

Article

Optimal Bidding Strategy for Horizontal Pumped Storage-Wind-Solar Hybrid Systems in Day-Ahead Markets: A Hybrid Uncertainty Modeling Approach

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Abstract

This paper addresses the multi-source uncertainties faced by horizontal pumped storage-wind-solar (HWS) hybrid systems in the day-ahead market by proposing a hybrid stochastic-robust optimization model for bidding and scheduling. The model employs a scenario-based method to capture the randomness of wind and solar power output, utilizes Information Gap Decision Theory (IGDT) to handle the epistemic uncertainty in runoff inflow forecasting, and constructs a price-acceptance probability function based on historical statistics to characterize the market mechanism. By maximizing the system's tolerable uncertainty immunity gap, the model co-optimizes generation schedules, pumped-storage operation, and market bids while ensuring that revenue under the worst-case inflow scenario does not fall below a predefined threshold. Simulation results based on an actual project in Hubei Province demonstrate that the proposed method effectively balances revenue and risk, showing significant advantages in both revenue stability and robustness compared to the system before retrofitting. This study provides practical decision-making support for hybrid systems with horizontal pumped storage participating in electricity markets.

Keywords: bidding strategy; day-ahead markets; horizontal pumped storage-wind-solar; IGDT; uncertainty modeling



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1. Introduction

1.1. Background

Against the backdrop of global energy transition and the “dual-carbon” goals, the installed capacity of renewable energy, represented by wind and solar power, has been growing at a consistently high rate [1]. The integration and accommodation of these renewable sources have become a core issue in the evolution of power systems. Electricity market reforms provide a key mechanism for the efficient allocation of clean energy [2]; however, the inherent intermittency and variability of wind and solar output expose their independent participation in market competition to significant revenue risks and penalty pressures [3]. In this context, hydro-wind-solar multi-energy complementary systems, which leverage the flexible regulation capabilities of hydropower to mitigate renewable

energy fluctuations, are seen as an effective pathway to enhance the accommodation capacity of new energy sources and market competitiveness.

In recent years, horizontal pumped storage has emerged as a promising technology for hybrid system upgrades, offering advantages such as flexible layout and minimal environmental impact. However, when integrated systems participate in the day-ahead market, they face multi-faceted uncertainties: stochastic renewable output [4], difficult-to-quantify inflow forecasting errors [5], and volatile market prices [6]. Traditional scheduling methods cannot adequately coordinate market risks with operational constraints to develop strategies that balance profitability and robustness. Therefore, constructing a robust optimization model capable of simultaneously handling these uncertainties and co-optimizing dispatch and bidding decisions remains a critical and unresolved challenge in both research and practice.

1.2. Literature Review

Building upon the aforementioned research background, the core challenges facing hybrid systems participating in electricity markets are primarily manifested as the coupled problem of “multi-source uncertainty management” [7,8] and “coordinated optimization in complex market environments.” To this end, this paper will systematically review existing research from three dimensions: “the theoretical evolution of uncertainty modeling”, “the methodological development of bidding strategies for HWS systems”, and “emerging market mechanisms”.

(1) Uncertainty Modeling

How to accurately and efficiently characterize uncertainties and construct decision models adapted to them is a core issue for enhancing the economic efficiency, reliability, and flexibility of system operation.

Uncertainty modeling methods can be classified into probabilistic and uncertainty set models. Probabilistic approaches generate multiple scenarios to characterize uncertainty, as seen in stochastic electricity market analyses [9,10], long-term system control [11], and risk propagation modeling [12,13]. However, such methods face computational challenges due to scenario dimensionality [14], addressed through decomposition algorithms [15] and parallel computing [12].

In contrast, uncertainty set models do not require precise probability distributions but define the fluctuation range of uncertain variables to optimize against worst-case scenarios. Robust Optimization is commonly used to handle uncertainties such as electricity prices [16] and wind power output [17]. IGDT extends this approach by providing both risk-averse and opportunity-seeking strategies, offering more flexible risk management capabilities [18,19].

In recent years, the use of ambiguity sets to characterize uncertainties has gained popularity, enabling finer modeling of distribution features while avoiding reliance on precise distribution assumptions. For instance, Liu et al. [20] developed a moment-based ambiguity set incorporating mean, variance, and skewness to better capture wind and solar output uncertainties, transforming the model into a tractable MILP through linear decision rules. Similarly, Jin et al. [21] proposed a Wasserstein-metric-based ambiguity set within a two-stage distributionally robust optimization framework for hydropower-integrated peak shaving, achieving an effective balance between operational reliability and economy.

In summary, the selection of uncertainty modeling methods depends critically on the source of uncertainty, the type of available information, the risk preferences of decision-makers, and the computational complexity requirements of the problem.

(2) Bidding Strategies

The core of bidding strategies for hydro-wind-solar complementary systems participating in the electricity market lies in leveraging the flexibility of hydropower to mitigate

the uncertainty of wind and solar power output, while maximizing revenue through coordinated optimization in the market. Zhang et al. [22] proposed a stochastic model for multi-timescale and cross-provincial bidding, identifying “solar during the day, wind during the night” and “wind in winter, solar in summer” complementarities while quantifying key transmission capacity thresholds. Yu et al. [23] focused on optimizing bidding curves for cascade hydropower plants across multiple regional markets, proposing an improved interval programming method to balance conservatism and robustness in uncertainty management. Xiao et al. [24] proposed a multi-stage bidding-clearing-scheduling framework with Shapley value-based revenue-sharing, boosting renewable profits and reducing water curtailment. Li et al. [25] innovatively balanced bidding strategies with the “future utility” of reservoir storage, establishing a two-layer nested optimization model that simultaneously considers current profits and long-term storage value. Both studies by Tan et al. [26,27] employed a hybrid model combining data-driven approaches and IGDT to address uncertainties in wind and solar generation as well as market prices: Ref. [26] focused on multi-stage day-ahead and intraday bidding, while Ref. [27] incorporated historical data-driven clustering of competitor bidding scenarios into a bi-level model, enhancing the adaptability of the strategy.

To address competitive dynamics in the market, researchers have developed various analytical frameworks. Shafiekhani et al. [28] employed bi-level mathematical programming with equilibrium constraints to characterize the strategic bidding of a virtual power plant in joint energy and ancillary service markets, solving for the Nash equilibrium. Shafiekhani et al. [29] further applied IGDT to develop a bidding model for renewable-based virtual power plants under three operational modes: risk-neutral, risk-averse, and risk-seeking. Nazari and Ardehali [30] investigated the coordinated bidding of a wind-pumped storage-thermal system, quantifying its profit improvement in energy and reserve markets along with the impact of carbon emission prices. Abedinia et al. [31] approached from the demand side, proposing a hybrid robust-stochastic bidding model for large industrial consumers that accounts for wind and solar uncertainties.

However, existing studies often portray market clearing mechanisms in an overly idealized or indirect manner. They either treat hydro-wind-solar systems as price-takers (e.g., Jurasz et al. [32]), completely disregarding the impact of their own bids on the market, or employ more sophisticated but computationally intensive bi-level/equilibrium models (e.g., Shafiekhani et al. [28], Ilak et al. [33]), which still rely on indirect inferences.

(3) Emerging Market Mechanisms

Beyond wholesale markets, decentralized market architectures have gained increasing attention in recent years. Peer-to-peer energy trading allows prosumers to directly exchange energy with one another (Zhou et al. [34]), transforming traditional consumers into active prosumers within local communities (Manchalwar et al. [35]). Unlike centralized markets, P2P trading requires distributed coordination mechanisms. Zhan et al. [36] proposed a two-stage framework integrating P2P trading with voltage regulation services, while Xiong et al. [37] designed both day-ahead and real-time P2P trading mechanisms.

The implementation of decentralized markets increasingly relies on blockchain technology to address trust and settlement challenges. Wang et al. [38] developed a blockchain-enabled smart grid system, and Nazari and Biswas [39] demonstrated blockchain applications in retail energy trading. Du et al. [40] showed that blockchain-based P2P trading can increase producer revenue while reducing consumer costs. However, such systems also introduce challenges including active power losses and transaction costs P and Salam [41]. Regarding privacy and accountability mechanisms, Erdayandi et al. [42] proposed a privacy-preserving billing protocol, while Tushar et al. [43] designed coalition formation games for strategic participation in P2P trading.

Recent advancements have extended blockchain applications toward more granular architectures for electricity rights and yield instruments. Vieira and Zhang [44] developed smart contract frameworks for time-discrete market clearing using double auctions. Haryani et al. [45] proposed semi-fungible tokens (ERC-1155) for divisible and traceable Renewable Energy Certificates. Evdokimov et al. [46] developed time-stamped ERC-1155 order books encoding price and delivery slots, plus IndexYT for yield token liquidity. Seven et al. [47] demonstrated cross-chain VPP trading using Avalanche and DEXs. These works show blockchain enables fine-grained rights allocation and derivative-like products relevant to our hourly bidding context.

