

## Article

# Analytical Calculation Method for Power Supply Reliability Indices in Distribution Networks Based on Extended Component Fault Modeling

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## Abstract

Existing reliability assessment methods for distribution networks insufficiently characterize the impacts of faults in key components other than branches, and they often suffer from high computational complexity and repeated searches when identifying fault-affected areas. To address these issues, this paper proposes an analytical method for calculating reliability indices in distribution networks based on extended component fault modeling. First, the fault modeling objects in distribution networks are extended to multiple types of key components, including branches, sectionalizing switches, ring main units, and distribution transformers. Second, a spatial correlation matrix is constructed to describe the topology of the distribution system, source-to-load power supply paths, and the positional relationships of switching components. On this basis, an extended component fault impact correlation matrix is derived to characterize the spatial correlations between different types of component faults and the affected load nodes. Finally, through a single algebraic operation between the extended component fault-impact correlation matrix and the reliability parameter vector, explicit analytical expressions of both load-level and system-level reliability indices are obtained, thereby avoiding the repeated iterative searches required in the fault enumeration process of existing reliability assessment methods. Case study results show that the proposed method improves the evaluation efficiency by one order of magnitude while maintaining result accuracy. It can also explicitly quantify the impacts of faults in branches, sectionalizing switches, ring main units, and distribution transformers on system reliability, thereby more accurately reflecting the actual level of secure and reliable operation of the distribution network. This method can provide theoretical support and a methodological basis for reliability assessment and planning of complex distribution systems.



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## 1. Introduction

As a critical link between the power system and end users, the distribution network directly affects the continuity of electricity consumption and the quality of urban power supply [1]. With the increasing complexity of distribution network structures, key components such as switching devices, distribution transformers, and ring main units (RMUs) play an increasingly important role in fault isolation, load transfer, and service restoration [2]. Their

operating states have therefore become major factors affecting system reliability. Existing reliability assessment methods generally treat branch faults as the primary fault source, while the analytical modeling of fault impacts from the aforementioned critical non-branch components remains insufficient. As a result, these methods have difficulty fully capturing the effects of multiple types of equipment faults on the outage states of load nodes, which may cause the calculated reliability indices to deviate from actual operating conditions [3]. Therefore, it is necessary to develop an explicit analytical calculation method for reliability indices that accounts for the impacts of faults in multiple types of key components, thereby improving the accuracy of distribution network reliability assessment.

A variety of methods have been developed for evaluating reliability indices of distribution systems, and the related research has become relatively mature. These methods can be broadly classified into simulation-based methods [4–6] and analytical methods [7,8]. Simulation-based methods simulate stochastic component fault events by randomly sampling the fault states of system equipment and subsequently assess their impacts on service interruptions at both the system and customer levels. For example, Reference [9] adopts a non-sequential Monte Carlo simulation method to sample the states of distribution system components and statistically derive the interruption frequency and interruption duration of load points under different fault states. Reference [10] further accounts for the chronological characteristics of component failures, repairs, and operating states, and calculates system reliability indices based on sequential Monte Carlo simulation. Reference [11] incorporates enhanced sampling strategies, such as Latin hypercube sampling and variance reduction, into conventional Monte Carlo simulation to reduce the number of random samples and improve the computational efficiency of reliability assessment.

In the modeling process, the above methods usually treat branch faults as the sole fault source and obtain reliability indices by statistically analyzing outage impacts under different branch-fault scenarios. However, their results mainly rely on sample statistics, leading to a trade-off between computational accuracy and computational time. Moreover, it is difficult for these methods to establish explicit analytical relationships among component fault types, fault impact scopes, and load outage states. When it is necessary to further distinguish the impact mechanisms of different types of component faults on load nodes, simulation-based methods remain limited in interpretability and modeling capability [12].

Analytical methods can directly establish quantitative relationships between component fault parameters and reliability indices and thus constitute an important category of methods for distribution network reliability assessment. Their core idea is to enumerate all component fault events, analyze the impact of each fault on loads, form a set of system fault modes, and finally obtain the reliability indices of load nodes and the overall system. For example, Reference [13] proposed a network-equivalent method, in which branch feeders are represented as series components of the main feeder, thereby simplifying the reliability analysis of the system main feeder. References [14,15] mapped the impacts of faults in components outside the minimum path onto the minimum path, searched for the minimum paths between sources and loads in the system, and finally traversed the remaining nodes to obtain reliability indices. Reference [16] partitioned the distribution network according to the switches at both ends of each line and analyzed the impacts of faults in different line sections on load nodes. Reference [17] derived a fault correlation matrix for each standard reliability calculation unit with  $M$  sections and  $N$  tie switches, thereby enabling analytical calculation of the reliability indices of a single feeder. On this basis, Reference [18] constructed a network-wide spatial correlation matrix and obtained load-level and system-level reliability indices for passive distribution systems based on matrix algebraic operations.

Based on the above analysis, existing analytical reliability assessment methods mostly focus on single-line faults as the analysis object, while insufficiently considering the impacts of faults in key components such as sectionalizing switches, RMUs, and distribution transformers. Therefore, a unified analytical modeling framework for multiple types of component faults has not yet been established. Neglecting the impacts of such component faults in reliability assessment may cause the calculated reliability indices to deviate from actual operating conditions. Moreover, when the distribution system is large in scale, with numerous and diverse devices, the computational efficiency of the aforementioned methods decreases significantly [19].

To address these issues, this paper proposes an analytical calculation method for power supply reliability indices in distribution networks based on extended component fault modeling. The main contributions are summarized as follows:

(1) An extended component fault modeling framework covering branches, switches, RMUs, and distribution transformers is developed, which overcomes the limitation of traditional reliability assessment methods that consider only branch faults and thus more accurately reflects the actual level of secure and reliable operation of distribution networks.

(2) Based on the existing branch fault–impact correlation matrix, the scope of fault objects is further extended to construct a network-wide extended component fault–impact correlation matrix, which explicitly characterizes the spatial correlation relationships between different types of component faults and load nodes, thereby providing a matrix-model basis for the subsequent explicit calculation of reliability indices.

