

Article

Why Bother on Model Complexity?—A Consistent Analytic Model for Power Output Prediction of Offshore Wind Farms

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Abstract

The present work concerns the further development of a simplified approach for predicting the average power output of offshore wind farms developed by Sørensen and Larsen. In this paper a correction factor, accounting for the finite size of a wind farm, was averaged over all operating wind speeds before deriving the convolution of the production of the wind farm wind turbines over the site wind speed distribution. In the present work, the wind-speed-dependent correction factor is redefined and consistently included in the wind speed Weibull distribution convolution. In addition, a simple model of the dependence of sea surface roughness on wind speed is introduced to refine the derived correction factor. Finally, a revised version of the simplified Geostrophic Drag Law has been derived to improve the dependence of the magnitude of the geostrophic wind speed on the friction Rossby number. The derived improved model is validated against a comprehensive set of quality-ensured full-scale production data from six different operating wind farms covering a diverse segment of Danish waters and further benchmarked against a series of commonly used state-of-the-art engineering simulation models. The comparison of computed results to actual monthly wind farm production data over a series of years shows mean prediction deviations and maximum standard deviations of the errors of about 4% and 8%, respectively. This is very much in accordance with predictions from the other engineering models against which the present model was benchmarked. The advantage of the present model, however, is its simplicity, the demand of only very few input data, and its low computational time, which is an order of magnitude lower than other engineering models.

Keywords: offshore wind farms; resource assessment; wind farm boundary layer model; model validation; model inter-comparison



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1. Introduction

Energy supply security is challenged by geopolitical and local political conflicts—particularly when it comes to fossil fuels. The global warming of our planet is caused by fossil fuels at an ever increasing pace to an extent that recent research warns that our planet is rapidly approaching the ‘tipping point’, where this process becomes irreversible [1], and the global climate system undergoes severe changes. Without changes in fossil emissions, a collapse of the Atlantic meridional overturning circulation—a major tipping element in the climate system—is estimated to occur around mid-century.

All this together calls for a radical change in the composition of the future energy source portfolio worldwide towards sustainable and renewable energy sources; otherwise, we will inevitable face profound negative consequences for natural systems, the human condition of life, and economies. The reaction of the major political players has moved towards incentives to promote the ‘green transition’ [2]. Among the most promising alternative energy sources are solar and wind energy [3], and offshore wind energy stands out as one of the most competitive renewable energy sources, and which is also scalable to the extent needed. GWEC Market Intelligence projections indicate that over a ten-year period (2022–2031), more than 315 GW of new offshore wind capacity will be installed, resulting in a global offshore wind capacity of 370 GW by the end of 2031 [4].

The cost of energy (COE) of wind farms stands out as the most important single factor in the deployment of wind in the energy system. Reliable estimates of COE are thus needed, and this requires fully integrated models of site-specific energy production as well as expenses in terms of wind farm capital costs, operation costs, and maintenance costs.

High-fidelity simulations of annual energy productions of wind farms are computationally demanding, as they require simulations of the wake-affected flow field inside the wind farms for a large range of ambient wind speed and direction conditions. Such advanced computing tools for atmospheric flows solve the full field equations, which, in principle, may cope with all wind farm flow situations, including meso-scale interactions. However, due to the computational burden, such tools are not useful for power production predictions in practice.

Medium fidelity platforms such as TopFarm [5], HAWC2Farm [6], Dynamiks [7], and FastFarm [8] exist, but even these fall short as too elaborate and computationally intensive when it comes to power predictions associated with the future large-scale deployment of offshore wind energy. Therefore, wind power developers are generally forced to rely on simplified engineering wake models to reduce the computational burden. Such models are usually based on analytical *single-wake* approaches, which are subsequently combined with a wake superposition receipt to establish the wind farm flow field. One exception is the linearized RANS flow solver Fuga which, despite being a full blown *linearized* CFD solver, is extremely fast and able to simulate the complete wind farm flow field with computer resources comparable to what is required by the simplistic engineering models based on aggregated single-wake models.

An even faster but also an even more simplistic approach is to model the wind farm flow field as a fully developed wind farm array boundary layer, which assumes that the wind farm is so large—in principle infinite—that the wind field inside the wind farm is in equilibrium with the flow field of the atmospheric boundary layer. Based on this idea, Sørensen and Larsen developed a simplistic model platform [9], which meets the demands on affordable computational resources that prevents higher-order models being used to model the complex wake-affected wind farm flow field for the full range of operational conditions needed.

The production part of the simplistic model model is based on a fully developed wind farm array boundary layer model assuming a two-level roughness model consisting of, respectively, the standard surface roughness and large-scale roughness elements consisting of the wind farm wind turbines (WTs). This approach was originally developed by Templin [10] and Newman [11] to determine the impact of large-scale utilization of wind power on the available wind resources. The method was later refined by Frandsen [12] and subsequently validated against large eddy simulations by Calaf et al. [13].

In connection with the development of the simplistic model in [9], the boundary layer model was further adapted to a *finite size* wind farm, and the Weibull distribution, describing the wind resource at a specific site, was transformed from the ambient inflow

distribution to the infinite wind farm boundary layer flow distribution, which is the decisive distribution needed for the correct estimation of the infinite wind farm energy production. This model has been well received in the wind energy community and, e.g., used to define a new very useful concept—the wind farm wind factor—in [14] as well as to explore the potential of massive exploitation of wind power in the North Sea [15].

The present paper focuses on further refinement, consolidation, and validation of the production part of the simplistic model complex, which was recently improved and successfully validated against historical full-scale data from six wind farms in [16]. In this paper, the correction factor, accounting for the finite size of a wind farm, was *averaged over wind speeds* before deriving the convolution of the production of wind farm WTs over the wind speed variable. Compared to that paper, the present paper redefines the wind-speed-dependent correction factor and consistently includes this factor in the Weibull convolution. In addition, a simple model of the dependence of sea surface roughness on wind speed is introduced to refine the simulated correction factor. Finally, the simplified version of the Geostrophic Drag Law used in [9] has been revised to gain accuracy; i.e., specifically to improve the dependence of the geostrophic wind magnitude on the friction Rossby number. The evaluation of the present improved model involves both benchmarking against state-of-the-art engineering models and validation against a diverse set of historical full-scale wind farm production data covering a segment of Danish waters.

The paper is structured as follows: In Section 2, we develop the refined wind farm production estimator. The focus is on adapting production from an *infinite* wind farm, where the WTs can be considered to be operating within a fully developed wind farm boundary layer, to production from a *finite* wind farm, where WTs in the wind farm ‘periphery’ can be considered as operating under undisturbed ambient boundary layer conditions. Thus, the improvement of the model aims to balance the finite wind farm production best possible between contributions associated with, respectively, the fully developed wind farm boundary layer and the fully developed ambient boundary layer. The best possible balance between these boundary layers, within the framework of the simplistic model concept, is based on detailed comprehensive simulations using the *linear* RANS code Fuga [17], including wind farm blocking effects and an (offshore) wind-speed-dependent turbulence intensity (TI). The refined wind farm production estimator developed is summarized in Section 3. Section 4 presents the production estimates for a range of wind farms in Danish waters. The predicted wind farm energy productions obtained from the developed simplistic model are benchmarked against predictions from a range of state-of-the-art engineering production models and further validated against historical monthly energy production data from available offshore wind farms using mean wind speed and direction distributions derived from the New European Wind Atlas [18]. In the discussion following in Section 5, the outcome of the model validation is put in perspective relative to alternative state-of-the-art engineering models. In Section 6, we conclude and outline the main findings of the study. The paper ends with six appendices containing, respectively, mathematical details of the model, characteristics of Legendre polynomials, details of a simplistic WT model, an innovative approximation of the Geostrophic Drag Law, an empirically based model describing offshore turbulence dependence on wind speed, and the main characteristics of the analyzed wind farms.

2. Model Refinement

The basic presumption of a *fully developed* wind farm array boundary layer model, on which the simplistic model resides, in principle, requires an infinite wind farm. This is obviously not fulfilled in practice, where the flow regime close to the edge of the wind farm in the upstream wind direction can be characterized as a *transition regime*. In this regime,

the fully developed ambient boundary layer transforms into the fully developed boundary layer of the wind farm. To account for this transition, a correction factor for finite wind farm production was introduced in [9] and later improved in [16]. The correction factor approach utilizes the fact that the wind farm boundary layer model does not estimate the production of each wind turbine, but the aggregated production of the entire wind farm. This is also reflected in the topology of the wind farm defined in the boundary layer model, which is in general different from the topology of the wind farm in question and can be considered as a representative ‘mean’ wind farm topology.

The effect of this flow transition regime may be simplified to depend on the number of wind turbines, the turbine inter-spacing, and the ambient mean wind speed. We will use the Fuga linearized CFD RANS solver developed by Ott et al. [17] to condense these dependencies into a surrogate—i.e., a three-dimensional mathematical surface. Being a full-blown *linear* CFD solver, Fuga, contrary to standard engineering models, computes the wake-affected wind farm flow field directly in a consistent manner and accounts for flow features like the flow field speed-up around WTs. Conventional engineering models, on the other hand, are typically *single-wake* models coupled with a wake superposition algorithm, which is not an integrated part of the wake model.

Fuga is formulated in a mixed spectral domain, which facilitates extremely fast solutions based on a system of look-up tables, where some are general and others are wind-turbine-specific. The drag of the rotor is, in the present case, modeled as an actuator disc representing a uniform drag force. For further details on Fuga, we refer to Section 4.2.4. The same overall approach was followed in [16], however, simplifying the CFD simulations by neglecting wind farm *blockage* and, moreover, assuming a representative offshore TI covering all wind speeds. The present use of Fuga to improve the description of the wind farm flow transition regime includes both wind farm blockage effects and a wind-speed-dependent offshore TI. Moreover, the correction factor, developed in [16], was averaged over wind speeds before entering the convolution integral expressing the average wind farm production over all wind speeds. In the present model refinement, an analytical integration of the average wind farm production over all wind speeds—with the integral kernel including the WT power curve, the correction factor, and the Weibull mean wind speed probability density function—is devised.

Offshore TI depends on mean wind speed, upstream roughness conditions, measuring height, and thermal convection. This affects the Fuga CFD simulations, as increased turbulence leads to faster wake recovery and vice versa. Offshore upstream surface roughness, responsible for the mechanically generated part of the turbulence, is dictated by the capillary waves of the sea [19] and may thus be characterized by the wind speed. The aim is here to quantify TI in terms of wind speed only, meaning that for a population of turbulence standard deviations, variations in the upstream fetch conditions and the atmospheric stability will implicitly appear as variability in the turbulence standard deviation. An empirical model for wind-speed-dependent TI is presented in Appendix E, while the Fuga setup and the associated derivation of the refined transition correction factor are addressed in Section 2.2. The model also accounts for typical average wind farm losses over the year split in the three categories: availability, electrical losses, and turbine performance.

2.1. Reformulation of Weibull Convolution

The expressions for the average power production of a WT is formulated in [9] as a convolution of the wind turbine power curve with the mean wind speed probability density

function (pdf) expressed in terms of a two-parameter Weibull distribution, $f_U(U; \lambda, k)$. The pdf of a Weibull-distributed random variable is given by

$$f_U(U; \lambda, k) = \begin{cases} \frac{k}{\lambda} \left(\frac{U}{\lambda} \right)^{k-1} e^{-\left(\frac{U}{\lambda} \right)^k} & \text{for } U \geq 0 \\ 0 & \text{for } U < 0 \end{cases} \quad (1)$$

where U is a realization of a mean wind speed stochastic variable U , $k > 0$ is the Weibull shape parameter, and $\lambda > 0$ is the Weibull scale parameter.

In the original model formulation, the correction factor was assigned the constant value 3, which resulted in predicted results within 10% compared to the actual production data. This is comparable to the performance of more complicated models [9,16], which require more detailed input as well as larger computational efforts.

When refining the correction factor to depend on the *inflow* wind speed, this has to be accounted for in the integrals leading to the average power production of a finite wind farm, where the ABL boundary layer is only in equilibrium over a part of the wind farm. Intuitively, it makes sense to let the correction factor additionally depend on the number of wind farm WTs (the correction factor must vanish asymptotically as the number of WTs approaches infinity) and the WT internal spacing (the correction factor must approach one asymptotically as the WT internal spacing approaches infinity).

In analogy with the original formulation in [9] (cf. Equation (27)), we will express the finite wind farm production as a weighted sum of contributions from WTs located within the fully developed wind farm boundary layer and of WTs located inside the undisturbed ambient boundary layer, respectively.

The correction factor, which expresses the weighting of, respectively, equilibrium wind farm boundary layer and ambient boundary layer conditions for a finite wind farm, can be defined in various ways. Contrary to the original formulation given in [9], the revised formulation presented here has well-defined intuitive asymptotic properties and is given by

$$P_f(N_T, S, U_a) = (1 - a(N_T, S, U_a)) P_{if}(U_a, S) + a(N_T, S, U_a) P_s(U_a); \quad 0 \leq a(N_T, S, U_a) \leq 1 \quad (2)$$

where $P_f(N_T, S, U_a)$ is the production of a representative WT in a finite size wind farm, $P_{if}(U_a, S)$ is the production of the same WT in the fully developed wind farm boundary layer, and $P_s(U_a)$ is the production of an analog solitary WT. The parameters U_a , S , and N_T are, respectively, the undisturbed ambient mean wind speed, the WT spacing measured in rotor diameters, and the number of WTs in the finite wind farm.

Rearranged, Equation (2) gives the following expression for the finite wind farm correction factor:

$$a(N_T, S, U_a) = \frac{P_f(N_T, S, U_a) - P_{if}(U_a, S)}{P_s(U_a) - P_{if}(U_a, S)} \quad (3)$$

A priori, we expect the following asymptotic properties of the correction factor:

- $\lim_{N_T \rightarrow \infty} a(N_T, S, U_a) = 0$, since the number of WTs in the ‘wind speed transition regime’ is proportional to $\sqrt{N_T}$, while the number of WTs operating in the fully developed wind farm boundary layer is proportional to N_T ;
- $\lim_{N_T \rightarrow 1} a(N_T, S, U_a) = 1$, as ambient conditions prevail for solitary WTs;
- $\lim_{S \rightarrow \infty} a(N_T, S, U_a) = 1$, since the wind farm WTs, under such conditions, can be considered to consist of solitary WTs. This consideration is, however, not relevant for realistic wind farm layouts and will not be pursued further in the analysis of the correction factor to follow;

- If U_a is sufficiently above the rated wind speed such that $P_s(U_a) - P_{if}(U_a, S) = 0$, then all wind farm WTs run at rated power, and $a(N_T, S, U_a) \equiv 1$ (numerator and denominator in Equation (3)) takes identical values in this limit;
- Transition to the above stated limit values is conjectured to be strictly monotonic.

Since the correction factor in the revised formulation is *wind-speed-dependent*, the reformulated average WT power production model results, in addition to the convolution integrals in the original formulation, in evaluation of integrals of the following form:

$$P_y(N_T, S) = \int_{U_i}^{U_o} P_{wt}(U) a(N_T, S, U_a) f_U(U; \lambda, k) dU \quad (4)$$

with $P_{wt}(U)$ being the WT production at the (reference) wind farm wind speed U —i.e., expressing the WT power curve—and U_a being the undisturbed ambient wind speed *associated with wind speed* U at the location of the WT in question. U_i and U_o are, respectively, the WT cut-in and cut-out wind speeds, and $a(N_T, S, U_a)$ is the correction factor for a wind farm consisting of N_T WTs uniformly spaced with the normalized spacing distance S . The implicit relation between U and U_a in Equation (4) will be turned into an explicit relation when the specific integrals are to be evaluated.

To obtain closed-form expressions of the involved production integrals, we assume a factorization facilitating the following power function expansion of the weighting factor:

$$a(N_T, S, U_a) = \sum_{i=0}^M \xi_i(N_T, S) U_a^{p_i} \quad (5)$$

where the exponent $p_i \in \mathbb{R}$ may, or may not, degenerate to a natural number for a given value of the counting variable i . Further, we adopt the WT power curve formulation from [9]; i.e.,

$$P_{wt}(U) = \begin{cases} \alpha U^3 + \beta & \text{for } U_i \leq U < U_r \\ P_G & \text{for } U_r \leq U < U_o \end{cases} \quad (6)$$

with U_r denoting the rated wind speed of the WT in question.

Combining Equations (2) and (4)–(6), four arche integral types result as building blocks for the closed-form solution in addition to the original convolution integrals formulated in [9]—two integrals associated with the *ambient* equilibrium boundary layer conditions below and above rated wind speed, respectively, and two integrals associated with the *fully developed* wind farm boundary layer conditions, respectively, below and above rated wind speed. In the following, the lower indices ‘s1’ and ‘s2’ refer to, respectively, production of solitary WTs (i.e., referring to equilibrium ambient boundary layer conditions) below and above rated wind speed, whereas the lower indices ‘f1’ and ‘f2’ refer to production of WTs operating under fully developed wind farm boundary layer conditions below and above rated wind speed, respectively.

Reformulating Equation (2) as

$$P_f(N_T, S, U_a) = a(N_T, S, U_a) P_s(U_a) - a(N_T, S, U_a) P_{if}(U_a, S) + P_{if}(U_a, S) \quad (7)$$

and subsequently introducing the Weibull convolution, Equation (4), as well as representing the WT power production according to Equation (6), we get six separate terms to determine the average power production of a representative *finite* wind farm WT:

$$P_y(N_T, S) = P_{y,s1}(N_T, S) + P_{y,s2}(N_T, S) - P_{y,f1}(N_T, S) - P_{y,f2}(N_T, S) + P_{y,f1}^{orig} + P_{y,f2}^{orig} \quad (8)$$

where $P_{y,f1}^{orig}$ and $P_{y,f2}^{orig}$ denote, respectively, the power production of a WT operating below and above the rated wind speed within a *fully developed* wind farm boundary layer as derived in the original paper [9].

In the following, we present closed-form analytical expressions for each of the four new arche integral types appearing in Equation (8).

$$P_{y,s1}(N_T, S) = \alpha \sum_{i=0}^M \xi_i(N_T, S) \int_{U_i}^{U_r} U_a^{p_i+3} \frac{k}{\lambda} \left(\frac{U_a}{\lambda} \right)^{k-1} e^{-\left(\frac{U_a}{\lambda}\right)^k} dU_a + \beta \sum_{i=0}^M \xi_i(N_T, S) \int_{U_i}^{U_r} U_a^{p_i} \frac{k}{\lambda} \left(\frac{U_a}{\lambda} \right)^{k-1} e^{-\left(\frac{U_a}{\lambda}\right)^k} dU_a \quad (9)$$

which, using the transformation $t := \left(\frac{U_a}{\lambda}\right)^k$, can be reduced to the following closed-form solution (see Appendix A for more details):

$$P_{y,s1}(N_T, S) = \alpha \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i+3} \left[\Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_i}{\lambda}\right)^k\right) \right] + \beta \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_i}{\lambda}\right)^k\right) \right] \quad (10)$$

where $\Gamma(\star, \star)$ denotes the (lower) incomplete Gamma function ([20], formula 6.5.1). Similarly, again with reference to Appendix A for details, we have

$$P_{y,s2}(N_T, S) = P_G \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_o}{\lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) \right] \quad (11)$$

The Weibull distribution representing the mean wind conditions *inside* an infinite wind farm (i.e., under fully developed wind farm boundary layer conditions) differs from the Weibull distribution referring to ambient conditions. Using the results derived in [9] and in Appendix D, this effect can, in the below rated wind speed regime (see Equation (A7)), be accounted for by modifying the Weibull scale parameter from λ to $\epsilon_1 \lambda$, where ϵ_1 is given by

$$\epsilon_1 := \frac{1 + \frac{\gamma}{\delta}}{1 + \frac{\gamma}{\kappa} \sqrt{\frac{\pi C_{T,r}}{8 S^2} + \left(\frac{\kappa}{\delta}\right)^2}} \quad (12)$$

in which κ is the von Kármán constant (≈ 0.41), $C_{T,r}$ is the WT thrust coefficient at the rated wind speed and

$$\gamma := \ln\left(\frac{G}{f' h}\right), \quad f' := f e^{A_*}, \quad \delta := \ln\left(\frac{h}{z_0}\right), \quad f := 2 \Omega \sin \phi \quad (13)$$

with G denoting the geostrophic wind speed, f being the Coriolis parameter expressed in terms of the rotational speed of the earth, Ω , and the latitude, ϕ , $A_* \equiv 2.0$, h is the height to which the Weibull distribution refers, and z_0 is the terrain roughness.

