

Article

Multi-Objective Optimization and Analysis of Two-Degree-of-Freedom Flux-Switching Permanent Magnet Machine

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Abstract

Two-degree-of-freedom flux-switching permanent magnet (2DoFFSPM) machines can directly drive loads to realize rotary, linear, and helical motions. They therefore provide a compact solution for multi-dimensional direct-drive systems. However, the double-salient stator–mover structure and end effects cause large torque and thrust ripples, reducing motion stability and positioning accuracy. This paper presents a multi-objective optimization of a 2DoFFSPM machine under helical operation. Average torque, torque ripple, average thrust, and thrust ripple are selected as optimization objectives. Four key structural parameters are selected as optimization variables. The Taguchi method is used to construct the orthogonal design. Three-dimensional finite-element analysis (3D FEA) is then performed under helical operation to evaluate the optimization objectives. The optimized design reduces torque ripple and thrust ripple by 47.3% and 48.38%, respectively, and increases average thrust by 8.1%, with only a 1.1% decrease in average torque. These results demonstrate that the proposed method achieves a balanced improvement in output capability and motion stability under helical operation.

Keywords: two-degree-of-freedom machine; helical motion; flux-switching; permanent magnet; Taguchi optimization; 3D finite-element analysis

1. Introduction

Compared with two-degree-of-freedom (2DoF) drive systems constructed using single-degree-of-freedom (SDoF) machines, 2DoF machines can directly drive loads to realize linear, rotary, and helical motions [1,2], thereby eliminating complex intermediate transmission mechanisms. As a result, they offer a compact structure, fast dynamic response, and excellent motion performance. Owing to these advantages, 2DoF machines have attracted increasing attention in multidimensional direct-drive systems [3,4], as well as in emerging low-power smart systems [5]. These trends further motivate the development of compact and efficient 2DoF machine topologies.

Currently, 2DoF machines are mainly classified into three major topologies: induction-type [6,7], switched reluctance-type [8,9], and permanent magnet (PM)-type [10,11]. Induction-type machines have simple structures and low cost. However, their power



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density and efficiency are relatively low, which limits their application in heavy-load systems. Switched reluctance-type machines have robust structures and good fault tolerance. However, they usually suffer from large torque/thrust ripples and high acoustic noise. PM-type machines have higher power density and higher efficiency and have therefore become an important research direction. Nevertheless, conventional surface-mounted PM machines with magnets on the mover have low PM utilization and relatively high material cost.

Two-degree-of-freedom flux-switching permanent magnet (2DoFFSPM) machines represent a unique topology of PM-type 2DoF machines. In this topology, the PMs and armature windings are placed on the stator side, while the mover has a simple salient-pole structure. This arrangement improves structural reliability and is beneficial for heat dissipation. It also provides high power density and good electromagnetic performance [12,13]. However, the double-salient stator–mover structure produces strong flux concentration. The end effect is also significant. These factors lead to large torque and thrust ripples. In addition, the coupling between rotary and linear motions further affects the stability and reliability of the machine.

Structural parameter optimization is an effective way to reduce torque/thrust ripples and improve the overall performance of electrical machines. Commonly used optimization methods include the genetic algorithm (GA), particle swarm optimization (PSO), and the Taguchi method [14–16]. GA and PSO have strong global optimization capability. However, they usually require a large number of finite-element simulations, especially for three-dimensional machine models. This results in high computational cost. In contrast, the Taguchi method uses orthogonal arrays to arrange multi-parameter and multi-level experiments. It can reduce the number of required simulations while retaining useful statistical information. Therefore, it is suitable for the rapid optimization of machines with strongly coupled design parameters.

The Taguchi method has been widely used in machine design. For conventional SdoF machines, it has been combined with genetic algorithms to optimize surface-mounted permanent magnet synchronous motors, reducing the cogging torque from 6.7846 mNm to 0.943 mNm and improving the efficiency from 76.21% to 78.45% [17]. For 2DoF machines, the Taguchi method has also been applied to both linear and rotary motion optimization. In the linear motion domain, it reduced thrust ripple by 6.1% and improved the thrust-to-mass ratio by 5.8% for an axial-magnetized dual-stator 2DoF permanent magnet machine [18]. In the rotary motion domain, it was used to suppress edge effects in a 2DoF direct-drive induction machine, leading to a 61.3% reduction in mover losses and a 13% increase in rated efficiency [19]. In addition, the Taguchi method has been extended to spherical motors, where the air-gap flux density and torque were increased by 8.7% and 15.6%, respectively [20].

Although these studies have demonstrated the effectiveness of the Taguchi method, multi-objective optimization of 2DoFFSPM machines under helical operation still needs further investigation. In helical operation, the rotary and linear electromagnetic fields interact with each other. Therefore, the design parameters affect torque, thrust, and their ripples in a coupled manner. A comprehensive optimization strategy is required to balance output capability and motion stability.

In this paper, a 2DoFFSPM machine is investigated under helical operation. First, the topology, operating principle, main design parameters, and three-dimensional finite-element model (3D FEM) of the machine are introduced. Then, four key geometrical parameters are selected as optimization variables, while average torque, torque ripple, average thrust, and thrust ripple are chosen as optimization objectives. An $L_{25}(5^4)$ orthogonal array is constructed based on the Taguchi method, and the performance of different pa-

parameter combinations is evaluated by three-dimensional finite-element analysis (3D FEA). The contribution ratios of the design parameters are further quantified through analysis of variance. Finally, a comprehensive optimal parameter combination is determined using a weighted evaluation function, and the effectiveness of the optimized scheme is verified by simulation.

The remainder of this paper is organized as follows. Section 2 introduces the topology, operating principle, main design parameters, and 3D FEM of the investigated 2DoFFSPM machine. Section 3 presents the multi-objective optimization procedure based on the Taguchi method, including parameter selection, orthogonal experimental design, simulation analysis, and comparison of optimization schemes. Section 4 summarizes the main conclusions of this study.

