

Article

A Novel Method to Calculate Water Influx Parameters and Geologic Reserves for Fractured-Vuggy Reservoirs with Bottom/Edge Water

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Abstract: In practical oilfield production, the phenomenon of water influx typically shortens the water-free recovery period of wells, leading to water flooding and causing a sharp decline in the production well yields, bringing great harm to production. Water invasion usually occurs as a result of the elastic expansion of the water as well as the compaction of the aquifer pore space. However, it can be due to the special characteristics of fractured-vuggy reservoirs such as non-homogeneity and the discrete distribution of the pore spaces. It is challenging to use traditional seepage flow theories to analyze the characteristics of water influx. Also, reservoir numerical simulation methods require numerous parameters which are difficult to obtain, which significantly reduces the accuracy of the results. In this study, considering the driving energy for water influx, a water influx characteristic model was obtained by fitting a graph plate. Subsequently, an iterative calculation method was used to simultaneously obtain water influx volume and OOIP. The aquifer to hydrocarbon ratio was determined by fitting the water influx curve with the graphic plate. Results show that the calculation method is sensitive to the values of reservoir pressure and the crude oil formation volume factor. After applying the method to one field case, it was discovered that water influx performance can be characterized into two types, i.e., linear water influx and logarithmic water influx. In the early stages, the water influx rate of logarithmic water influx is greater compared to linear water influx. However, the volume and energy of waterbody are limited, and the water invasion phenomenon occurs almost exclusively within a short period after the invasion. On the other hand, the volume of waterbody invaded by linear water influx is larger, and it can maintain a stable rate of water influx. The results of the study can provide theoretical support for the waterbody energy evaluation and dynamic analysis of water influx, as well as the control and management of water in these types of reservoirs.

Keywords: water influx; material balance; OOIP; aquifer to hydrocarbon ratio



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1. Introduction

More than half of the global oil and gas resources are stored in carbonate reservoirs, which, therefore, has become one of the most important research targets in the oil and gas industry [1–4]. Among these carbonate reservoirs, fractured-vuggy carbonates are one of the most significant types, and they are widely distributed worldwide [5–7]. For example, fractured-vuggy carbonate reservoirs with large reserves have been discovered in China's Tarim Basin, such as the Harahatang area, Fumang oil field, and Shunbei oil field [8–11].

However, the production of fractured-vuggy carbonate reservoirs is quite different from the conventional sandstone reservoirs. Moreover, two fractured-vuggy carbonate

reservoirs in the same block can behave differently [12,13]. For some fractured-vuggy carbonate reservoirs, they are connected to a waterbody. During the production process, water invasion readily occurs, which obviously will influence the production [14,15]. Therefore, it is of significance to master the dynamics of water influx.

To date, studies on water invasion have focused on homogeneous reservoirs [16–19]. However, due to the random distribution of the pore space, fractured-vuggy carbonate reservoirs are quite different from homogeneous reservoirs [20]. This renders some conventional methods inapplicable.

In the past, the calculation of water influx in oil and gas reservoirs often relied on analytical methods. Based on whether the water influx rate changes with time, the seepage characteristics and conditions of water invading the reservoir, and the different data required for calculation, methods for calculating water influx can be divided into steady-state models [21,22], unsteady-state models [23,24], and pseudo-steady-state models [25]. The abovementioned methods require static geological parameters of the reservoir and water to select appropriate models for calculation. However, these parameters are often difficult to obtain, leading to challenges in implementing the calculation methods mentioned above. Additionally, using analytical methods to calculate water influx also presents issues such as complex formulas, tedious processes, and large computational requirements.

Numerical simulation methods require obtaining a detailed geological model and introducing numerous difficult-to-determine parameters. Normal geological exploration methods are similarly difficult to apply [26,27]. This is particularly challenging for fractured-vuggy carbonate reservoirs with highly heterogeneous and spatially discrete distributions.

Therefore, employing material balance methods eliminates the need to consider the geometric shape of the reservoir and does not require the physical parameters of the reservoir rock. It relies solely on dynamic production data, measured static pressure data, and necessary fluid analysis information to estimate OOIP and predict water influx [28–30]. Some scholars have studied the phenomenon of water invasion in oil and gas reservoirs through material balance equations. Chen Jun et al. [31] use the difference between the P/Z of a water-driven gas reservoir and a closed gas reservoir to calculate water influx and OGIP. Yue Shijun et al. [32] combined the principles of material balance with fluid percolation theory to calculate the water influx and dynamic reserves of sandstone gas reservoirs. However, the strong heterogeneity and random spatial distribution of fissure-type reservoirs make traditional percolation theories difficult to apply. Tan Xiaohua et al. [33] established a material balance method, considering water influx phenomena and obtained characteristic diagrams for water-influenced gas reservoirs. Based on the existing research, Xiong Yu et al. [34] proposed a new material balance equation which is applicable to condensate gas reservoirs. They considered the differences of composition between produced and injected fluids and the effect of water influx. Additionally, Yong Li et al. [35] have conducted identification and prediction of water invasion phenomena in fractured-vuggy carbonate reservoir.

However, there is relatively limited application of the characteristics of fractured-vuggy carbonate reservoirs with bottom/edge water in material balance equations. Making the material balance method applicable to oil reservoirs, computing water influx characteristic parameters without percolation theory, and determining specific parameters such as water influx rate, OOIP, and aquifer to hydrocarbon ratio using only dynamic production data and readily available parameters remain challenging issues in current research.