Contrary to the Fuga production estimates formulated in Section 2.2, we will not include the offshore surface roughness variability with wind speed in the simplistic model, but consider the surface roughness constant with a value corresponding to the *Weibull weighted* wind-speed-dependent turbulence intensity illustrated in Figure A3; i.e.,

$$\overline{TI} = \int_{U_i}^{U_o} TI(U_a) f_U(U_a; \lambda, k) dU_a \quad (14)$$

Assuming neutral atmospheric stratification, Panofsky et al. ([21], Table 7.1) offers the following empirical relationship:

$$\sigma_u = 2.39 u_* \quad (15)$$

where σ_u is the standard deviation of the along wind turbulence component, and u_* is the friction velocity. For neutral atmospheric stratification, the logarithmic wind speed shear model applies, and with reference to the WT hub height, h , we therefore obtain the following relationship connecting the characteristic site turbulence intensity, $\overline{TI}$, and the associated characteristic site roughness, $\bar{z}_0$:

$$\bar{z}_0 := \frac{h}{\exp(2.39 \kappa / \overline{TI})} \quad (16)$$

We will use $\bar{z}_0$ to refine the formulation of the equilibrium offshore wind farm boundary layer; i.e., to reformulate the δ -variable in Equation (13) as

$$\delta = \ln\left(\frac{h}{\bar{z}_0}\right) \quad (17)$$

and therefore to use this variable in the computation of the ϵ_1 -variable in Equation (12) and the ϵ_2 -variable in Equation (23).

Analogously to the formulation of WT power production under ambient equilibrium boundary layer conditions below the rated wind speed—and again referring to Appendix A for details—the WT power production, under *fully developed* wind farm boundary layer conditions below the rated wind speed, can be expressed by the following closed-form solution:

$$\begin{aligned} P_{y,f1}(N_T, S) = & \alpha \sum_{i=0}^M \xi_i(N_T, S) \epsilon_1^3 \lambda^{p_i+3} \left[\Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_r}{\epsilon_1 \lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_i}{\epsilon_1 \lambda}\right)^k\right) \right] \\ & + \beta \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\epsilon_1 \lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_i}{\epsilon_1 \lambda}\right)^k\right) \right] \end{aligned} \quad (18)$$

In ([9], Appendix A), it is proved that the functional relationship between the ambient mean wind speed, U_a , and the associated mean wind speed in the fully developed wind farm boundary layer, U_f , is *monotonic*, which means that there exists a one-to-one transformation between U_a and U_f . Therefore, the following relationship applies [22]:

$$f_{U_f}(U_f) dU_f \equiv f_{U_a}(U_a) dU_a \quad (19)$$

The expansion of $a(N_T, S, U_a)$ in Equation (5) fortunately refers to the ambient mean wind speeds U_a . Note, that in case of U_f appearing as a factor to the wind farm wind speed Weibull distribution, the integral leading to the above rated wind farm WT production cannot be expressed in closed form. This is because there is an *implicit* relationship between U_f and U_a , which means that an explicit expression of the variable U_f as a function of U_a cannot be derived. We thus have

$$\sum_{i=0}^M \xi_i(N_T, S) U_a^{p_i} f_{U_f}(U_f) dU_f = \sum_{i=0}^M \xi_i(N_T, S) U_a^{p_i} f_{U_a}(U_a) dU_a \quad (20)$$

whereby

$$\begin{aligned} \sum_{i=0}^M \xi_i(N_T, S) \int_{U_{f,r}}^{U_{f,o}} U_a^{p_i} f_{U_f}(U_f) dU_f &= \sum_{i=0}^M \xi_i(N_T, S) \int_{U_f=U_{f,r}}^{U_{f,o}} U_a^{p_i} f_{U_a}(U_a) dU_a \\ &= \sum_{i=0}^M \xi_i(N_T, S) \int_{U_r/\epsilon_2(U_r)}^{U_o/\epsilon_2(U_o)} U_a^{p_i} f_{U_a}(U_a) dU_a \end{aligned} \quad (21)$$

Noticing the equivalence between Equations (11) and (21), we finally have

$$\begin{aligned} P_{y,f2}(N_T, S) = \\ P_G \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i + k}{k}, \left(\frac{U_o}{\epsilon_2(U_o)\lambda}\right)^k\right) - \Gamma\left(\frac{p_i + k}{k}, \left(\frac{U_r}{\epsilon_2(U_r)\lambda}\right)^k\right) \right] \end{aligned} \quad (22)$$

where $\epsilon_2(U_f)$ is given by (cf. [9] and Appendix D)

$$\epsilon_2(U_f) = \frac{1 + \frac{\gamma}{\delta}}{1 + \frac{\gamma}{\kappa} \sqrt{\frac{\pi C_{Tx}}{8 S^2} \left(\frac{U_r}{U_f}\right)^{3.2} + \left(\frac{\kappa}{\delta}\right)^2}} \quad (23)$$

With reference to Equation (2) and with the formulation of the arche integral types—i.e., Equations (10)–(22)—in place, the finite wind farm power production, P_{WF} , can finally be formulated as

$$\begin{aligned} P_{WF}(N_T, S) = \\ N_T \left(P_{y,s1}(N_T, S) + P_{y,s2}(N_T, S) - P_{y,f1}(N_T, S) - P_{y,f2}(N_T, S) + P_{y,f1}^{orig} + P_{y,f2}^{orig} \right) \end{aligned} \quad (24)$$

The integrals expressing $P_{y,f1}^{orig}$ and $P_{y,f2}^{orig}$ in closed form are evaluated in [9] as, respectively,

$$\begin{aligned} P_{y,f1}^{orig} = \alpha(\epsilon_1 \lambda)^3 \left[\Gamma\left(\frac{k+3}{k}, \left(\frac{U_r}{\epsilon_1 \lambda}\right)^k\right) - \Gamma\left(\frac{k+3}{k}, \left(\frac{U_i}{\epsilon_1 \lambda}\right)^k\right) \right] \\ + \beta \left[\exp\left(-\frac{U_i}{\epsilon_1 \lambda}\right)^k - \exp\left(-\frac{U_r}{\epsilon_1 \lambda}\right)^k \right] \end{aligned} \quad (25)$$

and

$$P_{y,f2}^{orig} = P_G \left[\exp\left(-\frac{U_r}{\epsilon_2(U_r)\lambda}\right)^k - \exp\left(-\frac{U_o}{\epsilon_2(U_o)\lambda}\right)^k \right] \quad (26)$$

The next step is to derive explicit expressions for the factorization of $a(N_T, S, U_a)$ in Equation (5); i.e., to derive specific values of the exponents p_i and specific fits to the coefficient surfaces $\xi_i(N_T, S)$. This requires a huge number of wind farm flow field simulations, and we use the extremely fast linear CFD code Fuga for this purpose.

2.2. Fuga-Based Recalibration of the Finite Wind Farm Approach

The ‘infinite wind farm’ is an abstraction, and consequently the basic assumption behind the fully developed wind farm array boundary layer approach modeling of the wake-affected flow inside a (finite) wind farm is violated except for the WTs located ‘sufficiently deep’ inside the wind farm relative to the inflow direction. In the flow regime at—and close to—the edge(s) of the wind farm in the upstream direction, a flow field transition takes place, in which the ambient boundary layer equilibrium gradually transforms into the fully developed wind farm array boundary layer. The impact of this transition boundary layer zone was in the original model [9] compensated by introducing an em-

pirically determined *wind-speed-independent* correction factor, $\tilde{a}$, and the finite wind farm average WT power production, P_f , accordingly calculated as

$$P_f = (N_T - \tilde{a}\sqrt{N_T}) P_{if} + \tilde{a}\sqrt{N_T} P_s \quad (27)$$

where P_{if} is the average power production of a WT located within the fully developed wind farm boundary layer, and P_s is the average power production of a solitary WT located within the fully developed ambient boundary layer. The undisturbed ambient site inflow wind climate is assumed identical for these two WTs. These weighting factors were predominantly rooted in superficial geometrical considerations. Because the introduced correction factor, $\tilde{a}$, was independent of the wind speed, the Weibull convolution, leading to the average WT power production, is not affected by the original correction factor. This is changed in the present reformulation of the finite farm correction, where the new correction factor correctly depends on the wind speed.

The simplistic model complex does not account for details of the actual wind farm *layout*, but simplifies a given wind farm topology by assuming a quadratic wind farm layout with an identical ‘average’—or characteristic—row/column inter-spacing defined by

$$S = \frac{\sqrt{A}}{D(\sqrt{N_T} - 1)} \quad (28)$$

where the spacing, S , is measured in rotor diameters, D , and A denotes the area of the wind farm. Within this framework, setting the correction factor to $\tilde{a} = 3$, being the ‘crude’ estimate in [9], corresponds to stating that the average number of WTs subject to free flow field conditions amounts to three-quarters of the turbines located along the circumference of the wind farm. However, this simple geometric interpretation is misleading, as the original correction factor, in addition to compensating for the lack of equilibrium within the flow transition regime, also accounted for additional energy losses unrelated to the flow conditions, such as wind farm availability, electrical losses, and WT mis-performance.

For the present remodeling of the finite wind farm correction, the redefined correction factor will focus on accounting for the effect of the flow *transition regime* only, and an additional wind farm *efficiency factor* is included to account for wind farm availability, electrical losses, and WT mis-performance. We will now concentrate on the detailed modeling of the improved a -correction factor, expressing the relative importance of the transition boundary layer flow regime. This correction factor depends on the *undisturbed ambient* inflow mean wind speed as well as the wind farm topology. In the context of the simplistic model, the topology of the wind farm is condensed into WT spacing and the number of WTs of the wind farm.

Turbulence affects viscous diffusion within the wake-affected flow regime, and to include this aspect and assess the sensitivity of the Fuga simulations to turbulence intensity, a model for offshore wind-speed-dependent surface roughness is derived in Appendix E. Thus, the expressions derived in Equations (A40)–(A44) and illustrated in Figure A3 will define the inflow ambient turbulence condition for the Fuga simulations. However, turbulence build-up inside the wind farm, as, e.g., described in Calaf et al. [13], will not be taken into account.

Wind farm *blocking* effects—or upstream wind farm induction—have attracted increasing attention in recent years as an important factor in wind farm performance assessment—see, e.g., [23]. Therefore, blocking is included in the present Fuga simulations. Wind farm blocking is associated with upstream induction caused collectively by the wind farm WTs. In Fuga, this feature is accounted for by using an iterative procedure, which, regarding computational speed, is fully justified because of Fuga’s extreme

computational speed. First, a conventional Fuga computation is performed. This is a parabolic scheme that starts with the most upstream WTs exposed to the ambient flow field and subsequently gradually includes downstream WTs, when the aggregated effect of wakes from the upstream WTs is computed. In the following iterative course, the above computational scheme is repeated, however, now additionally accounting for upstream induction from all the Fuga actuator discs collectively constituting the wind farm flow resistance. This iterative procedure is repeated until convergence is achieved—typically after 10–25 iterations with an acceptable accuracy requirement. For more details on the blocking feature, we refer to [24].

To proceed, we need to define an appropriate *calibration case* accounting for inflow conditions, wind farm topology, and WT characteristics. To establish a generic setup, it is appropriate to assume a *uniform* wind direction rose; i.e., all inflow directions occur equally frequently. This is a natural choice, because such specific information is irrelevant in the many cases where the physical wind farm topology diverges from the fictitious squared wind farm layout assumed for the present simplistic framework. Adopting this simplification further allows for computationally reductions, as symmetries can be utilized with the defined fictitious wind farm topology. Thus, consistent with the formulation of the simplistic wind farm model, the wind farm topology used to determine the enhanced correction factor, $a(N_T, S, U_a)$, is taken as a quadratic wind farm with WTs arranged in perpendicular ‘rows’ and ‘columns’ with uniform distances, S . This finite size wind farm, consisting of N_T WTs, is exposed to a horizontally uniform inflow field with a logarithmic vertical shear (i.e., neutral stratification) defined by the WT hub height velocity U_a and the offshore TI resulting from Equations (A40) and (A44) in Appendix E. To properly specify the vertical shear with reference to U_a , the surface roughness, z_0 , is needed. This can be approximated from Equation (A40) using the weighted measuring height from, respectively, the Vindeby and the Gedser campaigns—i.e., $z = (37.5 \times 5015 + 30.0 \times 22419) \text{ m} / 27434 = 31.4 \text{ m}$ (cf. Appendix E for details). We therefore have [25]

$$\hat{\mu}_\sigma = \frac{\psi\kappa}{\ln(z/z_0)} = \frac{0.96}{\ln(z/z_0)} \quad (29)$$

or

$$z_0 = \frac{31.4\text{m}}{\exp(0.96/\hat{\mu}_\sigma)} \quad (30)$$

where ψ is the scaling constant relating the friction velocity and standard deviation of the longitudinal turbulence component (≈ 2.4) [21], and the von Kármán constant $\kappa \approx 0.41$.

Referring to the redefined finite wind farm WT production weighting, specified in Equations (2) and (3), we have

$$a(N_T, S, U_a) = \frac{P_f(N_T, S, U_a) - P_{if}(U_a, S)}{P_s(U_a) - P_{if}(U_a, S)}$$

where the parameter dependencies, resulting from the revised definition of the correction factor, have been included for clarity. The WT production referring to the infinite wind farm, $P_{if}(U_a, S)$, results directly from the equilibrium wind farm boundary layer model and depends, for obvious reasons, not on N_T . The solitary WT production, $P_s(U_a)$, results from the equilibrium ABL layer, and the finite wind farm WT production, $P_f(N_T, S, U_a)$, results from Fuga simulations. The *average* WT production for *finite* wind farms, conditioned on N_T , U_a , and S , is simulated with a wind farm size resolution of $\Delta(\sqrt{N_T}) = 5$ with the wind farm size ranging from $\sqrt{N_T} = 5$ to $\sqrt{N_T} = 50$. This is believed to cover all realistic wind farm sizes and, as mentioned in Section 2.1, the correction factor will asymptotically approach zero as the number of WTs goes to infinity.

Adopting a *uniform* inflow direction distribution, the Fuga simulations can, for symmetry reasons, be limited to a 45° inflow sector as, e.g., the one shown in Figure 1. The finite wind farm production is, in this context, considered as the finite wind farm production *averaged* over the wind farm inflow direction sector. More specifically, the underlying set of Fuga simulations is defined by a direction resolution of 1°, a wind speed resolution of 0.5 m/s within the wind speed range [4 m/s; 25 m/s], and a spacing resolution of 1D within the range [3D, 13D]. In total, this amounts to 227,700 Fuga simulations.

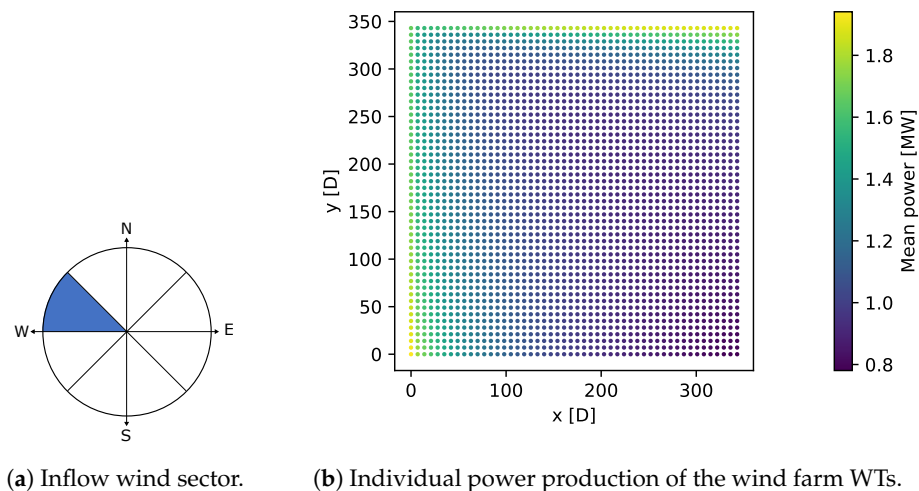


Figure 1. Fuga-simulated mean power for the example finite wind farm case consisting of 2,500 WTs. Inflow wind speed is 11 m/s, spacing 7D, and inflow directions are within the sector 270–315°.

The last thing to consider to complete the setup for the Fuga CFD simulations is the wind farm *WT characteristics*. In the original formulation of the minimalistic wind farm model [9], a simple generic WT model was formulated, which was later improved—with particular focus on performance in the above rated wind speed regime—by van der Laan et al. [26], where all details on the applied data can be found. This refined WT model with a minor, but important, modification (cf. Appendix B) is now implemented in the platform, expressing the essential thrust and power coefficient WT characteristics, constituting the WT model, as respectively

$$C_T(U) = \begin{cases} C_{T,r} & \text{for } U_i \leq U < U_r \\ C_{T,r} (U_r/U)^{3.2} & \text{for } U_r \leq U < U_o \end{cases} \quad (31)$$

and

$$C_P(U) = \begin{cases} \frac{C_{P,r}}{1 - \left(\frac{U_i}{U_r}\right)^3} \left[1 - \left(\frac{U_i}{U}\right)^3\right] & \text{for } U_i \leq U < U_r \\ C_{P,r} (U_r/U)^3 & \text{for } U_r \leq U < U_o \end{cases} \quad (32)$$

where U_i is the WT cut-in wind speed, U_o is the WT cut-out wind speed, and subscript r denotes the value at 'rated' conditions. To avoid numerical instabilities, the discontinuous step in $C_{T,r}$ from 0 to $C_{T,r}$ at U_i is, in the numerical handling, smoothed by multiplying $C_{T,r}$ with the factor $\frac{\arctan(100(U-U_i))}{\pi} + 0.5$. If not otherwise stated, the values $C_{T,r} = 0.75$ and $C_{P,r} = 0.48$ from [9] can be used in this context.

This simple generic WT model works surprisingly well, as shown in Figure 2, where the performance of the model is validated against detailed aerodynamic data from ten WTs ranging in size, in terms of rotor diameters, from 27 m to 240 m, and including both

research reference WTs (3 WTs) and real commercial WTs (7 WTs). In terms of rated power, this portfolio of WTs covers nearly three orders of magnitude.

