

Article

Fault Location in Distribution Networks Using Apparent-Inductance-Based Algorithm [†]

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Abstract

Accurate fault location is essential for rapid service restoration in distribution networks. However, modern active distribution networks (ADNs) with high penetration of distributed energy resources (DERs) challenge conventional methods through multi-source fault contributions, bidirectional power flows, and converter-limited fault currents. This paper presents a time-domain fault location method for both passive distribution networks (PDNs) and ADNs, based on a three-sample apparent inductance estimator that uses local voltage and current measurements. The estimator exploits the strong correlation between line inductance and fault distance, with reduced sensitivity to fault resistance compared with classical impedance approaches. Its performance was evaluated on a 22 kV, 20 km distribution feeder, covering three fault types, four fault resistance levels (5–500 Ω), four fault locations, EN 50160 standard-compliant harmonic distortion, and DER penetration levels from 0 to 80%. Under ideal sinusoidal conditions, relative location errors remained below 2% for low-resistance faults. In ADNs, the method achieved errors below 5% for low-resistance faults across all fault types, with accuracy decreasing for high-resistance faults at high DER penetration. A sensitivity analysis confirmed robustness across the tested simulation conditions, covering the SNR, load current, THD, and DER penetration.

Keywords: apparent inductance; active distribution networks; distribution systems; distributed energy resources; fault location; fault resistance

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1. Introduction

Faults in electrical distribution networks are a major cause of power outages and reliability degradation in modern power systems. Recent statistics indicate that around 90% of power system outages result from distribution network disturbances [1]. Rapid and accurate fault location is, therefore, essential for efficient service restoration and continuous supply quality. The increasing deployment of digital relays, monitoring devices, and synchrophasor technologies has enhanced system observability. However, their application in distribution systems remains challenging due to feeder characteristics such as

high resistance-to-reactance (R/X) ratios, unbalanced operation, and variable loading conditions, compounded by the multi-lateral topology of most feeders, which introduces multiple parallel current paths that complicate impedance-based measurements [2]. These characteristics significantly limit the effectiveness of classical impedance-based fault location techniques that were originally developed for transmission networks.

These challenges are further reinforced by the ongoing structural transformation of distribution networks driven by the large-scale integration of distributed energy resources (DERs), including photovoltaic systems, wind turbines, battery storage units, and small-scale generators, increasingly connected at the medium- and low-voltage levels [3]. This evolution from traditionally passive distribution networks (PDNs) with unidirectional power flow to active distribution networks (ADNs) with bidirectional flows and intermittent generation improves system flexibility and sustainability, but also fundamentally alters the fault response. Multi-source fault current contributions, dynamic voltage profiles, and converter-limited fault currents challenge the protection and monitoring schemes that were originally designed for radial feeders [4–6].

Fault location techniques are commonly classified into impedance-based, travelling-wave-based, and data-driven approaches. Traditional single-ended impedance methods are attractive for their simplicity and compatibility with existing infrastructure [4,7], but exhibit poor accuracy in the presence of high fault resistance, waveform distortion, and measurement noise [8]. Time-domain formulations that exploit transient voltage and current information have been proposed to improve robustness [9,10], while steady-state matching and distance-metric approaches address complex feeder topologies [11]. More recently, inductance-oriented strategies have been introduced, exploiting the fact that the inductive component of the line impedance remains strongly correlated with distance even when resistive effects and fault resistance introduce significant uncertainty [12]. Travelling-wave methods can achieve high accuracy but require high-bandwidth sensors, precise time synchronization, and careful handling of wave reflections, limiting their large-scale deployment [13–15]. Data-driven approaches based on machine learning and graph-based models offer promising performance but often require extensive training data and raise concerns regarding generalization across different operating conditions [16,17]. Comprehensive reviews and industry standards summarize the advantages and practical limitations of these approaches [18,19].

Despite these advancements, many existing methods require high-frequency sampling, multiple synchronized measurement points, or detailed system models, limiting their applicability in real-world networks [20]. In ADNs specifically, the apparent impedance seen at a single terminal no longer corresponds directly to the physical line segment between the substation and the fault, as inverter-based resources limit and control their fault current output, leading to nontraditional current waveforms [5,6,17]. Recent studies have highlighted that protection systems in high-inverter-penetration networks may experience miscoordination, underreaching or overreaching of distance locators, and reduced sensitivity to high-resistance faults [6,21].

The proposed apparent inductance-based fault location method provides a promising alternative in this context. By directly exploiting the inductive relationship between voltage and current, they operate in the time domain, require only local measurements, and can be implemented using a small number of samples, making them well-suited for both passive and active distribution networks. The inductive component of the line impedance remains strongly correlated with distance even under adverse conditions [12]. In ADNs, however, the apparent inductance observed at the substation is also influenced by local DER injections, which may bias the inductance estimation and the inferred fault dis-

tance. To date, most studies addressing DER impacts have focused on protection coordination and relay performance rather than on the specific implications for inductance-based fault location accuracy [5,6,17,21].

This paper develops, validates, and applies a three-sample apparent inductance estimator for fault location in distribution networks. The estimator incorporates median filtering and diagnostic bounding to suppress false estimates caused by transients and noise. Its performance is evaluated across a range of fault types, fault locations, fault resistance levels, and EN 50160-compliant harmonic distortion [10].

The main contributions of this paper are summarized as follows:

- Algorithm application to passive and active networks: The apparent inductance-based fault location algorithm is systematically evaluated in the context of PDNs under not only the fundamental frequency but also EN 50160 standard-compliant harmonic distortions. The algorithm application in ADNs highlights how DER penetration alters the physical meaning of the apparent inductance used for distance-to-fault estimation.
- Signal processing for robust estimation: A three-sample apparent inductance estimator incorporating adaptive moving-average filtering, IQR-based outlier rejection, and median-based stable-window aggregation is developed and validated, providing reliable inductance estimation under EN 50160 standard-compliant harmonic distortion without dedicated harmonic compensation.
- Quantitative impact assessment: The influence of fault proximity to the measurement point, system harmonic distortion, and DER penetration levels on the voltage and current measurements, as well as on fault location accuracy, is quantified across various fault types and fault resistances.
- Practical design guidance for protection engineers: This study identifies the operating conditions under which the apparent inductance-based method remains reliable and those under which accuracy degrades significantly, providing quantitative performance boundaries that directly guide the adaptation of fault location schemes to the evolving architecture of active distribution networks with high DER penetration.

2. Method for Apparent Inductance-Based Fault Location in DS

2.1. Method Description

The apparent inductance-based algorithm belongs to the family of differential equation algorithms, which represent an important theme in line relaying. This algorithm estimates the fundamental frequency components of voltage and current to determine the impedance to the fault [22], as shown in Figure 1, where a single-line diagram of a faulted medium-voltage feeder is represented.

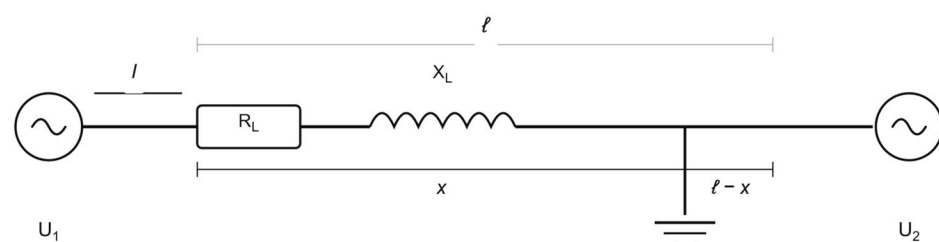


Figure 1. Single-line representation of a medium-voltage distribution line.

Differential equation algorithms, in contrast, are based on a system model and can be viewed as parameter estimation problems. Considering the single-phase model case from the faulted line shown in Figure 1, the differential equation relating the voltage and current seen by the relay can be written as shown in (1) [23,24]:

$$u(t) = Ri(t) + L \frac{di(t)}{dt} \quad (1)$$

Since $u(t)$ and $i(t)$ are measured, the line resistance R and apparent L_{app} can be estimated, and the fault distance d_f can be computed. To avoid numerical differentiation of measured quantities, ref. [22] integrates (1) over two consecutive intervals, yielding (2) and (3).

$$\int_{t_0}^{t_1} u(t)dt = R \int_{t_0}^{t_1} i(t)dt + L_{app}[i(t_1) - i(t_0)] \quad (2)$$

$$\int_{t_1}^{t_2} u(t)dt = R \int_{t_1}^{t_2} i(t)dt + L_{app}[i(t_2) - i(t_1)] \quad (3)$$

Approximating the integrals with the trapezoidal rule (4) and assuming a uniform sampling interval Δt , Equations (2) and (3) become (5) in terms of samples at k , $k+1$, and $k+2$.

$$\int_{t_0}^{t_1} u(t)dt = \frac{\Delta t}{2} [u(t_1) + u(t_0)] = \frac{\Delta t}{2} [u_1 + u_2] \quad (4)$$

$$\begin{bmatrix} \frac{\Delta t}{2}(i_{k+1} + i_k) & (i_{k+1} - i_k) \\ \frac{\Delta t}{2}(i_{k+2} + i_{k+1}) & (i_{k+2} - i_{k+1}) \end{bmatrix} \begin{bmatrix} R \\ L_{app} \end{bmatrix} = \begin{bmatrix} \frac{\Delta t}{2}(u_{k+1} + u_k) \\ \frac{\Delta t}{2}(u_{k+2} + u_{k+1}) \end{bmatrix} \quad (5)$$

The three voltage and current samples are sufficient to estimate the line resistance R and apparent inductance L_{app} [25], as shown in (6) and (7).

$$R = \frac{[(u_{k+1} + u_k)(i_{k+2} - i_{k+1}) - (u_{k+2} + u_{k+1})(i_{k+1} - i_k)]}{(i_{k+1} + i_k)(i_{k+2} - i_{k+1}) - (i_{k+2} + i_{k+1})(i_{k+1} - i_k)} \quad (6)$$

$$L_{app} = \frac{\Delta t}{2} \frac{[(i_{k+1} + i_k)(u_{k+2} + u_{k+1}) - (i_{k+2} + i_{k+1})(u_{k+1} - u_k)]}{(i_{k+1} + i_k)(i_{k+2} - i_{k+1}) - (i_{k+2} + i_{k+1})(i_{k+1} - i_k)} \quad (7)$$

2.2. The Proposed Apparent Inductance Algorithm

The proposed fault-location algorithm estimates the distance to the fault by evaluating the apparent inductance of the distribution feeder section between the measuring point and the fault. The method operates entirely in the time domain and relies on three successive samples of local voltage and current measurements, enabling fast response without the need for phasor estimation or sequence decomposition [26,27]. Let $u_a(t)$ denote the pre-fault phase-A voltage measured at the feeder head, and $u_{af}(t)$, the corresponding during-fault voltage following fault inception. The instantaneous voltage drop along the faulted feeder section is defined as shown in (8):

$$u_{drop}(t) = u_a(t) - u_{af}(t) \quad (8)$$

which represents the apparent voltage consumed by the feeder impedance up to the fault location. This quantity inherently captures both resistive and inductive effects of the line segment under fault conditions.

A model-based voltage drop is also computed using the lumped RL feeder model as shown in (9)

$$u_{RL} = R_{line}i_a(t) + L_{line} \frac{di_a(t)}{dt} \quad (9)$$

where $i_a(t)$ is the instantaneous phase-A current, and R_{line} and L_{line} denote the resistance and inductance of the feeder section between the relay location and the fault. The voltage u_{drop} or u_{RL} serves as a model-based reference that allows diagnostic comparison and validation of the estimator behavior. Both are evaluated to assess robustness against measurement noise, DC offsets, and modeling uncertainties.

The apparent inductance is computed from three consecutive samples (k , $k + 1$, and $k + 2$) using the closed-form algebraic estimator in (7), which requires no numerical differentiation or frequency-domain processing and is inherently immune to the exponentially decaying DC offset present after fault inception.

2.3. Remarks on Applicability, Assumptions, and Limitations

The proposed method is particularly well-suited to distribution networks, where short line lengths, high R/X ratios, and non-transposed conductors limit the effectiveness of conventional phasor-based or impedance-relay approaches. By operating directly in the time domain and relying solely on local measurements, the algorithm enables fast and reliable fault location using measurements that are readily available in modern digital relays and power quality recorders.

Nevertheless, the method assumes a predominantly series RL representation of the faulted feeder section and neglects shunt capacitance, which is acceptable for short- and medium-length distribution feeders [2]. The accuracy of the distance estimate depends on the correctness of the feeder inductance parameter and may be affected by severe waveform distortion, high harmonic content, or significant infeed from downstream sources.