This paper applies the equations of material balance based on the mechanism of water invasion due to pressure drop in the waterbody caused by pressure drop in the reservoir, compaction of the pore space, and elastic expansion of water in the waterbody. Unlike analytical methods that require many parameters that are difficult to obtain, involve cumbersome processes, and demand extensive calculations, the new calculation method that combines the theory of water invasion elasticity with the material balance equation is not only applicable to fractured-vuggy reservoirs but also requires only readily available production data. By using a simple iterative calculation method, one can accurately

obtain key parameters such as water influx rate, OOIP, and aquifer to hydrocarbon ratio simultaneously. Through this method, we derive a graphical representation to illustrate how water influx volume varies with cumulative oil production across different aquifer to hydrocarbon ratios, grounded in the mechanisms of water influx and considering factors such as pore space reduction and water expansion in a waterbody. Afterwards, water influx characteristic models of the target reservoir were obtained. Subsequently, based the material balance equation, we established an iterative calculation method to calculate the water influx rate and OOIP, fitting the water influx curve with the graph plate to solve the aquifer to hydrocarbon ratio. Finally, a set of novel computational methods has been developed to simultaneously solve multiple water influx characteristic parameters. This efficient and straightforward calculation method offers feasible applications for actual oilfield operations. And our results provide theoretical support for the water influx dynamic analysis and waterbody energy evaluation, as well as water control and management of fractured-vuggy carbonate oil reservoirs with active bottom/edge water.

2. Establishment of Water Influx Characteristic Model

If there are bottom or edge waters connected to a fractured-vuggy carbonate reservoir, the water invasion could be caused by the reduction of pore space and the water expansion in the waterbody, when the fluid pressure declines as production proceeds (as shown in Figure 1). The combined effects of the contraction of rock pores and the elastic expansion of the water in the waterbody cause the occurrence of water invasion.

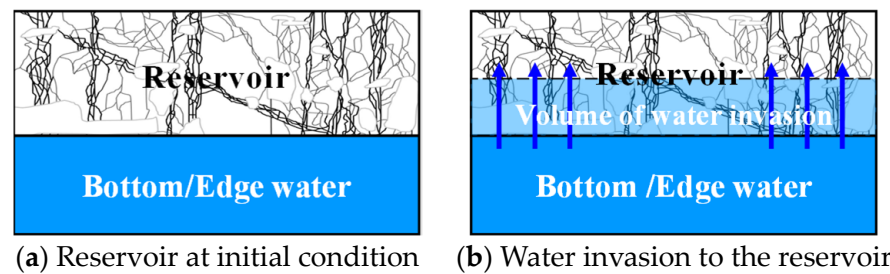


Figure 1. Schematic of water invasion as fluid pressure declines.

In this section, based on the fundamental principles of water invasion described above, the relationship between net water influx and the cumulative oil production under various aquifer water/hydrocarbon ratios will be derived. In addition, by fitting the curves in a graph plate, the relationship between the water influx and the cumulative oil production is obtained. From this, the water influx characteristic model is also established.

2.1. Calculation of Pore Space Reduction Volume and Water Expansion Volume in Waterbody

The isothermal water compressibility coefficient is defined as relative volume change as response to pressure changes.

$$C_w = \frac{1}{V_w} \frac{\Delta V_w}{\Delta p}, \quad (1)$$

where C_w is isothermal compression factor of water, MPa^{-1} ; V_w is water volume in the waterbody, Δp is stratigraphic pressure change, MPa; and ΔV_w is the volume change of water in the waterbody when the stratigraphic pressure changes by Δp .

The differential form of the above equation is

$$C_w = \frac{dV_w}{V_w dp}. \quad (2)$$

At the initial condition, the formation pressure p is represented by p_i ; water volume in the waterbody V_w is represented by V_{wi} . By integrating both sides of the equation, the following equation is obtained:

$$\int_p^{p_i} C_w dp = \int_{V_w}^{V_{wi}} \frac{1}{V_w} dV_w. \quad (3)$$

The integral is obtained by taking the logarithm of both sides;

$$V_w = V_{wi} \cdot e^{C_w(p_i-p)} \quad (4)$$

When pressure declines from initial pressure p_i to p , the water volume changes in the waterbody due to expansion can be derived, as shown in the following equation:

$$\Delta V_w = V_w - V_{wi} = V_{wi} [e^{C_w(p_i-p)} - 1]. \quad (5)$$

Similarly, the rock pore compressibility coefficient is defined below.

$$C_p = -\frac{1}{V_p} \frac{\Delta V_p}{\Delta p}, \quad (6)$$

where C_p is pore compression factor, MPa^{-1} ; V_p is pore volume in the waterbody, Δp is stratigraphic pressure change, MPa ; and ΔV_p is the volume change of the pore in the waterbody when the stratigraphic pressure changes by Δp .

The differential form of the above equation is

$$C_p = -\frac{1}{V_p} \frac{dV_p}{dp}. \quad (7)$$

Same as above, p_i and V_{pi} used here indicate the formation pressure and waterbody pore volume at the initial condition. Integrating both sides of the above equation give rise to

$$\int_p^{p_i} C_p dp = -\int_{V_p}^{V_{pi}} \frac{1}{V_p} dV_p. \quad (8)$$

Taking the logarithm of the above equation, the waterbody pore volume at the current V_p is derived as

$$V_p = V_{pi} \cdot e^{-C_p(p_i-p)}. \quad (9)$$

The reduced waterbody pore volume ΔV_p can be calculated using the following equation:

$$\Delta V_p = V_p - V_{pi} = V_{pi} [1 - e^{-C_p(p_i-p)}]. \quad (10)$$

As is well known, as production proceeds, the formation pressure declines from p_i to p ; subsequently, water in the waterbody invades into the reservoir. And the water influx comprises two components, i.e., the water volume caused by the waterbody volume expansion ΔV_{we} and the water volume caused by the waterbody pore volume reduction ΔV_{pe} . Here, it is assumed that the waterbody pore space is fully saturated with water, and the volume is represented by V_e . Then, the following two equations are derived:

$$\Delta V_{we} = V_e [e^{C_w(p_i-p)} - 1] \quad (11)$$

$$\Delta V_{pe} = V_e [1 - e^{-C_p(p_i-p)}]. \quad (12)$$

2.2. Derivation of the Relationship between the Net Water Influx and Cumulative Oil Production

As stated previously, the total water influx W_e can be written as

$$W_e = \Delta V_{we} + \Delta V_{pe} = V_e \left[e^{C_w(p_i - p)} - e^{-C_p(p_i - p)} \right]. \quad (13)$$

