

Article

Safety-Aware Allocation of Hybrid PV-WT Generation in Unbalanced Feeders Incorporating Load-Following Errors and Power Unbalance Ratio Limits

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Abstract

Existing DG allocation studies commonly rely on static or balanced feeder assumptions, leaving two practical issues insufficiently addressed: the hourly mismatch between renewable outputs and feeder demands and diesel-backup phase-imbalance safety in unbalanced networks. This paper presents a safety-aware allocation framework for hybrid photovoltaic (PV) and wind turbine (WT) systems in unbalanced three-phase feeders. The methodology explicitly accounts for 24 h generation–load coordination and diesel backup operating limits through two post-load-flow indicators: load-following error (LFE), which measures the hourly mismatch between aggregate distributed generation (DG) outputs and a load-proportional target, and power unbalance ratio (PUR), which limits diesel-unit phase-power imbalance to 10% during dispatch. The constrained siting and sizing problem is solved using a genetic algorithm (GA) on the IEEE 37-bus feeder under realistic diurnal load, solar, and wind profiles. While an unconstrained allocation achieves 64.57% active-power loss reduction, it exceeds the adopted diesel PUR screening threshold. Enforcing the PUR constraint yields a feasible operating state with 60.87% loss reduction, retaining 94.27% of the unconstrained benefit. Robustness checks across 30 independent runs and GA/PSO/ACO benchmarking confirm that the adopted GA provides the lowest dispersion and highly repeatable feasible outcomes. The results show that the framework improves energy efficiency, voltage regulation, and daily coordination while satisfying the adopted diesel phase-power screening criterion under severe feeder asymmetry.

Keywords: photovoltaic systems; wind power generation; diesel backup generators; distributed generation allocation; unbalanced distribution systems; load-following error; power unbalance ratio



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1. Introduction

Integrating photovoltaic (PV) and wind turbine (WT) distributed generation (DG) resources into distribution feeders is central to renewable electricity expansion because feeder losses, voltage quality, and operational flexibility depend strongly on where and how variable resources are connected [1,2]. Recent studies have addressed closely related directions, including renewable DG and storage planning in reconfigurable distribution systems [3], radial distribution-system reconfiguration and techno-economic planning [4,5], and voltage regulation screening in distribution networks [6]. Yet feeder-level planning is still often based on static or balanced approximations, even though renewable portfolios are deployed on three-phase networks with persistent phase asymmetry and time-varying net demand. Consequently, hybrid DG allocations that appear promising under simplified analysis can still leave two practical issues unresolved on actual unbalanced feeders.

Feeder-level daily coordination is still seldom quantified directly. PV and WT output rarely follow the feeder's diurnal load shape, while common indicators, such as capacity factor, curtailment ratio, and penetration level, reflect utilization or installed volume rather than the hour-by-hour mismatch between aggregate renewable production and demand. Planners therefore still lack a compact feeder-level measure of when a candidate portfolio over- or undersupplies the feeder across a full operating day, even when conventional metrics appear to be favorable [1].

Diesel-backup feasibility under phase asymmetry is also rarely screened explicitly in planning studies. When diesel support is retained to cover renewable variability on strongly unbalanced feeders, phase-to-phase load differences can impose unequal loading on the rotating machine. Sustained operation under such conditions induces negative-sequence effects, accelerates stator heating, and may trigger protective operation [7,8]. Accordingly, diesel support is treated here as limited operational backup for the renewable portfolio rather than as a primary supply source.

The recent literature on DG allocation in distribution feeders can be grouped into three broad categories: static siting, sizing, or hosting-capacity studies on balanced or simplified feeders, with evaluation centered mainly on loss, voltage, or PV accommodation indices [9,10]; studies that extend the analysis to unbalanced feeders, network reconfiguration, or multiple resource technologies but still omit an explicit feeder-level 24 h coordination metric and diesel-backup safety screening [4,11,12]; and time-varying or operationally richer planning studies that consider loading levels or dynamic operational strategies without jointly evaluating hybrid PV-WT portfolios against rotating-machine safety [2,13,14]. Related microgrid-dispatch research has also advanced multi-objective operational optimization with dynamic constraints, for example, through PBI-based optimization combined with reinforcement learning for frequency-aware microgrid dispatch [15]. Such work motivates richer operational assessments, while the present study focuses on the siting and sizing of hybrid DG units on an unbalanced feeder with explicit LFEs and diesel-PUR screening.

Taken together, these studies indicate a gap: Existing approaches frequently under-represent feeder asymmetry, omit direct quantification of daily coordination, or evaluate portfolios without explicitly screening for diesel-generator phase imbalance. Table 1 summarizes these methodological differences.

Table 1. Comparative analysis of representative DG allocation methodologies and the proposed framework.

Study	Unbalanced Analysis	Realistic Profile	Multi-DG Types	LFE Metric	PUR Constraint	24 h Analysis
Sharma et al. (2024) [9]	✗	✗	✗	✗	✗	✗
Painuli et al. (2025) [2]	✓	✓ ^a	✗	✗	✗	✓
Eid & Alsafrani (2024) [16]	✓	✓	✗	✗	✗	✓
Ali et al. (2022) [11]	✓	✗	✓	✗	✗	✓
Madhusudhan et al. (2021) [17]	✗	✗	✗	✗	✗	✗
Nazih et al. (2025) [12]	✓	✗	✓	✗	✗	✓
This Study	✓	✓	✓	✓	✓	✓

^a In Painuli et al. [2], the realistic profile is limited to National Household Travel Survey (NHTS) 2017 EV charging data; the present study instead uses sectoral 24 h feeder load profiles. ✓ indicates that the feature is explicitly included in the cited study, whereas ✗ indicates that it is not explicitly included. The LFE and PUR columns refer to the specific feeder-level daily coordination metric and diesel phase-power safety screen formulated in this study. Related concepts such as renewable utilization, curtailment, penetration limits, and negative-sequence protection exist in the literature; however, the compared studies do not use these two indicators as explicit planning–screening quantities for hybrid PV-WT-diesel allocation in an unbalanced 24 h feeder model.

As summarized in Table 1, a DG portfolio that performs well under simplified assumptions may still exhibit poor daily matching or unsafe diesel loading once unbalanced 24 h operation is considered. The table is therefore intended to identify the methodological gap addressed here, rather than to imply that previous studies ignored all aspects of renewable coordination or power-quality protection. To address this gap, this paper develops a GA-based framework for hybrid PV-WT allocation in unbalanced distribution networks, with explicit evaluation of temporal coordination, diesel-backup phase imbalance, and solution repeatability. The main contributions are as follows:

- Load-following error (LFE) metric with load-proportional target penetration ($\lambda_{\text{target}}(h) = 0.40 \cdot P_{\text{load}}(h) / P_{\text{load}}^{\text{peak}}$) that quantifies the hour-by-hour deviation between the total DG output and a demand-scaled generation target over the 24 h horizon.
- Power unbalance ratio (PUR) constraint that limits diesel-unit phase-power imbalance to 10%, turning rotating-machine safety into an explicit screening condition for unbalanced operation.
- Integrated unbalanced 24 h framework that combines realistic load profiles, hybrid PV and WT allocation, and diesel backup within a three-phase feeder so that loss reduction, voltage support, temporal coordination, and diesel-backup safety are evaluated within a single optimization setting.
- Repeatability assessment using 30 independent runs and GA/PSO/ACO benchmarking to distinguish isolated best values from consistently feasible performance.

The contribution therefore lies in a screening framework that evaluates hybrid DG portfolios on existing unbalanced feeders while preserving quantitative evidence on daily coordination, diesel-backup safety, and solution repeatability.

The remainder of this paper is organized as follows. Section 2 describes the methodology, Section 3 presents the case study, Section 4 reports and discusses the results, and Section 5 concludes this paper.

2. Materials and Methods

This section presents the load-flow model, optimization formulation, and GA implementation adopted for DG allocation in the unbalanced feeder.

2.1. Load Flow Analysis

Accurate modeling of inter-phase coupling is essential in unbalanced distribution networks. Inherent load asymmetry and delta-connected elements can produce phase-specific voltage deviations that must be retained if DG allocation is to be screened credibly. To capture these effects, a three-phase unbalanced Newton–Raphson (NR) load-flow formulation is adopted. The feeder admittance model is assembled using full phase-coupling blocks so that mutual coupling and asymmetric network behavior are preserved explicitly throughout the solution process.

