

Article

Operational Cost Optimization of Rural LV Distribution Systems Considering Power Quality Constraints

Xiaoying Sun ¹, Xianglin Chen ¹, Xiangyu He ¹, Chenghong Gu ² and Xinsong Zhang ^{1,*}

¹ School of Electrical Engineering and Automation, Nantong University, Nantong 226019, China; 2312320013@stmail.ntu.edu.cn (X.S.); 2412320024@stmail.ntu.edu.cn (X.C.); 2412310016@stmail.ntu.edu.cn (X.H.)

² Department of Electrical and Electronic Engineering, University of Bath, Bath BA2 7AY, UK; cg277@bath.ac.uk

* Correspondence: zhang.xs@ntu.edu.cn

Abstract

High penetration of residential photovoltaic (PV) systems frequently causes voltage violations and three-phase unbalance in rural low-voltage distribution systems (LVDS). These issues directly result in large-scale PV curtailment and significantly increase the operating costs of the rural LVDS. To address these issues, this paper proposes an operational cost optimization model and a corresponding solution method for the rural LVDS, explicitly considering power quality constraints. The decision variables of the model include the three-phase active and reactive power outputs of the distributed PV inverters and the energy storage system (ESS). The optimization objective is to minimize the total electricity cost of the rural LVDS, incorporating the operational cost of the ESS, subject to power quality constraints such as voltage limits and three-phase unbalance margins. Given the non-convex nature of the original model, second-order cone relaxation (SOCR) is employed to reformulate it as a convex optimization problem, which can be solved efficiently and robustly to global optimality. Simulation results based on real-world case studies demonstrate the effectiveness of the proposed method in mitigating voltage violations, reducing three-phase unbalance, and enabling economical operation. Overall, the proposed approach facilitates the high-penetration integration of distributed PV in the rural LVDS.

Keywords: residential photovoltaics; rural low-voltage distribution systems; power quality; second-order cone programming; operational cost

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1. Introduction

1.1. Context and Motivation

Against the backdrop of the global transition toward a low-carbon energy structure, photovoltaic (PV) generation has emerged as a crucial pillar for achieving the carbon peaking and carbon neutrality goals [1,2], owing to its clean, renewable nature, and continuously declining costs. As the end of 2025, the cumulative installed capacity of PV generation in China reached 1200 GW, representing a year-on-year growth of 35%. Specifically, centralized and distributed PV accounted for 670 GW and 530 GW, respectively [3]. Characterized by the advantages of on-site energy consumption and local load balancing

[4,5], residential PV systems have become increasingly prevalent in rural low-voltage distribution systems (LVDS), thereby emerging as one of the primary application scenarios for distributed PV development.

Compared with their urban counterparts, rural LVDS have long been characterized by single-phase connections and the uneven spatiotemporal distribution of loads [6,7], rendering power quality issues such as three-phase unbalance and voltage violations far more pronounced. With the large-scale integration of PV systems, voltage violations at the points of interconnection and their adjacent nodes could be significantly exacerbated. Furthermore, the single-phase integration of residential PVs may further aggravate the three-phase unbalance [8–10], thereby triggering PV curtailment and escalating the overall operational costs of the distribution network.

1.2. Literature Review and Research Gap

Addressing power quality issues and mitigating PV curtailment in rural LVDS are crucial for minimizing the overall operational costs of the system. Reference [11] incorporated a dynamic power-reference response module into the control strategy of distributed residential PV inverters, thereby mitigating PV curtailment through the regulation of voltage deviations. By formulating voltage unbalance constraints as second-order cone constraints, refs. [12,13] proposed a two-stage multi-objective Volt-Var control model, which reduces PV curtailment by optimizing the voltage profile. Reference [14] proposed a coordinated active-reactive power control strategy utilizing residential PV inverters to reduce PV curtailment by enhancing the voltage quality of the LVDS. Furthermore, reference [15] developed a coordinated active and reactive power voltage regulation strategy based on a measurement-derived sensitivity matrix, successfully diminishing PV curtailment while achieving precise voltage control. To minimize network losses and voltage deviations, reference [16] presented an adaptive control strategy for residential PVs that dynamically adjusts power outputs based on predicted nodal voltages, consequently alleviating PV curtailment. Finally, reference [17] introduced a strategy for the day-ahead centralized optimization and tuning of inverter P-V and Q-V curve parameters; during the real-time autonomous control stage, each inverter employs local droop control, which ultimately limits PV curtailment.

The rapid advancement of battery energy storage systems (BESS) offers an emerging technological alternative for addressing power quality issues, alleviating PV curtailment, and reducing overall operational costs in LVDS. Reference [18] proposed a voltage control strategy based on sensitivity analysis. By regulating the power injected by energy storage, this method effectively suppresses voltage deviations and minimizes active power losses. Reference [19] developed a cost-driven BESS optimization framework for LVDS, which significantly cuts system network losses while enhancing both voltage regulation capabilities and system reliability. Furthermore, a proportional coordinated voltage control scheme integrating BESS and PV inverters was designed in [20,21], improving system voltage quality while simultaneously achieving state of charge (SOC) equalization among the energy storage units. To regulate voltage and mitigate PV curtailment concurrently, ref. [22] introduced a two-stage voltage control approach: the first stage regulates the BESS based on sensitivity analysis, followed by the second stage which dispatches the reactive power of the PV inverters.

To clearly illustrate the research gap and highlight the contributions of this paper, a comparison between the proposed approach and existing studies is summarized in Table 1. Despite the significant progress made in the aforementioned studies, a critical review reveals that most existing methods fail to simultaneously address economic operation and comprehensive power quality enhancement (including both voltage regulation and unbalance mitigation).

Table 1. Comparison of the proposed method with existing literature.

Reference	Main Objective	Control Devices	Voltage Regulation	Three-Phase Unbalance Mitigation
[11]	Voltage	PV	Yes	No
[12]	Power losses, voltage	PV (reactive)	Yes	No
[13]	Power losses, voltage	PV, CB, OLTC, SVC	Yes	Yes
[14]	PV curtailment, voltage	PV	Yes	No
[15]	PV curtailment	PV	Yes	No
[16]	Power losses, voltage	PV	Yes	No
[17]	Power losses, voltage, curtailed active power of PVs	PV	Yes	No
[18]	Voltage	PV, BESS	Yes	No
[19]	Cost	PV, BESS	Yes	No
[20]	Power losses, voltage	PV, BESS	Yes	No
[21]	Cost, voltage	PV, BESS	Yes	No
[22]	PV curtailment, voltage	PV, BESS	Yes	Yes
Proposed	Cost, voltage	PV, BESS	Yes	Yes

As shown in Table 1, studies [11–17] primarily focus on voltage regulation, power losses, or PV curtailment using PV inverters or traditional devices. However, they generally neglect economic objectives (e.g., operational costs) and the integration of BESS. On the other hand, while studies [18–22] incorporate BESS to improve voltage profiles or reduce costs, they still exhibit limitations. For instance, studies such as [18–21] overlook the severe three-phase unbalance inherent in rural LVDS. Although a few studies, such as [13,22], successfully address three-phase unbalance, they do not incorporate comprehensive economic optimization, and study [13] relies solely on traditional regulation devices alongside PVs rather than BESS.

This oversight in the existing literature is critical, as ignoring phase asymmetry can lead to inaccurate cost estimations and unfeasible dispatch strategies in highly unbalanced networks. Therefore, the proposed method bridges this gap by simultaneously optimizing daily operational costs, voltage regulation, and three-phase unbalance through the coordinated control of PV and BESS.

The main reasons behind this trend of overlooking joint optimization are threefold. First, there is an inherent conflict between economic efficiency and power quality enhancement. Mitigating voltage violations and three-phase unbalance typically requires frequent operation of energy storage systems (incurring conversion losses and degradation) or PV active power curtailment (reducing electricity sales revenue), making it challenging to balance both aspects. Second, integrating complex power quality constraints into an economic dispatch model significantly increases mathematical complexity. The inclusion of three-phase unbalanced AC power flow and discrete BESS operating states transforms the model into a highly non-linear and non-convex Mixed-Integer Nonlinear Programming (MINLP) problem, which is computationally demanding to solve. Third, traditional economic optimization models often assume balanced network conditions. However, rural LVDS are characterized by severe phase asymmetry due to single-phase loads and PVs, necessitating high-dimensional three-phase modeling that further complicates the joint optimization framework.

1.3. Main Contributions and Paper Organization

As discussed above, the existing literature largely fails to provide a comprehensive framework that simultaneously addresses operational costs and power quality in rural LVDS. To address this, the main research question of this study is formulated as follows:

How can we effectively coordinate distributed PV inverters and BESS to minimize the comprehensive operational costs of rural LVDS, while strictly guaranteeing power quality under high PV penetration and severe load asymmetry? The underlying hypothesis of this work is that by integrating phase-decoupled power flow, three-phase PV active/reactive power support, and BESS dispatch into a unified mathematical framework, the system can achieve a global economic optimum without violating voltage and three-phase unbalance limits.

To bridge this research gap, this paper proposes an operational cost optimization model for rural LVDS that rigorously incorporates power quality constraints. Specifically, the decision variables encompass the three-phase active and reactive power outputs of distributed PV inverters, alongside the charging and discharging power of the BESS. The objective function aims to minimize the total operating cost of the LVDS, including BESS operational costs, while the constraints explicitly account for voltage limits and three-phase unbalance. Given that the original formulation is a non-convex optimization problem, it is mathematically reformulated into a Mixed-Integer Second-Order Cone Programming (MISOCP) model, enabling efficient computation via the Gurobi solver. Finally, comprehensive case studies are conducted using field data from a practical rural LVDS in China. Simulation results demonstrate the superior performance of the proposed method in mitigating voltage violations, alleviating three-phase unbalance, and reducing operational costs, thereby highlighting its substantial practical engineering value.