The proposed method is developed and validated on a single radial feeder, which represents the most common configuration in medium-voltage distribution networks and provides a clean basis for systematic performance characterization. In multi-branch networks, faults occurring on downstream laterals or on the main feeder beyond a junction point introduce additional current infeed paths that alter the apparent impedance seen at the measurement point. Specifically, for a fault on a lateral, the load current of the healthy branches continues to flow through the measurement point, adding to the measured current and causing the estimator to underestimate the apparent inductance and, therefore, the fault distance. For a fault on the main feeder beyond a junction, the lateral loads act as parallel impedance paths, similarly distorting the current measurement and introducing a systematic bias in the distance estimate. The magnitude of this bias depends on the number of branches, their impedances, and the load distribution, making it network-specific. A thorough analysis of these effects and the development of compensation strategies for multi-branch topologies are identified as a priority direction for future work.

3. Distribution System Case Study and Modeling

3.1. Distribution Line Model

The system shown in Figure 1 is implemented as a 22 kV, 50 Hz, 20 km distribution line with AlFe ACSR DOG conductors. Its model was implemented, simulated, and tested using MATLAB codes and MATLAB/Simulink version R2023b, as shown in Figure 2. Various types of faults were considered to occur in the system at 0.04 s, including single-line-to-ground, line-to-line, double-line-to-ground, three-phase, and three-phase-to-ground faults. The faults were tested at actual fault locations of 3 km, 7 km, 12 km, and 18 km. The impact of fault resistance, as well as the presence of 3rd, 5th, 7th, and 9th harmonics, was also evaluated.

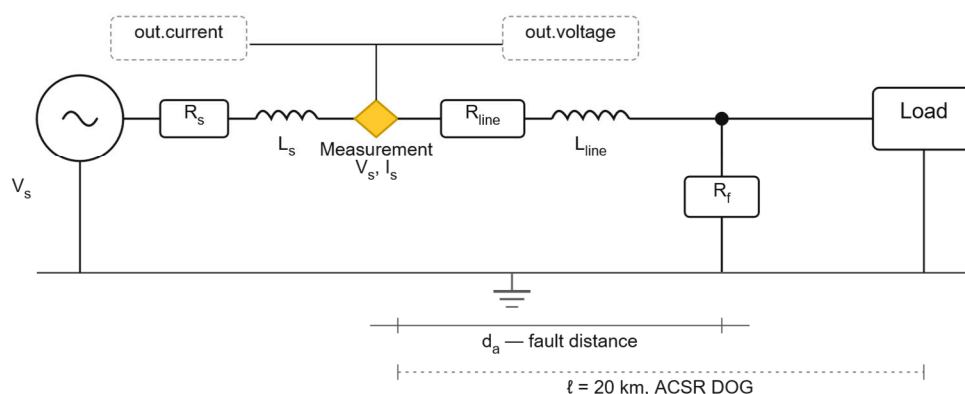


Figure 2. Faulted distribution line model.

The source is a 22 kV, 50 Hz three-phase voltage source feeding a load of 8 MW active and 2.6295 Mvar reactive power. The typical line parameters for the ACSR DOG conductors are given in Table 1.

Table 1. Typical parameters of an ACSR DOG (AlFe) 22 kV overhead distribution line.

Parameter	Positive Sequence (1)	Zero Sequence (0)
Resistance, R (Ω/km)	0.30–0.32	0.7–1.2
Inductance, L (mH/km)	1.1–1.3	3.5–4.5
Reactance, X, at 50 Hz (Ω/km)	0.35–0.41	1.1–1.4
Capacitance ($\mu\text{F}/\text{km}$)	0.008–0.012	0.003–0.005

3.2. Specific Operating Aspects Tested

This section examines the operating conditions that affect the accuracy of the apparent inductance-based estimator. These include the vicinity and remoteness of the fault with respect to the measuring relay, the impact of the fault resistance levels and the presence of harmonics. To be more realistic, various effects faced in real-life distribution systems, such as measurement noise and instrument transformer inaccuracies, are considered.

The distance between the measuring device and the fault is a key factor in the analysis. If the fault occurs closer to the measuring point, the faulted loop is too small with low impedance. This results in the sudden voltage collapse to nearly zero, causing the fault current to rise to very high values. Meanwhile, the fault occurring at the remote end with respect to the measuring unit position is accompanied by a slight decrease in voltage and an increase in current.

Fault resistance also significantly affects protection systems, and fault locators in particular. The impact may differ depending on the system's earthing method: solidly earthed, resistance earthed, compensated, or unearthed network. In any case, when a fault occurs with a certain fault resistance, this resistance is in series with the impedance of the faulted portion of the line and increases the impedance. The physical line inductance per unit length is not affected by fault resistance, as it is a geometric property of the conductor. However, the apparent inductance estimated from voltage and current samples in the time domain at the measurement point is sensitive to fault resistance through its effect on the fault current magnitude and the resulting signal quality of the voltage drop measurement. This sensitivity is quantified in Section 4.2. The third aspect evaluated is harmonic

distortion. The 3rd, 5th, 7th, and 9th harmonics, which are the most commonly found in distribution networks, are considered. Equations (10) and (11) describe the impact of the harmonic of order h in the system, which amplifies the signal by $h\omega$ after differentiation.

$$u_h(t) = Ri_h(t) + L \frac{di_h(t)}{dt} \quad (10)$$

$$\frac{di_h}{dt} = \sum_h h\omega I_h \cos(h\omega t) \quad (11)$$

The harmonic level limits provided by the standard EN 50160 related to voltage disturbances and voltage characteristics in public distribution systems are considered, and the robustness of the proposed algorithm estimator is evaluated. These limits are expressed as percentages of the nominal voltage at the supply terminals and are 5%, 6%, 5%, and 1.5% for the 3rd, 5th, 7th, and 9th harmonics, respectively [10].

3.3. Proposed Methodology Overview in ADNs

The apparent inductance-based algorithm is here extended and evaluated for ADNs with varying DER penetration levels, following four stages: (i) system modeling under active network conditions, (ii) signal acquisition and conditioning, (iii) apparent inductance and fault location estimation, and (iv) performance assessment.

Unlike conventional passive feeders, ADNs are characterized by multi-source fault current contributions and bidirectional power flows, which alter the voltage and current waveforms measured at the substation. The method is applied to a 22 kV distribution feeder equipped with converter-based DERs connected downstream of the measurement point, as illustrated in Figure 3. Various case scenarios are tested involving different fault types and fault resistances and a range of DER penetration levels. For each case, the algorithm estimates the fault distance using locally measured voltages and currents, thus preserving the single-ended nature of the method. This enables a direct assessment of how network activeness affects the apparent inductance concept.

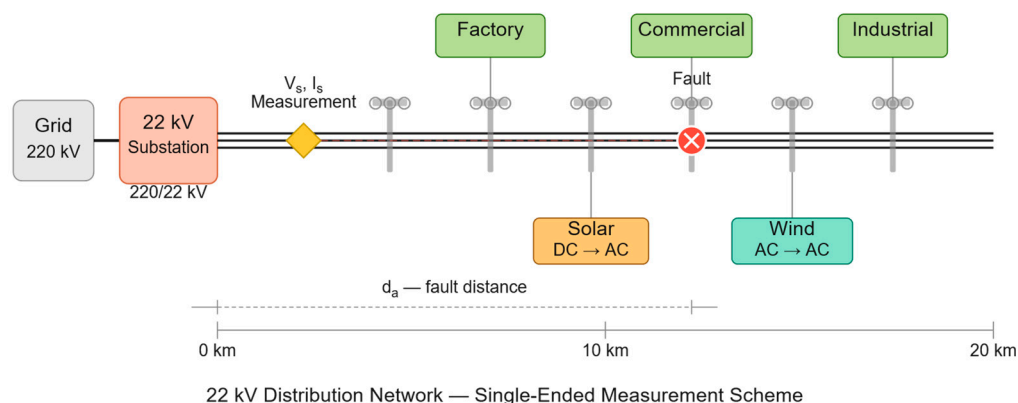


Figure 3. Active distribution network of 22 kV: physical layout.

The system comprises a utility grid connection (left), a 220/22 kV substation, and a 20 km distribution line with distribution poles. Three loads (factory, commercial, and industrial) and two DER installations (solar and wind) are connected along the line. Different types of faults are simulated at 10 km from the substation. Voltage and current measurements are taken at the grid side. The line conductors are ACSR (AlFe), with the same parameters as shown in Table 1.

3.4. Modeling of the Active Distribution Network

The feeder is modeled as a series combination of resistance and inductance, with parameters derived from ACSR DOG (AlFe) conductor data. The upstream grid is represented by a Thevenin equivalent consisting of a voltage source and internal impedance (R_s , L_s). The fault is modeled as a shunt branch with resistance R_f located at a known distance d_a from the substation.

DERs are modeled as controlled current sources connected at the downstream end of the feeder. Their contribution during faults is scaled according to the penetration level, defined as the ratio between the DER-rated power and feeder load demand. The considered penetration levels of 0%, 20%, 50%, and 80% reflect typical operating conditions of active distribution systems in different case scenarios.

During faults, DER fault current contribution is assumed to be limited and controlled, consistent with the behavior of inverter-based resources [28,29]. This reflects a key ADN characteristic: fault currents no longer scale linearly with impedance, directly affecting inductance-based estimation.

The modeling of DERs as controlled current sources with fixed current-limiting behavior reflects the dominant operating mode of grid-connected inverters during fault conditions, as mandated by grid codes such as IEEE 1547 and IEC 61727. The fixed current-limiting model represents a conservative scenario in terms of current magnitude, since real inverters typically inject less than the rated current limit during fault transients. The results reported, related to different fault locations and various DER penetration levels, therefore, provide a conservative upper bound on the DER-induced location bias for a given penetration level. A systematic study of alternative control modes, including reactive-current-priority LVRT strategies and droop-controlled inverters, is identified as a direction for future work.

3.5. Signal Acquisition and Conditioning

The voltage and current at the substation are sampled at a fixed frequency of 10 kHz. Measurement noise and instrument transformer ratio errors are added to emulate realistic protection system conditions. A signal-to-noise ratio (SNR) of 65 dB for the voltage and 63 dB for the current is considered. The maximum error of 0.2% of the actual CT current and 0.1% of the actual PT voltage is considered. However, for change-based measurements, systematic constant errors do not affect the process outcome. The measured signals are processed through a short moving-average filter to mitigate high-frequency noise while preserving the fundamental transient dynamics required for time-domain estimation. This adaptive filtering ensures smooth apparent inductance convergence and stable numerical derivatives.

3.6. Apparent Inductance Estimation Modeling in Active Distribution Networks

Following the formulation in Section 2.3, the governing equation is given in (1). The equivalent circuit of the medium-voltage distribution line in question is depicted in Figure 4.

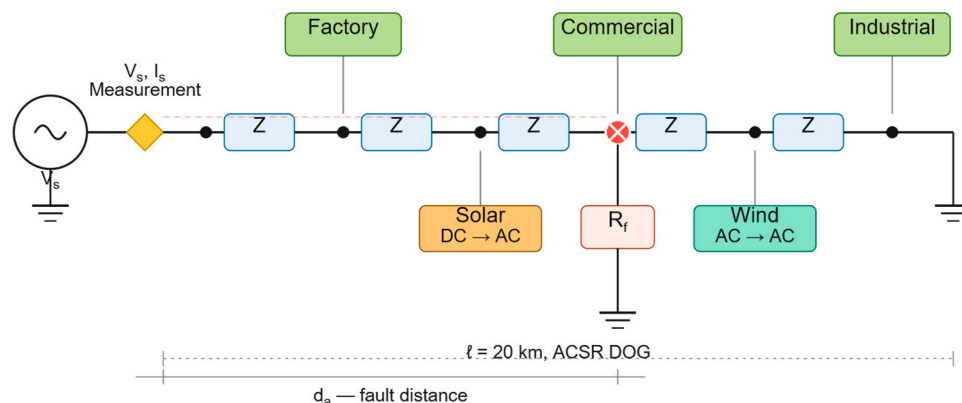


Figure 4. Equivalent circuit: 22 kV distribution network.

Using three consecutive samples of voltage and current, the apparent inductance is estimated through a discrete-time formulation derived from the trapezoidal integration rule [22,25] following the formulation in (6) and (7). This formulation is equally applied in the ADN case to isolate performance variations attributable to DER penetration.

In the presence of DER sources downstream of the measurement point, the fault current I_f at the fault location is the sum of the substation infeed I_{sub} and the DER contribution I_{DER} : $I_f = I_{sub} + I_{DER}$. The apparent inductance estimator uses only I_{sub} and the substation voltage and, therefore, estimates an apparent inductance that corresponds to a shorter fault distance than the true distance. The DER contribution reduces I_{sub} below I_f , while the voltage drop at the substation reflects the full fault current path, causing the estimator to systematically underestimate the fault distance. This bias grows with the DER penetration level.

3.7. Fault Location Estimation

To enhance robustness, the apparent inductance is computed over a during-fault time window after transient settling. Outliers are removed using an interquartile range (IQR) criterion [30], and the median value of the remaining samples is selected as the representative inductance estimate, L_{app} . The estimated fault distance d_f is obtained as shown in (12):

$$d_f = \frac{L_{app}}{L'} \quad (12)$$

where L_{app} denotes the line apparent inductance, and L' denotes the inductance per unit length of the feeder.