(3) Based on a single algebraic calculation between the extended component fault–impact correlation matrix and the reliability parameter vector, explicit analytical expressions of network-wide load-level and system-level reliability indices are derived, avoiding repeated network searches during the fault enumeration process and substantially improving computational efficiency while maintaining result accuracy.

The remainder of this paper is organized as follows. Section 2 establishes four types of network-wide spatial correlation matrices, providing the matrix-based modeling foundation for subsequent calculations. Section 3 constructs the extended component fault impact correlation matrix to characterize the impacts of multiple types of component faults on the supply states of load points. Section 4 proposes an explicit analytical calculation method for distribution network reliability indices based on the extended component fault impact correlation matrix. Section 5 verifies the correctness and effectiveness of the proposed method using a standard test system and a typical double-loop distribution system. Finally, conclusions and future work are presented.

## 2. Extended Component Fault Objects and Construction of Spatial Correlation Matrices in Distribution Networks

Distribution networks are characterized by a large number of sectionalizing and tie switches as well as complex reconfigurable supply paths. Therefore, matrix-based modeling for reliability assessment can effectively reduce repeated topology searches and is well suited to distribution networks with numerous nodes, wide service areas, and complex reconfigurable paths.

Accordingly, this paper first defines an extended component fault object set for distribution networks and then constructs spatial correlation matrices to describe the topological relationships among nodes, branches, switching devices, and load points. These matrices provide the modeling basis for subsequent identification of fault impact types and explicit analytical calculation of reliability indices.

### 2.1. Definition of the Extended Component Fault Object Set

Let the topology of a distribution network be represented as:

$$G = (\Omega_N, \Omega_B) \quad (1)$$

where  $\Omega_N$  denotes the set of nodes, and  $\Omega_B$  denotes the set of branches.

To overcome the limitation of conventional reliability assessment methods that consider only branch faults, this paper extends the fault sources in distribution networks from branch components to an extended component set. The extended component fault source set is defined as:

$$\Omega_F = \Omega_B \cup \Omega_{SS} \cup \Omega_{RMU} \cup \Omega_T \quad (2)$$

where  $\Omega_{SS}$ ,  $\Omega_{RMU}$ , and  $\Omega_T$  denote the sets of sectionalizing switches, RMUs, and distribution transformers, respectively. The corresponding numbers of components are denoted by  $N_{SS}$ ,  $N_{RMU}$ , and  $N_T$ .

### 2.2. Construction of Spatial Correlation Matrices for Distribution Networks

This section establishes the network-wide node branch correlation matrix (NBIM), power supply path matrix (PSPM), sectionalizing switch matrix (SSM), and tie switch matrix (TSM) to characterize the spatial correlations among nodes, branches, and switching devices in the distribution system, thereby providing a matrix-based modeling foundation for the explicit analytical calculation of reliability indices. The elements of NBIM, PSPM, SSM, and TSM are defined in Equations (3)–(6), respectively.

$$NBIM_{i,j} = \begin{cases} 1, & \text{node } i \text{ is the sending - end node of branch } j \\ -1, & \text{node } i \text{ is the receiving - end node of branch } j \\ 0, & \text{node } i \text{ not connected to branch } j \end{cases} \quad (3)$$

$$PSPM_{i,j} = \begin{cases} 1, & \text{branch } i \text{ lies on the power supply path of node } j \\ 0, & \text{branch } i \text{ does not lie on the power supply path of node } j \end{cases} \quad (4)$$

$$SSM_{i,j} = \begin{cases} 1, & \text{sectionalizing switch exists between branch } i \text{ and node } j \\ 0, & \text{no sectionalizing switch exists between branch } i \text{ and node } j \end{cases} \quad (5)$$

$$TSM_{i,j} = \begin{cases} 1, & \text{tie switch exists downstream of branch } j \\ 0, & \text{no tie switch exists downstream of branch } j \end{cases} \quad (6)$$

## 3. Construction of the Extended Component Fault Impact Correlation Matrix

The indices commonly used for power supply reliability assessment of distribution networks include system average interruption frequency index (SAIFI), system average interruption duration index (SAIDI), and energy not supplied (ENS). To ensure the computational tractability of the explicit calculation model for reliability indices, the following assumptions are made in this paper with reference to [8]:

- (1) This paper mainly considers single permanent faults of components in distribution networks, including branches, sectionalizing switches, RMUs, and distribution transformers. Transient faults and reclosing processes are beyond the scope of this study.
- (2) Fault events of different components are assumed to be mutually independent. Considering that the annual failure rates of distribution equipment are generally

low, simultaneous faults of multiple components are not considered in this paper, nor are multiple cascading failures caused by factors such as extreme weather or external damage.

- (3) It is assumed that protective devices, sectionalizing switches, and tie switches can operate correctly. After a fault occurs, the operating logic of circuit breakers and sectionalizing switches is as follows: the circuit breaker at the outlet of the feeder where the fault is located trips first, disconnecting the faulted feeder section and causing all load nodes on the feeder to be interrupted. Then, the sectionalizing switches at both ends of the faulted branch are opened to isolate the fault, and the circuit breakers in non-faulted areas are reclosed to restore power supply to loads in these areas. Finally, after the fault is repaired, the sectionalizing switches on both sides of the faulted branch are closed to restore normal power supply to all loads in the system.

Based on the above assumptions, this section constructs an extended component fault–impact correlation matrix to characterize the spatial correlation relationships between different types of component faults and the outage states of load points.

### 3.1. Classification of Fault Impact Types

To uniformly describe the impacts of different component faults on the supply states of load points, this paper classifies extended component fault impacts into four types, namely types a, b, c, and d.

A type-a fault impact indicates that a component fault causes a load point to lose power supply, and the supply cannot be restored within a short time through fault isolation or load transfer via tie switches. The affected load can be restored only after the faulted component is repaired. The interruption duration associated with this type of fault impact is mainly determined by the repair time of the faulted component.