Although these emerging market models operate at different scales than the wholesale market considered in this paper, they share common challenges in uncertainty management and strategic decision-making.

1.3. Motivation and Contributions

To address the absence of an integrated day-ahead bidding model for HWS systems that simultaneously accounts for stochastic renewable generation, epistemic inflow uncertainty, and an explicit market acceptance proxy—while remaining computationally tractable under realistic operational constraints—this paper proposes an IGDT-based robust bidding and dispatch model for HWS systems participating in the day-ahead electricity market.

The contributions of this paper are threefold, each directly addressing a key component of the aforementioned gap:

1. **Hybrid Uncertainty Modeling Framework:** This paper proposes a hybrid framework that integrates scenario-based stochastic programming for renewables, robust optimization for hydrological uncertainty, and statistically-derived market acceptance functions. This enables differentiated uncertainty treatment based on data characteristics, offering a realistic representation of real-world decision-making.
2. **Practical IGDT Implementation for Hydropower Systems:** This work develops a practical IGDT formulation specifically tailored for hydropower scheduling by establishing time-dependent, asymmetric uncertainty sets that reflect the physical characteristics of runoff forecasting errors, significantly enhancing practical applicability.
3. **Comprehensive Decision Support for HWS Systems:** The model provides a unified framework for co-optimizing dispatch and bidding under multiple uncertainties. By simultaneously scheduling generation, pumping, and market offers while ensuring hydrological robustness, it offers actionable insights for HWS operators in competitive markets.

The paper is organized as follows: Section 2 introduces horizontal pumped storage technology. Section 3 presents the hybrid uncertainty modeling framework. Section 4 develops the stochastic-IGDT bidding model and describes the solution methodology. Section 5 provides case studies, and Section 6 concludes the paper.

2. Overview of HWS System

2.1. Horizontal Pumped Storage Technology and Its Advantages

Pumped storage is the most mature large-scale energy storage technology (as shown in Figure 1), serving critical functions like peak shaving and frequency regulation [48]. Traditional vertical-shaft pumped storage relies heavily on natural topography for head formation (as shown in Figure 1a), leading to remote sites, high costs, and long construction cycles. Recently, horizontal pumped storage has emerged as a novel alternative. Its key feature is the horizontal arrangement of core equipment, integrated with underground reservoirs or tunnels (as shown in Figure 1b). Compared with traditional vertical-shaft pumped storage, horizontal pumped storage demonstrates the following significant advantages:

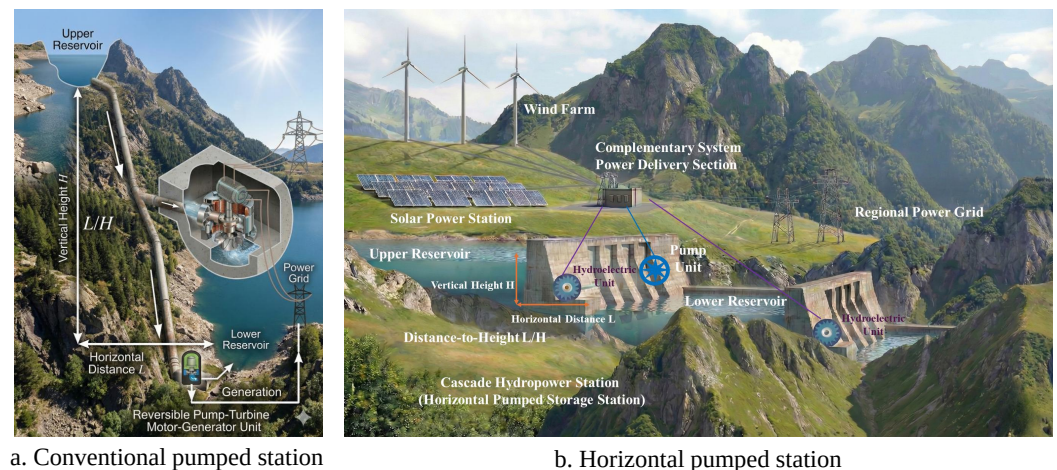


Figure 1. Comparison of conventional pumped station and horizontal pumped station.

(1) Broad Site Adaptability

It has relatively lower requirements for topography and head, effectively overcoming the geographical constraint of traditional pumped storage that relies on mountainous sites.

(2) Minimal Environmental Impact

It eliminates the need for large-scale dam construction and extensive upper reservoirs, significantly reducing ecological impact.

(3) Lower Investment

The engineering structure is relatively simplified, which helps shorten construction periods and reduce initial investment costs.

Horizontal pumped storage can enhance traditional cascade hydropower stations with storage capability through various approaches, such as “adding reversible units,” “retrofitting existing reversible units,” or “adding pumping stations.” Therefore, HPS provides an innovative technical pathway for energy storage in renewable-energy-rich areas or topographically constrained regions. This paper focuses on the retrofitting approach of adding pumping stations.

2.2. HWS System Architecture

The HWS system constructed in this study is an integrated entity that combines physical coupling with market coordination. It primarily consists of the following core components:

(1) Wind Farms and Solar Stations

Serving as the main primary energy input units of the system. Their output is intermittent and uncertain, representing one of the primary sources of both revenue and risk for the system.

(2) Horizontal Pumped Storage Power Station

Acting as the rapid-response and energy buffer unit of the system. It enables intraday energy transfer—pumping and storing water during periods of low electricity prices or surplus renewable generation, and generating power during peak price periods or when renewable output is insufficient. This mechanism achieves arbitrage and enhances the overall economic efficiency of the system.

(3) Electricity Market Interface

The HWS system participates as a unified entity in the day-ahead energy market. Decision-makers must collaboratively optimize the generation schedule of conventional hydropower, the pumping/generating states and power of the horizontal pumped storage, and formulate an optimal market bidding curve, based on multi-faceted uncertainty fore-

casts for wind, solar, inflow, and electricity prices. This aims to maximize the system's total expected revenue while ensuring operational robustness under uncertain conditions.

3. Uncertainty Modeling

3.1. Extraction of Typical Scenarios for Wind and Solar Based on Morphological Similarity

The stochasticity and spatiotemporal correlation of wind and solar constitute one of the core uncertainties faced by HWS systems. Direct optimization using all historical data or Monte Carlo random sampling leads to the curse of dimensionality in computation, and the generated scenario set may contain significant redundancy, making it difficult to characterize the typical spatiotemporal morphological features of the output. Traditional clustering methods (e.g., K-means) typically rely on Euclidean distance, which requires strict alignment of sequences. However, renewable power output often exhibits significant temporal phase shifts and local deformations. Euclidean distance amplifies such differences, resulting in the misclassification of morphologically similar but time-shifted curves into distinct categories.

To address this, this section introduces a hierarchical clustering method based on Dynamic Time Warping (DTW). By allowing non-linear and flexible alignment of the time axis, DTW can effectively capture the essential similarity of curves at the morphological level, aligning better with the physical characteristics of renewable power output. The specific implementation steps are as follows:

1. **Data Preprocessing:** Historical renewable power output data is collected, divided into daily or specified time intervals, and normalized to the $[0, 1]$ range to eliminate differences in installed capacity and focus on curve morphology.
2. **Distance Matrix Calculation:** Based on the DTW algorithm, the morphological distance between all pairs of output curves is computed to construct a distance matrix.
3. **Hierarchical Clustering:** An agglomerative hierarchical clustering algorithm is employed, using the DTW distance matrix as input to construct a clustering dendrogram.
4. **Scenario Extraction and Probability Assignment:** The optimal number of clusters is determined using metrics such as the silhouette coefficient. The central curve of each cluster is selected as a typical output scenario, and the proportion of samples in that cluster is assigned as the occurrence probability of the scenario.

This method can extract a concise and highly representative set of typical scenarios from vast historical data, providing structured, high-quality uncertainty inputs for subsequent optimization models. This significantly enhances computational efficiency while ensuring model accuracy.

Wind and solar are clustered separately using the above procedure, yielding two independent scenario sets with probabilities θ_w^i ($i = 1, \dots, N_w$) and θ_s^j ($j = 1, \dots, N_s$), respectively. Assuming independence between wind and solar power outputs, the joint scenarios ζ for the stochastic optimization are formed as the Cartesian product of these two sets. Each joint scenario represents a unique combination (wind scenario i , solar scenario j) with probability $\theta_\zeta = \theta_w^i \times \theta_s^j$.

