

Article

Research on a Multilayer Two-Dimensional Equivalent Thermal Analysis Method for an Integrated Transmitting Antenna Cold Plate

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Abstract

This study proposes a multilayer equivalent thermal analysis method, based on flow boundary layer theory, considering the computational problem of three-dimensional (3D) conjugate heat transfer analysis of a transmitting antenna cold plate. To cut down computational overhead, a stratification strategy is introduced to build an equivalent model, such that the 3D conjugate heat transfer problem can be transformed into a multilayer two-dimensional (2D) heat transfer (M2DHT) problem. For the iterative analysis of the M2DHT, the equivalent heat transfer coefficient was introduced as the link between layers. Based on the Prandtl boundary layer theory, convective heat transfer coefficients of the boundary layer with temperature-independent fluid properties were deduced, further improving the accuracy of the 2D conjugate heat transfer analysis. The multilayer equivalent analysis method addresses the shortcomings of the heat transfer coefficient calculation through tedious 3D analysis. With well-set proper convective heat boundaries configured, iterative computation via M2DHT is utilized to resolve the surface temperature field of the cold plate. Numerical simulations demonstrate that the presented approach accurately characterizes temperature distributions while substantially boosting computational efficiency in thermal evaluations of cold plates for transmitting antennas.

Keywords: integrated transmitting antenna; conjugate heat transfer; equivalent thermal analysis method; equivalent heat transfer coefficient



Academic Editor: Mahmoud Bourouis

Received: 14 May 2026

Revised: 15 June 2026

Accepted: 25 June 2026

Published: 2 July 2026

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1. Introduction

The microwave transmitting antenna, as a core component of the microwave wireless power transmission (MWPT) system, relies on its high-power emission capability to ensure proper system operation. However, due to the inherent efficiency limitations of its active devices, approximately 50% of the input power is converted into waste heat, significantly degrading antenna performance [1,2]. Therefore, the thermal design for the active devices of the transmitting antenna is crucial, as it not only achieves effective heat dissipation but also optimizes system energy efficiency to ensure maximum power utilization and optimal overall performance of the MWPT system. Cold plates are often used as a powerful cooling means for transmitting antennae due to their high cooling efficiency and good uniformity [3–5]. In the past decade, studies on cold plates have mainly

focused on methods to obtain better heat dissipation performance using size and shape optimization methods [6–8]. Some scholars applied the topology optimization method to the structural design of cooling channels to obtain the optimal flow path [6,9–14]. Studies concerning dimensional and geometric optimization for cold plates generally rely on full three-dimensional (3D) numerical models. However, research on cold plate topology optimization design has mostly been conducted on two-dimensional (2D) models and very rarely on 3D topology optimization. Notably, the number of meshes for the 3D heat dissipation model of the cold plate is considerably large, which increases the simulation time significantly. When conducting design optimization, the calculation efficiency problem significantly affects the optimization process. Nonetheless, the accuracy of the analysis is also significantly reduced because the simple 2D model cannot accurately describe the surface temperature of the 3D model. Therefore, achieving a 2D equivalent heat dissipation analysis for a 3D model is particularly important when the calculation efficiency and solving accuracy become very important. The thermal design of a cold plate requires massive parametric analysis, working condition traversal and structural optimization in practical engineering. Engineers usually need to carry out hundreds or even thousands of repeated simulations to screen optimal structural schemes, flow parameters and layout forms. In such scenarios, the cumulative computing cost of the full 3D model will become extremely high.

There are two primary processing strategies for two-dimensional equivalent thermal analysis of cold plates. The first approach performs direct two-dimensional calculations while overlooking the inherent disparities between 2D and three-dimensional simulation results. The second scheme employs a multilayer framework that takes the dimensional discrepancies of 2D and 3D analysis into full consideration. The idea of directly using 2D analysis to replace 3D analysis is mainly adopted by the teams of Dede [9], Tadayoshi Matsumori [10], and Adriano A. Koga [11]. Since the difference between the 2D and 3D analyses is neglected, 2D analysis results do not match those of the 3D analysis. Therefore, only 2D analysis results were used as an estimate, and 3D simulation analysis was required after the 2D analysis was completed. The strategy for the multilayer model is mainly adopted by the teams of Caroline McConnell [12], Tijs Van Oevelen [6], J.H.K. Haertel [13], and Shi Zeng [14]. The relevant details are elaborated below. First, the 3D model is layered based on its characteristics. Subsequently, a heat exchange model between different layers is established. Finally, approximate results of the 3D model analysis are obtained through the analysis of the 2D equivalent model. The key step in this strategy is to establish a connection between different layers. In the work of Georg Pingen et al., an adiabatic-heat transfer boundary condition was constructed for the connection between different layers: for a solid–solid contact, the heat flow is completely transmitted, whereas, for a fluid–solid contact, there is no heat transfer [12]. Although their method does not match the actual situation, it is crucial as an initial work. Tijs Van Oevelen and Martine Baelmans adopted the averaging method. They reduced the three-dimensional geometry to a two-dimensional formulation by averaging the governing thermal equations along the thickness direction and subsequently derived numerical solutions via analysis on the simplified planar model [6]. The biggest shortcoming of this approach is that it can only be applied to the middle layer and does not consider the heat conduction effect of the upper and lower substrates in the 3D model. The method used by Haertel et al. was similar to that used by Georg Pingen et al. They divided the 3D model into a design layer and a solid substrate layer and improved the setting of the heat transfer boundary conditions between different layers. They believed that there is also heat flow transmitted at the fluid–solid contact surface and introduced the concept of the heat transfer coefficient [13]. However, there is no further explanation on how to determine the heat transfer coefficient. Additionally,

Shi Zeng et al. first mentioned the accuracy of the equivalent analysis of 2D models [14]. They adopted the concept of multilayers to simplify the 3D model and established the relationship between different layers using the heat transfer coefficient. The averaged convective coefficient was formulated using extracted simulation data of the 3D geometric model. Their study has significantly improved the analysis accuracy of the 2D equivalent model. However, its shortcoming is that the heat transfer coefficient is based on simulation results of the 3D model. Performing a prior 3D model simulation remains necessary. Tao Zeng et al. developed a transient equivalent analysis 2D model to study the instantaneous heat dissipation performance of a heat sink [15]. Zhao et al. extended the work of Shi Zeng et al. and Tao Zeng et al. to a three-layer model and improved thermal accuracy by considering the heat conduction of the top layer [16]. Based on the work of Bugra Kanargi et al. and Tao Zeng et al., Huang et al. conducted further research on the 2D equivalent analysis of microchannels. Considering that the viscous dissipation of the outside surface caused by the top and bottom will significantly impact the fluid flow in the microchannel, they introduced an out-of-plane viscous dissipation term in the 2D equivalent analysis equation to further improve the accuracy of the 2D equivalent analysis method of the microchannel [17]. However, the aforementioned authors used equivalent heat transfer coefficients obtained using 3D model calculations. Recently, Yan et al. noticed that fluid viscous and inertial forces are of the same order at low Reynolds numbers and improved the accuracy of the 2D equivalent analysis model considering the effect of the interlayer viscous force [7]. While the work of Yan et al. eliminates 3D model calculations, it is only restricted to the low Reynolds number flow in microchannels, rather than regular flow channels and high Reynolds numbers.

To address this issue, this study proposed a multilayer equivalent analysis method for conjugate heat transfer analysis. First, considering the heat conduction effect of the upper and lower substrates, the cold plate is layered and modeled according to the main heat transfer way of the cold plate. Second, for the M2DHT problem, the convective layer is divided into the mainstream and boundary layer regions based on the Prandtl boundary layer theory. The convective thermal coefficient corresponding to the boundary layer is derived, which eliminates the reliance on a complete 3D model. Subsequently, the heat transfer boundary conditions are constructed as the link of the relationship. Finally, a multilayer 2D equivalent model is established. This method can accurately predict the actual thermal performance and enable reliable 2D equivalent analysis.