2. Structure and Operating Principle

2.1. Topological Structure

The investigated 2DoFFSPM machine consists of an arc-shaped rotary stator, an arc-shaped linear stator, and a salient-pole mover, as illustrated in Figure 1. Similar to conventional flux-switching permanent magnet machines, both arc-shaped stators are composed of stator cores, PMs, and armature windings. As shown in Figure 1a, the core segments and PMs of the arc-shaped rotary stator are alternately arranged along the circumferential direction. Axial slots are opened on the cores, and the PMs are tangentially magnetized with opposite magnetization directions between adjacent PMs. For the arc-shaped linear stator in Figure 1b, the cores and PMs are sequentially arranged along the axial direction with circumferential slots on the cores. The mover adopts a solid structure, and both axial and circumferential slots are machined on its surface to form an array of salient-pole teeth as depicted in Figure 1c. The arc-shaped rotary stator and arc-shaped linear stator are circumferentially connected in parallel, and the overall structure of the machine is presented in Figure 1d.

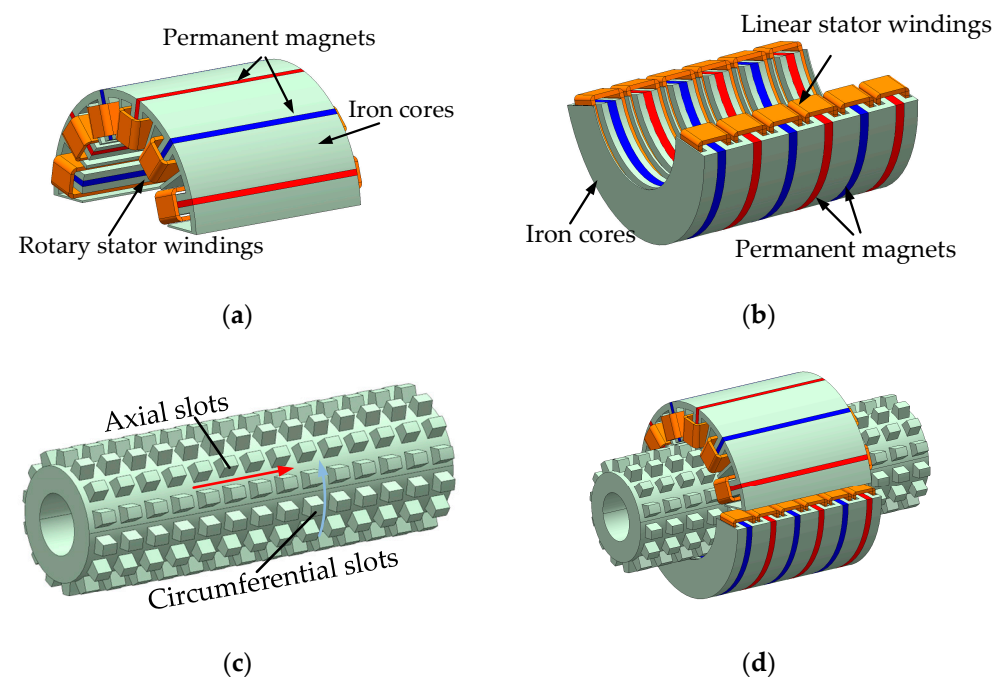


Figure 1. Topological structure of the 2DoFFSPM machine. (a) Arc-shaped rotary stator. (b) Arc-shaped linear stator. (c) Salient-pole mover. (d) Overall structure.

2.2. Operating Principle and Main Parameters

The 2DoFFSPM machine is essentially a special type of flux-switching machine. Therefore, it follows the conventional flux-switching operating principle. That is, the flux, or flux linkage, linked by the winding turns changes its direction and magnitude according to the relative position between the mover salient poles and the stator teeth, thereby generating an induced electromotive force with alternating amplitude and phase.

To explain the operating principle of the 2DoFFSPM machine more clearly, the salient-pole teeth arranged along the axial direction on the mover surface are defined as axial tooth rows. Each axial tooth row consists of several salient-pole teeth distributed in the axial direction. Similarly, the salient-pole teeth arranged along the circumferential direction on the mover surface are defined as circumferential tooth rows. Each circumferential tooth row consists of several salient-pole teeth distributed in the circumferential direction, as shown in Figure 2.

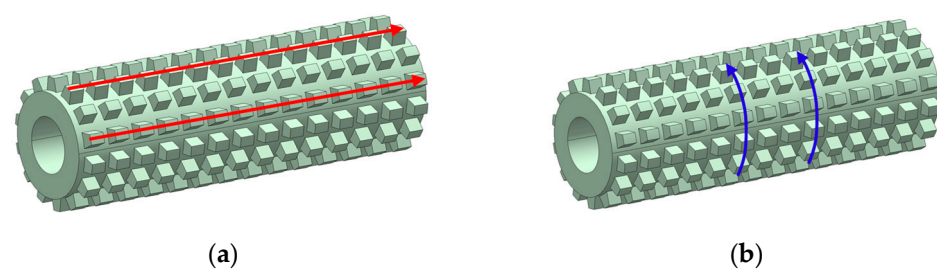


Figure 2. Mover tooth-rank distribution of the 2DoFFSPM machine. (a) Axial tooth array. (b) Circumferential tooth array.

When the mover of the 2DoFFSPM machine interacts with the arc-shaped rotary stator, the machine can be regarded as a flux-switching rotary machine. Taking any two adjacent cores of the arc-shaped rotary stator, together with the neighboring permanent magnets, rotary winding coil, and an axial tooth row of the mover, as an example, the magnetic circuit established between the mover axial tooth row and the permanent magnets of the arc-shaped rotary stator passes through four key positions sequentially during rotary motion.