By combining the mechanism of water influx with elasticity theory, a relationship has been established between the water influx rate and the pressure drop in fractured-vuggy oil reservoirs. However, since waterbody pore space V_e is difficult to obtain, the concept of aquifer to hydrocarbon ratio has been introduced.

The aquifer–hydrocarbon ratio n is defined as the ratio of waterbody pore volume to hydrocarbon pore volume,

$$n = \frac{V_e}{V_a}. \quad (14)$$

Substituting Equation (14) into Equation (13) gives rise to

$$W_e = n \cdot V_a \left[e^{C_w(p_i - p)} - e^{-C_p(p_i - p)} \right]. \quad (15)$$

Consequently, a relationship has been established between the water influx rate and the pressure drop in fractured-vuggy reservoirs under different aquifer to hydrocarbon ratios. Furthermore, initial pore volume V_a can be calculated from the OOIP (original-oil-in-place) N as shown below:

$$V_a = \frac{NB_{oi}}{1 - S_{wi}}, \quad (16)$$

where N is OOIP, m^3 ; B_{oi} is the crude oil volume factor at the initial state; and S_{wi} is the initial water saturation.

Substituting Equation (16) into Equation (15) gives the following relationship:

$$W_e = n \cdot \frac{NB_{oi}}{1 - S_{wi}} \cdot \left[e^{C_w(p_i - p)} - e^{-C_p(p_i - p)} \right]. \quad (17)$$

Subtract the produced water volume from above, and then the net water influx is derived.

$$W_e - W_p B_w = n \cdot \frac{NB_{oi}}{1 - S_{wi}} \cdot \left[e^{C_w(p_i - p)} - e^{-C_p(p_i - p)} \right] - W_p B_w, \quad (18)$$

where W_p is the produced volume at the surface condition, and B_w is the water formation volume factor.

Equation (18) can also be written as

$$\frac{W_e - W_p B_w}{NB_{oi}} = \frac{n}{1 - S_{wi}} \cdot \left[e^{C_w(p_i - p)} - e^{-C_p(p_i - p)} \right] - \frac{W_p B_w}{NB_{oi}}. \quad (19)$$

Here, aquifer to hydrocarbon ratio n , initial water saturation S_{wi} , OOIP N , and initial oil formation volume factor B_{oi} are constants. Therefore, net water influx volume is a function of the well production. In this way, Equation (19) can be represented by the following equation:

$$W_e - W_p B_w = f(N_p), \quad (20)$$

where N_p is the cumulative oil production.

Thereby, curve fitting was performed between the net water influx and cumulative oil production data. It was found that the relationship falls into the following two categories (represented by Equations (21) and (22)):

$$W_e - W_p B_w = f(N_p) = a \cdot N_p \quad (21)$$

$$W_e - W_p B_w = f(N_p) = a \cdot \ln N_p + b, \quad (22)$$

where a and b are both constants.

In addition, the curve fitting results are shown in Figure 2, and the correlation coefficient R^2 are above 0.9.

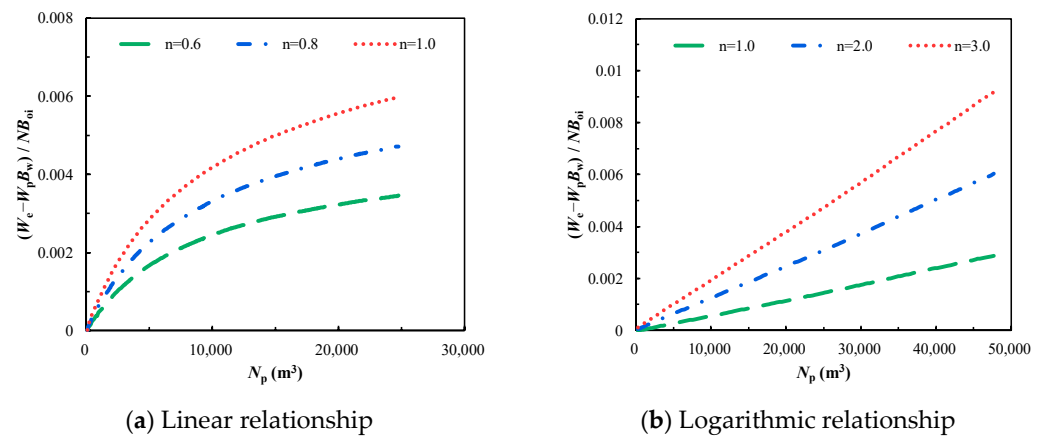


Figure 2. Curve fitting results between net water influx and cumulative oil production.