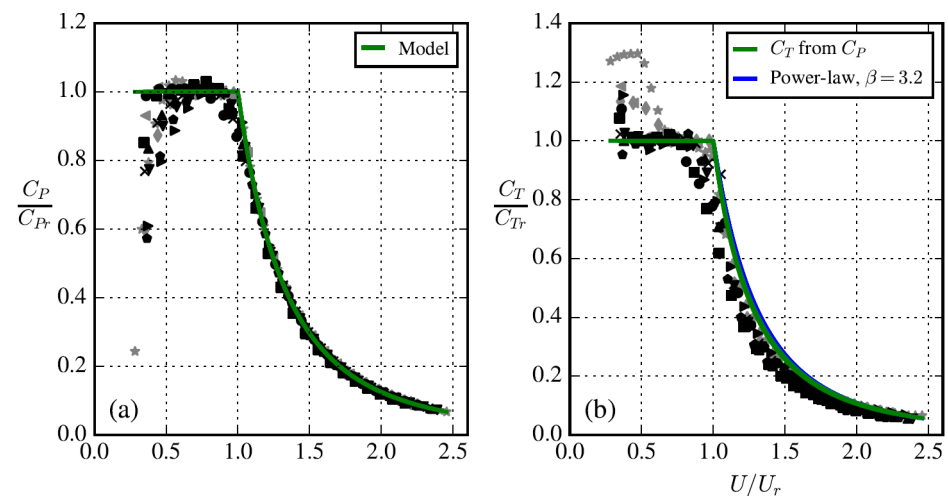


Figure 2. Comparison of (a) normalized wind turbine power coefficient, C_P , and (b) normalized wind turbine thrust coefficient, C_T , of the generic wind turbine model with aerodynamic data of existing (black markers) and reference (grey markers) wind turbines. This figure is, with a single change, reproduced from [26] with permission from the authors of this paper.

We use the simplistic generic WT for the Fuga simulation cases, and based on the excellent performance of this WT model and the collapse of WT characteristics when expressed in normalized coordinates, there is (luckily) no need for including a variety of test-case WTs for recalibration of the finite farm correction factor.

As a sanity check of the suite of finite wind farm simulations, it has been verified that $P_f(N_T, S, U_a) \leq P_s(U_a)$ holds true for all values of N_T , U_a , and S . The equality in this relation occurs when U_a is high enough to ensure WT inflow above rated wind speed for all wind farm WTs for a given spacing S .

The set of Fuga simulations is used to determine the population of $a(N_T, S, U_a)$ values from Equation (3) needed to establish an empirical expression for the three-dimensional hyperplane, which describes the dependence of the finite-farm correction factor on the variables N_T , S , and U_a , respectively. One of the dependencies—i.e., the dependence on the ambient wind speed U_a —can be incorporated directly in the closed-form Weibull convolution (c.f. Section 2.1) with appropriate factorization, whereas the remaining two dependencies will remain as fitted two-dimensional hyperplanes—cf. Equation (5).

However, deriving the $a(N_T, S, U_a)$ fitting population from Equation (3) is not without challenges. This is because this simple equation is numerically ill conditioned around the wind farm rated wind speed, which is illustrated in Figure 3. Around the rated wind farm wind speed, the nominator and denominator in Equation (3) become very small, causing numerical uncertainties, which in turn causes a -‘outliers’. Therefore, we will tie the population of $a(N_T, S, U_a)$ values used for the a -fitting to the conjectured asymptotic properties of the correction factor; i.e., specifically that the transition to the above rated wind farm wind speed, where all wind farm WTs are running at rated power, is conjectured to be strictly monotonic. This is achieved simply by eliminating the a -values that do not meet this requirement from the a -fitting population, which is illustrated in Figure 3 below.

For a wind farm consisting of 50 WTs, the resulting set of $a(N_T, S, U_a)$ as a function of the ambient inflow wind speed is illustrated in Figure 4 for the investigated span of WT inter-spacings.

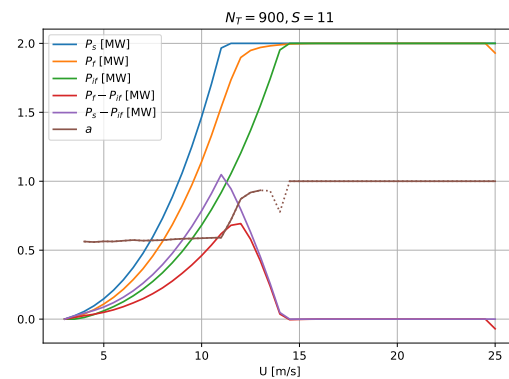


Figure 3. Elements of $a(N_T, S, U_a)$. The blue curve represents power production of a solitary WT, the orange curve represents the Fuga-simulated production of a finite wind farm turbine, and the green curve represents power production of a WT within the equilibrium wind farm boundary layer. The red and purple curves represent, respectively, the nominator and denominator in the a -expression. Around rated wind farm wind speed, the nominator and denominator in Equation (3) become very small causing numerical uncertainties, which in turn causes a -‘outliers’, as seen in the dotted part of the brown curve.

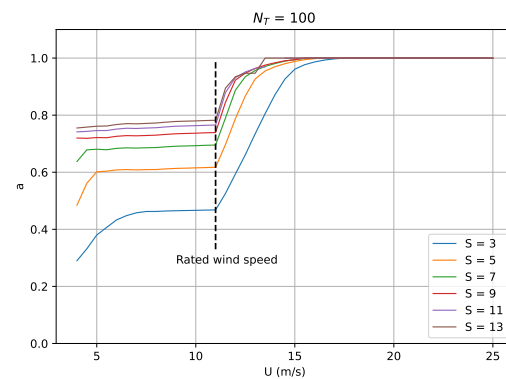


Figure 4. The a factor as a function of wind speed for different spacings after removing segments with negative gradients in the above-rated wind speed regime.

Surface fitting can be performed in many ways—among the most important are cubic spline, Kriging, and least squares polynomial surface fitting. Cubic spline is known to be inferior to Kriging. Kriging with polynomial trend surfaces is mathematically identical to generalized least squares polynomial surface fitting, but since we aim for *closed-form* expressions of the two-dimensional hyperplanes $\xi_i(N_T, S)$, which facilitate evaluation without interpolation, the least squares polynomial fit is preferred over Kriging in this context, because Kriging results in a vast amount of weight factors, which is expressed less elegantly in closed form.

2.3. Coefficient Hyperplane Fitting

It turns out that the nature of the requested factorization of, respectively, U_a and $\xi_i(N_T, S)$ differs significantly from the below rated to the above rated wind speed regime as illustrated in Figure 4. It is therefore convenient to split the requested fitting into these two regimes. However, a given WT may have a rated wind speed different from the rated speed of the simplistic WT on which the simulated a -values are based. In order to account for this effect for a WT with *arbitrary rated wind speed*, we rewrite the wind speed variable as

$$U_a = \frac{U_r}{\tilde{U}_r} \tilde{U}_a \quad (33)$$

where $\tilde{U}_r := 11\text{ m/s}$ is the rated speed of the simplistic WT used to generate the $a(N_T, S, U_a)$ hyperplanes. This stretching/compression of the wind speed coordinate (i.e., a linear affinity of the wind speed) is consistent with the analysis in [26] leading to collapse of WT power and thrust curves (cf. Figure 2).

Least squares polynomial fits require a generic fitting surface to be defined. The ‘optimal’ generic fitting surface will be determined iteratively, and factors such as simplicity (i.e., number of terms) and accuracy of the fit will be the guiding stars in this respect. Initially, we specify the expansion given in Equation (5) as

$$a(N_T, S, U_a) = \sum_{i,j,k=0}^{M_i, M_j, M_k} \zeta_{ijk} N_T^{p_i} S^{q_j} \tilde{U}_a^{r_k} \quad (34)$$

since for the simplistic WT, $U_r = \tilde{U}_r$. In the general case, $p_i, q_j, r_k \in \mathbb{R}$, and $r_k \in \mathbb{R}$ can be handled in closed form with respect to the Weibull convolution to follow. However, it turns out that the most convenient in the present case is covered by $r_k \in \mathbb{N}$, whereas a previous attempt [27], used $r_k \in \mathbb{Z}$.

The fitting of the correction factor within the above rated wind speed regime is more demanding than the below rated counterpart, since the a hyperplanes turn out to be somewhat more curved in this regime. However, the same class of summation kernels for the generic fitting expressions is chosen for these fits, but in particular, the number of expression terms will deviate between the fits covering, respectively, the below and above rated regimes.

As the parameter intervals are confined—i.e., $N_T \in [25; 2500]$, $S \in [3D; 13D]$, and $\tilde{U}_a \in [4\text{ m/s}; 25\text{ m/s}]$ —we aim for functional expressions with *compact support*. The upper part of the wind speed regime—specifically, the $\tilde{U}_a$ wind speed interval $[20\text{ m/s}; 25\text{ m/s}]$ —is easily handled, as all WTs turn out to have sufficient wind for rated production, which in turn means that $a \equiv 1$ in this wind speed regime.

To reduce the number of fitting parameters in the respective wind speed regimes, $[4\text{ m/s}; 11\text{ m/s}]$ and $[11\text{ m/s}; 20\text{ m/s}]$, as much as possible, and to ease the fitting procedure, we formulate the fitting surfaces in terms of sets of orthogonal polynomials. Since the polynomials are orthogonal, we can fit the polynomial coefficients sequentially one-by-one, which eases the numerical treatment. One such attractive polynomial family is Legendre polynomials (an alternative could be a wavelet expansion in terms, e.g., the Ricker wavelet family). In order to utilize the orthogonality properties of Legendre polynomials, we define scaled variants of the independent variables that recast these into variables belonging to the interval $[-1; 1]$. Because the fitting procedure is split between below and above rated regimes, we need two sets of rescaled variables. These are defined as

$$\begin{aligned} n_T(N_T) &:= \frac{2\sqrt{N_T} - 55}{45} \\ s(S) &:= \frac{2S - 16}{10} \\ \tilde{u}_a(\tilde{U}_a) &:= \begin{cases} \frac{2\tilde{U}_a - 15}{7} & \text{for } 4 \leq \tilde{U}_a < 11 \\ \frac{2\tilde{U}_a - 31}{9} & \text{for } 11 \leq \tilde{U}_a < 20 \end{cases} \end{aligned} \quad (35)$$

where $n_T \in [-1; 1]$, $s \in [-1; 1]$, and $\tilde{u}_a \in [-1; 1]$ for $N_T \in [25; 2500]$, $S \in [3; 13]$, and $\tilde{U}_a \in [4; 11]$ or $\tilde{U}_a \in [11; 20]$.

With these definitions, we have created expansion functions that are orthogonal within the $(N_T, S, \tilde{U}_a)$ parameter space considered, thus facilitating identification of the ‘optimal’ set of $\{M_i, M_j, M_k\}$ among sets of all possible selections of $\{M_i, M_j, M_k\}$. In the

following, the dependence of the scaled parameters on the physical parameters defined in Equation (35) will be omitted for clarity.

In the normalized functional space, defined in Equation (35), we express the generic fitting surface as

$$a(N_T, S, \tilde{U}_a) \equiv a_n(n_T, s, \tilde{u}_a) = \sum_{i=0}^{M_i} \sum_{j=0}^{M_j} \sum_{k=0}^{M_k} \zeta_{ijk} P_i(n_T) P_j(s) P_k(\tilde{u}_a) \quad (36)$$

where $a_n(\star, \star, \star)$ denotes the correction factor expressed in normalized variables, $M_i \in \{0, 1, 2, 3, 4, 5\}$, $M_j \in \{0, 1, 2, 3, 4, 5\}$, and $M_k \in \{0, 1, 2, 3, 4, 5\}$. Note, that since $k \in \{0, 1, 2, 3, 4, 5\}$, the velocity decomposition in Equation (5) is expressed in terms of Legendre polynomials of order up to 5 and thus eventually involves velocity terms with integer polynomial degrees up to order 5. With this setting, least squares fits are performed for all possible selections of $\{M_i, M_j, M_k\}$ using the error metric

$$\begin{aligned} \Pi_o(M_i, M_j, M_k) = & \iiint \left[\sum_{i=0}^{M_i} \sum_{j=0}^{M_j} \sum_{k=0}^{M_k} \zeta_{ijk} P_i(\delta(n_T - n_T^{(k)})) P_j(\delta(s - s^{(k)})) P_k(\delta(\tilde{u}_a - \tilde{u}_a^{(k)})) \right. \\ & \left. - a_n(\delta(n_T - n_T^{(k)}), \delta(s - s^{(k)}), \delta(\tilde{u}_a - \tilde{u}_a^{(k)})) \right]^2 dn_T ds d\tilde{u}_a \end{aligned} \quad (37)$$

with $P_n(\star)$ denoting the Legendre polynomial of degree n , and where the integration spans over the interval $[-1; 1]$ for each of the three normalized variables, thus spanning the entire Fuga simulations space. $\delta(\star)$ is the Dirac function, ζ_{ijk} is the set of all free parameters to be determined, and $\{(n_T^{(k)}, s^{(k)}, \tilde{u}_a^{(k)})\}$ is the set of all $(n_T, s, \tilde{u}_a)$ ‘grid points’, in which $a_n(n_T, s, \tilde{u}_a)$ has been evaluated by means of Fuga simulations.

The set of linear system equations for the least squares fit is defined by

$$\frac{\partial}{\partial \zeta_{ijk}} \Pi_o(M_i, M_j, M_k) = 0 \quad (38)$$

Among the set of least squares fits, representing the available selections of $\{M_i, M_j, M_k\}$, the ‘optimal’ fit is chosen balancing the considerations regarding accuracy against complexity of the fit; i.e., the number of terms in the fitting surface. With respect to accuracy, attention is paid both to the error metric (37) and the maximum deviation between fit and $a_n(n_T^{(k)}, s^{(k)}, \tilde{u}_a^{(k)})$.

Based on these considerations, the fit of the correction parameter in the below rated wind speed regime, corresponding to spanning dimensions $\{M_i, M_j, M_k\} = \{3, 3, 2\}$, is selected. For the above rated regime, the fit corresponding to the spanning dimensions $\{M_i, M_j, M_k\} = \{2, 3, 4\}$, is selected. The motivation for these choices is illustrated in Figure 5.

For obvious reasons, few terms (i.e., low M) are preferred over many terms (i.e., large M), and the balancing of ‘small’ improvement and number of terms in the expansion is based on a judgment. This is illustrated in Figure 5, where the error metric has been plotted as a function of the number of terms in the a -factorization. The selected ‘sweet spots’ for the simplistic model are indicated by red dots. Furthermore, the model with Legendre orders $\{M_i, M_j, M_k\} = \{5, 5, 5\}$ has been used as a reference for comparison.

A good metric for assessing the quality of these fits is to compare the Fuga AEP simulations with the AEP predictions resulting from the simplistic model for identical

squared example wind farms. The result of this comparison is shown in Figure 6 for both the selected and the more accurate model.

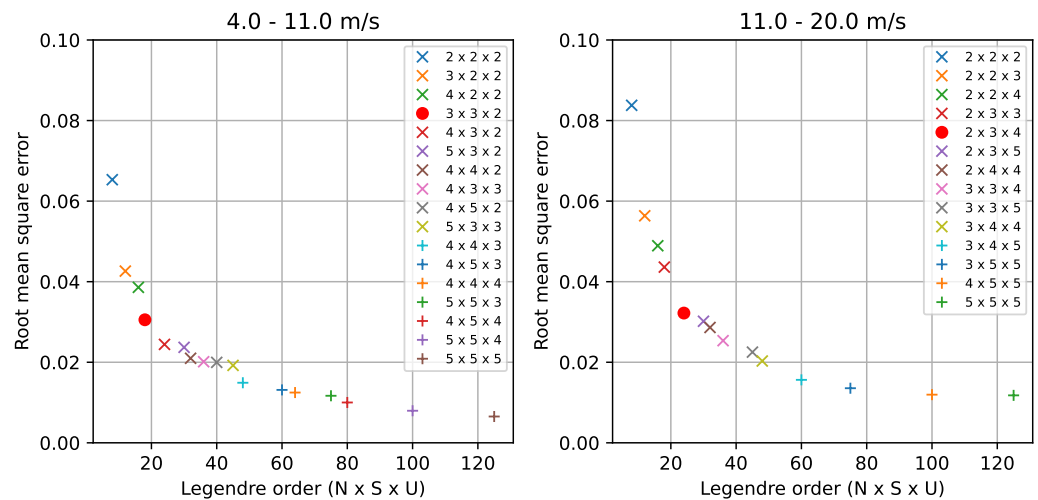


Figure 5. Root mean square error of the fit as a function of the sum of orders of the Legendre polynomials (including the constant term) used for the a -factorization. For a target error of around 3%, the fit with Legendre orders $\{M_i, M_j, M_k\} = \{3, 3, 2\}$ are used for the below rated wind speed regime, while the fit with Legendre orders $\{M_i, M_j, M_k\} = \{2, 3, 4\}$ are used for the above rated wind speed regime. The selected sets of Legendre polynomials are marked with red dots.

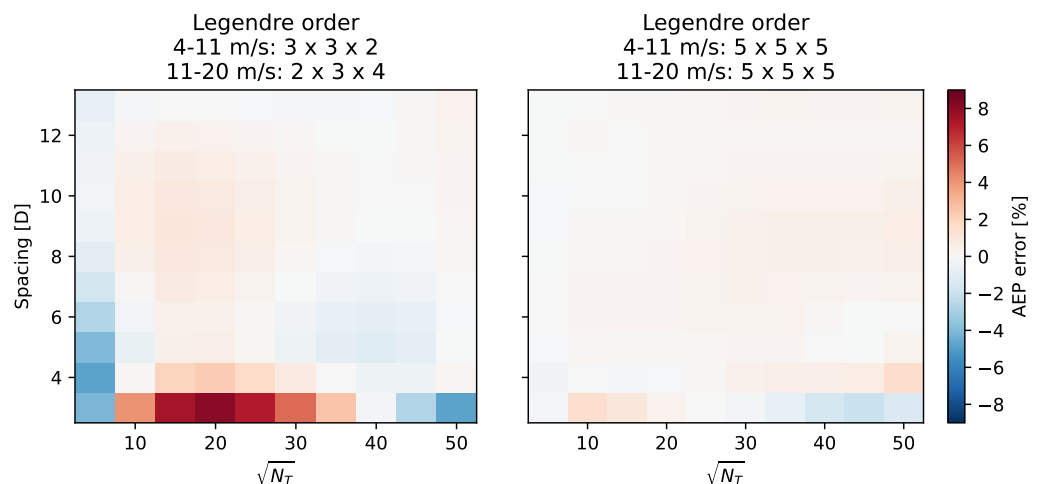


Figure 6. AEP of calibration input (i.e., the Fuga-simulated squared wind farm) versus model predicted AEP of the same wind farm.

As seen, the inclusion of more Legendre terms improves the accuracy of the AEP with respect to the Fuga-simulated AEP for wind farms with Cartesian squared topology. However, this has only a limited effect on the accuracy when applied to typical wind farm layouts; i.e., WT spacings larger than 3D and number of WTs exceeding 25. Moreover, when compared to actual production data, the higher fitting accuracy did not produce better results. The simpler fit was therefore selected for the present simplistic model.

Referring to the factorization of $a(N_T, S, U_a)$ defined in Equation (5), we have that $M_k = 2$ for the below rated wind speed regime, and $M_k = 4$ for the above rated wind speed regime. Therefore, the velocity factorization involves terms with integer polynomial degrees spanned by $\{0, 1\}$ and $\{0, 1, 2, 3\}$, respectively. The optimal set of Legendre polynomials used in the fitting of $a(N_T, S, U_a)$ is listed in Appendix C.