At each iteration, the active- and reactive-power mismatches are linearized with respect to voltage magnitudes and phase angles, the resulting Jacobian system is solved, and the state variables are updated until $\|\Delta \mathbf{P}\|, \|\Delta \mathbf{Q}\| < 1 \times 10^{-6}$ p.u. In this implementation, the state correction is obtained from the sparse linear system $\mathbf{J} \Delta \mathbf{x} = -\mathbf{F}$ using sparse lower–upper (LU) factorization [18]. This formulation preserves the coupled three-phase structure within a single numerical framework and avoids masking severe phase-specific deviations through balanced or aggregated approximations.

Several iterative techniques are available for distribution-level load flow, including Gauss–Seidel, current-injection, and backward/forward sweep formulations. The NR solver is used here because the load flow is invoked once per candidate solution per hour within the fitness-evaluation loop, and the optimization screens diesel-unit candidates using phase-resolved PUR values. Under these conditions, maintaining a fully coupled representation is prioritized over the computational savings of simplified formulations.

2.2. GA Implementation

The GA is adopted as the main optimizer because the proposed allocation problem is mixed-integer, non-convex, and non-smooth: the chromosome combines discrete siting decisions, discrete technology labels, and continuous per-phase active/reactive DG capacities, while the objective is evaluated through repeated nonlinear unbalanced load-flow simulations over 24 h. This does not imply that only a GA can be used. Rather, GA is selected because its real-coded representation, tournament selection, block crossover, and adaptive feasible mutation can accommodate the 253-variable mixed chromosome and penalty-based feasibility screening without requiring derivative information. The later GA/PSO/ACO comparison is used to verify this choice empirically under an identical computational budget.

2.2.1. Objective Function

The objective function G is defined as follows:

$$\min G = k_1 \cdot (1 - \text{APLRI}) + k_2 \cdot \text{VDI} + k_3 \cdot P_{\text{pen}} + k_4 \cdot \text{LFE} + k_5 \cdot N_{\text{DG}} \quad (1)$$

where $(1 - \text{APLRI})$, VDI , P_{pen} , LFE , and N_{DG} denote residual active-power loss, voltage deviation, DG penetration, the daily load-following error, and the DG unit count, respectively.

Min–max normalization is used to place all objectives on the range $[0, 1]$:

$$f_i^{\text{norm}} = \frac{f_i - f_i^{\min}}{f_i^{\max} - f_i^{\min}} \quad (2)$$

The normalized multi-objective function becomes

$$\min G_{\text{norm}} = \sum_{i=1}^5 k_i \cdot f_i^{\text{norm}} \quad (3)$$

Equal weights ($k_i = 0.2, \sum k_i = 1$) prevent scale bias.

2.2.2. Performance Indices

Active Power Loss Reduction Index (APLRI)

The APLRI quantifies the reduction in active-power losses achieved by the DG installation, and it is defined as

$$\text{APLRI} = \frac{P_{\text{loss}}^{\text{base}} - P_{\text{loss}}^{\text{DG}}}{P_{\text{loss}}^{\text{base}}} \quad (4)$$

where $P_{\text{loss}}^{\text{base}}$ and $P_{\text{loss}}^{\text{DG}}$ are the total active-power losses before and after the DG installation, respectively.

VDI

For unbalanced three-phase systems, the VDI is calculated as [19]

$$\text{VDI} = \sum_{i=1}^{N_{\text{BUS}}} \sum_{\phi \in \{A,B,C\}} \frac{|V_{\text{nom}} - V_{i,\phi}|}{V_{\text{nom}}} \quad (5)$$

where V_{nom} represents the nominal voltage (1 p.u.), $V_{i,\phi}$ denotes the voltage magnitude at bus i for phase $\phi \in \{A, B, C\}$, and N_{BUS} represents the total number of buses.

Unlike balanced models that assume $V_A = V_B = V_C$, this formulation sums deviations across phases independently ($3N_{\text{BUS}}$ terms), enabling the identification of severely affected phases for targeted DG placement.

LFE

LFE measures how closely aggregate DG outputs track the load-proportional target over the 24 h horizon, penalizing both over-generation at light loads and under-generation at higher demand.

The LFE for each hour h is formulated as a normalized error metric:

$$\text{LFE}_h = \frac{|\Delta P(h)|}{\max(P_{\text{target}}(h), P_{\text{min}})} \quad (6)$$

where $\Delta P(h) = P_{\text{DG,total}}(h) - P_{\text{target}}(h)$, and $P_{\text{min}} = 0.0001$ MW prevents division by zero.

The total DG generation at hour h is computed as

$$P_{\text{DG,total}}(h) = \sum_{i=1}^{N_{\text{DG}}} [P_{\text{DG},i}^A(h) + P_{\text{DG},i}^B(h) + P_{\text{DG},i}^C(h)] \quad (7)$$

where $P_{\text{DG},i}^A(h)$, $P_{\text{DG},i}^B(h)$, and $P_{\text{DG},i}^C(h)$ represent the active power outputs of DG unit i on phases A, B, and C at hour h , respectively, and N_{DG} denotes the total number of installed DG units.

The target DG generation at hour h depends on the total system load and the desired penetration ratio:

$$P_{\text{target}}(h) = P_{\text{load,total}}(h) \times \lambda_{\text{target}}(h) \quad (8)$$

where $P_{\text{load,total}}(h)$ represents the total system load at hour h , which is calculated as

$$P_{\text{load,total}}(h) = P_{\text{load,Y}}(h) + P_{\text{load,\Delta}}(h) \quad (9)$$

where $P_{\text{load,Y}}(h)$ and $P_{\text{load,\Delta}}(h)$ represent the total Y-connected and delta-connected loads at hour h , respectively.

Target penetration scales with load, $\lambda_{\text{target}}(h) = 0.40 \cdot P_{\text{load,total}}(h) / P_{\text{load}}^{\text{peak}}$, preventing over-generation during low-load periods [1].

The overall LFE is the daily average of the hourly values: $\text{LFE} = \frac{1}{24} \sum_{h=1}^{24} \text{LFE}_h$. Equal weighting ensures unbiased cross-scenario comparisons.

DG Penetration Level

DG penetration measures the ratio of the total active power generated by DG units to the total active power demand in the distribution network [20]:

$$P_{\text{pen}} = \frac{\sum_{i=1}^{N_{\text{DG}}} P_{\text{DG},i}}{\sum_{j=1}^{N_{\text{L}}} P_{\text{load},j}} \quad (10)$$

where $P_{\text{DG},i}$ represents the active power generated by DG unit i , and $P_{\text{load},j}$ denotes the active power demand at load bus j .

Including P_{pen} in the objective discourages excessive penetration. In addition, penetration is constrained to $\leq 40\%$ as the adopted planning limit. The 40% value is used as a controlled planning cap rather than as a universal hosting-capacity value. It was selected to keep this study in a moderate-to-high DG penetration range and to preserve a fair comparison among the five scenarios under the same planning policy. The fact that Cases 4 and 5 operate close to this cap indicates that the cap materially influences the final hybrid solutions and that the reported solutions should be interpreted as performance under a specified penetration policy, not as unconstrained hosting-capacity maxima.

2.2.3. Constraints

These constraints ensure feasibility and safety. Hard constraints (voltage, power factor (PF), and PUR) use adaptive penalties; soft constraints are incorporated into the objective function [21].

Equality Constraints

Per-phase power balance for each $\phi \in \{A, B, C\}$ and hour h [22]:

$$P_{\text{Grid}}^{\phi}(h) + \sum_{i=1}^{N_{\text{DG}}} P_{\text{DG},i}^{\phi}(h) = \sum_{j=1}^{N_{\text{L}}} P_{\text{load},j}^{\phi}(h) + P_{\text{loss}}^{\phi}(h), \quad (11a)$$

$$Q_{\text{Grid}}^{\phi}(h) + \sum_{i=1}^{N_{\text{DG}}} Q_{\text{DG},i}^{\phi}(h) = \sum_{j=1}^{N_{\text{L}}} Q_{\text{load},j}^{\phi}(h) + Q_{\text{loss}}^{\phi}(h), \quad (11b)$$

Per-phase formulation is essential, as phase-to-phase load differences reach 50% to 100% in the IEEE 37-bus test system [23], which motivates a phase-resolved representation of the feeder.