3. Calculation of Water Influx and OOIP

3.1. Establishment of Material Balance Equation

For fractured-vuggy carbonate reservoirs with an active waterbody, natural energies that drive oil production not only come from oil expansion and elastic drive of bottom/edge water but also from the expansion of the original water in the pore space and the energy released by the deformation of the reservoir rock. This is especially true for ultra-deep fractured-vuggy carbonate reservoirs with high reservoir pressure. Therefore, the material balance equation of fractured-vuggy carbonate with edge/bottom water is written as [30,36,37]

$$N_p B_o = N(B_o - B_{oi}) + W_e - W_p B_w + N B_{oi} \frac{C_w S_{wi} + C_f}{1 - S_{wi}} (p_i - p), \quad (23)$$

where N_p represents the cumulative oil production, m^3 ; B_o represents oil formation volume factor, m^3/m^3 ; N represents original oil in place, m^3 ; B_{oi} represents initial oil formation volume factor, m^3/m^3 ; W_e represents cumulative water influx, m^3 ; W_p represents cumulative water production, m^3 ; B_w represents water formation volume factor, m^3/m^3 ; C_w represents isothermal compression factor of water, MPa^{-1} ; S_{wi} represents irreducible water saturation, dimensionless; C_f represents formation rock compressibility, MPa^{-1} ; p_i represents initial reservoir pressure, MPa ; and p represents average reservoir pressure, MPa .

Dividing both sides of the equation by $(N \cdot B_{oi})$ gives the following equation:

$$1 - \frac{C_w S_{wi} + C_f}{1 - S_{wi}} (p_i - p) - \frac{W_e - W_p B_w}{N B_{oi}} = \frac{B_o}{B_{oi}} \left(1 - \frac{N_p}{N} \right). \quad (24)$$

As per the definition of the oil formation volume factor, it refers to the ratio of the volume of crude oil at reservoir conditions to the volume of oil at standard conditions.

$$B_o = \frac{V_o}{V_s}, \quad (25)$$

where V_o represents the volume of oil at reservoir conditions, and V_s represents the volume of oil at standard conditions.

Equation (25) can be rewritten as

$$\frac{V_o}{V_{oi}} \cdot \frac{V_{oi}}{V_s} = \frac{V_o}{V_d} \cdot \frac{V_d}{V_{oi}} \cdot B_{oi}, \quad (26)$$

where V_{oi} represents the oil volume at the initial reservoir pressure, V_d represents the oil volume at the bubble point pressure, and B_{oi} is the oil formation volume factor at initial conditions. And the following relationship is derived:

$$B_o = \frac{V_o}{V_d} \cdot \frac{V_d}{V_{oi}} \cdot B_{oi}. \quad (27)$$

By conducting laboratory PVT analysis, the initial oil formation volume factor B_{oi} and the relationship between pressure and oil volume can be determined. Therefore, the oil formation volume factor can be expressed as a function of the reservoir pressure.

$$\frac{B_o}{B_{oi}} = g(p). \quad (28)$$

Substituting Equation (28) into Equation (24) yields

$$1 - \frac{C_w S_{wi} + C_f}{1 - S_{wi}} (p_i - p) - \frac{W_e - W_p B_w}{N B_{oi}} = g(p) \left(1 - \frac{N_p}{N} \right). \quad (29)$$

Dividing both sides of the equation by $(p_i - p)$ gives the following equation:

$$\frac{1}{p_i - p} \cdot \frac{N_p}{N} \cdot g(p) - \frac{W_e - W_p B_w}{N B_{oi}} \frac{1}{p_i - p} - \frac{C_w S_{wi} + C_f}{1 - S_{wi}} = \frac{1}{p_i - p} g(p) - \frac{1}{p_i - p} \quad (30)$$

Combined with the relationship between net water influx and cumulative oil production obtained in the previous section, it can be derived that

$$\frac{1}{p_i - p} \cdot [g(p) - 1] = \frac{1}{N} \cdot \left[g(p) N_p - \frac{f(N_p)}{B_{oi}} \right] \cdot \frac{1}{p_i - p} - \frac{C_w S_{wi} + C_f}{1 - S_{wi}}. \quad (31)$$

The above equation can be regarded as a linear equation in the form of

$$y = m \cdot X - b, \quad (32)$$

if assuming

$$Y = \frac{1}{p_i - p} \cdot [g(p) - 1] \quad (33)$$

$$m = \frac{1}{N} \quad (34)$$

$$X = \left[g(p) N_p - \frac{f(N_p)}{B_{oi}} \right] \cdot \frac{1}{p_i - p}. \quad (35)$$

Thereby, the OOIP N can be calculated from the slope m as shown in Equation (34).

Furthermore, by analyzing a large body of actual field data, researchers have found that there is a correlation between the sum of the effective compressibility and the net water influx and the cumulative oil production [37]. Here, we use λ to represent the correlation coefficient λ , and then it gives the following equation:

$$\frac{C_w S_{wi} + C_f}{1 - S_{wi}} (p_i - p) + \frac{W_e - W_p B_w}{N B_{oi}} = \lambda \cdot N_p. \quad (36)$$

Substituting Equation (36) into Equation (29) yields

$$1 - \lambda \cdot N_p = g(p) \left(1 - \frac{N_p}{N} \right). \quad (37)$$

The above equation can also be written as

$$\lambda = \frac{1}{N_p} - g(p) \left(\frac{1}{N_p} - \frac{1}{N} \right). \quad (38)$$

Substituting actual pressure and production data into Equation (38) yields a series of λ . According to Lajda's criterion, the outliers are eliminated. Then the mean and the standard deviation of λ are calculated as

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}. \quad (39)$$

To guarantee the accuracy of our calculation, the λ with the values outside the range of $[\lambda - \sigma, \lambda + \sigma]$ are eliminated. The remaining λ are used to calculate the average λ^* , and then Equation (36) becomes

$$\frac{C_w S_{wi} + C_f}{1 - S_{wi}} (p_i - p) + \frac{W_e - W_p B_w}{NB_{oi}} = \lambda^* \cdot N_p. \quad (40)$$

Combining Equations (20) and (40) gives the following equation:

$$\frac{C_w S_{wi} + C_f}{1 - S_{wi}} (p_i - p) + \frac{f(N_p)}{NB_{oi}} = \lambda^* \cdot N_p. \quad (41)$$

Equation (41) can also be rewritten as

$$f(N_p) = \left[\lambda^* \cdot N_p - \frac{C_w S_{wi} + C_f}{1 - S_{wi}} (p_i - p) \right] NB_{oi}. \quad (42)$$

3.2. Equation Solving

As described in Section 2.2, the relationship between the net water influx and cumulative oil production falls into two categories, i.e., linear and logarithmic relationships. For these two types, the equation solving methods are similar. Here, we use a logarithmic type as an example.