3.8. Performance Assessment Strategy

The proposed methodology is validated through a comprehensive set of simulation scenarios reflecting realistic DS operating conditions. Performance is evaluated using the following metrics: the estimated distance to fault location, d_f , and the actual fault location, D_a . The absolute error and relative error are respectively given in (13) and (14):

$$X_e = |d_f - D_a| \quad (13)$$

$$P_e = \left| \frac{d_f - D_a}{D_a} \right| \times 100 \quad (14)$$

The assessment covers fault type, fault resistance (low- and high-impedance), fault location, EN 50160-compliant harmonics, and DER penetration levels. The overall approach is summarized in Figure 5.

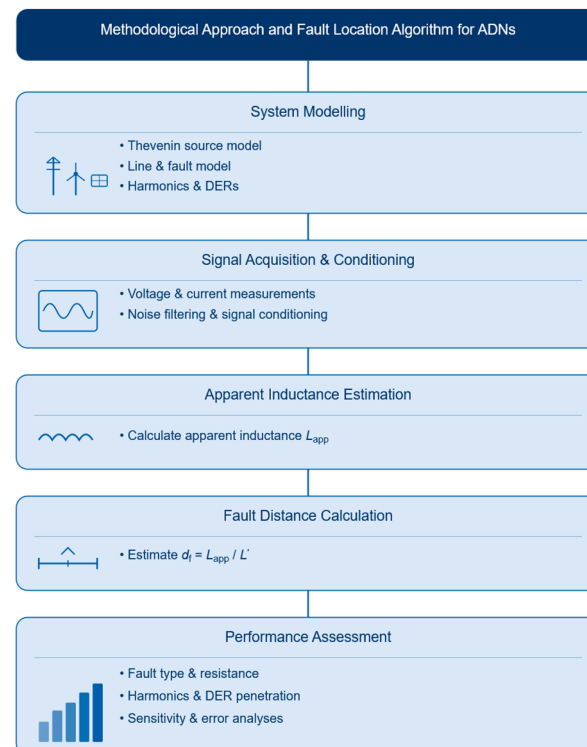


Figure 5. Methodological approach and apparent inductance algorithm.

4. Simulation Results and Discussion

4.1. Fault Simulation in Different Working Conditions

Figure 6 illustrates the voltage and current waveforms recorded at the measurement point for two representative fault scenarios that bound the range of conditions studied in this work. Figure 6a shows a three-phase fault at the remote end of the feeder. The voltage sag is moderate, and the fault current surge is relatively small, reflecting the significant line impedance between the measurement point and the fault. The apparent inductance estimated from these measurements is, therefore, high, correctly indicating a fault far from the substation. Figure 6b shows a double-line-to-ground fault near the measurement point. The voltage collapses sharply, and the fault current surge is large, as little line impedance separates the relay from the fault. The apparent inductance is correspondingly low, indicating a close fault. Voltage and current waveforms illustrating fault proximity effects are shown in Figure 6, including (a) a three-phase fault at the remote end and a double-line-to-ground fault near the relay (b).

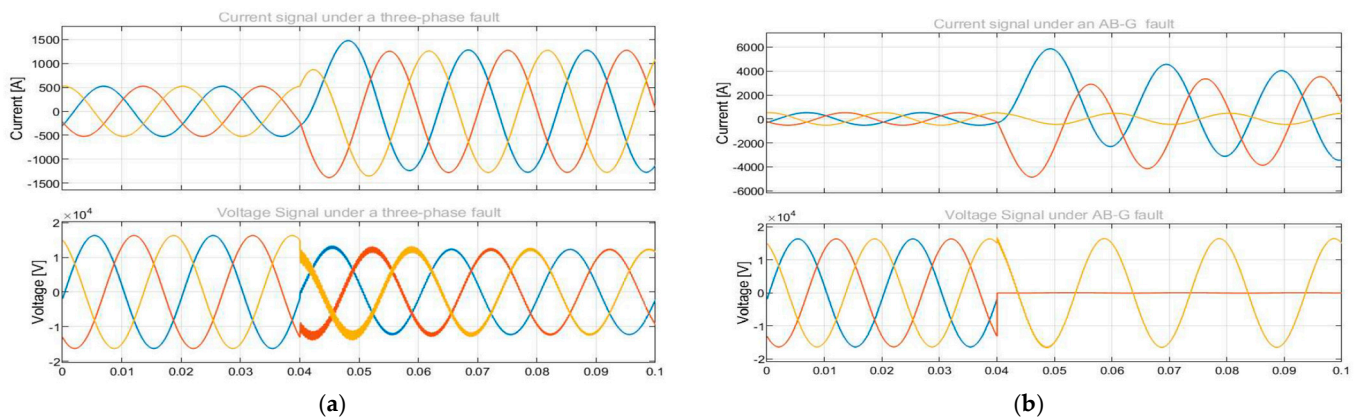


Figure 6. Voltage and current waveforms: (a) three-phase fault far from the measuring unit; (b) double-line-to-ground fault near the measurement point. Each colored line refers to a different phase.

These two cases illustrate the fundamental operating principle of the apparent inductance algorithm: the closer the fault, the smaller the estimated inductance, and the more severe the voltage collapse and current surge observed at the relay. This proximity-to-inductance relationship holds under ideal sinusoidal conditions and forms the basis for the systematic performance evaluation conducted in Sections 4.2 to 4.4, where the effects of fault resistance, harmonic distortion, and DER penetration on this relationship are progressively introduced and quantified.

4.2. Fault Location with the Three-Sample Apparent Inductance Algorithm at Fundamental Frequency

A representative case in Figure 7 shows the algorithm applied to a single-phase fault at 12 km with an R_f of 5Ω under sinusoidal conditions. The estimated inductance of 14.174 mH corresponds to a fault location of 11.8515 km, with a relative error of 1.238%.

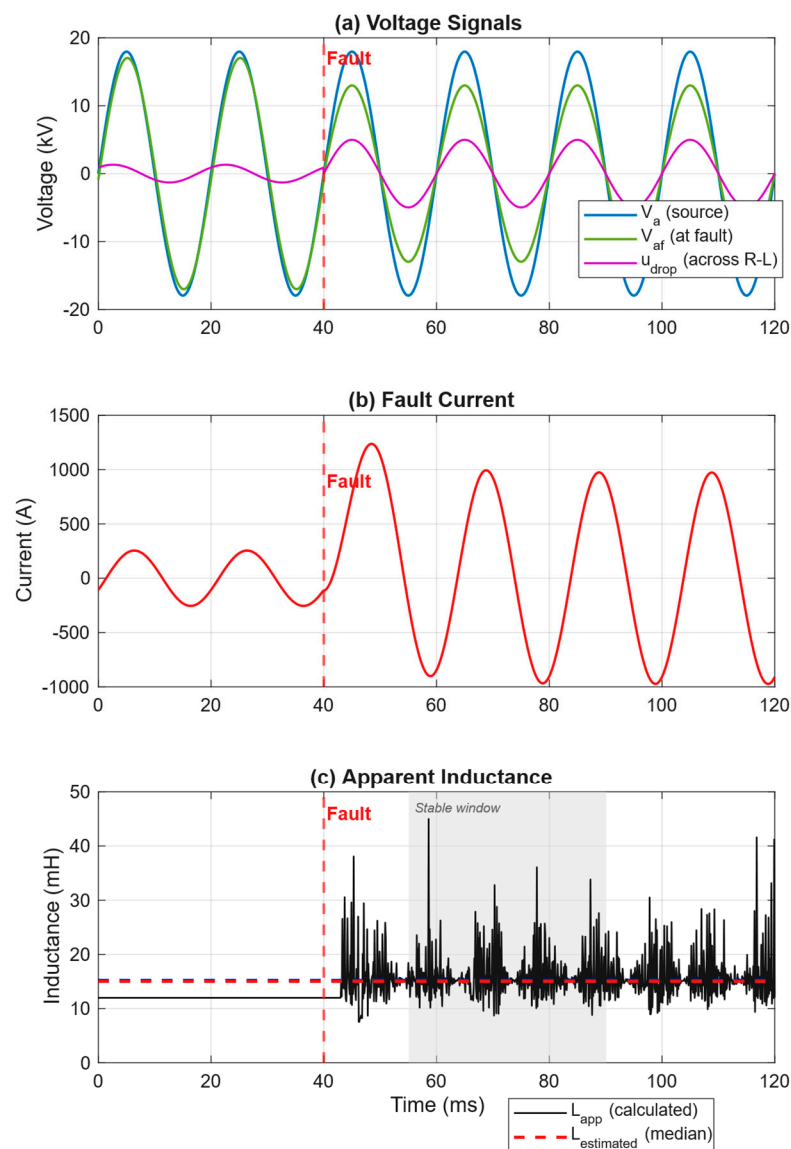


Figure 7. Apparent inductance algorithm fault location under PDN (50 Hz).

For a comprehensive analysis of the system that considers three fault types (A-G, AB, and ABC-G), three actual fault distances (2 km, 10 km, and 19 km) and two different fault resistances (a low R_f of 5 Ω and a high R_f of 500 Ω) under ideal sinusoidal conditions, the proposed apparent-inductance-based fault location method demonstrates consistently high accuracy across all tested fault types, resistances, and locations. For low-resistance faults, the relative error remains very small for all scenarios, confirming the strong convergence properties of the inductance estimator and its weak sensitivity to operating point variations. Even for high-resistance faults, where the fault current magnitude is significantly reduced, the enhanced algorithm keeps errors within admissible ranges. The comprehensive analysis results are summarized in Table 2 and compared in Figure 8.

Table 2. Comprehensive analysis of the system under fundamental frequency.

Fault	R_f (Ω)	D_a (km)	d_f (km)	X_e (km)	P_e (%)
AG	5	2	1.9711	0.0289	1.445
AG	500	2	1.8911	0.1089	5.447

AG	5	10	9.8585	0.1415	1.415
AG	500	10	9.0223	0.9777	9.777
AG	5	19	18.6628	0.3372	1.775
AG	500	19	18.1739	0.8261	4.348
AB	5	2	1.9868	0.0132	0.661
AB	500	2	2.0013	0.0013	0.063
AB	5	10	9.8949	0.1051	1.051
AB	500	10	9.3999	0.6001	6.001
AB	5	19	18.7140	0.2860	1.505
AB	500	19	17.4278	1.5722	8.274
ABCG	5	2	1.9888	0.0112	0.560
ABCG	500	2	1.8130	0.1870	9.349
ABCG	5	10	9.8443	0.1557	1.557
ABCG	500	10	9.9113	0.0887	0.887
ABCG	5	19	18.9121	0.0879	0.463
ABCG	500	19	18.1475	0.8525	4.487

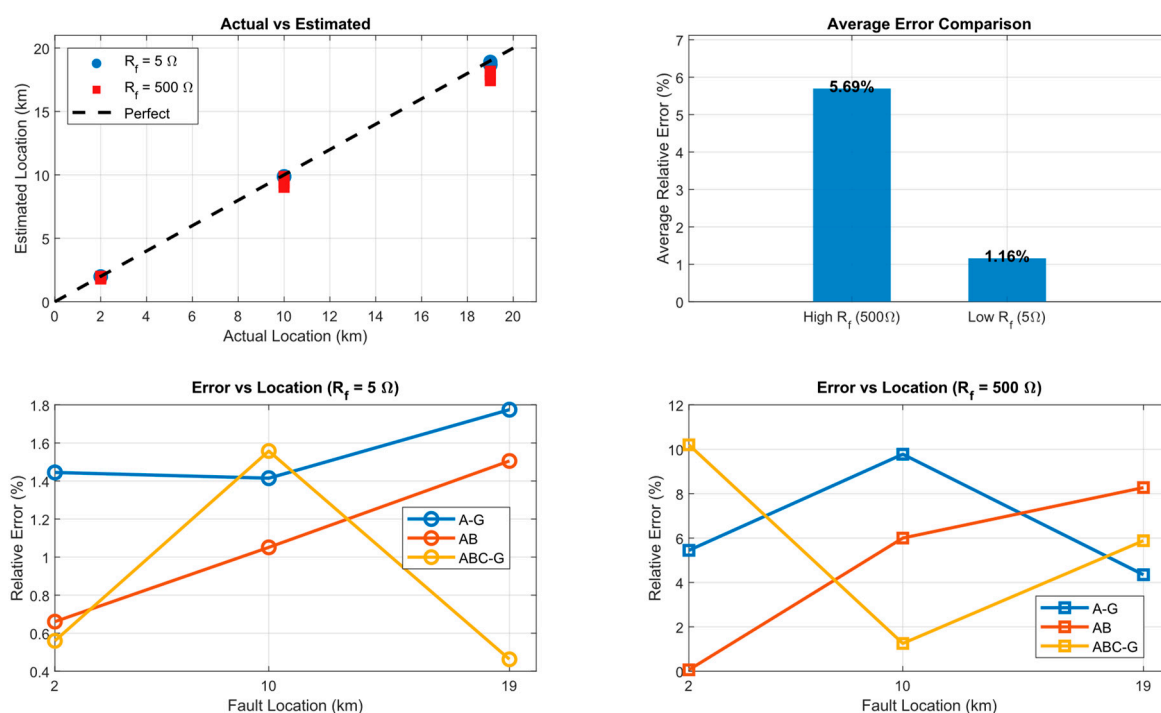


Figure 8. Comprehensive analysis result comparison under 50 Hz.

