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Decentralized Multi-Area Economic Dispatch in Power Systems Using the Consensus Algorithm

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Abstract: A multi-area power system requires coordination to enhance reliability and reduce operating costs. Economic dispatch in such systems is crucial because of the uncertainties associated with variable loads and the increasing penetration of renewable energy sources. This paper presents a hierarchical consensus algorithm designed to determine the economic dispatch in a multi-area power system, accounting for the uncertainties in load and renewable generation. The proposed algorithm, which utilizes distributed agents, operates across three levels. Level 1 coordinates all areas, while levels 2 and 3 form a leader–follower consensus algorithm for overall economic dispatch. Breadth-first search is employed to identify the leader agent within each area. To address the uncertainties in loads and renewable generation, Monte Carlo simulations are performed. The efficacy of the proposed method is validated using the IEEE 39-bus and 118-bus systems, as well as a realistic 1968-bus power system in Taiwan. The traditional equal lambda method is employed to verify that the proposed approach is suitable for multi-area power systems using distributed computation.

Keywords: breadth-first search; consensus algorithm; decentralized operation; economic dispatch; multi-agent system; uncertainty



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1. Introduction

Power systems are becoming larger because system demand is increasing, as is the penetration of renewable generation. Some technical issues related to the operation of a modern bulk smart grid include the following: (i) the high-dimensionality of parameters and variables results in low computational efficiency; (ii) heavily transmitted data in a wide-area measurement system leads to potential latency; (iii) a cloud-based data warehouse requires high-cost maintenance and is vulnerable to cyber-attacks; and (iv) an independent system operator (ISO) is required to carry out economic dispatch neutrally. The problem of economic dispatch (ED) is the most fundamental and crucial power system-related problem because it affects power balance and operational cost. Thus, distributed or decentralized algorithms must be developed to solve the large-scale multi-area economic dispatch (MAED) problem [1]. Decentralized algorithms for ED can be categorized into (i) decomposition methods and (ii) consensus-based algorithms. They are detailed as follows.

Lai et al. decomposed a power system into areas to generate subproblems with the addition of tie-line power-flow variables [1]. Li et al. used a decentralized approach that was based on a modified generalized Benders decomposition, in which the locally optimal cost function of each area was introduced [2]. Guo presented a primal decomposition method based on critical region projection, defining a sub-problem for each area, in which internal generation was a decision variable and the boundary phase angles were coupling variables [3]. Wu presented a transformation-based MAED, which preserved information privacy in each area [4].

The number of consensus algorithms (CAs) greatly exceeds the number of decomposition methods. CAs are generally associated with a multi-agent system (MAS). Xue et al. used a modified distributed consensus-based algorithm with power restriction factors to

obtain the real-time ED decision of a microgrid [5]. Yan et al. studied a network-constrained ED problem by using distributed power flow and ratio consensus as fundamental tools [6]. Elsayed et al. used a three-stage flooding-based CA to carry out ED [7]. Zhang developed a collaborative CA for decentralized ED with a communication network [8]. Yang et al. used a minimum-time CA to solve ED [9]. Wang et al. employed a sub-gradient-based CA recovery frequency, incorporating a criterion of equal incremental cost [10]. Yang et al. presented a CA that required each generator to be aware of only its own cost parameters and its neighbors' virtual incremental costs [11]. Wang et al. used a distributed DC optimal power flow approach to solve MAED, which was coordinated using consensus variables (i.e., phase angles on boundary buses) [12]. Hamdi developed a CA in MAS to solve ED with a practical communication network [13]. Chen and Zhao used a CA to deal with ED that managed the distributed resources in microgrids to realize a plug-and-play property [14]. Hao decoupled an MAED problem into several parallel sub-problems by the primal-dual principle-based MAS [15]. Liu used an alternating direction method of multipliers, the projected gradient method, and the average CA to deal with ED [16]. Liu et al. used a decentralized collaborative MAS-based CA, which was embedded in genetic algorithms to develop a bi-level model, to solve ED [17]. Wang et al. used incremental cost at each bus as a consensus variable and the local mismatch between total demand and generation as a feedback variable to solve ED [18]. Wang derived algebraic conditions related to feedback gains, cost coefficients, and some constants derived from the geometric convergence of a CA [19]. Cheng proposed a framework of system dynamics, adversaries, and the cybersecurity of consensus-based distributed ED [20]. Fu et al. optimized unit commitment using a deep reinforcement learning model and then solved the ED problem using a CA [21]. Yan et al. used a secure scheme for CA-based ED that involved the Paillier cryptosystem with time-varying network weights [22]. Bai used a CA to estimate global information in a distributed fashion and leveraged saddle point dynamics to find the optimal solution of ED [23].

Two recent review articles on MAED problems can be found in [24,25], where various methods, uncertainties, and future directions are discussed.

Based on the above descriptions, existing methods have at least one of the following limitations:

- (i) Almost all CAs ignored real power loss [1–6,8–20,22]; thus, the ED problem was greatly simplified without considering incremental loss.
- (ii) Only DC power flows were implemented if line flow constraints were considered [1–6,8,10,12]. The impacts of heavy loads on voltages were completely ignored.
- (iii) Many papers neglected both real power loss and line flow constraints [5,9–11,13–15,17–22].
- (iv) When the leader–follower CA was utilized, the leader agent was arbitrarily selected [8,17,23].
- (v) The uncertainties in load and renewables were not addressed [1–23], especially in bulk smart grids [1–4,6,22] and microgrids [10,14,17–20].

This paper presents a novel leader–follower CA to study MAED, addressing the aforementioned research gaps. The contributions of the proposed method are summarized as follows:

- (i) A three-level hierarchical CA is presented, considering real power loss and incremental loss, which were ignored in [1–6,8–20,22].
- (ii) AC power flow studies are conducted to estimate tie-line flows among areas; however, only DC power flows were implemented in [1–6,8,10,12] and line flow constraints were neglected in [5,9–11,13–15,17–22].
- (iii) In the CA, the leader agent, which was arbitrarily selected in [8,17,23], is determined by breadth-first search (BFS) to reduce the computational time.
- (iv) The uncertainties in loads and renewables are implemented by Monte Carlo simulation. The works in [1–23] did not address this critical issue.