At the first key position, the magnetic reluctance of the magnetic circuit is minimized, the flux amplitude reaches its maximum value, and the permanent-magnet flux linkage linked by the coil reaches its positive maximum, which corresponds to one extreme position of the flux-switching process. At the second key position, the magnetic reluctance becomes maximum, the flux amplitude decreases to its minimum value, and the permanent-magnet flux linkage linked by the coil becomes zero. At the third key position, the magnetic reluctance is minimized again, the flux amplitude reaches its maximum value, and the permanent-magnet flux linkage linked by the coil reaches its negative maximum. At the fourth key position, the magnetic reluctance becomes maximum again and the flux linkage returns to zero, which completes one flux-switching cycle for the rotary winding. According to Faraday's law of electromagnetic induction, as well as the consistency and complementarity of the windings in flux-switching machines, a sinusoidal no-load back electromotive force is generated in the rotary windings.

Similarly, when the mover of the 2DoFFSPM machine performs linear motion, the magnetic circuit established between the circumferential tooth rows of the mover and the permanent magnets of the arc-shaped linear stator also passes through the above four key positions sequentially. During this process, the flux linkage of the linear winding coils changes successively from positive maximum to zero, then to negative maximum and zero again, which results in a sinusoidal no-load back electromotive force in the linear windings.

Helical motion can be spatially decomposed into circumferential rotary motion and axial linear motion. Therefore, when the mover of the 2DoFFSPM machine performs

helical motion, it can be regarded as the simultaneous rotary motion of the axial tooth rows and linear motion of the circumferential tooth rows on the mover surface. Based on the aforementioned no-load back electromotive force generation mechanisms of the rotary and linear parts, sinusoidal no-load back electromotive forces are generated simultaneously in both winding sets, which explains the basic operating principle of the 2DoFFSPM machine under helical motion.

Main parameters of the 2DoFFSPM machine are listed in Table 1.

Table 1. Parameters of the 2DoFFSPM machine.

Parameters	Rotary Value	Linear Value
Stator outer diameter	120 mm	120 mm
Stator inner diameter	72 mm	72 mm
Stator length	100 mm	114 mm
Stator arc angle	180 deg	150 deg
Mover outer diameter	69 mm	69 mm
Mover length	224 mm	224 mm
Number of mover poles	14	14
Permanent magnet height	24 mm	24 mm
Number of PM poles	6	6
Number of winding turns	180	120
Slot fill factor	47.2	51.8
Winding connection	Y-type	Y-type

2.3. Finite Element Model

To conduct subsequent electromagnetic analysis of the 2DoFFSPM machine, a 3D FEM is established using the Infolytica MagNet 7.8.3 simulation software. The mesh generation result and magnetic field distribution of the model are shown in Figure 3.

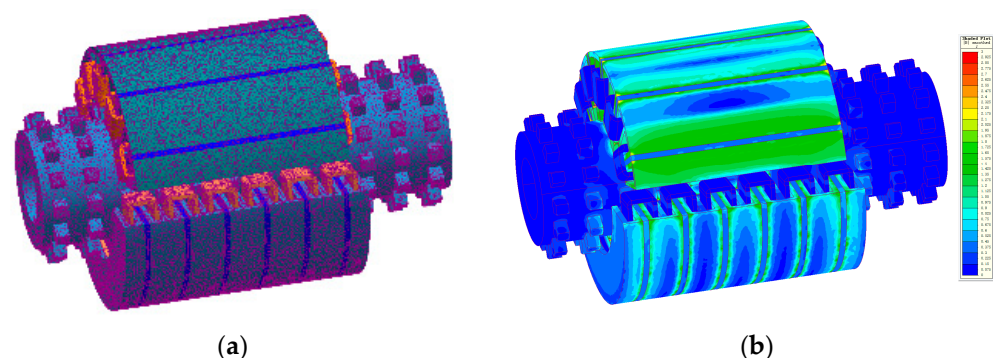


Figure 3. 2DoFFSPM machine model. (a) Meshing. (b) Field distribution.

Figure 3a presents the detailed mesh division of the model. Generally, the machine model adopts a relatively fine meshing scheme, and mesh refinement is performed in the air gap region to improve the simulation accuracy under actual operating conditions. In addition, as can be seen from the magnetic field distribution in Figure 3b, magnetic saturation only occurs at the edge of the stator salient teeth, and almost no saturation appears in other regions of the machine.

3. Multi-Objective Optimization

3.1. Taguchi Method

The Taguchi method is a robust design approach widely used for multi-objective engineering optimization. It focuses on controlling variability during the product design stage. This effectively improves the robustness of product performance. This method uses

orthogonal arrays to design experiments. It can get comprehensive test information with a small number of runs. Compared with traditional full factorial designs, it significantly shortens test cycles and improves analysis efficiency. In the field of machine design and optimization, the Taguchi method can quantify the influence of multiple structural parameters on performance indicators at the same time. It maintains high analysis accuracy while reducing computational costs and development cycles. It is an economical and efficient method for machine optimization design. The Taguchi method used in this work follows three core steps: determination of optimization parameters and objectives, simulation and analysis, and comparison of optimization schemes.

It should be noted that the Taguchi method used in this study is employed as an efficient parameter-screening and optimization approach rather than a Pareto-based multi-objective optimization algorithm. Therefore, the final comprehensive scheme is determined using a weighted evaluation function to achieve an engineering compromise among torque, thrust, torque ripple, and thrust ripple.

3.2. Optimization Parameters and Objectives Determination

The average torque, average thrust, torque ripple, and thrust ripple are selected as the quantitative optimization objectives because they directly evaluate the output capability and motion stability of the 2DoFFSPM machine, while also providing the numerical response values required for the Taguchi-based comprehensive evaluation.