3.2. Uncertainty Modeling of Runoff Inflow Using IGDT

To address the intraday forecast errors in runoff inflow, this section employs the IGDT for characterization. IGDT does not require assuming a probability distribution for the uncertain parameter; instead, it describes the possible fluctuation range through an uncertainty set and performs robust decision-making based on this. The actual inflow $\tilde{I}_t$ in period t is defined to belong to the following interval-based set:

$$U(\alpha) = \{i_t : i_t \in [\hat{i}_t(1 - \alpha), \hat{i}_t(1 + \alpha)]\} \quad (1)$$

This set describes that the actual inflow sequence i_t may lie within a “pipe” centered on the predicted sequence $\hat{i}_t$. The width of the pipe is uniformly controlled by α . $\alpha = 0$ indicates complete confidence in the prediction (no uncertainty); $\alpha > 0$ indicates that prediction errors are considered.

3.3. Modeling of Price Acceptance Probability Based on Historical Data Statistics

In the day-ahead market, whether a generator’s bid is accepted depends directly on its price relative to the market clearing price. To internalize this market mechanism within the optimization model, this section assumes that historical clearing prices can reflect the statistical patterns of market prices. By statistically analyzing historical clearing price data for the same time periods and fitting their probability distributions, a price acceptance probability function is constructed to quantify the likelihood that any given bid will be accepted by the market. The specific modeling steps are as follows:

1. Data Preprocessing: Historical day-ahead market clearing price data is collected to form clearing price sequences λ_t for each time period.
2. Distribution Fitting: For each trading period, a probability distribution is fitted to its historical clearing price samples. Distributions such as the normal distribution, log-normal distribution, or generalized Pareto distribution may be selected to account for both general patterns and extreme price events.
3. Function Definition: Based on the fitted cumulative distribution function $F_t(x)$, the price acceptance probability function is defined as:

$$\psi(\lambda_t) = 1 - F_t(\lambda_t) \quad (2)$$

The probability that bid λ_t is accepted equals the probability that the historical clearing price is lower than that bid.

This method is directly based on observable, publicly available market data, avoids complex assumptions, and offers the advantages of transparency and ease of implementation. The resulting probability values can be directly embedded into the subsequent robust optimization model as key inputs for calculating expected revenue, thereby effectively incorporating the uncertainty of market clearing into the decision-making framework.

4. A Hybrid Stochastic-IGDT Bidding Model with Market Clearing Risks

4.1. Overall Framework and Concept of the Model

The core of this study is to establish an integrated robust optimization framework that combines physical operation with market decision-making. The model employs IGDT as its overarching architecture. Its fundamental logic is to obtain the most robust strategy—achieving “immunity” against unknown risks—by maximizing the range of uncertainty perturbations the system can withstand, without relying on precise probability distributions. Within this framework: Inflow uncertainty is described by the IGDT set $U(\alpha)$; the system must be immune to any fluctuation within this set. Wind-solar randomness is captured by a scenario set and its associated probabilities; the system must satisfy the revenue requirement in expectation across all possible scenarios. Market-clearing uncertainty is internalized via a price-acceptance probability function, which transforms clearing risk into expected-revenue calculations.

In essence, the framework performs a “stress test” on the stochastic uncertainties (wind/solar and market) within the strictest boundaries of epistemic uncertainty (inflow), thereby producing a set of dispatch and bidding strategies that guarantee performance under multiple uncertainties.

4.2. The Proposed IGDT-Based Model

4.2.1. Objective Function

$$\max \alpha \quad (3)$$

$$\text{s.t. } \min_{i_t \in U(\alpha)} \sum_{\xi \in \Xi} \theta_{\xi} R_{\xi}(\lambda_t) \geq R_c \quad (4)$$

$$R_c = \gamma R_0 \quad (5)$$

$$\text{all physical and operational constraints} \quad (6)$$

Equation (3) seeks to maximize the uncertainty gap α . It reflects the core philosophy of the IGDT model, which is to find an operational strategy capable of withstanding the greatest possible deviation of uncertain parameters—such as forecast errors for inflow—while still guaranteeing that a minimum acceptable level of performance is achieved. Equation (4) defines the robustness requirement. It ensures that the expected system revenue remains above a predefined critical threshold R_c , even under the worst-case realization of uncertainties within the expanded set $U(\alpha)$. R_c is the product of the deterministic revenue R_0 and the reduction coefficient γ ($\gamma < 1$). γ is a reduction coefficient set by the decision-maker according to risk preference. A smaller γ indicates that the decision-maker is willing to sacrifice more revenue in exchange for stronger robustness. This is the central safeguard in the IGDT robustness formulation, linking the uncertainty budget α directly to a guaranteed performance level. All decision variables must adhere to the complete set of physical laws and engineering limits governing the hybrid system, as shown in Equation (6). These detailed constraints, presented next, ensure the feasibility and safety of the proposed operation schedule.

4.2.2. Constraints

(1) Revenue Calculation

$$R_{\xi} = \sum_{t \in \mathbb{T}} P_{t,\xi}^{\text{hwsp}} \lambda_t \psi(\lambda_t) \Delta t \quad \forall \xi \in \Xi \quad (7)$$

The revenue in a given scenario ξ and time period is calculated as the product of the system's net power output offered to the market, the market bidding price, the probability of the offer being accepted at that price, and the duration of the period. This formulation explicitly integrates the market mechanism, via the price acceptance probability function $\psi(\cdot)$, into the financial model of the plant.

(2) Total System Output

$$P_{t,\xi}^{\text{hwsp}} = P_t^{\text{hydro}} + P_{t,\xi}^{\text{wind}} + P_{t,\xi}^{\text{solar}} - P_t^{\text{pump}} \quad \forall t \in \mathbb{T}, \forall \xi \in \Xi \quad (8)$$

This equation defines the total net power output of the hydropower-wind-solar-pumped storage (HWSP) system available for market sale. It deducts the power consumed by the pumping units, which represents a load on the system.

(3) Water Balance

$$V_{t+1} = V_t + (i_t - g_t - s_t + \pi_t) \Delta t \quad \forall t \in \mathbb{T} \quad (9)$$

The fundamental continuity equation for the reservoir, dictating that the change in storage volume over a time step equals the net inflow.

(4) Discharge Constraint

$$q_t = g_t + s_t - \pi_t \quad \forall t \in \mathbb{T} \quad (10)$$

The total discharge flowing out of the dam and passing the power plant tailrace is defined here. It is the sum of water passed through the turbines and any spillage, reduced by the portion of that downstream flow which is subsequently pumped back upstream.

(5) Power Generation Function

$$P_t^{\text{hydro}} = 9.81 \times 10^{-6} \rho_w \eta^{\text{hydro}} q_t h_t^{\text{hydro}} \quad \forall t \in \mathbb{T} \quad (11)$$

This is the physical equation governing hydropower production.

(6) Forebay Water Level and Reservoir Storage Constraint

$$Z_t = Z(V_t) \quad \forall t \in \mathbb{T} \quad (12)$$

A nonlinear, monotonic relationship links the water surface elevation at the dam (forebay level) to the volume of water stored in the reservoir.

(7) Tailrace Water Level and Discharge Constraint

$$l_t = l(q_t) \quad \forall t \in \mathbb{T} \quad (13)$$

The water level immediately downstream of the power plant (tailrace level) is not constant but rises with increasing total discharge due to backwater effects.

(8) Net Head Constraint of Hydropower

$$h_t^{\text{hydro}} = \frac{Z_t + Z_{t+1}}{2} - l_t - h_{\text{loss}} \quad \forall t \in \mathbb{T} \quad (14)$$

The net head effective for power generation is the average forebay water level across the time step (approximating the dynamic process), minus the tailwater level, and further reduced by hydraulic losses in the penstocks and intake structures. Net head is a primary determinant of generation efficiency and output.

(9) Pump Water Extraction Constraint

$$P_t^{\text{pump}} = \frac{9.81 \times 10^{-6} \rho_w h_t^{\text{pump}} \pi_t}{\eta^{\text{pump}}} \quad \forall t \in \mathbb{T} \quad (15)$$

The power required to operate the pumped storage units is calculated similarly to generation, but in reverse. It is proportional to the volume flow rate being pumped and the required pumping head, and inversely proportional to the combined efficiency of the pump and motor.

(10) Net Head Constraint of Pump

$$h_t^{\text{pump}} = \frac{Z_t + Z_{t+1}}{2} - l_t + h_{\text{loss}} \quad \forall t \in \mathbb{T} \quad (16)$$

The pumping head is also the difference between the average forebay level and the tailwater level. However, for pumping, the hydraulic losses in the conduits represent an additional energy burden that must be overcome, so they are added to the geodetic head difference.