The remainder of this paper is organized as follows: Section 2 formulates the operational cost optimization model for the rural LVDS, detailing the power flow model, objective function, constraints, and the corresponding model solution method. Section 3 presents the case studies and validation, including the system setup, simulation results, a comprehensive comparative analysis, and a sensitivity analysis to demonstrate the robustness of the proposed strategy. Finally, Section 4 draws the conclusions.

2. Rural LVDS Operational Cost Optimization Model and Solution

2.1. Power Flow Model

LVDS typically adopt a three-phase four-wire configuration with a radial topology, as illustrated in Figure 1:

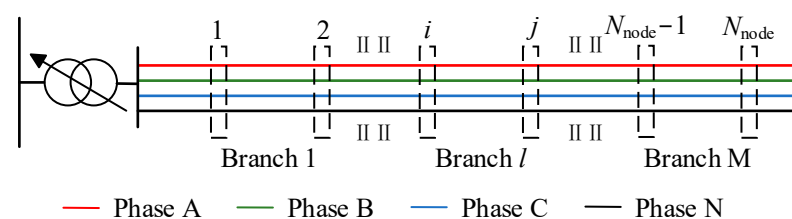


Figure 1. Typical structure of the LVDS.

In Figure 1, the presented LVDS comprise N_{node} nodes and M branches. Due to the extensive integration of single-phase loads and distributed PVs, severe three-phase unbalance issues frequently arise. Consequently, a three-phase power flow model is required to accurately characterize the operational state of the system. In rural LVDS, feeders are typically short and utilize overhead conductors with relatively large phase spacing, making the inter-phase mutual coupling much weaker than the self-impedance effects. Therefore, the inter-phase mutual impedance is neglected in this study to decouple phase-wise power flows. Although this simplification may introduce minor deviations in voltage calculations under severe phase coupling, it avoids non-convex cross-phase terms, thereby enabling the Second-Order Cone Programming (SOCP) relaxation applied in this study.

Ultimately, this simplification offers a practical trade-off between computational tractability and modeling accuracy for macro-level operational optimization, and its effectiveness has been widely supported in existing studies [23]. Taking the operational state of phase φ on branch l at time t as an example, a multi-period approximately decoupled DistFlow branch model is established [24], as depicted in Figure 2.

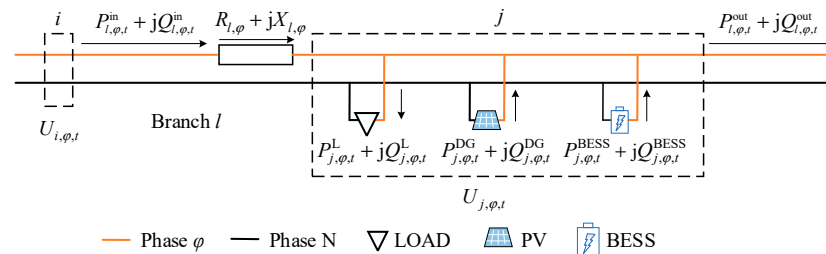


Figure 2. DistFlow branch model for phase φ on branch l in the LVDS.

In Figure 2, let i and j denote the indices of the sending-end and receiving-end nodes of branch l , respectively. Accordingly, the power flow model for branch l can be defined as:

$$\begin{cases} P_{l,\varphi,t}^{\text{in}} - R_{l,\varphi} I_{l,\varphi,t}^2 - P_{j,\varphi,t}^L + A_{j,\varphi}^{\text{DG}} P_{j,\varphi,t}^{\text{DG}} + A_{j,\varphi}^{\text{BESS}} P_{j,\varphi,t}^{\text{BESS}} = P_{l,\varphi,t}^{\text{out}} \\ Q_{l,\varphi,t}^{\text{in}} - X_{l,\varphi} I_{l,\varphi,t}^2 - Q_{j,\varphi,t}^L + A_{j,\varphi}^{\text{DG}} Q_{j,\varphi,t}^{\text{DG}} + A_{j,\varphi}^{\text{BESS}} Q_{j,\varphi,t}^{\text{BESS}} = Q_{l,\varphi,t}^{\text{out}} \end{cases} \quad (1)$$

$$U_{i,\varphi,t}^2 - U_{j,\varphi,t}^2 - 2(R_{l,\varphi} P_{l,\varphi,t}^{\text{in}} + X_{l,\varphi} Q_{l,\varphi,t}^{\text{in}}) + I_{l,\varphi,t}^2 (R_{l,\varphi}^2 + X_{l,\varphi}^2) = 0 \quad (2)$$

$$I_{l,\varphi,t}^2 = \frac{(P_{l,\varphi,t}^{\text{in}})^2 + (Q_{l,\varphi,t}^{\text{in}})^2}{U_{i,\varphi,t}^2} \quad (3)$$

In (1), the power balance equations for branch l are presented, where $i, j \in \{1, 2, \dots, N_{\text{node}}\}$ denote the indices of the sending-end and receiving-end nodes of branch l ; $l \in \{1, 2, \dots, M\}$ is the branch index; $\varphi \in \{A, B, C\}$ denotes the phase indicator; $P_{l,\varphi,t}^{\text{in}}$, $Q_{l,\varphi,t}^{\text{in}}$ and $P_{l,\varphi,t}^{\text{out}}$, $Q_{l,\varphi,t}^{\text{out}}$ are the active and reactive power flowing into and out of phase φ of branch l at time t . $R_{l,\varphi}$ and $X_{l,\varphi}$ represent the resistance and reactance of phase φ of branch l ; $I_{l,\varphi,t}$ is the corresponding current magnitude at time t . For node j on phase φ at time t , $P_{j,\varphi,t}^L$ and $Q_{j,\varphi,t}^L$ denote the load demands; $P_{j,\varphi,t}^{\text{DG}}$ and $Q_{j,\varphi,t}^{\text{DG}}$ denote the active and reactive power outputs of the distributed generator (DG); $P_{j,\varphi,t}^{\text{BESS}}$ and $Q_{j,\varphi,t}^{\text{BESS}}$ represent the power injected by the energy storage system (ESS); $A_{j,\varphi}^{\text{DG}}$ and $A_{j,\varphi}^{\text{BESS}}$ are binary indicator variables equal to 1 if the corresponding device is connected, and 0 otherwise. Furthermore, (2) formulates the voltage balance equation, where $U_{i,\varphi,t}$ and $U_{j,\varphi,t}$ denote the voltage magnitudes at nodes i and j . Finally, (3) defines the calculation of the current magnitude for phase φ on branch l at time t .

2.2. Operational Cost Optimization Model

To reduce the operational expenses of rural LVDS, this paper proposes an operational optimization model. By jointly optimizing the active and reactive power outputs of distributed PV systems and the charging/discharging power of the BESS, the model aims to

minimize the comprehensive electricity cost of the system while satisfying operational constraints such as voltage limits and three-phase unbalance.

2.2.1. Objective Function

Considering that high PV penetration frequently causes reverse power flow during midday, and that the pricing mechanisms for purchasing electricity from and selling electricity to the upstream grid are typically asymmetric, this paper establishes a refined model for the energy exchange cost. On this basis, the optimization objective is defined as minimizing the total daily operational cost of the LVDS. The objective function mainly consists of two components: the energy exchange cost with the upstream grid and the operational loss cost of the BESS. The specific objective function is formulated as:

$$\min F = \sum_{t=1}^T (C_t^{\text{grid}} + C_t^{\text{BESS}}) \quad (4)$$

where F represents the total daily operational cost of the LVDS and T is the total number of scheduling periods. In this study, the length of each scheduling period is set to 15 min, and thus $T = 96$. C_t^{grid} denotes the energy exchange cost with the upstream grid at time t and C_t^{BESS} denotes the operational loss cost of the ESS at time t .

To accurately evaluate the economic performance under bidirectional power flow, the energy exchange cost C_t^{grid} is decomposed into two components: the electricity purchasing cost and the electricity selling revenue. Specifically, the expressions for C_t^{grid} and C_t^{BESS} in (4) are given by:

$$C_t^{\text{grid}} = (P_t^{\text{in}} \rho_{1,t} - P_t^{\text{out}} \rho_2) \Delta t \quad (5)$$

$$C_t^{\text{BESS}} = P_t^{\text{BESS}} \rho_3 \Delta t \quad (6)$$

where P_t^{in} and P_t^{out} represent the active power purchased from and sold to the upstream grid at time t ; P_t^{BESS} is the total charging and discharging power of the ESS at time t ; $\rho_{1,t}$, ρ_2 and ρ_3 denote the time-of-use electricity purchasing price, the unit selling price of surplus PV power, and the unit energy loss cost during ESS operation at time t [25–27]; and Δt is the duration of the time step, which is set to 0.25 h.

The expressions for P_t^{in} and P_t^{out} in (5) are given by:

$$P_t^{\text{in}} = P_t^{\text{L}} + P_t^{\text{loss}} + P_t^{\text{BESS}} - P_t^{\text{DG}} \quad (7)$$

$$P_t^{\text{out}} = P_t^{\text{DG}} - P_t^{\text{L}} - P_t^{\text{loss}} - P_t^{\text{BESS}} \quad (8)$$

where P_t^{L} , P_t^{DG} and P_t^{loss} denote the total load demand, the total distributed PV output, and the total network losses at time t . As indicated by (4)–(8), an increase in the system's power demand leads to a corresponding rise in the operational cost. Therefore, while achieving economic operation, the proposed model simultaneously promotes renewable energy accommodation.