The remainder of this paper is organized as follows. The second section introduces the establishment of the multilayer equivalent analysis model, which includes the stratification strategy, analysis process of the multilayer equivalent model, and derivation of equivalent heat transfer coefficient. The numerical calculations and corresponding discussions are illustrated in Section 3, and the final conclusions are provided in Section 4.

2. Multilayer Two-Dimensional Equivalent Modeling

The integrated transmitting antenna is illustrated in Figure 1. It mainly consists of an antenna array, T/R modules (power devices) and a liquid cooling cold plate. The antenna array is arranged at the top. The T/R modules are mounted on the radio frequency (RF) circuit board, and the integrated assembly of the two is placed in the middle. The liquid cooling cold plate is located at the bottom.

In the integrated transmitting antenna, the liquid cooling cold plate serves two key functions. It acts as a carrier to support the antenna array, integrated circuits and relevant modules. More importantly, it performs heat dissipation to keep power devices such as T/R modules operating within the normal temperature range.

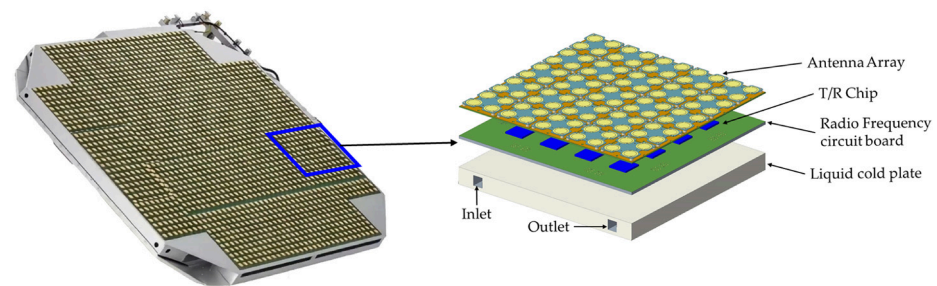


Figure 1. Schematic diagram of integrated transmitting antenna.

2.1. Stratification Strategy

As conduction and convection coexist in cold plate interiors, conjugate heat transfer simulations become mandatory. Based on its structural features, the cold plate is divided into four functional layers: device layer, cover layer, convection layer and floor layer. The layered structure is shown in Figure 2.

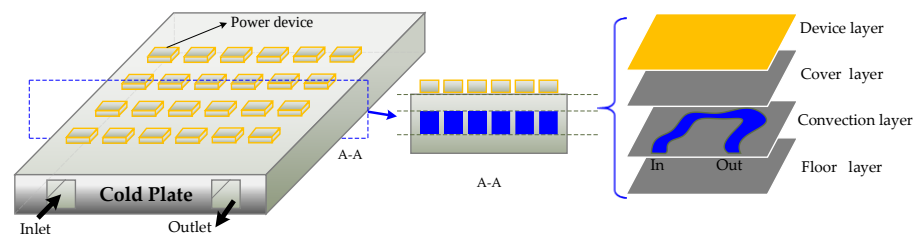


Figure 2. Layered diagram of cold plate.

Figure 2 shows that the device and cover layers are both heat conductive and in direct contact. Therefore, the device and cover layers could be combined as a single first layer. In this layer, the upper surface is the heat source location surface, and the lower surface is the heat exchange surface. Since the lower surface is in contact with not only the solid portion of the middle layer but also the fluid, both conduction and convection heat transfer occur in the heat exchange surface. The second layer is the conjugate heat transfer layer in which heat dissipation by conduction and convection heat transfer is involved. In addition, the upper surface of the second layer is in contact with the first layer, the lower surface is in contact with the floor layer, and both surfaces exhibit conduction and convection heat transfer. Consistent with the first layer, thermal conduction dominates the third layer. By contrast, convection takes place on its upper boundary, matching the position of the internal flow channel.

Based on the layered structure, appropriate boundary conditions are defined to integrate individual layers into a complete analytical model. The upper surface of Layer 1 can be set as the heat flux boundary where the heat resource is located. Since concurrent solid–solid and solid–fluid contacts appear on two interfaces: Layer 1's bottom surface paired with Layer 2's upper surface, and Layer 2's lower surface paired with Layer 3's top surface, an equivalent heat transfer coefficient is introduced to establish the convection boundary for building up the heat transfer of different surfaces. The determination of the equivalent heat transfer coefficient is specified in Section 2.3.

2.2. Analyzing Procedure

For the simulation process, the first layer is initially analyzed to obtain the temperature distribution. Then the analysis result of the first layer is transmitted to the second layer through boundary conditions. Next, conjugate heat transfer analysis is performed on the second layer, and the analysis result of the second layer is transmitted to the first and third

layers through boundary conditions. The third layer is then analyzed. The temperature distribution between the first and second layers, as well as between the second and third layers, is compared. If the temperature error is within a limited range, the analysis is completed. Otherwise, the foregoing iterative procedures will be circulated continuously until the temperature of each layer reaches a steady state, and the temperature error is within a limited range. The simulation process is illustrated in Figure 3.

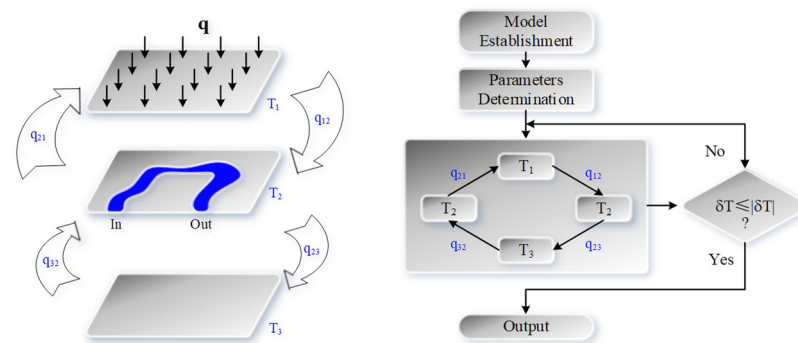


Figure 3. Schematic of the multilayer equivalent simulation process.

2.3. Equivalent Heat Transfer Coefficient

From the perspective of fluid mechanics, a thin flow layer forms adjacent to solid walls. Restricted by the wall no-slip boundary condition and mainstream flow velocity characteristics, the fluid exhibits a steep velocity gradient along the wall-normal direction, where viscous forces cannot be neglected. L. Prandtl's boundary layer theory states that, for low-viscosity fluids, viscous effects only require consideration within zones featuring substantial cross-flow velocity gradients, namely the near-wall domains adjacent to solid surfaces. The fluid in the main flow area can be treated as inviscid [18]. Considering the structural features of cold plates, the vertical height of flow channels is far smaller than their planar dimensions. Accordingly, the fluid flow within the second layer can be simplified to the flow confined between two parallel plates accompanied by boundary layers. Moreover, limited channel length prevents the formation of fully developed flow within the cold plate. Therefore, this article focuses on the developing flow. Furthermore, this research only covers laminar boundary layer flow over flat plates. Accordingly, we further hypothesize that both velocity and temperature profiles within the boundary layer follow quadratic curves. Furthermore, velocity and temperature profiles within the mainstream domain are presumed uniform along the channel height direction, as shown in Figure 4. Consequently, solving the equivalent heat transfer coefficient at the fluid–solid interface can be transformed into calculating the boundary layer convective heat transfer coefficient.

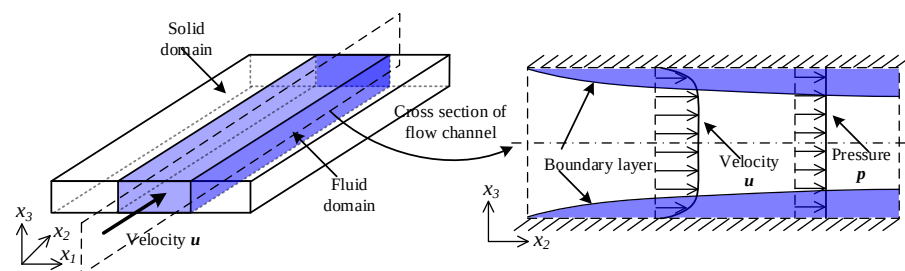


Figure 4. Velocity and pressure profile distribution assumptions of the conjugate heat transfer layer.