(11) Boundary Constraints

$$V^{\min} \leq V_t \leq V^{\max} \quad \forall t \in \mathbb{T} \quad (17)$$

$$Z^{\min} \leq Z_t \leq Z^{\max} \quad \forall t \in \mathbb{T} \quad (18)$$

$$q^{\min} \leq q_t \leq q^{\max} \quad \forall t \in \mathbb{T} \quad (19)$$

$$g^{\min} \leq g_t \leq g^{\max} \quad \forall t \in \mathbb{T} \quad (20)$$

$$\pi^{\min} \leq \pi_t \leq \pi^{\max} \quad \forall t \in \mathbb{T} \quad (21)$$

$$P_t^{\text{hydro}} \leq N^{\text{hydro}} \quad \forall t \in \mathbb{T} \quad (22)$$

$$P_t^{\text{pump}} \leq N^{\text{pump}} \quad \forall t \in \mathbb{T} \quad (23)$$

$$Z_0 = Z^{\text{initial}} \quad (24)$$

$$Z_{\|\mathbb{T}\|} = Z^{\text{final}} \quad (25)$$

These equations collectively define the safe and feasible operational limits for each component of the hybrid energy system. Equations (17)–(20) specify the upper and lower bounds for reservoir storage, water level, total discharge, and generation discharge, respectively, to ensure the safety and compliance of reservoir operation. Equation (21) constrains the pumping flow within the technical capacity of the pumping equipment. Equations (22) and (23) represent physical capacity limits, ensuring that hydropower output does not exceed its installed capacity and that pumping power remains within its rated capacity, respectively. Finally, Equations (24) and (25) set the water level boundary conditions at the beginning and end of the scheduling horizon. The former anchors the optimization to a known initial state, while the latter defines a specific target water level at the period's end to ensure cyclic operation or meet long-term scheduling objectives. Together, these constraints form the hard boundaries for the safe, stable, and compliant operation of the system.

4.3. Solution Methodology

The proposed IGDT-based optimization model is originally a bi-level problem. By representing reservoir inflow using the worst-case expectation under the IGDT framework (i.e., $i_t = (1 - \alpha)\hat{i}_t$), it is reduced to a single-level deterministic non-convex nonlinear problem, solved by Particle Swarm Optimization (PSO) [49].

Based on domain experience and relevant literature references, the PSO parameters are configured as follows: the swarm size is set to 50 to balance global exploration capability and computational efficiency; a linearly decreasing inertia weight strategy is adopted, with an initial weight $\omega_{\text{start}} = 0.9$ and a final weight $\omega_{\text{end}} = 0.4$, enhancing global search ability in early iterations and facilitating local exploitation in later stages; both the cognitive and social learning factors are set to 2.0, consistent with standard PSO configurations; the termination criterion is a maximum of 200 iterations, beyond which preliminary tests show the objective value has stabilized.

Regarding constraint handling, feasibility checks are performed on the decision variables of all particles after each iteration update: for reservoir water level violations, values exceeding the upper bound $Z^{\max}$ are clipped to $Z^{\max}$, and values below the lower bound $Z^{\min}$ are clipped to $Z^{\min}$; other physical variables (e.g., generation discharge, pumping flow, power output) are similarly repaired by clipping to their respective bounds. Subsequently, each particle is checked against coupled constraints (reservoir continuity and terminal water level). Infeasible particles are either repaired via spillage adjustments (with spillage recorded) or discarded if no feasible repair exists, ensuring only feasible particles remain.

5. Case Study

5.1. Case Settings

This study takes a typical water conservancy hub in Hubei Province, China, as the research object. The hub possesses multi-year regulation capability. After undergoing horizontal pumped storage retrofitting, it forms a HWS multi-energy complementary system together with surrounding wind farms and solar power stations. The total installed capacity of the system is 2150 MW, consisting of 350 MW of conventional hydropower, 1200 MW of wind power, and 600 MW of solar power. The conventional hydropower unit is equipped with an added pumping station, forming a 600 MW pumping unit.

The actual data used in this case are sourced as follows: the reservoir inflow is based on operational forecast data; the wind power, solar power, and day-ahead market prices are all derived from three-year (2022–2024) real historical data, reflecting actual climate patterns and market volatility. The model is programmed in Python 3.9, using NumPy and SciPy for numerical computations, and custom-coded PSO. Plotting is done with Matplotlib (version: 3.10.6). To mitigate the effects of randomness, each experimental case is independently repeated ten times, and the run with the optimal objective function value is taken as the final result. The solving environment is the Windows 11 operating system, with an Intel(R) Core(TM) i7-13850HX CPU (2.10 GHz) and 32 GB of RAM.

A comparative model is designed to comprehensively evaluate the performance of the proposed scheme: a conventional hydro-wind-solar (CHWS) hybrid system without the horizontal pumped storage retrofit. Considering the inflow characteristics and electricity price fluctuation patterns of the studied case in Hubei Province, experiments are conducted under two typical hydrological scenarios—summer flood season and autumn flood season—to provide an in-depth analysis of the impact of energy storage retrofitting on the system's market revenue and operational robustness under different inflow conditions.

5.2. Uncertainty Result

5.2.1. Wind and Solar

The clustering results of wind and solar power are shown in Figure 2 and Table 1. Hierarchical clustering based on the DTW distance matrix was performed, with the optimal number of clusters determined by silhouette coefficients. Both wind and solar were clustered into 2 categories, with silhouette scores of 0.68 (wind) and 0.72 (solar), indicating good clustering quality. Wind and solar are clustered separately to preserve their temporal characteristics and output patterns.

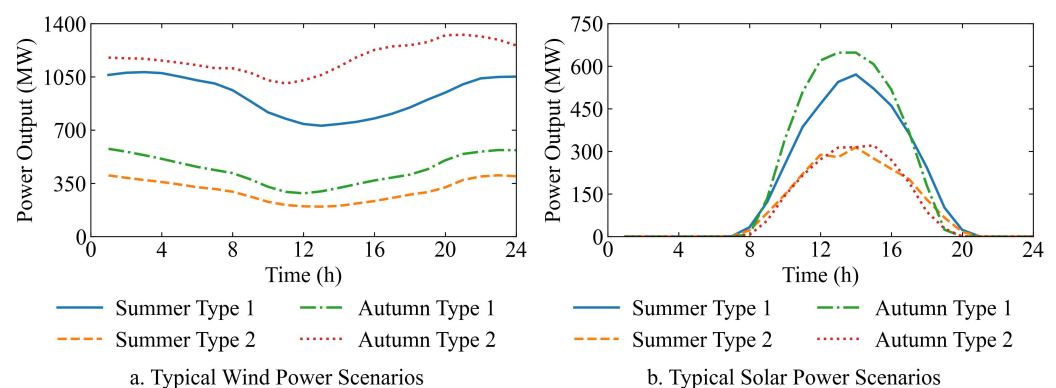


Figure 2. Clustering results of wind and solar power.

The clustering results reveal distinct characteristics for each energy source. Wind power shows significant volatility and pronounced diurnal patterns, with output peaks often occurring at night. In contrast, solar power generation is entirely concentrated during

daylight hours, with zero output at night. As a result, its variability depends largely on weather conditions within the daytime window.

Scenario probabilities are assigned based on the proportion of historical samples in each cluster, as detailed in Table 1. The two wind scenarios represent high-wind and low-wind conditions, while the two PV scenarios represent clear-sky and cloudy-sky conditions, collectively capturing the typical operational patterns of renewable generation at the study site.

Table 1. Scenario probabilities for wind and solar.

Generation Type	Scenario	Rate
Wind	Summer Type 1	30.43%
	Summer Type 2	69.57%
	Autumn Type 1	75.82%
	Autumn Type 2	24.18%
Solar	Summer Type 1	67.39%
	Summer Type 2	32.61%
	Autumn Type 1	67.03%
	Autumn Type 2	32.97%

5.2.2. Price Acceptance Probability Function

The empirical distribution of historical clearing prices exhibits an S-shaped nonlinear pattern, characterized by high probability density in the mid-price range and lower density at both extremes. To capture this pattern with a smooth, analytically tractable function, we adopt the Sigmoid function as the fitting basis. Although the Particle Swarm Optimization (PSO) solver does not require gradient information, a continuously differentiable acceptance-probability function helps maintain a smooth objective-function landscape, which stabilizes the search process and improves convergence reliability. The fitted Sigmoid function is given in Equation (26).

$$\psi(\lambda_t) = 1 - \frac{1}{1 + e^{-k_t(\lambda_t - \lambda_{t,0})}} \quad (26)$$

This method achieves data dimensionality reduction (using two parameters, k_t and λ_0 , to characterize a large volume of historical data) and endows the parameters with clear physical meanings (λ_0 represents the reference price, and k_t represents price sensitivity). Due to space limitations, the specific parameters for each period and their corresponding R^2 values are provided in Appendix A.