A type-b fault impact indicates that a component fault causes a short-term interruption at a load point; however, after fault isolation is completed, the load point can be restored through its original primary supply path.

A type-c fault impact indicates that a component fault causes a load point to lose its primary supply path. After fault isolation is completed, the load point must be transferred to an alternative supply source through a tie switch to restore power supply.

A type-d fault impact indicates that a component fault does not cause an interruption at the corresponding load point; that is, the fault event has no impact on the supply state of that load point.

### 3.2. Definition of the Extended Component Fault Impact Correlation Matrix

#### 3.2.1. Branch Fault Impact Correlation Matrix

For branch faults, their impact types on load points are mainly determined by three factors: first, whether the faulted branch lies on the primary supply path of the load point; second, whether fault isolation can be achieved by sectionalizing switches after the fault; and third, whether load transfer through tie switches is available after fault isolation.

When faulted branch  $i$  is located upstream of load point  $j$ , that is, when branch  $i$  lies on the supply path of load point  $j$ , the faulted branch and the load point belong to the same feeder. In this case, the fault impact type is either type-a or type-c. If there is no downstream tie connection capable of supporting load transfer, the fault impact is classified as type-a. Otherwise, if a downstream tie connection is available for load transfer, the fault impact is classified as type-c.

When faulted branch  $i$  does not lie on the supply path of load point  $j$ , branch  $i$  and load point  $j$  may belong to the same feeder or to different feeders. In this case, the fault impact

type is either type-b or type-d. If a sectionalizing switch exists between faulted branch  $i$  and load point  $j$ , the fault impact is classified as type-b. If no sectionalizing switch exists between them, branch  $i$  and load point  $j$  are regarded as belonging to different feeders, and the fault impact is classified as type-d.

Based on the above analysis, the impact type of a branch fault can be jointly identified using the PSPM, SSM, and TSM. Let variables  $e$ ,  $f$ , and  $g$  denote the values of the corresponding elements in the PSPM, SSM, and TSM, respectively. When analyzing the impact of a fault in branch  $i$  on load point  $j$ , the corresponding fault impact type can be determined according to the values of  $\text{PSPM}_{i,j} = e$ ,  $\text{SSM}_{i,j} = f$ , and  $\text{TSM}_{i,j} = g$ .

To correspond to the four fault impact types, namely type-a, type-b, type-c, and type-d impacts, four branch fault impact matrices are defined as  $\text{B\_FICM}^a$ ,  $\text{B\_FICM}^b$ ,  $\text{B\_FICM}^c$ , and  $\text{B\_FICM}^d$ . The unified expression of the branch fault impact correlation matrices is given in Equation (7).

$$\text{B\_FICM}^k \in \{0, 1\}^{N_B \times N_L}, k \in \{a, b, c, d\} \quad (7)$$

According to the logical relationships among PSPM, SSM, and TSM, the four types of branch fault impacts are identified by Equations (8)–(11).

$$\text{B\_FICM}^a = \text{PSPM} \circ \text{SSM} \circ (\text{ONE} - \text{TSM}) \quad (8)$$

$$\text{B\_FICM}^b = ((\text{ONE} - \text{PSPM}) \circ \text{SSM}) \circ (\text{ONE} - \text{TSM}) \cup ((\text{ONE} - \text{PSPM}) \circ \text{SSM} \circ \text{TSM}) \quad (9)$$

$$\text{B\_FICM}^c = \text{PSPM} \circ \text{SSM} \circ \text{TSM} \quad (10)$$

$$\text{B\_FICM}^d = ((\text{ONE} - \text{PSPM}) \circ (\text{ONE} - \text{SSM})) \circ (\text{ONE} - \text{TSM}) \cup ((\text{ONE} - \text{PSPM}) \circ (\text{ONE} - \text{SSM}) \circ \text{TSM}) \quad (11)$$

where  $\circ$  denotes the Hadamard product, i.e., element-wise multiplication between matrices of the same dimension, and ONE denotes an all-ones matrix.

### 3.2.2. Sectionalizing Switch Fault Impact Correlation Matrix

In the distribution network model considered in this paper, sectionalizing switches are installed at both ends of each branch to facilitate the isolation of faulted branches and the restoration of supply to unfaulted areas. Therefore, when a fault occurs in the sectionalizing switch at either end of a branch, its impact range should not be identified solely according to the position of the individual switch. Instead, it should be determined by the position of the corresponding branch in the distribution network topology and its impact relationship with load points.

Based on the above analysis, this paper aggregates the faults of sectionalizing switches at both ends of a branch and characterizes them at the branch level. Let the sectionalizing switches at the two ends of branch  $i$  be denoted as  $\text{SS}_i^1$  and  $\text{SS}_i^2$ , respectively. When a fault occurs in either  $\text{SS}_i^1$  or  $\text{SS}_i^2$ , it is regarded as a sectionalizing switch fault event associated with branch  $i$ . The impact type of this fault event at the load-point level is consistent with that of a fault in branch  $i$ .

Therefore, the impact type of the sectionalizing switch fault associated with branch  $i$  on load point  $j$  can be directly determined by the impact type of the fault in branch  $i$  on load point  $j$ . Accordingly, to correspond to the four fault impact types, namely type-a, type-b, type-c, and type-d impacts, four sectionalizing switch fault impact correlation matrices are defined as  $\text{SS\_FICM}^a$ ,  $\text{SS\_FICM}^b$ ,  $\text{SS\_FICM}^c$ , and  $\text{SS\_FICM}^d$ , respectively.

The unified expression of the sectionalizing switch fault impact correlation matrices is given in Equation (12).

$$SS\_FICM^k \in \{0, 1\}^{N_{SS} \times N_L}, k \in \{a, b, c, d\} \quad (12)$$

For the sectionalizing switches at both ends of branch  $i$ , their failure rates are denoted by  $\lambda_{SS,i}^1$  and  $\lambda_{SS,i}^2$ , respectively, and their average repair times are denoted by  $r_{SS,i}^1$  and  $r_{SS,i}^2$ , respectively.