The rest of this paper is organized as follows. Section 2 provides basic concepts in graph theory and preliminaries about the consensus algorithm. Section 3 presents the proposed method for solving the MAED problem, which incorporates BFS. Section 4 presents the results of simulating the IEEE 30- and 118-bus systems as well as a realistic 1968-bus system in Taiwan. Section 5 draws conclusions and makes recommendations for future research.

2. Background

Consensus algorithms have been applied to solve many engineering problems, such as power loss minimization [26], power converter control [27], voltage regulation in distribution systems [28], and optimal battery control [29]. This section provides the backgrounds of graph theory and the commonly used first-order consensus algorithm.

2.1. Graph Theory

A graph is defined as $G \equiv \{V, E\}$, where V is a set of n vertices and E is a set of edges. A graph is undirected if every $(i, j) \in E$. The set of neighbors of vertex i can be defined as $N_i(E) \equiv \{j \in V | (i, j) \in E\}$. For an undirected graph, the elements a_{ij} in an adjacency matrix A , which means the connectivity of a graph, are defined as $a_{ii} = 0$ and $a_{ij} = a_{ji} > 0$ if vertex j is connected to vertex i , and $a_{ij} = 0$ otherwise. The degree matrix D of a graph is a diagonal matrix, of which the i th diagonal element is $\sum_{j \in N_i} a_{ij}$. The Laplacian matrix L of a graph is defined as $L \equiv D - A \equiv [\zeta_{ij}]$ [30–33].

$$\zeta_{ii} = \sum_{i \neq j} a_{ij} \quad (1)$$

$$\zeta_{ij} = -a_{ij} \quad (2)$$

The Laplacian matrix is symmetric positive semi-definite.

2.2. First-Order Consensus Algorithm

The consensus algorithm has many applications to power systems, including in reactive power control [34–36], droop control [37,38], and negative-sequence impedance control [39]. The widely used first-order consensus algorithm is introduced here. Let $x_i \in \mathbb{R}$ be a state at a vertex i . x_i represents an incremental cost multiplied by its penalty factor in a power system. A network reaches a consensus if and only if $x_i = x_j$ for all i, j . If a fixed time is required for data to travel between vertices, then the consensus network dynamics can be modeled as a discrete-time dynamic system, as follows [23].

$$x_i[t+1] = \sum_{j=1}^N d_{ij} x_j[t], \quad i = 1, \dots, N \quad (3)$$

where t is the discrete time index and

$$d_{ij} = |\zeta_{ij}| / \sum_{j=1}^N |\zeta_{ij}| \quad i = 1, \dots, N \quad (4)$$

where the symbol $|\zeta_{ij}|$ denotes the absolute value of ζ_{ij} . The CA is implemented using MAS herein. Each agent comprises a local controller (LC) and a consensus manager (CM). The LC frequently reports its local message to the CM, which coordinates neighboring agents via a communication system. The LC obeys the latest instruction from its CM to adjust the present state. This process iterates until the CA converges.

2.3. Security-Constrained Economic Dispatch

The security-constrained ED (SCED) problem involving AC power flow can be formulated as follows.

$$\min_{PG_m} \sum_{m \in G} f_m(PG_m) \quad (5)$$

$$\text{Subject to } P_j(V, \theta) = 0 \quad j \in Nb \quad (6)$$

$$Q_j(V, \theta) = 0 \quad j \in Nb \quad (7)$$

$$P_\ell(V, \theta) \leq \overline{P}_\ell, \quad \ell \in L \quad (8)$$

$$PG_m^{\min} \leq PG_m \leq PG_m^{\max} \quad m \in G \quad (9)$$

where $f_m(PG_m)$ is the cost function of generator m , which generates power PG_m . Equations (6) and (7) are the real and reactive power flow equations at bus j , respectively. The terms V and θ represent the vectors of voltage magnitudes and phase angles, respectively. Equation (8) specifies the ℓ th line flow constraint with the maximum $\overline{P}_\ell$. Equation (9) signifies the maximum and minimum values of power generation at bus m . The terms G , Nb , and L are the sets of generators, buses, and lines, respectively. When the optimal solution of ED is obtained, the following conditions are satisfied [9,14]:

$$\frac{df_m(PG_m)}{dPG_m} = \lambda \text{ for } PG_m^{\min} \leq PG_m \leq PG_m^{\max} \quad (10)$$

$$\frac{df_m(PG_m)}{dPG_m} < \lambda \text{ for } PG_m = PG_m^{\max} \quad (11)$$

$$\frac{df_m(PG_m)}{dPG_m} > \lambda \text{ for } PG_m = PG_m^{\min} \quad (12)$$

where

$$\lambda = \frac{1}{\left(1 - \frac{\partial PL}{\partial PG_m}\right)} \frac{df_m(PG_m)}{dPG_m} \quad (13)$$

where $df_m(PG_m)/dPG_m$ is the incremental cost (IC). The term $\partial PL/\partial PG_m$ is the incremental loss (IL), which adjusts the allocation of the power generation at generator m by considering the loss that is caused by distances between this generator and load centers. The symbol λ is the Lagrange multiplier, which is the consensus variable herein.

Let both m and n be indices of generators; PG_m and PG_n are power generations from generators m and n , respectively. The loss of real power in an area can be expressed in terms of loss coefficients, as shown in Equation (14), called as Kron's loss formula. This loss formula can be utilized to calculate the IL.

$$PL = \sum_{m \in G} \sum_{n \in G} PG_m B_{mn} PG_n + \sum_{m \in G} B_{m0} PG_m + B_{00} \quad (14)$$

where the parameters B_{mm} , B_{m0} are the second-order and first-order loss coefficients, respectively. The parameter B_{00} is a constant. The selection of B_{mm} , B_{m0} , and B_{00} is generally based on the system load level.