The relationship between the inflow speed variables defined in Equation (33) and the fitted a - and ζ -surfaces can be expressed as

$$\tilde{a}(N_T, S, \tilde{U}_a) = \sum_{i=0}^M \tilde{\zeta}_i(N_T, S) \tilde{U}_a^{p_i} = \sum_{i=0}^M \tilde{\zeta}_i(N_T, S) \frac{\tilde{U}_r^{p_i}}{\tilde{U}_r^{p_i}} \frac{U_r^{p_i}}{\tilde{U}_r^{p_i}} \tilde{U}_a^{p_i} \quad (39)$$

or

$$a(N_T, S, U_a) = \sum_{i=0}^M \left(\tilde{\zeta}_i(N_T, S) \frac{\tilde{U}_r^{p_i}}{\tilde{U}_r^{p_i}} \right) U_a^{p_i} := \sum_{i=0}^M \tilde{\zeta}_i(N_T, S) U_a^{p_i} \quad (40)$$

where $\tilde{a}$ and $\tilde{\zeta}$ refer to the ambient velocity parameter $\tilde{U}_a$.

With reference to physical *ambient* velocities U_a , the resulting fits of the ζ -surfaces are given in the following, where the ζ -surfaces associated with the below and the above rated wind speed regime are denoted by, respectively, $\zeta^{(f1)}$ and $\zeta^{(f2)}$. These are defined by, respectively,

$$\begin{aligned} \zeta_0^{(f1)}(N_T(n_T), S(s)) &\equiv 0.500 - 0.140n_T + 0.100P_2(n_T) \\ &\quad + [0.130 - 0.100n_T - 0.053P_2(n_T)]s \\ &\quad + [-0.005 + 0.139n_T + 0.024P_2(n_T)]P_2(s) \end{aligned} \quad (41)$$

$$\begin{aligned} U_r \zeta_1^{(f1)}(N_T(n_T), S(s)) &\equiv 0.034 - 0.051n_T - 0.023P_2(n_T) \\ &\quad + [0.044 + 0.157n_T + 0.023P_2(n_T)]s \\ &\quad + [-0.055 - 0.146n_T - 0.011P_2(n_T)]P_2(s) \end{aligned} \quad (42)$$

and

$$\begin{aligned} \zeta_0^{(f2)}(N_T(n_T), S(s)) &\equiv -7.363 - 1.495n_T \\ &\quad + [-4.151 - 6.027n_T]s \\ &\quad + [4.422 + 3.572n_T]P_2(s) \end{aligned} \quad (43)$$

$$\begin{aligned} U_r \zeta_1^{(f2)}(N_T(n_T), S(s)) &\equiv 15.767 + 2.126n_T \\ &\quad + [10.313 + 13.335n_T]s \\ &\quad + [-9.789 - 7.224n_T]P_2(s) \end{aligned} \quad (44)$$

$$\begin{aligned} U_r^2 \zeta_2^{(f2)}(N_T(n_T), S(s)) &\equiv -9.909 - 0.950n_T \\ &\quad + [-7.863 - 9.413n_T]s \\ &\quad + [6.891 + 4.668n_T]P_2(s) \end{aligned} \quad (45)$$

$$\begin{aligned} U_r^3 \zeta_3^{(f2)}(N_T(n_T), S(s)) &\equiv 2.074 + 0.128n_T \\ &\quad + [1.900 + 2.149n_T]s \\ &\quad + [-1.556 - 0.975n_T]P_2(s) \end{aligned} \quad (46)$$

with $N_T \in [25; 2.500]$, $S \in [3; 13]$, and $1 \leq U_a \leq (20U_r)/(11\tilde{U}_r)$. The factors in the ζ_i -plane expressions relate to the reformulated wind speed coordinate given in Equation (33), and the implicit normalization of the equations with $\frac{1}{\tilde{U}_r}$ raised to the power of the associated ambient mean wind speed factor is performed to facilitate the lowest possible number of necessary digits in numerical factors. Note that around rated wind speed, the derived fit with continuous gradients is likely to better represent a real WT than the simplistic WT, due to the ‘softening’ of the WT shoulder region for real WTs.

The production of an arbitrary finite wind farm WT in the below and above rated wind speed regimes, respectively, is listed in the following sections.

2.3.1. Below Rated WT Production

Depending on the time span, to which the mean wind speed Weibull distribution refers, the average yearly or monthly power production of a WT operating below rated wind speed within a finite wind farm can be formulated as the sum of two parts—one part accounting for power production within the wind speed regime between cut-in and rated *ambient* wind speed, and another part accounting for power production within the wind regime above rated *ambient* wind speed and up to rated wind farm wind speed, since rated wind farm wind speed corresponds to a somewhat higher ambient wind speed. Thus,

$$\begin{aligned}
 P_{y,f1}(N_T, S) = & \alpha \sum_{i=0}^M \xi_i^{(f1)}(N_T, S) \epsilon_1^3 \lambda^{p_i+3} \left[\Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_i}{\epsilon_1 \lambda}\right)^k\right) \right] \\
 & + \beta \sum_{i=0}^M \xi_i^{(f1)}(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_i}{\epsilon_1 \lambda}\right)^k\right) \right] \\
 & + \alpha \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \epsilon_1^3 \lambda^{p_i+3} \left[\Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_r}{\epsilon_1 \lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) \right] \\
 & + \beta \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\epsilon_1 \lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) \right]
 \end{aligned} \quad (47)$$

since p_i (cf. Equation (40)) in the optimization process was justified to equal i . We refer to Appendix A for more details.

2.3.2. Above Rated WT Production

For a WT subjected to above rated wind speed conditions, the yearly/monthly production of a WT operating within a finite wind farm is also given by the sum of two parts—in this case, with one part accounting for the wind speed regime above rated wind farm wind speed and up to wind farm wind speed $\frac{20U_r}{11} := \phi U_r$, and the second part accounting for the wind farm wind speed regime beyond ϕU_r and up to the cut-out wind speed, U_o .

$$\begin{aligned}
 P_{y,f2}(N_T, S) = & + P_G \sum_{i=0}^3 \xi_i^{(f2)}(N_T, S) \lambda^i \left[\Gamma\left(\frac{i+k}{k}, \left(\frac{\phi U_r}{\epsilon_2(\phi U_r)\lambda}\right)^k\right) - \Gamma\left(\frac{i+k}{k}, \left(\frac{U_r}{\epsilon_1 \lambda}\right)^k\right) \right] \\
 & + \mathcal{H}(U_o - \phi U_r) P_G \left[\exp\left(-\left(\frac{\phi U_r}{\epsilon_2(\phi U_r)\lambda}\right)^k\right) - \exp\left(-\left(\frac{U_o}{\epsilon_2(U_o)\lambda}\right)^k\right) \right]
 \end{aligned} \quad (48)$$

with $p_i = i$, $\phi := \frac{20}{11}$, and where $\mathcal{H}(\star)$ denotes the Heaviside function, which ensures that the last term only contributes to the above-rated energy production in case $U_o > \phi U_r$. Again, we refer to Appendix A for more details on the derivation of the closed form solutions.

Summing up, the power production from a finite wind farm, P_{WF} , can be estimated using the developed simplistic framework by combining each of the six terms in Equation (24) with their explicit formulations given in Equations (25) and (26) and in Equations (41)–(48).

3. Model Summary

The improved closed-form simplistic wind farm power prediction model is based on two *infinite* ABL boundary layer models and a transition regime between these that facilitate the estimation of finite wind farms. The two involved boundary layer models are, respectively, the conventional neutral boundary layer over flat terrain with a logarithmic

shear layer and a double logarithmic wind farm boundary layer representing the horizontal homogeneous boundary layer over an infinite wind farm. An infinite wind farm is an abstraction, and therefore the flow transition regime is needed to facilitate realistic *finite* wind farm power production estimates.

The transition flow regime is formulated as a weighed sum of the flat terrain and the wind farm boundary layers. The required weight is a function depending on (1) the number of WT in the wind farm; (2) the spacing of the wind farm WTs, where *homogeneous spacing* is assumed; and (3) the undisturbed wind farm inflow speed. In addition to homogeneous spacing, the simplistic wind farm layout is assumed to be a squared wind farm. In case the number of wind farm turbines, N_T , is not in the form where $\sqrt{N_T} \in \mathbb{N}$, the weighting factor is based on interpolated values, since this function is represented by a closed-form hyperplane. The weighting function is derived by fitting a mathematical hyperplane to a huge number of CFD simulations using the linear CFD code Fuga, thus spanning the relevant space of, respectively, number of farm WTs, turbine spacings, and inflow wind speeds. This parameter space is believed to cover realistic wind farm conditions with wind speeds in the range [3; 25] m/s, WT spacings limited to $S \in [3; 13]$ rotor diameters, and the number of WTs limited to $N_T \in [25; 2500]$.

These ranges have been determined from experience and ‘common sense’. Regarding the specific choice for the U_a range, we have associated this with normal cut-in and cut-out turbine design wind speeds. The spacing range has been chosen to cover what is seen in contemporary wind farm layouts, with the Lillgrund wind farm being a very densely populated wind farm with inter-spacings down to $3.3D$. The most common inter-spacing for existing wind farms is in the range of $5D$ to $9D$. To our knowledge, no wind farm has spacings above $13D$. Finally, for the number of wind farm turbines, we have simply chosen a maximal number of turbines (i.e., 2500) that is unlikely to be exceeded by a *single* wind farm. The lower limit expresses, in our opinion, a threshold of what can be characterized as a ‘conventional’ wind farm. Within the described parameter space, the Fuga simulations account for wind farm *blocking* and, as offshore conditions are assumed, also for the dependence of the *sea surface roughness* on wind speed.

Where possible, the fitted hyperplanes, $a(N_T, S, U_a)$, have been normalized to facilitate the lowest possible number of necessary digits in the numerical factors without compromising the accuracy. To investigate the quality of the resulting hyperplanes, Fuga simulations of squared wind farms of various sizes and WT inter-spacings have been benchmarked with analog predictions resulting from the revised simplistic model used at the same wind farms, and a close to perfect match has resulted—cf. Figure 6.

A natural next step is to check the influence of the topology idealization in the simplistic model using the Fuga platform. However, this is not straightforward using the full-scale wind farms analyzed in this paper for various reasons. One thing is that a basic requirement is that the number of WTs in the wind farm must be of the form $\sqrt{N_T} \in \mathbb{N}$ to facilitate the construction of an analog squared wind farm in the sense of the simplistic model. This is only satisfied for one of the selected wind farms (i.e., the Hornsrev3 wind farm). Another important aspect is that the wind farm topology and the site inflow direction probability density function are intimately associated, but the inflow direction statistics are not accounted for in the simplistic model for obvious reasons. Therefore, a one-to-one Fuga comparison of the topology effect cannot be performed using the available wind farms.

An alternative—less direct—approach to justify approximating the wind farm topology with a square-shaped wind farm (for the present text cases) is to acknowledge the inseparable character of topology and inflow direction statics and investigate these as one combined effect. First, with reference to Figure 6, we note that the simplistic model performance is shown to be within the order $\pm 2\%$ of the target Fuga simulation of wind farms with identical (squared)

topology for the entire range of (allowable) spacings and wind farm sizes (the WT spacings are in the range $[3; 13]D$, and the number of WTs is in the range $[15; 2500]$). With this in mind, we next focus on the combined effect of simplifying the topology and inflow statistics by comparing the *mean* deviation between AEP recordings and AEP predictions referring to, respectively, Fuga and the simplistic model. The results are listed in Table 1 below.

Table 1. Mean deviations between AEP recordings and AEP predictions referring to, respectively, Fuga and the simplistic model.

Wind Farm	Fuga	Larsen 2026
Rødsand1	9.18%	8.09%
Rødsand2	4.14%	4.35%
Hornsrev1	7.55%	6.23%
Hornsrev2	0.24%	1.36%
Hornsrev3	−2.6%	−1.83%
Anholt	0.02%	−0.31%

From this table, we observe that the listed prediction deviations are within $\pm 1.4\%$ in all cases. The Rødsand2, Hornsrev2, and Anholt wind farms are the wind farms with topologies that deviate the most from a squared layout. The listed deviations associated with these wind farms are of the same order of magnitude as the deviations associated with the remaining wind farms, which indicates that the topology approximation combined with the inflow simplification (i.e., excluding conditioning the wind speed statistics on inflow direction) in these particular cases is justified, but in addition, also indicates that the topology approximation in isolation is of minor importance.

The Geostrophic Drag Law takes a prominent place in the model formulation, and an improved approximation has been developed for use with the simplistic model improving its dependence on the Coriolis parameter.

Finally, it should be mentioned that the model distinguishes correctly between the Weibull-based wind resource estimate associated with, respectively, the conventional ABL and the infinite wind farm boundary layer.

4. Results

To assess the improvements to the originally developed minimalistic model [9], and to cross-validate the improved minimalistic model against other state-of-the-art engineering models, we compare model predictions to actual production data from a wide range of Danish offshore farms.

4.1. Available Data

For the validation and benchmarking, we have chosen the Anholt, the two Rødsand, and the three Horns Rev offshore wind farms, from which monthly wind farm production data are publicly available at <https://ens.dk/analyser-og-statistik/data-oversigt-over-energisektoren> (accessed on 30 January 2026). These wind farms represent nearly two decades of experience with offshore wind energy, and they are quite different when it comes to location, wind resources, size, topology, and WTs. The layouts of the six wind farms are shown in Figure 7, and a brief description of each of these can be found in Appendix F.

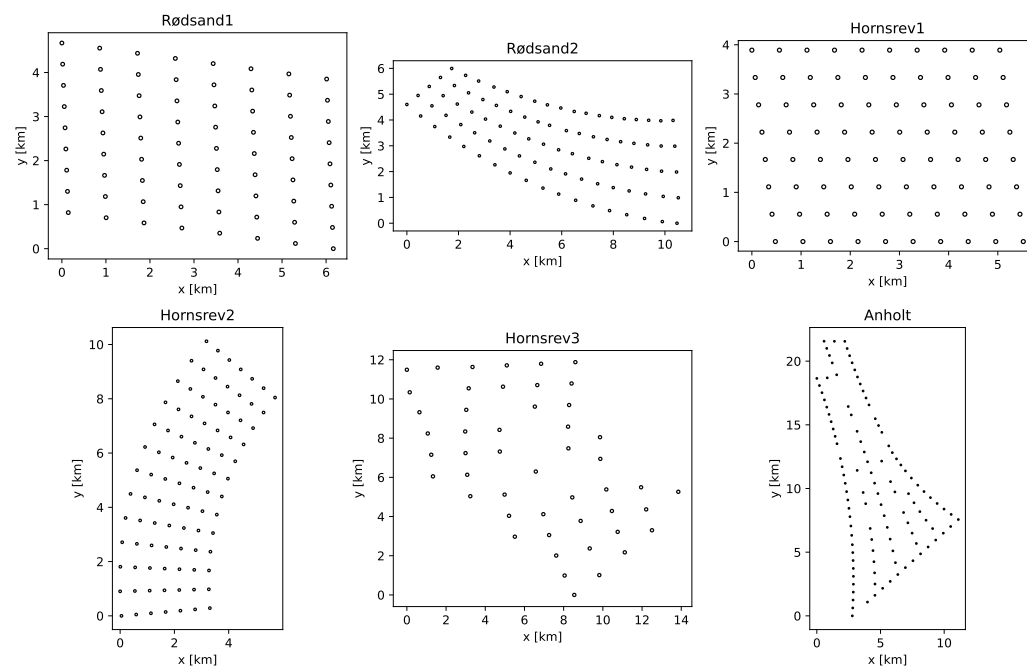


Figure 7. Layout of the six offshore wind farms used for the model validation.

For model-based estimation of wind farm AEP, both wind-farm-specific data and site wind climate data are needed.

The key data for each of the wind farms are summarized in Table 2, which shows the number of WTs, the rotor diameter, the hub height, and the rated power of each turbine as well as the area of the individual wind farms (calculated as the convex hull around the WT positions), the average normalized distance between the turbines determined using Equation (28), and the year of commissioning.

Table 2. Wind farm data from Energistyrelsen, <https://ens.dk/media/3531/download> (accessed on 30 January 2026).

Site	N_T	Diameter [m]	Height [m]	Power [MW]	Area [km ²]	S [D]	Commissioned
Rødsand1	72	82.4	68.8	2.3	23.0	7.8	2003
Rødsand2	90	92.6	80	2.3	34.8	7.5	2010
Hornsrev1	80	80	70	2.0	19.6	7.0	2002
Hornsrev2	91	93	55	2.3	35.7	7.5	2009
Hornsrev3	49	164	105	8.3	100.3	10.2	2018
Anholt	111	120	81.6	3.6	118.9	9.5	2012

When it comes to wind climate data, the site wind speed Weibull distributions are needed as input to the simplistic model. For the classic engineering models, both Weibull and histogram representation of the wind speed statistics apply. Since the wind farm production data are available with one-month resolution, we will base the simplistic model predictions on Weibull distributions covering each individual month. The monthly energy production of the respective wind farms is shown in Figure 8.

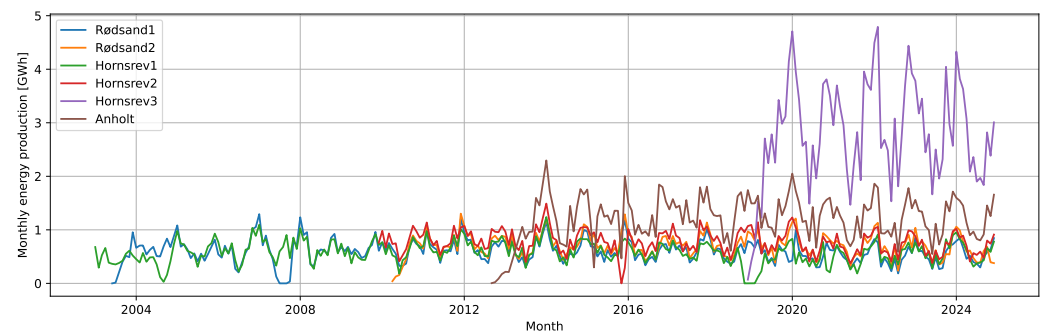


Figure 8. Monthly energy production from the analyzed wind farms covering the time slot ranging from 2003 to 2025.

To obtain the required site Weibull distributions, wind speed time series are extracted at the center wind farm locations from the New European Wind Atlas [18]. The time series data are subsequently interpolated to the relevant WT hub heights—wind direction by linear interpolation and wind speed by log-space linear interpolation. The Weibull scale and shape parameters are then obtained by fitting the Weibull distribution to the full wind speed time series of each month; i.e., without considering the wind direction. In total, this sums to 920 individually estimated Weibull distributions.

One month is a relative short time span for a good Weibull fit compared to a time span of one year, and larger statistical deviations are consequently expected because of the reduction in the population of observations. However, since the requested Weibull distributions relate to a 360° sector, the statistical uncertainty on these is comparable with the statistical uncertainty on the traditional yearly Weibull distributions conditioned on inflow sector. An example of an estimated one-month Weibull distribution is shown in Figure 9.