Furthermore, the expressions for P_t^{L} , P_t^{DG} , P_t^{loss} and P_t^{BESS} in (7) are formulated as:

$$P_t^{\text{L}} = \sum_j^{N_{\text{node}}} \sum_{\varphi} P_{j,\varphi,t}^{\text{L}} \quad (9)$$

$$P_t^{\text{DG}} = \sum_j^{N_{\text{node}}} \sum_{\varphi} A_{j,\varphi}^{\text{DG}} P_{j,\varphi,t}^{\text{DG}} \quad (10)$$

$$P_t^{\text{loss}} = \sum_l^M \sum_{\varphi} R_{l,\varphi} I_{l,\varphi,t}^2 \quad (11)$$

$$P_t^{\text{BESS}} = \sum_j^{N_{\text{node}}} \sum_{\varphi} A_{j,\varphi}^{\text{BESS}} (D_{j,\varphi,t}^{\text{char}} P_{j,\varphi,t}^{\text{char}} - D_{j,\varphi,t}^{\text{disc}} P_{j,\varphi,t}^{\text{disc}}) \quad (12)$$

where $P_{j,\varphi,t}^{\text{char}}$ and $P_{j,\varphi,t}^{\text{disc}}$ represent the charging and discharging power of the ESS at node j , phase φ , at time t ; $D_{j,\varphi,t}^{\text{char}}$ and $D_{j,\varphi,t}^{\text{disc}}$ denote the corresponding charging and discharging states. Specifically, $D_{j,\varphi,t}^{\text{char}}$ and $D_{j,\varphi,t}^{\text{disc}}$ are binary indicator variables, where 1 indicates that the ESS is in the corresponding operating state and 0 otherwise, subject to $D_{j,\varphi,t}^{\text{char}} + D_{j,\varphi,t}^{\text{disc}} \leq 1$.

2.2.2. Constraints

The constraints of the proposed operational optimization model for the LVDS include power flow constraints, PV operating constraints, ESS operating constraints, network operational constraints, system security constraints, and three-phase unbalance constraints. These are detailed as follows.

1. Power Flow Constraints

The three-phase power flow in the LVDS must satisfy (1) and (2).

2. PV Operating Constraints

The distributed PV systems are integrated into the network via inverters, and each phase can be regulated independently. During operation, the PV systems must satisfy the following constraints:

$$0 \leq P_{j,\varphi,t}^{\text{DG}} \leq P_{j,\varphi,t}^{\text{DG,max}} \quad (13)$$

$$-P_{j,\varphi,t}^{\text{DG,max}} \tan \phi_{j,\varphi} \leq Q_{j,\varphi,t}^{\text{DG}} \leq P_{j,\varphi,t}^{\text{DG,max}} \tan \phi_{j,\varphi} \quad (14)$$

where $P_{j,\varphi,t}^{\text{DG,max}}$ is the maximum active power of the PV inverter at node j , phase φ , at time t , and $\phi_{j,\varphi}$ is the corresponding power factor angle of the PV inverter at node j , phase φ .

3. ESS Operating Constraints

For the ESS units connected to the LVDS, each phase is also independently controllable. During operation, the ESS must satisfy the following constraints:

$$\begin{cases} 0 \leq D_{j,\varphi,t}^{\text{char}} P_{j,\varphi,t}^{\text{char}} \leq P_{j,\varphi}^{\text{char,max}} \\ 0 \leq D_{j,\varphi,t}^{\text{disc}} P_{j,\varphi,t}^{\text{disc}} \leq P_{j,\varphi}^{\text{disc,max}} \\ D_{j,\varphi,t}^{\text{char}} + D_{j,\varphi,t}^{\text{disc}} \leq 1 \end{cases} \quad (15)$$

$$\begin{cases} S_{j,\varphi}^{\text{min}} \leq S_{j,\varphi,t} \leq S_{j,\varphi}^{\text{max}} \\ S_{j,\varphi,t_0} = S_{j,\varphi,t_n} \end{cases} \quad (16)$$

where $P_{j,\varphi}^{\text{char,max}}$ and $P_{j,\varphi}^{\text{disc,max}}$ represent the maximum charging and discharging power limits of the ESS at node j , phase φ ; $S_{j,\varphi,t}$ is the SOC at time t ; $S_{j,\varphi}^{\text{min}}$ and $S_{j,\varphi}^{\text{max}}$ denote

the minimum and maximum allowable SOC limits; S_{j,φ,t_0} and S_{j,φ,t_n} are the initial and final SOC values of the scheduling day at node j , phase φ .

(15) gives the adjustable range of the charging and discharging power of the ESS and introduces two state variables, $D_{j,\varphi,t}^{\text{char}}$ and $D_{j,\varphi,t}^{\text{disc}}$, subject to $D_{j,\varphi,t}^{\text{char}} + D_{j,\varphi,t}^{\text{disc}} \leq 1$. This guarantees that at any given moment, the per-phase ESS operates strictly in one of three states: charging, discharging, or idle. (16) specifies the allowable range of the ESS SOC and further imposes a daily cycle constraint, i.e., the SOC at the beginning of the 24 h scheduling horizon is equal to that at the end.

4. Network Operational Constraints

To conform to the practical physical operating characteristics, P_t^{in} and P_t^{out} of the LVDS at time t must satisfy:

$$P_t^{\text{in}} - P_t^{\text{out}} = P_t^{\text{L}} - P_t^{\text{DG}} + P_t^{\text{loss}} + P_t^{\text{BESS}} \quad (17)$$

$$P_t^{\text{in}} \geq 0 \quad (18)$$

$$P_t^{\text{out}} \geq 0 \quad (19)$$

5. System Security Constraints

To ensure the secure operation of the LVDS, the nodal voltage magnitude and branch current magnitude must satisfy the following constraints:

$$U_i^{\min} \leq U_{i,\varphi,t} \leq U_i^{\max} \quad (20)$$

$$I_{l,\varphi,t} \leq I_l^{\max} \quad (21)$$

where $U_i^{\min}$ and $U_i^{\max}$ represent the lower and upper bounds of the voltage magnitude at node i ; and $I_l^{\max}$ is the maximum allowable current magnitude flowing through branch l .

6. Three-phase Unbalance Constraints

To guarantee the power quality of the LVDS, the three-phase voltage unbalance factor (VUF) must satisfy:

$$0 \leq \varsigma_{i,t} \leq \varsigma_{\max} \quad (22)$$

where $\varsigma_{\max}$ denotes the upper limit of the VUF, and $\varsigma_{i,t}$ is the actual VUF at node i at time t .

2.2.3. Model Solution

The proposed operational cost optimization model for the rural LVDS, which incorporates power quality constraints, contains nonlinear and non-convex terms arising from the AC power flow equations, as well as discrete integer variables representing the operating states of the ESS. Therefore, the original problem is a typical MINLP problem. Since MINLP is generally NP-hard, directly applying heuristic algorithms or interior-point methods usually cannot guarantee global optimality and may result in low computational efficiency [28]. Additionally, while alternative convexification techniques such as Semi-Definite Programming (SDP) [29] relaxation could theoretically provide tighter bounds, they impose prohibitive computational burdens when dealing with integer variables.

To achieve efficient and reliable computation while balancing modeling accuracy, this paper adopts an approximately decoupled DistFlow branch power flow model. By

introducing auxiliary variables, such as the squared voltage magnitudes and squared current magnitudes [30], the original nonlinear objective function is transformed into a linear form. Meanwhile, the original non-convex power flow equations are relaxed into convex second-order cone inequalities, thereby converting the original problem into a computationally tractable MISOCP problem. The model is implemented in the YALMIP modeling toolbox and solved using the Gurobi solver.

The conic part of the reformulated model can be written in the standard SOCP form [31]:

$$\|A_i x = b_i\|_2 \leq c_i^T x + d_i, i = 1, 2 \cdots n \quad (23)$$

where $x \in R^n$ represents an n -dimensional decision vector, and $x \in R^n$, $b_i \in R^m$, $c_i \in R^n$, and $d_i \in R$ denote constant parameters of appropriate dimensions.

As can be seen from (7) and (8), the network loss term in the objective function contains nonlinear squared terms. Therefore, to linearize the objective function, an auxiliary variable representing the squared branch current magnitude is introduced:

$$\tilde{I}_{l,\varphi,t} = I_{l,\varphi,t}^2 \quad (24)$$

Substituting (24) into (11) eliminates the squared term of P_t^{loss} , yielding:

$$P_t^{\text{loss}*} = \sum_l \sum_{\varphi} R_{l,\varphi} \tilde{I}_{l,\varphi,t} \quad (25)$$

Accordingly, (7) and (8) can be reformulated as:

$$P_t^{\text{in}} = P_t^{\text{L}} + P_t^{\text{loss}*} + P_t^{\text{BESS}} - P_t^{\text{DG}} \quad (26)$$

$$P_t^{\text{out}} = P_t^{\text{DG}} - P_t^{\text{L}} - P_t^{\text{loss}*} - P_t^{\text{BESS}} \quad (27)$$