3. Proposed Method

The distributed computation that is proposed in this paper is based on MAS. Without loss of generality, the topology of the communication system is assumed to be the same as the topology of the power system [14,23]. The following mathematical formulation is derived by assuming that each bus (vertex in a graph or agent in a MAS) consists of a generator and a demand [3,13,14,23].

3.1. Leader in the Consensus Algorithm

The convergence of the CA depends on the connectivity of a system. This can be examined by estimating the eigenvalue of the Laplacian matrix L , which contains information on network performance. Specifically, the second smallest eigenvalue of the Laplacian matrix is a measure of the convergence speed of a CA. Accordingly, if the power system (the graph) is very large or complex, then the coordination among agents becomes very poor because the consensus variables of agents far from the center of the system are updated slowly. Since this work concerns a large-scale power system, the leader–follower CA is adopted to avoid the aforementioned issue.

The first step in developing the leader–follower CA is to identify the leader agent in a MAS. Widely used search algorithms include uninformed depth-first search (DFS) and breadth-first search (BFS). Depth-first movement through a tree may skip the levels at which the goal vertex appears and waste a long time exhaustively exploring lower parts of the tree [40]. Restated, DFS will not necessarily find the shortest path. BFS, on the other hand, will find the shortest path between the starting vertex and any other goal vertex. Thus, BFS is adopted herein to identify the leader agent in the CA because the topology of the communication system is assumed to be the same as the topology of the bulk power system.

BFS aims to find the goal (generator) agent among all agents (vertices) at a given level (depth) before using the successors of those agents to push on. Restated, all the agents are expended at a given depth in a search tree before any agent at the next level is expended. Algorithm 1 presents the pseudo-code of the non-recursive implementation of BFS [41,42].

Algorithm 1. Pseudo-code of BFS.

Initialization: queue=[]
 state=root_node;
While (true);
 if goal is (state), then return SUCCESS;
 else, add_to_back_of_queue (successors (state));
 if queue is [], then report FAILURE;
 state=queue [0];
 remove_first_item from (queue);
End while.

In Algorithm 1, the pseudo-code initializes an empty queue and the start state first. The statement “state=queue [0]” implies that the “state equals the first item in the queue” [42].

The aforementioned queue in Algorithm 1 is a first-in-first-out (FIFO) queue, ensuring that the agents that are visited first will be expended first. The FIFO queue places all newly generated successors at the end of the queue, indicating that shallow agents are expended before deeper agents [42].

The total number of agents between a presumed leader agent and all goal agents is defined as “the length of a path”. The presumed agent with the shortest path (in terms of total number of agents) is the leader agent in this graph. If two presumed leader agents have the same path, then the one with more neighboring agents is identified as the final leader agent.

3.2. Three-Level Hierarchical CA

A hierarchical three-level CA is proposed herein to solve a large MAED problem. Level 1 updates λ in each area using information of its own real power imbalance and λ_s from neighboring areas; the leader agents in all areas ($s = 1, 2, \dots, S$) at level 2 receive (send) λ (power loss/tie-line flow) from (to) their agents in their corresponding areas at level 1, as shown in Figure 1. The leader agent at level 2 sends its λ to all follower agents in the corresponding area; the follower agents update their λ_s to estimate their power generations at level 3. The follower agents ($I = 1, 2, \dots, N$) in area s at level 3 coordinate with one another by considering tie-line flow and λ_s , as shown in Figure 2.

In order to enhance the convergence performance at level 1, a control term is augmented to Equation (3) by considering the mismatch between supply and demand in an area. The consensus variable λ_s at level 1 is thus updated using Equation (15).

$$\lambda_s[t + 1] = \sum_{\eta=1}^{ns} d_{s\eta} \lambda_{\eta}[t] + \Delta \lambda_s \quad (15)$$

where n_s denotes the number of neighboring areas for area s .

$$\Delta\lambda_s = \frac{\Delta P_s}{\sum \frac{\partial PG_{sm}}{\partial \lambda_s}} = \frac{\Delta P_s}{\sum \frac{\gamma_{sm}(1-B_{m0})+B_{mm}\beta_{sm}}{2(\gamma_{sm}+\lambda_s B_{mm})^2}} \quad (16)$$

where ΔP_s is the power imbalance in area s ; PG_{sm} is the power generation of generator m in area s ; and β_{sm} and γ_{sm} are the first- and second-order coefficients in the cost function for generator m in area s , respectively. More specifically, $\Delta\lambda_s$ in Equations (15) and (16) was expressed by $\epsilon_s \Delta P_s$ in previous works [9,13,18], where the symbol ϵ_s denotes a sufficiently small positive constant [9] affecting the convergence rate of the algorithm [13,18]. This work utilizes $\left(\sum \frac{\partial PG_{sm}}{\partial \lambda_s}\right)^{-1}$ to replace ϵ_s by adaptively changing its value during iterations. Because $PG_{sm} = \frac{\lambda_s - \beta_{sm} - \lambda_s B_{m0}}{2(\gamma_{sm} + \lambda_s B_{mm})}$, according to Equation (13), the expression in the denominator in Equation (16) can be obtained.