Under the assumption that faults in the sectionalizing switches at both ends of a branch are mutually independent and that their failure rates are sufficiently small, the equivalent failure rate of the sectionalizing switch fault event associated with branch  $i$  is calculated according to Equation (13).

$$\lambda_{SS,i} = \lambda_{SS,i}^1 + \lambda_{SS,i}^2 \quad (13)$$

The equivalent repair time of the sectionalizing switch fault event is further calculated by Equation (14).

$$r_{SS,i} = \frac{r_{SS,i}^1 \lambda_{SS,i}^1 + r_{SS,i}^2 \lambda_{SS,i}^2}{\lambda_{SS,i}^1 + \lambda_{SS,i}^2} \quad (14)$$

### 3.2.3. Construction of the Ring Main Unit Fault Impact Correlation Matrix

As important primary equipment in medium-voltage distribution networks, RMUs are used for feeder sectionalization, load connection, and ring-network power supply. Their operating states directly affect the reliability of the associated load points. When an RMU fault occurs, the load points connected to or controlled by the RMU are generally difficult to restore within a short time through conventional sectionalizing isolation or load transfer via tie switches. Without considering measures such as spare bays, bypass supply, or rapid equipment replacement, the affected loads typically can be restored only after the RMU is repaired. Therefore, the impact of an RMU fault on its associated load points is classified as a type-a fault impact in this paper.

To describe the correspondence between RMU faults and load points, the RMU fault impact correlation matrix is defined as  $RMU\_FICM^a$ . The unified expression of the RMU fault impact correlation matrix is given in Equation (15).

$$RMU\_FICM^a \in \{0, 1\}^{N_{RMU} \times N_L} \quad (15)$$

### 3.2.4. Construction of the Distribution Transformer Fault Impact Matrix

Unlike branch faults, distribution transformer faults do not involve sectionalizing isolation or load transfer along the supply path. Without considering measures such as standby power sources on the low-voltage side, rapid connection of mobile transformers, or load transfer through low-voltage interconnection, the load points supplied by a faulted distribution transformer can be restored only after the faulted equipment is repaired. Therefore, in this paper, the impact of a distribution transformer fault on its supplied load points is classified as a type-a fault impact.

To describe the supply correspondence between distribution transformers and load points, the distribution transformer fault impact correlation matrix is defined as  $T\_FICM^a$ . The unified expression of the distribution transformer fault impact correlation matrix is given in Equation (16).

$$T\_FICM^a \in \{0, 1\}^{N_T \times N_L} \quad (16)$$

#### 4. Analytical Calculation Model for Distribution Network Reliability Indices

After the fault impact correlation matrices of branches, sectionalizing switches, RMUs, and distribution transformers are constructed separately, they can be vertically concatenated according to the component order in the extended component fault source set  $\Omega_F$ , thereby obtaining the network-wide extended component fault impact correlation matrix.

The matrix corresponding to type-a fault impacts can be expressed as:

$$C\_FICM^a = \begin{bmatrix} B\_FICM^a \\ SS\_FICM^a \\ RMU\_FICM^a \\ T\_FICM^a \end{bmatrix} \quad (17)$$

The matrix corresponding to type-b fault impacts can be expressed as:

$$C\_FICM^b = \begin{bmatrix} B\_FICM^b \\ SS\_FICM^b \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

The matrix corresponding to type-c fault impacts can be expressed as:

$$C\_FICM^c = \begin{bmatrix} B\_FICM^c \\ SS\_FICM^c \\ 0 \\ 0 \end{bmatrix} \quad (19)$$

Let the row vector of extended component failure rates be defined as  $\lambda_F = [\lambda_B, \lambda_{SS}, \lambda_{RMU}, \lambda_T]$ , let the row vector of average repair times of extended components be defined as  $r_F = [r_B, r_{SS}, r_{RMU}, r_T]$ , let the row vector of load demands at load points be defined as  $P_L = [P_1, P_2, \dots, P_{NL}]$ , where  $t_{ss}$  denotes the operation time of sectionalizing switches, and  $t_{ts}$  denotes the operation time of tie switches. Based on the extended component fault impact correlation matrices and the reliability parameter vectors, the reliability indices of the distribution network can be explicitly calculated.

The load-level reliability indices, including interruption frequency, interruption duration, and energy not supplied, are calculated according to Equations (20)–(22).

$$\lambda^{LP} = \lambda \times (C\_FICM^a + C\_FICM^b + C\_FICM^c) \quad (20)$$

$$r^{LP} = r \circ \lambda \times C\_FICM^a + t_{ss} \cdot \lambda \times C\_FICM^b + t_{ts} \cdot \lambda \times C\_FICM^c \quad (21)$$

$$ens^{LP} = r^{LP} \circ P_L \quad (22)$$

Equation (20) is used to calculate the average annual interruption frequency of load nodes. Since type-a, type-b, and type-c failures all result in interruptions at load nodes, the interruption frequency of each load node can be obtained by summing the three failure–impact correlation matrices and multiplying the result by the component failure-rate vector  $\lambda$ .

Equation (21) is used to calculate the average annual interruption duration of load nodes. Different failure types correspond to different restoration mechanisms: type-a failures require component repair, so the interruption duration is related to the repair time  $r$ ; type-b failures can be restored after the sectionalizing switch operates, so the interruption duration is related to  $t_{ss}$ ; and type-c failures can be restored through load transfer via the

tie switch, so the interruption duration is related to  $t_{ts}$ . Therefore, the average annual interruption duration of each load node can be obtained by separately calculating and summing the interruption durations associated with the three failure types.

Equation (22) is used to calculate the energy not supplied at load nodes. It is given by the product of the average annual interruption duration  $r^{LP}$  of each load node and the corresponding load demand  $P_L$ .