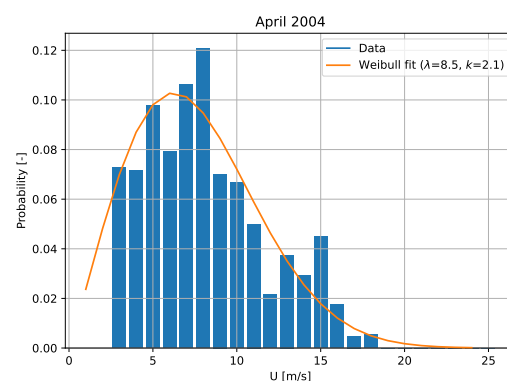


Figure 9. Wind speed histogram distribution and Weibull fit for April 2004 at the Rødsand1 site.

Regarding the state-of-the-art engineering wake models included in Section 4.2 where, contrary to the simplistic model, the actual wind farm topology is modeled, we include aspects of the wind farm inflow direction in the production estimates to generate the most accurate predictions possible. The underlying wind speed statistics, conditioned on inflow direction, are in this case represented in terms of histograms based on binning of the monthly wind speed data population in, respectively, 1 m/s and 1° bins and used as input for these models.

4.2. Available Models

Apart from being validated with field data, the simplistic model has been benchmarked against a selection of state-of-the-art engineering wake models. In the following, we briefly

introduce the prediction models used for this comparison. For more details on the models, we refer to the documentation in PyWake [24].

4.2.1. The N.O. Jensen Wake Model

This model is the oldest and simplest wake model [28]. The model exists in various variants and is widely used in the industry. Assuming a top-hat velocity deficit, it combines one-dimensional momentum theory with the assumption of a linear expansion of the wake, resulting in simple analytical equations for the normalized velocity deficit, which is scaled with the wind speed at the source turbine including wake effects from upstream WTs. The only unknown parameter of the model is the expansion coefficient, which essentially depends on the turbulence intensity (TI). In the present work, we employ the wake expansion factor $k = 0.38 \text{ TI} + 4e^{-3}$ found in [29]. The local TI is calculated from the ambient flow conditions and the wake-added turbulence modeled as described in [30]. Alternatively, the wake expansion factors developed in [31] can be used. The model further applies linear superposition of multiple wakes.

4.2.2. The Gaussian Deficit Model

This model is based on the mass and momentum conservation combined with the assumption of a self-similar Gaussian profile for the wake deficit [32]. As in the case of the N.O. Jensen model, this results in a general expression for the velocity deficit, in which the wake expansion is a function of the *local* TI. The model uses the same wake superposition method, TI-dependent expansion factor, and wake-added turbulence model as the N.O. Jensen model.

4.2.3. TurbOPark

This model follows the basics of the Gaussian deficit model, but applies a non-linear wake expansion that is determined by the local TI in the wake. The used model includes a Gaussian overlap model to compute the rotor-average deficit and a surface model that mirrors the wake at the surface. The model is developed by Ørsted and has been calibrated to match the production of Ørsted's portfolio of wind farms [33]. As for all the models used in the present work, the predicted power production is multiplied by a loss factor, even though some of the loss is probably already included in the calibration.

4.2.4. Fuga

Fuga is based on superposition of solutions to the linearized turbulent Navier–Stokes equations with the rotors in the wind farm represented by actuator discs, and the Reynolds stresses represented by a ‘simple’ eddy viscosity-type of closure with the prescribed eddy viscosity $K = \kappa u_* z$, where κ denotes the von Kármán constant, u_* is the friction velocity, and z is the height above terrain. This simple closure works surprisingly well for wakes; actually, better than standard $k - \epsilon$ and mixing length closures. The equations are formulated in a mixed spectral domain, which facilitates extremely fast solutions based on a system of look-up tables, where some are general, and others are WT specific. These tables are subsequently used to determine the velocity field behind a single solitary turbine. Fuga can be run in either a 2D or a 3D mode (i.e., corresponding to a 2D or 3D formulation of the rotor actuator disc). Since the infinite boundary layer model by nature is a 2D model accounting for the horizontal homogeneous wind farm flow conditions at hub height, Fuga is, for the present purpose, executed in the 2D mode. Fuga was developed at Risø by Ott et al. [34] and is considered as one of the fastest full field models for wind farm computations. In the present work, it is employed to determine refined values for the a -parameter in the simplistic model (see Section 2.2) as well as a predictive tool for wind farm power estimation.

4.3. Model Validation

We aim to explore the full potential of each of the investigated models. Regarding the state-of-the-art engineering wake models listed in Section 4.2 where, contrary to the simplistic wind farm boundary layer model, the actual wind farm topology is modeled, we will consistently include aspects of wind farm inflow direction in the monthly energy production estimates. This is done by simulating each wind farm for different wind speeds and wind directions using the simple generic WT model, Equations (31) and (32). Subsequently, the simulated power is weighted with the corresponding histogram bin probability and the number of hours per month.

Thus, in summary, for the model benchmarking, wind farm topology and inflow direction aspects are utilized in the state-of-the-art engineering wake models, while the simplistic model works without this information. Moreover, it should be mentioned that a wind farm availability loss of 9% [16,35] has been included in all model predictions, except for the original simplistic model, where the availability loss is included in the finite wind farm correction factor of $a = 3$.

The quality of the available full-scale monthly production data is in general good, but outliers due to cable break down, curtailment etc. exists. To evaluate the capability of the models to simulate normal wind farm operation, these outliers have been filtered by the following approach: The monthly wake loss is estimated by subtracting the actual wind farm production from the approximated production without wake effects (based on the power curves, undisturbed inflow data from the New European Wind Atlas [18] and the number of wind turbines). A Gaussian function is fitted to the wake loss histogram and observations with wake loss larger than the Gaussian mean plus two standard deviations (i.e., the 95% fractile) are discarded (see Figure 10).

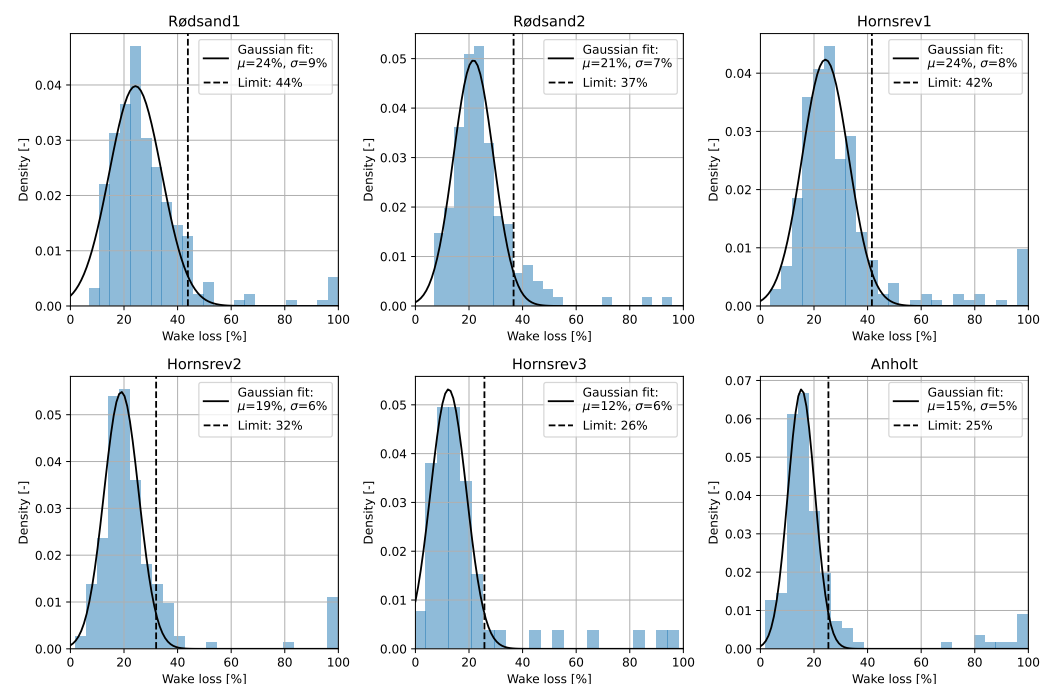


Figure 10. Actual wake losses and the wake loss limit used to filter the production data. The vertical dotted lines represent the Gaussian mean (μ) plus two standard deviations (2σ); i.e., the 95% quantiles.

Using the filtered production data, a one-to-one comparison of the respective model predicted production estimates against actual production is performed. The result is presented in Figure 11, giving both the error distributions and the error means represented by the distribution-specific horizontal lines.

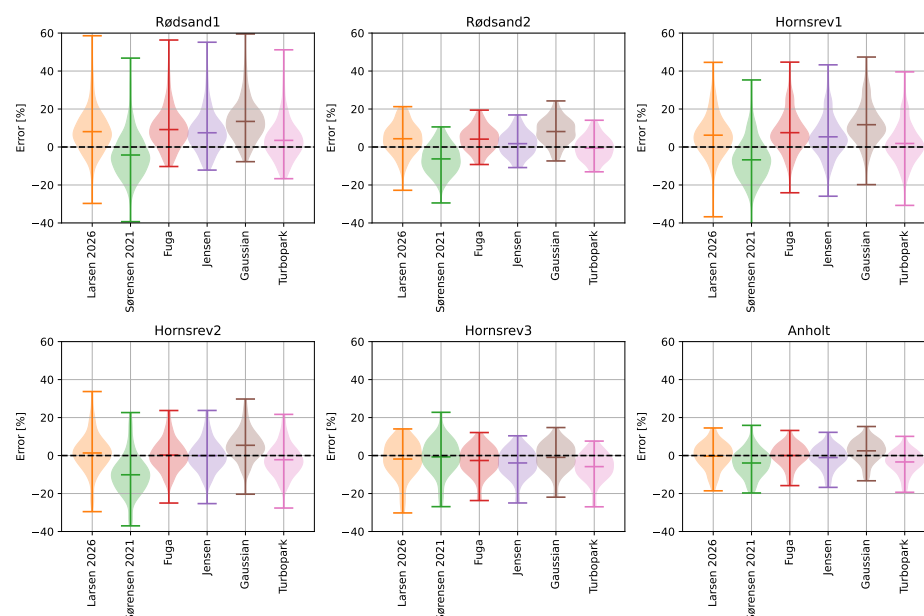


Figure 11. Monthly prediction error distribution for each of the investigated models [9].

Averaging across all wind farms and all production months investigated, the mean error and the error standard deviation for the individual models are presented in Figure 12. Altogether, this amounts to 920 independent observations.

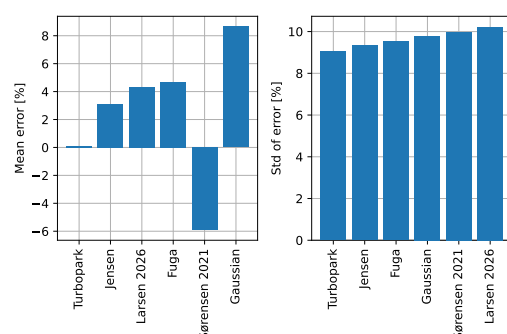


Figure 12. Mean and standard deviation of the errors across all months and wind farms [9].

Apart from model prediction performance, the required CPU resources are of interest. For the investigated six models, their respective CPU needs are shown in Figure 13 for the six investigated wind farms.

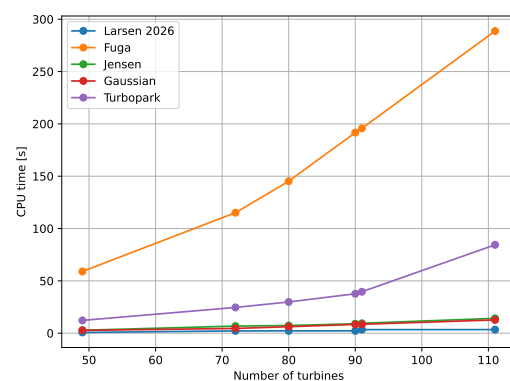


Figure 13. CPU resources required for each of the five analyzed wind farm power prediction models. The six dots represent the six investigated wind farms.

5. Discussion

The results include both aspects of model benchmarking and model validation against full-scale data.

5.1. Model Benchmarking

Relevant aspects of inter-model comparisons include requested input, computational resources, and model capabilities—e.g., accounting for neighboring wind farms.

With respect to model input, it is obvious that the simplistic model requires a very modest set of input parameters only in comparison with the other wind farm prediction platforms—i.e., the area of the wind farm, the latitude of the wind farm site, the number of WTs within the wind farm, the unconditional site wind speed Weibull distribution, the nameplate WT capacity, the WT rotor diameter, and the WT hub height. Thus, no information about the topology of the wind farm is needed.

The classic engineering wind farm prediction tools can be used with the same modest set of inputs, e.g., by assuming a Cartesian equally spaced topology and uniform wind direction distribution (as done when deriving the calibration factor for the simplistic model). Typically, however, the wind-direction-dependent wind speed statistics and the actual wind farm topology are available and used to improve the accuracy. This is also the case in the present study. Furthermore, the classic engineering models are applied using the raw inflow histograms, describing wind speed statistics conditioned on wind direction, thus bypassing the Weibull fitting and related uncertainty, which is unavoidable when running the simplistic model. Finally, it should be mentioned that detailed WT information in terms of WT power and thrust curves provided by the wind turbine manufacturer can be utilized by the classic engineering wind farm prediction tools, but in this study, the simple generic wind turbine model, expressed in Equations (31) and (32), is used for all models.

Regarding computational resources, it is seen from Figure 13 that the CPU requirements of the simplistic model are the lowest of all the investigated models and are furthermore invariant with respect to the number of WTs within a wind farm. This is contrary to all the classic engineering wind farm models, which have a theoretical runtime in the order of magnitude $O(N_T^2)$. For wind farms with sizes of the order of 100 WTs, the CPU resources requested by Fuga and TurboPark are orders of magnitudes larger than the CPU resources needed by the simplistic model.

Regarding the model capabilities, the simplistic wind farm platform is, more or less, bounded to single wind farm production predictions, whereas the classic engineering models have the flexibility to simulate clusters of closely spaced (individual) wind farms.

5.2. Validation of the Simplistic Model

Model prediction accuracy goes hand in hand with the model validation and links to the results presented in Figures 11 and 12.

The main take-away from Figure 11 is that the error distribution for all models approximately displays the same variability, and that the variability in the prediction error spread seems more wind-farm-specific than model-specific. Also individual model biases tend to be more wind-farm-dependent than model-dependent, which may indicate that the applied filter used on the actual production data is not completely adequate.

Referring to Figure 12, it is seen that the revised simplistic model shows a clear performance improvement compared to the originally developed simplistic model in [9]. Across all data, the revised simplistic model predicts with an accuracy only slightly less than the linear CFD model Fuga to which it is ‘calibrated’ with regard to the finite-farm correction factor. In terms of the mean estimate, the performance is clearly better than the Gaussian model but less accurate than the N.O. Jensen model and TurboPark. However,

the differences are small considering the inherent uncertainties hidden in a wind farm availability factor of 9%, which in the present study is considered a ‘general’ number, whereas in real life these additional external losses will vary from wind farm to wind farm. As discussed by Cabioch [36] and Gylling [37], these losses depend on factors such as the electrical components and the grid at the point of common coupling (electrical losses), the characteristics of the WT rotor (turbine effectiveness), and the availability of the wind farm WTs (due to component breakdown and wear). For the present study, we have assumed a generic 9% loss due to lack of wind-farm-specific information on wind farm availability, etc. With reference to the two above mentioned references, the aggregated generic loss of 9% is composed of loss of availability (5.0%), electrical losses (2.0%), and WT performance losses (2.0%). If, for instance, the true average loss for these wind farms should have been 11% then, under these circumstances, the N.O. Jensen, the Fuga, and the revised simplistic model would be the best-performing modeling platforms in terms of accuracy. Also, if an alternative better-performing RANS turbulence closure model was developed for Fuga, this model, and thus, in turn, the revised simplistic model would gain in accuracy.

Comparing the error-prediction distribution for Rødsand1 and Rødsand2 shows that the spread is considerably larger for Rødsand1 than for Rødsand2. This is most likely associated with the commissioning of the Rødsand2 wind farm in 2011, seven years after the commissioning of Rødsand1. For the dominant westerly inflow directions, the Rødsand1 wind farm is located only 3 km downstream of Rødsand2—see Figure 14a. As illustrated in Figure 14b, the wake losses of Rødsand1 increase significantly after Rødsand2 becomes operational. This is also reflected in the mean losses described by the associated Gaussian fits, where the mean loss increases from 20% in the period 2003–2009 to 27% in the period 2011–2023.

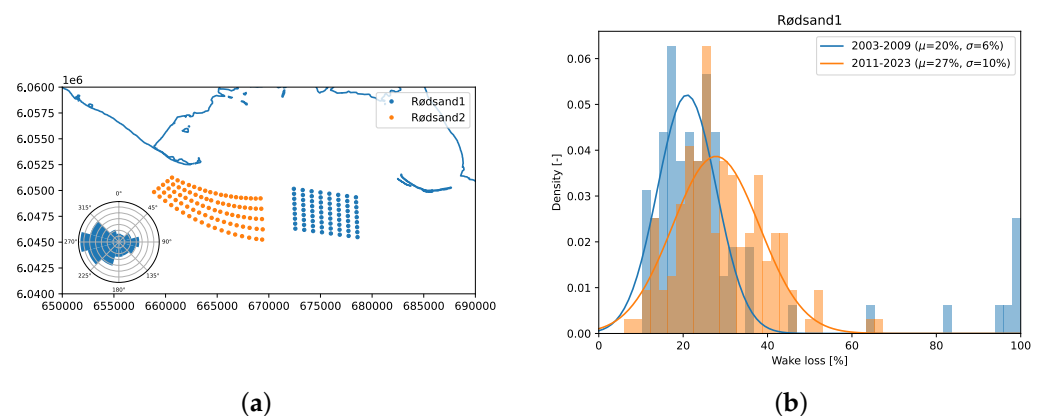


Figure 14. Rødsand 1 is located around 3 km downstream of Rødsand2 for the most-dominating wind directions. After inauguration of the Rødsand2 wind farm, the average wake loss of Rødsand1 increased from 20% to 27%. (a) Layout of the Rødsand1 and Rødsand2 wind farms along with the local inflow wind direction distribution. (b) Wake loss density distribution for the Rødsand1 wind farm split on the periods, respectively, before and after inauguration of the Rødsand2 wind farm.

Because the site statistics refer to the wind climate on the site *without* presence of wind farms, the consequence is that the most reliable model predictions of the Rødsand1 production are associated with the time span from its inauguration in 2003 and up to the inauguration of the Rødsand2 wind farm in 2011. Therefore, in addition to a larger error spread, a model over-prediction over the entire operational period extending from 2003 to 2023 should be expected for Rødsand1. This is also indicated for most of the models in Figure 11.

6. Conclusions

We have developed an improved closed-form simplistic wind farm power prediction model based on two infinite ABL boundary layer models and a transition regime between these facilitating estimation of finite wind farms. The model requires only a very limited set of input data.

Considering all its assumptions, the revised simplistic model works surprisingly well when benchmarked against five state-of-the-art wind farm power prediction platforms and validated against a vast amount of quality-ensured real production data from six wind farms, which vary significantly in terms of size, topology, and WT sizes.

When it comes to the standard deviation of the prediction error, all models perform similarly, with their respective coefficients of variation ranging between 9% and 10%. Combining this information with the assumption of a generic loss factor of 9%, which ideally should be a wind-farm-specific value, it is hard to decide which model is the most accurate on a rational basis. If, e.g., the wind farm average losses should have been 11%, then, under these circumstances, the N.O. Jensen, the Fuga, and the revised simplistic model would be the best-performing modeling platforms in terms of accuracy. Thus, in conclusion, the investigated models—except the original simplistic model and the Gaussian model—perform equally well. This is remarkable, as the classical engineering models include detailed modeling of the wind farm topology along with detailed inflow statistics in a histogram representation conditioned on inflow direction, whereas for the simplistic model only a direction-independent Weibull wind speed distribution is available.