It can be seen from (1)–(3) that squared terms also exist in the power flow constraints. Therefore, the non-convex parts of the power flow constraints need to be convexified. Based on (24), an auxiliary variable representing the squared nodal voltage magnitude is further introduced:

$$\tilde{U}_{l,\varphi,t} = U_{l,\varphi,t}^2 \quad (28)$$

Substituting the auxiliary variables defined in (24) and (28) into the power flow (1)–(3) yields the following linearized expressions:

$$\begin{cases} P_{l,\varphi,t}^{\text{in}} - R_{l,\varphi} \tilde{I}_{l,\varphi,t} - P_{j,\varphi,t}^{\text{L}} + A_{j,\varphi}^{\text{DG}} P_{j,\varphi,t}^{\text{DG}} + A_{j,\varphi}^{\text{BESS}} P_{j,\varphi,t}^{\text{BESS}} = P_{l,\varphi,t}^{\text{out}} \\ Q_{l,\varphi,t}^{\text{in}} - X_{l,\varphi} \tilde{I}_{l,\varphi,t} - Q_{j,\varphi,t}^{\text{L}} + A_{j,\varphi}^{\text{DG}} Q_{j,\varphi,t}^{\text{DG}} + A_{j,\varphi}^{\text{BESS}} Q_{j,\varphi,t}^{\text{BESS}} = Q_{l,\varphi,t}^{\text{out}} \end{cases} \quad (29)$$

$$\tilde{U}_{i,\varphi,t} - \tilde{U}_{j,\varphi,t} - 2(R_{l,\varphi} P_{l,\varphi,t}^{\text{in}} + X_{l,\varphi} Q_{l,\varphi,t}^{\text{in}}) + \tilde{I}_{l,\varphi,t} (R_{l,\varphi}^2 + X_{l,\varphi}^2) = 0 \quad (30)$$

$$\tilde{I}_{l,\varphi,t} = \frac{(P_{l,\varphi,t}^{\text{in}})^2 + (Q_{l,\varphi,t}^{\text{in}})^2}{\tilde{U}_{i,\varphi,t}} \quad (31)$$

For the non-convex equality constraint in (31), the second-order cone relaxation (SOCR) technique is adopted to relax it into the following convex inequality constraint:

$$\tilde{I}_{l,\varphi,t} \geq \frac{(P_{l,\varphi,t}^{\text{in}})^2 + (Q_{l,\varphi,t}^{\text{in}})^2}{\tilde{U}_{i,\varphi,t}} \quad (32)$$

This constraint can be further rewritten in the standard second-order cone form:

$$\left\| \begin{array}{c} 2P_{l,\varphi,t}^{\text{in}} \\ 2Q_{l,\varphi,t}^{\text{in}} \\ \tilde{I}_{l,\varphi,t} - \tilde{U}_{i,\varphi,t} \end{array} \right\|_2 \leq \tilde{I}_{l,\varphi,t} + \tilde{U}_{i,\varphi,t} \quad (33)$$

By transforming the nonlinear variables in the objective function into linear ones and relaxing the non-convex constraints, the original problem can be converted into a MISOCP model, formulated as:

$$\left\{ \begin{array}{l} \min F = \sum_{t=1}^T (C_t^{\text{grid}} + C_t^{\text{BESS}}) \\ \text{s.t. Eq. (5)(6)(9)(10)(12)(13)(14)} \\ \quad (15)(16)(17)(18)(19)(20)(21) \\ \quad (22)(25)(26)(27)(29)(30)(33) \end{array} \right. \quad (34)$$

Since the rural LVDS studied in this paper has a typical radial topology, and the objective function is strictly increasing with respect to the squared branch current magnitude, the sufficient conditions for the exactness of the second-order cone relaxation are satisfied. Therefore, the inequality in (33) holds as an equality at the optimal solution, which guarantees that the relaxed solution is also an exact physically feasible solution to the original AC power flow equations [32].

3. Case Studies and Validation

3.1. Setup and System Description

To verify the effectiveness of the proposed strategy, simulation case studies are conducted using field data from a practical 37-node 0.4-kV rural test system in China. It should be noted that due to the physical voltage drop limits at the 0.4 kV level, the spatial scale of a single rural low voltage distribution system is inherently constrained. Therefore, this 37-node test system is highly representative of the typical scale and ‘weak grid’ characteristics (e.g., high R/X ratio) of real-world rural distribution areas. The topology of this test system is illustrated in Figure 3. The voltage at the low-voltage side of the distribution transformer is set to 1.03 p.u. The line parameters are configured according to the IEEE European Low Voltage Test Feeder standard [33]. According to the Chinese National Standards GB/T 12325-2008 and GB/T 15543-2008, the allowable voltage deviation range during normal operation is 0.93–1.07 p.u., and the upper limit of the VUF is 2% [34,35]. When solving the resulting MISOCP model, the optimality gap is set to 1%.

In the test LVDS, a distributed ESS is installed at node 19 and connected independently to each phase. The ESS capacity of each phase is 5 kW/25 kWh, with a charging/discharging efficiency of 95%. Residential distributed PV systems are connected at eight nodes, and their connection schemes and installed capacities are shown in Figure 4. Loads are connected at all nodes, and the load connection schemes and maximum capacities are shown in Figure 5.

As shown in Figure 5, the system includes a mixture of single-phase, two-phase, and three-phase loads, and the load capacities vary significantly across nodes. This distribution reflects the typical operating characteristics of LVDS, including uneven spatial load distribution and local phase asymmetry.

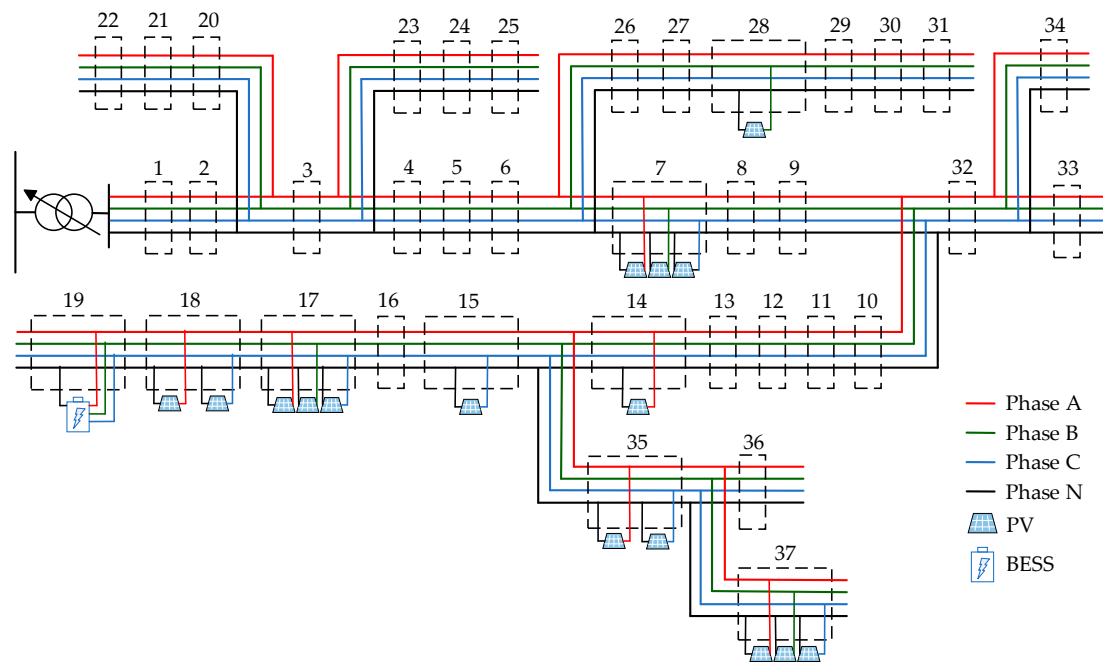


Figure 3. Topology of the 37-node rural test system.

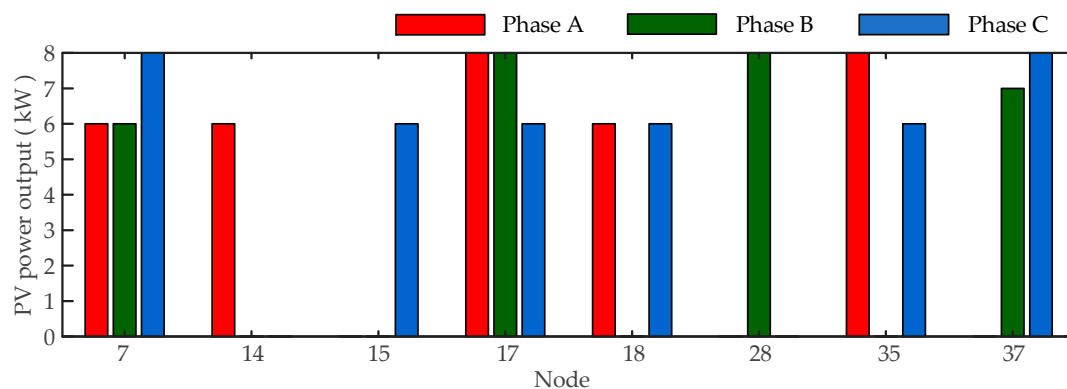


Figure 4. Nodal locations and installed capacities of the distributed PV systems.

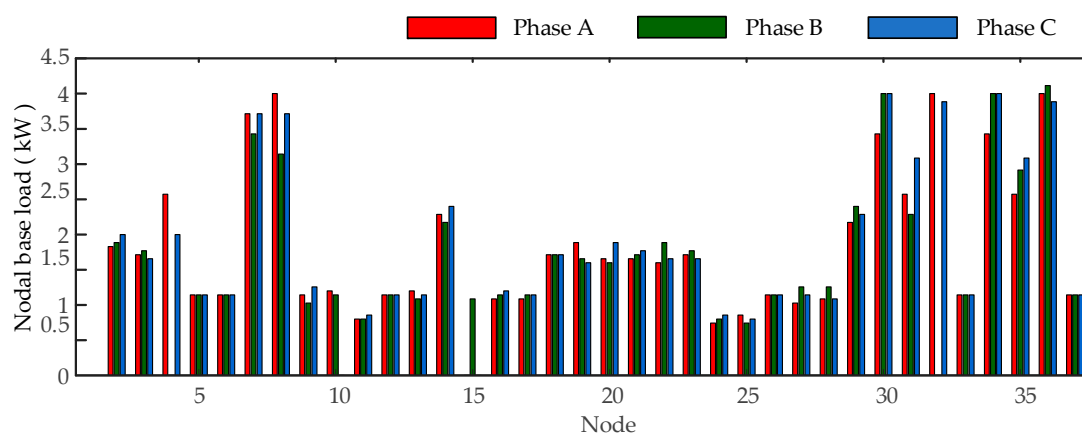


Figure 5. Phase connection schemes and peak capacities of the nodal base loads.

