

Article

Probabilistic Design Method for Aircraft Thermal Protective Layers Based on Surrogate Models

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Abstract: In this study, a probabilistic method was proposed for an aircraft's thermal protective layers. The uncertainties of material properties, geometric dimensions, and incoming flow environments were considered for the design inputs. To accelerate the design efficiency, Latin hypercube sampling and surrogate models were built based on finite element method calculations to enhance the simulation efficiency. Thus, the Monte Carlo method can be implemented with such a fast simulation method and produce a massive number of samples for the uncertainty quantification and sensitivity analysis, exploring their impact on the back temperature of the thermal protection layer. Compared to the deterministic method with the extreme deviation design, the probabilistic design yields a weight reduction of 15.61%. This indicates that probabilistic design is an efficient approach to enhance the performance of aircraft and reduce the overall weight of the aircraft. The general goal of this study is to provide a new design method for the coating film of thermal protection systems by considering multiple sources of uncertainties.

Keywords: probabilistic design; thermal protection system; surrogate models; uncertainty



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1. Introduction

The concept of Martian immigration has received more and more attention and discussion in recent years. The repeatable Mars launcher is the key technology to realize this concept. It sends the Mars probe into a predetermined orbit and then re-enters the atmosphere. In this process, it will experience severe aerodynamic heating. The thermal protection system (TPS) is one of the most critical systems for the aircraft of Mars launcher, ensuring the structural integrity of the aircraft in high-temperature and harsh aerodynamic thermal environments. The reliability and lightweight design of the thermal protection system are crucial for the aircraft. Thermal protective layers are one of the most essential parts of the TPS of the aircraft.

In the design process of the thermal protection system, the most critical issue is to rationally design the structural form, heat-resistant materials, and their dimensions while reducing the system weight to ensure the thermal insulation effect. Various uncertainties, such as flight path deviation, data measurement errors, material property dispersion, system processing and assembly deviations, and the external load environments are inevitable in the actual design process [1–3]. In the robust thermal protection system design, traditional methods usually assume that all factors that may affect aircraft safety occur simultaneously without considering the likelihood of such occurrences [4–7]. Within the current trend of lightweight and integrated development of thermal protection systems, this design approach is excessively conservative and ineffective. Probabilistic design methods quantitatively assess random uncertain parameters using a probabilistic approach. Given uncertain input information, these methods estimate the uncertainty of output responses. Probabilistic design methods can effectively reduce system weight while ensuring reliability [8].

Researchers have studied the probabilistic design approach for thermal protection systems in great detail in the past few years. Howell et al. [9] used the Monte Carlo method to consider parameter uncertainty in heat conduction and compared it with traditional worst-case design methods. This method can more reasonably set design margins and safety factors, reduce design conservatism, and lay the foundation for probabilistic design with uncertainty. Mazzaracchio et al. [10] proposed a statistical method based on Monte Carlo technology. An uncertainty and sensitivity analysis was conducted on the initial dimensions of the aircraft ablation thermal protection system to estimate the confidence level attributed to the selected insulation layer thickness. Chen et al. [11] conducted a Monte Carlo sensitivity analysis and uncertainty analysis on the thermal protection system of high-speed aircraft. To determine the size margin of the thermal protection system, the probability relationship between the thickness margin of the protection system and the temperature of the underlying material was obtained. Weaver et al. [12] studied the effects of inflow velocity, temperature, density, and collision coefficient on the aerodynamic heat and flow field of aircraft using CFD numerical simulation methods. Dec et al. [4] coupled a physics-based analysis of flight trajectory, incoming flow velocity, temperature, density, and materials into a Monte Carlo analysis to accurately quantify system margins. The impact of materials, incoming flow velocity, and flight trajectory on determining the size of thermal protection systems were examined.

NASA has researched the probabilistic design of the ablative thermal protection system for the Mars Science Laboratory (MSL) [13,14]. This research considers uncertainties in re-entry trajectory, aerothermal environment, and material properties when entering the Martian atmosphere. It proposes a probabilistic-based method for designing the thickness of the thermal protection coating. Numerical simulations and analyses indicate that, with the adoption of the probabilistic design, the thickness of the thermal protection system can be reduced from 2.29 inches to 1.39 inches, achieving a 40% decrease and yielding significant benefits. Researchers have also conducted research on ablation analysis considering uncertainties [15,16]. Deng et al. [17] have presented a thermal reliability assessment method based on importance sampling, effectively addressing the issue of the low efficiency of Monte Carlo methods in analyzing low failure probability problems. Zhang et al. [18] have characterized the stochastic process of aerothermal loads and, considering three categories of uncertainties including thermal loads, material properties, and geometric parameters, established a surrogate model for predicting the system's thermal response and conducted a probabilistic analysis of a multilayer nonablative thermal protection system. Havey et al. [19] combined aerothermal analysis with the ballistic forming process to determine the shape of the incoming trajectory based on the temperature characteristics of the thermal protection system and studied the influence of ballistic parameters on the minimum thermal load of the aircraft. Bose et al. [20] proposed Monte Carlo uncertainty and sensitivity analysis techniques to study the effects of reaction rate constants, vibration chemical coupling parameters, vibration relaxation time, and transport properties on aerothermal heat. Hollis et al. [21] performed Navier–Stokes calculations on the proposed Mars smart lander, extracted the boundary layer edge characteristics, and made aerothermal predictions for the Mars smart lander.