4.3. Fault Location with the Three-Sample Apparent Inductance Algorithm Under Harmonics

The algorithm was applied to the system under EN 50160-compliant third, fifth, seventh, and ninth harmonics [10]. Figure 9 illustrates a representative case: a single-phase fault at 12 km, with an R_f of 5 Ω , yielding an estimated location of 11.725 km (relative error of 2.29%), only marginally higher than the 50 Hz baseline (1.238%).

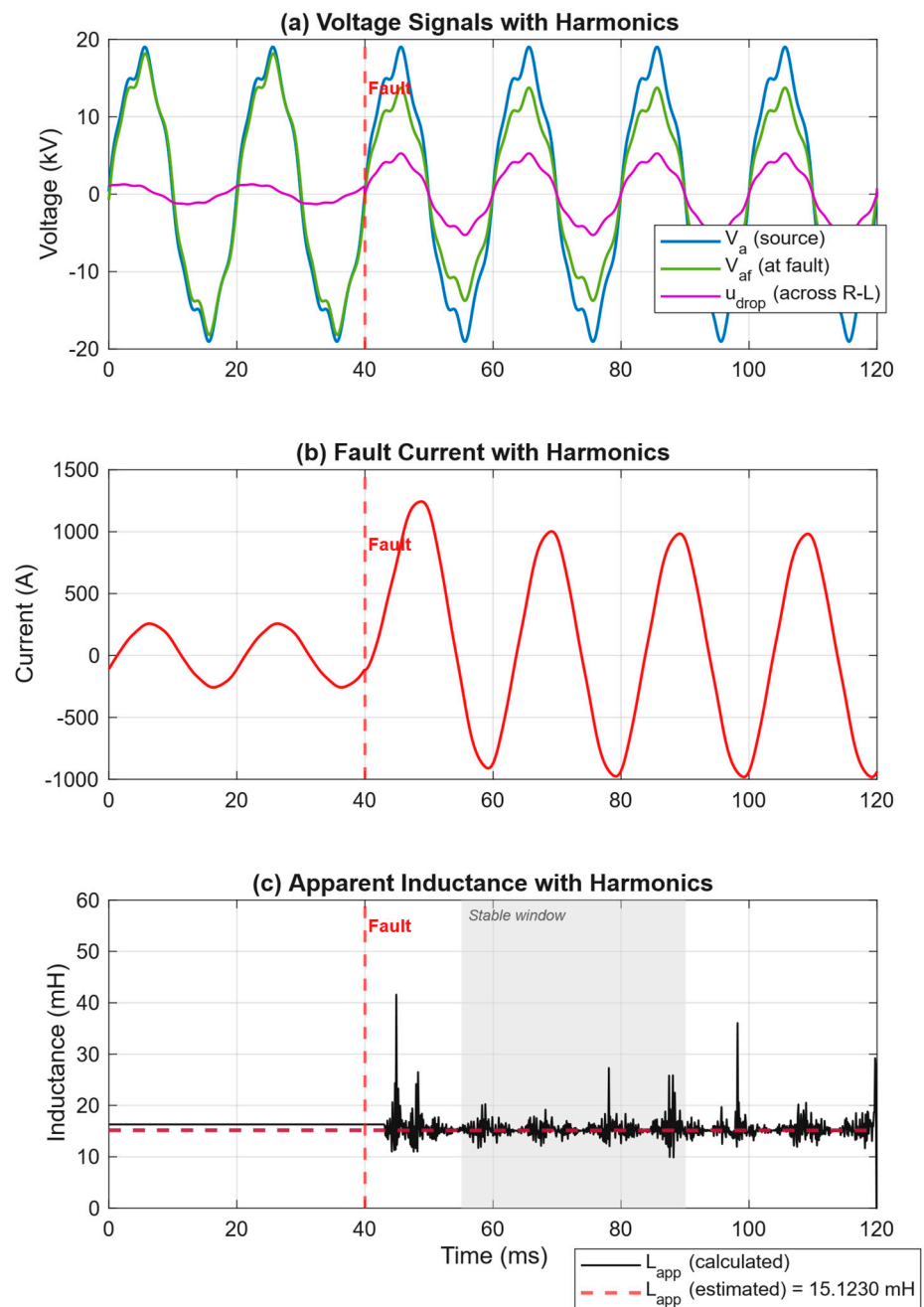


Figure 9. Apparent inductance algorithm fault location under harmonics.

The comparison of the voltage harmonic spectrum before and after fault inception shows that the harmonic content remains essentially bounded within the EN 50160 limits, even under fault conditions. While the fault introduces a noticeable increase in the fundamental and low-order harmonic magnitudes due to the abrupt change in network impedance, no abnormal harmonic amplification is observed. This confirms that the fault primarily affects the signal amplitude rather than its spectral composition, validating the use of harmonic-robust signal processing and supporting the reliability of the proposed fault-location algorithm in harmonically distorted but standards-compliant environments. This is depicted in Figure 10.

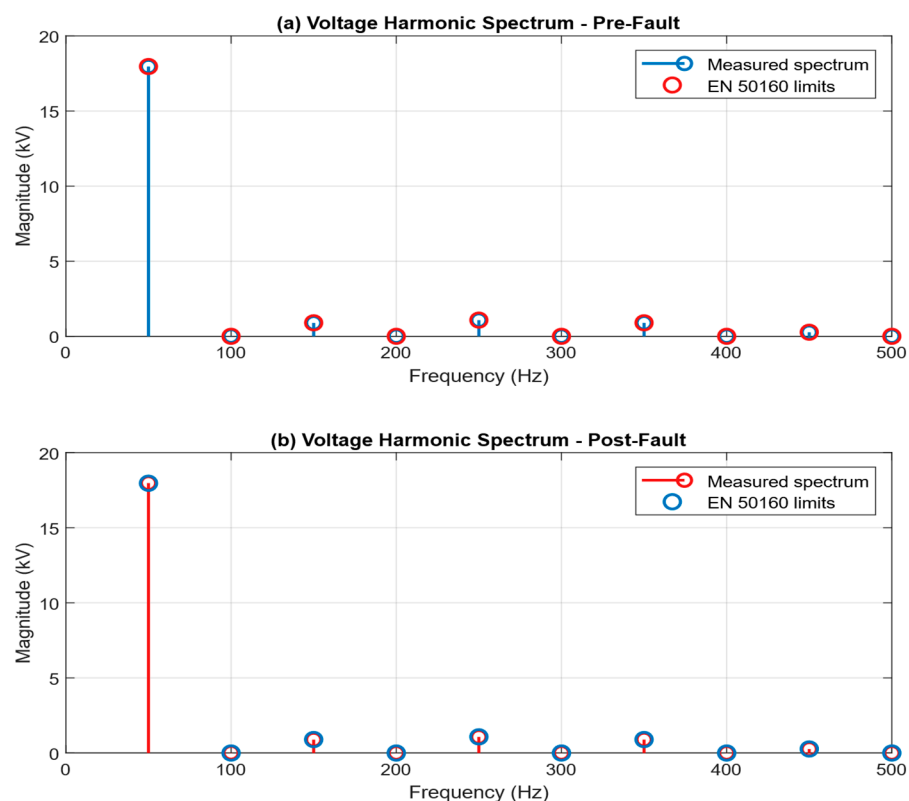


Figure 10. Voltage harmonic spectrum comparison.

The comprehensive analysis conducted for the system with EN 50160-compliant harmonics is presented in this subsection, and the results are summarized in Table 3. While the presence of harmonics slightly increases the variability in voltage and current measurements, the resulting impact on location accuracy remains limited across all tested conditions.

Table 3. Comprehensive analysis of the system under EN 50160 standard-compliant harmonics.

Fault	R_f (Ω)	D_a (km)	d_f (km)	X_e (km)	P_e (%)
AG	5	2	1.9716	0.0284	1.418
AG	500	2	1.6543	0.3457	17.283
AG	5	10	9.7986	0.2014	2.014
AG	500	10	9.2395	0.7605	7.605
AG	5	19	18.5393	0.4607	2.425
AG	500	19	19.5264	0.5264	2.771
AB	5	2	1.9777	0.0223	1.113
AB	500	2	1.970	0.0030	0.149
AB	5	10	9.8682	0.1318	1.318
AB	500	10	9.3457	0.6543	6.543
AB	5	19	18.7627	0.2373	1.249
AB	500	19	17.0524	1.9476	10.250
ABCG	5	2	1.9792	0.0208	1.041
ABCG	500	2	1.8302	0.1698	8.488
ABCG	5	10	9.9036	0.0964	0.964
ABCG	500	10	9.9027	0.0973	0.973
ABCG	5	19	18.7821	0.2179	1.147
ABCG	500	19	17.8648	1.1352	5.975

For high-resistance faults, a moderate increase in the estimation error is observed, especially for ground-involved faults at short distances; however, the enhanced algorithm incorporating adaptive filtering and resistance-inductance compensation keeps errors within acceptable bounds. Overall, the results confirm that realistic harmonic distortion does not compromise the reliability of the method, validating its suitability for practical deployment in modern distribution networks operating under non-ideal power-quality conditions. The analysis results are compared in Figure 11.

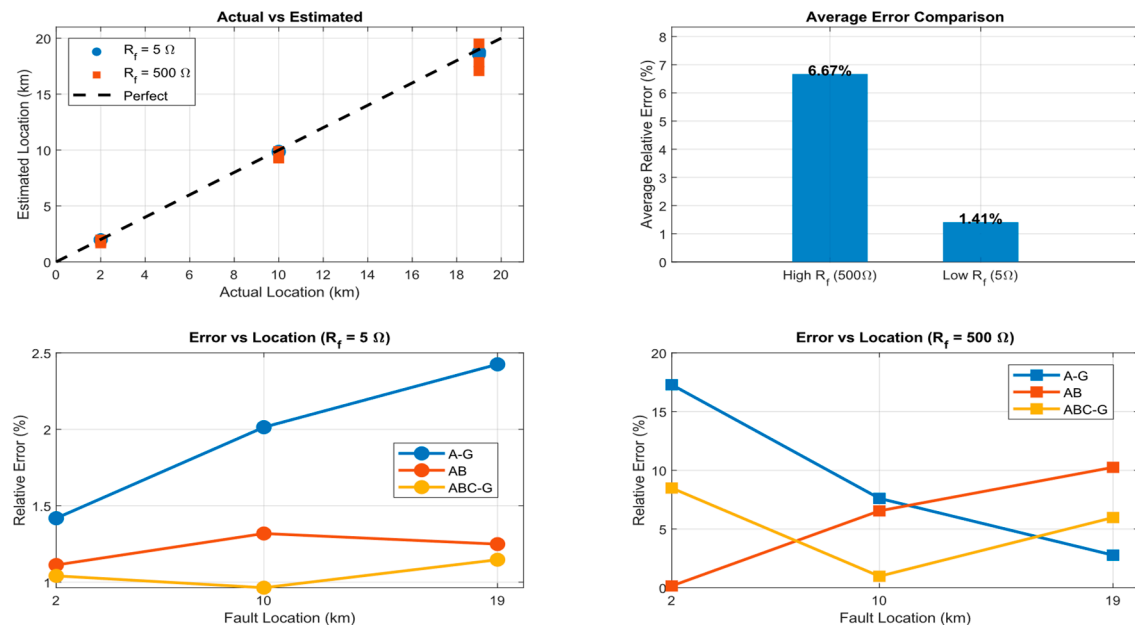


Figure 11. EN 50160 standard-compliant comprehensive analysis result comparison.

4.4. Comparison Between the Comprehensive Analysis Under Fundamental Frequency and Harmonics

Figure 12 compares the relative errors under 50 Hz and EN 50160-compliant harmonic conditions. Most data points cluster near the unity line, confirming accuracy preservation under typical power quality distortion. High-resistance ground faults show the largest sensitivity; in several scenarios, the harmonic case yields comparable or slightly improved accuracy due to adaptive filtering and median-based aggregation.

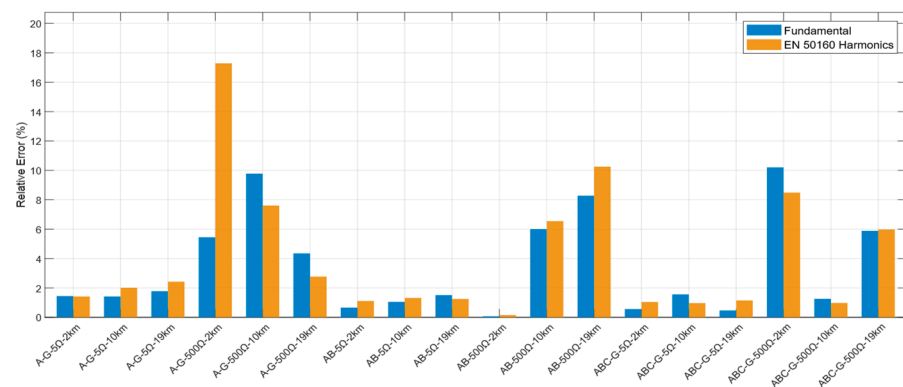


Figure 12. Comparison between the comprehensive analysis for the system under fundamental frequency and that under EN 50160-compliant harmonics.