The following assumptions are considered in the analysis of the heat transfer and fluid flow of the cold plates:

- (1) The coolant flow is incompressible, single-phase and steady-state.
- (2) The coolant flow exhibits laminar flow characteristics.
- (3) The coolant and cold plate materials are constant, homogeneous and isotropic.
- (4) Heat loss, body forces, and gravity do not have any effect.
- (5) No slip between coolant and wall.

2.3.1. Fluid–Solid Interfacial Equivalent Heat Transfer Coefficient

For flow over a flat plate [19], the critical Reynolds number for the transition from laminar to turbulent flow is approximately $Re_x = 3 \times 10^4$. In pipe flow [19], the maximum Reynolds number for stable laminar flow is 2100. Therefore, this study takes 2100 as the upper Reynolds number limit for the validity of the laminar flow assumption. Within the boundary layer, fluid velocity rises from zero up to the mainstream velocity, while pressure stays uniform parallel to the wall. For steady, incompressible laminar flow, the governing equations describing boundary layer convective heat transfer are written as [19]:

$$\frac{\partial u}{\partial x_2} + \frac{\partial v}{\partial x_3} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x_2} + v \frac{\partial u}{\partial x_3} = -\frac{1}{\rho} \frac{dp}{dx_2} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x_3^2} \right) \quad (2)$$

$$\rho C_p \left(u \frac{\partial T}{\partial x_2} + v \frac{\partial T}{\partial x_3} \right) = k_f \frac{\partial^2 T}{\partial x_3^2} \quad (3)$$

$$u = 0, \quad \partial u / \partial x_3 = \tau_w / \mu \quad \text{at } x_3 = 0 \quad (4)$$

$$u = u_\infty, \quad \partial u / \partial x_3 = 0 \quad \text{at } x_3 = \delta \quad (5)$$

$$T = T_w \quad \text{at } x_3 = 0 \quad (6)$$

$$T = T_\infty, \quad \partial T / \partial x_3 = 0 \quad \text{at } x_3 = \delta_T \quad (7)$$

where u denotes the streamwise velocity, v denotes the wall-normal velocity, p denotes for static pressure, ρ denotes the fluid density, μ denotes the fluid dynamic viscosity, k_f denotes the fluid thermal conductivity, C_p denotes the fluid specific heat capacity at constant pressure, τ_w denotes the shear stress at the wall, u_∞ corresponds to the mainstream velocity, δ denotes the hydrodynamic boundary layer thickness, and δ_T represents the thermal boundary layer thickness.

Under the premise that the velocity profile within the laminar boundary layer follows a quadratic function, the explicit expression of boundary layer velocity distribution can be derived by applying the corresponding boundary conditions.

$$\frac{u}{u_\infty} = 2 \left(\frac{x_3}{\delta} \right) - \left(\frac{x_3}{\delta} \right)^2, \quad 0 \leq x_3 \leq \delta \quad (8)$$

where x_3 represents the channel height and δ denotes the hydrodynamic boundary layer thickness.

Since this study focuses on the effect of the boundary layer on heat transfer, the relationship between the thickness of the flow boundary layer and the flow distance can be obtained by derivation. The detailed derivations are provided in Appendix A.

$$\delta = \left(\frac{30\mu x_2}{\rho u_\infty} \right)^{1/2} \quad (9)$$

where μ denotes the fluid dynamic viscosity, x_2 is the flow distance, ρ denotes the fluid density, and u_∞ corresponds to the mainstream velocity.

Once the hydrodynamic boundary layer thickness is solved, we define the thickness ratio between the hydrodynamic boundary layer and the thermal boundary layer as $\xi = \delta_T / \delta$. Based on the assumption that the temperature distribution in the laminar boundary layer satisfies a quadratic curve, the specific form of the temperature distribution can be derived as:

$$\theta = T - T_w, \theta_\infty = T_\infty - T_w \quad (10)$$

$$\theta = 2\theta_\infty \left(\frac{x_3}{\delta_T} \right) - \theta_\infty \left(\frac{x_3}{\delta_T} \right)^2, \quad 0 \leq x_3 \leq \delta_T \quad (11)$$

Substituting Equations (8)–(11) into Equation (3) and performing integration over the thermal boundary layer's normal direction, the thermal boundary layer thickness can then be derived analytically. Detailed derivations are provided in Appendix A.

$$\delta_T = \left(\frac{4k_f}{5\mu C_p} \right)^{1/3} \delta = \left(\frac{4k_f}{5\mu C_p} \right)^{1/3} \left(\frac{30\mu x_2}{\rho u_\infty} \right)^{1/2} \quad (12)$$

where ρ is the fluid density, μ is the fluid dynamic viscosity, k_f is the fluid thermal conductivity, C_p is the fluid specific heat capacity at constant pressure, and u_∞ is the velocity in the main flow area.

According to the definition for third-type thermal boundary conditions, which incorporate the convection heat transfer coefficient, the following relation holds:

$$-k_f \left(\frac{\partial T}{\partial x_3} \right)_w = h_f (T_w - T_f) \quad (13)$$

Substituting Equations (11) and (12) into Equation (13), the boundary layer convective heat transfer coefficient is derived as:

$$h_f = 2k_f \left(\frac{4k_f}{5\mu C_p} \right)^{-1/3} \left(\frac{30\mu x_2}{\rho u_\infty} \right)^{-1/2} \quad (14)$$

2.3.2. Solid–Solid Interfacial Equivalent Heat Transfer Coefficient

To analyze the solid–solid interfacial equivalent heat transfer coefficient, the same analysis strategy of fluid–solid interfacial equivalent heat transfer coefficient is adopted. For consistency with the fluid–solid interface analysis, we thereby select a contact region whose thickness matches the thermal boundary layer thickness of the fluid–solid interface for subsequent derivation.

As illustrated in Figure 5, the dimension along the thickness direction is far smaller than those in the other directions. Accordingly, nearly all heat flux propagates solely through the thickness direction. Based on the Fourier law of heat conduction, the heat flux density transferred in the thickness direction can be expressed as [19]:

$$q = -k_s \left(\frac{\partial T}{\partial x_3} \right) = k_s \frac{T_{w1} - T_{w2}}{\delta_T} \quad (15)$$

Based on the definition of third-kind boundary conditions that incorporate the convective heat transfer coefficient, Equation (15) can be converted by substituting Equation (15) into Equation (13) to obtain the equivalent heat transfer coefficient of the solid–solid surface:

$$h_s = \frac{k_s}{\delta_T} \quad (16)$$

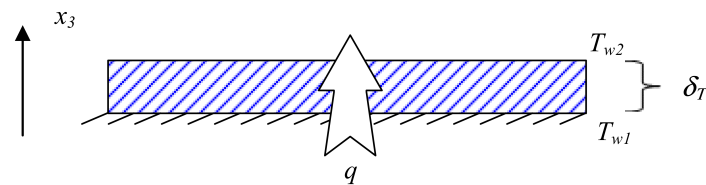


Figure 5. Schematic of solid–solid surface heat transfer.

2.4. Establishment of Governing Equations at Each Layer

Since there is only thermal conduction in Layer 1, the dimension in the thickness direction is much smaller than those in the other two directions, which applies similarly to the thermal resistance. Thus, the heat is mainly transferred along the thickness direction. Here, we used the averaging method adopted by Tijs Van Oevelen and Martine Baelmans [6]. By averaging the heat transfer governing equation of Layer 1 in the thickness direction, the reduced-dimensional governing equation of the heat transfer of Layer 1 can be obtained as:

$$\nabla_2 \cdot k_s \nabla_2 [T_T]_{H_T} + \frac{q_s}{H_T} - \frac{q_{iT}}{H_T} = 0 \quad (17)$$

where $\nabla_2 = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j}$, $[T_T]_{H_T}$ is the average temperature field in the layer along the thickness direction, q_s is the heat flux density generated by the power device, q_{iT} is the heat flux transmitted from Layer 1 to Layer 2, and H_T indicates the thickness of Layer 1.