The study by Villanueva et al. [22] introduces a dynamic design space partitioning approach for optimizing problems with expensive evaluations and multiple local optima, emphasizing the identification of multiple local optima throughout the design process. Guo et al. [23] concentrate on the thermomechanical optimization of metallic thermal protection systems under aerodynamic heating conditions. Kim et al. [24] investigate the computational simulation of lightning strikes on aircraft and the design of lightning protection systems. Reeve et al. [25] propose a framework for incorporating uncertainty analysis into the conceptual design and evaluation of aircraft thermal management systems, employing Monte Carlo methods to address probabilistic inputs of crucial system performance metrics. Kciuk et al. [26] delve into the design and modeling of intelligent building offices and thermal comfort based on probabilistic neural networks. Meanwhile, Cao et al. [27]

present a novel methodology for aircraft engine performance reliability design using deep neural network (DNN)-based surrogate models. Together, these papers provide a comprehensive theoretical foundation and practical guidance for the probabilistic design of aircraft thermal protection systems.

Predicting the aerodynamic heating that the aircraft will experience is the first step in designing the thermal protective layer system. There are currently many obstacles in the way of accurately predicting aerodynamic heating through numerical calculations and wind tunnel tests; however, engineering algorithms offer effective prediction techniques. Coupling engineering algorithms with finite element thermal analysis will further improve the accuracy of engineering algorithms and provide more accurate temperature fields for the thermal protective layers of the aircraft.

This study compared traditional and probabilistic designs for a three-dimensional thermal protection layer for a Mars launcher aircraft using single-layer high-temperature-resistant insulation coating materials. The finite element method and aerothermal engineering algorithms were used in this approach. The probabilistic method mainly quantifies the uncertainties of material properties, geometric dimensions, and incoming flow environments. The Latin hypercube sampling method was used to generate samples. Sensitivity analysis based on the Monte Carlo method and the Sobol index method was conducted. The results of this study can rapidly and accurately provide preliminary design solutions for the thermal protection layers of the aircraft.

2. Model and Methods

2.1. Finite Element Model Verification

In this work, the model of the aircraft for Mars launching is a common three-stage rocket. The specific geometric dimensions of the spacecraft are shown in the figure below. As shown in Figure 1, the typical geometric model of the spacecraft is simplified into five parts: the nose cone, the cones, and the three-stage cylinders.

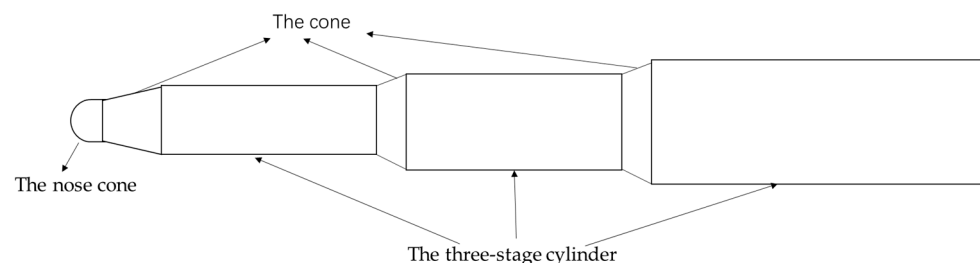


Figure 1. Simplified geometric model.

Considering the different engineering algorithms of the thermal environment for different geometric shapes, as shown in Figure 1, the typical geometric model of the spacecraft is simplified into five parts: the nose cone, the cones, and the three-stage cylinders (as shown in Figure 2). The different divisions were used for the simulation of the finite element method (FEM).

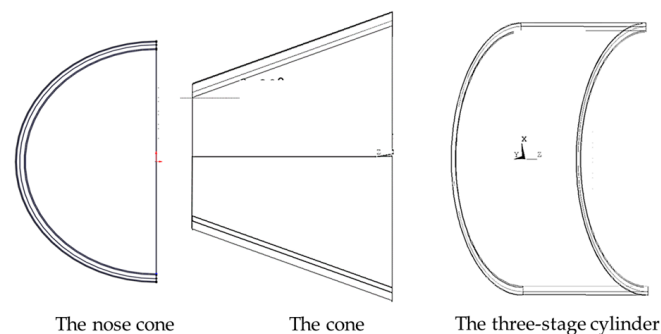


Figure 2. Different divisions of geometric model for FEM simulation.

Engineering algorithms are relatively simple and effective, with high prediction accuracy for commonly used geometric shapes. Various engineering algorithms for aerodynamic heating have been developed. Based on the basic equation of the high-speed boundary layer of an axisymmetric body, the Fay–Riddell formula provides the stagnation point heat flux of a three-dimensional blunt-headed axisymmetric body under laminar boundary layer, thermochemical equilibrium, and fully catalytic wall conditions. The stagnation point heat flux was calculated by the following equation:

$$q_s = 0.763 \text{Pr}^{-0.6} \left(\frac{\rho_w \mu_w}{\rho_s \mu_s} \right)^{0.1} \left(\rho_s \mu_s \frac{du}{dx} \right)^{0.5} \left(1 + (Le^{0.52} - 1) \frac{h_d}{h_s} \right) (h_s - h_w) \quad (1)$$

Here, Pr is the Prandtl number, Le is the Lewis number, Rn is the radius of the nose cone, h is enthalpy, and the subscripts w and s in the equation represent the physical quantities at the wall and stagnation point, respectively.