Finally, coming to computational requirements, the simplistic model stands out, compared to the other models, with CPU consumption invariant with respect to the number of wind farm turbines and, for farm sizes of around 100 WTs, decades less than the Fuga and TurboPark platforms.

In a future perspective, the simplistic model can be generalized to account for ABL stability by adjusting the two-layer infinite wind farm boundary layer model by replacing the neutral logarithmic shear layers with the shear layers resulting from Monin–Obukhov Similarity Theory (MOST). In addition, the introduction of an entrainment model will be considered as well as investigating the sensitivity of the finite wind farm correction calibration in Section 2.2 on the TI wind speed dependence by applying both types of TI estimates; i.e., based on the mean or the mode of the wind speed standard deviation distribution, respectively.

Inclusion of effects of shadowing upstream wind farms is less promising, since the wake loss behind an upstream-located wind farm is not horizontally homogeneous, but depends on the downstream distance, and a wind farm, in general, covers a substantial downstream area.

In summary—and coming back to the heading of this paper that refers to an easily implementable *closed-form* analytical formulation of wind farm production, which only requires very few input data—the revised simplistic model is recommended for production estimates of wind farms with a predominant 2D-type of topology and which is not located in the downstream wake of upstream wind farms with reference to the predominant site wind direction. The prediction horizon must be long enough to justify a Weibull description of the site wind resource. The present analysis has shown that a prediction horizon down to, at least, one month can be justified.

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Data Availability Statement: The wind farm layout, wind turbine parameters, and monthly wind farm production data of the investigated six wind farms are publicly available at <https://ens.dk/analyser-og-statistik/data-oversigt-over-energiesektoren> (accessed on 30 January 2026). The updated simplistic model as well as the reference models are available in PyWake [24].

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Nomenclature

The following abbreviations are used in this manuscript:

Name	Symbol	Units
Grid points ($n_T, s, \tilde{u}_a$)	$\{(n_T^{(k)}, s^{(k)}, \tilde{u}_a^{(k)})\}$	-
Range (set of exponents)	$\{M_i, M_j, M_k\}$	-
Incomplete (lower) Gamma function	$\Gamma(\star, \star)$	-
Cumulative Gaussian distribution	$\Phi(\star)$	-
Error metric	$\Pi_o(M_i, M_j, M_k)$	-
Rotational speed of the Earth	Ω	rad/s
Wind farm area	A	m ²
Empirical constant (Geostrophic Drag Law)	A_*	-
Empirical constant (Geostrophic Drag Law)	B_*	-
Power coefficient	$C_P(U)$	-
Power coefficient (rated)	$C_{P,r}$	-
Thrust coefficient	$C_T(U)$	-
Thrust coefficient (rated)	$C_{T,r}$	-
Rotor diameter	D	m
Log-normal distribution	$F_\sigma(\star \star)$	-
Geostrophic wind speed	G	m/s
Heaviside function	$\mathcal{H}(\star)$	-
Eddy viscosity	K	m ² /s
Number of turbines	N_T	-
Probability	$P(\star)$	-
Finite wind farm production	$P_f(N_T, S, U_a)$	MW
Wind turbine production at rated	P_G	MW
Power production (infinite wind farm boundary layer)	$P_{if}(U_a, S)$	MW
Legendre polynomial (degree n)	$P_n(\star)$	-
Power production (solitary WT)	$P_s(U_a)$	MW
Power production of finite wind farm	$P_{WF}(N_T, S)$	MW
Wind turbine power curve	$P_{wt}(U)$	MW
Production integral (finite WF)	$P_y(N_T, S)$	MW
Production integral (WT; infinite BL; below rated)	$P_{y,f1}(N_T, S)$	MW
Production integral (WT; infinite BL; above rated)	$P_{y,f2}(N_T, S)$	MW
Production integral (WT; ambient BL; below rated)	$P_{y,s1}(N_T, S)$	MW
Production integral (WT; ambient BL; above rated)	$P_{y,s2}(N_T, S)$	MW
Power production (WT; infinite wind farm; below rated)	$P_{y,f1}^{orig}$	MW
Power production (WT; infinite wind farm; above rated)	$P_{y,f2}^{orig}$	MW
Inter-turbine spacing	S	D
Turbulence intensity	TI	-
Mean wind speed	U	m/s
Ambient undisturbed mean wind speed	U_a	m/s

Wind speed at hub height	U_h	m/s
Wind speed at hub height (ambient)	$U_{h,0}$	m/s
Cut-in wind speed	U_i	m/s
Cut-out wind speed	U_o	m/s
Rated wind speed	U_r	m/s
Mean wind speed (time span T)	U_T	m/s
Corrected ambient mean wind speed	$\tilde{U}_a$	m/s
Rated speed of reference model (11 m/s)	$\tilde{U}_r$	m/s
Characteristic TI	$\overline{TI}$	-
Correction constant (finite wind farms)	$a(N_T, S, U_a)$	-
Correction constant (finite wind farms—old model)	$\tilde{a}$	-
Distribution parameter	$a_1(U_T)$	-
Distribution parameter	$a_2(U_T)$	-
Correction constant (normalized variables)	$a_n(\star, \star, \star)$	-
Factor in WT power curve description	α	MW/(m/s) ³
Term in WT power curve description	β	MW
Wind farm drag coefficient	c_t	-
Auxiliary coefficient (wind farm BL)	δ	-
Dirac function	$\delta(\star)$	-
Auxiliary coefficient (wind farm BL)	ϵ_1	-
Auxiliary coefficient (wind farm BL)	$\epsilon_2(U_f)$	-
Coriolis parameter	f	rad/s
Normalized Coriolis parameter	f'	-
Weibull pdf	$f_U(U; \lambda, k)$	-
Gravitational acceleration	g	m/s ²
Auxiliary coefficient (wind farm BL)	γ	-
Hub height/reference height	h	m
Shape parameter (Weibull)	k	-
Von Kármán constant	κ	-
Scale parameter (Weibull)	λ	m/s
Gaussian mean	μ	-
Mean (log-normal distribution)	μ_σ	m/s
Scaled N_T	$n_T(N_T)$	-
Exponent in the factorization of $a(N_T, S, U_a)$	p_i	-
Exponent in the factorization of $a(N_T, S, U_a)$	q_j	-
Exponent in the factorization of $a(N_T, S, U_a)$	r_k	-
Air density	ρ	kg/m ³
Scaled S	$s(S)$	-
Gaussian standard deviation	σ	-
Standard deviation (time span T)	σ_T	m/s
Standard deviation (longitudinal turbulence)	σ_u	m/s
Scaling constant relating u_* and σ_u	ψ	-
Transformation variable	t	-
Shear stress	τ	-
Friction velocity	u_*	m/s
Scaling factor	ζ	m/s
Factor ($a(N_T, S, \tilde{U}_a)$ factorization)	ζ_{ijk}	-
Height above terrain/measuring height	z	m
Roughness height	z_0	m
Roughness height (wind farm)	z_{wf}	m
Mode (log-normal distribution)	η_σ	m/s
Characteristic z_0	$\bar{z}_0$	-
Upper wind-speed factor $20/\tilde{U}_r$	θ	-
Site latitude	ϕ	deg

Scaled $\tilde{U}_a$	$\tilde{u}_a(\tilde{U}_a)$	-
Factor in the factorization of $a(N_T, S, U_a)$	$\xi_i(N_T, S)$	-
Factor (factorization below rated)	$\xi_i^{(f1)}(N_T, S)$	-
Factor (factorization above rated)	$\xi_i^{(f2)}(N_T, S)$	-

Appendix A

This appendix outlines details in the derivation of closed-form solutions to four arche integral types describing the *Weibull-weighted* production of WTs located both within the ambient equilibrium boundary layer and in the fully developed wind farm boundary layer. The most tricky integrals are those associated with the fully developed wind farm boundary layer, and, in fact, closed-form solutions are only possible here because ϵ_1 , related to the below rated regime, is independent of the farm velocity, U_f , and, equally important, because the integral kernel related to the above rated regime does not explicitly involve U_f but only its differential dU_f .

We will take advantage of the above-described small ‘window of opportunities’ and derive closed-form integral solutions for both the ambient equilibrium and wind farm boundary layers. For each of these equilibrium boundary layer conditions, the expressions for the WT production are divided between below rated and above rated operational conditions.

With the nomenclature introduced in Section 2.1. The correction factor is in Section 2.3 approximated as a polynomial expansion, but, if convenient, the integrals to follow can equally well be brought on closed-form, with the correction factor expanded in terms of power functions, however, with the limitation that powers, p , must satisfy $p > -k$, where k denotes the relevant Weibull shape parameter.

We have the following expression for the WT production associated with *ambient boundary layer equilibrium* conditions *below rated* wind speed:

$$\begin{aligned}
 P_{y,s1}(N_T, S) &= \alpha \sum_{i=0}^M \xi_i(N_T, S) \int_{U_i}^{U_r} U_a^{p_i+3} \frac{k}{\lambda} \left(\frac{U_a}{\lambda} \right)^{k-1} e^{-\left(\frac{U_a}{\lambda}\right)^k} dU_a \\
 &\quad + \beta \sum_{i=0}^M \xi_i(N_T, S) \int_{U_i}^{U_r} U_a^{p_i} \frac{k}{\lambda} \left(\frac{U_a}{\lambda} \right)^{k-1} e^{-\left(\frac{U_a}{\lambda}\right)^k} dU_a \\
 &= \frac{\alpha k}{\lambda} \sum_{i=0}^M \xi_i(N_T, S) \int_{U_i}^{U_r} U_a^{p_i+3} \left(\frac{U_a}{\lambda} \right)^{k-1} e^{-\left(\frac{U_a}{\lambda}\right)^k} dU_a \\
 &\quad + \frac{\beta k}{\lambda} \sum_{i=0}^M \xi_i(N_T, S) \int_{U_i}^{U_r} U_a^{p_i} \left(\frac{U_a}{\lambda} \right)^{k-1} e^{-\left(\frac{U_a}{\lambda}\right)^k} dU_a
 \end{aligned} \tag{A1}$$

Next, we introduce the transformation $t := \left(\frac{U_a}{\lambda}\right)^k$ and get

$$\begin{aligned}
 P_{y,s1}(N_T, S) &= \frac{\alpha k}{\lambda} \frac{\lambda}{k} \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i+3} \int_{U_a=U_i}^{U_r} t^{\frac{p_i+3}{k}} e^{-t} dt \\
 &\quad + \frac{\beta k}{\lambda} \frac{\lambda}{k} \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i} \int_{U_a=U_i}^{U_r} t^{\frac{p_i}{k}} e^{-t} dt \\
 &= \alpha \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i+3} \int_{U_a=U_i}^{U_r} t^{\left(\frac{p_i+3}{k}-1\right)} e^{-t} dt \\
 &\quad + \beta \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i} \int_{U_a=U_i}^{U_r} t^{\left(\frac{p_i}{k}-1\right)} e^{-t} dt
 \end{aligned} \tag{A2}$$

Introducing the definition of the incomplete Gamma function, $\Gamma(\star, \star)$, Equation (A2) finally reduces to the following closed-form expression:

$$P_{y,s1}(N_T, S) = \alpha \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i+3} \left[\Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_i}{\lambda}\right)^k\right) \right] + \beta \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_i}{\lambda}\right)^k\right) \right] \quad (\text{A3})$$

For the *above rated* WT production under *ambient* equilibrium conditions, we have the following formulation:

$$P_{y,s2}(N_T, S) = P_G \sum_{i=0}^M \xi_i(N_T, S) \int_{U_r}^{U_o} U_a^{p_i} \frac{k}{\lambda} \left(\frac{U_a}{\lambda}\right)^{k-1} e^{-\left(\frac{U_a}{\lambda}\right)^k} dU_a = \frac{P_G k}{\lambda} \sum_{i=0}^M \xi_i(N_T, S) \int_{U_r}^{U_o} U_a^{p_i} \left(\frac{U_a}{\lambda}\right)^{k-1} e^{-\left(\frac{U_a}{\lambda}\right)^k} dU_a \quad (\text{A4})$$

Applying the transformation leading to Equation (A2) yields, in analogy with the β -term in Equation (A2), the following closed-form expression:

$$P_{y,s2}(N_T, S) = P_G \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i} \int_{U_a=U_r}^{U_o} t^{\left(\frac{p_i+k}{k}-1\right)} e^{-t} dt = P_G \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_o}{\lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) \right] \quad (\text{A5})$$

We now turn to WTs located in the fully developed *wind farm boundary layer*, where the following expression for the average WT power production applies for the *below rated* wind speed regime:

$$P_{y,f1}(N_T, S) = \alpha \sum_{i=0}^M \xi_i(N_T, S) \int_{U_i}^{U_r} U_a^{p_i} U_f^3 f_{U_f}(U_f) dU_f + \beta \sum_{i=0}^M \xi_i(N_T, S) \int_{U_i}^{U_r} U_a^{p_i} f_{U_f}(U_f) dU_f \quad (\text{A6})$$

where $f_{U_f}(\star)$ is the mean wind speed Weibull distribution associated with the farm wind climate. Note that, contrary to Equation (A1), the integration variable is now the mean wind speed inside an ‘infinite’ wind farm, U_f . The mean wind speeds at the hub height inside and outside the wind farm are two interrelated stochastic variables. Specifically, the transformation between these two is given by [9]

$$U_f = \begin{cases} \epsilon_1 U_a & \text{for } U_i \leq U_f < U_r \\ \epsilon_2(U_f) U_a & \text{for } U_r \leq U_f \leq U_o \end{cases} \quad (\text{A7})$$

where ϵ_1 is defined in Equation (12), and $\epsilon_2(U_f)$ is defined in Equation (23).

In ([9], Appendix A), it is proved that the functional relationship between the ambient mean wind speed, U_a , and the associated mean wind speed within the fully developed

wind farm boundary layer, U_f , is *monotonic*, which means that there exists a one-to-one transformation between U_a and U_f . Therefore, the following relationship applies [22]:

$$f_{U_f}(U_f) dU_f \equiv f_{U_a}(U_a) dU_a \quad (\text{A8})$$

where $f_{U_a}(\star)$ is the mean wind speed Weibull distribution associated with the ambient wind climate, which is specified a priori for the site.

Combining Equations (A7) and (A8), we obtain the following relationship between the ambient and the wind farm Weibull distributions associated with the *below rated* wind speed regime:

$$f_{U_f}(U_f) = f_{U_a}(U_a) \frac{dU_a}{dU_f} = \frac{f_{U_a}(U_a)}{\epsilon_1} = \frac{f_{U_a}(U_f/\epsilon_1)}{\epsilon_1} \quad \text{for } U_f \leq U_r \quad (\text{A9})$$

Introducing Equations (A7) and (A9) into Equation (A6) we obtain

$$\begin{aligned} P_{y,f1}(N_T, S) &= \alpha \sum_{i=0}^M \xi_i(N_T, S) \int_{U_i}^{U_r} \left(\frac{U_f}{\epsilon_1}\right)^{p_i} U_f^3 \frac{k}{\epsilon_1 \lambda} \left(\frac{U_f}{\epsilon_1 \lambda}\right)^{k-1} e^{-\left(\frac{U_f}{\epsilon_1 \lambda}\right)^k} dU_f \\ &\quad + \beta \sum_{i=0}^M \xi_i(N_T, S) \int_{U_i}^{U_r} \left(\frac{U_f}{\epsilon_1}\right)^{p_i} \frac{k}{\epsilon_1 \lambda} \left(\frac{U_f}{\epsilon_1 \lambda}\right)^{k-1} e^{-\left(\frac{U_f}{\epsilon_1 \lambda}\right)^k} dU_f \\ &= \alpha \sum_{i=0}^M \frac{\xi_i(N_T, S)}{\epsilon_1^{p_i}} \int_{U_i}^{U_r} U_f^{p_i+3} \frac{k}{\epsilon_1 \lambda} \left(\frac{U_f}{\epsilon_1 \lambda}\right)^{k-1} e^{-\left(\frac{U_f}{\epsilon_1 \lambda}\right)^k} dU_f \\ &\quad + \beta \sum_{i=0}^M \frac{\xi_i(N_T, S)}{\epsilon_1^{p_i}} \int_{U_i}^{U_r} U_f^{p_i} \frac{k}{\epsilon_1 \lambda} \left(\frac{U_f}{\epsilon_1 \lambda}\right)^{k-1} e^{-\left(\frac{U_f}{\epsilon_1 \lambda}\right)^k} dU_f \end{aligned} \quad (\text{A10})$$

To proceed, we introduce the transformation $t := \left(\frac{U_f}{\epsilon_1 \lambda}\right)^k$ and obtain

$$\begin{aligned} P_{y,f1}(N_T, S) &= \alpha \sum_{i=0}^M \frac{\xi_i(N_T, S)}{\epsilon_1^{p_i}} (\epsilon_1 \lambda)^{p_i+3} \int_{U_f=U_i}^{U_r} t^{\left(\frac{p_i+k+3}{k}-1\right)} e^{-t} dt \\ &\quad + \beta \sum_{i=0}^M \frac{\xi_i(N_T, S)}{\epsilon_1^{p_i}} (\epsilon_1 \lambda)^{p_i} \int_{U_f=U_i}^{U_r} t^{\left(\frac{p_i+k}{k}-1\right)} e^{-t} dt \\ &= \alpha \sum_{i=0}^M \xi_i(N_T, S) \epsilon_1^3 \lambda^{p_i+3} \int_{U_f=U_i}^{U_r} t^{\left(\frac{p_i+k+3}{k}-1\right)} e^{-t} dt \\ &\quad + \beta \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i} \int_{U_f=U_i}^{U_r} t^{\left(\frac{p_i+k}{k}-1\right)} e^{-t} dt \end{aligned} \quad (\text{A11})$$

which, in terms of the incomplete Gamma function, can be recast into the following closed-form expression:

$$\begin{aligned} P_{y,f1}(N_T, S) &= \\ &\quad \alpha \sum_{i=0}^M \xi_i(N_T, S) \epsilon_1^3 \lambda^{p_i+3} \left[\Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_r}{\epsilon_1 \lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_i}{\epsilon_1 \lambda}\right)^k\right) \right] \\ &\quad + \beta \sum_{i=0}^M \xi_i(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\epsilon_1 \lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_i}{\epsilon_1 \lambda}\right)^k\right) \right] \end{aligned} \quad (\text{A12})$$

With regard to fitting of the $\xi_i(N_T, S)$ hyperplanes, it turns out to be convenient to split the below rated wind speed regime into two parts—one part accounting for the wind speed

regime up to rated *ambient* wind speed, $\xi_i^{(f1)}(N_T, S)$, and another part accounting for the wind speed regime between rated ambient and rated wind farm wind speeds, $\xi_i^{(f2)}(N_T, S)$ —see Figure 4 as well as Equations (41) and (42) and Equations (43)–(46). With reference to Equation (A12), we therefore obtain:

$$\begin{aligned}
 P_{y,f1}(N_T, S) = & \alpha \sum_{i=0}^M \xi_i^{(f1)}(N_T, S) \epsilon_1^3 \lambda^{p_i+3} \left[\Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_i}{\epsilon_1 \lambda}\right)^k\right) \right] \\
 & + \beta \sum_{i=0}^M \xi_i^{(f1)}(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_i}{\epsilon_1 \lambda}\right)^k\right) \right] \\
 & + \alpha \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \epsilon_1^3 \lambda^{p_i+3} \left[\Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_r}{\epsilon_1 \lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k+3}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) \right] \\
 & + \beta \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\epsilon_1 \lambda}\right)^k\right) - \Gamma\left(\frac{p_i+k}{k}, \left(\frac{U_r}{\lambda}\right)^k\right) \right]
 \end{aligned} \quad (A13)$$

Finally, for the fully developed *wind farm boundary layer* in the *above rated* wind speed regime, we have the following expression for the average power WT production:

$$P_{y,f2}(N_T, S) = P_G \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \int_{U_r}^{U_o} U_a^{p_i} f_{U_f}(U_f) dU_f \quad (A14)$$

Using Equation (A8), we reformulate Equation (A14) as

$$\begin{aligned}
 P_{y,f2}(N_T, S) &= P_G \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \int_{U_f=U_r}^{U_o} U_a^{p_i} f_{U_a}(U_a) dU_a \\
 &= P_G \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \int_{U_f=U_r}^{U_o} U_a^{p_i} \frac{k}{\lambda} \left(\frac{U_a}{\lambda}\right)^{k-1} e^{-\left(\frac{U_a}{\lambda}\right)^k} dU_a \\
 &= P_G k \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \lambda^{p_i-1} \int_{U_f=U_r}^{U_o} \left(\frac{U_a}{\lambda}\right)^{p_i+k-1} e^{-\left(\frac{U_a}{\lambda}\right)^k} dU_a
 \end{aligned} \quad (A15)$$

In arriving at Equation (A15), which eventually leads to a closed-form solution for the above rated average WT power production within a fully developed wind farm boundary layer, we benefit from having the polynomial expansion of the correction factor, $a(N_T, S, U_a)$, expressed in terms of *ambient* mean wind speeds U_a . In case of U_f appearing as a factor to the wind farm Weibull distribution, $f_{U_f}(\star)$, the integral leading to the above rated wind farm WT power production cannot be expressed in closed form. This is because, contrary to the below rated wind speed regime (cf. Equation (A7)), there is an *implicit* relationship between U_f and U_a in the above rated wind speed regime.