Considering the small geographical area of the LVDS, the case study assumes that solar irradiance conditions and user electricity consumption patterns are basically identical. Therefore, the per-unit profiles of phase loads and distributed PV outputs are as-

sumed to be the same at all nodes. The corresponding temporal profiles are shown in Figure 6.

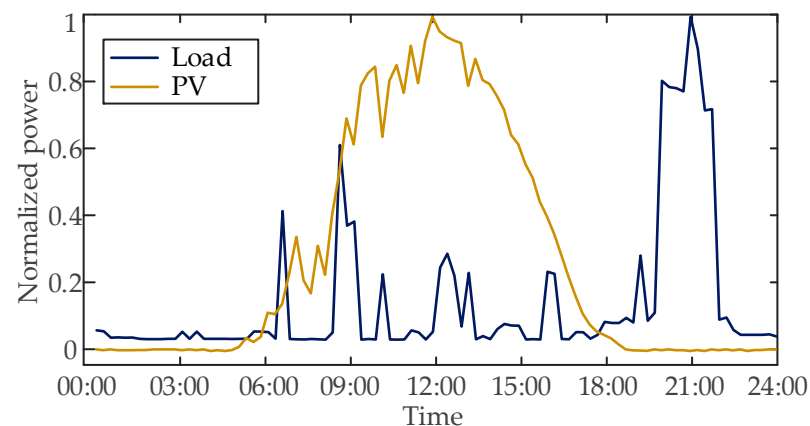


Figure 6. Normalized daily profiles of load demand and PV output in the LVDS.

3.2. Results

Initially, based on the aforementioned temporal profiles of load demand and distributed PV output, a 24 h three-phase unbalanced power flow analysis is carried out for the test LVDS.

Figure 7a–c show the spatiotemporal distribution of nodal voltages in the test LVDS. In the figure, regions from yellow to red indicate upper-limit voltage violations, while dark blue regions indicate lower-limit voltage violations. The statistical results show that voltage violations in the test LVDS are relatively severe. From a temporal perspective, among the 96 time periods in a day, voltage upper-limit and lower-limit violations in at least one phase occur in 29 periods (30.21%) and 3 periods (3.12%), respectively. From a spatial perspective, among the 37 nodes in the network, 18 nodes (48.65%) and 12 nodes (32.43%) experience upper-limit and lower-limit voltage violations, respectively.

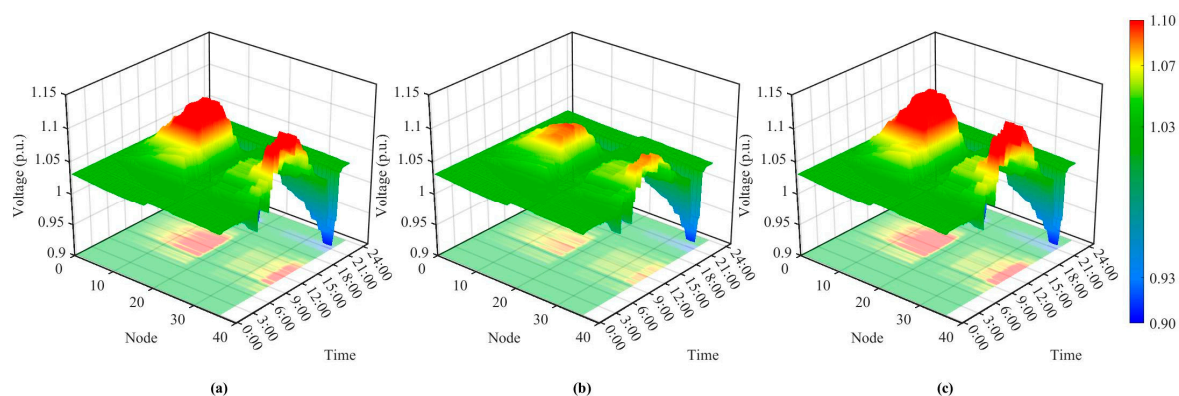


Figure 7. Spatiotemporal distribution of the initial nodal voltages. (a) Phase A; (b) Phase B; (c) Phase C.

Further analysis of the spatiotemporal distribution shows that during 10:00–16:00, due to the coincidence of peak PV output and low load demand, widespread upper-limit voltage violations occur in the LVDS. In contrast, during the nighttime period from 20:00 to 22:00, lower-limit voltage violations appear at some end nodes because of the absence of PV output and the evening load peak.

To further examine the spatial distribution of voltage violations, the operating snapshot at 12:00, when the most severe overvoltage occurs during the day, is selected for de-

tailed analysis, as shown in Figure 8. The statistical results indicate that at this time, 16 nodes (43.24% of the total nodes) experience voltage violations in at least one phase. In particular, nodes 10–19 and 35–37 even exhibit simultaneous upper-limit violations in all three phases. Among them, node 19 shows the most severe voltage rise, with the maximum phase voltage reaching 1.15 p.u.

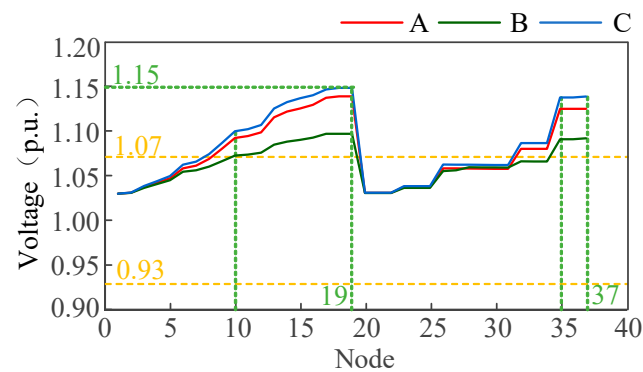


Figure 8. Initial nodal voltages at 12:00.

The above results indicate that under the combined effect of high PV penetration and light-load conditions, significant reverse power flow can lead to severe upper-limit voltage violations. This not only threatens the secure and stable operation of the LVDS, but also highlights the necessity of introducing voltage control strategies.

In addition, due to the combined effects of single-phase distributed PV integration and asymmetric load characteristics, the test LVDS also suffer from severe three-phase voltage unbalance. Figure 9 shows the spatiotemporal distribution of the VUF at each node in the test LVDS. In the figure, regions from yellow to red indicate that the VUF exceeds the allowable limit. The statistical results show that, from a temporal perspective, among the 96 time periods in a day, there are 21 periods (21.9%) in which at least one node has a VUF exceeding the limit. From a spatial perspective, among the 37 nodes in the network, 10 nodes (27.03%) experience VUF violations.

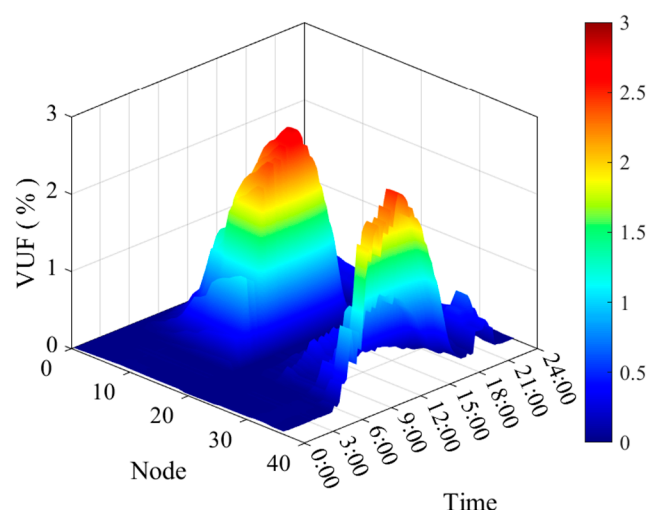


Figure 9. Spatiotemporal distribution of the initial VUF.

To further investigate its spatial distribution, the 12:00 operating snapshot is again selected for analysis, as shown in Figure 10. The statistical results indicate that at this time, 10 nodes (27.03% of the total nodes) have VUF values exceeding the safety limit. Specifi-

cally, nodes 13–19 and 35–37 show significant unbalance violations, among which node 19 exhibits the most severe three-phase unbalance, with a VUF of up to 2.78%.

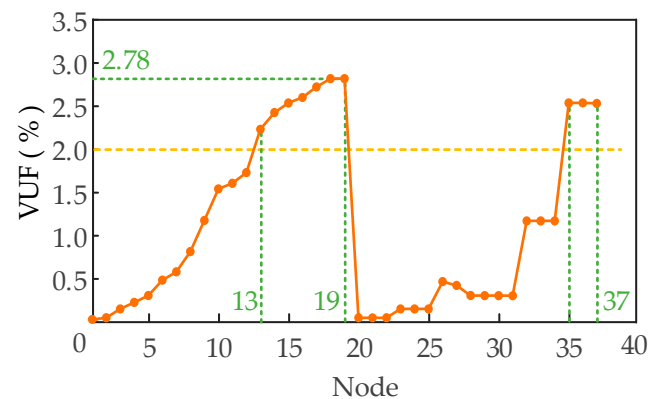


Figure 10. Initial VUF at 12:00.