For the cone, using the modified Newtonian pressure distribution and isentropic outflow conditions, the heat flux density on the surface of the cone is deduced by the following equation:

$$\frac{q_{wl}}{q_{ws}} = A(\theta_c) \frac{\frac{x'}{R_N}}{\left[B(\theta_c) + \left(\frac{x'}{R_N} \right)^3 \right]^{0.5}}, \quad (2)$$

where x' is the distance along the surface measured from the vertex of the cone.

$$A(\theta_c) = \left(\sin^2 \theta_c - \frac{\sin^2 \theta_c + 1}{\gamma_\infty Ma_\infty^2} \right)^{\frac{1}{2}} \cdot \sqrt{\frac{\pi}{2} - \theta_c} \cdot \frac{\sqrt{3}}{2} \quad (3)$$

$$B(\theta_c) = \frac{\frac{3}{16}}{\sin^2 \theta_c \left[\sin^2 \theta_c - \frac{\sin^2 \theta_c - 1}{\gamma_\infty Ma_\infty^2} \right]} \cdot \left[\frac{D(\theta)}{\theta} \right]_{\theta=\frac{\pi}{2}-\theta_c} - \cot^3 \theta_c \quad (4)$$

$$\frac{q_{wl}}{q_{ws}} = A(\theta_c) \frac{\frac{x'}{R_N}}{\left[B(\theta_c) + \left(\frac{x'}{R_N} \right)^3 \right]^{0.5}} \quad (5)$$

The heat flux density of a nonstationary circular plate can be calculated using the flat plate turbulence formula as follows:

$$q_{ox} = 0.332 \rho_e u_e \text{Pr}^{-2/3} \left(\frac{\rho^* \mu^*}{\rho_e \mu_e} \right)^{0.5} \text{Re}_x^{-0.5} (h_r - h_w) \quad (6)$$

For each division of the simplified modes, a heat conduction model is used for heat transfer design and analysis. The simplified heat conduction model is shown in Figure 3.

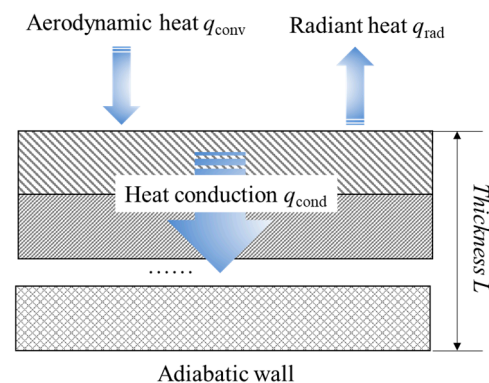


Figure 3. Simplified model of heat transfer for TPS.

In the FEM simulation, the heat conduction equation can be expressed as follows:

$$\rho c \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x} \left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial T}{\partial z} \right) \quad (7)$$

The outside boundary conditions for aerodynamic heating on the outside surface wereset as

$$k_x \frac{\partial T}{\partial x} n_x + k_y \frac{\partial T}{\partial y} n_y + k_z \frac{\partial T}{\partial z} n_z = h(T_a - T) \quad (8)$$

Neglecting the thermal radiation effect on the lower surface, the inner surface was set as an adiabatic wall with boundary conditions of

$$k_x \frac{\partial T}{\partial x} n_x + k_y \frac{\partial T}{\partial y} n_y + k_z \frac{\partial T}{\partial z} n_z = 0 \quad (9)$$

In the simulation calculation process, the initial distribution of thermal protection temperature is generally set as uniform

$$T(x, y, z, 0) = T^0(x, y, z) \quad (10)$$

where ρ is the material density; c is the specific heat capacity of the material; τ is time; k_x , k_y , and k_z are the thermal conductivity of the material along the three main directions of the object; n_x , n_y , and n_z are the direction cosine of the normal outside the boundary; h is the convective heat transfer coefficient; and T is the temperature and also a function of time and coordinates.

The aerothermal load is an essential input for the design of the thermal protective layers of the aircraft. The NASATND-5450 report provides detailed research and extensive experiments on the sphere-cone-shaped warhead, offering a substantial amount of reliable experimental data for aerothermal calculations. The warhead model was taken from the report as the research object. In this study, a verification analysis of the computational methods for aerothermal engineering was conducted with experimental data from the NASATND-5450 report as the basis. The relative errors were within 5%, which indicated the accuracy of the stagnation point heat flux density obtained in this work. The simulated values of the cone obtained using engineering algorithms were also compared with experimental data with all errors within 10%. The experimental values are listed in Table 1.

Table 1. Comparison of engineering algorithms and experimental values.

Region	Calculated Value	Experimental Value	Error
Stagnation heat flux (kW/m ²)	693.8	670	3.55%
	227.08	215.7	5.2%
Blunt cone heat flux ratio	0.1302	0.122	6.7%
	0.063287	0.0664	4.68%
	0.058	0.0636	8.86%
	0.0537	0.0581	7.54%

2.2. Deterministic Simulation Modeling

The simulation model was first designed with the traditional design and analysis methods for the thermal protection system. Figure 1 gives the main steps of the simulation. The specific process was as follows:

- The surface thermal loads on the aircraft, based on flight altitude, speed, atmospheric environment, navigation parameters, etc., were determined.
- The structural forms and material types were determined based on the external surface heat flux distribution and internal surface temperature limits used in various parts of the thermal protection system for the aircraft.

- (c) With the determined basic structures and materials of the thermal protection system, the dimensions (mainly thickness) of the thermal protective layers were determined by heat transfer analysis as follows:

$$h(x, y, z) = f(Q(t), MAT) \quad (11)$$

Here, $h(x, y, z)$ is the heat flux passing through the protective structure, $Q(t)$ is the aerothermal input, and MAT is material properties.

In the analysis of the temperature field for the thermal protection structure, the back temperature limits of the thermal protection system were performed.

- (d) Considering the impact of uncertainties on the thermal protection performance, a reasonable safety margin was necessarily set on the nominal thickness to obtain the final dimensions and weight of the thermal protection layer. The design should meet the back temperature and weight limits.
- (e) After the preliminary design of the thermal protection system, an assessment and verification was conducted.

In this paper, during the preliminary design, classic empirical formulas were used for the aerodynamic heat calculation, as shown in Figure 4. Heat flux data were calculated from the incoming flow data per second.