Inequality Constraints

The voltage magnitude on each phase is constrained to the adopted ANSI C84.1-2020 range of 0.9 p.u. to 1.1 p.u. [24]:

$$V_{\min} \leq |V_{i,\phi}| \leq V_{\max} \quad \forall i \in \{1, \dots, N_b\}, \forall \phi \in \{A, B, C\} \quad (12)$$

where $V_{\min}$ and $V_{\max}$ are the adopted voltage limits, and $|V_{i,\phi}|$ is the phase-voltage magnitude at bus i . Enforcing the constraint per phase preserves the unbalanced formulation and avoids masking violations through phase averaging:

- **DG Penetration Limits:** The total DG capacity should not exceed a certain percentage of the total system capacity to prevent overloading and maintain system stability:

$$\frac{\sum_{i=1}^{N_{DG}} P_{DG,i}}{\sum_{j=1}^{N_L} P_{load,j}} \leq \lambda_{\max} \quad (13)$$

where $\lambda_{\max} = 0.40$ (40% maximum penetration). This cap is used here as a controlled planning reference limit that bounds aggregate DG penetration and keeps the comparison focused on temporal mismatch and diesel phase-unbalance effects. Because the optimized penetrations in Cases 4 and 5 are 39.58% and 39.64%, respectively, they lie only 0.42 pp and 0.36 pp below the adopted cap. The constraint is therefore near-binding and materially influences the final hybrid solutions. The reported APLRI values should be interpreted as the best solutions found under the adopted 40% planning policy.

- **DG Capacity Limits:** Constraints on DG sizes ensure that generated power remains within specified operating bounds:

$$\begin{aligned} P_{DG,\min} &\leq P_{DG,i} \leq P_{DG,\max} \\ Q_{DG,\min} &\leq Q_{DG,i} \leq Q_{DG,\max} \end{aligned} \quad (14)$$

where $P_{DG,\min}$, $P_{DG,\max}$, $Q_{DG,\min}$, and $Q_{DG,\max}$ denote the adopted active- and reactive-power limits.

- **PF Limits:** DG units must operate within specified PF limits to maintain system stability and power quality per IEEE Std 1547-2018 requirements [8]. In the case study, the minimum acceptable DG power factor is set to $PF_{DG,\min} = 0.87$, while $PF_{DG,\max} = 1.00$. Both lagging and leading reactive-power directions are allowed provided that the magnitude of the resulting PF remains within this interval. The PF is calculated as follows:

$$PF_{DG,i} = \frac{P_{DG,i}}{\sqrt{P_{DG,i}^2 + Q_{DG,i}^2}} \quad (15)$$

The PF must satisfy:

$$PF_{DG,\min} \leq PF_{DG,i} \leq PF_{DG,\max} \quad (16)$$

where $PF_{DG,\min}$ and $PF_{DG,\max}$ are the allowable PF bounds. Numerically, this gives $0.87 \leq PF_{DG,i} \leq 1.00$ for each active DG phase used in the optimization.

- **Load-Following Constraints:** To ensure proper DG operation throughout the 24 h period and prevent system instability, hourly penetration constraints are enforced:

$$\frac{P_{DG,\text{total}}(h)}{P_{\text{load,total}}(h)} \leq \lambda_{\max} \quad \forall h \in \{1, \dots, 24\} \quad (17)$$

This hourly cap prevents over-generation during low-demand periods.

- **DG Availability Constraints:** For renewable DG types, generation is limited by resource availability:

For PV units,

$$P_{PV,i}(h) \leq P_{PV,i}^{\text{rated}} \times CF_{PV}(h) \quad (18)$$

where $CF_{PV}(h)$ is the PV capacity factor at hour h , derived from the Photovoltaic Geographical Information System's (PVGIS) solar irradiance data [25].

For WT units:

$$P_{WT,i}(h) \leq P_{WT,i}^{\text{rated}} \times CF_{WT}(h) \quad (19)$$

For diesel generators:

$$P_{\text{diesel},i}^{\min} \leq P_{\text{diesel},i}(h) \leq P_{\text{diesel},i}^{\max} \quad (\text{when operating}) \quad (20)$$

- PUR Constraint for Diesel Generators:

In unbalanced networks, diesel generators experience phase power asymmetry that increases thermal stress, mechanical vibration, and the likelihood of protection-related issues [7]. This study adopts a 10% phase-imbalance threshold for the diesel units considered here. The threshold is used as a conservative planning screen intended to limit exposure to strongly unbalanced diesel dispatch; it is not presented as a direct numerical limit prescribed by either standard. The PUR quantifies this imbalance:

$$P_{\text{avg}} = \frac{P_A + P_B + P_C}{3} \quad (21)$$

$$\Delta P_{\max} = \max(|P_A - P_{\text{avg}}|, |P_B - P_{\text{avg}}|, |P_C - P_{\text{avg}}|) \quad (22)$$

The PUR for diesel generator i is computed using a numerically stable formulation that prevents division by zero during low-power or non-dispatch operation:

$$\text{PUR}_{\text{diesel},i}(\%) = \begin{cases} \frac{\Delta P_{\max}}{P_{\text{avg}}} \times 100, & \text{if } P_{\text{avg}} > P_{\min,\text{op}} \\ 0, & \text{if } P_{\text{avg}} \leq P_{\min,\text{op}} \end{cases} \quad (23)$$

where $P_{\min,\text{op}} = 0.01$ MW is the operational threshold. When $P_{\text{avg}} \leq P_{\min,\text{op}}$, the PUR is defined as zero (the generator is not dispatched). The 10% threshold is therefore interpreted here as a conservative planning criterion for the diesel units considered. This constraint applies exclusively to diesel generators because PV and WT systems are converter-based and are not subject to rotating-machine mechanical stress.

Physically, enforcing the PUR limit promotes more balanced phase loading and can reduce negative-sequence stress in the stator winding. However, PUR is defined based on phase active powers, whereas the negative-sequence current ratio is obtained from the phase-current phasors; therefore, no universal conversion between the two quantities is assumed. For the adopted ABC phase sequence, a detailed machine-protection assessment should calculate $I_1 = (I_A + aI_B + a^2I_C)/3$ and $I_2 = (I_A + a^2I_B + aI_C)/3$, where $a = e^{j2\pi/3}$, and report $|I_2|/|I_1|$. In the present planning framework, the optimization constraint is enforced directly on PUR.

2.2.4. GA Parameters

The GA was executed with a population size of 1000 for 200 generations, using tournament selection, two-point crossover, and an adaptive feasible mutation function.

2.3. GA Design and Population Generation

The initial GA population consists of chromosomes that encode DG location, type, and per-phase active and reactive power capacity.

2.3.1. Chromosome Encoding

The GA optimization employs a real-coded chromosome structure specifically designed to represent complete DG allocation solutions for unbalanced three-phase distribution networks. Unlike balanced system encodings that assume symmetrical phase loading and utilize single aggregate power variables, this formulation explicitly encodes per-phase active and reactive power for each DG unit, enabling optimization to exploit phase-specific characteristics and address voltage and current unbalance.

Each chromosome $\mathbf{x} \in \mathbb{R}^{1+7N}$ encodes DG locations, per-phase capacities (separate values for phases A, B, and C), and technology types, where N denotes the number of candidate buses, *excluding the slack bus*. For the IEEE 37-bus feeder used here, this gives $N = 36$ and therefore a 253-dimensional search space. The expanded dimensionality ($1 + 6N$ power variables + N type variables) compared to balanced encodings ($1 + 2N + N$) reflects the necessity of independent phase control in unbalanced systems. The chromosome structure is defined as

$$\mathbf{x} = [b, \mathbf{p}_1^T, \mathbf{q}_1^T, \dots, \mathbf{p}_N^T, \mathbf{q}_N^T, \mathbf{t}^T]^T \quad (24)$$

where the encoding components are illustrated compactly in Figure 1.