These performance metrics are most significantly affected by the following parameters: axial length of the rotary stator, circumferential angle of the linear stator, geometric parameters of the mover salient pole, air gap thickness, stator slot width and armature winding turns. Four key geometrical parameters are selected as the optimization variables in this study: the axial length of the rotary stator L_{sr} , the circumferential angle of the linear stator L_{sl} , the circumferential angle of the mover salient pole M_{sr} , and the axial length of the mover salient pole M_{sl} . These parameters were selected based on electromagnetic mechanism, engineering feasibility, and computational-cost considerations. Specifically, L_{sr} and L_{sl} represent the main axial and circumferential geometrical features of the rotary and linear stators, respectively, while M_{sr} and M_{sl} describe the corresponding mover-side salient-pole features interacting with the stator structures. These parameters directly affect the air-gap permeance distribution, flux-modulation effect, end effect, and electromagnetic coupling between rotary and linear motions. Since the proposed 2DoFFSPM machine requires time-consuming 3D FEA calculations, only these key structural parameters were selected to balance optimization accuracy and computational cost.

The cross-sectional views of the selected optimization parameters are shown in Figure 4.

The optimization targets are to maximize torque and thrust, while minimizing torque ripple and thrust ripple. Five levels are set for each parameter based on its initial design value. The specific value ranges and corresponding levels are listed in Tables 2 and 3.

Table 2. Ranges of design parameters.

Ranges	L_{sr} (mm)	L_{sl} (deg)	M_{sr} (deg)	M_{sl} (mm)
Minimum value	98	136	2	2
Maximum value	114	162	14	10

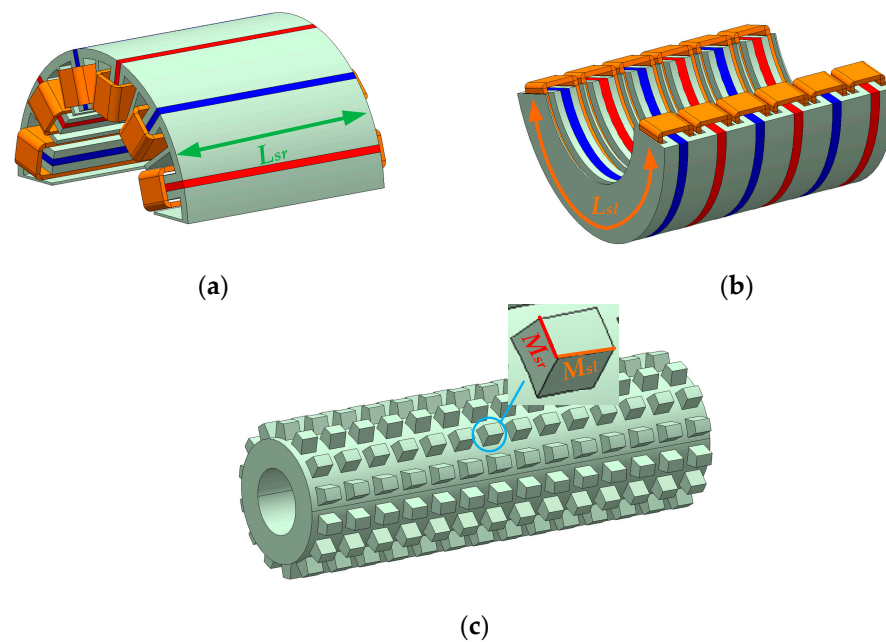


Figure 4. Schematic diagram of the selected optimization parameters. (a) L_{sr} . (b) L_{sl} . (c) M_{sr} and M_{sl} .

Table 3. Level of design parameters.

Item	L_{sr} (mm)	L_{sl} (deg)	M_{sr} (deg)	M_{sl} (mm)
Level 1	98	136	2	2
Level 2	102	142.5	5	4
Level 3	106	149	8	6
Level 4	110	155.5	11	8
Level 5	114	162	14	10

3.3. Simulation and Analysis

Following this, four key parameters are selected for optimization: rotary stator axial length, linear stator circumferential angle, mover salient pole axial length, and mover salient pole circumferential angle. The Taguchi experimental design includes 4 factors with 5 levels each. A full factorial design would require 625 simulation runs, which is extremely time-consuming for 3D FEA. In contrast, the $L_{25}(5^4)$ orthogonal array reduces the number of simulations while retaining sufficient information for main-effect analysis.

All simulations are performed under the helical operation condition of the machine, with the mover running at a rotational speed of 214 r/min and a linear speed of 0.8 m/s. The coupling effects between rotary and linear motions are taken into account throughout the simulation process. The armature windings are supplied with three-phase alternating current with an effective value of 4 A. The detailed simulation results are listed in Table 4.

Based on the data obtained from the aforementioned orthogonal tests, the formulas for calculating the mean value and variance of each optimization objective at each parameter level are presented below.

$$\bar{y}(k) = \frac{1}{m} \sum_{t=1}^m y_{k,t} \quad (1)$$

$$S = \frac{1}{m} \sum_{t=1}^m [\bar{y}(t) - \bar{y}]^2 \quad (2)$$

where k denotes the parameter level value, $y_{k,t}$ refers to the experimental result of the t -th test run under level k , and m represents the total number of experiments at level k .

Table 4. Summary of orthogonal test results.