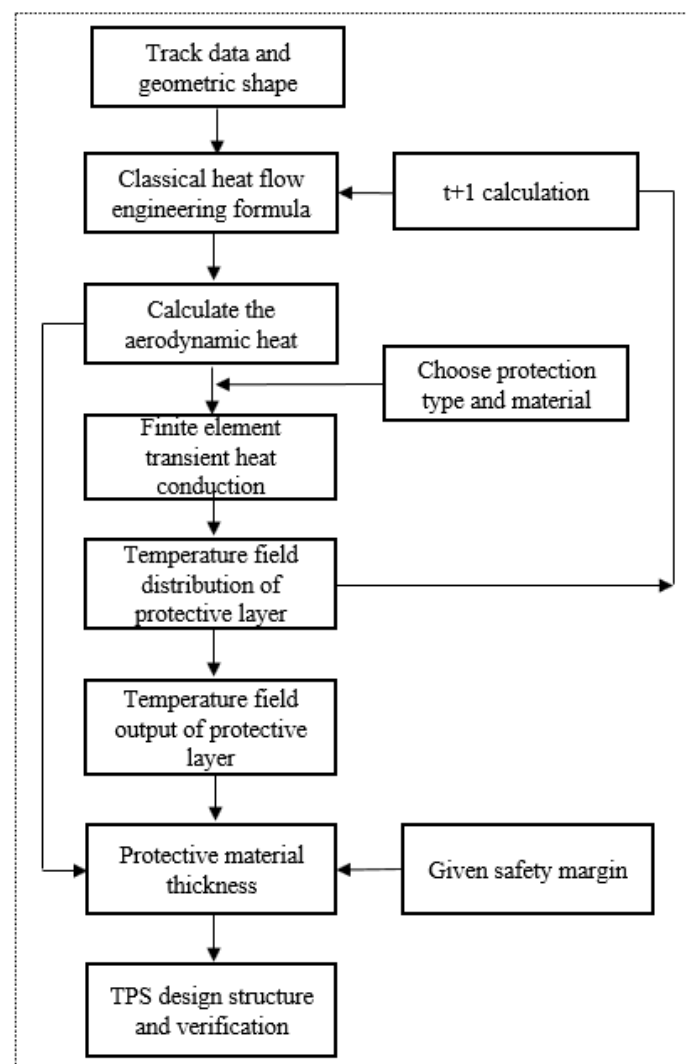


Figure 4. Deterministic design process diagram.

The layers for the aircraft were designed as a common three-stage rocket. Considering the heat flux calculation for the aerodynamic heat, the aircraft is divided into five different regions: the stagnation point region, the spherical region (assuming the central angle 30° – 90°), the cone region, and the first-, second-, and third-stage cylindrical regions. Each region's maximum heat flux values were calculated by adopting a relatively conservative design approach to preserve safety margins for the next stage of the TPS design. These calculated values were then applied to the surfaces of the corresponding geometric areas. The resulting heat flux curves for the stagnation point of the spherical head, the spherical head surface, the cone, and the cylindrical plates are shown in Figure 5.

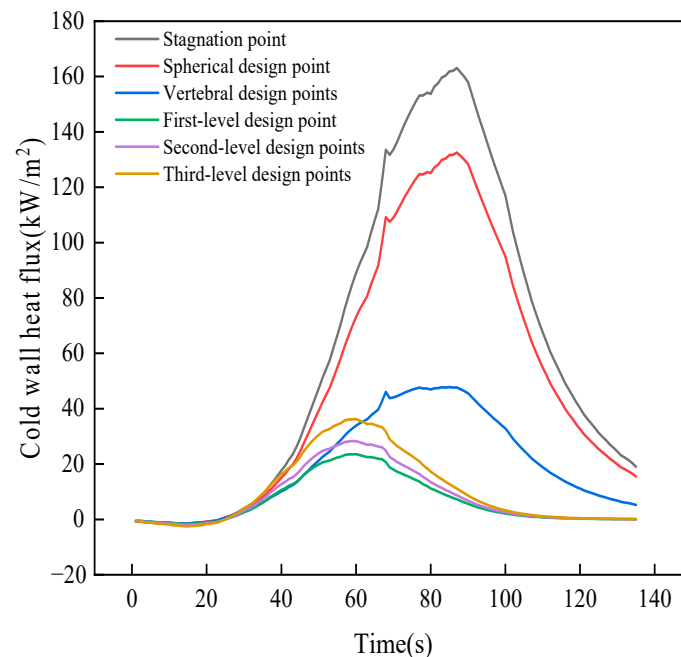


Figure 5. Maximum heat flux density curves in different regions.

2.3. Probabilistic Simulation Modeling

Various uncertainties, such as flight trajectory deviations, material property variations, analysis model errors, system assembly deviations, external load environments, and other unknown uncertainties were considered. The probabilistic design and reliability assessment process for aircraft TPSs are illustrated in Figure 6, which includes three main steps: deterministic design, probabilistic design, and reliability assessment.

Firstly, initial geometric dimensions and material properties were determined in the deterministic design for the thermal protection system.

Secondly, the uncertainties, regarding their sources and distributions, were introduced into the analysis model in the probabilistic design process.

Subsequently, finite element analysis or surrogate models (such as response surface methods) were used for thermal reliability analysis.

During the probability analysis, it is crucial to identify the uncertainty sources, ranges, and distributions and to estimate their parameters reasonably. The uncertainty parameters are usually assumed to follow a normal distribution with given mean values and standard deviations.

The next step involves incorporating these uncertainty parameters into the analysis model, usually achieved through finite element parametric modeling with the engineering practice.

Subsequently, the probability parameters of the system temperature field were obtained through Monte Carlo simulation. Typically, the highest temperature ($T_{\max}$) of the aircraft's cold structure is selected as the output variable in the design process.

$$T_{\max} = \Gamma(s_1, s_2 \dots) \quad (12)$$

In this equation, Γ represents the mapping relationship between random inputs and outputs, while $s_1, s_2, \dots$ represent random input variables.