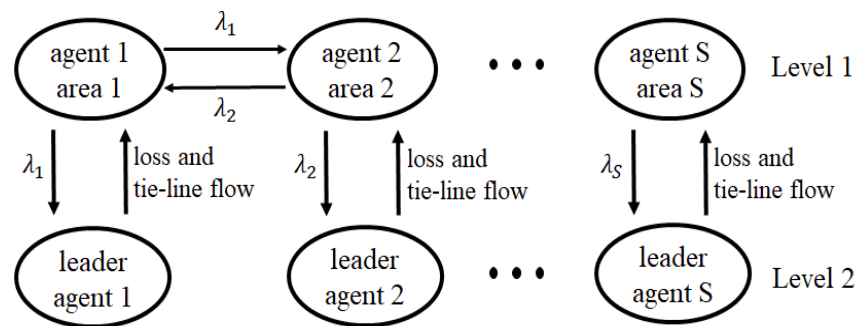


Figure 1. Interaction between levels 1 and 2.

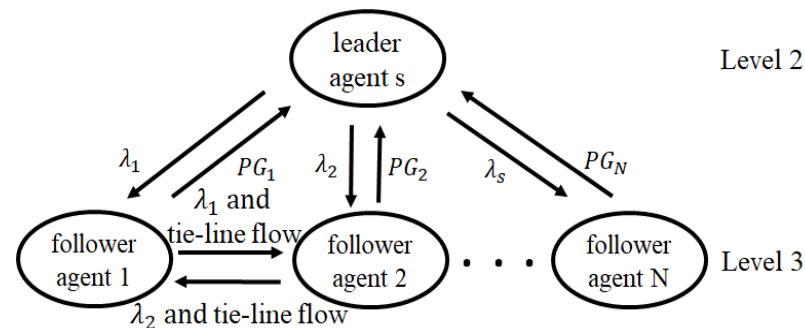


Figure 2. Interaction between levels 2 and 3.

Let n_j be the number of neighboring agents for follower agent i . The consensus variable λ_i of the follower agent i at level 3 is updated using Equation (17) as per Equation (3).

$$\lambda_i[t+1] = \sum_{\zeta=1}^{n_j} d_{i\zeta} \lambda_{\zeta}[t] \quad (17)$$

Because both Equations (15) and (17), which represent the updates of consensus variables during iterations at levels 1 and 3, respectively, are derived from Equation (3), the proposed method ensures convergence.

3.3. Flowchart and Implementation of the Proposed Method

Suppose that MAED involves six areas in a power system. “Matlabpool local” in MATLAB 2019a can be used to activate six out of eight cores in a CPU. Each core conducts an independent ED in a single area by using the “parfor” command that carries out iterative loops for Monte Carlo simulations involving uncertainty. Accordingly, each core can perform distributed computing for a single area. One more core is needed to deal

with the interactions among these six areas. When the distributed computation ends, the “matlabpool close” command is used to close down all cores in the CPU.

MATLAB can only control the PSS/E power flow program indirectly via Python library paths (sys.path.append and os.environ), with “import psspy” in the developed code.

Blocks A, B, C, and D in Figure 3 indicate distributed computation in an area using a single core. Other parallel blocks that are like A, B, C, and D are not shown in Figure 3. Blocks A, B, C, and D also involve one Monte Carlo simulation run. One thousand Monte Carlo simulations are carried out in an iteration to estimate the impact of uncertainty on the power flow results. Reactive power generation is examined during the iterations. If the reactive power generation of a generator violates its limits, then this generator is changed to a load bus with constant real/reactive injections.

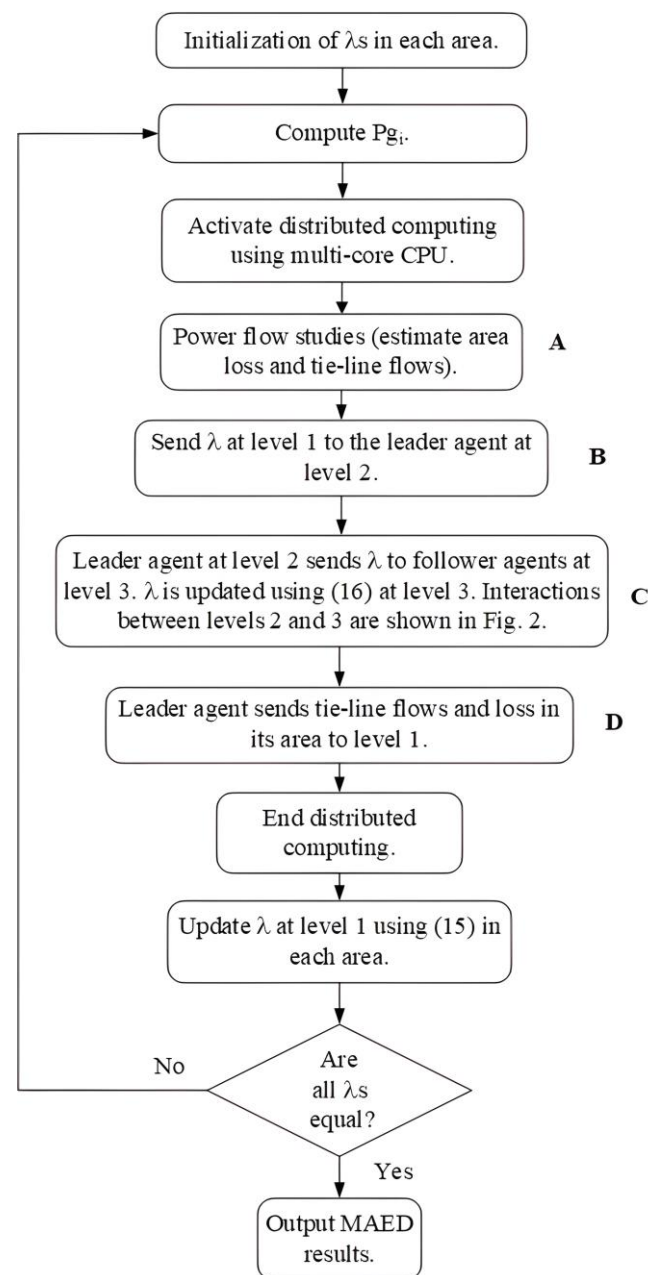


Figure 3. Flowchart of the proposed method.