5.3. Bidding Result and Analysis

5.3.1. Sensitivity Analysis

To investigate the impact of risk preference on decision-making, this study selects four typical reduction coefficients ($\gamma = 97.5\%, 95.0\%, 92.5\%, 90.0\%$) for a parametric analysis of the critical revenue threshold R_c . These values are empirically determined based on engineering practice and parametric analysis requirements, consistent with the common IGDT practice of employing multi-level reduction coefficients to illustrate risk preference variations. Najafi et al. [16], Rezaei et al. [18], Mazidi et al. [19], Tan et al. [26], Shafiekhani et al. [29], closely aligned with the acceptable revenue loss range (5–10%) indicated by dispatch decision-makers in our cooperative enterprise surveys. This section will provide a detailed discussion across three dimensions: robustness trade-offs, multi-energy complementary scheduling mechanisms, and market bidding responses.

Figure 3 illustrates the trend of the system uncertainty gap with respect to the reduction coefficient under two typical hydrological scenarios: the summer flood season and the autumn flood season.

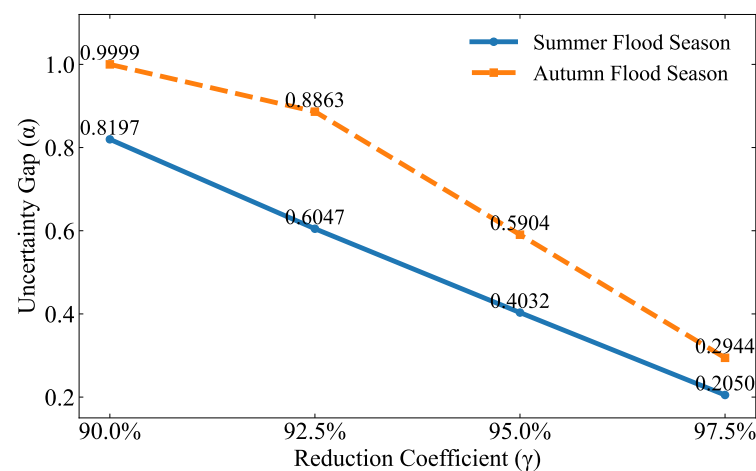


Figure 3. The relationship of uncertainty gap and reduction coefficient.

(1) Inverse Relationship Between Revenue and Robustness

Whether during the summer or autumn flood season, the uncertainty gap significantly decreases as the reduction coefficient increases. When it is set to 97.5% (i.e., decision-makers pursue a higher guaranteed revenue), the system can only tolerate a relatively small range of inflow forecast errors (e.g., a summer flood season gap of 0.2050). Conversely, when it is relaxed to 90.0%, the system's immunity to uncertainty is greatly enhanced (the summer flood season gap increases to 0.8197). This quantifies the "revenue-risk" trade-off inherent in scheduling decisions: if decision-makers are willing to sacrifice part of the potential revenue expectation, the model can reserve more operational flexibility to withstand more severe inflow fluctuation risks.

(2) Seasonal Robustness Differences

Under the same risk preference level, the uncertainty gap during the autumn flood season consistently outperforms that of the summer flood season. This indicates that under autumn hydrological conditions, the system possesses stronger regulation elasticity and anti-interference capability. This may be attributed to the relatively stable baseflow characteristics in autumn and a higher adequacy of system capacity in handling the flow rates typical of this season.

5.3.2. Scheduling Strategy and Market Response Analysis

Figures 4–6 jointly illustrate the system's physical scheduling strategies and market response behaviors during the summer and autumn flood seasons. By analyzing three figures together, the system's "buy-low, sell-high" arbitrage logic becomes clearly evident.

First, the system exhibits high sensitivity to day-ahead market electricity price signals. **Low-Price Energy Storage:** As seen in Figures 4 and 6a, during low-price periods in the early morning of the summer flood season, horizontal units operate intensively in pumping mode. Consequently, the reservoir water level rises significantly, converting low-cost electrical energy into stored potential energy. **High-Price Power Generation:** During peak price periods, the pumped-storage units cease pumping and, in coordination with conventional units, focus on power generation, leading to a rapid decline in water level. This inter-temporal energy shifting not only maximizes economic revenue through market arbitrage but also effectively absorbs the wind power prevalent during nighttime hours.

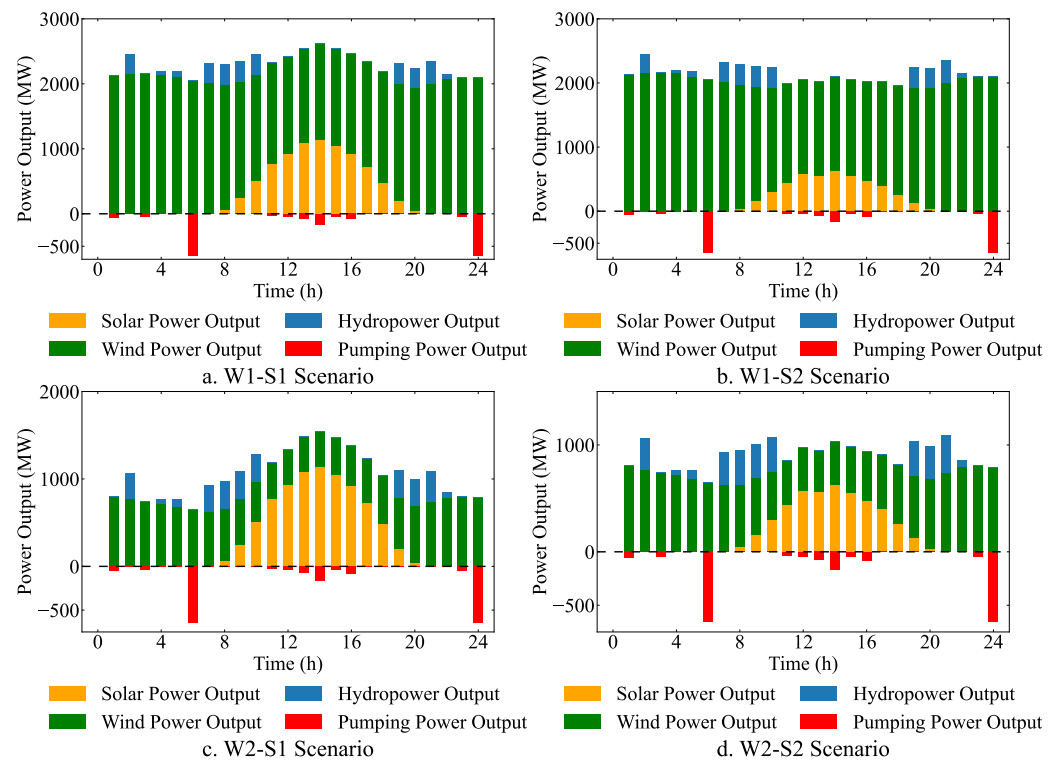


Figure 4. Power output of various generation sources during the summer flood season ($\gamma = 97.5\%$).

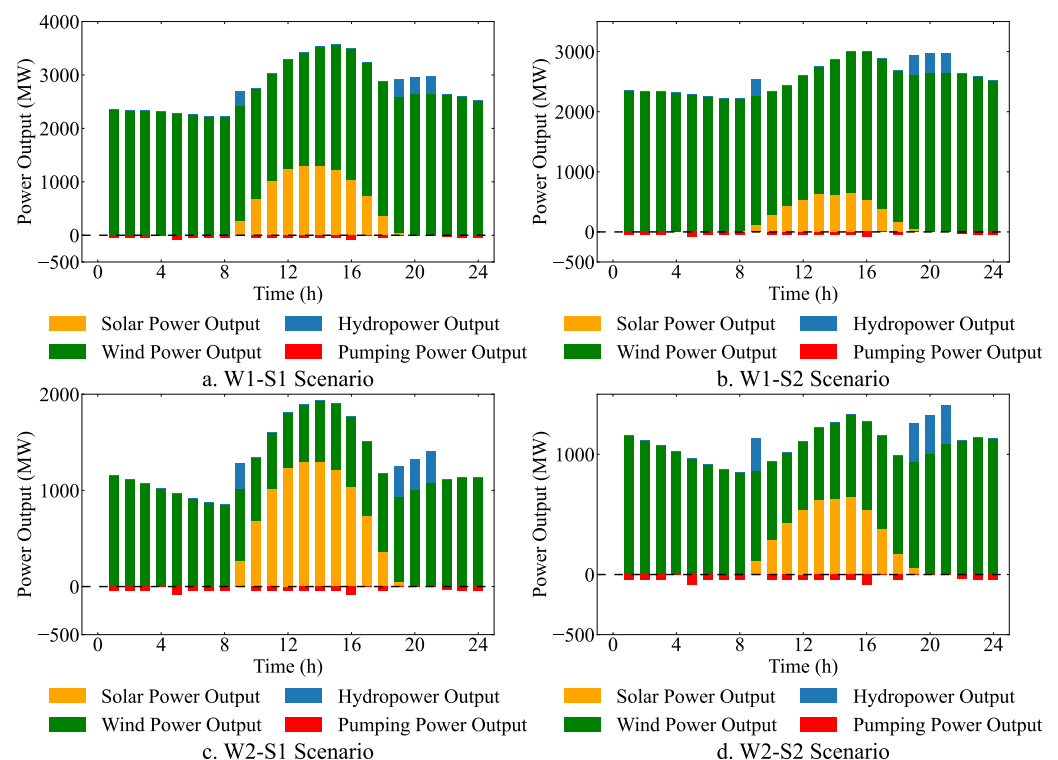


Figure 5. Power output of various generation sources during the autumn flood season ($\gamma = 97.5\%$).