During the probabilistic design process, defining a reasonable reliability range is essential. If the system meets the reliability requirements, such as the back temperature of the aircraft, the design process concludes. Otherwise, the controllable parameters of the system were adjusted, and a reassessment of reliability was conducted. It should be noted that excessively high system reliability also indicates a deviation from the design requirements, suggesting an oversized margin. In such cases, dimension adjustments should be required to reduce the system weight.

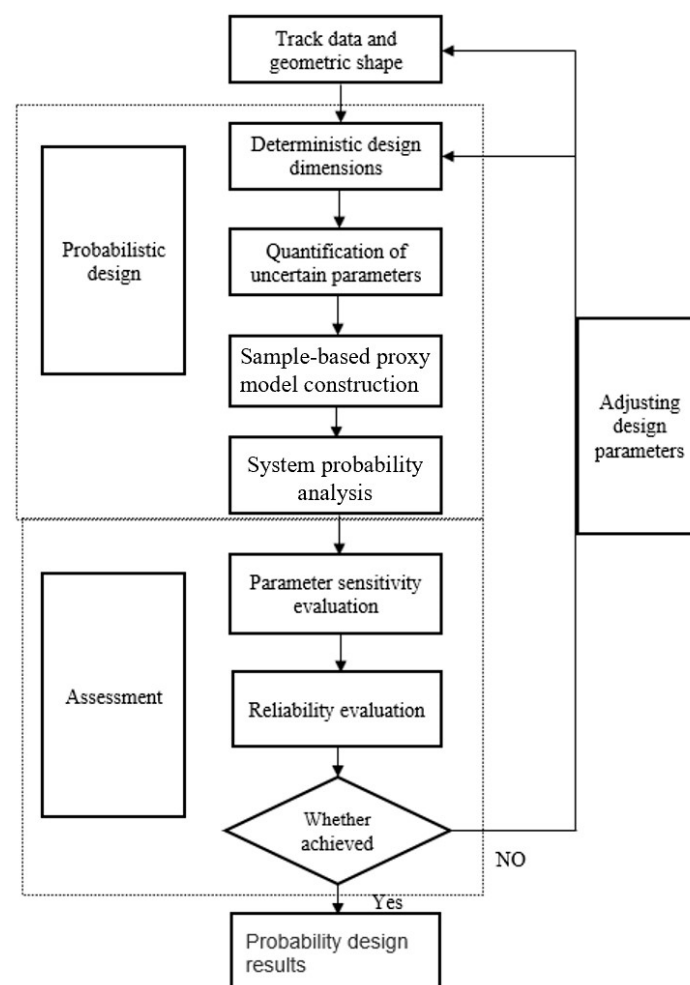


Figure 6. Probability design flowchart.

The finite element model proposed in this work was validated using the heat flux data from the reference [28]. A comparison graph of the back temperature curve was obtained through simulation by ANSYS Fluent 17.0. The maximum error of the comparison does not exceed 1.35%, and the simulation results fit well with the data in the reference [28]. The comparison is presented as shown in Figure 7.

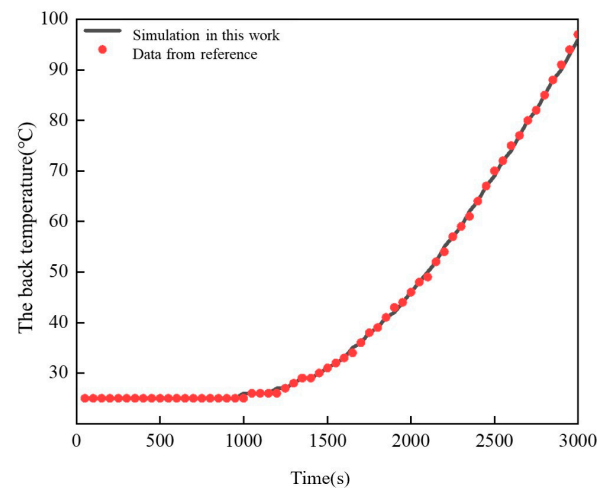


Figure 7. Validation of model in this work.

3. Results and Discussion

The differences between deterministic and probabilistic design methods were compared in this work.

3.1. Deterministic Determination

The traditional TPS employs two methods for determining margin. The first method involves the superposition of worst-case scenarios, where all parameters were set to the worst-case values, and the required layer thickness of the TPS was calculated. The second method was the square root of the sum of squares (RSS) approach, summarizing the uncertainties mentioned above to calculate the necessary thickness.

In this work, the deterministic model was first established to give the initial values of the TPS layers. All the uncertainties were selected based on the worst-case scenario for the limitations of the back temperature for the TPS. Finite element analysis was then performed. The initial dimensions of the aircraft thermal protection layer are listed in Table 2.

Table 2. Initial dimensions of deterministic design based on the worst-case.

Input Parameter	Variable Name	Unit	Mean Value
Thickness of ball head stagnation region	D_1	mm	8
Thickness of nonstationary area of ball head	D_2	mm	8
Cone area thickness	D_3	mm	7
Thickness of the third-level cylindrical region	D_4	mm	5
Thickness of the second-level cylindrical region	D_5	mm	5
Thickness of the first-level cylindrical region	D_6	mm	5

The engineering calculation method that has been verified was used for the determination of aerodynamic heat. In the calculation, the initial temperature was set as 27 °C. The thermal flux was obtained for the first-stage region. The heat flux was then applied to the surface of the thermal protection layer. Finite element transient heat conduction analysis was performed to obtain the temperature field distribution. The process was iteratively repeated to obtain the temperature history curve for the interface of the TPS's protective layer over the first 135 s of the launcher. The temperature history curves for each stage region are illustrated in Figure 8.