Number	L_{sr}	L_{sl}	M_{sr}	M_{sl}	Torque (Nm)	Torque Ripple (%)	Thrust (N)	Thrust Ripple (%)
1	98	136	2	2	1.435	18.68	20.03	16.49
2	98	142.5	5	4	2.69	22.32	34.16	18.46
3	98	149	8	6	3.52	16.84	43.39	12.72
4	98	155.5	11	8	3.78	8.32	43.98	16.07
5	98	162	14	10	3.28	19.02	34.36	31.10
6	102	136	5	6	3.59	24.92	31.68	25.47
7	102	142.5	8	8	4.34	21.17	34.44	30.20
8	102	149	11	10	4.41	17.11	28.22	35.35
9	102	155.5	14	2	1.55	16.66	49.94	28.94
10	102	162	2	4	2.30	13.08	28.83	20.78
11	106	136	8	10	5.1	20.44	22.62	42.55
12	106	142.5	11	2	1.95	25.49	40.49	35.5
13	106	149	14	4	2.38	16.73	59.28	28.1
14	106	155.5	2	6	3.13	11.09	27.25	13.97
15	106	162	5	8	4.40	9.46	32.17	20.07
16	110	136	11	4	3.04	19.98	46.87	22.94
17	110	142.5	14	6	2.97	19.60	55.36	13.19
18	110	149	2	8	3.92	19.03	22.94	21
19	110	155.5	5	10	5.25	13.96	21.48	32.55
20	110	162	8	2	2.08	14.98	39.86	28.24
21	114	136	14	8	3.51	19.11	43.92	44.76
22	114	142.5	2	10	4.72	22.44	15.53	47.21
23	114	149	5	2	2.07	13.4	29.68	37.74
24	114	155.5	8	4	3.30	10.08	46.28	42.22
25	114	162	11	6	3.81	6.82	54.97	44.57

Subsequently, the torque, torque ripple, thrust and thrust ripple data in Table 4 are split and counted in accordance with Equations (1) and (2). The mean values, variances and the proportion of the total variance are calculated, and all computed results are compiled into Table 5. Based on these data, Figure 5 presents the main effect curves of individual factors versus optimization objectives to visually demonstrate the influence level of each structural parameter.

From the above, the mover salient pole axial length is the dominant factor for torque, responsible for 84.8% of the total variance. It also shows a non-negligible influence on thrust, contributing 35.1%. Regarding torque ripple, the linear stator circumferential angle plays the most significant role, accounting for 81% of the variance. Thrust is primarily governed by the mover salient pole circumferential angle, which contributes 61.1% of the total variance. As for thrust ripple, the rotating stator axial length has a notable effect, with a contribution ratio of 65.7%.

Table 5. Summary of mean values and variances.

Parameters	Level	Torque (Nm)	Torque Ripple (%)	Thrust (N)	Thrust Ripple (%)
L_{sr}	1	2.94	17.03	35.18	18.97
	2	3.24	18.59	34.62	28.15
	3	3.39	16.64	36.36	28.04
	4	3.45	17.51	37.30	23.58
	5	3.48	14.37	38.08	43.30

Table 5. Cont.

Parameters	Level	Torque (Nm)	Torque Ripple (%)	Thrust (N)	Thrust Ripple (%)
Variance		0.0393	0.0002	1.6447	0.0067
Variance contribution ratio (%)		3.6	9.52	1.3	65.69
L_{sl}	1	3.336	20.63	33.03	30.44
	2	3.335	22.20	35.99	28.91
	3	3.262	16.62	36.70	26.98
	4	3.402	12.02	37.78	26.75
	5	3.175	12.67	38.04	28.95
Variance		0.0060	0.0017	3.2438	0.0002
Variance contribution ratio (%)		0.6	80.96	2.5	1.96
M_{sr}	1	3.102	16.86	22.92	23.89
	2	3.60	16.81	29.84	26.86
	3	3.67	16.70	37.32	28.66
	4	3.40	15.54	40.98	30.88
	5	2.74	18.23	48.57	29.22
Variance		0.1185	0.0001	78.7399	0.0006
Variance contribution ratio (%)		11	4.76	61.1	5.88
M_{sl}	1	1.82	17.84	36.00	29.38
	2	2.74	16.44	43.08	26.50
	3	3.40	15.85	42.53	21.98
	4	3.99	15.42	35.49	26.42
	5	4.55	18.59	24.44	37.75
Variance		0.9136	0.0001	45.2419	0.0027
Variance contribution ratio (%)		84.8	4.76	35.1	26.47

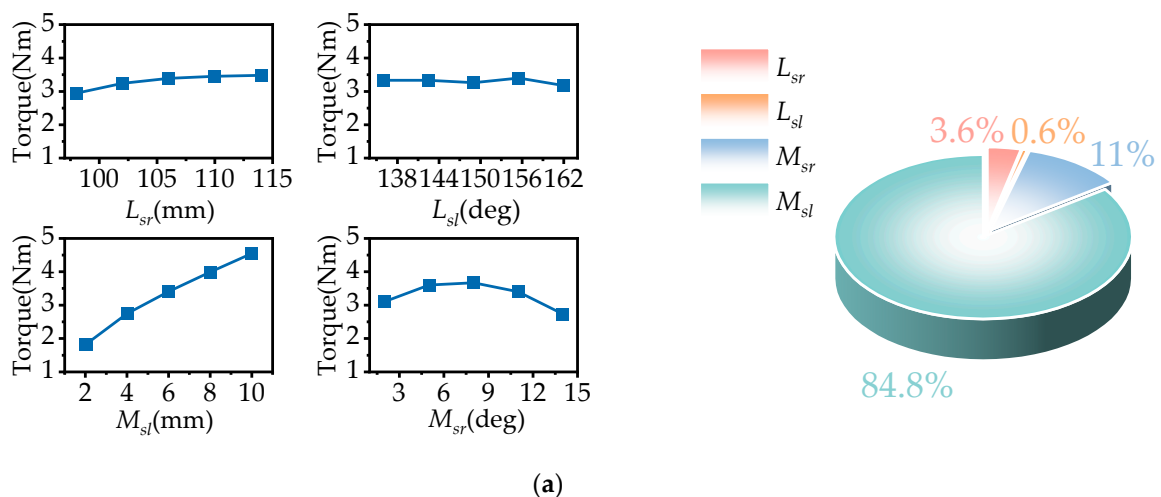


Figure 5. Cont.

