high-tiered carbon taxes, but further research is needed to explore whether there is a more effective classification standard.

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