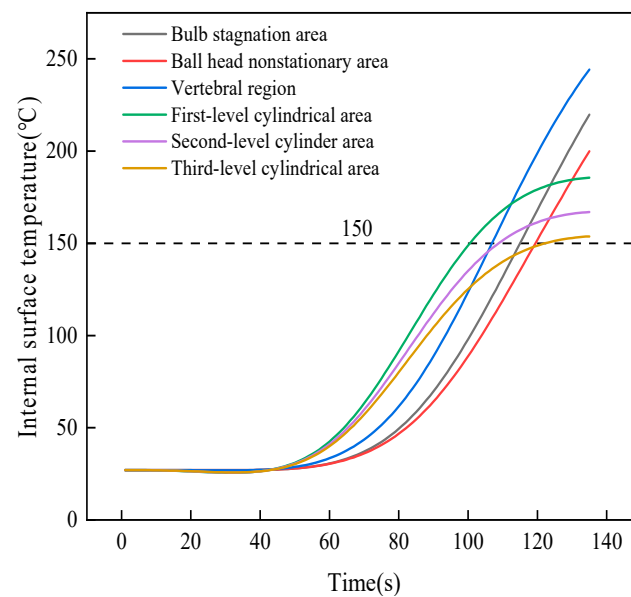


Figure 8. Initial surface temperature history curve.

The tolerance temperature was set at 150 °C for the safety of the onboard equipment, represented as the black dashed line in Figures 8 and 9, which indicates the temperature limit on the aircraft's cold structure. If the maximum of the calculated temperature on the inner surface exceeds the limit temperature line, it is necessary to redesign the thermal protection scheme. This can be achieved by increasing the thickness of the coating. Since the back temperature is monotonically related to the layer thickness of the TPS, an iterative design process using finite element analysis (FEM) was performed combined with an optimization algorithm. The process continued until the inner surface temperature dropped below the temperature limitation. As seen in Figure 9, the maximum back temperature of the design was precisely 150 °C and not less than 149.5 °C.

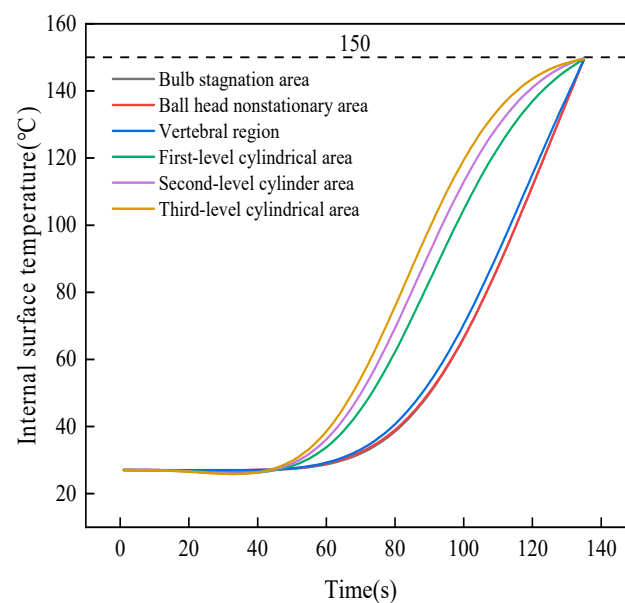


Figure 9. Time history curves of optimal thickness corresponding to back temperature.

The optimal thickness of the optimally designed coating layer of the TPS determined with the deterministic method is shown in Table 3.

Table 3. Optimal thickness of the coating layer of TPS for the traditional design.

Input Parameter	Variable Name	Unit	Mean Value
Thickness of ball head stagnation region	D_1	mm	9.3
Thickness of nonstationary area of ball head	D_2	mm	8.97
Cone area thickness	D_3	mm	8.83
Thickness of the third-level cylindrical region	D_4	mm	6.05
Thickness of the second-level cylindrical region	D_5	mm	5.55
Thickness of the first-level cylindrical region	D_6	mm	5.15

3.2. Probabilistic Designation

Based on the probabilistic design process mentioned in Section 2.3, the deterministic model is established for finite element analysis with each uncertainty at its mean value, as shown in Table 4. According to reliability requirements, the uncertainties, including flight trajectory deviations, material property variations, analysis model errors, system assembly deviations, and external load environments, were taken into account. The thermal protection system was set to achieve a reliability of 99.999%, with the internal system tolerating a temperature setting at 150 °C in the deterministic method.

Table 4. Uncertainty parameters for the probabilistic designation.

Input Parameter	Variable Name	Distribution Type	Mean Value	Standard Deviation, %
Thickness of ball head stagnation region	D_1 , mm	Gaussian distribution	8	1
Thickness of nonstationary area of ball head	D_2 , mm		8	1
Cone thickness	D_3 , mm		7	1
First-level cylinder thickness	D_4 , mm		5	1
Secondary cylinder thickness	D_5 , mm		5	1
Third-level cylinder thickness	D_6 , mm		5	1
Coating density	DENS, kg m ⁻³	Gaussian distribution	560	1
Coating specific heat capacity	C , J kg ⁻¹ K ⁻¹		1510	1
Coating thermal conductivity	k , W m ⁻¹ K ⁻¹		0.1	1
Coating emissivity	EMIS	Uniform distribution	0.8	1
Flight speed coefficient	X_{SU}		1	1
Incoming flow density coefficient	X_{SM}		1	1
Incoming temperature coefficient	X_{ST}		1	1
Ball head radius	Rn , m		1	1
Planck number	Pr		0.721	0.679
Lewis number	Le		1.442	1.358
Lewis number weight	α		0.5356	0.5044

The uncertainty in the aerodynamic heat calculations was from uncertainties in atmospheric environmental parameters and errors introduced during the database interpolation process. The uncertainty in the design related to the trajectory is caused mainly by orbit, orbital maneuvering, and the uncertainty in recognizing characteristic numbers in the formulas. In many cases, these uncertainties can only be estimated rather than precisely determined.