4.5. Fault Location with the Three-Sample Apparent Inductance Algorithm in ADNs

The proposed fault location algorithm was evaluated in an active distribution network, where the impact of DERs on signal measurements and the fault location accuracy were assessed. Three representative case scenarios consisting of A-G with an R_f of $5\ \Omega$ at 0% and 20% of DER penetration, as well as ABC-G with an R_f of $500\ \Omega$ at 80% DER, are presented in Figures 13–15. A comprehensive set of scenarios was analyzed for three actual fault distances, 2 km, 10 km, and 19 km, consistent with the fault locations evaluated in the PDN analysis. Three fault types (A–G, AB, and ABC–G) and two fault resistances ($5\ \Omega$, standing for low-impedance faults, and $500\ \Omega$, representing high-impedance faults) were applied under different DER penetration levels (0%, 20%, 50%, and 80%). The graphical representation of the single-phase low-impedance fault with 0% DER penetration is shown in Figure 13. Some of the preliminary results of this section were presented in a conference paper referenced in [31].

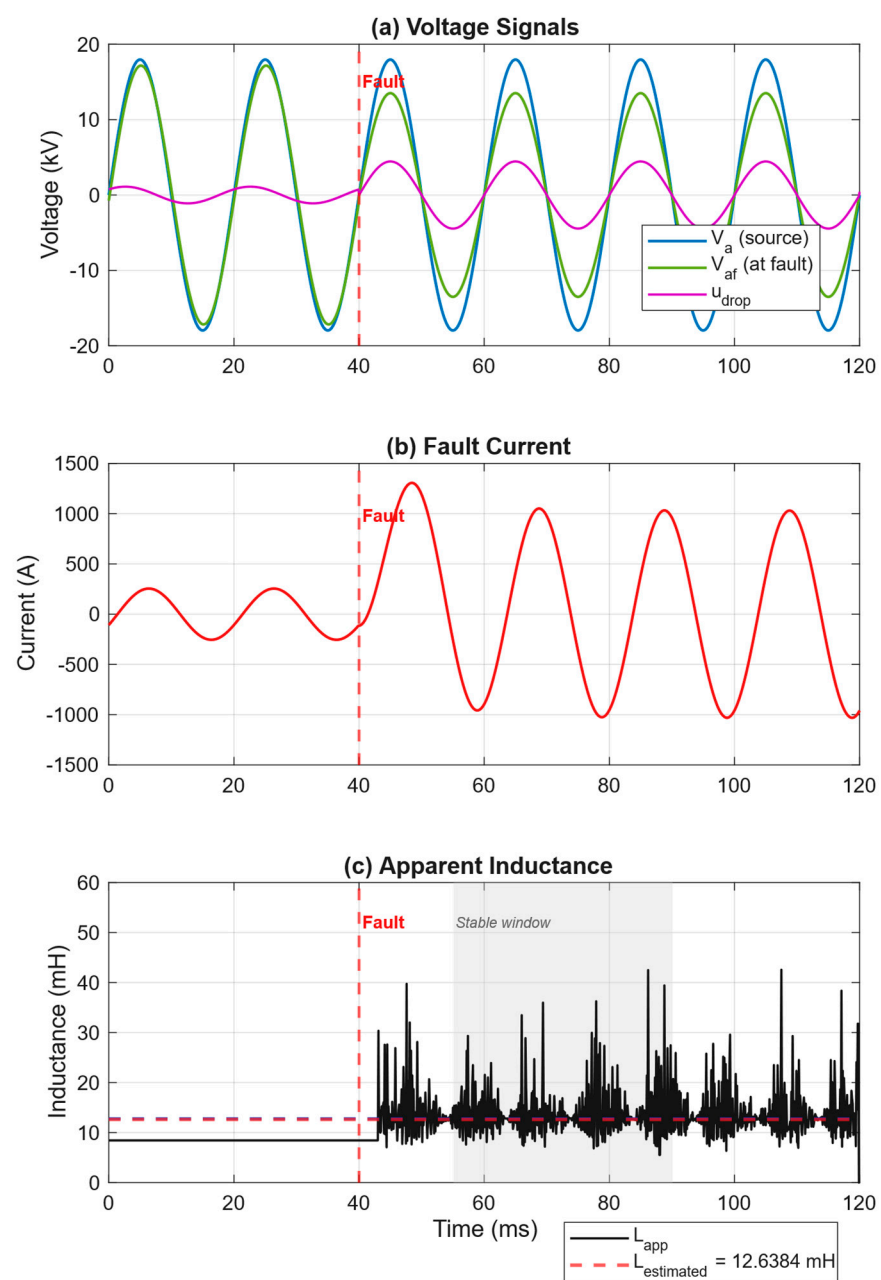


Figure 13. Single-phase fault, with R_f of $5\ \Omega$ at 10km, with 0% DER penetration.

The apparent inductance algorithm for fault location, applied in a distribution network with 0% DER penetration, under a single-phase low-impedance fault (R_f of $5\ \Omega$) located at 10km, demonstrates higher performance, with a relative error of 1.246%. The easiest fault location case among all scenarios tested at 0% DER penetration was ABC-G ($5\ \Omega$), with a relative error of 0.018%, as shown in Table 4. The results of a single-phase low-impedance fault (R_f of $5\ \Omega$) at 20% DER penetration are depicted in Figure 14.

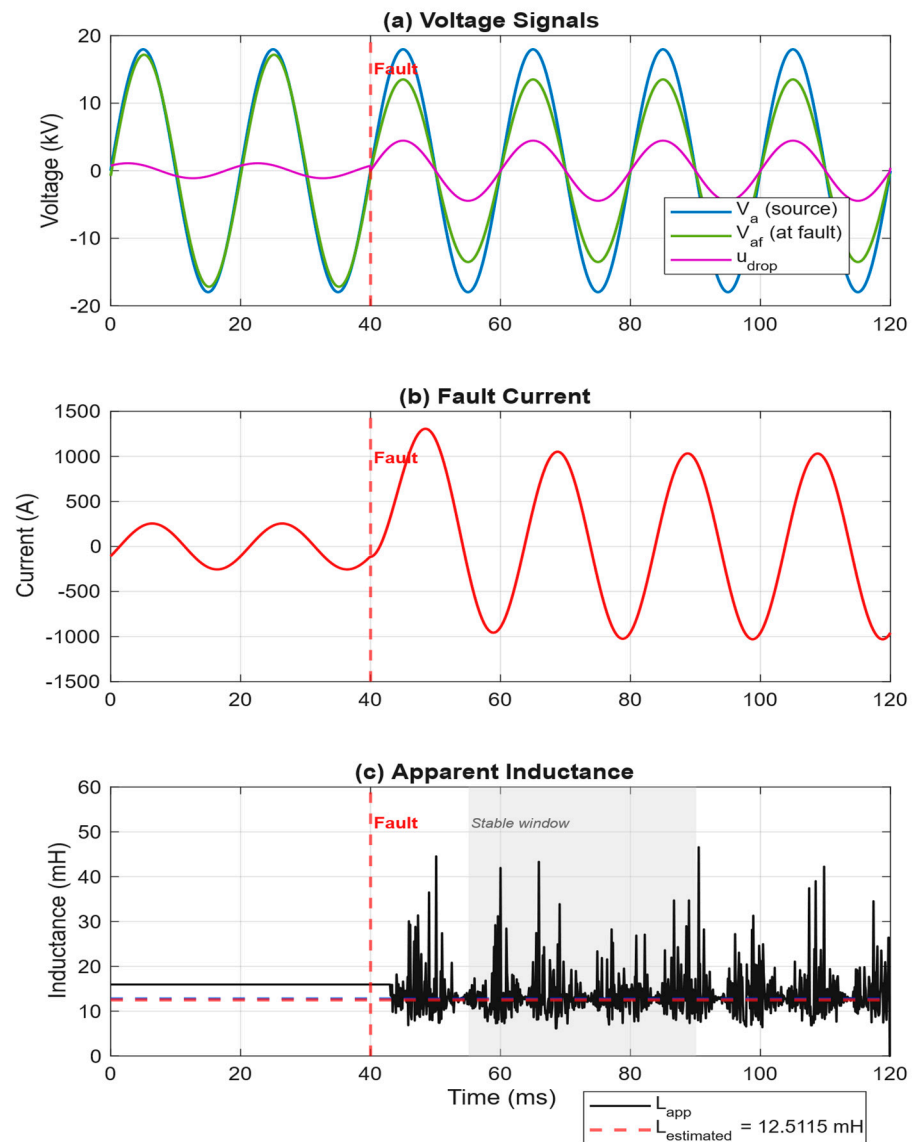


Figure 14. Single-phase fault, with R_f of $5\ \Omega$ at 10 km, with 20% DER penetration.

The relative error exhibited by the proposed fault location algorithm for the case of A-G ($5\ \Omega$), with 20% DER penetration, is 2.246%, which is higher than that of the same fault type at 0% DER penetration (1.246%), and remarkably lower than that of the same fault at 50% and 80% DER penetration levels, which are respectively 3.246% and 4.246%. This demonstrates the impact of increasing intermittent DER penetrations on the accuracy of the fault location algorithm in distribution systems.

Another aspect that significantly affects the accuracy of the apparent inductance estimation and, consequently, the fault location determination is the fault resistance. In the context of the present work, two cases in terms of fault resistance are considered: low-impedance faults and high-impedance faults. The examples of low-impedance faults were discussed above (Figures 13 and 14). In Figure 15, an example of a high-impedance three-phase fault (R_f of 500 Ω) occurring at 10 km along an ADN with 80% DER penetration is depicted. The low fault current signal due to the high fault resistance, the signal alteration, and the multi-source nature of this fault due to the high penetration of the DER result in a relative error of 23.774%. Across all three fault distances, the worst-case conditions were A-G at 500 Ω with DER penetration of 20% or above, reaching a relative error of 30%.

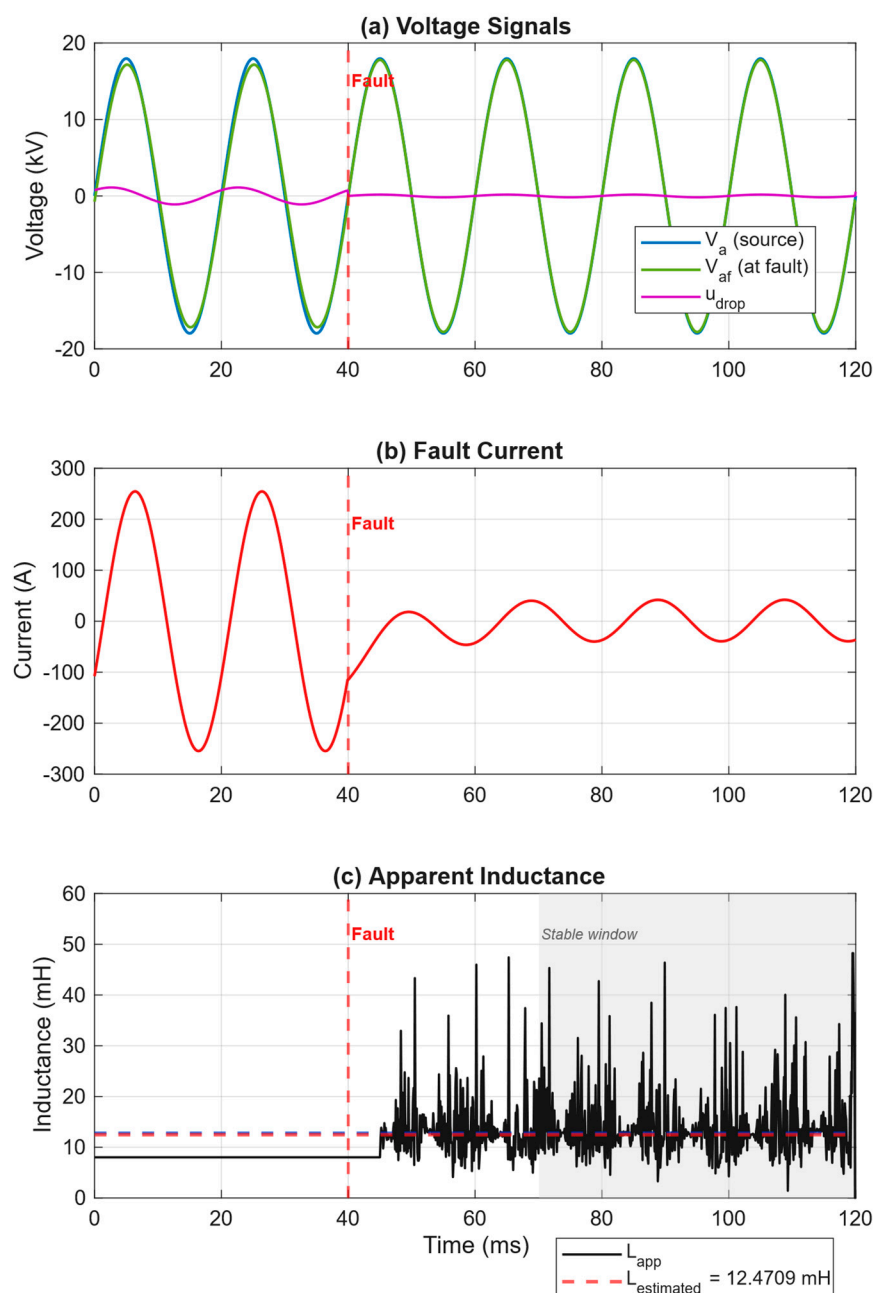


Figure 15. Three-phase fault, with R_f of 500 Ω at 10 km, with 80% DER penetration.