4. Simulation Results

In this paper, the IEEE 39- and 118-bus systems [43], as well as the Taiwan power system, are used as test systems. The IEEE 39-bus system includes ten generators supplying a total system demand of 6097.1 MW. The IEEE 118-bus system, divided into five areas, has a total system demand of 3733.07 MW. The Taiwan power system consists of 1968 buses across six power supply areas, with a peak load of 39.72 GW and 64 online generators. The traditional SCED method [44] is employed to validate the applicability of the proposed method. When implementing the traditional SCED method, the power systems are considered in their intact (single area) conditions. The results obtained from the traditional SCED method are used to verify the optimality achieved by the proposed method.

4.1. Identification of Leader Agent Using BFS

In this section, the IEEE 39-bus system, shown in Figure 4, is used to evaluate the performance of BFS in identifying the leader agent, which can reduce computational effort. Since the multiple areas are not crucial issues in the identification of the leader agent, the IEEE 39-bus system is not partitioned into areas in this testing. All buses are firstly considered as candidate presumed leader agents. As described in Section 3.1, the first step is to seek the shortest path (in terms of the number of agents) for a presumed leader agent. As shown in Figure 5, the paths of buses 3, 4, and 15~18 to the goal agents (all generator buses) are the shortest (that is, six agents). The second step is to examine the maximum number of neighboring buses (in terms of the number of agents) from step 1. The numbers of neighboring buses for presumed leaders at buses 3, 4, and 15~18 are 3, 3, 2, 5, 3, and 2, respectively. Thus, bus 16 is the leader agent in the IEEE 39-bus system.

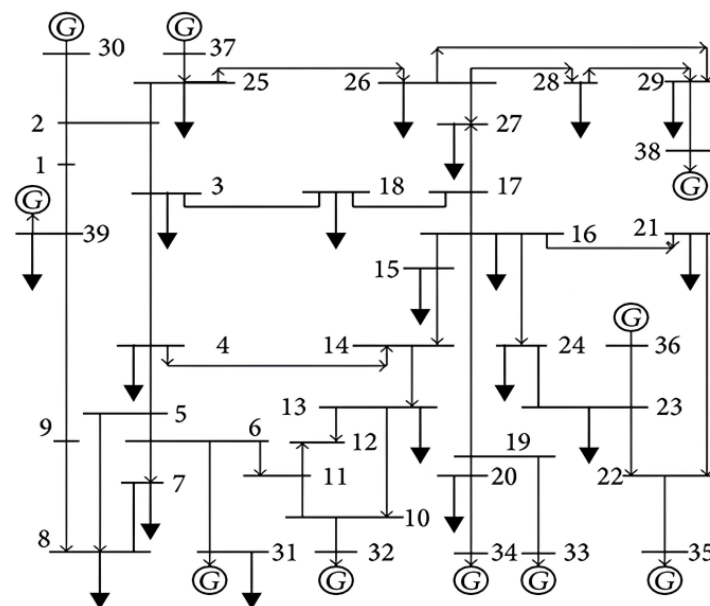


Figure 4. One-line diagram of the IEEE 39-bus system.

Figure 5 also illustrates the number of iterations for each presumed leader agent. It can be found that the number of iterations is lowest (222) if the leader agent is at bus 16; it is highest (1115) if the presumed leader agent is at bus 38. This result verifies the applicability of the proposed BFS to identify the leader agent.

Tables 1 and 2 show the optimal results obtained by the proposed CA and traditional SCED methods [44]. These two methods yield the same optimal results. Most of the ten generators reach their maximum power generation except for generators at buses 31, 32, and 34.

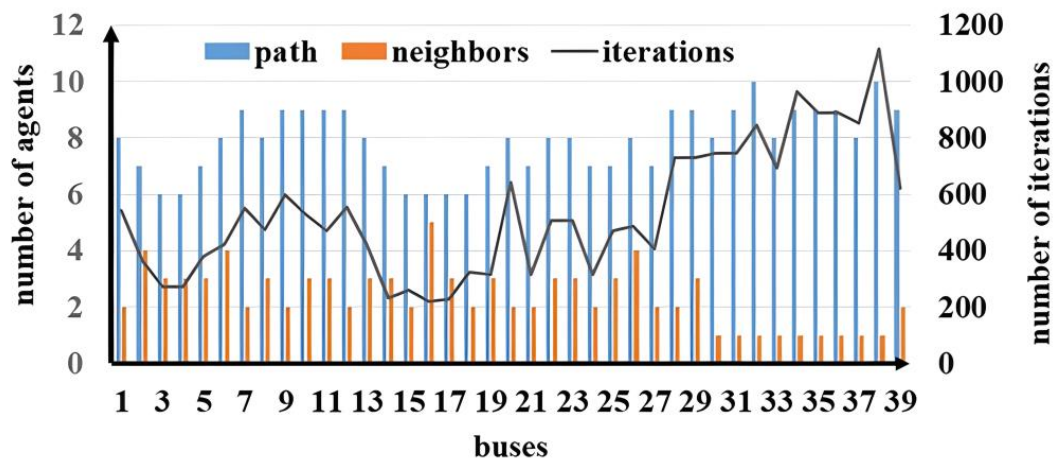


Figure 5. Results of BFS in the IEEE 39-bus system.

Table 1. Optimal results obtained by the two methods (39-bus system).

Method	Proposed Method	Traditional SCED Method [44]
System load (MW)		6097.1
Total loss (MW)	46.8995	46.8985
Incremental cost (\$/MWh)	17.5443	17.5436
Generation cost (\$/h)	66,268.34	66,266.62
Total generation (MW)	6143.9995	6143.9985

Table 2. Optimal power generations (39-bus system).