With the deterministic analysis, a rough estimate of the influence on the temperature by various uncertainties was made, and the preliminary design size parameters were determined. According to reliability requirements, considering various uncertainties, the TPS of the aircraft should achieve a reliability of 99.999% with the tolerated temperature of 150 °C for a safe environment of equipment inside the aircraft. The design parameters were incorporated into a deterministic model using parametric modeling. The parametric model was established by considering the uncertainties from various factors mentioned above. The maximum back temperature per second was taken as the output parameter. The sample calculations were performed by Latin hypercube sampling. Three sets of normal

distribution samples with a standard deviation of 0.05 were generated using the Latin hypercube sampling method, with a total of 100 samples in each group. The sample results were fitted to a response surface, and simulations were carried out 100,000 times using the response surface. The accuracy of model fitting is mainly measured by the goodness of fit R^2_{Adj} and relative errors of L_2 . R^2_{Adj} is the correlation coefficient excluding the items included in the regression equation, and the better model fit is

$$R^2_{Adj} = 1 - \left[\left(\frac{SS_{residual}}{df_{residual}} \right) / \left(\frac{SS_{residual} + SS_{model}}{df_{residual} + df_{model}} \right) \right] \quad (13)$$

where $SS_{residual}$ is the sum of the squared residuals, SS_{model} is the sum of the squared terms of the model, and $df_{residual}$ and df_{model} represent the degrees of freedom of the residual and the degrees of freedom of the model, respectively. The relative error L_2 , which is the degree of difference between two sets of numbers, is calculated by

$$L_2 = \left\| (y_{pred} - y_{true}) / y_{true} \right\|^2, \quad (14)$$

where y_{pred} is the predicted or estimated value, y_{true} is the true value, and $\| \cdot \|^2$ denotes the L_2 paradigm (Euclidean paradigm).

In this work, the effects of the number of training samples and the number of polynomials on the fitting results of the response surface agent models were also considered. A total of four different response surfaces were constructed, namely, linear polynomial response surfaces for the training data $N = 10$ and $N = 15$ and nonlinear polynomial (quadratic) response surfaces for the training data $N = 10$ and $N = 15$. The goodness of fit and relative errors are given in Table 5.

Table 5. Results of fitting accuracy of the response surfaces.

Parameters	Linear Polynomial Response Surfaces		Nonlinear Polynomial Response Surfaces	
	$N = 10$	$N = 15$	$N = 10$	$N = 15$
R^2_{Adj}	0.991800	0.993251	0.999713	0.999966
L_2	0.011842	0.011021	0.002216	0.000787

From Table 5, the values of R^2_{Adj} and L_2 are optimal for the nonlinear polynomial (quadratic) response surface with $N = 15$, indicating that better results can be obtained with more training data and higher order polynomials. However, linear polynomials with less data can also give good prediction results. In addition, the amount of data or the order should be kept in a proper range to avoid problems such as overfitting.

The probability distribution curve of the maximum temperature parameter (T_{max}) of the inner surface was then determined from the statistical method. The finite element method (FEM) was used in conjunction with optimization algorithms. The optimal thickness is continuously sought and plotted as a curve, as shown in Figure 10.

In the probabilistic designation, the increase in coating thickness was employed to enhance system reliability. The reliability of the TPS was set as 99.999% with the consideration of various uncertainties, which meets the design requirements of the aircraft. The dimensions of the design with the probabilistic method are listed in Table 6 as follows.

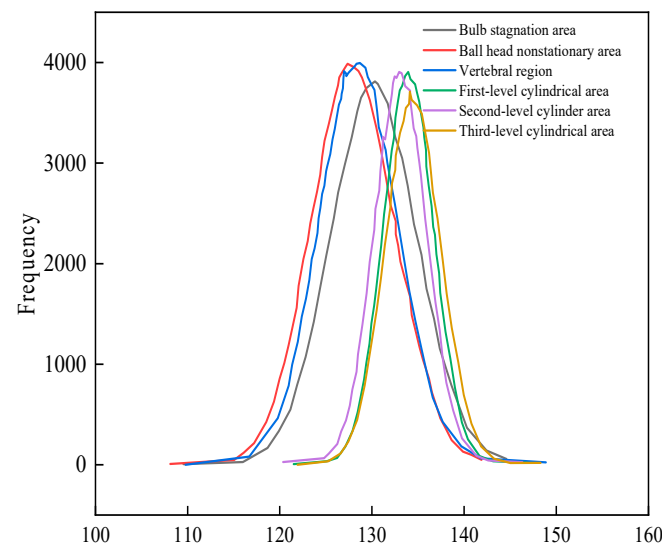


Figure 10. Probability distribution of optimal thickness back temperature for probability design.

Table 6. Optimal dimensions for system probability design.

The Designed Parameter of Coating Layers	Variable Name	Unit	Mean Value
Thickness of ball head stagnation region	D_1	mm	8
Thickness of nonstationary area of ball head region	D_2	mm	7.75
Cone area thickness	D_3	mm	7.55
Thickness of the third-level cylindrical region	D_4	mm	5.1
Thickness of the second-level cylindrical region	D_5	mm	4.7
Thickness of the first-level cylindrical region	D_6	mm	4.3

Following the consideration of the supposed dispersity of uncertainties in the third-stage region and its impact on the thermal reliability of the system, different standard deviations of uncertainty factors, namely, 2.5% and 5%, were set for the probabilistic designation. The results in Table 7 indicate that with an increase in the dispersity of uncertainties, the system's reliability decreases from 99.9999% to 80.6177%. Therefore, to enhance the thermal reliability of the system, actions such as controlling machining precision and reducing machining errors should be taken, especially for the parameters that significantly affect the effectiveness of thermal protection.