Heat transfers through Layer 1 and enters the convection layer by conduction and convection. Since both conductive and convective heat transfer coexist within Layer 2, a conjugate heat transfer analysis is necessary. Following the laminar boundary layer assumption adopted in this work, the velocity and temperature profiles of the mainstream outside the boundary layer are uniform along the channel height. Thus, governing equations for reduced-dimensional conjugate heat transfer analysis can be obtained by averaging the governing equations of conjugate heat transfer in the main flow area.

$$\nabla_2 \cdot [\mathbf{u}]_{H_M} = 0 \quad (18)$$

$$\rho [\mathbf{u}]_{H_M} \cdot \nabla_2 [\mathbf{u}]_{H_M} - \mu \nabla_2 \cdot (\nabla_2 [\mathbf{u}]_{H_M} + \nabla_2 [\mathbf{u}]_{H_M}^T) + \nabla_2 [p]_{H_M} = 0 \quad (19)$$

$$\rho C_p \nabla_2 \cdot [\mathbf{u}]_{H_M} [T_M]_{H_M} - \nabla_2 \cdot k_f \nabla_2 [T_M]_{H_M} + \frac{q_{iT}}{H_M} - \frac{q_{iB}}{H_M} = 0 \quad (20)$$

where ρ is the fluid density, μ is the fluid dynamic viscosity, C_p is the fluid specific heat capacity at constant pressure, k_f is the fluid thermal conductivity, p is the 2D pressure, $[\mathbf{u}]_{H_M}$ is the average velocity field in the thickness direction, $[T_M]_{H_M}$ is the average temperature field in the thickness direction, q_{iT} is the heat flux from the first layer to the second layer, q_{iB} is the heat flux from the second layer to the third layer, and H_M is the thickness of the second layer.

Through the analysis of the boundary layer, the fluid close to the wall is decelerated owing to the action of the viscous force, such that the boundary layer loses part of the flow. In addition, the main flow area is raised by the extrusion thickness of the boundary layer, as shown in Figure 6. Therefore, when reduced-dimensional governing equations of the convective layer conjugate heat transfer analysis are used, the inlet flow velocity must be corrected accordingly.

The extrusion thickness of the boundary layer can be expressed as:

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dx_3 \quad (21)$$

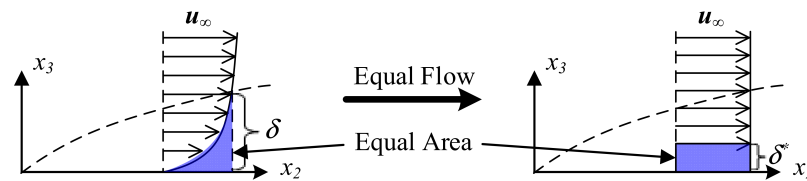


Figure 6. Schematic of the extrusion thickness of the boundary layer.

Substituting Equation (8) into Equation (21), the extrusion thickness of the boundary layer can be obtained as $\delta^* = \delta/3$.

According to the principle of equal flow, owing to the existence of the extrusion thickness, the reduction in the cross-section causes the inlet velocity of the main flow area to be slightly higher than the original average inlet velocity. The correction equation is as follows:

$$U^* = U \frac{H_M}{H_M - 2\delta^*} \quad (22)$$

where U^* denotes the corrected inlet velocity of the main flow area, U denotes the original average flow velocity at the inlet, H_M denotes the thickness of the second layer, and δ^* denotes the extrusion thickness of the boundary layer.

The condition for Layer 3 is the same as that for Layer 1, where only thermal conduction occurs. Therefore, referring to the processing method of the first layer, the governing equation of the heat transfer analysis of this layer is averaged along the thickness direction, and the reduced-dimensional governing equation can be expressed as:

$$\nabla_2 \cdot k_s \nabla_2 [T_B]_{H_B} + \frac{q_{iB}}{H_B} = 0 \quad (23)$$

where $\nabla_2 = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j}$, $[T_B]_{H_B}$ is the average temperature field in the layer along the thickness direction, q_{iB} is the heat flux density transferred from Layer 2 to Layer 2, and H_B is the thickness of Layer 3.

Governing equations and boundary conditions for each layer are presented in Table 1. To simplify the expression of equations, the average operator of the temperature field and the velocity field were hidden in the formulas later in this study.

Table 1. Governing equations and boundary conditions of each layer.

	Governing Equations	Boundary Conditions
The first layer	$\nabla_2 \cdot k_s \nabla_2 T_T + \frac{q_s}{H_T} - \frac{q_{iT}}{H_T} = 0$	$-\mathbf{n} \cdot k \nabla_2 T_T = 0$ at $\partial\Omega$ $q_{iT} = h_f(T_T - T_M)$ at $\Omega_{solid \& fluid}$ $q_{iT} = h_s(T_T - T_M)$ at $\Omega_{solid \& solid}$
The second layer	$\nabla_2 \cdot \mathbf{u} = 0$ $\rho \mathbf{u} \cdot \nabla_2 \mathbf{u} - \mu \nabla_2 \cdot (\nabla_2 \mathbf{u} + \nabla_2 \mathbf{u}^T) + \nabla_2 p = 0$ $\rho C_p \nabla_2 \cdot \mathbf{u} T_M - \nabla_2 \cdot k_f \nabla_2 T_M + \frac{q_{iT}}{H_M} - \frac{q_{iB}}{H_M} = 0$	$\mathbf{u} = U^*$, $T_M = T_{inlet}$ on Γ_{inlet} $p = 0$, $-\mathbf{n} \cdot k_f \nabla_2 T_M = 0$ on Γ_{outlet} $-\mathbf{n} \cdot k_s \nabla_2 T_M = 0$ at $\partial\Omega$ $q_{iT} = h_f(T_T - T_M)$ at $\Omega_{solid \& fluid}$ $q_{iT} = h_s(T_T - T_M)$ at $\Omega_{solid \& solid}$ $q_{iB} = h_f(T_M - T_B)$ at $\Omega_{solid \& fluid}$ $q_{iB} = h_s(T_M - T_B)$ at $\Omega_{solid \& solid}$
The third layer	$\nabla_2 \cdot k_s \nabla_2 T_B + \frac{q_{iB}}{H_B} = 0$	$-\mathbf{n} \cdot k \nabla_2 T_B = 0$ at $\partial\Omega$ $q_{iB} = h_f(T_M - T_B)$ at $\Omega_{solid \& fluid}$ $q_{iB} = h_s(T_M - T_B)$ at $\Omega_{solid \& solid}$

Where ρ is the fluid density, μ is the fluid dynamic viscosity, C_p is the fluid-specific heat capacity at constant pressure, k_s and k_f are the solid and fluid thermal conductivities, respectively. p is the 2D pressure, $\mathbf{u}$ is the average velocity field in the thickness direction, T_T , T_M , and T_B are the average temperature fields of the first, second, and third layers

along the thickness direction, respectively. H_T , H_M , and H_B are the thicknesses of the first, second, and third layers, respectively. q_s is the heat flux density generated by the power device, q_{iT} is the heat flux from the first layer to the second layer, and q_{iB} is the heat flux from the second layer to the third layer.

3. Numerical Experiments and Discussion

3.1. Verifying the Validity of the Proposed Model and Methodology

In order to demonstrate the proposed model and methodology, S-type and Z-type cold plates were used as examples. The cold plate size was $100\text{ mm} \times 80\text{ mm} \times 10\text{ mm}$, the cross-section of the flow channel is $6\text{ mm} \times 6\text{ mm}$, and uniform heat flux was applied to the upper surface of the cold plate. The Reynolds number of the entrance is 600. Details are shown in Figure 7.