Second, the system exhibits periodicity of reservoir operation. The water level process shown in Figure 6 indicates that after a 24-h “pump-generate” cycle, the final water level accurately returns to the target level. This demonstrates that the proposed model can strictly ensure the medium-to-long-term water balance of the reservoir while executing high-frequency market arbitrage operations, achieving a unification of short-term economic efficiency and long-term resource sustainability.

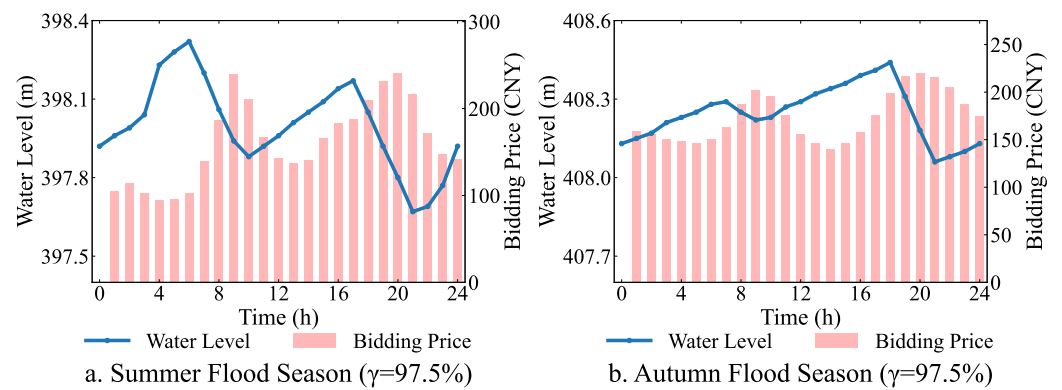


Figure 6. Water level and bidding price of summer and autumn flood season.

5.4. Model Comparison

Table 2 shows that there are significant differences in robustness between conventional hydropower and horizontal pumped storage during the summer flood season. The uncertainty gap (α) of CHWS system is generally low, indicating that during the summer and autumn flood season, it is constrained by strong hydraulic limitations, exhibits poor regulation flexibility, is highly sensitive to fluctuations in market prices or inflow, and has weak risk resistance.

Table 2. Comparison of HWS system and PHWS system.

Season	Type	γ	α
summer flood season	HWS system	97.5%	0.2050
		95.0%	0.4032
		92.5%	0.6047
		90.0%	0.8197
	CHWS system	97.5%	0.0100
		95.0%	0.2746
		92.5%	0.3760
		90.0%	0.5391
autumn flood season	HWS system	97.5%	0.2944
		95.0%	0.5904
		92.5%	0.8863
		90.0%	0.9999
	CHWS system	97.5%	0.1892
		95.0%	0.2919
		92.5%	0.3852
		90.0%	0.4456

In contrast, HWS system demonstrates exceptional robustness. Under the same revenue expectation (e.g., $\gamma = 0.900$), its uncertainty tolerance ($\alpha = 0.8197$) is significantly higher than that of CHWS system ($\alpha = 0.5391$). This highlights the decisive role of pumped storage in enhancing the robustness of multi-energy complementary systems in complex market environments. Leveraging its bidirectional conversion characteristics, pumped storage ensures that the system can meet expected economic performance indicators even under highly volatile conditions.

6. Conclusions

This study addresses the multiple uncertainty challenges faced by a hybrid hydro-wind-solar system with horizontal pumped storage in the day-ahead electricity market,

proposing a robust bidding and dispatch optimization method based on hybrid uncertainty modeling. The main conclusions are as follows: (1) A hybrid uncertainty framework is developed, combining scenario-based modeling for renewables, IGDT for runoff inflow, and a statistical price-acceptance function for market mechanisms. (2) An IGDT-based robust optimization model is established to maximize the system's tolerable uncertainty range while guaranteeing revenue above a critical threshold under worst-case inflow. (3) The model provides a practical decision-support tool that co-optimizes dispatch and bidding, effectively balancing economic performance and robustness in real-world case studies.

This study not only provides a methodology and tools for the market operation of HWS systems, but its core idea of “hybrid modeling, robust decision-making” can also be extended to other decision-making scenarios in new power systems with high proportions of renewable energy and coexisting multiple types of uncertainties.

This study has several limitations: (1) the system is assumed to be a price-taker, neglecting the impact of its own bids on market-clearing prices; (2) the price-acceptance function is derived from historical statistics and may not capture future changes in market structure; (3) intra-day adjustments, ancillary service markets, and other multi-stage trading mechanisms are not considered. Future work will incorporate bilevel game-theoretic models, dynamic market-mechanism modeling, and multi-timescale coordinated bidding.

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Data Availability Statement: The numerical examples presented in this paper are derived from a real-world engineering application. Owing to proprietary constraints, the underlying data cannot be shared. Nevertheless, the associated computational code may be provided upon reasonable request.

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Nomenclature

The following abbreviations and symbols are used in this manuscript:

Abbreviations

HWS	horizontal pumped storage-wind-solar
¥	Chinese currency unit

Sets & Indices

ξ, Ξ	scenario index and set
$t, \mathbb{T}$	hour index and set

Variables

$U()$	expanded uncertainty set
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α	uncertainty gap
$\hat{i}_t, i_t$	forecast/actual value of inflow volume at period t (m ³)
$\psi(\lambda_t)$	price acceptance probability function at bid price λ_t
λ_t	electricity price (¥/kWh)
θ_ξ	probability of scenario ξ
R_ξ	total revenue of scenario ξ (¥)
R_c	critical performance (¥)
R_0	expected performance under certain scenarios (¥)
γ	risk-aversion parameter (¥)
$p_{\xi,t}^{hws}$	total system output of scenario ξ at period t (MW)
Δt	time step (3600 s, equivalent to 1 h)
$p_{t,\xi}^{wind}, p_{t,\xi}^{solar}$	wind/solar output of scenario ξ at period t (MW)
p_t^{hydro}, p_t^{pump}	hydropower output/pumping power at period t (MW)
ρ_w	water density (equals 1000kg/m ³)
V_t, V_{t+1}	reservoir storage at period $t/t + 1$ (m ³)
g_t, s_t, q_t, π_t	generation/spillage/total discharge/pumping flow at period t (m ³ /s)
$\eta^{hydro}, \eta^{pump}$	generating/pumping efficiency
h_t^{hydro}, h_t^{pump}	net/pumping head at period t (m)
Z_t	water level at period t (m)
l_t	tailrace level at period t (m)
h_{loss}	head loss (m)
V^{min}, V^{max}	storage bounds (m ³)
Z^{min}, Z^{max}	water level bounds (m)
q^{min}, q^{max}	discharge bounds (m ³ /s)
g^{min}, g^{max}	generation discharge bounds (m ³ /s)
π^{min}, π^{max}	pump discharge bounds (m ³ /s)
N^{hydro}, N^{pump}	hydropower installed capacity/pumping rated capacity (MW)
$ \mathbb{T} $	The cardinality of set $\mathbb{T}$
$Z^{initial}, Z^{final}$	initial/final water level (m)

Appendix A. Price Acceptance Probability Function

The main parameters of the price acceptance function for each period—autumn flood season and summer flood season—are shown in Table A1.