Chromosome Encoding Structure ($N = 2$ candidate buses, excluding slack)

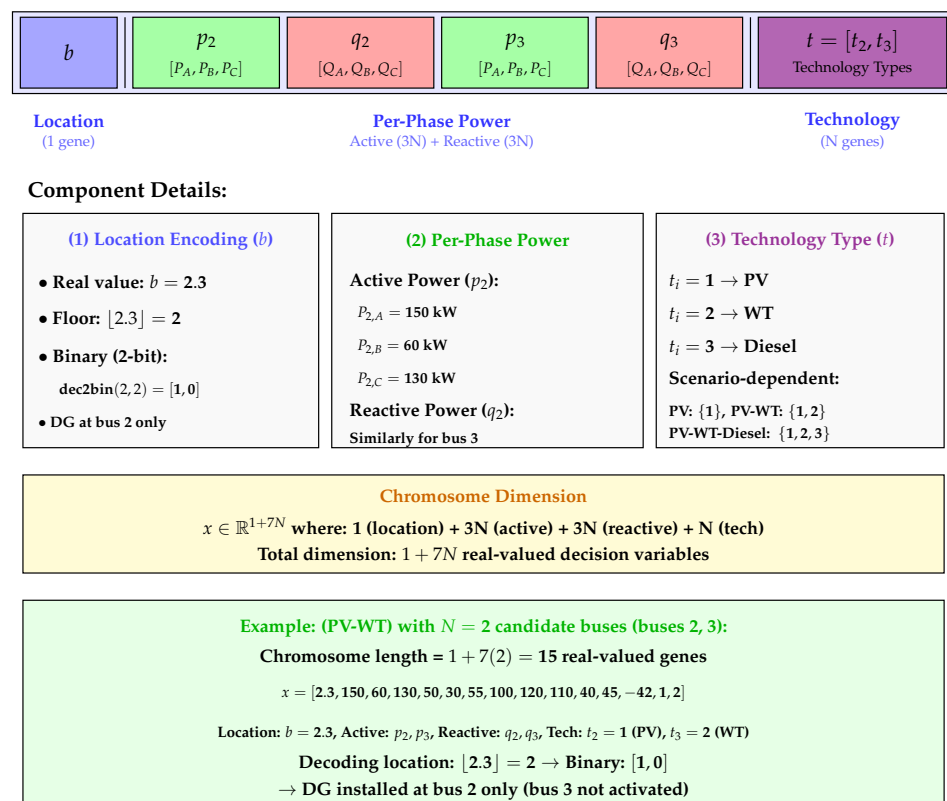


Figure 1. Chromosome encoding structure and decoding process for an unbalanced feeder ($N = 2$ candidate buses, excluding the slack bus), explicitly separating phase-specific capacities and technology identifiers.

The chromosome components are defined as follows, where each component controls a distinct aspect of DG siting, sizing, and technology selection:

- (1) **Location:** Continuous gene b acts as a surrogate for one of the 2^N possible siting patterns. For interpretation, $\lfloor b \rfloor$ is read as an integer in $[0, 2^N - 1]$ and converted to an N -bit binary vector $\mathbf{B} \in \{0, 1\}^N$, where each bit indicates whether the corresponding candidate bus is activated.
- (2) **Per-Phase Power:** The vectors $\mathbf{p}_i = [P_{i,A}, P_{i,B}, P_{i,C}]^T$ and $\mathbf{q}_i = [Q_{i,A}, Q_{i,B}, Q_{i,C}]^T$ collect the active- and reactive-power ratings assigned to DG unit i on phases A, B, and C, respectively. When candidate bus i is not activated by the location gene, these entries remain inactive in the decoded solution. This phase-resolved representation allows the GA to size injections independently across the three phases and to retain feeder asymmetry during chromosome decoding and fitness evaluation.

- (3) **Technology:** The discrete variable $t_i \in \{1, 2, 3\}$ identifies the DG technology assigned to candidate bus i , where 1, 2, and 3 denote PV, WT, and diesel, respectively. This gene determines which resource model, availability profile, and operating constraints are applied to that unit during the 24 h simulation.

2.3.2. Enhanced Population Initialization

To sample the high-dimensional search space ($1 + 7N$ dimensions) while maintaining a sufficient density of feasible solutions, the initial population (N_{pop} chromosomes) is generated using a hybrid topology-aware sampling strategy. Random initialization often leads to heavily clustered or highly infeasible states in constrained spaces, which delays convergence [26,27]. The adopted initialization strategy mitigates this through three interacting mechanisms:

(1) **Multi-Strategy Distribution:** The generation matrix is partitioned among four sampling techniques. Latin Hypercube Sampling (30%) stratifies each decision variable to prevent clustering and provide broad search-space coverage. Sobol Sequences (20%) inject low-discrepancy quasi-random points to fill unrepresented gaps and improve high-dimensional uniformity. Uniform Random Sampling (40%) preserves stochastic diversity across the boundaries, while explicit boundary exploration (10%) sets parameters at their upper and lower technical limits.

(2) **Topology-Aware Sparsity Filter:** Translating a single continuous gene b into an array of discrete siting indicators creates a rugged decoding landscape, where adjacent values can yield differing activation combinations. To guide the solver toward realistic grid reinforcements, a sparsity filter restricts the initialization density. Independent of the initial sampling method, 70% of the chromosomes are constrained so that only a targeted fraction (20% to 60%) of the candidate buses are assigned active DG units. This rejects dense or near-empty configurations that would violate operational constraints in early generations. The remaining 30% of the population is left randomized to prevent premature convergence [27–29].

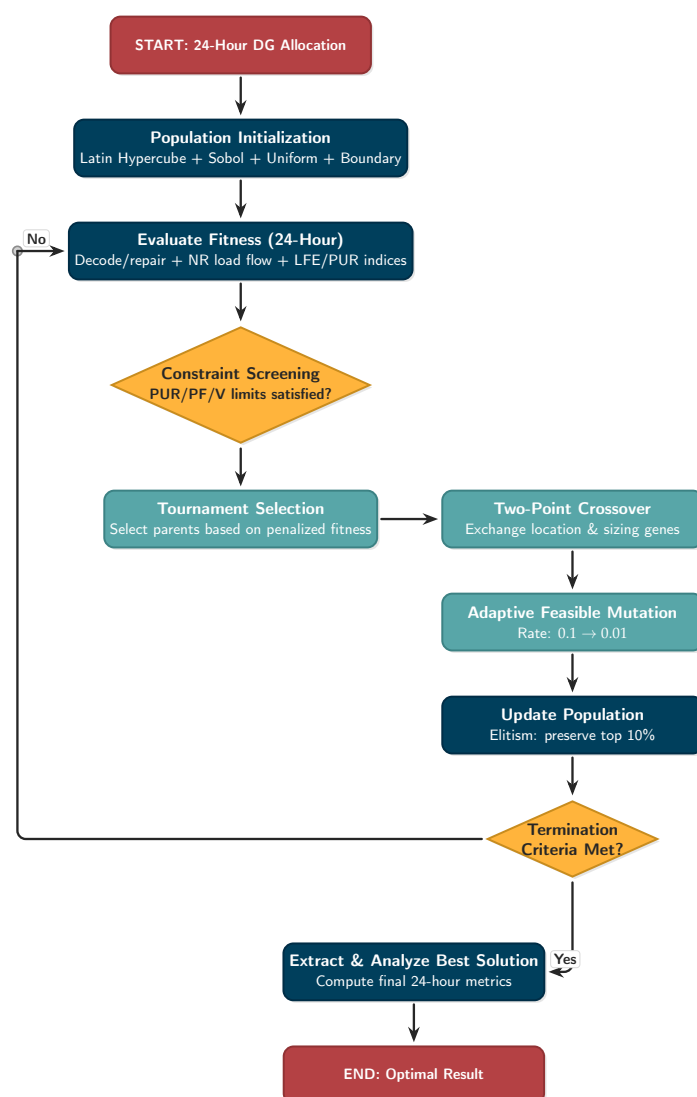
(3) **Jacobian-Based Bus Pre-Screening:** A composite sensitivity index is computed for each candidate bus from the Newton–Raphson Jacobian over the 24 h operating profile. The index reflects an incremental reduction in three-phase active-power losses and the corresponding improvement in voltage magnitude under a small active-power injection, averaged over the day. Candidate buses are then ranked, and the topology-aware portion of the initial population is biased toward the highest-ranked subset, while the remaining randomized portion preserves search diversity.

2.4. Algorithm Procedure

The algorithm follows a standard GA workflow initialized by the topology-aware sampling strategy. For each chromosome, fitness evaluation sweeps the 24 h horizon, executing the three-phase NR load flow hourly to accumulate operating metrics, daily LFE, and maximum diesel PUR. An explicit feasibility-screening step then enforces the operational PUR, PF, and voltage limits via fitness penalties. Subsequent operations include tournament selection, two-point crossover, and adaptive feasible mutation, with physical variable bounds repaired after each generation. Table 2 lists the adopted GA settings, and Figure 2 summarizes the computational sequence.

Table 2. Genetic algorithm configuration parameters.

Parameter	Value	Purpose
Population Size	1000	Maintains diversity in the 253-variable search space
Max Generations	200	Allows deep exploration
Elite Count	100 (10%)	Preserves top solutions (elitism)
Max Stall Generations	15	Prevents premature convergence
Selection	Tournament (Size = 4)	Balances exploration vs exploitation
Crossover Function	Two-Point	Better for long chromosomes
Crossover Fraction	0.8	High recombination rate
Mutation Function	Adaptive Feasible	Dynamic rate (0.1 → 0.01)

**Figure 2.** Flowchart of the adopted genetic algorithm for 24 h DG allocation, including explicit post-evaluation constraint screening using the PUR, PF, and voltage limits.