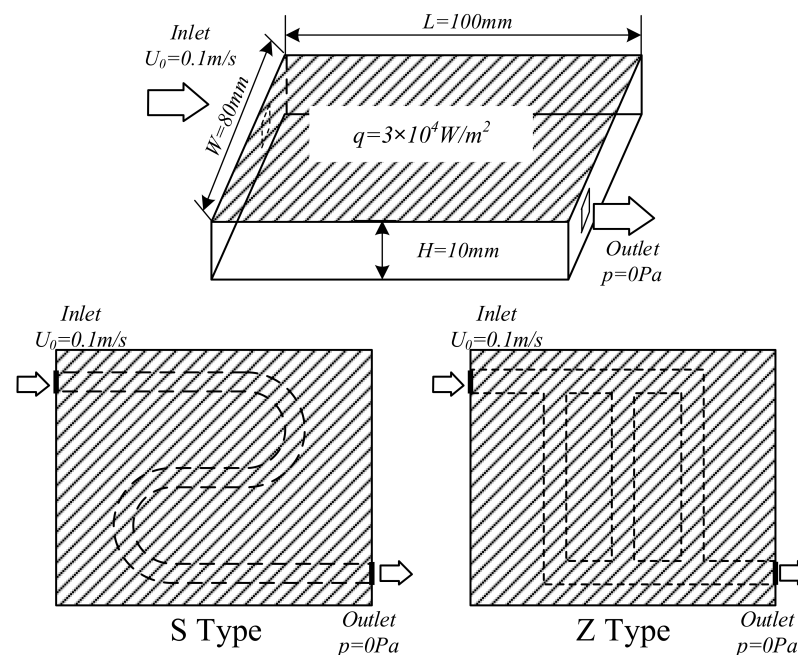


Figure 7. Simulation Model.

Parameters for the simulation analysis are described in Table 2:

Table 2. Thermal simulation analysis parameters.

Model	Size [mm]	Inlet Section [mm]	Inlet Velocity [m/s]	Inlet Temperature [K]	Outlet Pressure [Pa]	External Heat Source [W/m ²]
3D model	100 × 80 × 10	6 × 6	0.1	293.15	0	30,000
2D equivalent model	100 × 80	–	0.1	293.15	0	30,000

Material properties are given in Table 3.

Table 3. Material Properties [20,21].

Material	k [W/m × K]	C_p [J/kg × K]	ρ [kg/m ³]	μ [N × s/m ²]
Water	0.599	4182	998.2	0.001
Aluminum	202.4	871	2719	–

For S-type and Z-type cold plates, the calculated Reynolds number is 598.92, satisfying the laminar flow assumption. Finite element meshing was performed based on the built geometric model. For both S-type and Z-type cold plates, the 3D model adopts a mixed mesh dominated by tetrahedra, where a two-layer prism mesh is used as the boundary layer at the interface between the flow channel and the solid; the 2D model adopts a mixed mesh mainly composed of triangles, with two layers of quadrilateral mesh used as the boundary layer at the interface between the channel and the solid. Additionally, to facilitate the rapid transmission of data, different layers in the 2D model shared the same grids. The schematic diagram of grid division is shown in Figure 8.

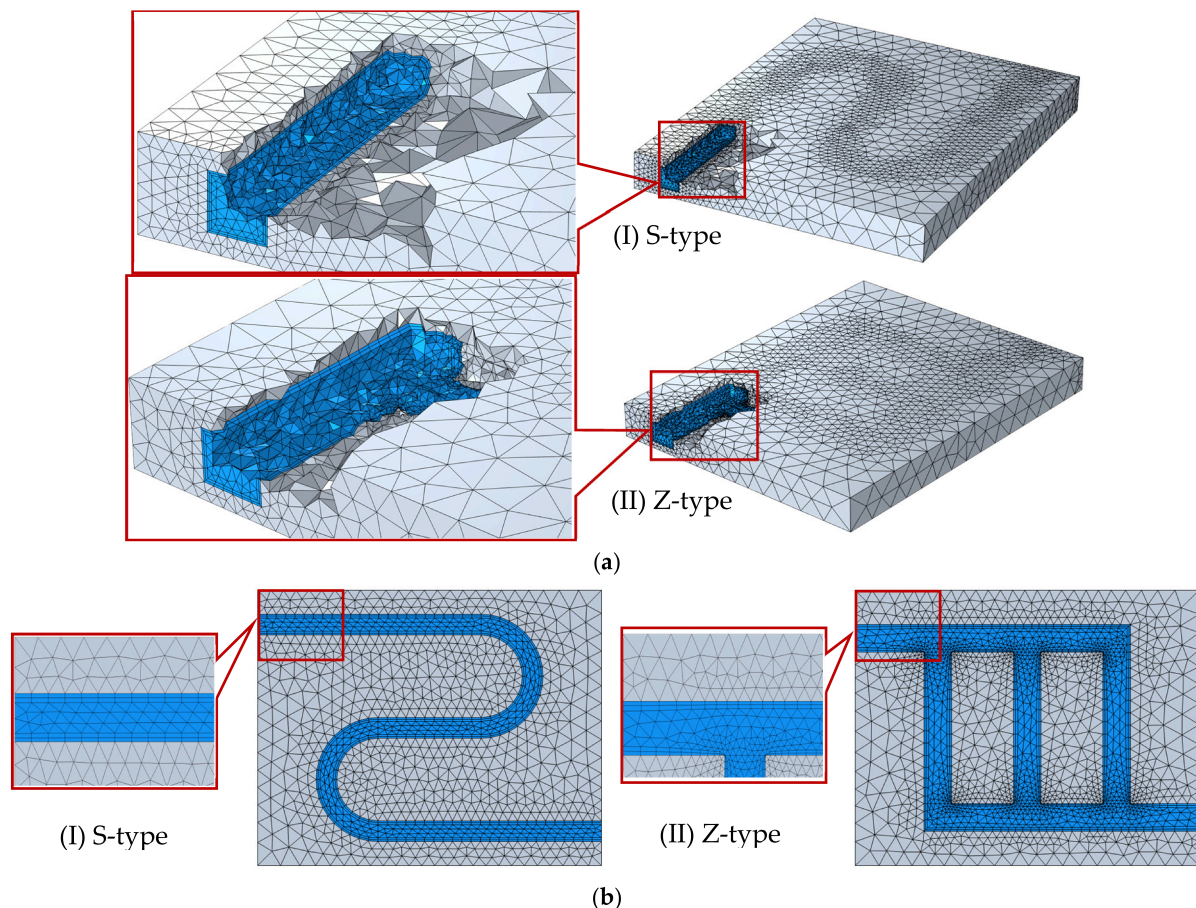


Figure 8. Grid and partial enlargement of the analysis model: (a) 3D model with the enlargement of local channel; (b) 2D model with the enlargement of local channel.

Firstly, to ensure the accuracy of the 3D analysis results, we conducted a grid independence check. As heating electronic devices are directly installed on the surface of the cold plate, the installing surface temperature of the cold plate directly reflects the temperature of the electronic devices. Thus, this study has been focused on the temperature of the installing surface. The maximum temperature and the average temperature of the installing surface are selected as the index to perform the grid independence study. Figure 9 shows the result of the grid independence study.

As presented in Figure 9, a grid independence verification is carried out with four sets of mesh quantities ranging from 80,000 to 400,000 cells, corresponding to different mesh sizes. The influences of mesh configurations on the peak temperature and volume-averaged temperature are investigated. For the S-shaped cold plate, a mesh of 210,151 cells is selected for subsequent numerical simulations, where the relative errors of peak temperature and average temperature are controlled within 0.21% and 0.25%, respectively. An identical

mesh convergence procedure is conducted for the Z-shaped cold plate, and the final mesh count of 217,038 cells is adopted, yielding relative deviations no greater than 0.07% and 0.01% for peak and average temperature separately.