Severe three-phase voltage unbalance not only increases network losses and additional transformer temperature rise, but may also lead to serious safety hazards such as excessive neutral current. To simultaneously address the above power quality issues, namely voltage violations and three-phase unbalance, the proposed operational cost optimization model with power quality constraints is applied to perform global optimization for the typical daily operating condition of the test LVDS.

Figure 11a–c show the spatiotemporal distribution of nodal voltages after optimization using the proposed model. The simulation results show that during the 24 h scheduling horizon, the large-area voltage violations observed in the initial state are completely eliminated, and the three-phase voltages at all nodes are maintained within the safe operating range of 0.93–1.07 p.u.

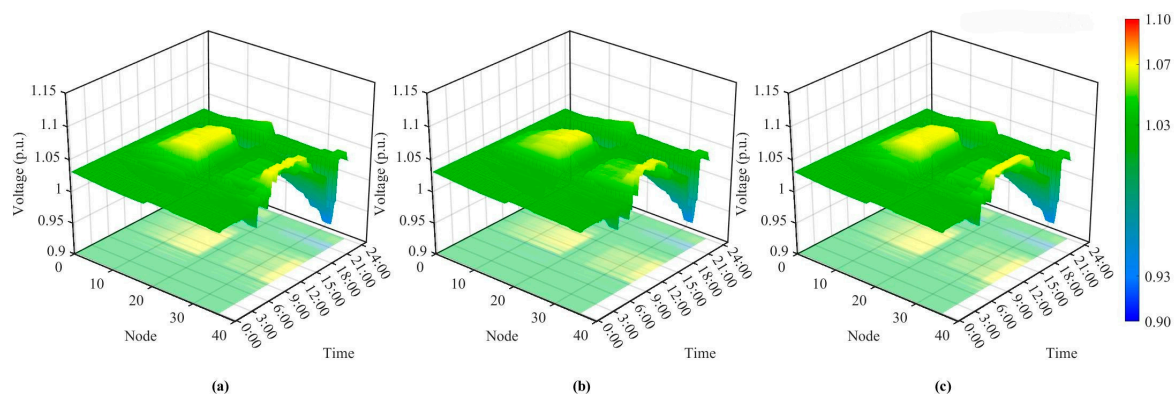


Figure 11. Spatiotemporal distribution of the optimized nodal voltages. (a) Phase A; (b) Phase B; (c) Phase C.

Taking the 12:00 operating snapshot, which initially experiences the most severe overvoltage, as an example, Figure 12 shows that after applying the proposed strategy, the nodal voltages throughout the LVDS are effectively regulated. The voltage rise caused by PV reverse power flow is fully suppressed, and all voltage constraints are satisfied.

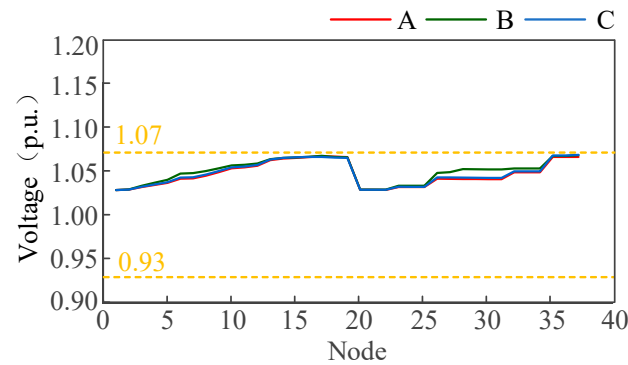


Figure 12. Optimized nodal voltages at 12:00.

For the three-phase unbalance issue, Figure 13 shows the spatiotemporal distribution of the optimized VUF at each node. The simulation results indicate that, after regulation by the proposed model, the VUF at all nodes is significantly reduced over the entire spatial and temporal range and remains within the 2% limit specified by the national standard.

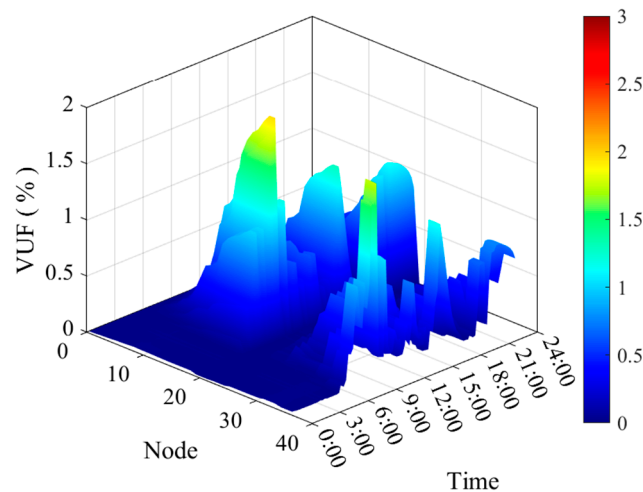


Figure 13. Spatiotemporal distribution of the optimized VUF.

The 12:00 operating snapshot is also selected for analysis. As shown in Figure 14, the severe unbalance distortion that previously reached 2.78% is successfully suppressed, and the VUF at all nodes is reduced to within 2%, satisfying the national standard.

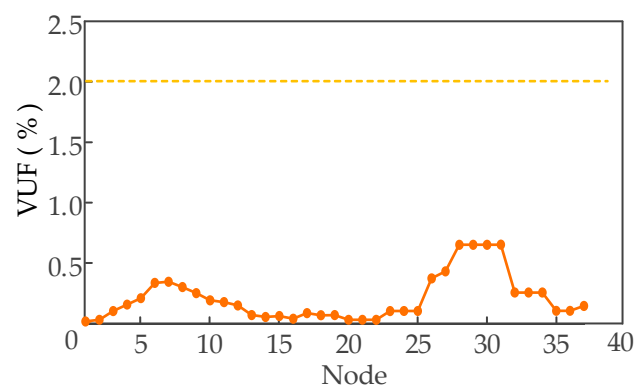


Figure 14. Optimized VUF at 12:00.

From the above comparative results in both temporal and spatial dimensions, it can be concluded that under high-PV-penetration conditions, the proposed optimization

model can not only eliminate voltage violations on a system-wide scale, but also effectively mitigate three-phase unbalance, thereby providing support for the safe and stable operation of the LVDS.

Figure 15a–d compare the total PV active power outputs of each phase and the three-phase total before and after optimization. The shaded areas represent PV curtailment during the typical day. Around noon, when PV output is high and the system load is low, the system mitigates voltage violations by actively curtailing PV active power.

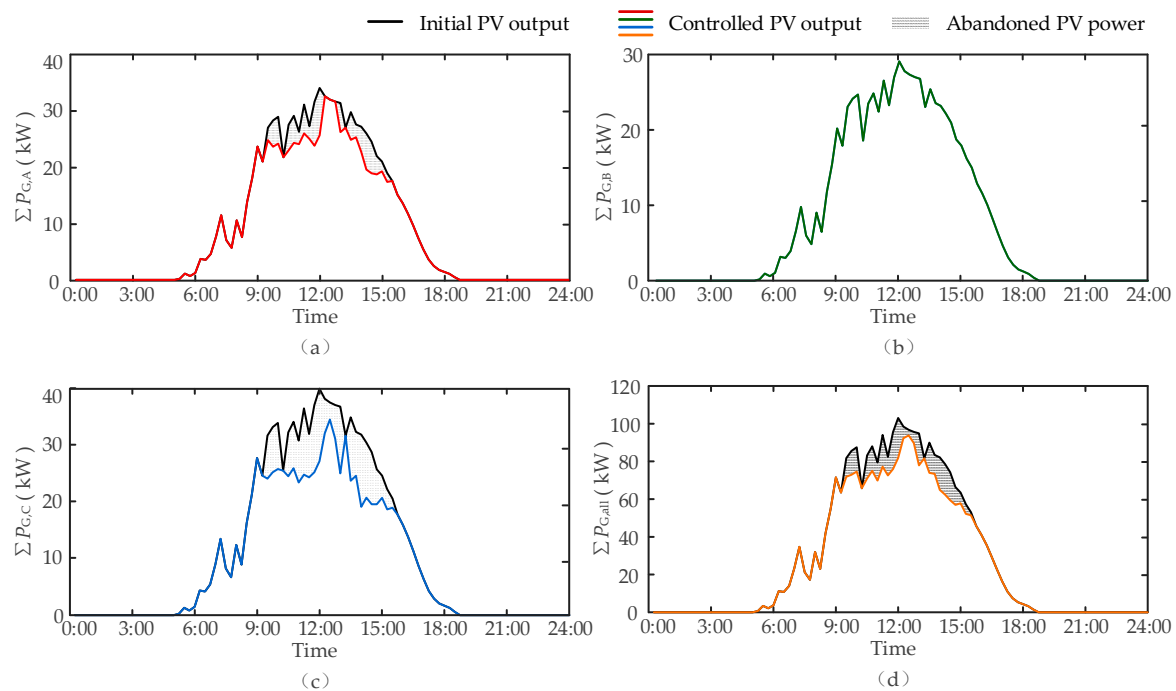


Figure 15. Total PV active power outputs of individual phases and the three-phase total before and after optimization. (a) Phase A; (b) Phase B; (c) Phase C; (d) Three-phase aggregate.

Figure 16a–d show the time-series profiles of total PV reactive power for each phase and the three-phase total before and after optimization. Around noon, although reactive power injection increases significantly, its effect on nodal voltage regulation remains relatively limited. In contrast, during the nighttime load peak, when PV active power output is zero, the PV inverters provide reactive power support for the system voltage. Overall, because the distribution lines have a high resistance-to-reactance ratio ($R/X > 10$), the voltage sensitivity to reactive power is relatively low, which limits the overall effectiveness of reactive power regulation.