3. Case Study

3.1. System Description

The IEEE 37-bus system [30] is adopted as the test feeder because it provides a well-established benchmark for unbalanced distribution studies. It is a three-wire delta configuration operating at 4.8 kV line-to-line, with underground lines, mixed loads, and severe

phase unbalance. The total feeder demand is 2.457 MW and 1.201 MVar. On a phase basis, the asymmetry is pronounced: Phase C carries 1091 kW, compared with 727 kW on Phase A and 639 kW on Phase B, yielding a 1.71:1 C-to-B active-power ratio. This imbalance helps explain the PUR violations later observed in the diesel-unit results. A voltage-control device connected at bus 701 maintains the bus voltage at 1.0437 p.u., 1.0250 p.u., and 1.0345 p.u. for phases A, B, and C, respectively, to emulate realistic operating conditions.

Loads are represented by a composite ZIP load model so that feeder demand varies with local voltage rather than being treated as purely constant power. The active-power component is expressed as follows:

$$P(V) = P_0 \left[k_Z \left(\frac{V}{V_{\text{nom}}} \right)^2 + k_I \left(\frac{V}{V_{\text{nom}}} \right) + k_P \right] \quad (25)$$

where V is the local voltage magnitude, P_0 is the nominal active power at $V = V_{\text{nom}}$, and $k_Z + k_I + k_P = 1$. Reactive power follows an analogous formulation. Retaining this voltage-dependent load representation improves the realism of both loss calculations and voltage-sensitivity assessment relative to a pure constant-power model.

3.2. DG Modeling

This study considers three DG technologies. For completeness, the adopted PV, WT, and diesel-backup models are summarized below using realistic meteorological inputs from PVGIS for Ras Gharib, Egypt [25].

PV outputs, denoted by P_{pv} , are modeled using the standard temperature-dependent equation [31]:

$$P_{\text{pv}} = P_{\text{STC}} \frac{I_s}{1000} [1 + \gamma(T_c - 25)] \quad (26)$$

where P_{STC} is the PV rated power at standard test conditions (STC) ($P_{\text{STC}} = 250$ W), I_s is the solar irradiance in W m^{-2} , $\gamma = -0.0045$ is the temperature coefficient, and T_c is the cell temperature. The latter is estimated from the NOCT model as [31]:

$$T_c = T_a + \frac{T_{\text{NOCT}} - 20}{800} I_s \quad (27)$$

where $T_{\text{NOCT}} = 46$ °C, and T_a is the ambient temperature. Together, Equations (26) and (27) capture the reduction in PV output as cell temperature rises above the STC reference value.

WT output, denoted by P_{wt} , is represented by the standard four-part wind-turbine power curve [32]:

$$P_{\text{wt}}(v) = \begin{cases} 0, & v < v_{ci} \\ P_r \frac{v^3 - v_{ci}^3}{v_r^3 - v_{ci}^3}, & v_{ci} \leq v < v_r \\ P_r, & v_r \leq v < v_{co} \\ 0, & v \geq v_{co} \end{cases} \quad (28)$$

where $P_r = 500$ W is the rated power of the IstaBreeze i-500W wind turbine (Altinel Enerji Dış Tic. Ltd. Şti., İstanbul, Türkiye), v is the wind speed, $v_{ci} = 3$ m s⁻¹ is the cut-in speed, $v_r = 16$ m s⁻¹ is the rated speed, and $v_{co} = 17$ m s⁻¹ is the cut-out speed. Under this model, the WT produces no power below cut-in or above cut-out, ramps up between the cut-in and rated speeds, and delivers rated power between v_r and v_{co} .

The ratings $P_{\text{STC}} = 250$ W and $P_r = 500$ W denote the individual PV-module and WT-turbine specifications used in the resource models. In the allocation problem, each PV or WT candidate represents an aggregated installation at the selected bus, so the optimized

DG capacity corresponds to the total installed array or turbine-group rating rather than to a single module or a single turbine.

Diesel backup units are modeled here as dispatchable rotating backup sources. In the present study, their operating limits follow diesel-based standby-service classification under ISO 8528-1 [33]. Each diesel unit is restricted to a scheduled 3 h dispatch window from 19:00 to 22:00, corresponding to three hourly intervals during the evening demand peak; outside this window, the unit remains offline ($P_{\text{diesel}}(h) = 0$). This operating assumption keeps the diesel-backup model consistent with standby-service practice and ensures that performance metrics and screening limits, including PUR and minimum-loading requirements, are evaluated only during active service. The diesel window is therefore treated as an exogenous standby-service policy in this planning study and not as a dynamically optimized unit-commitment variable. Optimizing the backup schedule jointly with DG siting and sizing would require additional operational states, start-stop costs, and fuel/emission dispatch objectives. Detailed fuel-consumption and thermodynamic-efficiency modeling is therefore not embedded in the optimization loop; the fuel and direct-emission quantities reported later are post-processing screening indicators derived from the optimized dispatch profiles.

3.3. Load Profile Generation

Twenty-four-hour load profiles were generated using the R package *standardlastprofile* (<https://CRAN.R-project.org/package=standardlastprofile> (accessed on 7 July 2026)) for the residential (H0), commercial (G0–G6), and agricultural (L0–L2) sectors. The profiles were normalized to [0, 1] to preserve relative variations, and the resulting overall system profile, shown in Figure 3, captures the temporal demand variability across these sectors.

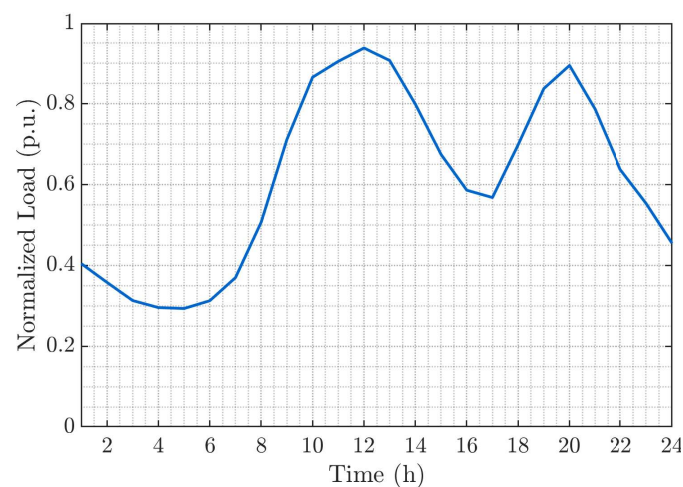


Figure 3. Normalized 24 h load profile for the IEEE 37-bus system showing residential, commercial, and agricultural demand patterns.

3.4. Simulation Cases

Five optimization scenarios were developed to evaluate DG technology combinations and operational constraints:

- Case 1: Allocation of PV-only DG units.
- Case 2: Allocation of WT-only DG units.
- Case 3: Allocation of hybrid PV and WT DG units.
- Case 4: Allocation of PV, WT, and diesel DG units without considering PUR.
- Case 5: Allocation of PV, WT, and diesel DG units with the PUR constraint ($\text{PUR} \leq 10\%$) for diesel generators, as defined in (23).

4. Results and Discussion

This section evaluates the GA-based framework on the IEEE 37-bus system across five DG allocation cases, advancing from single-technology benchmarks to hybrid portfolios with diesel backup. This progression isolates unconstrained technical performance from PUR-compliant deployment. After establishing a no-DG base case, the analysis covers technology-level comparisons, the Case 4/Case 5 compliance trade-off, algorithm benchmarking, and implementation implications. Post-optimization checks confirmed that scenario rankings were unaffected by normalization, so all metrics are reported in their original units. Unless noted otherwise, the tables and figures present the best feasible run from the 30-run set.

4.1. Statistical Validation

Each scenario was executed 30 times with different random seeds. Across scenarios, the observed dispersion is low, indicating that the reported best-run values are not isolated outliers. Table 3 extends this repeatability check across Cases 1–5 for the adopted GA, while Table 10 shows that the adopted GA also maintained the lowest coefficient of variation among the tested solvers for Case 5.

Table 3. Run-to-run repeatability of the adopted GA over 30 independent runs.