Introducing the transformation $t := \left(\frac{U_a}{\lambda}\right)^k$ in (A15), we obtain

$$\begin{aligned}
 P_{y,f2}(N_T, S) &= P_G k \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \lambda^{p_i-1} \int_{U_f=U_r}^{U_o} \left(t^{1/k}\right)^{p_i+k-1} e^{-t} \frac{\lambda}{k} \left(\frac{U_a}{\lambda}\right)^{1-k} dt \\
 &= P_G \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \lambda^{p_i} \int_{U_f=U_r}^{U_o} t^{\frac{p_i+k-1}{k}} t^{\frac{1-k}{k}} e^{-t} dt \\
 &= P_G \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \lambda^{p_i} \int_{U_f=U_r}^{U_o} t^{p_i/k} e^{-t} dt
 \end{aligned} \quad (A16)$$

In analogy with Equations (A2) and (A3), the closed-form solution to Equation (A16) can be formulated as

$$P_{y,f2}(N_T, S) = P_G \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i + k}{k}, \left(\frac{U_o}{\epsilon_2(U_o)\lambda}\right)^k\right) - \Gamma\left(\frac{p_i + k}{k}, \left(\frac{U_r}{\epsilon_2(U_r)\lambda}\right)^k\right) \right] \quad (\text{A17})$$

where $\epsilon_2(U_f)$ is defined by Equation (23).

In the *above rated* wind farm wind speed regime, the Fuga simulations, based on the simplistic WT defined by Equations (31) and (32), show that the weighting factor, $a(N_T, S, U_a)$, with a good approximation, equals 1 beyond $U_a = 20$ m/s (cf. Figure 4), which equals $\frac{20U_r}{11}$ for an arbitrary WT (cf. Equation (33)). Therefore, the average power production of a WT operating in this wind speed regime is given by the sum of two parts—one that accounts for the wind speed regime up to the wind farm wind speed $\frac{20U_r}{11} := \theta U_r$, and another part that accounts for the wind speed regime beyond θU_r and up to the wind farm cut-out wind speed, U_o . Thus,

$$P_{y,f2}(N_T, S) = P_G \sum_{i=0}^M \xi_i^{(f2)}(N_T, S) \lambda^{p_i} \left[\Gamma\left(\frac{p_i + k}{k}, \left(\frac{\theta U_r}{\epsilon_2(\theta U_r)\lambda}\right)^k\right) - \Gamma\left(\frac{p_i + k}{k}, \left(\frac{U_r}{\epsilon_1\lambda}\right)^k\right) \right] + \mathcal{H}(U_o - \theta U_r) P_G \left[\exp\left(-\left(\frac{\theta U_r}{\epsilon_2(\theta U_r)\lambda}\right)^k\right) - \exp\left(-\left(\frac{U_o}{\epsilon_2(U_o)\lambda}\right)^k\right) \right] \quad (\text{A18})$$

where the Heaviside function, $\mathcal{H}(\star)$, has been introduced to ensure correct scaling of the wind speed regime between rated wind farm wind speed and U_o for all (realistic) values of U_r .

Appendix B

With reference to Equation (6), the explicit expression for the power coefficient, C_P , of the simplistic WT is derived in the following. We have

$$P_{wt}(U) = P_G \left(\frac{U^3 - U_i^3}{U_r^3 - U_i^3} \right) \quad (\text{A19})$$

The power coefficient at the rated wind speed is given by

$$C_{P,r} = \frac{P_G}{\frac{1}{2}\rho U_r^3 A} \quad (\text{A20})$$

with P_G being WT production at the rated wind speed, ρ being the air density, and A being the rotor area. By combining Equations (A19) and (A20), we obtain

$$P_{wt}(U) = \frac{\frac{1}{2}\rho U_r^3 A C_{P,r}}{U_r^3 - U_i^3} (U^3 - U_i^3) \quad (\text{A21})$$

whereby

$$C_P(U) = \frac{P_{wt}(U)}{\frac{1}{2}\rho U^3 A} = \frac{U_r^3 C_{P,r}}{U_r^3 - U_i^3} \frac{(U^3 - U_i^3)}{U^3} = \frac{C_{P,r}}{1 - \left(\frac{U_i}{U_r}\right)^3} \left[1 - \left(\frac{U_i}{U}\right)^3 \right] \quad (\text{A22})$$

Appendix C

The relevant set of Legendre polynomials used in the fitted expressions for the ξ -hyper planes is given below. Based on Rodrigues' formula, the relevant Legendre polynomials are given by

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n \quad (\text{A23})$$

and we find

$$\begin{aligned} P_0(x) &= 1 \\ P_1(x) &= x \\ P_3(x) &= \frac{1}{2} (3x^2 - 1) \\ P_3(x) &= \frac{1}{2} (5x^3 - 3x) \end{aligned} \quad (\text{A24})$$

Appendix D

To facilitate an explicit expression for u_* in terms of the geostrophic wind speed, we need to rely on an approximate expression of the Geostrophic Drag Law. In this appendix, the simplistic version of the Geostrophic Drag Law used in [9] will be refined and re-calibrated, in order to improve the dependence of the geostrophic wind magnitude on the friction Rossby number and thus, in turn, the dependence on the site latitude. The latitude dependence of the approximation expressed in Equation (A26) is manifested through dependence of A_* on the latitude—see Figure A1. Assuming inertial and viscous forces to be small (i.e., low Rossby and Ekman numbers) relative to, respectively, Coriolis, pressure, and friction forces, the Geostrophic Drag Law for neutral atmospheric stratification may be formulated as [38]

$$\frac{G}{u_*} = \frac{1}{\kappa} \sqrt{\left(\ln \left(\frac{u_*}{f z_0} \right) - A \right)^2 + B^2} = \frac{1}{\kappa} \sqrt{\left(\ln \left(\frac{G}{f z_0} \right) - \ln \left(\frac{G}{u_*} \right) - A \right)^2 + B^2} \quad (\text{A25})$$

where G denotes the magnitude of the geostrophic wind, f is the Coriolis parameter (cf. Section 2.1), z_0 is the terrain roughness length, u_* is the friction velocity, and A and B are constants that, in neutral stratification, often are taken as $A = 1.8$ and $B = 4.5$ [39].

The approximate expression for Equation (A25) given in [40] and used for production estimates in [9] reads

$$\frac{G}{u_*} = \frac{1}{\kappa} \left(\ln \left(\frac{G}{f z_0} \right) - A_* \right) \quad (\text{A26})$$

with A_* being an empirical constant, which in the model of Frandsen [12] takes the value 4. Now, comparing the left-hand-sides of Equations (A25) and (A26) as functions of the logarithm to the friction Rossby number, $\ln(G/(f z_0))$, for various values of A_* , it is observed that the gradient of the approximate expression differs from the gradient of the full formulation in Equation (A25)—see Figure A1.

In response to this observation, we suggest the following reformulated approximation to Equation (A25):

$$\frac{G}{u_*} = \frac{B_*}{\kappa} \left(\ln \left(\frac{G}{f z_0} \right) - A_* \right) \quad (\text{A27})$$

where A_* and B_* are constants, which can be calibrated to $(A_*, B_*) = (2.0, 0.85)$. We thereby obtain a linear approximation (in the logarithm to the friction Rossby number) that closely follows the Geostrophic Drag Law for neutral atmospheric stratification expressed in Equation (A25). This is illustrated in Figure A2.

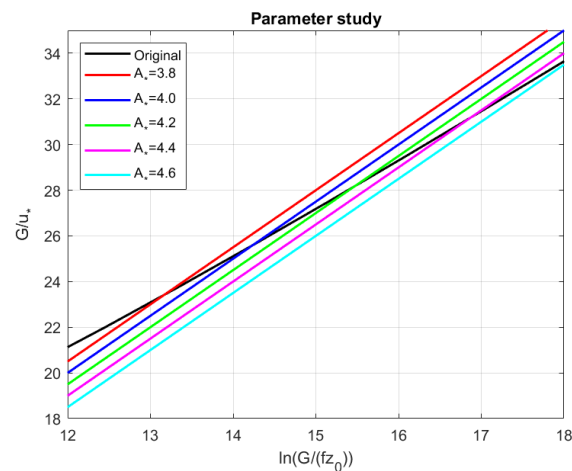


Figure A1. Geostrophic wind normalized by friction velocity as a function of the friction Rossby number for various values of A_* (referring to Equation (A26)). Black line refers to Equation (A25).

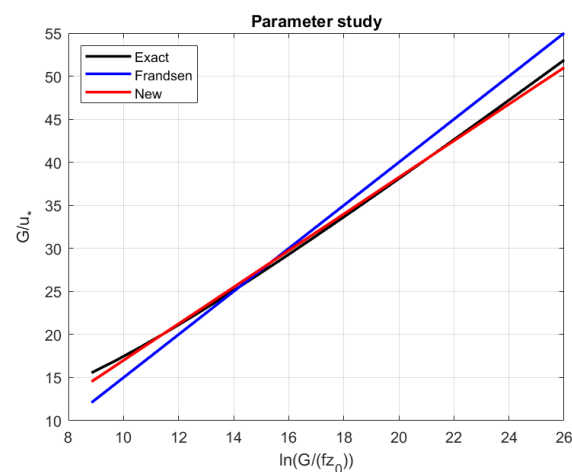


Figure A2. Geostrophic wind normalized by friction velocity as a function of the friction Rossby number. Black curve refers to Equation (A25); blue line refers to Equation (A26); and red line refers to Equation (A27).

In order to proceed, we briefly recap the concept of friction velocity. The friction velocity—or shear velocity—is a fluid mechanics concept often used in meteorology, in which a shear stress is reformulated in units of velocity. It is defined as

$$u_* = \sqrt{\frac{\tau}{\rho}} \quad (\text{A28})$$

where τ is the shear stress in an arbitrary layer of fluid, and ρ is the density of the fluid.

In meteorology, the friction velocity is used to define the logarithmic wind shear of a flow over flat and homogeneous terrain under neutral stratification. This is also the use we will make of the concept in the present context, where two logarithmic shear layers are defined—one inner layer describing the flow conditions associated with the flat and homogeneous ground, defined in terms of the roughness height $z_{0,i}$, and an outer layer reaching beyond the WT hub height and characterized by the combined roughness of the terrain and the WTs.

From momentum conservation, the difference in shear stress above, τ_{wf} , and below hub height, τ_0 , is counteracted by the total thrust per unit area exerted on the flow by the WTs, which can be formulated as follows:

$$\tau_{wf} - \tau_0 = \frac{\frac{1}{2}\rho C_T \frac{\pi}{4} D^2 U_h}{S^5 D^2} = \rho \frac{\pi C_T U_h^2}{8 S^2} = \rho c_t U_h^2 \quad (\text{A29})$$

with

$$c_t := \frac{\pi C_T}{8 S^2} \quad (\text{A30})$$

Expressed in terms of friction velocities, we have using Equations (A28) and (A30):

$$u_{*,wf}^2 = u_{*,0}^2 + c_t U_h^2 \quad (\text{A31})$$

with indices '0' and 'wf' indicating inner and wind farm flow field layers, respectively. Thus,

$$u_{*,wf} = \sqrt{u_{*,0}^2 + c_t U_h^2} = U_h \sqrt{\kappa^2 \left(\ln \left(\frac{h}{z_0} \right) \right)^{-2} + c_t} \quad (\text{A32})$$

The outer wind farm layer friction velocity can also be formulated in terms of the geostrophic wind speed, G , using the reformulated Geostrophic Drag Law approximation given in Equation (A27).

$$u_{*,wf} = \frac{\kappa G}{B_* \left[\ln \left(\frac{G}{f' z_{wf}} \right) - A_* \right]} = \frac{\kappa G}{B_* \left[\ln \left(\frac{G}{f' z_{wf}} \right) \right]} \quad (\text{A33})$$

with

$$f' := f e^{A_*} \quad (\text{A34})$$

Thus, combining Equations (A32) and (A33), the hub height mean wind velocity inside a fully developed wind farm boundary layer is given by

$$U_h = \frac{\kappa G}{B_* \left[\ln \left(\frac{G}{f' z_{wf}} \right) \right] \sqrt{\kappa^2 \left(\ln \left(\frac{h}{z_0} \right) \right)^{-2} + c_t}} \quad (\text{A35})$$

To close this equation, an expression for the roughness height z_{wf} is needed. This is obtained requiring continuity of the outer and inner flow conditions at hub height; i.e.,

$$\begin{aligned} z_{wf} &= h \exp \left(-\frac{u_{*,0}}{u_{*,wf}} \ln \left(\frac{h}{z_0} \right) \right) \\ &= h \exp \left(-\frac{u_{*,0}}{\sqrt{u_{*,0}^2 + c_t U_h^2}} \ln \left(\frac{h}{z_0} \right) \right) \\ &= h \exp \left(-\frac{\kappa}{\sqrt{\frac{u_{*,0}^2}{U_h^2} + c_t}} \right) \\ &= h \exp \left(-\frac{\kappa}{\sqrt{\kappa^2 \left(\ln \left(\frac{h}{z_0} \right) \right)^{-2} + c_t}} \right) \end{aligned} \quad (\text{A36})$$

which introduced in Equation (A35) finally gives

$$\begin{aligned}
 U_h &= \frac{\kappa G}{B_* \left[\ln \left(\frac{G}{f'h} \right) - \ln z_{wf} \right] \sqrt{\kappa^2 \left(\ln \left(\frac{h}{z_0} \right) \right)^{-2} + c_t}} \\
 &= \frac{\kappa G}{B_* \left[\ln \left(\frac{G}{f'h} \right) + \frac{\kappa}{\sqrt{\kappa^2 \left(\ln \left(\frac{h}{z_0} \right) \right)^{-2} + c_t}} \right] \sqrt{\kappa^2 \left(\ln \left(\frac{h}{z_0} \right) \right)^{-2} + c_t}} \\
 &= \frac{\kappa G}{B_* \left[\ln \left(\frac{G}{f'h} \right) \sqrt{\kappa^2 \left(\ln \left(\frac{h}{z_0} \right) \right)^{-2} + c_t + \kappa} \right]}
 \end{aligned} \tag{A37}$$

with $A_* \equiv 2.0$ and $B_* \equiv 0.85$. For $c_t = 0$, we observe that z_{wf} equals z_0 using Equation (A36), and thereby that Equation (A33) degenerates to the conventional Geostrophic Drag Law associated with surface roughness z_0 .

Compared to the description of the infinite wind farm boundary layer in [9], we note that the introduction of B_* and the recalibration of A_* essentially represents a scaling of the infinite wind farm wind speed, U_h , and therefore does not compromise the basis for the coordinate transformation between the wind farm wind speed and the ambient wind speed used to derive the closed-form integral expressions. This requires the gradient of the mean wind speed of the fully developed wind farm boundary layer, U_h , with respect to the undisturbed ambient mean wind speed, $U_{h,0}$, to be positive in the above rated wind speed regime, which is proven in ([9], Appendix B).

From (A37), we see that the definition of the auxiliary variable, γ , is invariant with respect to the redefinition of the approximate Geostrophic Drag Law. Using Equations (A37) and (A38), it is further seen that the definitions of, respectively, ϵ_1 and $\epsilon_2(U_f)$ in Equations (12) and (23) are invariant with respect to the redefinition of the drag law, which in turn guaranties that $dU_{h,0}/dU > 0$ as alternatively derived above. Hence, the effect of the reformulated Geostrophic Drag Law only enters into force in the (iterative) determination of the geostrophic wind speed, G .

Equation (A37) expresses the farm flow at the hub height in terms of the geostrophic wind speed. However, in most cases, only the *undisturbed* mean wind speed $U_{h,0}$ at a given height h is known for a specific site. To overcome this, we can determine the associated geostrophic wind speed simply by setting $c_t = 0$ and solving Equation (A37) for G . Thus,

$$U_{h,0} = \frac{\kappa G}{B_* \left[\ln \left(\frac{G}{f'h} \right) \sqrt{\kappa^2 \left(\ln \left(\frac{h}{z_0} \right) \right)^{-2} + \kappa} \right]} \tag{A38}$$

This is a transcendental equation, which cannot be transformed into an algebraic equation and therefore has to be solved iteratively; e.g., using the Newton–Raphson method.

Appendix E

In this appendix, the distributions of turbulence standard deviations, conditioned on the mean wind speed, are quantified by fitting the measured data to logarithmic Gaussian distributions, which have been proven to be adequate for this purpose [41].

Offshore sites are far more homogeneous than onshore sites, as the majority of offshore sites (i.e., excluding sites populated with floating wind turbines) are likely to be associated with shallow water areas. This in turn opens up a simple and relative general description of the variability of an offshore turbulence intensity as a function of the mean wind speed. The present section presents such a model. Based on a huge amount of 10 min shallow

water offshore wind speed statistics, originating from Vindeby (5015 recordings measured at level 37.5 m) and Gedser (22,419 recordings measured at level 30.0 m) full-scale recording campaigns, the variability of the standard deviation of the horizontal wind speed is quantified in terms of a set of probabilistic models conditioned on the mean wind speed.