Bus	Proposed Method (MW)	Traditional SCED Method (MW) [44]
30	800	800
31	720.69	720.65
32	501.04	501.01
33	560	560
34	412.27	412.24
35	700	700
36	600	600
37	550	550
38	550	550
39	750	750

4.2. Verification of B-Coefficients in Loss Formula

The peak demand in the Taiwan power system is used to test the accuracy of the B coefficients in the loss formula. Sixty-four thermal generators are online. The losses that are computed using the B coefficients are compared with those obtained using the PSS/E (version 32) software package. As shown in Table 3, the losses obtained by both methods are almost the same for the original data (100%); the error is 0.04%. When the bus loads increase or decrease, the errors increase. However, in the worst case in Table 3, the error is 1.94%, which is associated with 95% of the original demand. This result verifies that the traditional B coefficients are applicable if the online thermal generators and the system topology are unchanged.

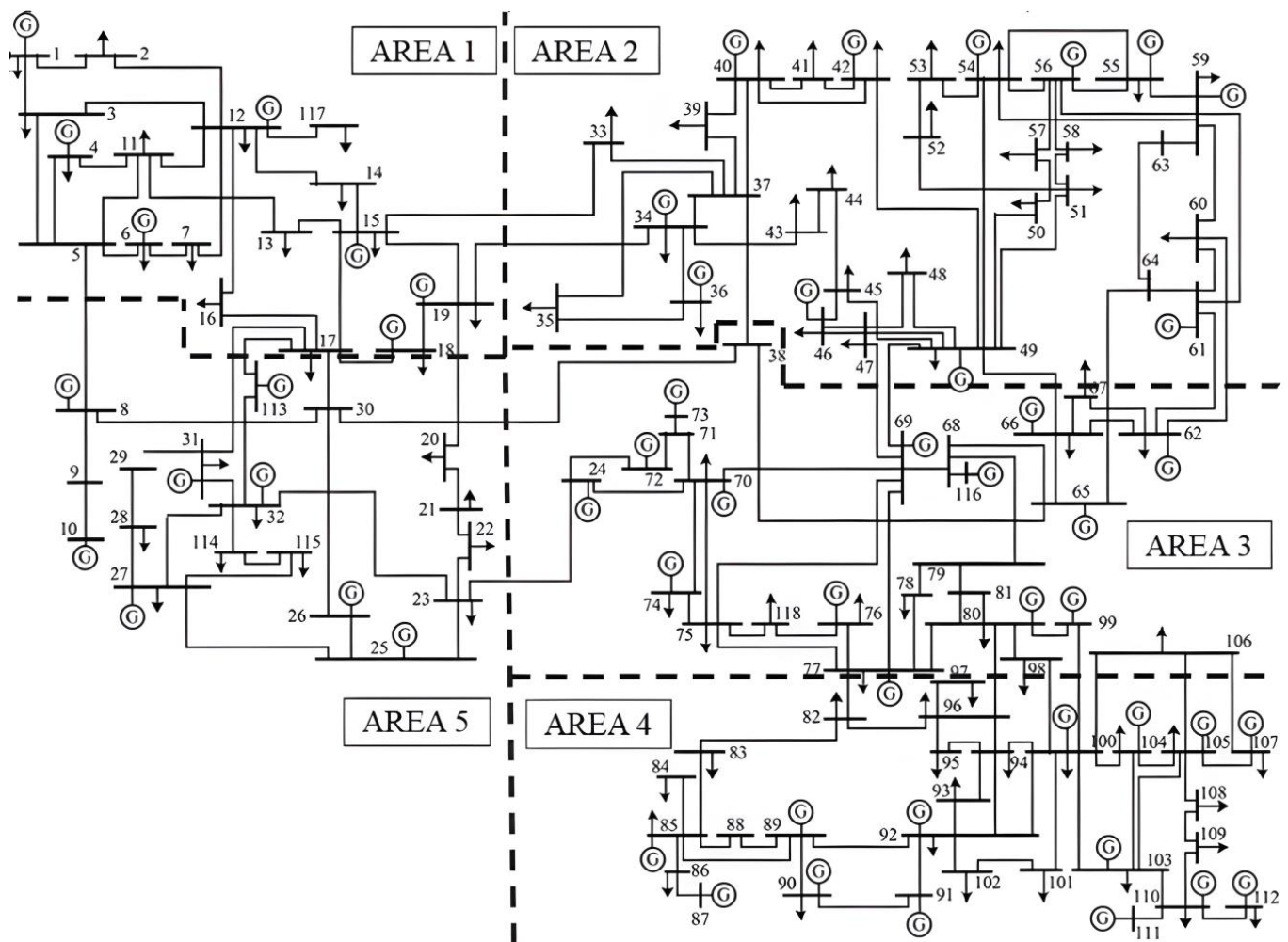
Table 3. Accuracy of B coefficients in loss formula.

Percentage	Load (MW)	MW Loss by PSS/E	MW Loss By B coefficient	Error (%)
105%	41704	583.4	591.6	1.41
103%	40910	555.8	559.9	0.73
100%	39718	516.5	516.7	0.04
97%	38527	480.9	475.2	−1.12
95%	37733	455.0	446.2	−1.94

Based on the results in Table 3, the sensitivity (error%/load%) is estimated to be 33.5% ($=1.41 - (-1.94)/(105 - 95)$). If a maximum MW loss error of $\pm 5\%$ is acceptable, the system load must be within the range of 85.07~114.92% of the original system load.

4.3. Simulation Results of the IEEE 118-Bus System

The IEEE 118-bus system is used to demonstrate the use of the proposed consensus algorithm in solving MAED. This system is decomposed into five areas, as shown in Figure 6. It consists of 54 generators and 91 load buses, in which the total demand is 3733.07 MW.