Scenario	Mean APLRI (%)	Standard Deviation (%)	CoV (%)	95% CI
Case 1 (PV-only)	45.66	0.081	0.177	[45.63, 45.69]
Case 2 (WT-only)	46.64	0.084	0.181	[46.61, 46.67]
Case 3 (PV-WT hybrid)	54.11	0.072	0.133	[54.08, 54.13]
Case 4 (PV-WT-diesel)	64.52	0.098	0.152	[64.48, 64.56]
Case 5 (PV-WT-diesel with PUR)	60.85	0.044	0.072	[60.84, 60.87]

Note: CoV is computed as $100 \times s / \bar{x}$, and the 95% CI is the two-sided t interval for mean APLRI across 30 runs.

4.2. Base-Case System Performance

Table 4 summarizes the base-case system metrics without DG integration. The feeder exhibits a total daily loss of 7963.13 kWh, average hourly loss of 331.80 kW, and peak loss of 547.55 kW. The hourly loss profile shows high variability, indicated by a standard deviation of 130.84 kW. Furthermore, the baseline average voltage deviation is 0.0616 p.u., and the overall VDI is 54.69%, with phase contributions of 65.20%, 43.22%, and 55.64% for Phases A, B, and C, respectively, confirming substantial feeder unbalance.

Table 4. Base-case system performance metrics without DG integration.

Performance Metric	Value
Total Daily Energy Loss (kWh)	7963.13
Average Hourly Loss (kW)	331.80
Peak Hourly Loss (kW)	547.55
Minimum Hourly Loss (kW)	160.56
Standard Deviation (kW)	130.84
Average Voltage Deviation (p.u.)	0.0616
Total VDI (%)	54.69
VDI Phase A (%)	65.20
VDI Phase B (%)	43.22
VDI Phase C (%)	55.64

4.3. Cross-Scenario Comparison of DG Technology Mixes

Hybrid configurations outperform single-technology approaches. PV-WT (Case 3) achieves a best-run APLRI of 54.15%, exceeding PV-only (45.75%) and WT-only (46.68%) by 8.40 pp and 7.47 pp, respectively. This result is consistent with temporal complementarity between the two resources: PV contributes primarily during midday, whereas WT provides support into the evening hours. The corresponding voltage improvement increases to 44.19%, compared with 36.64% for PV-only and 37.21% for WT-only.

4.3.1. Overall Performance Metric Comparison

Table 5 consolidates the key performance indicators across all scenarios, enabling direct quantitative comparisons. For space efficiency in tables and figures, scenarios are identified using abbreviated names (PV-only, WT-only, PV-WT hybrid, PV-WT-diesel, and PV-WT-diesel (with PUR)), corresponding to Cases 1–5 as defined previously.

Table 5. Comprehensive performance comparison across all DG allocation scenarios.

Scenario	Daily Loss (kWh)	Best APLRI (%)	Energy Saved (kWh)	Avg. Volt Dev. (p.u.)	Volt. Impr. (%)	Total VDI (%)	VDI-A (%)	VDI-B (%)	VDI-C (%)
Base Case	7963.13	–	–	0.0616	–	54.69	65.20	43.22	55.64
Case 1	4319.81	45.75	3643.32	0.0390	36.64	34.65	43.07	27.97	32.90
Case 2	4246.01	46.68	3717.12	0.0387	37.21	34.34	29.13	35.04	38.85
Case 3	3650.95	54.15	4312.18	0.0344	44.19	30.52	28.04	28.71	34.82
Case 4	2821.41	64.57	5141.72	0.0301	51.12	26.73	36.19	16.64	27.36
Case 5	3115.81	60.87	4847.32	0.0324	47.39	28.77	37.65	18.37	30.29

Case 4 records the highest technical performance across all evaluated scenarios, with 64.57% APLRI and 51.12% voltage improvement. Case 5 retains 60.87% APLRI and 47.39% voltage improvement while satisfying the adopted PUR limit. Case 4 therefore defines the unconstrained upper bound, whereas Case 5 is the deployable outcome once PUR compliance is imposed.

To complement the network-performance indicators with practical multi-energy quantities, Table 6 reports daily PV-WT energy, renewable-energy fraction, diesel energy, indicative diesel fuel consumption, and direct diesel CO₂ emissions for the five optimized cases. The calculation uses the same 24 h PV, WT, diesel, and load profiles used in the optimization. Renewable energy fraction is computed as E_{PV+WT}/E_{load} , where the daily load energy obtained from the scaled feeder profiles is 36.08 MWh. Diesel fuel use is reported as an indicative value using the assumed linear screening relation

$$F_{day} = \sum_{h=1}^{24} \sum_{i \in \mathcal{D}} u_i(h) [0.246P_i(h) + 0.0845P_{i,rated}],$$

where $u_i(h)$ is 1 when diesel unit i is online and 0 otherwise, $P_i(h)$ and $P_{i,rated}$ are in kW, and F_{day} is in L/day. This adopted, non-manufacturer-specific screening assumption is applied to each online diesel unit and then summed across active hours and units; it is not used as a detailed engine fuel curve or as an optimization objective. Direct diesel CO₂ is then estimated using a rounded approximate emission factor of 2.68 kg CO₂ per liter, consistent with the EPA gallon-based diesel CO₂ factor [34]. Avoided grid emissions are

not assigned a universal value because they depend on the location- and time-specific marginal generation mix.

Table 6. Post-processed daily renewable-energy, diesel-fuel, and direct-emission indicators.

Case	PV + WT Energy (MWh/day)	Renewable Energy Fraction (%)	Diesel Energy (MWh/day)	Diesel Fuel (L/day)	Direct Diesel CO ₂ (t/day)
Case 1	7.62	21.13	0.00	0.0	0.000
Case 2	8.45	23.43	0.00	0.0	0.000
Case 3	8.55	23.70	0.00	0.0	0.000
Case 4	7.99	22.16	0.12	38.5	0.103
Case 5	6.39	17.70	0.72	237.9	0.638

The values are post-processed from the optimized DG allocations and resource profiles; they are not additional optimization objectives. Renewable-energy fractions were calculated using the unrounded daily generation and load-energy values; displayed energies are rounded for presentation. Direct diesel CO₂ is reported only for the scheduled backup operation. Avoided grid emissions depend on the marginal grid-generation mix and are therefore not assigned a universal value in this feeder-screening study.

Table 6 also makes clear the operational cost of enforcing the diesel-PUR screen. In Case 5, the optimizer introduces a second diesel unit at bus 733 and distributes the scheduled diesel output across buses 712 and 733 with more balanced per-phase powers. The PUR-compliant solution therefore reduces phase-power imbalance but uses higher scheduled diesel energy than Case 4. This trade-off motivates future evaluations of storage, curtailment, demand response, and dynamic backup scheduling to reduce diesel reliance.

4.3.2. LFE Analysis

Table 7 presents the LFE metrics for all scenarios, quantifying how effectively each DG configuration maintains target penetration levels throughout the 24 h operational cycle. LFE values represent the mean relative deviation between actual DG penetration and the load-proportional target defined by (8) across hourly intervals, with peak hours defined as 9:00–14:00 and 19:00–22:00 (high-demand periods) and off-peak covering the remaining hours. An LFE of 1 indicates that the average absolute hourly mismatch equals the target itself; values below 1 indicate closer tracking, whereas values above 1 indicate that the average mismatch exceeds the target magnitude.

Table 7. LFE analysis across all DG allocation scenarios.

Scenario	Relative LFE (at Best Run)	Peak	Off-Peak	Max
Case 1 (PV)	1.46	0.91	1.78	4.40
Case 2 (WT)	1.22	0.49	1.63	2.93
Case 3 (PV-WT)	1.45	0.44	2.01	3.67
Case 4 (No PUR)	1.41	0.34	2.05	3.95
Case 5 (With PUR)	1.37	0.35	1.96	3.64

The LFE results reflect the expected influence of generation profiles on temporal coordination. Case 1 (PV) exhibits a high off-peak LFE of 1.78 because of the absence of nighttime generation. Case 2 (WT) achieves the lowest single-technology LFE of 1.22 because wind generation persists throughout the day. Combining PV and WT in Case 3 reduces the peak-hour LFE to 0.44 through resource complementarity, although the off-peak LFE rises to 2.01 when both sources produce during low-demand hours. Adding diesel backup in Case 4 further lowers the peak-hour LFE to 0.34, and enforcing the PUR

constraint in Case 5 yields a comparable overall LFE of 1.37 with the lowest maximum hourly LFE of 3.64. In practical terms, the Case 5 value of 1.37 means that the average hourly absolute mismatch is 137% of the target level, driven mainly by off-peak periods when renewable output and feeder demand are not perfectly aligned.