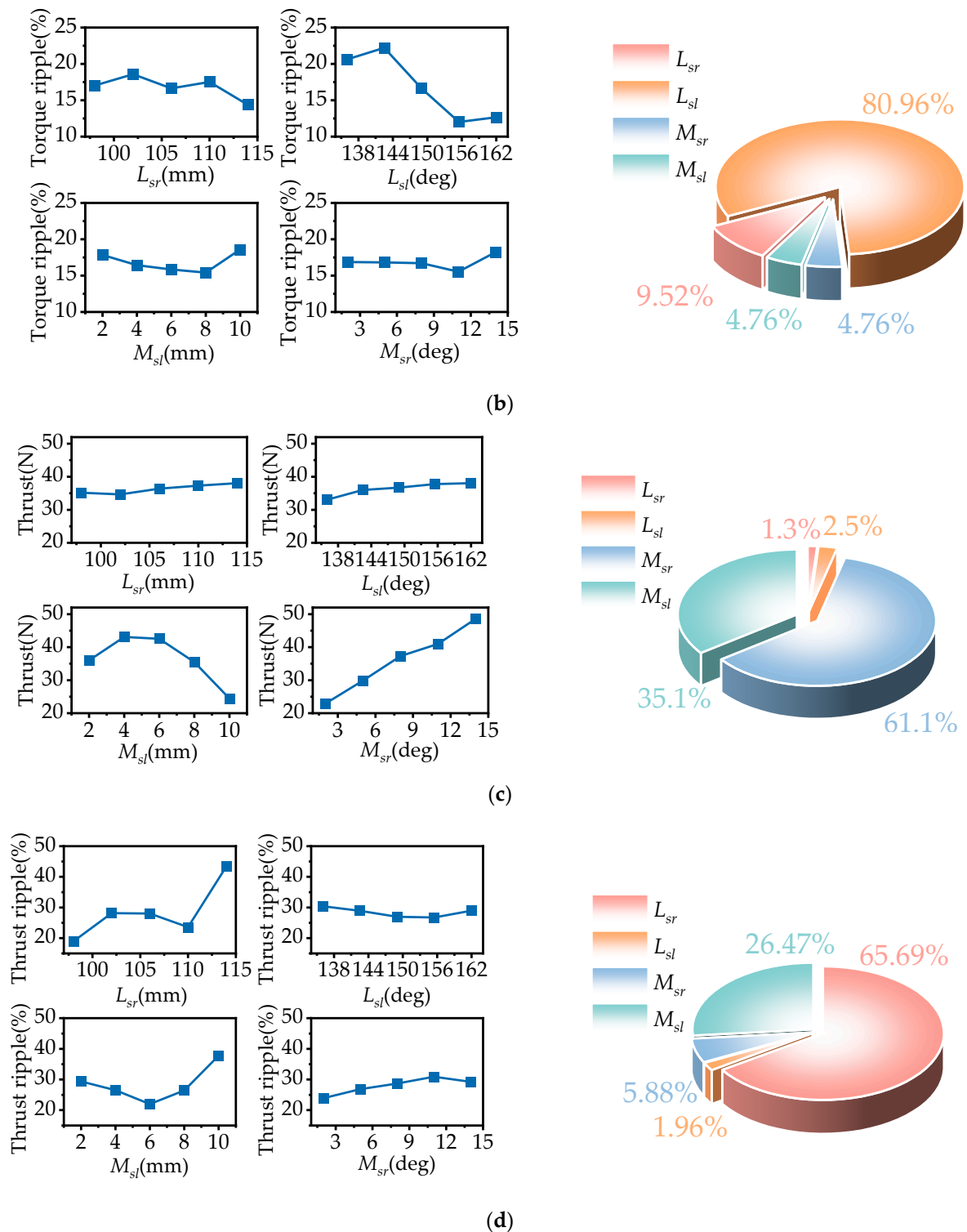


Figure 5. Main effects and contribution ratios of design factors on optimization objectives. (a) Torque. (b) Torque ripple. (c) Thrust. (d) Thrust ripple.

3.4. Comparison of Optimization Schemes

From the above analytical results, five tentative optimization schemes are obtained.

Scheme1 (M1): maximum torque configuration.

The axial length of the mover salient pole M_{sl} exerts the most significant influence on torque, so it is set to 10 mm. The axial length of the rotary stator L_{sr} is 114 mm, the

circumferential angle of the linear stator L_{sl} is 155.5 deg, and the circumferential angle of the mover salient pole M_{sr} is 8 deg.

Scheme2 (M2): minimum torque ripple configuration.

The circumferential angle of the linear stator L_{sl} is the dominant factor affecting torque ripple; thus, it is assigned a value of 155.5 deg. The axial length of the rotary stator L_{sr} is 114mm, the circumferential angle of the mover salient pole M_{sr} is 11 deg, and the axial length of the mover salient pole M_{sl} is 8 mm.

Scheme3 (M3): maximum thrust configuration.

The circumferential angle of the mover salient pole M_{sr} has the most pronounced impact on thrust, so it is set to 14 deg. The axial length of the mover salient pole M_{sl} is 4 mm, the circumferential angle of the linear stator L_{sl} is 162 deg, and the axial length of the rotary stator L_{sr} is 114 mm.

Scheme4 (M4): minimum thrust ripple configuration.

The axial length of the rotary stator L_{sr} is the primary factor influencing thrust ripple; thus, it is assigned a value of 98 mm. The circumferential angle of the linear stator L_{sl} is 155.5 deg, the axial length of the mover salient pole M_{sl} is 6 mm, and the circumferential angle of the mover salient pole M_{sr} is 2 deg.