**Figure 6.** Five areas in the IEEE 118-bus system.

The leader agents are identified by BFS to be buses 12, 49, 80, 92, and 32 for areas 1~5, respectively. The proposed method takes 79 iterations to converge to an optimal Lagrange multiplier (consensus variable) of 14.86 \$/MWh with a 0.001 \$/MWh tolerated variation between two consecutive two iterations. Figure 7 shows the convergence performance of consensus variables in the five areas. The same optimal Lagrange multiplier of 14.86 \$/MWh is also obtained using the traditional SCED method. The total generation costs obtained by the proposed method and the traditional SCED method are 59,141.29 \$/h and 59,141.37 \$/h, respectively.

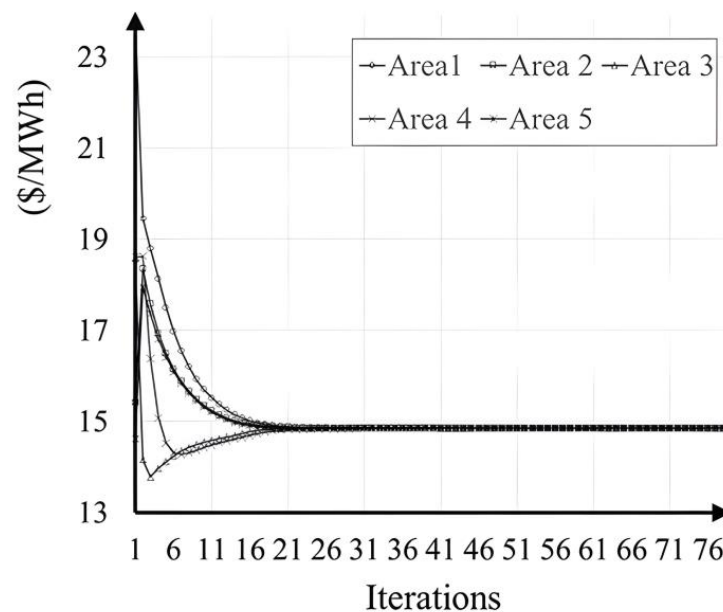


Figure 7. Convergence performance of consensus variables in 5 areas (the IEEE 118-bus system).

4.4. Simulation Results of the Taiwan Power System

In the Taiwan power system, the number of buses is 1968, and they are covered by six power supply areas—Taipei, Hsintao, Taichung, Jianan, Kaoping, and Huadong. The communication topology for agents at level 1 is assumed to be the same as the topology of the 345 kV system, while the communication structure for agents at levels 2 and 3 is assumed to be identical to the topology of the 161 kV system in the aforementioned six areas. A total of 824 agents are needed; Taipei has the largest number (208), while Hualian has the smallest number (21). In the Taiwan power system, the peak load is 39.72 GW, and the number of online generators is 64.

Six Lagrange multipliers (consensus variables) in six power supply areas converge to 3.798 \$/MWh by the proposed method and to 3.799 \$/MWh by the traditional SCED method. Figure 8 shows the convergence performance of six consensus variables. This result confirms that the proposed method is indeed applicable to a large-scale power system. The total generation costs obtained by the proposed method and the traditional SCED method are 152,812.53 \$/h and 152,850.87 \$/h, respectively.

4.5. Uncertainty in Loads and Renewables

The uncertainties in all bus loads in the IEEE 118-bus system and the uncertainties associated with both bus loads and renewable energy generation in the Taiwan power system are taken into account herein.

Traditional Distributed Stochastic Programming (DSP) [45] was utilized for restoring a multi-energy distribution system, involving joint district network reconfiguration under uncertainty. To address uncertainties from photovoltaic outputs and joint trading, energy management was tackled using the distributionally robust optimization (DRO) method [46]. Both DSP and DRO require reformulating the problem and introducing additional con-

straints in optimization. In contrast, this paper employs Monte Carlo simulation without altering the original problem formulation to investigate uncertainties stemming from bus loads and renewable power generations. Monte Carlo simulation is widely recognized as a benchmark for addressing uncertainties in various problem domains [47].

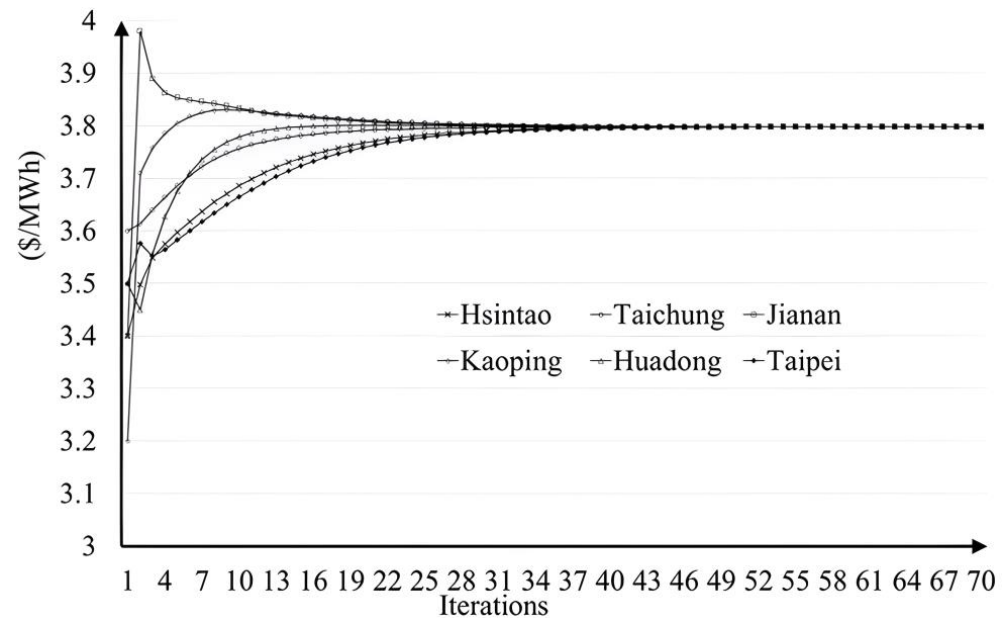


Figure 8. Convergence performance of consensus variables in 6 areas (the Taiwan power system).