Figure 17 shows the time-series profiles of the ESS SOC for each phase. Figure 18a–d show the time-series variations in ESS active power for each phase and the three-phase total. Around noon, as the system voltage gradually approaches the upper limit, the ESS enters the charging state to absorb excess power, thereby effectively alleviating overvoltage. During the nighttime load peak, the ESS switches to the discharging state to support the system voltage. These results demonstrate the important role of the ESS in peak shaving and valley filling, smoothing power fluctuations, and maintaining system power balance.

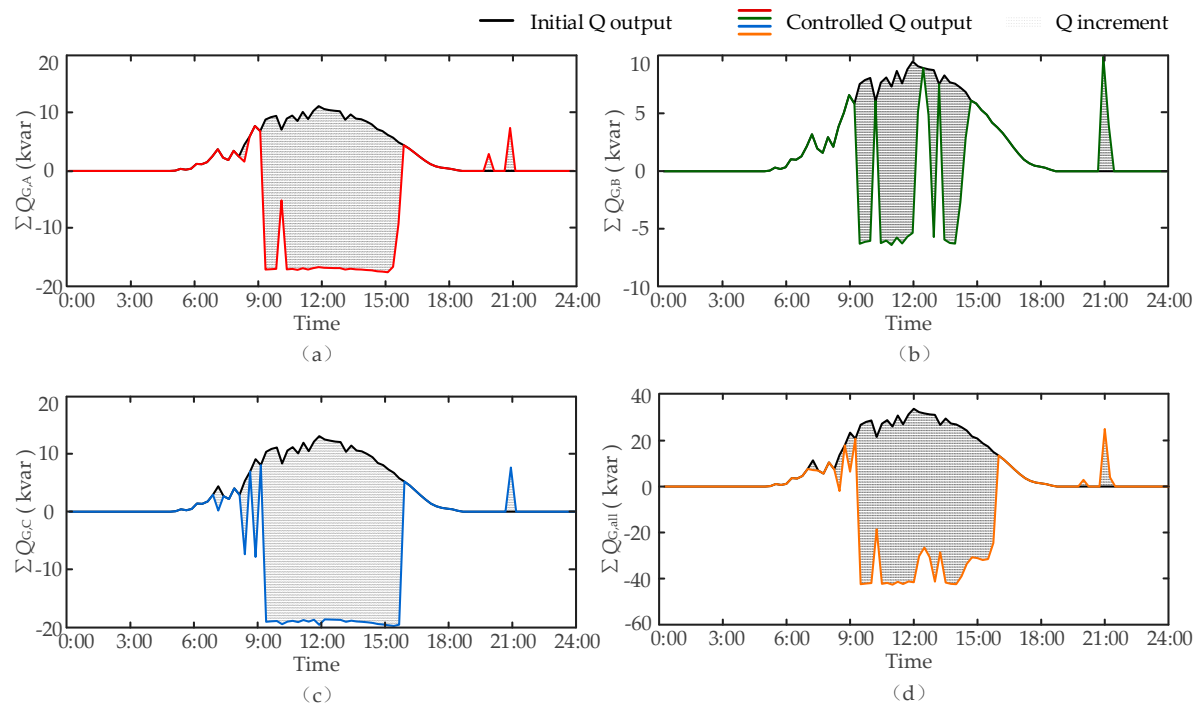


Figure 16. Total PV reactive power outputs of individual phases and the three-phase total before and after optimization. (a) Phase A; (b) Phase B; (c) Phase C; (d) Three-phase aggregate.

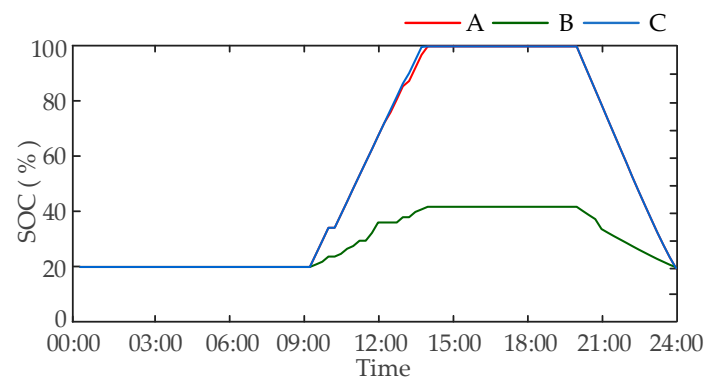


Figure 17. SOC of the ESS for each phase.

The daily operating cost of the test LVDS under the initial condition is 53.69 CNY. However, it should be noted that this initial state suffers from power quality issues, such as voltage violations and three-phase unbalance. After optimization using the proposed model, these issues are effectively resolved to ensure the safe and stable operation of the LVDS, which consequently leads to an increase in the daily operating cost to 71.21 CNY. A detailed comparison of key operational indicators is given in Table 2.

Table 2. Comparison of operational indicators before and after optimization.

Indicator	Initial Condition	After Optimization Using the Proposed Method
Daily network loss (kWh)	51.05	26.90
Daily PV curtailment (kWh)	0	66.53
ESS regulation energy (kWh)	0	91.25
Daily operational cost (CNY)	53.69	71.21

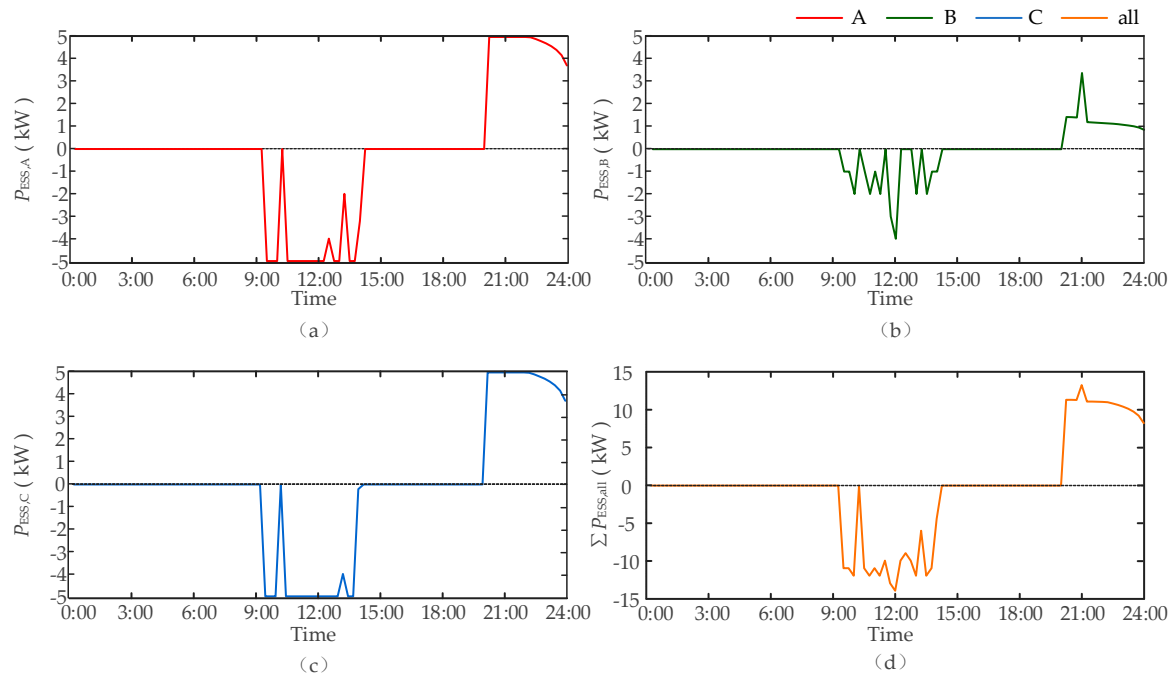


Figure 18. Time-series variations in ESS active power for individual phases and the three-phase total. (a) Phase A; (b) Phase B; (c) Phase C; (d) Three-phase total.

As shown in Table 2, the increase in operational cost is primarily attributed to the necessary PV curtailment and the energy-loss costs associated with ESS charging and discharging during the optimization process. Finally, regarding the computational performance, the proposed optimization model is solved in approximately 22.27 seconds on average. This demonstrates its high computational efficiency and suitability for practical day-ahead and intra-day scheduling in rural LVDS.

3.3. Comparative Analysis and Discussion

To further verify the effectiveness of the proposed method in mitigating three-phase voltage unbalance and improving the economic performance of the system, the following three regulation strategies are considered for comparison:

Strategy 1: An improved centralized coordinated control strategy [36]. Under this strategy, the PV inverters and ESS units at each node perform phase-wise adaptive regulation according to the sensitivity coefficients of the real-time voltage magnitudes of their connected phases. The control logic follows a multi-stage coordinated response mechanism: reactive power support first, ESS active power buffering second, and PV active power curtailment last.

Strategy 2: A conventional local autonomous droop control strategy. Under this strategy, the ESS units and PV inverters operate independently based on local voltage measurements without centralized coordination. Specifically, the ESS employs an active power-voltage droop mechanism to charge during voltage rises and discharge during voltage drops. Meanwhile, the PV inverters utilize a reactive power-voltage droop for local voltage support, followed by active power curtailment if the overvoltage persists.

Strategy 3: The comprehensive optimal regulation strategy proposed in this paper, which simultaneously considers power quality and operational economy.

Figure 19 shows the time-series variation in the maximum VUF among all nodes over the day under the three regulation strategies.