Scheme5 (M5): comprehensive optimal configuration.

The axial length of the rotary stator L_{sr} is set to 98 mm; the circumferential angle of the linear stator is set to L_{sl} 155.5 deg. According to the main-effect analysis, the first two high-priority levels of the mover salient-pole parameters are retained: the circumferential angle of the mover salient pole M_{sr} is set to 14 deg or 8 deg; the axial length of the mover salient pole M_{sl} is set to 6 mm or 10 mm. Accordingly, four candidate combinations, denoted as C1–C4, are constructed, as listed in Table 6.

Table 6. Candidate combinations constructed from the retained levels of M_{sr} and M_{sl} in Scheme M5.

Candidate	L_{sr} (mm)	L_{sl} (deg)	M_{sr} (deg)	M_{sl} (mm)
C1	98	155.5	14	10
C2	98	155.5	14	6
C3	98	155.5	8	10
C4	98	155.5	8	6

For the multi-objective optimization of the four performance metrics, higher torque and thrust, as well as lower torque and thrust ripples, are expected. Since these objectives have different units and optimization directions, a normalized weighted evaluation function is introduced for final decision-making, which is defined as follows.

$$\zeta = \left(\frac{T_i - T_{i\min}}{T_{i\max} - T_{i\min}} \times 0.5 + \frac{TR_{i\max} - TR_i}{TR_{i\max} - TR_{i\min}} \times 1.5 + \frac{F_i - F_{i\min}}{F_{i\max} - F_{i\min}} \times 0.5 + \frac{FR_{i\max} - FR_i}{FR_{i\max} - FR_{i\min}} \times 1.5 \right) \times 100 \quad (3)$$

where $T_{i\min}$, $TR_{i\max}$, $F_{i\min}$, $FR_{i\max}$ are the minimum torque, maximum torque ripple, minimum thrust, and maximum thrust ripple from the experimental data in Table 4, which are 1.435 Nm, 25.49%, 15.53 N, and 47.21%, respectively.

In Formula (3), the four performance indices are normalized to eliminate the influence of different units and magnitudes, and the weighting coefficients reflect the design preference of the comprehensive optimization. For the investigated 2DoFFSPM machine, ripple suppression is prioritized to improve operational stability and positioning accuracy under helical motion. Therefore, torque ripple and thrust ripple are assigned higher weighting coefficients of 1.5, while average torque and average thrust are assigned lower weighting coefficients of 0.5 to maintain acceptable output capability. The weighting ratio between

ripple-related indices and average output indices is thus 3:1, with equal weights used within each category.

To select the final configuration of Scheme M5, the 3D FEA performance results of the four candidates are substituted into Equation (3), and the corresponding comprehensive evaluation indices are summarized in Table 7.

Table 7. Performance comparison of the candidate combinations.

Candidate	Torque (Nm)	Torque Ripple (%)	Thrust (N)	Thrust Ripple (%)	ξ
C1	3.29	16.01	33.1	29.37	233.68
C2	2.59	16.09	60.59	9.87	359.28
C3	4.66	9.64	26	22.34	318.38
C4	3.51	8.29	45.44	9.46	389.92

As shown in Table 7, among the four candidate combinations, C4 obtains the highest comprehensive evaluation index of 389.92. Therefore, C4 is selected as the final parameter combination for Scheme M5. Scheme M5 is then compared with the Schemes M1–M4 in the following section.

The 3D FEA results of the five aforementioned optimization schemes are presented in Figure 6. The optimization effect of each scheme can be quantitatively evaluated using the comprehensive evaluation index ξ , and the calculated ξ values are summarized in Table 8.

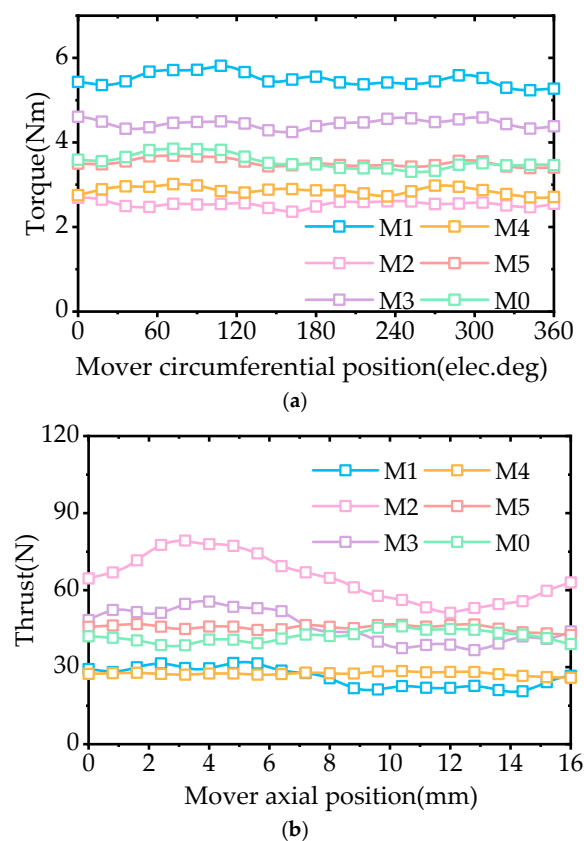


Figure 6. Output performance comparison. (a) Torque. (b) Thrust.

Table 8. Summary of comparative results.