By aggregating the reliability indices of individual load points, system-level reliability indices can be further obtained. In this paper, the system-level reliability indices SAIFI, SAIDI, and ENS are calculated according to Equations (23)–(25).

$$SAIFI = \lambda^{LP} \times \frac{n^T}{N_{cons}} \quad (23)$$

$$SAIDI = r^{LP} \times \frac{n^T}{N_{cons}} \quad (24)$$

$$ENS = r^{LP} \times (P_L)^T \quad (25)$$

where  $n^T$  represents a column vector in which the number of users at each load node is arranged in ascending order by index, and  $N_{cons}$  denotes the total number of users in the distribution network.

## 5. Case Study

To verify the correctness and effectiveness of the proposed method, this section conducts case studies from three aspects: accuracy validation, computational efficiency analysis, and application to a representative distribution system.

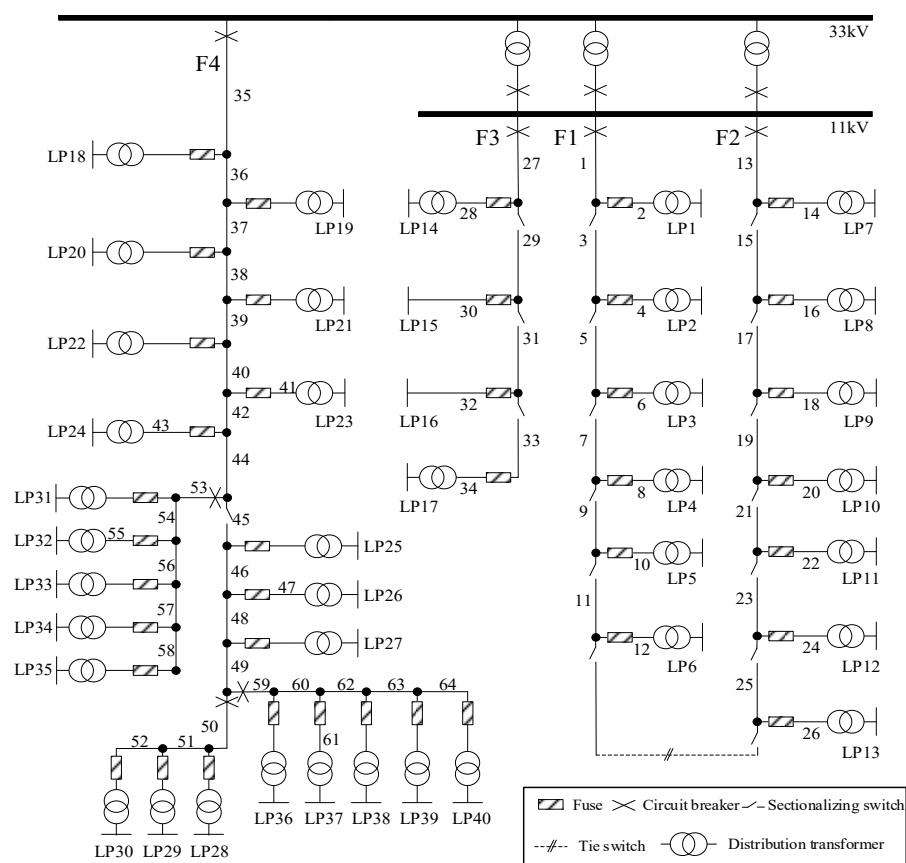
First, the IEEE RBTS Bus 6 benchmark system is selected, and the calculation results of the proposed method are compared with the benchmark results. Second, based on distribution systems of different scales, the computational time of the proposed method is compared with that of typical reliability assessment methods. Finally, a representative double-loop distribution system is used as the study object to analyze the impacts of faults in key components, such as sectionalizing switches, RMUs, and distribution transformers, on system reliability indices.

### 5.1. Algorithm Correctness Verification and Comparison of Computational Efficiency

The IEEE RBTS benchmark system is a widely used test system for evaluating distribution system reliability. The topology of the IEEE RBTS Bus 6 system is shown in Figure 1. The system contains 40 load points, 40 fuses, 38 distribution transformers, and 10 circuit breakers. The failure rate of distribution transformers is set to 0.015 failures/year, with a repair time of 200 h. The repair time for branch faults is set to 5 h, and the operating times of sectionalizing switches and tie switches are both set to 1 h. Component failure parameters are adopted from [7]. The calculation results are presented in Table 1.

The comparison results show that, except for minor differences attributable to the statistical precision reported in Reference [7], the reliability indices calculated by the proposed method are consistent with those reported in Reference [7]. This verifies the accuracy of the proposed reliability calculation method.

In this section, four distribution systems of different scales, namely 85-node, 137-node, 417-node, and 1080-node systems, are selected for reliability assessment. The computational efficiency of the proposed reliability assessment method is compared with that of three representative methods: the linear-programming-based assessment method [20], the Monte Carlo simulation method [5], and the network-topology-based analytical method [17].