The full results across all fault types (A–G, AB, and ABC–G), two fault resistances ($5\ \Omega$ and $500\ \Omega$), and DER penetration levels (0–80%) are summarized in Table 4 and compared in a graphical representation in Figure 16 and a bar chart in Figure 17.

In general, the comprehensive analysis results confirm that (1) higher-impedance faults are harder to locate than their corresponding low-impedance faults; (2) ADNs are more challenging for fault locators, and challenges increase tremendously with the increase in DER penetration; and (3) single-phase-to-ground faults are the hardest to locate due to the system imbalances and zero sequence bias, followed by the line-to-line faults for the same reasons, while three-phase faults are the easiest to locate due mainly to their symmetric aspect.

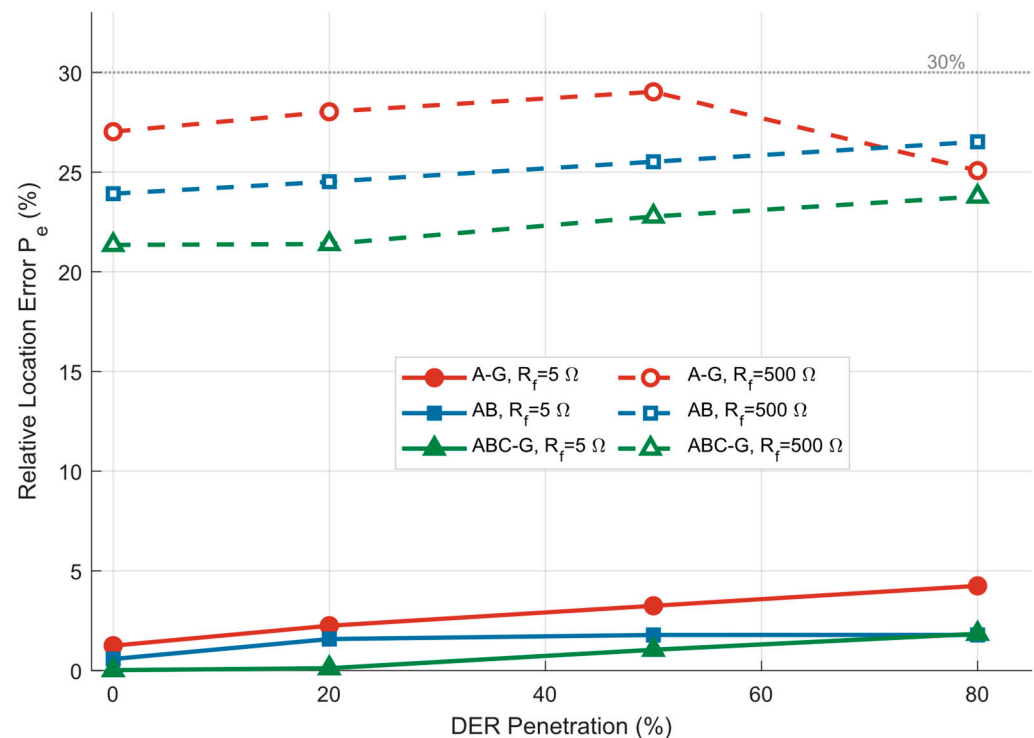


Figure 16. Comparison of different faults with R_f and different DER penetrations.

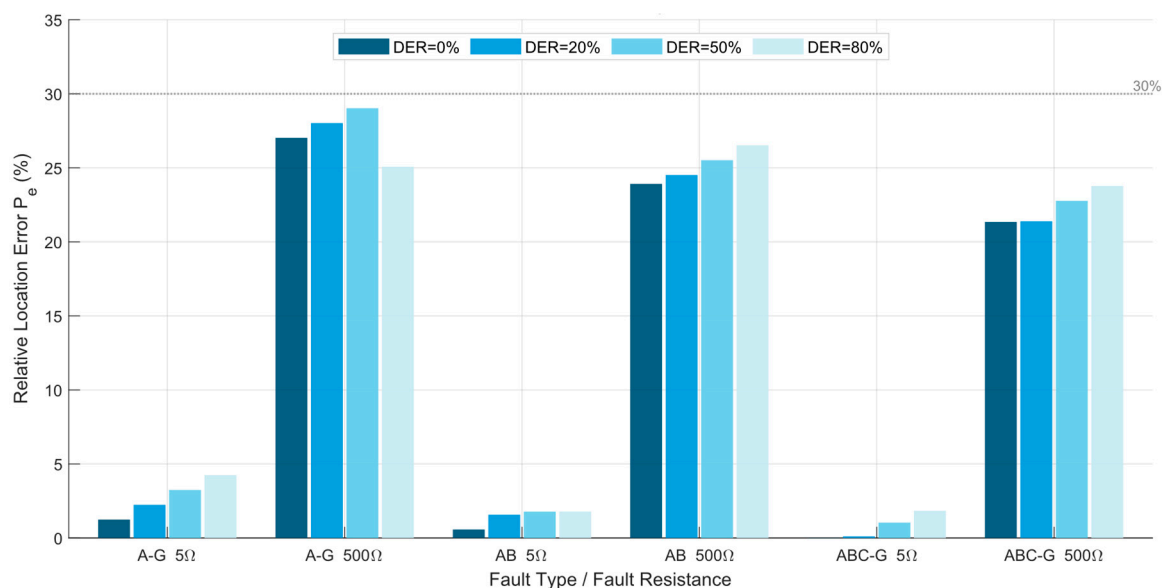


Figure 17. Chart comparison of different faults with R_f and different DER penetrations.

Figure 18 extends the fault location analysis to three fault distances (2 km, 10 km, and 19 km) across all fault types and both fault resistance levels, providing a comprehensive picture of how fault proximity interacts with DER penetration and fault resistance to affect estimator accuracy.

For low-impedance faults (R_f of 5 Ω , top row), the algorithm maintains reliable performance across all three locations, with relative errors remaining below 5% even at 80% DER penetration. The error increases monotonically with DER penetration at all locations, confirming that DER-induced current bias is the dominant degradation mechanism regardless of fault proximity. Faults at 19 km exhibit slightly higher errors than those at 10 km and 2 km, reflecting the weaker voltage-drop signal at the remote end of the feeder. The fault-type hierarchy of A-G > AB > ABC-G is consistently preserved across all locations and DER levels.

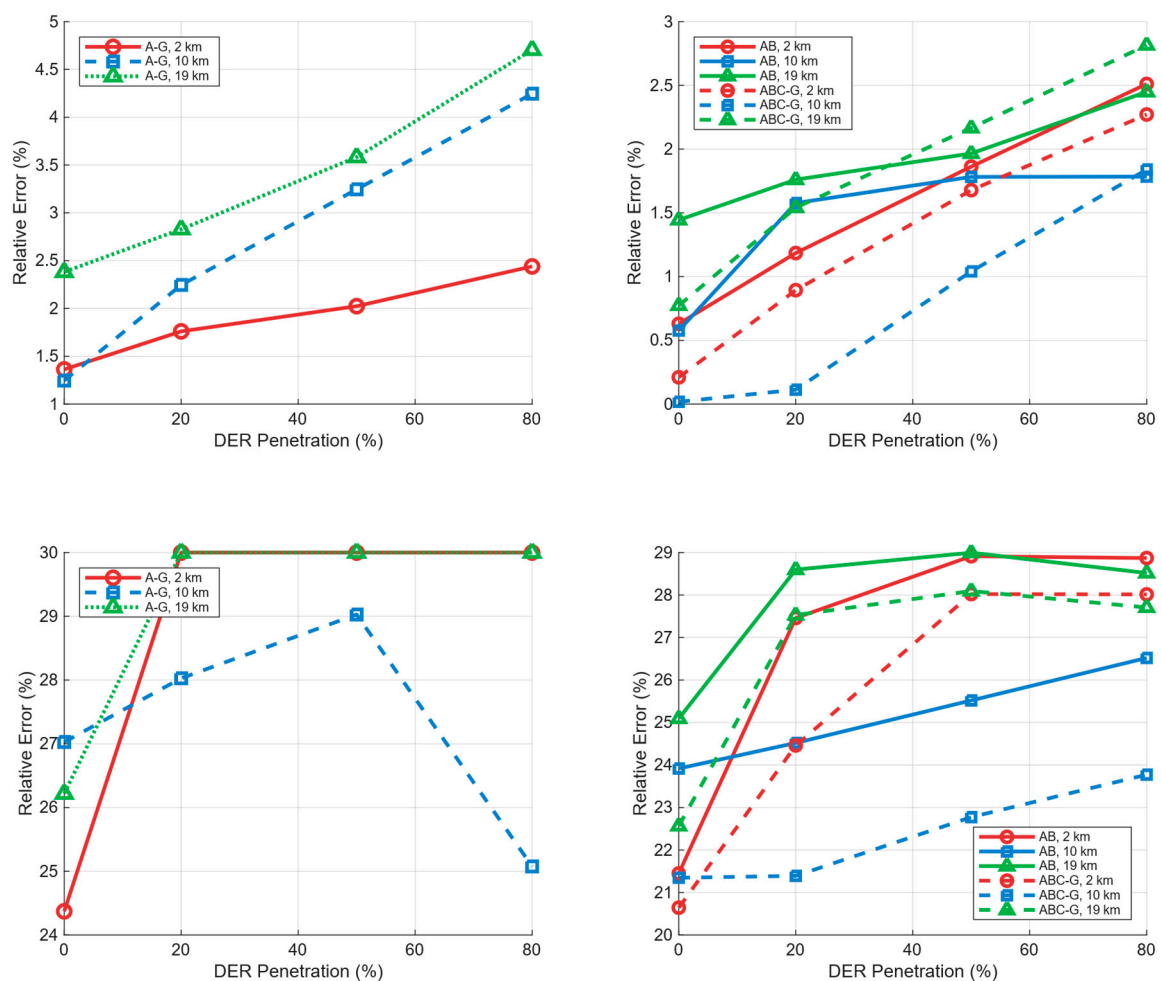


Figure 18. ADN fault location accuracy versus DER penetration for fault distances of 2 km, 10 km, and 19 km.

For high-impedance faults (R_f of 500 Ω , bottom row), the interaction between the fault distance and DER penetration is more pronounced. At low DER penetration, errors at 2 km are generally lower than those at 10 km and 19 km, since the stronger fault signal near the measurement point partially compensates for the high fault resistance. As DER penetration increases beyond 20%, the systematic DER-induced bias becomes dominant, and

errors at all three locations converge toward the practical accuracy limit, reaching relative errors of 29 to 30% under the worst-case conditions. Under these conditions, combining high-impedance single-phase faults with high DER penetration, the apparent inductance estimator reaches its operational boundary regardless of fault distance. In such scenarios, complementary protection measures would be required for reliable fault management. These may include neutral voltage displacement monitoring, which remains sensitive to high-impedance ground faults even when fault current is suppressed by DER current limiting; zero-sequence overcurrent elements with adaptive thresholds that account for DER penetration level; and fault passage indicators distributed along the feeder, which can narrow the faulted section independently of the substation measurement quality. The combination of the apparent inductance estimator for reliable low-to-medium resistance fault location with these complementary elements for high-resistance boundary conditions represents a practical protection strategy for high-DER active distribution networks.

Table 4. Different fault simulation results with different locations and levels of DER penetration.