Table A1. Price acceptance probability function parameters.

t	Autumn Flood Season			Summer Flood Season		
	k_t	$\lambda_{t,0}$	R^2	k_t	$\lambda_{t,0}$	R^2
0	8.48×10^{-2}	187.75	0.99	2.06×10^{-2}	113.71	0.97
1	8.36×10^{-2}	185.84	1.00	1.84×10^{-2}	118.99	0.97
2	8.57×10^{-2}	179.52	0.99	1.99×10^{-2}	103.48	0.94
3	8.36×10^{-2}	177.63	0.99	2.20×10^{-2}	97.94	0.96
4	7.95×10^{-2}	176.62	0.99	2.15×10^{-2}	98.90	0.95
5	8.72×10^{-2}	179.34	0.99	2.03×10^{-2}	107.11	0.95
6	8.01×10^{-2}	193.98	0.99	1.56×10^{-2}	154.00	0.98
7	4.58×10^{-2}	230.94	1.00	1.33×10^{-2}	216.94	0.98
8	2.82×10^{-2}	256.83	0.98	1.10×10^{-2}	284.97	0.99
9	3.04×10^{-2}	248.05	0.99	1.34×10^{-2}	255.90	0.99
10	3.30×10^{-2}	222.97	0.99	1.70×10^{-2}	202.72	0.99
11	3.76×10^{-2}	197.74	0.98	1.77×10^{-2}	170.98	0.99
12	3.70×10^{-2}	186.83	0.99	1.70×10^{-2}	159.60	0.99
13	3.83×10^{-2}	178.98	0.99	1.63×10^{-2}	157.02	0.99
14	4.08×10^{-2}	185.64	0.98	1.31×10^{-2}	177.44	0.99
15	3.94×10^{-2}	199.79	0.99	1.26×10^{-2}	205.54	0.99

Table A1. Cont.

t	Autumn Flood Season			Summer Flood Season		
	k_t	$\lambda_{t,0}$	R^2	k_t	$\lambda_{t,0}$	R^2
16	3.44×10^{-2}	222.06	0.99	1.53×10^{-2}	229.76	1.00
17	3.15×10^{-2}	251.11	0.99	1.60×10^{-2}	263.74	0.99
18	4.49×10^{-2}	265.56	0.98	1.56×10^{-2}	293.20	0.98
19	4.34×10^{-2}	269.38	0.94	1.15×10^{-2}	297.27	0.97
20	4.75×10^{-2}	262.66	0.93	1.34×10^{-2}	264.03	0.99
21	6.11×10^{-2}	245.82	0.97	1.99×10^{-2}	215.29	0.99
22	7.03×10^{-2}	223.32	0.99	1.85×10^{-2}	177.67	0.96
23	7.77×10^{-2}	208.51	1.00	1.94×10^{-2}	143.79	0.97

Appendix B. The Mean and Standard Deviation of the Ten Results

The mean and standard deviation of the ten results are shown in Table A2.

Table A2. The mean and standard deviation of the ten results.

Flood Season	$\gamma = 90.0\%$	$\gamma = 92.5\%$	$\gamma = 95.0\%$	$\gamma = 97.5\%$
Autumn	0.2919 ± 0.0025	0.5814 ± 0.0064	0.8764 ± 0.0099	0.9999 ± 0.0000
Summer	0.1953 ± 0.0097	0.3875 ± 0.0157	0.5813 ± 0.0235	0.7841 ± 0.0357

References

1. Zambon, R.C.; Asce, M.; Cicolani, M.D. Seasonal, Daily, and Hourly Optimal Reservoir Operations of the Brazilian Hydro–Wind–Solar–Thermal Power System. In Proceedings of the World Environmental and Water Resources Congress, Anchorage, AK, USA, 18–21 May 2025.
2. Li, Y.; Yu, X.; Yu, S.; Wang, T.; Liu, B.; Cheng, C. Emerging Markets Bring Changes in China’s Hydropower Plants: An Exploration of China’s Electricity Market Reform Policy. In Proceedings of the World Environmental and Water Resources Congress 2018, Minneapolis, MN, USA, 3–7 June 2018; pp. 105–119. [\[CrossRef\]](#)
3. Liu, S.; Yang, Q.; Cai, H.; Yan, M.; Zhang, M.; Wu, D.; Xie, M. Market Reform of Yunnan Electricity in Southwestern China: Practice, Challenges and Implications. *Renew. Sustain. Energy Rev.* **2019**, *113*, 109265. [\[CrossRef\]](#)
4. Kopyt, M.; Piotrowski, P.; Baczynski, D. Short-Term Energy Generation Forecasts at a Wind Farm—A Multi-Variant Comparison of the Effectiveness and Performance of Various Gradient-Boosted Decision Tree Models. *Energies* **2024**, *17*, 6194. [\[CrossRef\]](#)
5. Dires, F.G.; Amelin, M.; Bekele, G. Inflow Scenario Generation for the Ethiopian Hydropower System. *Water* **2023**, *15*, 500. [\[CrossRef\]](#)
6. Feng, H.; Zhang, Y.; Lan, Z.; Wang, K.; Wang, Y.; Chen, S.; Feng, C. Developing China’s Electricity Financial Market: Strategic Design of Financial Derivatives for Risk Management and Market Stability. *Energies* **2024**, *17*, 5854. [\[CrossRef\]](#)
7. Hakami, A.M.; Hasan, K.N.; Alzubaidi, M.; Datta, M. A Review of Uncertainty Modelling Techniques for Probabilistic Stability Analysis of Renewable-Rich Power Systems. *Energies* **2022**, *16*, 112. [\[CrossRef\]](#)
8. Jordehi, A.R. How to Deal with Uncertainties in Electric Power Systems? A Review. *Renew. Sustain. Energy Rev.* **2018**, *96*, 145–155. [\[CrossRef\]](#)
9. Aasgård, E.K.; Fleten, S.E.; Kaut, M.; Midthun, K.; Perez-Valdes, G.A. Hydropower Bidding in a Multi-Market Setting. *Energy Syst.* **2019**, *10*, 543–565. [\[CrossRef\]](#)
10. Kongelf, H.; Overrein, K.; Klæboe, G.; Fleten, S.E. Portfolio Size’s Effects on Gains from Coordinated Bidding in Electricity Markets: A Case Study of a Norwegian Hydropower Producer. *Energy Syst.* **2019**, *10*, 567–591. [\[CrossRef\]](#)
11. Zhang, Y.; Cheng, C.; Cai, H.; Jin, X.; Jia, Z.; Wu, X.; Su, H.; Yang, T. Long-Term Stochastic Model Predictive Control and Efficiency Assessment for Hydro-Wind-Solar Renewable Energy Supply System. *Appl. Energy* **2022**, *316*, 119134. [\[CrossRef\]](#)
12. Yang, Z.; Wang, Y.; Yang, K. The Stochastic Short-Term Hydropower Generation Scheduling Considering Uncertainty in Load Output Forecasts. *Energy* **2022**, *241*, 122838. [\[CrossRef\]](#)
13. Yang, Z.; Yang, K.; Wang, Y.; Su, L.; Hu, H. Multi-Objective Short-Term Hydropower Generation Operation for Cascade Reservoirs and Stochastic Decision Making under Multiple Uncertainties. *J. Clean. Prod.* **2020**, *276*, 122995. [\[CrossRef\]](#)
14. Liao, S.; Liu, H.; Liu, B.; Zhao, H.; Wang, M. An Information Gap Decision Theory-Based Decision-Making Model for Complementary Operation of Hydro-Wind-Solar System Considering Wind and Solar Output Uncertainties. *J. Clean. Prod.* **2022**, *348*, 131382. [\[CrossRef\]](#)