4.3.3. Network-Wide Voltage Margin Across Scenarios

Both phase-wise VDI (detailed in Table 5) and the average voltage deviation (Figure 4) show progressive improvement from the base case to the hybrid scenarios. The largest absolute VDI reduction occurs in Phase A, dropping from 65.20% in the base case to 36.19% in Case 4. While all optimized scenarios remain within the adopted limits, the minimum recorded bus voltage over the 24 h horizon in Case 5 is 0.9001 p.u. at Phase A of bus 740 (hour 18). Although technically feasible, this narrow margin above the lower limit suggests that providing local reactive support at the weakest downstream buses would be prudent in practical deployment.

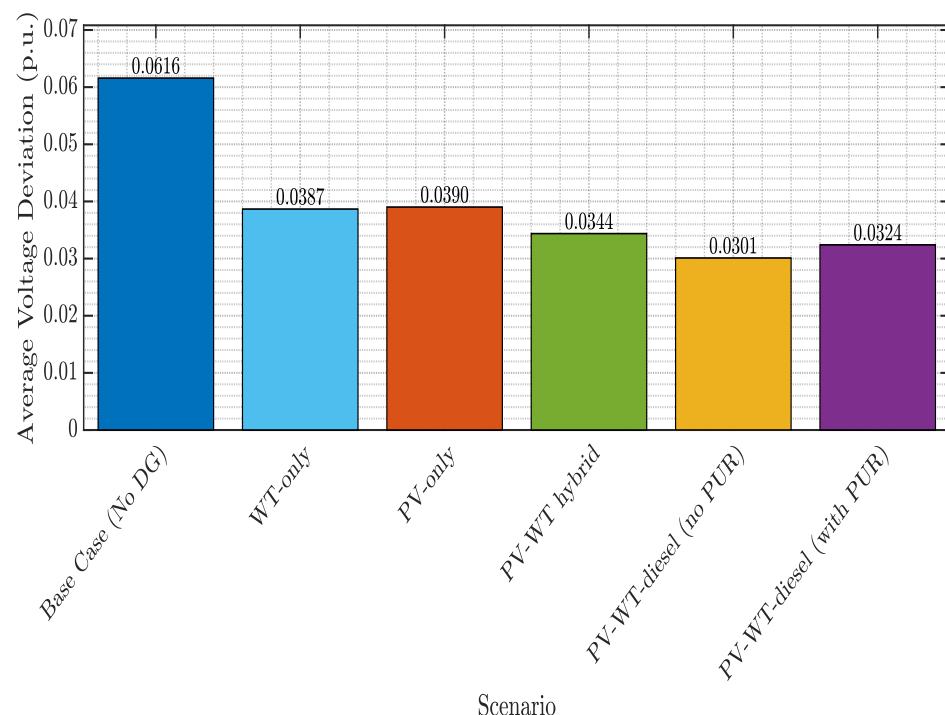


Figure 4. Average voltage deviation across all buses and hours for each DG allocation scenario (lower values indicate better voltage regulation).

4.4. Case 4 and Case 5: Compliance-Driven Reallocation

Table 8 compares the optimal DG allocations and phase-unbalance levels for the unconstrained Case 4 and the PUR-compliant Case 5. Without the PUR constraint, Case 4 achieves a best-run APLRI of 64.57% and a voltage improvement of 51.12%. However, the diesel unit at bus 712 operates at a PUR of 11.35%, which exceeds the study-specific 10% planning-screening threshold. When the PUR constraint is enforced in Case 5, both diesel units satisfy the adopted PUR threshold, and the solution yields a best-run APLRI of 60.87%, retaining 94.27% of the unconstrained loss-reduction benefit at a cost of 3.70 pp.

Table 8. Optimal DG allocation for Case 4 (without PUR constraint) and Case 5 (with PUR constraint).

Case	Bus	Type	P_a (kW)	P_b (kW)	P_c (kW)	Total (kW)	Avg PF	Max PUR ^a (%)
4	707	WT	138.82	132.33	130.81	401.95	0.905	–
	712	Diesel	11.48	14.41	12.94	38.84	0.925	11.35
	718	WT	40.69	25.37	31.79	97.85	0.931	–
	722	WT	25.49	12.00	14.55	52.04	0.929	–
	733	PV	60.20	60.81	53.05	174.06	0.921	–
	742	PV	67.47	77.58	62.80	207.86	0.903	–
Total (Case 4)	–	–	–	–	–	972.60	0.919	–
5	703	PV	64.31	60.57	75.62	200.50	0.901	–
	707	WT	89.15	95.66	81.11	265.92	0.907	–
	712	Diesel	16.45	18.53	16.70	51.68	0.979	7.55
	718	WT	30.62	40.25	33.85	104.72	0.914	–
	722	WT	15.78	10.10	11.65	37.53	0.970	–
	733	Diesel	64.83	61.43	61.97	188.23	0.912	3.33
	742	WT	66.49	39.03	19.90	125.42	0.920	–
Total (Case 5)	–	–	–	–	–	974.00	0.915	–

^a PUR monitored exclusively for diesel generators (rotating machinery). PV and WT systems are converter-based and are not subject to rotating-machine mechanical stress.

To achieve this compliance, the optimization engine structurally redistributes the DG portfolio. Case 4 deploys six units (972.60 kW and 39.58% penetration), with major WT capacities at buses 707, 718, and 722, PV capacities at buses 733 and 742, and a single diesel unit at bus 712. Case 5 redistributes a similar total capacity (974.00 kW and 39.64% penetration) across seven units by adding PV at bus 703 and a second diesel unit at bus 733. This redistribution reduces the worst diesel PUR from 11.35% in Case 4 to 7.55% and 3.33% for the two diesel units in Case 5.

Despite this necessary reallocation, the spatial voltage support remains highly consistent. Figure 5 compares the base case, the unconstrained Case 4 solution, and the PUR-compliant Case 5 solution at Hour 12 on a phase-resolved basis. Across all three phases, the largest departures from the baseline remain concentrated at electrically weak downstream buses (730–744). Case 4 provides the strongest uplift, whereas Case 5 remains only slightly lower, confirming that the performance penalty primarily reflects a slight reduction in absolute voltage uplift rather than degradation in the overall voltage-support pattern.