Substituting Equation (22) into Equation (42) gives the following equation:

$$a \ln N_p + \left\{ b - \left[\lambda^* \cdot N_p - \frac{C_w S_{wi} + C_f}{1 - S_{wi}} (p_i - p) \right] NB_{oi} \right\} = 0 \quad (43)$$

Regression with the well production data is performed to obtain the coefficients a and b . Then the net water influx is calculated using Equation (22).

In summary, more detailed steps employed in this study to calculate the OOIP N and net water influx volume ($W_e - W_p B_w$) are described below (also illustrated in Figure 3):

- ① Assign estimated values to coefficients a and b based on experience. Here $a = 3000$, $b = -20,000$ are used initially. Based on production data, Y and X are calculated using Equations (33) and (35);
- ② Regress X and Y with a linear relationship. Then the slope m is determined, and OOIP N is calculated using Equation (34);
- ③ Based on production data, a serial of λ is calculated using Equation (38). According to Lajda criterion, some abnormal values for λ are eliminated, and the average value of λ (denoted as λ^*) is calculated;
- ④ Perform logarithmic regression according to Equation (43), and new values for coefficients a and b are obtained;
- ⑤ Return to step ① and replace a and b with new values obtained from step ④ Repeat steps ①–④ until the values for a and b converge;

⑥ Substituting the obtained a and b from step ⑤ and N into Equation (22) to calculate the net water influx.

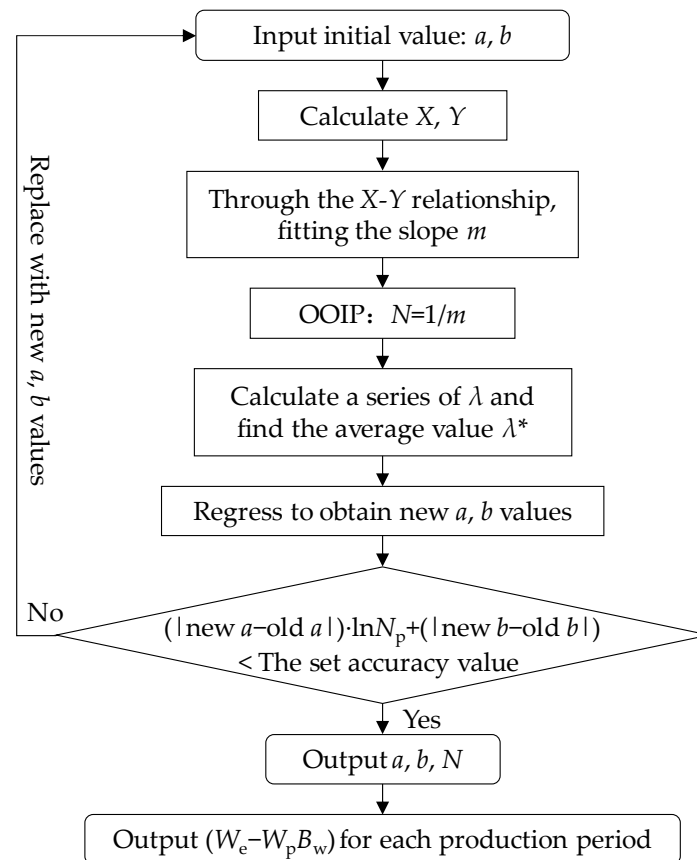


Figure 3. Flow chart for calculating OOIP and net water influx.

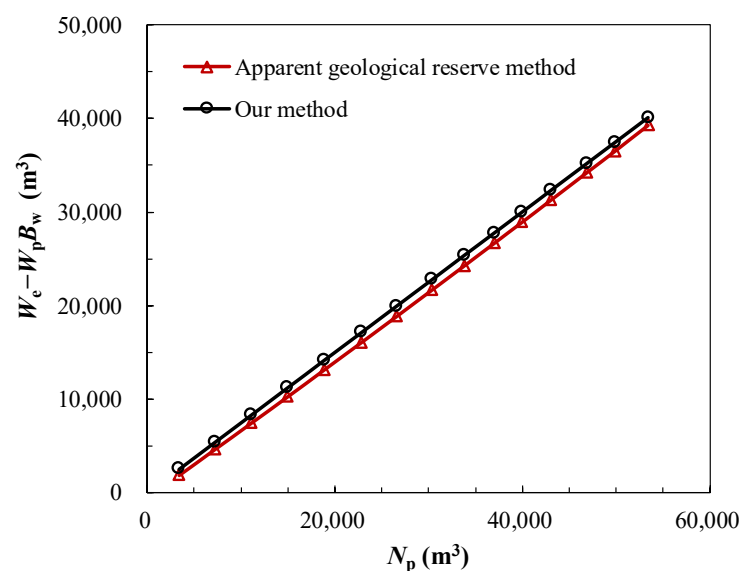
This novel method is applicable under the following conditions: (1) When reservoirs, blocks, or individual production wells are under the same formation pressure system. (2) In addition to the initial reservoir pressure, at least two stratigraphic hydrostatic gradient test points are required. (3) The more stratigraphic hydrostatic gradient test points, the higher the accuracy of the test results, and the higher the degree of reserve recovery, the more reliable the results of this method are for the calculation of OOIP and the water influx rate. (4) The selected dynamic production data must be time-continuous. For production downtime resulting from well shutdowns, missing values must be processed, for example, by interpolation, completions, or deletion of missing points. (5) This novel method is sensitive to the value of initial reservoir pressure and the test result of oil formation volume factor, so the convergence of the iterative calculation is affected by the above sensitivity factors to a certain extent.