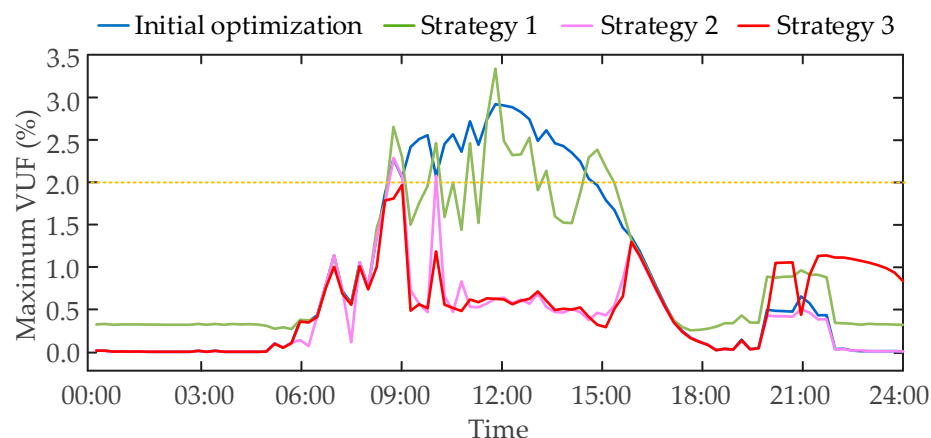


Figure 19. Comparison of the maximum VUF at each time step under different regulation strategies.

Figure 19 shows that, before regulation, the maximum daily VUF of the system is 2.78%. After applying Strategy 1, the maximum daily VUF not only fails to decrease below the specified limit of 2%, but instead increases to 3.22%. This indicates the limitations of the conventional sensitivity-based strategy, which regulates phases independently and may exacerbate unbalance under complex three-phase conditions. Under Strategy 2 (droop control), the maximum VUF is reduced to 2.14%. Although it improves the overall voltage profile, it still slightly exceeds the limit because local droop controllers lack the global awareness required to explicitly minimize the voltage differences among the three phases. In contrast, after applying Strategy 3 (the proposed method), the maximum VUF is effectively reduced to 1.96%, satisfying the national power quality standard of remaining below 2%.

These results indicate that the proposed method is more effective in mitigating three-phase voltage unbalance.

Table 3 compares the key operational indicators and total costs of the system under the three regulation strategies, including the relative percentage improvements achieved by the proposed method.

Table 3. Comparison of daily operational indicators and costs under different strategies.

Indicator	Strategy 1	Strategy 2	Strategy 3 (Proposed)	Improvement vs. S1 (%)	Improvement vs. S2 (%)
Daily network loss (kWh)	31.9	28.68	26.9	+15.67%	+6.21%
Daily PV curtailment (kWh)	135.66	274.71	66.53	+50.96%	+75.78%
ESS regulation energy (kWh)	120.18	74.94	91.25	-	-
Daily operational cost (CNY)	86.95	111.72	71.21	+18.10%	+36.26%

As shown in Table 3, compared with Strategy 1 and Strategy 2, the proposed method (Strategy 3) achieves significant reductions in daily network loss, daily PV curtailment, and overall operational cost. Specifically, compared to Strategy 1, the proposed method reduces network loss, PV curtailment, and operational cost by 15.67%, 50.96%, and 18.10%, respectively. When compared to Strategy 2, these improvements are even more pronounced, reaching 6.21%, 75.78%, and 36.26%, respectively.

From a technical perspective, these differences stem from the inherent control mechanisms of each strategy. Strategy 1 relies on local sensitivity coefficients, which can lead to excessive and uncoordinated ESS cycling (highest ESS regulation energy at 120.18 kWh) and suboptimal PV curtailment when dealing with complex three-phase coupling. Strategy 2 (droop control) strictly enforces local voltage limits without global awareness. Technically, to prevent overvoltage, local droop controllers aggressively curtail PV output, re-

sulting in massive PV curtailment (274.71 kWh) and the highest operational cost (111.72 CNY). In contrast, Strategy 3 utilizes a centralized MISOCP framework that globally optimizes power flows and explicitly coordinates the three phases. This global awareness allows the system to precisely dispatch the ESS (91.25 kWh) to buffer energy and minimize network losses, thereby maximizing PV accommodation and achieving the lowest overall cost (71.21 CNY) without violating power quality constraints.

These results demonstrate the advantages of the proposed method in improving PV accommodation capability, reducing regulation-related losses, and enhancing the overall operational economy of the system.

3.4. Sensitivity Analysis

To further validate the robustness and adaptability of the proposed method (Strategy 3), a comprehensive sensitivity analysis was conducted. Four key parameters were varied: energy prices ($\pm 20\%$), BESS capacity (-50% , $+50\%$), PV power profiles ($\pm 20\%$), and load profiles ($\pm 20\%$). The results are summarized in Table 4.

Table 4. Sensitivity analysis of the proposed method under varying system parameters.

Parameter	Variation	Daily Operational Cost (CNY)	Daily PV Curtailment (kWh)	Voltage Violations	Unbalance Violations
Base Case	Original data	71.21	66.53	None	None
Energy prices	-20%	56.97	66.53	None	None
	$+20\%$	85.46	66.53	None	None
BESS capacity	-50%	76.54	89.33	None	None
	$+50\%$	66.31	44.45	None	None
PV power	-20%	96.72	19.17	None	None
	$+20\%$	48.03	121.71	None	None
Load profiles	-20%	22.34	72.44	None	None
	$+20\%$	119.19	57.42	None	None

As shown in Table 4, the proposed method successfully maintains both voltage profiles and three-phase unbalance within safe limits (no voltage or unbalance violations) across all tested scenarios. This demonstrates its strong reliability in ensuring comprehensive power quality, even under severe parameter fluctuations. Economically, the daily operational cost and PV curtailment respond logically to parameter variations. For instance, a 50% reduction in BESS capacity leads to an increase in both cost and PV curtailment due to the weakened energy buffering capability. Conversely, an increase in load demand reduces PV curtailment as more solar power is consumed locally, though it increases the overall cost due to higher grid purchasing. Overall, the sensitivity analysis confirms that the proposed strategy remains highly effective, robust, and economically optimal under diverse operating conditions.

4. Conclusions

To address the severe power quality issues caused by high-penetration distributed PV integration in rural LVDS, this paper proposes an operational cost optimization model for rural LVDS that explicitly incorporates power quality constraints. Based on the case studies and comparative analysis, the main conclusions are summarized as follows:

- (1) In terms of model formulation and solution, the proposed model minimizes the total daily operational cost of the rural LVDS, including the charging and discharging losses of the ESS, while explicitly considering power quality constraints such as nodal voltage deviation and three-phase unbalance. The original non-convex nonlinear

problem is reformulated as an MISOCP model, which enables efficient global optimization. The case studies show that the proposed method can maintain nodal voltages at all time steps within the safe operating range of 0.93–1.07 p.u. and reduce the maximum VUF of the system to below the national standard limit of 2%. These results demonstrate that the proposed method can effectively ensure the safe and high-quality operation of rural LVDS under high PV penetration.

- (2) In terms of comprehensive performance and economic benefits, compared with both the improved centralized coordinated control strategy (Strategy 1) and the conventional droop control strategy (Strategy 2), the proposed method shows clear advantages. Under the same typical daily operating conditions, the proposed strategy not only overcomes the limitations of conventional strategies in suppressing three-phase unbalance and avoiding massive PV curtailment, but also significantly reduces operational costs. Specifically, compared to Strategy 1, the daily network loss, daily PV curtailment, and daily operational cost are reduced by approximately 15.67%, 50.96%, and 18.10%, respectively (with even greater cost and curtailment reductions compared to Strategy 2). These results verify that the proposed method can effectively balance cost control and PV accommodation, providing a promising practical solution for the stable and reliable operation of rural LVDS.
- (3) In terms of robustness and adaptability, the comprehensive sensitivity analysis confirms that the proposed strategy maintains its effectiveness under diverse operating conditions. Although variations in energy prices, BESS capacity, PV generation, and load demand significantly impact the operational costs and PV curtailment rates, the proposed method consistently ensures strict compliance with power quality constraints (i.e., no voltage or VUF violations) while achieving the optimal economic dispatch under all tested scenarios.

Finally, while the decoupled three-phase power flow model ensures computational efficiency, neglecting inter-phase mutual impedances may introduce slight calculation deviations under extreme phase coupling conditions. Future work will explore advanced convexification techniques or distributed algorithms to incorporate exact mutual coupling effects into the joint optimization framework. Additionally, since the current deterministic framework does not explicitly consider the inherent uncertainties of PV generation and load demand, forecast errors may affect real-time optimality. Specifically, under conditions of severe forecast deviations or extreme phase coupling, the current method might experience degraded performance and fail to produce the expected optimal results. Therefore, incorporating uncertainty management techniques, such as robust optimization or Model Predictive Control, will be a key focus of our future research. Furthermore, while this study focuses on the operational optimization of existing BESS assets, the sensitivity analysis reveals that BESS capacity significantly impacts daily costs. Therefore, future research will explore joint planning-and-operation co-optimization to determine the global optimal BESS sizing by balancing the massive initial capital expenditure against the long-term operational savings. Ultimately, coordinated optimization under market-based mechanisms, along with the exploration of BESS reactive power capabilities, will be investigated to comprehensively enhance both the power quality and economic operation of rural LVDS.

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Abbreviations

The following abbreviations are used in this manuscript:

PV	Photovoltaic
LVDS	Low-voltage distribution systems
ESS	Energy storage systems
SOCR	Second-order cone relaxation
BESS	Battery energy storage systems
SOC	State of charge
MINLP	Mixed-Integer Nonlinear Programming
MISOCP	Mixed-Integer Second-Order Cone Programming
DG	Distributed generator
VUF	Three-phase voltage unbalance factor
EV	Electric vehicle

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