**Figure 1.** Diagram of the IEEE RBTS Bus 6 system.

**Table 1.** Comparison of Reliability Indexes of the Load Nodes.

| Load Number | Reference [7]              |                      | This Paper                 |                      |
|-------------|----------------------------|----------------------|----------------------------|----------------------|
|             | $\lambda^{LP}$<br>(1/Year) | $r^{LP}$<br>(h/Year) | $\lambda^{LP}$<br>(1/Year) | $r^{LP}$<br>(h/Year) |
| 1           | 0.3303                     | 3.6662               | 0.3303                     | 3.6662               |
| 2           | 0.3433                     | 3.6923               | 0.3433                     | 3.6923               |
| 3           | 0.34                       | 3.715                | 0.34                       | 3.715                |
| 4           | 0.3303                     | 3.6662               | 0.3303                     | 3.6662               |
| 5           | 0.34                       | 3.676                | 0.34                       | 3.676                |
| 6           | 0.3303                     | 3.6793               | 0.3303                     | 3.6793               |
| 7           | 0.3693                     | 3.7052               | 0.3693                     | 3.7052               |
| 8           | 0.3725                     | 3.7605               | 0.3725                     | 3.7605               |
| 9           | 0.3725                     | 3.7215               | 0.3725                     | 3.7215               |
| 10          | 0.3595                     | 3.6565               | 0.3595                     | 3.6565               |
| 11          | 0.3693                     | 3.7572               | 0.3693                     | 3.7572               |
| 12          | 0.3595                     | 3.6955               | 0.3595                     | 3.6955               |
| 13          | 0.3693                     | 3.7052               | 0.3693                     | 3.7052               |
| 14          | 0.2425                     | 3.5785               | 0.2425                     | 3.5785               |
| 15          | 0.23725                    | 0.83525              | 0.2373                     | 0.8353               |
| 16          | 0.2405                     | 1.0075               | 0.2405                     | 1.0075               |
| 17          | 0.2425                     | 4.1375               | 0.2425                     | 4.1375               |
| 18          | 1.6725                     | 8.4015               | 1.6725                     | 8.4015               |
| 19          | 1.6725                     | 8.4015               | 1.6725                     | 8.4015               |
| 20          | 1.6725                     | 8.4015               | 1.6725                     | 8.4015               |
| 21          | 1.6725                     | 8.4015               | 1.6725                     | 8.4015               |
| 22          | 1.6725                     | 8.4015               | 1.6725                     | 8.4015               |
| 23          | 1.7115                     | 8.5965               | 1.7115                     | 8.5965               |

Table 1. Cont.

| Load Number | Reference [7]              |                      | This Paper                 |                      |
|-------------|----------------------------|----------------------|----------------------------|----------------------|
|             | $\lambda^{LP}$<br>(1/Year) | $r^{LP}$<br>(h/Year) | $\lambda^{LP}$<br>(1/Year) | $r^{LP}$<br>(h/Year) |
| 24          | 1.72125                    | 8.64525              | 1.7213                     | 8.6453               |
| 25          | 1.6725                     | 11.2875              | 1.6725                     | 11.2875              |
| 26          | 1.7115                     | 11.4825              | 1.7115                     | 11.4825              |
| 27          | 1.6725                     | 11.2875              | 1.6725                     | 11.2875              |
| 28          | 2.225                      | 14.05                | 2.225                      | 14.05                |
| 29          | 2.225                      | 14.05                | 2.225                      | 14.05                |
| 30          | 2.225                      | 14.05                | 2.225                      | 14.05                |
| 31          | 2.537                      | 12.724               | 2.537                      | 12.724               |
| 32          | 2.589                      | 12.984               | 2.589                      | 12.984               |
| 33          | 2.537                      | 12.724               | 2.537                      | 12.724               |
| 34          | 2.537                      | 12.724               | 2.537                      | 12.724               |
| 35          | 2.537                      | 12.724               | 2.537                      | 12.724               |
| 36          | 2.511                      | 15.48                | 2.511                      | 15.48                |
| 37          | 2.559                      | 15.723               | 2.559                      | 15.723               |
| 38          | 2.511                      | 15.48                | 2.511                      | 15.48                |
| 39          | 2.511                      | 15.48                | 2.511                      | 15.48                |
| 40          | 2.511                      | 15.48                | 2.511                      | 15.48                |

As shown in Table 2, the proposed method has the shortest computational time among the four distribution systems of different scales, demonstrating a clear advantage in computational efficiency. In the 85-node system, the computational time of the proposed method is 0.010 s, achieving a speedup of approximately 3.2 times over Reference [5], which is the second-fastest method at the same system scale. When the system scale increases to 1080 nodes, the proposed method requires only 0.591 s, whereas References [5,17] require 14.072 s and 8.862 s, respectively, further highlighting its advantage in computational efficiency. In addition, as the scale of the distribution system increases, the computational time of existing reliability assessment methods increases more significantly, whereas that of the proposed method grows more slowly, indicating better scalability to large-scale systems.

Table 2. Comparison of Computational Efficiency.

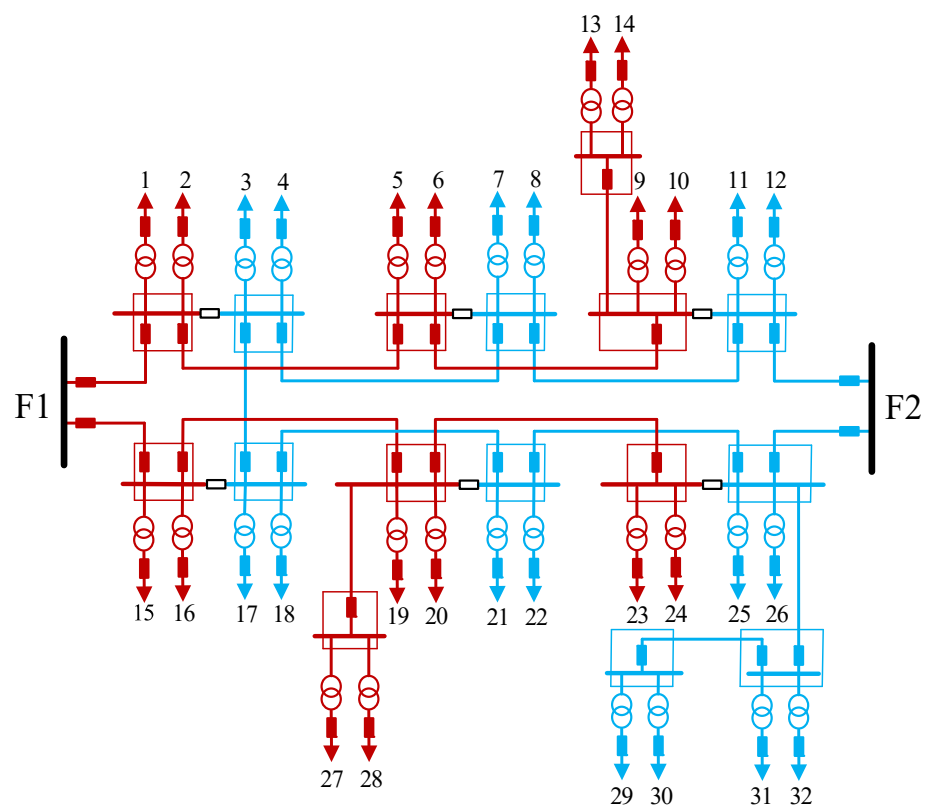
|                | 85-Node System | 137-Node System | 417-Node System | 1080-Node System |
|----------------|----------------|-----------------|-----------------|------------------|
| Reference [20] | 5.461          | 10.764          | 34.569          | 210.890          |
| Reference [5]  | 0.032          | 0.088           | 0.641           | 14.072           |
| Reference [17] | 0.061          | 0.145           | 1.003           | 8.862            |
| This paper     | 0.010          | 0.019           | 0.092           | 0.591            |