4. Method Validation

In this section, our developed method is validated by comparing with the already mature apparent geologic reserve method. The example used here is described in the below. At the initial condition, reservoir pressure $p_i = 71.15$ MPa, oil formation volume $B_{oi} = 2.5658$, oil density $= 0.8 \times 10^3$ kg/m³, formation water compressibility is 4.5×10^{-4} MPa⁻¹, the rock compressibility is 1×10^{-4} MPa⁻¹, and the initial water saturation $S_{wi} = 0$. The dynamic data and the calculated water influx using both methods are shown in Table 1 below. Comparison of the calculated net water influx from the apparent geological reserve method and our method is shown in Figure 4.

Table 1. Reservoir production data and the calculated water influx using both methods.

p (MPa)	B_o Dimensionless	N_p (m ³)	W_p (m ³)	Apparent Geological Reserve Method ($W_e - W_p B_w$) (m ³)	Our Method ($W_e - W_p B_w$) (m ³)	Relative Discrepancy (%)
70.82	2.567	3383.17	65.93	1868.52	2537.38	35.80
70.49	2.570	7182.86	143.28	4605.53	5387.15	16.97
70.15	2.572	11,091.32	247.27	7438.20	8318.49	11.83
69.82	2.574	14,909.24	323.10	10,222.23	11,181.93	9.39
69.47	2.576	18,864.04	396.96	13,123.74	14,148.03	7.80
69.13	2.579	22,810.20	486.52	16,036.85	17,107.65	6.68
68.81	2.581	26,528.77	550.13	18,798.32	19,896.58	5.84
68.47	2.583	30,343.55	600.51	21,647.77	22,757.66	5.13
68.18	2.585	33,766.67	635.50	24,218.92	25,325.00	4.57
67.89	2.587	36,996.19	673.29	26,657.00	27,747.14	4.09
67.64	2.588	39,949.24	702.11	28,896.87	29,961.93	3.69
67.37	2.590	43,033.64	732.16	31,247.07	32,275.23	3.29
67.03	2.592	46,869.24	783.10	34,184.92	35,151.93	2.83
66.77	2.594	49,871.80	826.23	36,496.51	37,403.85	2.49
66.46	2.596	53,431.04	896.08	39,250.11	40,073.28	2.10

**Figure 4.** Comparison of net water influx between our method and apparent geological reserve method.

From Table 1 and Figure 4, it can be seen that there is acceptable discrepancy between the apparent geological reserve method and our method. A more careful examination shows that, at the early stages of production, the discrepancy is relatively high, due to the relatively less water influx. However, as production proceeds, the discrepancy becomes negligible.

Furthermore, a plot with the apparent geological reserve N_p^p on the Y-axis and cumulative oil production N_p on the X-axis is shown in Figure 5. When the cumulative oil production $N_p = 0$, OOIP is obtained, OOIPs calculated using apparent geological reserve and our method are 310.15 and 316.67×10^6 m³, respectively. The difference is within 3%. Therefore, it concludes our developed method is reliable.

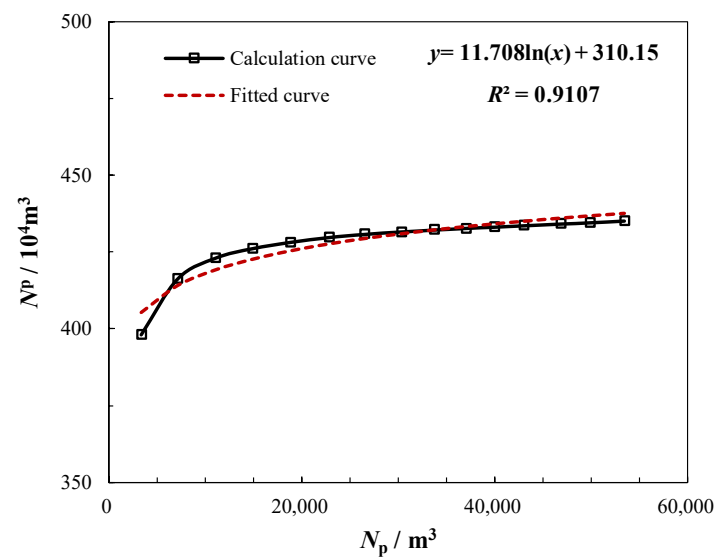


Figure 5. Apparent geological reserves versus cumulative oil production.

5. Application of the Developed Method to a Field Case

In this section, our developed method is applied to our target reservoir M. Table 2 summarizes some collected data for reservoir M. Table 3 lists the production data.

Table 2. Basic data for our target reservoir M.

Parameter	Value	Unit	Parameter	Value	Unit
p_{sc}	0.101325	MPa	B_{oi}	2.463	
T_{sc}	293	K	B_w	1.0344	
p_i	89.8	MPa	C_w	0.00045	MPa ⁻¹
T	423.48	K	C_f	0.0001	MPa ⁻¹
ρ_o	0.8×10^3	kg/m ³	S_{wi}	0	

Table 3. Production data of our target reservoir M.