Table 7. Impact of standard deviation of uncertainty on reliability of probabilistic designation.

Region	Standard Deviation of Uncertainty, %	Mean Back Temperature, °C	Standard Deviation of Back Temperature, °C	Reliability
Third-level circular plate	1	134.04	3.2599	99.9999%
region	2.5	135.8	8.1811	95.5589%
	5	136.05	16.33	80.6177%

For the improvement of the design of the TPS layers, a sensitivity analysis of the designing parameters was carried out. The sensitivity analysis was performed for input parameters as listed in Table 4. As the plate heat flux calculation formula does not include parameters, such as the Lewis number, the weight, and the head radius, these three parameters were not included in the analysis. The sensitivity results, shown in Figure 11,

show that the flight speed has the most significant impact on the back temperature of the TPS, which accounted for 31.24% and exhibited a positive correlation. This indicates that higher parameter values lead to higher maximum temperatures in the lower layer, resulting in poorer thermal protection system performance. In descending order of importance, the remaining parameters were the coating thickness, the coating specific heat capacity, the coating density, and the coating thermal conductivity. The other parameters were not sensitive to changes in the back temperature of the TPS.

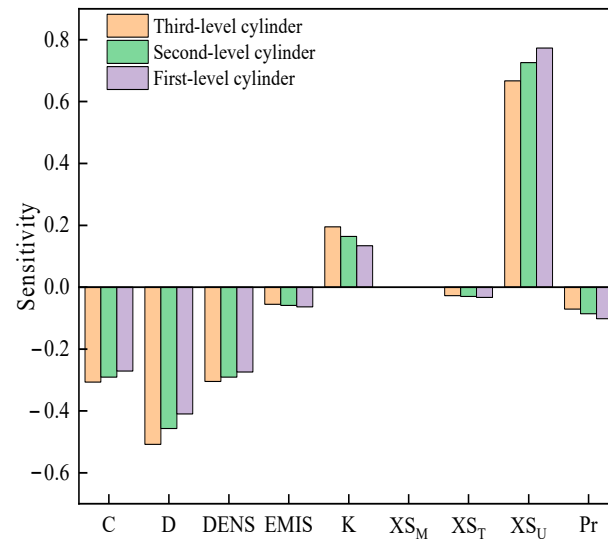


Figure 11. Sensitivity of cylindrical stagnation area.

A similar sensitivity analysis was also conducted for the head and cone stagnation areas, with results presented in Figure 12. The results show that coating thickness has the most significant negative impact, with a proportion of 30.02%. This implies that higher values of the coating thickness lead to lower maximum temperatures, indicating better thermal protection system performance. In descending order of importance, the other parameters were the flight speed, the coating-specific heat capacity, the coating density, the coating thermal conductivity, and the Prandtl number. The other parameters were not sensitive to changes in the back temperature of the TPS.

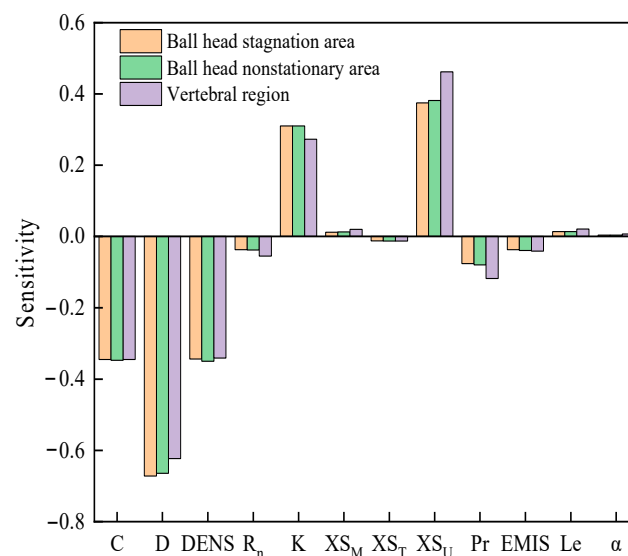


Figure 12. Sensitivity of the ball head and cone region.

3.3. Comparison

The weight results obtained from the traditional deterministic design process and the probabilistic design approach were compared. The weight of the coating layers of the TPS that was determined through the conventional deterministic design was 271.74 kg, while the probabilistic design method yielded a weight of 229.31 kg. The probabilistic design approach demonstrates a weight reduction of 15.61% compared with the traditional deterministic design. This significant reduction in weight highlights the efficiency of the probabilistic design method in enhancing the performance of the aircraft and minimizing the overall weight of the vehicle.

The probabilistic method allowed a more comprehensive consideration of uncertainties and variations in the designation, which led to a more optimized and reliable TPS. The results emphasized the advantages of the probabilistic method in the design and analysis of the TPSs of aircraft, contributing to improved performance and weight efficiency advancements.

4. Conclusions

In this work, a deterministic and probabilistic designation for the coating layers of TPSs was conducted for the aircraft of the Mars launcher. The main conclusions are as follows:

1. The developed engineering algorithm, combined with computational fluid dynamic (CFD) simulation methods, exhibits high accuracy.
2. The weight of the coating layer of the TPS obtained through the deterministic method is 271.74 kg with the extreme deviation design method, while the weight derived from the probabilistic method is 229.31 kg. Compared to the deterministic method with the extreme deviation design, the probabilistic design yields a weight reduction of 15.61%. This indicates that probabilistic design is an efficient approach to enhance the performance of aircraft and reduce the overall weight of the aircraft.

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