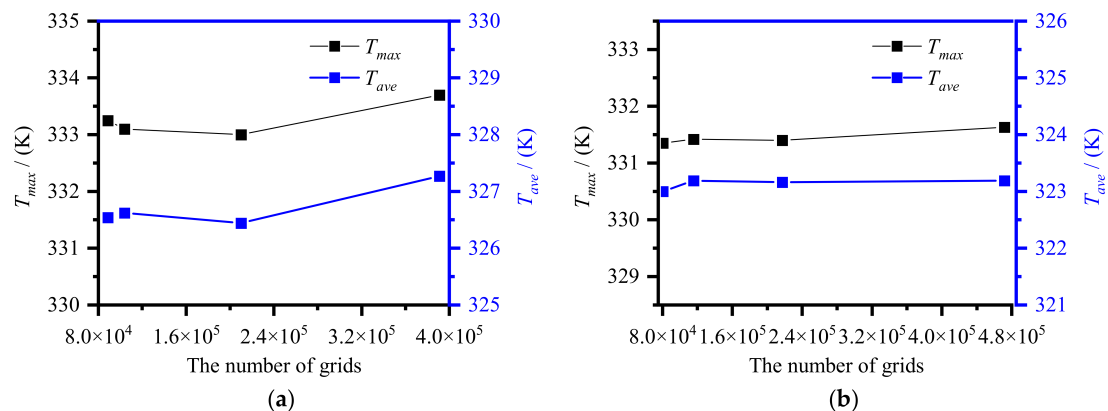


Figure 9. The result of grid independence study: (a) S-type; (b) Z-type.

The final mesh partitioning of different models is shown in Table 4 (3D model) and Table 5 (2D equivalent model).

Table 4. The mesh partitioning of 3D models.

Model	Channel Type	Element Number			Total Element Number
		Tetrahedra	Prism	Pyramid	
3D model	S-type	182,455	27,696	0	210,151
	Z-type	190,694	25,628	716	217,038

Table 5. The mesh partitioning of 2D equivalent models.

Model	Channel Type	Element Number		Total Element Number
		Triangle	Quad	
2D equivalent model	S-type	1926	372	2298
	Z-type	3191	484	3675

Temperature distributions of the 3D model and the 2D equivalent models are shown in Figures 10 and 11.

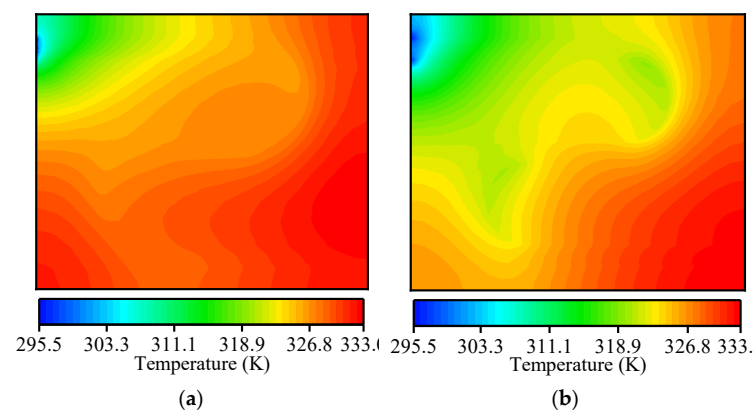


Figure 10. Temperature distribution of cold plate surface (S-type). (a) 3D model. (b) 2D equivalent model.

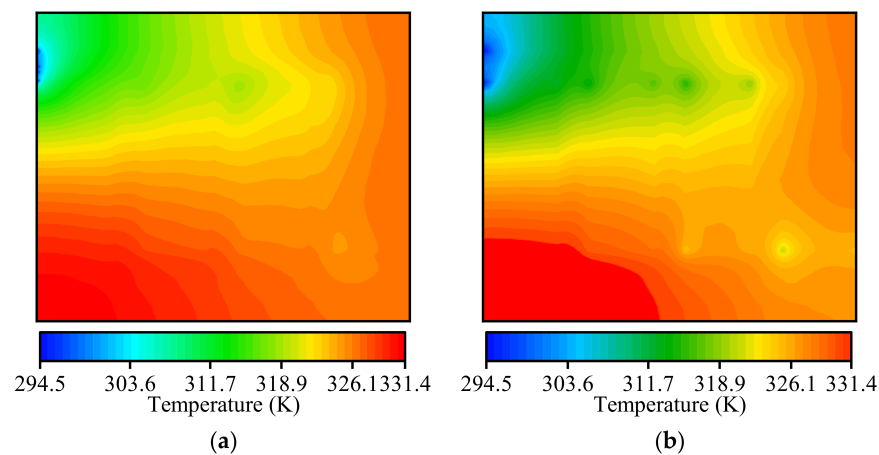


Figure 11. Temperature distribution of cold plate surface (Z-type). (a) 3D model. (b) 2D equivalent model.

To compare differences between the results of the 2D equivalent model and the 3D model, the highest temperature T_{max} , lowest temperature T_{min} , average temperature T_{ave} and the analysis time t were used. In addition, the root mean square error of the temperature RMS_T^E was used to express the accuracy of the 2D equivalent model.

$$RMS_T^E = \sqrt{\frac{1}{N} \sum_{i=1}^N (T_i^* - T_i)^2} \quad (24)$$

where N is the total number of nodes of the surface temperature, T_i^* is the node temperature obtained by the 2D equivalent model, and T_i is the node temperature obtained by the 3D model.

Numerical results are shown in Table 6.

Table 6. Comparison of temperature distribution analysis of S-type and Z-type cold plates.

Analysis Model		T_{max}/K	T_{min}/K	T_{ave}/K	RMS_T^E/K	$T_{max}-T_{min}/K$	t/s
S-type	3D model	333	295.5	326.4	-	37.5	386
	2D equivalent model	332.7	295.7	323.5	3.74	37	15
Z-type	3D model	331.4	294.5	323.1	-	36.9	564
	2D equivalent model	334.4	295.5	323.3	1.68	38.9	20

As shown in Figures 10 and 11, the temperature distribution of the 2D equivalent analysis model is in good agreement with that of the 3D analysis model. Furthermore, Table 6 shows that the root mean square error of temperature (RMS_T^E) is within 4K, and the errors of temperature range ($T_{max}-T_{min}$) is within 6%. These results show that the 2D equivalent model established in this study can provide a good equivalent representation of the 3D model.

3.2. Comparison of Results with Other Simulation Works

In addition, to verify the applicability of the method proposed here, two cases of conventional flow channel and micro flow channels are discussed. For conventional flow channels, our method is compared to the analytical model described in Ref. [15] (more details of the model can be found in Figure 6 and Tables 1 and 2 in Ref. [15]). The analysis results of the model of the conventional flow channel by the method of the reference and our proposed method are compared in Figure 12 and Table 7.

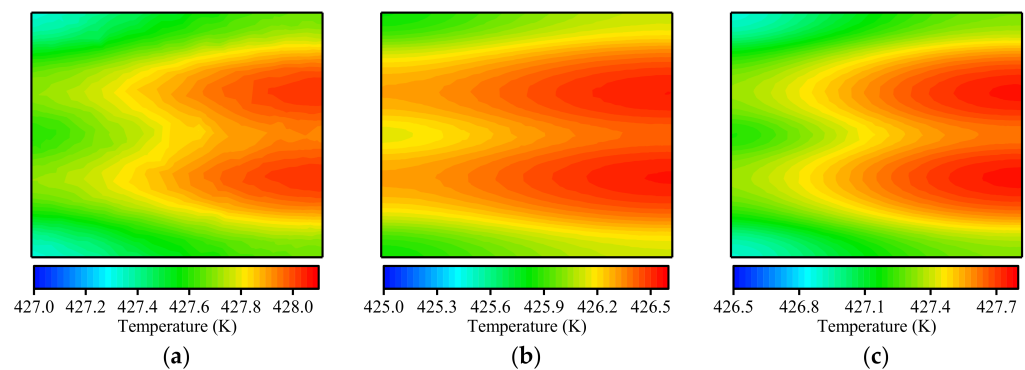


Figure 12. Comparison of temperature distribution results of different equivalent analysis methods for conventional flow channel. (a) 3D model. (b) Method of Ref. [15]. (c) Proposed method.

Table 7. Comparison of analysis results of different equivalent analysis methods with different flow channels.