Reference [20] incorporates fault restoration strategies and operational constraints into a linear programming model and calculates reliability indices through optimization. However, as the system scale increases, the number of decision variables and constraints grows simultaneously, resulting in a significant increase in solution time. The simulation-based method proposed in Reference [5] faces an inherent trade-off between computational accuracy and computational efficiency. Reference [17] defines an M-section and N-tie standard reliability calculation unit for distribution systems, within which the impacts of branch faults on load points are analyzed. Although this method can reduce the fault analysis range, it is applicable only to a single radial feeder and still requires repeated calculations for complex multi-feeder distribution systems.

In contrast, the proposed method constructs a network-wide C\_FICM to explicitly characterize the spatial correlation between any component fault in the distribution system and its impact on all load points. Through a single algebraic operation between the C\_FICM and reliability parameter vectors, both load-level and system-level reliability indices can be obtained in an explicit analytical form. Benefiting from the inherent parallelism of matrix operations, the proposed method significantly improves computational efficiency while maintaining result accuracy. Moreover, its efficiency advantage becomes more pronounced as the system scale increases.

### 5.2. Reliability Evaluation

A double-loop distribution system is used as the test case for the analytical calculation of reliability indices. The system topology is shown in Figure 2, and the component reliability parameters are listed in Table 3.



**Figure 2.** Diagram of the double-loop distribution system.

**Table 3.** Reliability Parameters.

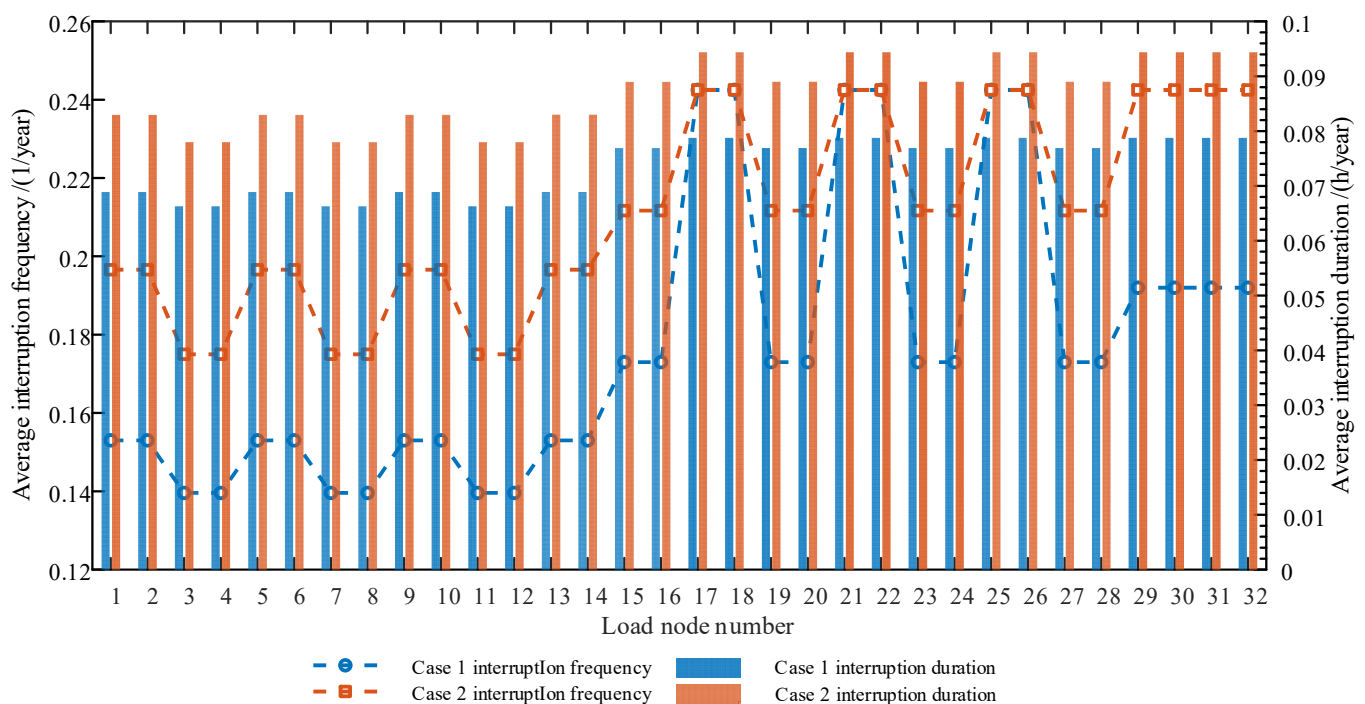
| No. | Parameter                    | Value                          |
|-----|------------------------------|--------------------------------|
| 1   | Branch failure rate          | 2.58 failures/(100 km·year)    |
| 2   | Branch repair time           | 12 h                           |
| 3   | Switch failure rate          | 0.12 failures/(100 units·year) |
| 4   | Switch repair time           | 6 h                            |
| 5   | Transformer failure rate     | 0.15 failures/(100 units·year) |
| 6   | Transformer repair time      | 6 h                            |
| 7   | Ring main units failure rate | 0.18 failures/(100 units·year) |
| 8   | Ring main units repair time  | 4 h                            |

To further analyze the influence of faults in key components other than branches on reliability assessment results, two calculation cases are designed for comparison. Case 1

considers only branch faults. Case 2 further accounts for faults in sectionalizing switches, RMUs, and distribution transformers on the basis of case 1. Based on the extended component fault impact correlation matrix constructed above, the system reliability indices under the two cases are calculated, and the results are listed in Table 4. The annual average interruption frequency and annual average interruption duration of each load point are shown in Figure 3.