Date	Cumulative Days of Production (d)	N_p (m ³)	W_p (m ³)	p (MPa)
15 September 2021	0	0	0	89.80
10 October 2021	25	1805.37	46.32	86.58
27 December 2021	50	3551.65	77.52	84.13
21 January 2022	75	5267.72	120.35	82.40
15 February 2022	100	7039.40	134.75	81.10
12 March 2022	125	8781.00	150.71	80.11
11 April 2022	150	10,599.23	166.06	79.27
15 May 2022	175	12,534.98	181.32	78.52
9 June 2022	200	14,598.91	214.4	77.83
4 July 2022	225	16,617.45	241.31	77.25
29 July 2022	250	18,478.59	278.89	76.78
23 August 2022	275	20,234.52	308.94	76.37
17 September 2022	300	21,937.77	344.84	76.01
12 October 2022	325	23,607.90	366.08	75.68
21 November 2022	350	25,123.31	400.86	75.40
16 December 2022	375	26,555.01	424.23	75.16

A plot of reservoir pressure versus cumulative oil production is shown in Figure 6. As can be seen, a sharp pressure drop occurs at the early stage. And in this stage, the

reservoir behaves like a constant volume reservoir without water influx. As production proceeds, water from the waterbody starts to invade into the reservoir, which slows down the reservoir pressure decline.

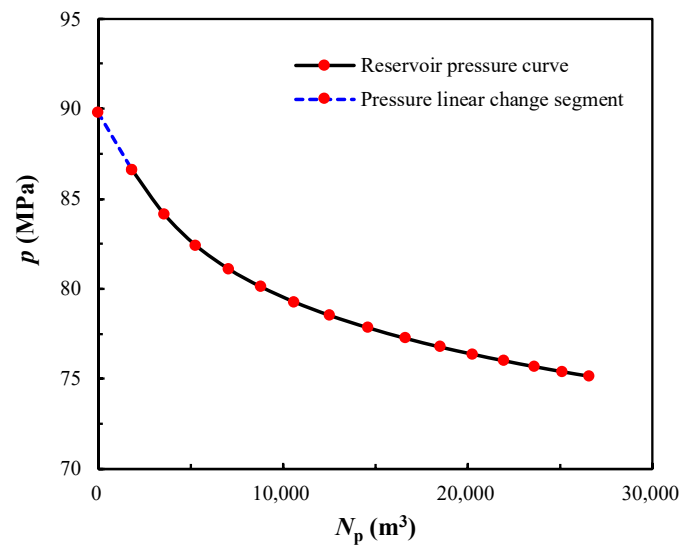


Figure 6. Reservoir pressure versus cumulative oil production.

In addition, from Figure 6, the time when water starts to invade the reservoir is determined, i.e., the time when the decline curve deviates from the linear trend. Based on the water influx conditions of the production wells in the study area, appropriate and different aquifer to hydrocarbon ratios should be set using Equations (21) and (22). Through function fitting, it was found that the curve in Figure 7 exhibits a logarithmic curve similar to that described by Equation (22).

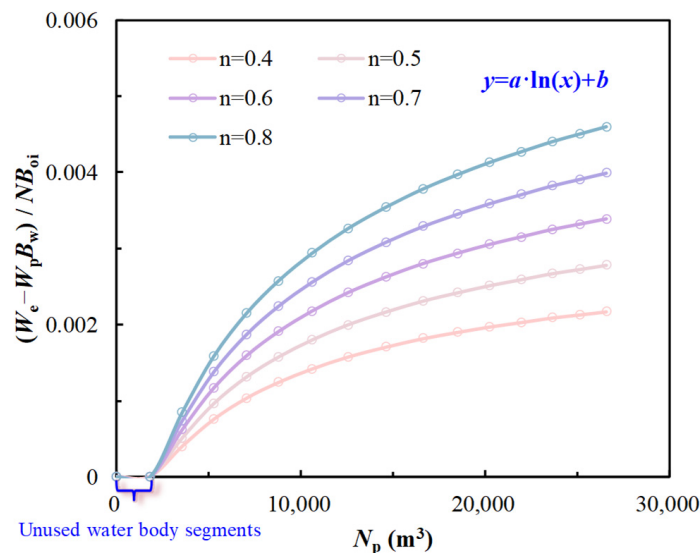


Figure 7. Curve of net water influx versus cumulative oil production.

As shown in Figure 8, the coefficients a and b gradually converge to certain values as the number of iterations increases, using the iterative solution method described in Section 3. Once the required precision is achieved, the values are output, resulting in $a = 2365.8$ and $b = -18,167.8$.

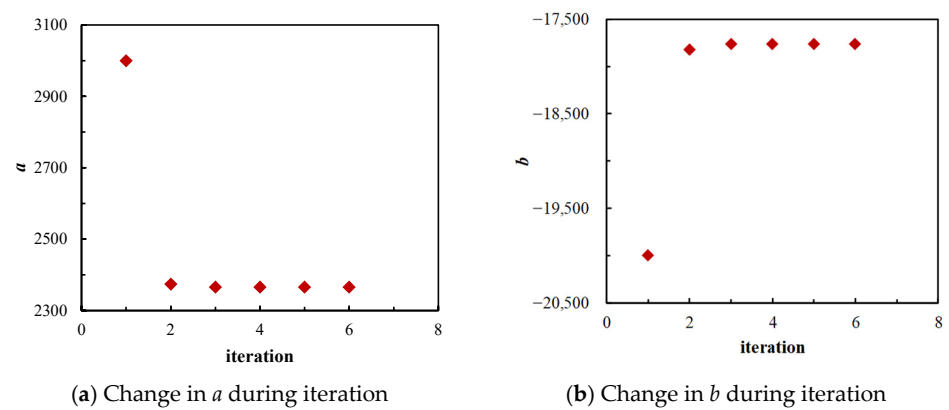


Figure 8. Graph of the change in a and b during the iteration.