D_a (km)	Fault	R_f (Ω)	DER (%)	d_f (km)	X_e (km)	Pe (%)
2	A-G	5	0	1.9728	0.0272	1.362
2	A-G	500	0	1.5126	0.4874	24.371
2	A-G	5	20	1.9648	0.0352	1.760
2	A-G	500	20	1.4000	0.6000	30.000
2	A-G	5	50	1.9595	0.0405	2.024
2	A-G	500	50	1.4000	0.6000	30.000
2	A-G	5	80	1.9512	0.0488	2.440
2	A-G	500	80	1.4000	0.6000	30.000
2	AB	5	0	2.0126	0.0126	0.630
2	AB	500	0	1.5710	0.4290	21.450
2	AB	5	20	2.0237	0.0237	1.185
2	AB	500	20	1.4508	0.5492	27.460
2	AB	5	50	2.0372	0.0372	1.862
2	AB	500	50	1.4217	0.5783	28.915
2	AB	5	80	2.0502	0.0502	2.511
2	AB	500	80	1.4226	0.5774	28.871
2	ABC-G	5	0	1.9958	0.0042	0.210
2	ABC-G	500	0	1.5871	0.4129	20.643
2	ABC-G	5	20	2.0179	0.0179	0.893
2	ABC-G	500	20	1.5108	0.4892	24.460
2	ABC-G	5	50	1.9664	0.0336	1.678
2	ABC-G	500	50	1.4395	0.5605	28.023
2	ABC-G	5	80	1.9546	0.0454	2.272
2	ABC-G	500	80	1.4397	0.5603	28.015

10	A-G	5	0	9.8754	0.1246	1.246
10	A-G	500	0	7.2972	2.7028	27.028
10	A-G	5	20	9.7754	0.2246	2.246
10	A-G	500	20	7.1972	2.8028	28.028
10	A-G	5	50	9.6754	0.3246	3.246
10	A-G	500	50	7.0972	2.9028	29.028
10	A-G	5	80	9.5754	0.4246	4.246
10	A-G	500	80	7.4924	2.5076	25.076
10	AB	5	0	9.9423	0.0577	0.577
10	AB	500	0	7.6082	2.3918	23.918
10	AB	5	20	9.8423	0.1577	1.577
10	AB	500	20	7.5480	2.4520	24.520
10	AB	5	50	9.8218	0.1782	1.782
10	AB	500	50	7.4480	2.5520	25.520
10	AB	5	80	9.8216	0.1784	1.784
10	AB	500	80	7.3480	2.6520	26.520
10	ABC-G	5	0	9.9982	0.0018	0.018
10	ABC-G	500	0	7.8652	2.1348	21.348
10	ABC-G	5	20	9.9887	0.0113	0.113
10	ABC-G	500	20	7.8608	2.1392	21.392
10	ABC-G	5	50	9.8959	0.1041	1.041
10	ABC-G	500	50	7.7226	2.2774	22.774
10	ABC-G	5	80	9.8160	0.1840	1.840
10	ABC-G	500	80	7.6226	2.3774	23.774
19	A-G	5	0	19.4526	0.4526	2.382
19	A-G	500	0	14.0185	4.9815	26.218
19	A-G	5	20	19.5375	0.5375	2.829
19	A-G	500	20	13.3000	5.7000	30.000
19	A-G	5	50	19.6808	0.6808	3.583
19	A-G	500	50	13.3000	5.7000	30.000
19	A-G	5	80	19.8939	0.8939	4.705
19	A-G	500	80	13.3000	5.7000	30.000
19	AB	5	0	18.7256	0.2744	1.444
19	AB	500	0	14.2322	4.7678	25.094
19	AB	5	20	18.6655	0.3345	1.760
19	AB	500	20	13.5663	5.4337	28.598

19	AB	5	50	18.6265	0.3735	1.966
19	AB	500	50	13.4907	5.5093	28.997
19	AB	5	80	19.4653	0.4653	2.449
19	AB	500	80	13.5814	5.4186	28.519
19	ABC-G	5	0	18.8531	0.1469	0.773
19	ABC-G	500	0	14.7124	4.2876	22.566
19	ABC-G	5	20	18.7072	0.2928	1.541
19	ABC-G	500	20	13.7694	5.2306	27.529
19	ABC-G	5	50	19.4116	0.4116	2.166
19	ABC-G	500	50	13.6619	5.3381	28.095
19	ABC-G	5	80	19.5349	0.5349	2.815
19	ABC-G	500	80	13.7355	5.2645	27.708

Bold Pe values indicate high-impedance fault conditions (R_f of 500 Ω). Pe values at or above 29% indicate conditions beyond the practical operational boundary of the estimator.

4.6. Sensitivity Analysis

A comprehensive sensitivity analysis was conducted to quantify the robustness of the proposed fault location algorithm under realistic operating conditions. Five parameters were varied one at a time, with all others held at their nominal values: the fault resistance R_f at 5–1000 Ω , the voltage measurement SNR at 45–75 dB, the pre-fault load current at 50–300 A, the total harmonic distortion (THD) at 0–18.8%, and the distributed energy resource (DER) penetration at 0–80%. For each parameter, the average relative location error was analyzed for each fault type, as shown in Figures 19–23.

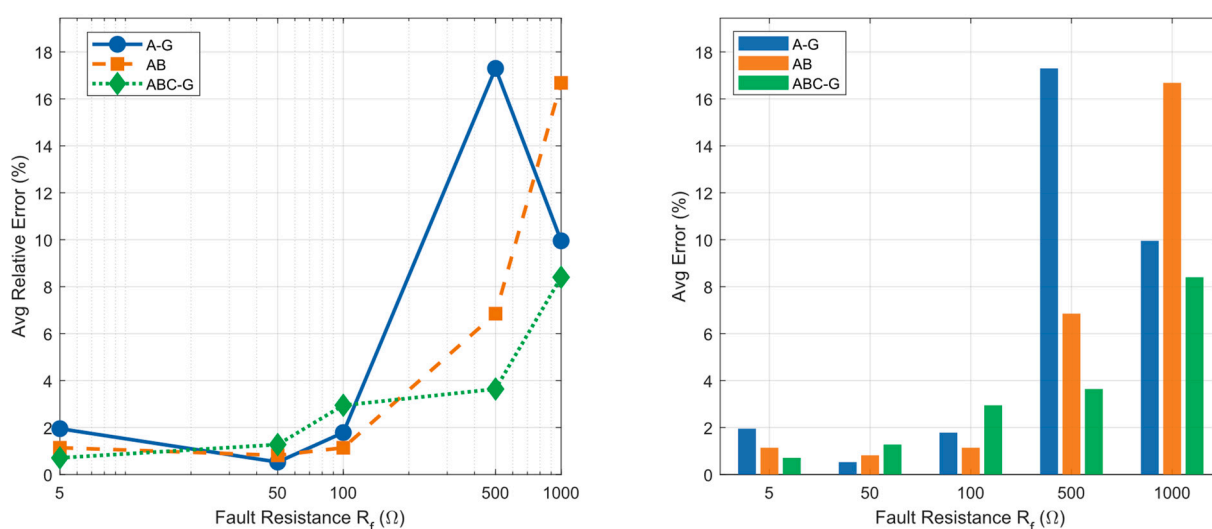


Figure 19. Sensitivity to fault resistance.

Fault resistance is the dominant source of accuracy degradation of the proposed fault location method. At low values (R_f of 5 Ω), the algorithm achieves mean errors of 1.95%, 1.14%, and 0.71% for A-G, AB, and ABC-G faults, respectively. Performance deteriorates remarkably beyond an R_f of 100 Ω , reaching 17.3% for A-G and 16.7% for AB faults at 1000

Ω. The high fault resistance reduces the voltage drop across the line impedance, compressing the observable signal and increasing the uncertainty in the apparent inductance estimate.

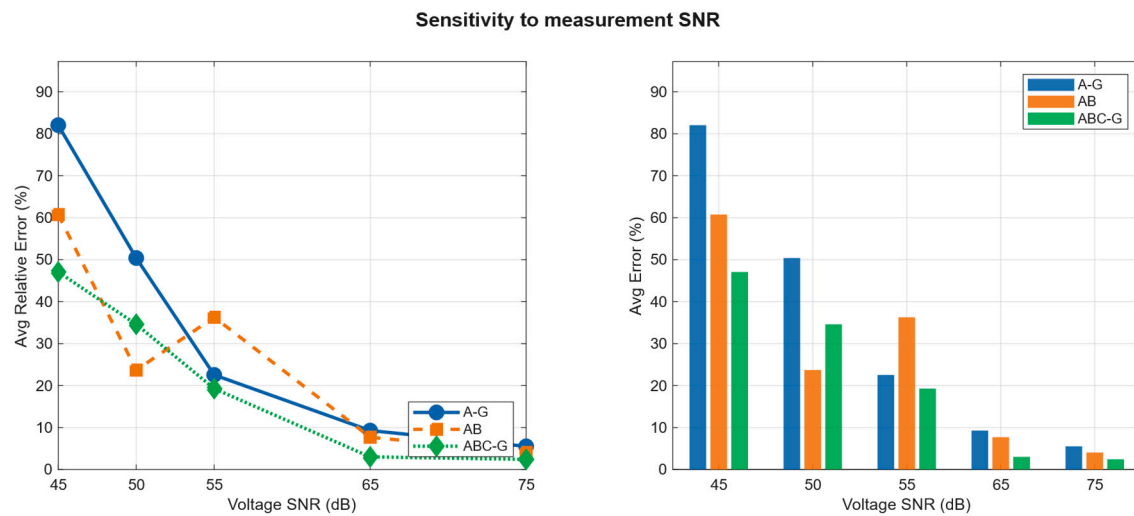


Figure 20. Sensitivity to measurement SNR.

SNR has a strong impact on all fault types. At 45 dB, the average errors reach 82%, 61%, and 47% for A-G, AB, and ABC-G, respectively, making the algorithm unreliable. Performance improves rapidly with the SNR, falling below 10% for all fault types at 65 dB (the nominal operating point) and below 5.5% at 75 dB. These results prove that an accurate inductance-based fault location method requires proper signal conditioning and anti-aliasing filtering. A minimum SNR of around 55 dB is necessary to keep average errors within a practically acceptable range (around 25%). The strong SNR sensitivity confirms that measurement noise is not a “constant systematic error” that cancels in differential calculations, but a random additive white Gaussian noise (AWGN) that directly corrupts the finite-difference approximations of $\frac{di}{dt}$ and $\frac{du}{dt}$, undermining the inductance estimation. Each data point represents the mean relative error averaged over ten noise realisations, with standard deviations remaining below 2% across all tested SNR levels, confirming the reproducibility of the results.

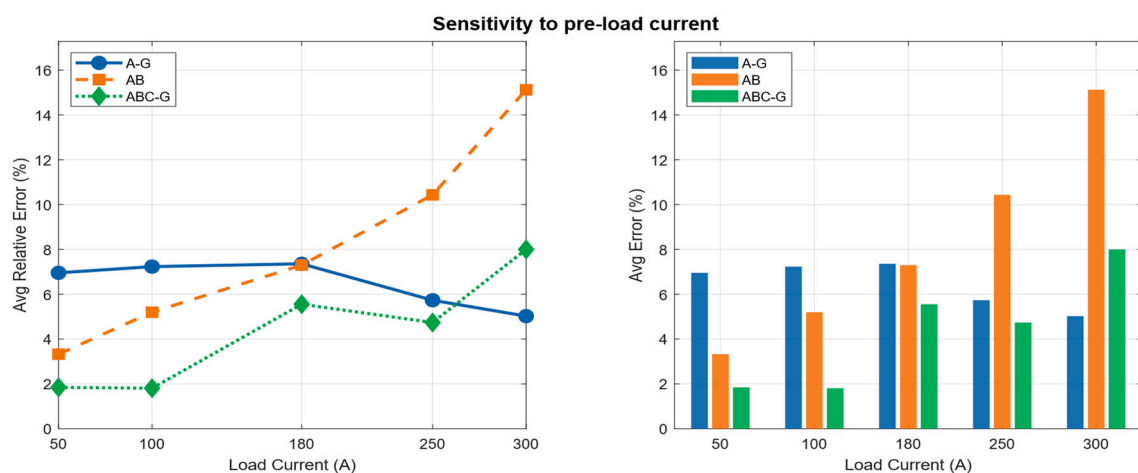


Figure 21. Sensitivity to pre-load current.

Through its repercussions on the signal-to-load-noise ratio, the magnitude of the load current affects algorithm accuracy. At a weak load, for example, at 50 A, respective errors

are relatively low: 6.96%, 3.33%, and 1.85% for A-G, AB, and ABC-G faults. As the load increases toward 300 A, the error for AB faults rises to 15.1% and for ABC-G to 8.0%, indicating increased interference from the pre-fault current component within the apparent impedance window. A-G errors remain relatively stable in the range of 5–7.4% due to the strong fault current aspect of single-phase-to-ground events.