Flow Channel	Model/Method	T_{max}/K	T_{min}/K	T_{ave}/K	RMS_T^E/K	t/s
Conventional flow channels	3D model	428.03	427.33	427.73	-	640
	Method of Ref. [15]	426.54	425.83	426.24	1.487	17
	Proposed method	427.76	426.89	427.39	0.3435	16
Micro flow channel	3D model	346.91	320.93	335.25	-	177
	Method of Ref. [17]	339.59	319.31	330.26	5.3986	47
	Proposed method	346.34	322.06	335.52	0.6797	47

For the micro flow channel, taking the analytical model in Ref. [17] as an example, the method proposed is compared to the method in the reference (more details on the model can be found in Figure 3 and Table 1 in Ref. [17]). It should be pointed out here that, because of the microchannel's small scale and the low Reynolds number, the effect of the outside surface viscous dissipation will have a greater impact on the fluid flow [17]. Therefore, the same processing method used in Ref. [17] was adopted here, and the out-of-plane viscous dissipation term was introduced into the 2D equivalent analysis equation (Equation (19)) [17]:

$$f = \frac{16\mu u}{H_M^2} \quad (25)$$

Analysis results of the model of the micro flow channel by the method of the reference and our proposed method are compared in Figure 13 and Table 7.

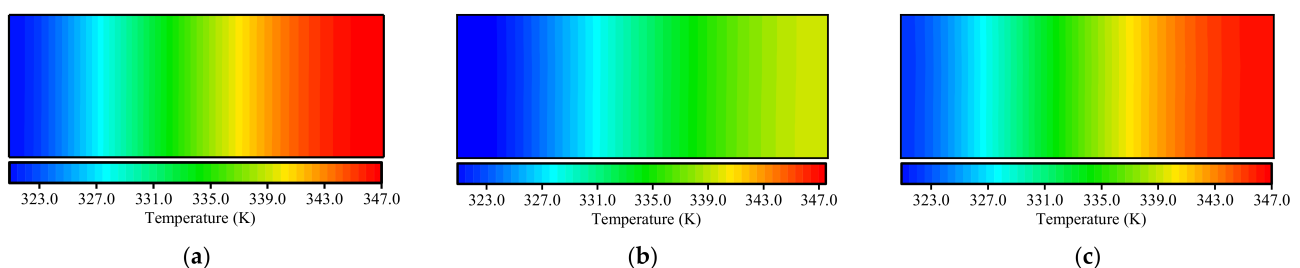


Figure 13. Comparison of temperature distribution results of different equivalent analysis methods for micro flow channel. (a) 3D model. (b) Method of Ref. [17]. (c) Proposed method.

Analysis results are sorted and summarized in Table 7. It can be seen from Figures 12 and 13 and Table 7 that the method proposed is applicable to the equivalent analysis of both conventional and micro flow channels. For conventional flow channel and micro flow

channel analysis cases, the proposed method can ensure that the root mean square error of the temperature RMS_T^E is within the range of 1 K. Compared with the methods in [15,17], our method further improves the accuracy of the 2D equivalent analysis while ensuring computational efficiency.

3.3. Comparison of Results with Other Experimental Work

To verify the consistency between the simulation results of the proposed method and experimental data, we conducted a comparative study with the work of Saleh, Z. M. and Jaffal, H. M [22]. The geometry of the cold plate and its internal flow channels, material properties and boundary conditions were all kept identical to those described in the reference paper. The cold plate has overall dimensions of 100 mm × 70 mm × 8 mm (length × width × thickness) and contains nine identical cooling channels, each 72 mm in length. The comparative analysis results are presented in Table 8.

Table 8. Comparison of temperature distribution analysis of the cold plate.

	Nusselt Number	Pressure Drop (Pa)	T _{max} /K	T _{min} /K
Simulation of Ref. [22]	7.34	149.11	328.70	296.00
Experiment of Ref. [22]	~6.85	~162.50	/	/
Proposed method	7.06	149.02	329.28	296.00

It can be observed from the results that, compared with the three-dimensional simulation data of the reference, the relative errors of the proposed method in this paper for the Nusselt number, pressure drop, maximum temperature and minimum temperature are 3.8%, 0.6%, 1.7% and 0.0%, respectively. When compared with the experimental measurements, the relative errors for the Nusselt number and pressure drop are 3.1% and 8.3%, respectively. The above results demonstrate that the predictions of the proposed method agree well with both the three-dimensional simulation and experimental results, which verifies the effectiveness of the presented method.

3.4. The Cold Plate Heat Transfer Analysis of the Transmitting Antenna

Finally, the multilayer 2D equivalent analysis method was applied to the cold plate heat transfer analysis of the transmitting antenna.

The integrated transmitting antenna is composed of multiple active subarrays. In this study, the cold plate of a single active subarray is selected for thermal analysis. Figure 14a is the simplified cold plate of a single module in the transmitting antenna. The heat generated by active components is transferred to the cold plate by means of heat conduction as the active components are installed directly on the cold plate. Then, the heat will be transferred to the cooling liquid by way of heat convection and finally be taken away by the cooling liquid. The overall dimension of the cold plate is 132 mm × 100 mm, on which 24 T/R modules (power devices) are arranged in a 4 × 6 layout. Each module has a heat dissipation power of approximately 4.5 W and a footprint of 6.67 mm × 6.67 mm × 2 mm, with a spacing of 15 mm between adjacent modules. The dimensions of the inlet and outlet are the same, both being 8 mm × 10 mm. The inlet flow velocity of the coolant is 0.1 m/s, and the inlet temperature is 293.15 K. A static pressure of 0 Pa is specified at the outlet. After calculation, the Reynolds number is 887.29, satisfying the laminar flow assumption. As the main heat dissipation way is the same as in the second section of this paper, the stratification strategy can be directly applied to build its equivalent model. Figure 14b is the equivalent 2D model with boundary conditions.

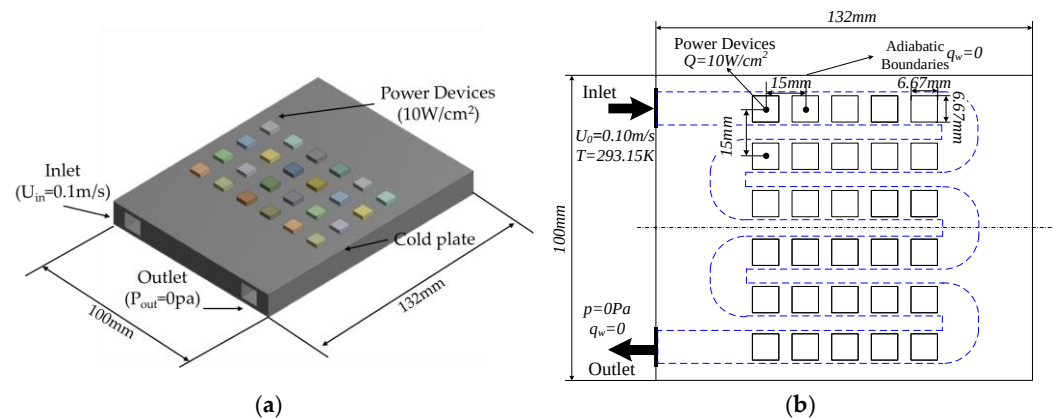


Figure 14. The cold plate of the transmitting antenna. (a) 3D model. (b) 2D equivalent.

Figure 15 shows that, for the overall temperature distribution, the 2D equivalent model is consistent with the 3D model. Analysis results for the 3D and 2D equivalent models are compared in Table 9.