After determining the coefficients a and b and substituting them into Equations (33) and (35), the values for X and Y are obtained. As shown in Figure 9, X vs. Y falls into a good linear relationship. According to Equation (34), the OOIP is determined by the reciprocal of the slope of the X – Y curve. After fitting and calculating, the value of N is found to be $692,073 \text{ m}^3$.

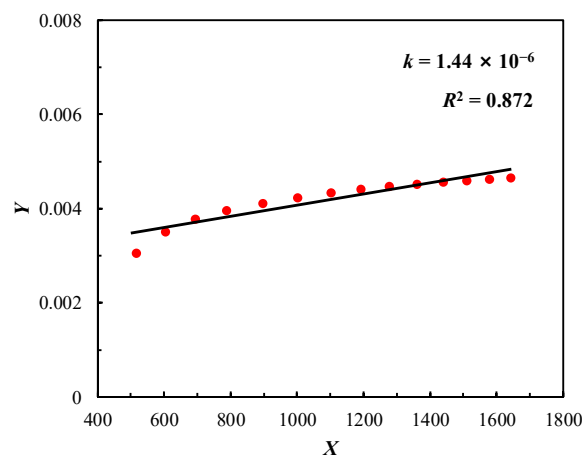


Figure 9. X versus Y .

Through regression, net water influx versus cumulative oil production was fitted into the curve as shown in Figure 10 below.

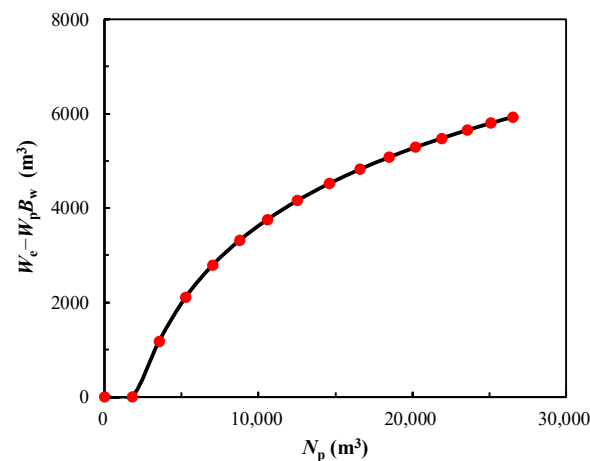


Figure 10. Water influx vs cumulative oil production for M.

Furthermore, to calculate the aquifer to hydrocarbon ratio, the net water influx $(W_e - W_p B_w)$ is converted into the form of $(W_e - W_p B_w) / NB_{oi}$. By fitting the actual production data into the chart, which describes the net water invasion term versus cumulative oil production under different aquifer to hydrocarbon ratios n , as shown in Figure 11 below, it yields $n = 0.6$.

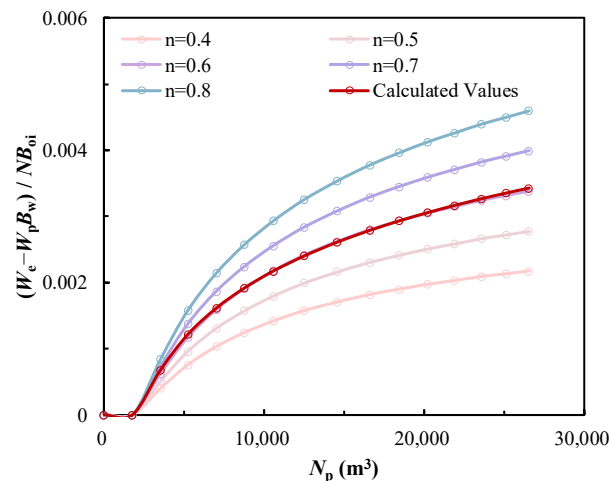


Figure 11. Chart for net water influx versus cumulative oil production for different aquifer to hydrocarbon ratio.

6. Conclusions

In this paper, a novel method based on the mechanism of water influx and the material balance equation is established to simultaneously solve water influx, OOIP, and aquifer to hydrocarbon ratios for fractured-vuggy carbonate reservoirs. After applying it to an actual fractured-vuggy carbonate reservoir with bottom water, the following points are pinpointed:

- (1) The calculation method is sensitive to the values of reservoir pressure and crude oil formation volume factor. Significant errors in reservoir static pressure tests and calculation errors in the oil formation volume factor may lead to the non-convergence of calculation results;
- (2) Water influx for fractured-vuggy carbonate reservoirs with bottom/edge water can be characterized into two types, logarithmic water influx and linear water influx. In the early stages of water influx, the water influx rate of logarithmic water influx often exceeds that of linear water influx;
- (3) For logarithmic water influx wells, the volume of the connected waterbody is relatively small, and the energy of the waterbody is weak. Water influx phenomena typically occur only within a short period after water movement. In contrast, for linear water influx wells, the volume of the connected waterbody is larger, and the energy of the waterbody can sustain a stable water influx rate.
- (4) The determination method of the aquifer to hydrocarbon ratio for fracture-vuggy carbonate reservoirs with bottom water, as well as related research findings on OOIP and the prediction of water invasion patterns, can provide theoretical support for the evaluation of waterbody energy, dynamic analysis of water invasion, and water control and management for such reservoirs.

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