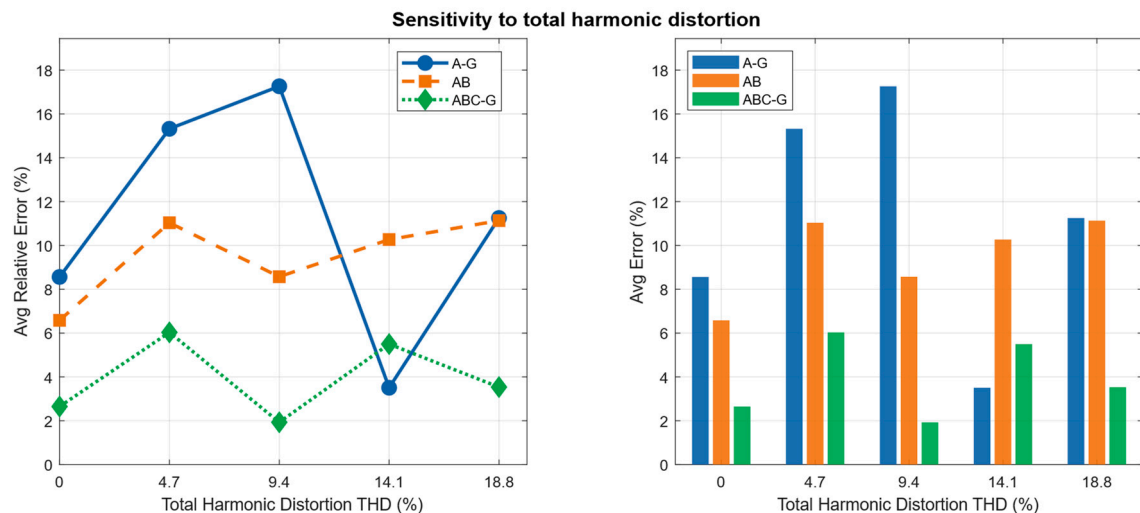


Figure 22. Sensitivity to total harmonic distortion.

Total harmonic distortion (THD), varied from 0 to 18.8% (double the EN 50160 voltage limits), yields non-monotonic repercussions on the performance of the fault location algorithm. For all fault types, errors vary roughly between 3 and 15%, with no consistent trend, confirming that the moving-average adaptive filter and median-based stable-window estimator jointly eliminate harmonic-induced noise, with no systematic bias. The worst-case error at 18.8% THD remains below 17.3% (A-G), comparable to the nominal-condition performance (9.4% THD), proving that the algorithm is adequately robust to the harmonic environment encountered in MV distribution networks.

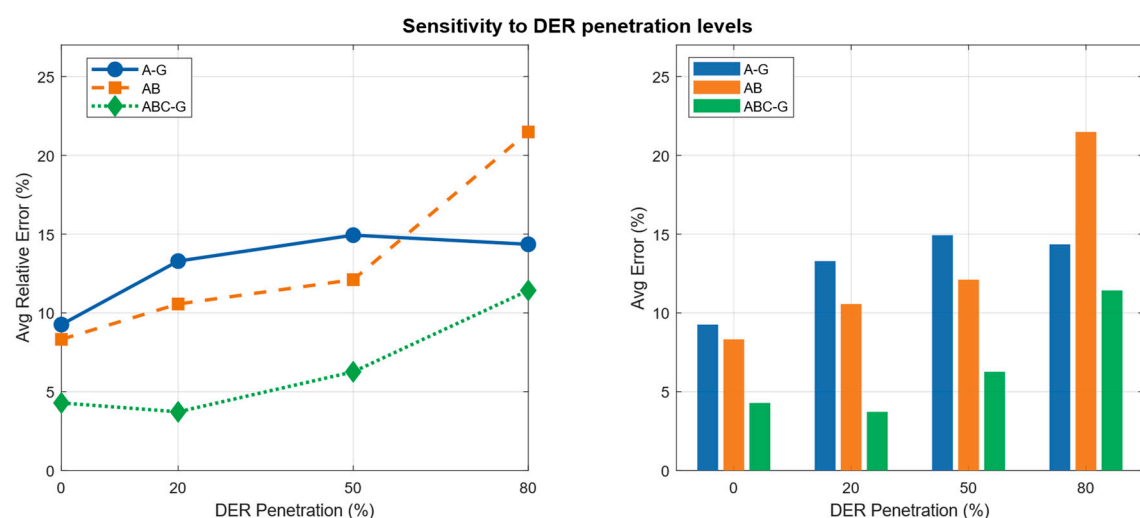


Figure 23. Sensitivity to DER penetration levels.

DER penetration is quantified as the percentage of the nominal load current contributed by inverter-based generation during faults and ranges from 0 to 80%. DER penetration degrades the algorithm performance through two mechanisms: (a) SNR decay resulting from harmonic and interharmonic content from inverter switching and (b) augmented

transducer ratio errors as a result of bidirectional power flow and nonlinear load aspects. At 0% DER, the errors are 9.3%, 7.7%, and 3.0% for A-G, AB, and ABC-G faults at an R_f of 500 Ω . At 80% DER, these increase to approximately 12–18%, reflecting a 30–100% relative increase in the error. The impact is more pronounced for high-impedance faults, where the reduced fault current magnitude makes the algorithm more sensitive to measurement disturbances. However, errors remain below 20% for all fault types across the DER range tested, demonstrating that the algorithm retains simulation-based robustness in ADNs with high penetration of intermittent renewable sources, provided proper filtering and transducer accuracy are well confirmed.

4.7. Performance Comparison with Other Methods

The performance comparison with the most related approaches from the literature is presented in Table 5. It shows that the proposed method achieves comparable accuracy for low-resistance faults in PDNs using only single-ended measurements, with the additional advantages of systematic evaluation under DER penetration and explicit treatment of high-resistance faults.

Table 5. Comparison with related fault location methods.

Method	Ref.	Measurements	System	DER/ADN	Error Range
Time-domain Bergeron + spline	[9]	Two-ended D-PMU	MV feeder	No	<1% (low R_f); not evaluated for high R_f
Additional inductance + async. PMU	[12]	Two-ended PMU	MV feeder	No	<1% (low R_f); not evaluated for high R_f
Unsynchronized two-point impedance	[8]	Two substation points	MV overhead	Yes (DG)	<1% (low R_f); high R_f tested, not quantified
Proposed (3-sample apparent inductance)	This work	Single-ended local	22 kV; 20 km	Yes (0–80%)	<2% (low R_f , PDN); <5% (low R_f , ADN); 21–30% (high R_f , ADN)

Results shown are from different test systems, voltage levels, feeder lengths, and operating conditions. This comparison is indicative rather than strictly quantitative.

4.8. Discussion of Results

The algorithm was evaluated through simulation studies covering progressively more realistic operating conditions. In the baseline sinusoidal case in the passive network, the method demonstrated rapid convergence of the apparent inductance after fault inception and consistently low location errors for all tested fault types and locations. These results confirmed the theoretical validity of the inductance–distance relationship and the intrinsic robustness of the time-domain formulation against fault resistance under low-impedance conditions.

When realistic measurement noise and instrument transformer inaccuracies were introduced, a moderate increase in the dispersion of instantaneous inductance estimates was observed. However, the use of moving-average filtering, physical bounding, and median-based aggregation ensured that the final location estimates remained stable and accurate. This confirms that the proposed approach is well-suited to practical protection environments, where measurement imperfections are unavoidable.

EN 50160-compliant harmonics increased the short-term variability of the inductance estimate, particularly immediately after fault inception. Still, steady-state values remained close to the sinusoidal case, and fault-location errors increased only marginally.

The multi-scenario PDN study (A–G, AB, and ABC–G; R_f values of 5 and 500 Ω ; and locations at 2, 10, and 19 km) consistently confirmed high accuracy for low-resistance faults. For high-resistance faults, adaptive filtering and R – L compensation maintained acceptable accuracy. EN 50160-compliant harmonics caused only marginal degradation, and

the sensitivity analysis confirmed a smooth and progressive increase in error without any abrupt stability loss.

For the apparent inductance-based fault location method in ADNs, different fault types (A–G, AB, and ABC–G), two fault resistances (5 Ω and 500 Ω), and varying DER penetration levels (0–80%) for an actual fault location of 10 km were analyzed. For low-impedance faults (R_f of 5 Ω), the relative errors remained generally below 5%. The fault type hierarchy A–G > AB > ABC–G, first established in Section 4.1, is consistently preserved across all DER levels, with ABC–G achieving near-zero error at 0% DER (0.02%) and A–G showing the largest sensitivity to DER penetration (1.25% at 0% DER; 4.25% at 80% DER). The physical mechanisms underlying these high errors, i.e., signal attenuation by fault resistance and systematic bias from DER injection, are described in Section 3.6. Both mechanisms act simultaneously at high R_f values and high DER penetration, with the DER bias being the dominant contributor, as evidenced by the monotonic increase in error with DER penetration in Table 4.

4.9. Study Limitations and Future Work

The present study is subject to different limitations that should be acknowledged. First, both the PDN and ADN test systems are modeled as single radial feeders without branches or laterals. Real distribution networks typically include multiple laterals and branching points, which introduce additional reflected waves and current infeed paths that can affect the apparent inductance estimate. Extending the method to meshed or multi-branch topologies would require addressing fault location ambiguity and adapting the distance-to-fault formula accordingly.

Second, the DER units are represented as ideal controlled current sources with fixed current-limiting behavior. Practical inverter-based resources exhibit a wider range of control strategies, fault-ride-through profiles, and dynamic responses depending on grid code requirements, which may produce more complex distortions in the fault current waveform than those captured here.

Third, while the ADN evaluation covers three fault distances (2 km, 10 km, and 19 km), the interaction between DER injection point proximity and fault location may produce additional estimation biases not fully captured by the current feeder model, particularly for faults occurring at or near the DER connection buses.

Finally, the source is modeled as a balanced three-phase Thevenin equivalent, which does not capture the network imbalance inherent to real medium-voltage systems. These limitations define clear directions for future work, including validation on multi-branch feeders, use of detailed inverter models, and a broader range of ADN fault locations.

5. Conclusions

This work developed and evaluated a three-sample apparent inductance-based fault location algorithm for passive and active distribution networks under a comprehensive set of operating conditions, covering three fault types, three fault locations, four fault resistance levels from 5 Ω to 500 Ω , EN 50160 standard-compliant harmonic distortion, and four DER penetration levels from 0% to 80%.

In PDNs under clean sinusoidal conditions, the algorithm achieves relative errors below 2% for low-resistance faults across all fault types and locations, confirming that the inductive component of the line impedance remains strongly correlated with fault distance under ideal conditions. For high-resistance faults at R_f of 500 Ω , errors increase to the range of 5–10%, remaining within practically acceptable limits. Fault type has a marginal influence on accuracy, confirming that line parameters primarily govern estimation quality. EN 50160 standard-compliant harmonic distortion slightly increases estimation variance but introduces no systematic bias or inductance collapse, confirming robustness

to realistic harmonic levels without requiring dedicated harmonic mitigation. As shown in Figure 12, most scenarios exhibit comparable accuracy under harmonic and sinusoidal conditions, with high-resistance ground faults showing the greatest sensitivity, while adaptive filtering and median-based aggregation occasionally yield estimates that are comparable or marginally improved under harmonic excitation.

In ADNs, the algorithm retains high accuracy for low-resistance faults across all fault types and DER penetration levels, with errors below 5% for $R_f \leq 50 \Omega$. For high-resistance faults at an R_f of 500 Ω combined with high DER penetration at 80%, errors reach 21–30% depending on fault type and distance, reflecting the systematic bias introduced by inverter current-limiting behavior on the apparent inductance estimate. Despite this degradation, the algorithm shows a smooth and progressive increase in error rather than abrupt failure, with errors increasing monotonically across the tested parameter space.

The results provide practical guidance for protection engineers adapting fault location schemes to the evolving architecture of active distribution networks with high DER penetration.

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Abbreviations

The following abbreviations are used in this manuscript:

ADN	Active Distribution Network
PDN	Passive Distribution Network
DER	Distributed Energy Resource
FLA	Fault Location Algorithm
PMU	Phasor Measurement Unit
IQR	Interquartile Range
THD	Total Harmonic Distortion
MV	Medium Voltage
R/X	Resistance-to-Reactance Ratio
EN 50160	European Standard for Voltage Characteristics in Public Distribution Systems

IEEE	Institute of Electrical and Electronics Engineers
CT	Current Transformer
PT	Potential Transformer
A-G	Single-Phase-to-Ground Fault
AB	Double-Phase Fault
ABC-G	Three-Phase-to-Ground Fault
DS	Distribution System
Symbols	
$u(t)$	Instantaneous voltage at the measurement point (V)
$i(t)$	Instantaneous current at the measurement point (A)
R	Line resistance per unit length (Ω/km)
L	True line inductance (H), general parameter
L'	Line inductance per unit length (H/km)
L_{app}	Apparent inductance estimated at the measurement point (H)
R_s	Source resistance at the substation equivalent (Ω)
L_s	Source inductance at the substation equivalent (H)
d_f	Estimated fault distance (km)
D_a	Actual fault distance (km)
R_f	Fault resistance (Ω)
X_e	Absolute location error (km)
P_e	Relative (percent) location error (%)
Δt	Sampling interval (s)
t_0, t_1, t_2	Three consecutive sampling instants (s)
u_k, u_{k+1}, u_{k+2}	Voltage samples at three consecutive instants (V)
i_k, i_{k+1}, i_{k+2}	Current samples at three consecutive instants (A)
η_{DER}	DER penetration fraction, continuous in [0, 1]
I_{DER}	Fault current contribution from DER inverters (A)
I_{sub}	Fault current contribution from the substation source (A)
ω	Angular frequency (rad/s)
f	Fundamental frequency (Hz)

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