The conditional cumulative log-normal distribution is expressed as

$$P(x \leq \sigma_T) \equiv F_\sigma(x|U_T) = \Phi\left(\frac{\ln x - a_1(U_T)}{a_2(U_T)}\right) \quad (\text{A39})$$

where $P(\star)$ denotes probability, $\Phi(\star)$ is the cumulative Gaussian distribution function, U_T and σ_T are, respectively, the mean wind speed and the standard deviation of the wind speed evaluated over time span T . The two distribution parameters $a_1(U_T)$ and $a_2(U_T)$ depend, as indicated, on U_T . Note that the realization variable, x , in Equation (A39) is restricted to positive values, which is consistent with a basic characteristic of standard deviation measures. From Equation (A39), $a_1(U_T)$ and $a_2(U_T)$ are seen to represent, respectively, the mean and standard deviation of $\ln x$.

For a log-normal distribution, we have the following expressions for the mean, μ_σ , and the mode, η_σ , of the wind speed standard deviation σ :

$$\begin{aligned} \mu_\sigma &= \exp\left(a_1(U_T) + \frac{a_2(U_T)^2}{2}\right) \\ \eta_\sigma &= \exp\left(a_1(U_T) - a_2(U_T)^2\right) \end{aligned} \quad (\text{A40})$$

Based on the data analysis in [42], two sets of *estimated* distribution parameters, $\hat{a}_1(U_T)$ and $\hat{a}_2(U_T)$, were obtained for, respectively, the Vindeby and Gedser offshore wind speed data using the normal scores plot method [43]. The following fitting expressions were obtained with lower indices ‘V’ and ‘R’ denoting fits related to Vindeby and Gedser data, respectively,

$$\begin{aligned} \hat{a}_{1,V}(U_{10}) &= -0.002657 U_{10}^2 + 0.17916 U_{10} - 1.8744 \\ \hat{a}_{2,V}(U_{10}) &= 29.628 \exp(-0.10042 U_{10} - 3.9311) \end{aligned} \quad (\text{A41})$$

and

$$\begin{aligned} \hat{a}_{1,R}(U_{10}) &= -0.002380 U_{10}^2 + 0.16283 U_{10} - 1.7913 \\ \hat{a}_{2,R}(U_{10}) &= 35.162 \exp(-0.0727 U_{10} - 4.005) \end{aligned} \quad (\text{A42})$$

where U_{10} denotes a 10-min mean wind speed.

A *characteristic site TI* can be defined in various ways. Based on the distribution of wind speed standard deviations, obvious choices are either the most likely turbulence intensity, η_σ , or the mean turbulence intensity, μ_σ . Referring to Equation (A40), *estimated* values of these characteristics, as based on the referred data analysis in [5], are given by

$$\begin{aligned} \hat{\mu}_\sigma &= \exp\left(\hat{a}_1(U_T) + \frac{\hat{a}_2(U_T)^2}{2}\right) \\ \hat{\eta}_\sigma &= \exp\left(\hat{a}_1(U_T) - \hat{a}_2(U_T)^2\right) \end{aligned} \quad (\text{A43})$$

where a circumflex indicates an estimate.

Inspection of Equations (A41) and (A42) indicates, as expected, that the wind climates at the two shallow water offshore sites are quite homogeneous, although differences exists mainly in the standard deviation of the log-normal distribution of the wind speed standard deviation.

To overcome the moderate differences in the estimated distribution parameters from the two sites, we define synthesized characteristic estimated parameters based on the weighted means of the expressions for $\hat{\alpha}_1(U_{10})$ and $\hat{\alpha}_2(U_{10})$, respectively. The weighting factors are selected as the relative number of the total number of available data series associated with each site; i.e., 5015/27,434 for the Vindeby estimates and 22,419/27,434 for the Gedser estimates. In doing that, we assume neglectable variability in TI values from the moderate difference in height reference for the two sets of data; i.e., 37.5 m and 30.0 m, respectively. The following results are obtained:

$$\begin{aligned}\hat{\alpha}_1(U_{10}) &= -0.00243 U_{10}^2 + 0.16582 U_{10} - 1.80649 \\ \hat{\alpha}_2(U_{10}) &= 34.15037 \exp(-0.07777 U_{10} - 3.99149)\end{aligned}\quad (\text{A44})$$

The resulting values for the characteristic offshore turbulence intensity as a function of the mean wind speed are illustrated in Figure A3 based on both the distribution mean and distribution mode values.

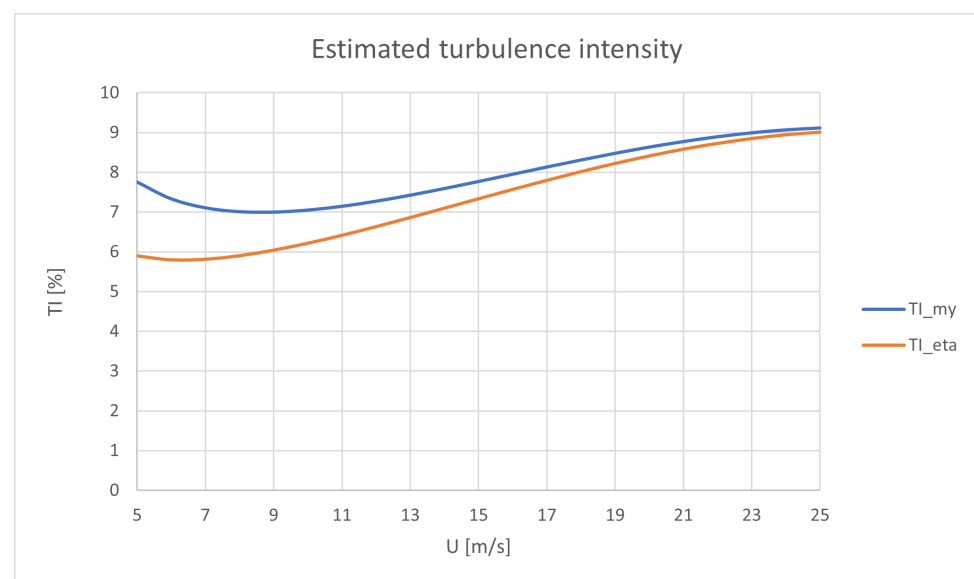


Figure A3. Estimated TI based on distribution mean ($\hat{\mu}_\sigma$) and distribution mode ($\hat{\eta}_\sigma$) values, respectively.

The turbulence standard deviation is, as mentioned, driven by surface friction and thermal convection. Thermal convection is dictated by the surface heat flux, which is independent of the wind speed. For offshore sites, this is contrary to the surface friction, which increase with increasing wind speed. Therefore the balance between convection and friction turbulence contributions depends on the wind speed. Convection is most important for lower wind speeds. For increasing wind speeds, the combined effect of convection and friction tends to approach the situation for neutral atmospheric conditions. For the present purpose, *mean* TI is considered the most representative value to cover the distribution as such, whereas the most likely event is more appropriate for a single event as an extreme.

Appendix F

The six wind farms introduced in Section 4.1 are quite different when it comes to farm size, WTs, topology, and location in Danish waters—see Figure 7. The main characteristics of each of these wind farms are given below.

Appendix F.1. Rødsand 1 Wind Farm

The Rødsand 1 wind farm is located 13 km south of Nysted in a shallow water area with water depths ranging between 6 m and 10 m. The farm consists of 72 Siemens

SWT 2.3-82 wind turbines. This wind farm was commissioned in 2003, and except for a 4.5 month period in 2007, it has been fully operational ever since.

Appendix F.2. Rødsand 2 Wind Farm

Rødsand 2 is located 3 km west of Rødsand 1. This wind farm consists of 90 Siemens SWT 2.3-93 wind turbines. The layout is a regular grid along 5 curve linear lines. The wind farm was commissioned in 2010 and has been fully operational ever since.

Appendix F.3. Horns Rev 1 Wind Farm

Horns Rev 1 is the first offshore wind farm erected in the North Sea. It is located 14–20 km west of Jutland at 6–14 m water depth. The farm consists of 80 Vestas V80-2.0MW turbines. The topology displays a regular layout that forms a nearly quadratic shape. The wind farm was put into operation in December 2002.

Appendix F.4. Horns Rev 2 Wind Farm

Horns Rev 2 is located on a shallow water area approximately 20 km NW of Horns Rev 1 with water depths ranging between 9 m and 17 m. The topology of Horns Rev 2 displays a curved shape with the turbines located at concentric arcs. The wind farm is populated by 91 Siemens SWT 2.3-93 wind turbines, and it was commissioned in September 2009.

Appendix F.5. Horns Rev 3 Wind Farm

Horns Rev 3 is located in the North Sea NE of Horns Rev 2 and 25–40 km west of Jutland at 11–19 m water depth. It is the newest of the three Horns Rev wind farms and equipped with 49 MHI Vestas V164-8.0 wind turbines. The farm layout appears less systematic compared to the other six wind farms. The scattered layout is the result of an optimization of the layout. This wind farm was decommissioned in August 2019.

Appendix F.6. Anholt Wind Farm

The Anholt wind farm is located at a shallow water area with a water depth of around 14 m in Kattegat between Djursland and Anholt. It has a capacity of 400 MW distributed on 111 Siemens 3.6 MW WTs. The wind farm has a fan-like topology and covers an area of 20 km in length and 5 km in width. This wind farm was fully operational in 2013.

References

1. Ditlevsen, P.; Ditlevsen, S. Warning of a forthcoming collapse of the Atlantic meridional overturning circulation. *Nat. Commun.* **2023**, *14*, 4254. <https://doi.org/10.1038/s41467-023-39810-w>.
2. International Renewable Energy Agency. *World Energy Transitions Outlook 2023: 1.5 °C Pathway*; International Renewable Energy Agency: Abu Dhabi, United Arab Emirates, 2023. Available online: www.irena.org (accessed on 29 June 2026).
3. IEA. *Integrating Solar and Wind*; IEA, Paris, France, 2024.
4. Global Wind Energy Council (GWEC). *Global Offshore Wind Report 2022*; Brussels: GWEC, 2022. Published 29 June 2022. Available online: <https://gwec.net/> (accessed on 29 June 2026).
5. Larsen, G.C.; Réthoré, P.E. TOPFARM—A Tool for Wind Farm Optimization. *Energy Procedia* **2013**, *35*, 317–324. <https://doi.org/10.1016/j.egypro.2013.07.184>.
6. Liew, J.; Göçmen, T.; Lio, A.; Larsen, G. Extending the dynamic wake meandering model in HAWC2Farm: A comparison with field measurements at the Lillgrund wind farm. *Wind Energy Sci.* **2023**, *8*, 1387–1402. <https://doi.org/10.5194/wes-8-1387-2023>.
7. Steiner, J.; Hodgson, E.L.; van der Laan, M.P.; Alcayaga, L.; Pedersen, M.; Andersen, S.J.; Larsen, G.; Réthoré, P.E. A multi-fidelity model benchmark for wake steering of a large turbine in a neutral ABL. *Wind Energy Sci. Discuss.* **2026**, *11*, 1679–1703. <https://doi.org/10.5194/wes-2025-200>.
8. Shaler, K.; Jonkman, J. FAST.Farm Development and Validation of Structural Load Prediction Against Large Eddy Simulations. *Wind Energy* **2020**, *24*, 428–449. <https://doi.org/10.1002/we.2581>.

9. Sørensen, J.N.; Larsen, G.C. A Minimalistic Prediction Model to Determine Energy Production and Costs of Offshore Wind Farms. *Energies* **2021**, *14*, 448. <https://doi.org/10.3390/en14020448>.
10. Templin, R.J. *An Estimate of the Interaction of Windmills in Widespread Arrays*; Technical Report, N.A.E. Report LTR-LA-171; National Research Council of Canada: Ottawa, ON, Canada, 1974.
11. Newman, B. The spacing of wind turbines in large arrays. *Energy Convers.* **1977**, *16*, 169–171. [https://doi.org/10.1016/0013-7480\(77\)90024-9](https://doi.org/10.1016/0013-7480(77)90024-9).
12. Frandsen, S. On the wind speed reduction in the center of large clusters of wind turbines. *J. Wind Eng. Ind. Aerodyn.* **1992**, *39*, 251–265. [https://doi.org/10.1016/0167-6105\(92\)90551-K](https://doi.org/10.1016/0167-6105(92)90551-K).
13. Calaf, M.; Meneveau, C.; Meyers, J. Large eddy simulation study of fully developed wind-turbine array boundary layers. *Phys. Fluids* **2010**, *22*, 015110. <https://doi.org/10.1063/1.3291077>.
14. Simão Ferreira, C.; Larsen, G.C.; Sørensen, J.N. A theoretical upper limit for offshore wind energy extraction. *Cell Rep. Sustain.* **2026**, *3*, 100573. <https://doi.org/10.1016/j.crsus.2025.100573>.
15. Sørensen, J.; Larsen, G. Assessment of energy production and costs associated with a massive exploitation of wind power in the North Sea. *J. Phys. Conf. Ser.* **2023**, *2505*, 012013. <https://doi.org/10.1088/1742-6596/2505/1/012013>.
16. Sørensen, J.N.; Garcia, A.M.I.; Larsen, G.C.; Pedersen, M.M.; Fournely, D. Extension and Validation of Minimalistic Prediction Model to Determine the Energy Production of Offshore Wind Farms. *J. Phys. Conf. Ser.* **2024**, *2767*, 092022. <https://doi.org/10.1088/1742-6596/2767/9/092022>.
17. Ott, S.; Nielsen, M. *Developments of the Offshore Wind Turbine Wake Model Fuga*; Number 0046 in DTU Wind Energy E; DTU Wind Energy: Roskilde, Denmark, 2014.
18. Hahmann, A.N.; Sile, T.; Witha, B.; Davis, N.; Dörenkämper, M.; Ezber, Y.; García-Bustamante, E.; González-Rouco, J.F.; Navarro, J.; Olsen, B.T.; et al. *New European Wind Atlas: Mesoscale Atlas*; DTU Wind Energy: Roskilde, Denmark, 2021. <https://doi.org/10.11583/DTU.14414096.v1>.
19. Hwang, P.A. Ocean Surface Roughness from Satellite Observations and Spectrum Modeling of Wind Waves. *J. Phys. Oceanogr.* **2022**, *52*, 2143–2158. <https://doi.org/10.1175/JPO-D-22-0043.1>.
20. Abramowitz, M.; Stegun, I.A. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Ninth Dover printing edition, ed.; Dover: New York, NY, USA, 1964.
21. Panofsky, H.A.; Dutton, J.A. *Atmospheric Turbulence. Models and Methods for Engineering Applications*; Wiley: New York, NY, USA, 1984.
22. Papoulis, A. *Probability, Random Variables, and Stochastic Processes*; McGraw-Hill: Columbus, OH, USA, 1991.
23. Bossanyi, E.; Bleeg, J. How do wind farm blockage and axial induction control interact? *J. Phys. Conf. Ser.* **2024**, *2767*, 092027. <https://doi.org/10.1088/1742-6596/2767/9/092027>.
24. PyWake. *Technical Report*; Technical University of Denmark: Roskilde, Denmark, 2018. Available online: <https://gitlab.windenergy.dtu.dk/TOFFARM/PyWake> (accessed on 29 June 2026).
25. Aagaard Madsen, H.; Larsen, G.; Larsen, T.; Troldborg, N.; Mikkelsen, R. Calibration and Validation of the Dynamic Wake Meandering Model for Implementation in an Aeroelastic Code. *J. Sol. Energy Eng.* **2010**, *132*, 041014. <https://doi.org/10.1115/1.4002555>.
26. van der Laan, M.P.; Andersen, S.J.; Réthoré, P.E.; Baungaard, M.; Sørensen, J.N.; Troldborg, N. Faster wind farm AEP calculations with CFD using a generalized wind turbine model. *J. Phys. Conf. Ser.* **2022**, *2265*, 022030. <https://doi.org/10.1088/1742-6596/2265/2/022030>.
27. Sørensen, J.N.; Larsen, G.C.; Pedersen, M.M. Validation of revised minimalistic model for predicting energy production of offshore wind farms. *J. Phys. Conf. Ser.* **2026**, *3224*, 032011. <https://doi.org/10.1088/1742-6596/3224/3/032011>.
28. Jensen, N. *A Note on Wind Generator Interaction*; Number 2411 in Risø-M; Risø National Laboratory: Roskilde, Denmark, 1983.
29. Niayifar, A.; Porté-Agel, F. Analytical Modeling of Wind Farms: A New Approach for Power Prediction. *Energies* **2016**, *9*, 741. <https://doi.org/10.3390/en9090741>.
30. Crespo, A.; Hernández, J. Turbulence characteristics in wind-turbine wakes. *J. Wind Eng. Ind. Aerodyn.* **1996**, *61*, 71–85. [https://doi.org/10.1016/0167-6105\(95\)00033-X](https://doi.org/10.1016/0167-6105(95)00033-X).
31. Machefaux, E.; Larsen, G.; Troldborg, N.; Gaunaa, M.; Rettenmeier, A. Empirical modeling of single-wake advection and expansion using full-scale pulsed lidar-based measurements. *Wind Energy* **2015**, *18*, 2085–2103. <https://doi.org/10.1002/we.1805>.
32. Bastankhah, M.; Porté-Agel, F. A new analytical model for wind-turbine wakes. *Renew. Energy* **2014**, *70*, 116–123. <https://doi.org/10.1016/j.renene.2014.01.002>.
33. Ørsted. *TurbOPark—A Turbulence Optimized Park Model*; Technical Report; Ørsted A/S: Fredericia, Denmark, 2024.
34. Ott, S.; Berg, J.; Nielsen, M. *Linearised CFD Models for Wakes*; Number 1772(EN) in Denmark; Forskningscenter Risø. Risø-R, Danmarks Tekniske Universitet, Risø Nationallaboratoriet for Bæredygtig Energi: Roskilde, Denmark, 2011.

35. Montes García, A.; Fournély, D.P.A. Extension of a Minimalistic Combined Production/Cost Model for Floating Wind Farms. Master's Thesis, Universitat Politècnica de Catalunya, Escola Tècnica Superior d'Enginyeria Industrial de Barcelona, Departament d'Enginyeria Química: Barcelona, Spain, 2023.
36. Cabioch, K. Improvement of Wind-Resource/Cost Model for Offshore Wind Farms. MSc Dissertation, DTU Wind Energy: Roskilde, Denmark, 2022.
37. Mortensen, N. 46200 Planning and Development of Wind Farms: Wind Resource Assessment Using the WAsP Software; Number 70 (ed.2) in DTU Wind Energy E; DTU Wind Energy: Roskilde, Denmark, 2015.
38. Berg, J.; Kelly, M.; Mann, J.; Nielsen, M. DTU 46100: Introduction to Micrometeorology for Wind Energy; DTU Wind Energy: Roskilde, Denmark, 2022.
39. Troen, I.; Lundtang Petersen, E. *European Wind Atlas*; Risø National Laboratory: Roskilde, Denmark, 1989.
40. Emeis, S.; Frandsen, S.T. Reduction of horizontal wind speed in a boundary layer with obstacles. *Bound.-Layer Meteorol.* **1993**, *64*, 297–305.
41. Ronold, K.; Larsen, G. Reliability-based design of wind-turbine rotor blades against failure in ultimate loading. *Eng. Struct.* **1999**, *22*, 565–574.
42. Larsen, G. Offshore fatigue design turbulence. *Wind Energy* **2001**, *4*, 107–120. <https://doi.org/10.1002/we.49>.
43. Ridgman, W.J. Statistical Methods, 8th edn, by G. W. Snedecor & W. G. Cochran. xx + 503 pp. Ames: Iowa State University Press (1989). ISBN 0 8138 1561 6. *J. Agric. Sci.* **1990**, *115*, 153.

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