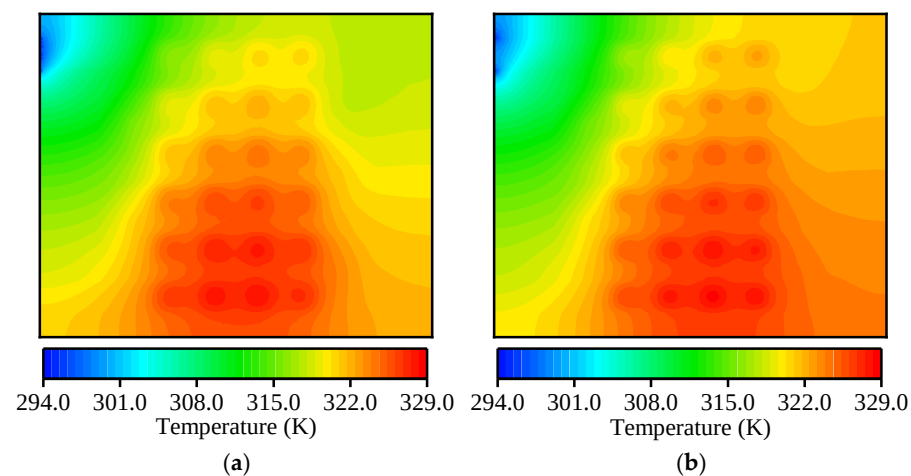


Figure 15. Comparison of the transmitting antenna cold plate analysis results. (a) Temperature distribution of the 3D model analysis. (b) Temperature distribution of the 2D equivalent model analysis.

Table 9. Comparison of temperature distribution analysis of transmitting antenna cold plate.

Model	T_{max}/K	T_{min}/K	T_{ave}/K	RMS_T^E/K	t/s
3D model	328.06	294.57	320.31	-	497
2D equivalent model	328.44	295.13	320.91	1.35	22

Table 9 shows that the 2D equivalent model reduces the calculation time from 497 s to 22 s with a root mean square error for the temperature of 1.35 K, which significantly improves the calculation efficiency. At the highest, lowest, and average temperatures, the analytical errors of the 2D equivalent model are all within 1 K, which also indicates that it has good equivalent accuracy.

4. Conclusions

In this study, a multilayer 2D equivalent analysis method was proposed for the thermal analysis of a 3D cold plate. Based on the stratification strategy and analysis process, the information exchange between layers is realized through the construction of the equivalent heat transfer coefficient. This has made it possible to obtain the temperature distribution of the cold plate through a multilayer loop iterative analysis. The correctness and applicability

of this method were verified using typical examples. Finally, the application of the proposed method to the transmitting antenna cold plate thermal analysis is presented. The following conclusions were drawn from this study.

1. According to the stratification strategy based on the main heat transfer way, a cold plate can be divided into heat conduction and heat convection layers. By setting heat transfer boundary conditions, the different layers are combined into a simple analysis model, so that the temperature distribution can be obtained by iterative analysis between layers.
2. Equivalent heat transfer coefficients of the solid–solid contact surface and the fluid–solid contact surface are different. For the fluid–solid contact surface, the influence of the fluid flow state on the heat exchange should be considered. Based on Prandtl’s boundary layer theory, the convective heat transfer coefficient at the fluid–solid interface is derived, which enhances the precision of the two-dimensional conjugate heat transfer analysis.
3. The multilayer equivalent analysis method eliminates the dependency on the 3D model. This overcomes the shortcomings of the previous calculation method of the heat transfer coefficient through 3D analysis. The correctness and effectiveness of the proposed model and methodology are demonstrated through numerical examples of S-type and Z-type cold plates. A comparison with different methods shows the superiority and versatility of the proposed method, which is applicable to both micro and conventional flow channels.
4. Compared to the 3D model simulation analysis, the application of the 2D equivalent model in the heat transfer analysis of the transmitting antenna cold plate shortens the calculation time from 497 s to 22 s with a root mean square error of temperature of 1.35 K, which significantly improves the calculation efficiency.

Author Contributions: S.Q.: Conceptualization, Methodology, Formal analysis, Writing—Original Draft. S.L.: Investigation, Formal analysis, Writing—Review and Editing. W.W.: Investigation, Writing—Review and Editing. X.T.: Investigation, Data Curation, Visualization. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded in part by the National Natural Science Foundation of China, grant number 52505278, in part by the Natural Science Basic Research Program of Shaanxi, grant number 2024JC-YBQN-0398, and the Guangdong Basic and Applied Basic Research Foundation, grant number 2023A1515110601.

Data Availability Statement: The data presented in this study are available on request from the corresponding author.

Acknowledgments: The authors would like to thank the reviewers for their comments and valuable suggestions.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

3D	Three-dimensional
2D	Two-dimensional
M2DHT	Multilayer two-dimensional heat transfer
MWPT	Microwave wireless power transmission

Appendix A

Appendix A.1

Detailed derivations of the fluid–solid surface equivalent heat transfer coefficients are given in this appendix. The velocity and pressure profile distribution assumptions of the conjugate heat transfer layer are shown in Figure 4. It is further assumed that the velocity and temperature distributions in the boundary layer are quadratic curve distributions. Moreover, the velocity and temperature distributions in the main flow area are assumed constant in the direction of the flow channel height. The approach used here is inspired by the reduction of the fluid dynamics equations of three-dimensional plane flows in [6,7].

Based on the assumption that the velocity distribution in the laminar boundary layer satisfies a quadratic curve, the velocity distribution in the boundary layer can be expressed as

$$\frac{u}{u_\infty} = a + b\left(\frac{x_3}{\delta}\right) + c\left(\frac{x_3}{\delta}\right)^2, \quad 0 \leq x_3 \leq \delta \quad (\text{A1})$$

By substituting boundary conditions (Equations (4) and (5)) into Equation (A1), the specific form of the velocity distribution can be obtained as Equation (8).

Since this work concentrates on the influence of the boundary layer on heat transfer, we integrate the boundary layer momentum differential equation (Equation (2)) across the thickness direction.

$$\rho \frac{d}{dx_2} \int_0^\delta (u_\infty - u) u dx_3 + \rho \frac{du_\infty}{dx_2} \int_0^\delta (u_\infty - u) dx_3 = \tau_w \quad (\text{A2})$$

Since the mainstream velocity remains uniform along the wall-normal direction: $du_\infty/dx = 0$, Equation (A2) can be simplified as follows:

$$\rho \frac{d}{dx_2} \int_0^\delta (u_\infty - u) u dx_3 = \tau_w \quad (\text{A3})$$

By substituting the velocity distribution (Equation (8)) into Equation (A3), the relationship between the thickness of the boundary layer and the flow distance can be obtained as

$$\delta = \left(\frac{30\mu x_2}{\rho u_\infty} \right)^{1/2} \quad (\text{A4})$$

where μ is the fluid dynamic viscosity, x_2 is the flow distance, ρ is the fluid density, and u_∞ is the velocity of the main flow area.

Appendix A.2

Based on the assumption that the temperature distribution in the laminar boundary layer satisfies a quadratic function, combined with boundary conditions (Equations (6) and (7)), the explicit form of the temperature distribution can be derived as:

$$T = T_w + \frac{2(T_\infty - T_w)}{\delta_T} x_3 + \frac{T_w - T_\infty}{\delta_T^2} x_3^2, \quad 0 \leq x_3 \leq \delta_T \quad (\text{A5})$$

Let $\theta = T - T_w$ and $\theta_\infty = T_\infty - T_w$, Equation (A5) can be rewritten as Equation (11).

By integrating the boundary layer energy differential equation (Equation (3)) across the thermal boundary layer thickness, the boundary layer energy integral equation can be obtained.

$$\frac{d}{dx} \int_0^{\delta_T} (T_\infty - T) u dx_3 = \frac{k_f}{\rho C_p} \left. \frac{\partial T}{\partial x_3} \right|_{x_3=0} \quad (\text{A6})$$

By substituting the velocity distribution (Equation (8)) and the temperature distribution (Equation (11)) into Equation (A6), the thickness of the thermal boundary layer can be obtained as follows:

$$\xi = \left(\frac{4k_f}{5\mu C_p} \right)^{1/3} \quad (\text{A7})$$

$$\delta_T = \left(\frac{4k_f}{5\mu C_p} \right)^{1/3} \delta = \left(\frac{4k_f}{5\mu C_p} \right)^{1/3} \left(\frac{30\mu x_2}{\rho u_\infty} \right)^{1/2} \quad (\text{A8})$$

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