**Table 4.** Reliability Indices for each case.

|        | SAIFI(1/Year) | SAIDI (h/Year) | ENS (kWh/Year) |
|--------|---------------|----------------|----------------|
| Case 1 | 0.1771        | 0.0735         | 11.7699        |
| Case 2 | 0.2106        | 0.0871         | 14.3356        |



**Figure 3.** Average interruption frequency and average interruption duration of load nodes.

As shown in Figure 3, within the same calculation case, the average annual interruption frequencies of load nodes on each feeder are consistent. However, the interruption frequencies in case 2 are generally higher than those in case 1. This is because case 2 further considers faults in sectionalizing switches, RMUs, and distribution transformers in addition to branch faults, thereby increasing the number of fault events that affect the supply states of load nodes. According to Equation (20), the interruption frequency of a load node is jointly determined by the failure rates of all components that lead to type-a, type-b, or type-c interruption impacts and their corresponding fault–impact relationships. Therefore, under the same case, feeders or load nodes with the same fault–impact range exhibit consistent interruption frequencies, whereas the inclusion of additional component fault types results in higher interruption frequencies at both load node and system levels.

As shown in Table 4, all system reliability indices obtained under case 2 are higher than those under case 1. When only branch faults are considered in case 1, the system SAIFI, SAIDI, and ENS are 0.1771 interruptions/year, 0.0735 h/year, and 11.7699 kWh/year, respectively. After further accounting for faults in sectionalizing switches, RMUs, and distribution transformers in case 2, these three indices increase to 0.2106 interruptions/year,

0.0871 h/year, and 14.3356 kWh/year, respectively. Compared with case 1, the SAIFI, SAIDI, and ENS of case 2 increase by approximately 18.92%, 18.50%, and 21.80%, respectively.

The main reason for the above differences is that faults in components such as sectionalizing switches, RMUs, and distribution transformers can also change the supply states of load points. Case 1 considers only branch faults, and its calculation results mainly reflect the impacts of line faults on load points after sectionalizing isolation, restoration through the original supply path, or load transfer via tie switches. However, in actual distribution network operation, sectionalizing switches, RMUs, and distribution transformers not only participate in fault isolation and supply-path switching but also play an important role in determining whether load points can maintain continuous power supply. In particular, when a ring main unit or distribution transformer fails, the affected loads are generally difficult to restore within a short time through conventional sectionalizing isolation or load transfer via tie switches. Instead, supply restoration often depends on the repair of the faulted equipment. Therefore, faults in these components increase the interruption duration of each load point and the corresponding energy not supplied.

In summary, case 1 underestimates the reliability indices because it neglects the impacts of faults in key components other than branches on load interruption states. In contrast, case 2 incorporates faults in branches, sectionalizing switches, RMUs, and distribution transformers into a unified calculation model, enabling an extended characterization of the impacts of multiple types of equipment faults on loads. Therefore, case 2 can more accurately reflect the actual reliability performance of distribution networks.

## 6. Conclusions

This paper develops a more refined analytical modeling framework for faults in conventional distribution equipment and proposes an analytical calculation method for distribution network power supply reliability indices based on extended component fault modeling. By constructing an extended component fault–impact correlation matrix, the proposed method characterizes the correlation relationships between faults in multiple types of components, including branches, sectionalizing switches, RMUs, and distribution transformers, and the outage states of load nodes. On this basis, explicit analytical calculation formulas for load-level and system-level reliability indices are derived, forming a distribution network reliability assessment method for multiple types of component faults. The main conclusions are as follows:

(1) The proposed method exhibits high computational efficiency. Case-study results for systems of different scales show that the proposed method requires shorter computational time in the 85-node, 137-node, 417-node, and 1080-node systems. In particular, in the 1080-node system, it achieves speedups of approximately 23.8 and 15.0 times over the Monte Carlo simulation method and the network-topology-based analytical method, respectively. As the system scale increases, its advantage in computational efficiency becomes more pronounced.

(2) Accounting for faults in multiple types of critical components can more accurately reflect the reliability level of distribution networks. After faults in multiple types of components, such as sectionalizing switches, RMUs, and distribution transformers, are considered, the system SAIFI, SAIDI, and ENS increase by approximately 18.92%, 18.50%, and 21.80%, respectively, compared with the case where only branch faults are considered. This indicates that ignoring faults in key components other than branches may lead to overly optimistic reliability assessment results.

(3) By incorporating components such as branches, sectionalizing switches, RMUs, and distribution transformers into the same fault–impact analysis framework, the contributions of different fault sources to the interruption frequency, interruption duration, and energy

not supplied of load nodes can be quantified. This provides a basis for weak-link identification, prioritization of equipment maintenance, and formulation of reliability improvement schemes in distribution networks.

It should be noted that the proposed method mainly performs analytical reliability calculation for independent single-component fault scenarios and does not further consider the impacts of multiple faults or fault correlations under extreme weather on reliability indices. Future work will extend the analytical reliability calculation method to complex fault scenarios.

In addition, the proposed method can be further extended to active distribution networks with distributed generation (DG) and battery energy storage systems (BESS). On the one hand, DG units and BESS can be added to the fault-source set as new fault objects, with corresponding rows added to the matrix to represent the impacts of faults in active devices on the supply states of load points. On the other hand, by considering the time-varying output of DG, the state of charge of BESS, and network connectivity, an active restoration capability matrix can be constructed to characterize the scope of islanded supply restoration provided by DG and BESS for de-energized loads after faults. In this way, the current static matrix framework can be extended into a time-varying reliability assessment framework applicable to active distribution networks.

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