Scheme	Torque (Nm)	Torque Ripple (%)	Thrust (N)	Thrust Ripple (%)	ξ
M1	5.49	10.53	26.04	45.86	267.53
M2	4.45	8.25	45.84	41.77	321.22
M3	2.54	13.18	64.59	43.57	280.51
M4	2.86	11.65	27.48	9.76	288.71
M5	3.51	8.29	45.44	9.46	389.92

As shown in Figure 6 and Table 8, M1, which takes torque as the primary optimization objective, achieves a maximum torque of 5.49 Nm, representing a 56.7% improvement over the original design. M2, which aims to minimize torque ripple, reduces the ripple to 8.25%, a 47.5% decrease compared with the original design. M3, which takes thrust as the primary optimization objective, obtains a maximum thrust of 64.59 N, an increase of 53.6% relative to the original design. M4, which targets the minimization of thrust ripple, lowers the ripple to 9.76%, a 49.15% reduction from the original design.

M5, which balances all four performance indicators, yields a torque of 3.51 Nm, a torque ripple of 8.29%, a thrust of 45.44 N, and a thrust ripple of 9.46%. Compared with the initial machine, it achieves significant improvements in three indicators: torque ripple is reduced by 47.3%, thrust is increased by 8.1%, and thrust ripple is decreased by 48.38%. After comprehensive evaluation using the optimal function value ξ , M5 demonstrates the best comprehensive performance in the multi-objective optimization.

Furthermore, to verify whether the optimized schemes indeed outperform the original 25 Taguchi orthogonal test samples in terms of comprehensive performance, the comprehensive evaluation function values for all original orthogonal test samples are further calculated. The calculation results are summarized in Table 9.

Table 9. Comprehensive evaluation indices of the orthogonal test samples and the initial design.

Number	ξ	Number	ξ
1	152.18	14	287.07
2	213.88	15	337.53
3	322.94	16	266.48
4	373.35	17	324.38
5	214.13	18	231.85
6	199.62	19	266.48
7	241.72	20	222.82
8	231.65	21	209.17
9	224.80	22	132.43
10	230.05	23	168.95
11	194.92	24	270.66
12	135.50	25	328.08
13	285.99	0	304.01

In Table 9, No. 0 represents the output performance of the initial prototype machine under identical simulation conditions, serving as the performance comparison baseline. The orthogonal test samples No. 3, 4, 15, 17 and 25, as well as the specialized optimization Schemes M2 and M5, all outperform this baseline in terms of comprehensive performance. The above data fully demonstrate that the parameters and levels selected in this orthogonal test are reasonable, and the test samples include high-quality schemes with comprehensive performance better than the baseline, verifying the effectiveness and feasibility of the

Taguchi method in the multi-objective optimization of the 2DoFFSPM machine studied in this paper.

Consequently, after comparing the performance of all orthogonal test samples and specialized optimization schemes, the optimal structural parameters of the 2DoFFSPM machine are finally determined as $L_{sr}(98.0)$, $L_{sl}(155.5)$, $M_{sr}(8.0)$ and $M_{sl}(6.0)$, which correspond to a rotary stator axial length of 98 mm, a linear stator circumferential angle of 155.5 deg, a mover salient pole circumferential angle of 8 deg, and a mover salient pole axial length of 6 mm, respectively.

4. Conclusions

This paper presents a Taguchi-based multi-objective optimization of a 2DoFFSPM machine under helical operation. Average torque, torque ripple, average thrust, and thrust ripple are selected as the optimization objectives. Four key geometrical parameters, namely L_{sr} , L_{sl} , M_{sr} , and M_{sl} are optimized using an $L_{25}(5^4)$ orthogonal array and 3D FEA. The main conclusions are summarized as follows:

- (1) The four design parameters have different dominant effects on machine performance. The linear stator circumferential angle L_{sl} has the strongest influence on torque ripple. The mover salient-pole circumferential angle M_{sr} mainly affects average thrust. The rotary stator axial length L_{sr} is the dominant factor for thrust ripple. The mover salient-pole axial length M_{sl} has the most significant effect on average torque.
- (2) Among the compared optimization schemes, M5 achieves the best comprehensive performance. The final optimized parameters are $L_{sr} = 98$ mm, $L_{sl} = 155.5$ deg, $M_{sr} = 8$ deg, and $M_{sl} = 6$ mm. Compared with the initial design, the optimized machine reduces torque ripple and thrust ripple by 47.3% and 48.38%, respectively, while increasing average thrust by 8.1%. The average torque decreases slightly by 1.1%, which is acceptable considering the significant ripple suppression and overall performance improvement.
- (3) The optimized design has practical engineering significance for 2DoF direct-drive applications. The reduction in torque and thrust ripples can improve motion smoothness, reduce vibration tendency, and enhance positioning accuracy under helical operation. The optimized structure also provides a better compromise between output capability and motion stability, which is important for compact rotary-linear drive systems used in precision actuation and advanced manufacturing.

Future work will focus on prototype manufacturing and experimental validation. The influence of air-gap tolerance, manufacturing errors, and assembly deviations will also be investigated. In addition, multi-physics optimization considering electromagnetic, thermal, and mechanical constraints will be carried out to further improve the reliability and engineering applicability of the 2DoFFSPM machine.

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Abbreviations

Symbol/Abbreviation	Full Name	Unit
2DoF	Two-degree-of-freedom	
SDoF	Single-degree-of-freedom	
SPM	Surface-mounted permanent magnet	
FEA	Finite-element analysis	
FEM	Finite-element method	
3D FEA	Three-dimensional finite-element analysis	
3D FEM	Three-dimensional finite-element model	
2DoFFSPM	Two-degree-of-freedom flux-switching permanent magnet	
GA	Genetic algorithm	
PSO	Particle swarm optimization	
L_{sr}	Axial length of the rotary stator	mm
L_{sl}	Circumferential angle of the linear stator	deg
M_{sr}	Circumferential angle of the mover salient pole	deg
M_{sl}	Axial length of the mover salient pole	mm
ξ	Comprehensive evaluation index	
k	Parameter level	
m	Number of experiments under each level	

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