15. Zhong, Z.; Fan, N.; Wu, L. A Hybrid Robust-Stochastic Optimization Approach for Day-Ahead Scheduling of Cascaded Hydroelectric System in Restructured Electricity Market. *Eur. J. Oper. Res.* **2023**, *306*, 909–926. [\[CrossRef\]](#)
16. Najafi, A.; Pourakbari-Kasmaei, M.; Jasinski, M.; Lehtonen, M.; Leonowicz, Z. A Medium-Term Hybrid IGDT-Robust Optimization Model for Optimal Self Scheduling of Multi-Carrier Energy Systems. *Energy* **2022**, *238*, 121661. [\[CrossRef\]](#)
17. Bafrani, A.A.; Rezazade, A.; Sedighizadeh, M. Robust Electrical Reserve and Energy Scheduling of Power System Considering Hydro Pumped Storage Units and Renewable Energy Resources. *J. Energy Storage* **2022**, *54*, 105310. [\[CrossRef\]](#)
18. Rezaei, N.; Ahmadi, A.; Khazali, A.; Aghaei, J. Multiobjective Risk-Constrained Optimal Bidding Strategy of Smart Microgrids: An IGDT-Based Normal Boundary Intersection Approach. *IEEE Trans. Ind. Inform.* **2019**, *15*, 1532–1543. [\[CrossRef\]](#)
19. Mazidi, M.; Monsef, H.; Siano, P. Design of a Risk-Averse Decision Making Tool for Smart Distribution Network Operators under Severe Uncertainties: An IGDT-inspired Augment ϵ -Constraint Based Multi-Objective Approach. *Energy* **2016**, *116*, 214–235. [\[CrossRef\]](#)
20. Liu, M.; Kong, X.; Lian, J.; Wang, J.; Yang, B. Distributionally Robust Coordinated Day-Ahead Scheduling of Cascade Pumped Hydro Energy Storage System and DC Transmission. *Appl. Energy* **2025**, *384*, 125449. [\[CrossRef\]](#)
21. Jin, X.; Liu, B.; Liao, S.; Cheng, C.; Zhang, Y.; Zhao, Z.; Lu, J. Wasserstein Metric-Based Two-Stage Distributionally Robust Optimization Model for Optimal Daily Peak Shaving Dispatch of Cascade Hydroplants under Renewable Energy Uncertainties. *Energy* **2022**, *260*, 125107. [\[CrossRef\]](#)
22. Zhang, Y.; Hu, X.; Wang, X.; Zhou, X.; Liu, Y.; Zhang, B.; Li, Y. Stochastic Bidding for Hydro–Wind–Solar Systems in Cross-Provincial Forward–Spot Markets: A Dimensionality-Reduced and Transmission-Aware Framework. *Energies* **2025**, *18*, 4222. [\[CrossRef\]](#)
23. Yu, S.; Zhang, J.; Cheng, C.; Shen, J. Bidding Optimization for Cascade Hydropower Plants in Multi-Regional Electricity Markets. *J. Clean. Prod.* **2024**, *435*, 140477. [\[CrossRef\]](#)
24. Xiao, Y.; Li, C.; Li, S.; Zhu, Q.; Chang, P.; Wu, T. Bidding-Clearing-Scheduling Coordinated Optimization for Cascade Hydro-Wind-Photovoltaic Alliances in Electricity Spot Markets: A Multi-Stage Framework. *Energy* **2025**, *335*, 138185. [\[CrossRef\]](#)
25. Li, X.; Liu, P.; Cheng, L.; Cheng, Q.; Zhang, W.; Xu, S.; Zheng, Y. Strategic Bidding for a Hydro-Wind-Photovoltaic Hybrid System Considering the Profit beyond Forecast Time. *Renew. Energy* **2023**, *204*, 277–289. [\[CrossRef\]](#)
26. Tan, Y.; Guan, L.; Huang, J.; Chen, L. A Multi-Stage Data-Driven IGDT-RO Model with Chance Compensation for Optimizing Bidding of RES Aggregator in Competitive Electricity Markets. *Int. J. Electr. Power Energy Syst.* **2023**, *154*, 109396. [\[CrossRef\]](#)
27. Tan, Y.; Guan, L. Hybrid Optimization for Collaborative Bidding Strategy of Renewable Resources Aggregator in Day-Ahead Market Considering Competitors' Strategies. *Int. J. Electr. Power Energy Syst.* **2023**, *145*, 108681. [\[CrossRef\]](#)
28. Shafiekhani, M.; Badri, A.; Shafie-khah, M.; Catalão, J.P. Strategic Bidding of Virtual Power Plant in Energy Markets: A Bi-Level Multi-Objective Approach. *Int. J. Electr. Power Energy Syst.* **2019**, *113*, 208–219. [\[CrossRef\]](#)
29. Shafiekhani, M.; Ahmadi, A.; Homaei, O.; Shafie-khah, M.; Catalão, J.P. Optimal Bidding Strategy of a Renewable-Based Virtual Power Plant Including Wind and Solar Units and Dispatchable Loads. *Energy* **2022**, *239*, 122379. [\[CrossRef\]](#)
30. Nazari, M.; Ardehali, M. Optimal Bidding Strategy for a GENCO in Day-Ahead Energy and Spinning Reserve Markets with Considerations for Coordinated Wind-Pumped Storage-Thermal System and CO₂ Emission. *Energy Strategy Rev.* **2019**, *26*, 100405. [\[CrossRef\]](#)
31. Abedinia, O.; Zareinejad, M.; Doranehgard, M.H.; Fathi, G.; Ghadimi, N. Optimal Offering and Bidding Strategies of Renewable Energy Based Large Consumer Using a Novel Hybrid Robust-Stochastic Approach. *J. Clean. Prod.* **2019**, *215*, 878–889. [\[CrossRef\]](#)
32. Jurasz, J.; Kies, A.; Zajac, P. Synergetic Operation of Photovoltaic and Hydro Power Stations on a Day-Ahead Energy Market. *Energy* **2020**, *212*, 118686. [\[CrossRef\]](#)
33. Ilak, P.; Kuzle, I.; Herencic, L.; Đakovic, J.; Ivan Rajsl, A. Market Power of Coordinated Hydro-wind Joint Bidding: Croatian Power System Case Study. *J. Mod. Power Syst. Clean Energy* **2022**, *10*, 531–541. [\[CrossRef\]](#)
34. Zhou, Y.; Wu, J.; Long, C.; Ming, W. State-of-the-Art Analysis and Perspectives for Peer-to-Peer Energy Trading. *Engineering* **2020**, *6*, 739–753. [\[CrossRef\]](#)
35. Manchalwar, A.D.; Patne, N.R.; Morey, C.D.; Pemmada, S. Prosumers and Retailers Based Decentralized Energy Trading Model in the Smart Grid Considering Network Constraints. *Int. J. Electr. Power Energy Syst.* **2024**, *160*, 110108. [\[CrossRef\]](#)
36. Zhan, B.; Feng, C.; Lin, Z.; Shao, X.; Wen, F. Peer-to-Peer Energy Trading among Prosumers with Voltage Regulation Services Provision. *Energies* **2023**, *16*, 5497. [\[CrossRef\]](#)
37. Xiong, X.; Qing, G.; Li, H. Blockchain-Based P2P Power Trading Mechanism for PV Prosumer. *Energy Rep.* **2022**, *8*, 300–310. [\[CrossRef\]](#)
38. Wang, R.; Chen, Y.; Li, E.; Che, L.; Xin, H.; Li, J.; Zhang, X. Joint Optimization of Energy Trading and Consensus Mechanism in Blockchain-Empowered Smart Grids: A Reinforcement Learning Approach. *J. Cloud Comput.* **2023**, *12*, 121. [\[CrossRef\]](#)
39. Nazari, M.H.; Biswas, A.K. Decentralized P2P Trading Based on Blockchain for Retail Electricity Markets. In Proceedings of the 2024 IEEE Power & Energy Society Innovative Smart Grid Technologies Conference (ISGT), Washington, DC, USA, 19–22 February 2024; pp. 1–5. [\[CrossRef\]](#)

40. Du, P.; Liu, Z.; Huang, B.; Jing, G.; Feng, L.; Yang, C. Blockchain Based Peer-To-Peer Energy Trading Between Wind Power Producer and Prosumers in Short-Term Market. *Front. Energy Res.* **2022**, *10*, 923292. [[CrossRef](#)]
41. P, M.; Salam, P.A. A Review of Peer-to-Peer Transaction Loss and Blockchain: Challenges and Drivers in the Roadmap to a Low-Carbon Future. *Front. Energy Res.* **2024**, *12*, 1397975. [[CrossRef](#)]
42. Erdayandi, K.; Cordeiro, L.C.; Mustafa, M.A. A Privacy-Preserving and Accountable Billing Protocol for Peer-to-Peer Energy Trading Markets. In Proceedings of the 2023 International Conference on Smart Energy Systems and Technologies (SEST), Mugla, Turkey, 4–6 September 2023; pp. 1–6. [[CrossRef](#)]
43. Tushar, W.; Saha, T.K.; Yuen, C.; Azim, M.I.; Morstyn, T.; Poor, H.V.; Bean, R. A Coalition Formation Game Framework for Peer-to-Peer Energy Trading. *Appl. Energy* **2020**, *261*, 114436. [[CrossRef](#)]
44. Vieira, G.; Zhang, J. Peer-to-Peer Energy Trading in a Microgrid Leveraged by Smart Contracts. *Renew. Sustain. Energy Rev.* **2021**, *143*, 110900. [[CrossRef](#)]
45. Haryani, I.; Aditya, I.A.; Nuraini, A.H.; Widiyanesti, S.; Alamsyah, A.; Nurhazizah, E.; Hakim, M.N.; Ruwani, T.; Arli, M.Z.D.; Girawan, N.D. Blockchain-Based Renewable Energy Certificate System in Indonesia. *Heliyon* **2025**, *11*, e42364. [[CrossRef](#)]
46. Evdokimov, V.; Kudin, A.; Chikhladze, V.; Artemchuk, V. A Blockchain Architecture for Hourly Electricity Rights and Yield Derivatives. *FinTech* **2025**, *5*, 2. [[CrossRef](#)]
47. Seven, S.; Yoldas, Y.; Soran, A.; Yalcin Alkan, G.; Jung, J.; Ustun, T.S.; Onen, A. Energy Trading on a Peer-to-Peer Basis between Virtual Power Plants Using Decentralized Finance Instruments. *Sustainability* **2022**, *14*, 13286. [[CrossRef](#)]
48. Blakers, A.; Stocks, M.; Lu, B.; Cheng, C. A Review of Pumped Hydro Energy Storage. *Prog. Energy* **2021**, *3*, 022003. [[CrossRef](#)]
49. Wang, X.; Li, Y.; Gong, S.; Hu, X.; Cheng, C. An Enhanced Micro-PSO Method to Deal with Asymmetric Electricity Markets Competition within Hydropower Cascade. *Appl. Energy* **2025**, *377*, 124235. [[CrossRef](#)]

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