The uncertainty is studied by considering each load with an interval of (mean value) $\times (1 \pm 5\%)$ MW at each bus in the IEEE 118-bus system. One thousand Monte Carlo simulations are run to obtain the Lagrange multiplier in each area in an iteration. Figure 9 shows the maxima (upper bounds) and the minima (lower bounds) of five Lagrange multipliers in five areas during the iterations. The upper and lower bounds converge to 14.94 and 14.68 \$/MWh, respectively. The required CPU time in this uncertainty study is 30,932.42 s, whereas the deterministic study in Section 4.3 requires only about 0.055 s.

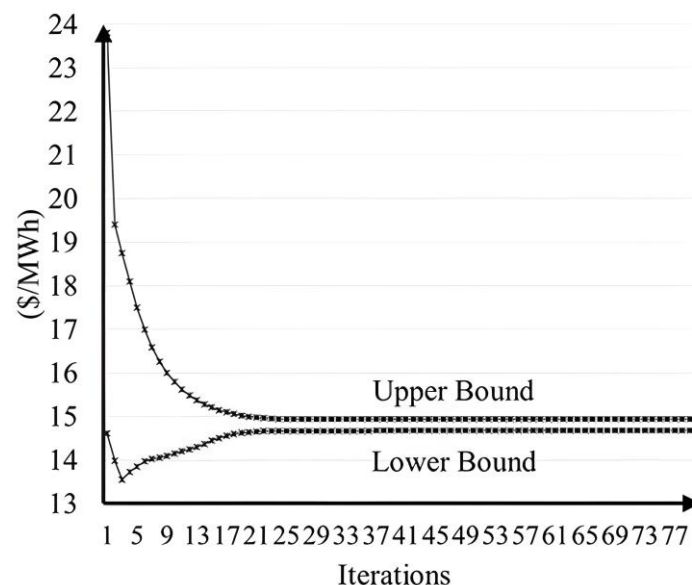


Figure 9. Upper/lower bounds of 5 consensus variables (the IEEE 118-bus system).

The uncertainty is also explored by considering power consumption/generation with an interval of (mean value) $\times (1 \pm 5\%)$ MW at each bus load, as well as solar power

generation (total mean of 1.68 GW with a variation of $\pm 5\%$ across all photovoltaic farms) and wind power generation (total mean of 282.09 MW with a variation of $\pm 5\%$ across all wind farms) in the Taiwan power system. One thousand Monte Carlo simulations are run to obtain the Lagrange multiplier in each area in an iteration. Figure 10 shows the upper and lower bounds of six Lagrange multipliers in six areas during the iterations. The upper and lower bounds converge to 4.114 and 3.613 \$/MWh, respectively. The required CPU time for this study with uncertainty is 81,683.2 s, whereas it is only approximately 0.307 s for the deterministic study in Section 4.4.

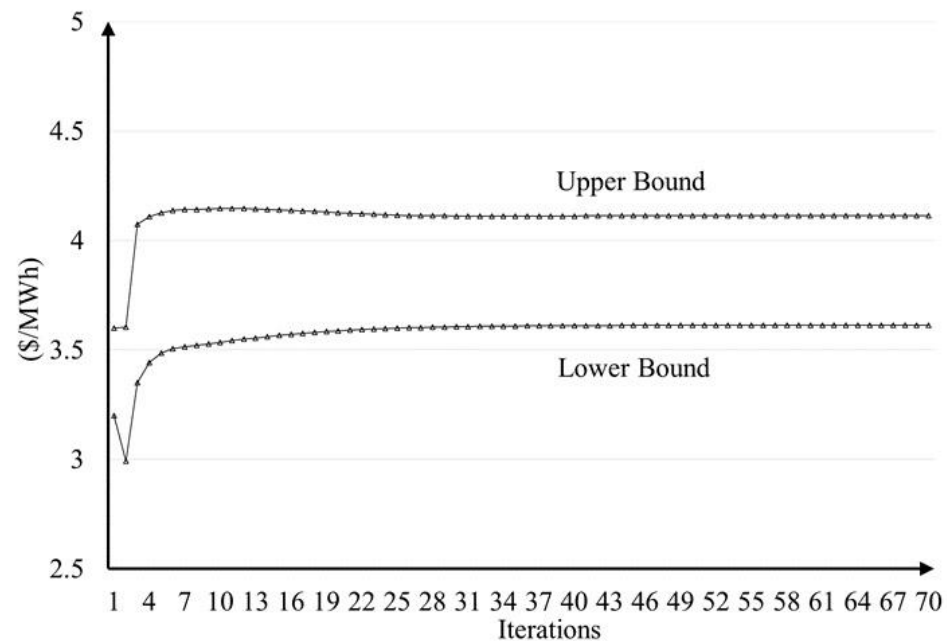


Figure 10. Upper/lower bounds of 6 consensus variables (the Taiwan power system).

5. Conclusions

A hierarchical consensus algorithm that has three levels for solving the problem of multi-area economic dispatch in a power system was presented. Level 1 coordinates all areas; levels 2 and 3 constitute a leader–follower consensus algorithm that uses distributed agents to achieve economic dispatch. The breadth-first search method was shown to correctly identify the leader agent in an area. The disadvantages of traditional methods, such as the lack of consideration of loss and an arbitrarily selected leader agent, are avoided in the proposed method. The results that were obtained using the IEEE 39- and 118-bus systems and a realistic 1968-bus Taiwan power system validated the proposed method with consideration of deterministic and uncertain loads/renewable generation. Future studies will utilize uncertainty methods, such as affine arithmetic models or probability-based methods, to reduce the computational effort when considering uncertainty. Additionally, the impact of latency caused by the communication system on the performance of the consensus algorithm will be investigated. These findings will be reported in future publications.

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