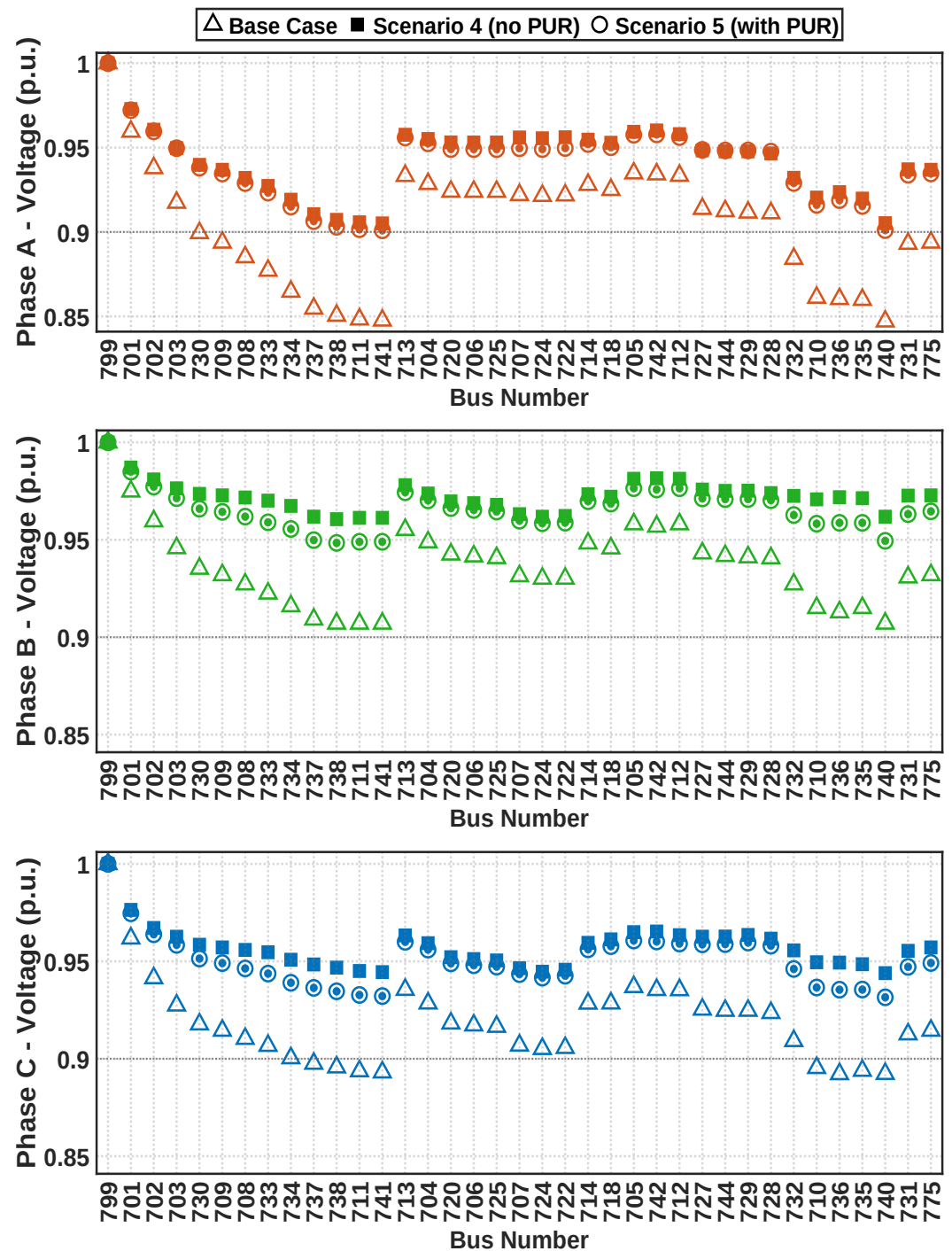


Figure 5. Phase-resolved peak-hour (12:00) voltage profiles across the 37 buses for the base case, Case 4, and Case 5 (0.9 p.u. to 1.1 p.u. limits).

4.5. Branch Thermal Utilization Verification

Thermal compliance is a basic feasibility requirement in DG planning because persistent branch overloading accelerates conductor aging and reduces the operating margin. To verify this aspect, branch-current magnitudes were evaluated for every scenario over the full 24 h horizon using the peak utilization factor (UF), defined as the maximum current magnitude normalized by conductor ampacity. In all scenarios, the UF remained below 100%. The highest value was 89.77% at hour 12 on branch 703–730 (Phase A) in the base case, while Cases 1–5 recorded peak UF values of 83.39%, 89.66%, 83.10%, 83.34%, and 89.63%, respectively.

Branch 703–730 consistently emerges as the limiting element across all scenarios, with peak utilization occurring at hour 12 or hour 20 depending on the DG mix. The retained margin of approximately 10% below conductor ampacity confirms that the studied penetration level remains thermally acceptable throughout the feeder. At the same time, the repeated identification of this branch as the governing element indicates that it would be the most likely candidate for reinforcement if DG penetration was increased further. This result supports the practical feasibility of the studied DG allocations under the thermal limits of the IEEE 37-bus feeder.

4.6. Algorithm Benchmarking and Repeatability Assessment

The GA performance was benchmarked against particle swarm optimization (PSO) and ant colony optimization (ACO) to verify the robustness of the allocation framework. Rather than claiming general algorithmic superiority, this comparison checks whether the highly constrained Case 5 solution is reproducible. All three solvers were programmed to optimize the same five-objective formulation, including the LFE and PUR constraints, under an identical computational budget of 1000 individuals, 200 iterations, and 30 independent runs.

A theoretical comparison is also useful because the three optimizers interact differently with the proposed chromosome and constraints. GA treats a candidate allocation as a recombinable chromosome, allowing location, size, phase, and technology blocks to be inherited together during crossover. PSO updates candidate vectors through velocity-like movements, which is efficient for continuous variables but requires repair or rounding when discrete siting and technology labels are embedded in the same vector. ACO is naturally suited to discrete path or selection problems; therefore, continuous per-phase DG sizes must be discretized or handled through additional local search. These differences are important here because the location gene creates a rugged decoding map, while PUR, PF, voltage, and hourly penetration checks introduce discontinuous penalty changes after each load-flow evaluation. Table 9 summarizes these theoretical distinctions before the numerical benchmarking results are discussed.

Table 9. Theoretical comparison of the benchmarked optimizers for the proposed allocation problem.

Method	Search Mechanism	Suitability to Mixed DG Allocation	Main Limitation in This Study
GA	Population recombination through selection, crossover, mutation, and elitism	Directly preserves DG blocks containing location status, per-phase powers, and technology labels	Requires careful population repair and penalty scaling to maintain feasibility
PSO	Coordinated movement of particles through personal and global best positions	Efficient exploration of continuous DG sizes and smooth objective regions	Discrete bus/type variables require rounding and repair, which may disturb convergence near feasibility boundaries
ACO	Probabilistic construction of solutions from pheromone-guided components	Strong for discrete siting and combinatorial selection	Continuous per-phase active/reactive sizing is less natural and requires discretization or hybrid handling

Table 10 confirms that all tested algorithms converge to feasible DG allocations, but the GA achieves better final solution quality and lower run-to-run variation. The GA reached the highest peak performance with a best-run APLRI of 60.87% and a voltage improvement of 47.39%, outperforming both PSO (60.78%) and ACO (60.70%). In terms of repeatability, the GA recorded a mean APLRI of 60.85% with a standard deviation of 0.04%. The corresponding coefficient of variation (CoV) is 0.07%, which is considerably lower than the dispersion observed for PSO (0.44%) and ACO (0.23%). This tight statistical clustering indicates repeatable convergence to a high-quality feasible region under the PUR and LFE limits. Nevertheless, a low CoV by itself is not a proof of global optimality; it can also occur when the initialization and repair operators repeatedly guide the population toward the same feasible basin. For this reason, the repeatability result is interpreted together with three safeguards: randomized seeds over 30 independent runs, mixed initialization strategies (Latin hypercube, Sobol, uniform random, and boundary exploration), and benchmarking against PSO and ACO under the same objective function and computational budget. The result should therefore be read as evidence of reproducible feasible performance for the adopted search design rather than as a mathematical guarantee of global optimality. Although PSO obtained a slightly lower relative LFE of 1.34 compared to 1.37 for the GA, this minor difference in load following is offset by the GA's measurable advantage in loss reduction and convergence stability. These statistics support reproducible feasible performance under the adopted search design rather than establishing global optimality.

Table 10. Algorithm performance comparison for Case 5 over 30 independent runs.

Performance Metric	GA (This Study)	PSO ^a	ACO ^b
Primary Objective (APLRI)			
Best APLRI (%)	60.87	60.78	60.70
Mean APLRI (%)	60.85	60.68	60.52
Std Dev (%)	0.04	0.26	0.14
CoV (%)	0.07	0.44	0.23
Voltage Regulation			
Volt. Impr. at Best Run (%)	47.39	47.12	47.14
Additional Metrics			
Relative LFE (at Best Run)	1.37	1.34	1.36

Note: All algorithms were programmed by the authors to optimize the full 5-objective function (incl. LFE/PUR). CoV = Coefficient of variation (Std Dev/Mean). Best performance in key metrics is bolded. ^a PSO implementation based on the principles from [11]. ^b ACO implementation based on the principles from [35].

4.7. Benchmarking Against Recent Literature

Direct APLRI comparisons across published studies require caution because the reported values are strongly influenced by load representation, objective-function structure, and operating constraints. Table 11 therefore benchmarks the present method by the overall assessment's scope rather than by the headline APLRI alone. Related studies have reported advances in voltage-stability screening and AVR-assisted DG allocation [36,37], bi-level operational dispatch in distribution networks [38], phase-resolved identification of unbalanced demand [39], integrated DG/protection allocation [13], DG allocation with reconfiguration under varying loading levels [40], PV hosting-capacity planning [41], unbalanced DER hosting-capacity enhancement [42], and resilience-oriented DG planning with dynamic operational strategies [14]. However, these studies do not report the combined

LFE-based daily coordination and PUR-based diesel-safety screening adopted here for hybrid PV–WT–diesel allocation.

Table 11. Benchmark comparison on the IEEE 37-bus system.

Study	Optimization Method	APLRI (%)	Load Profile	24 h Analysis	Operational Realism Screening (LFE/PUR)	Comparability ^a
Fathy et al. (2024) [43]	Memory-based GTO	94.03	Static (Implied)	✗	✗	Low
Ali et al. (2022) [11]	Hybrid PSO-SCO	92.75	Stochastic (96 h)	✓	✗	Medium
Benahcour et al. (2024) [44]	Improved GWO (IGWO)	80.66	Static (Implied)	✗	✗	Low
Eid & Alsafrani (2024) [16]	Dynamic AOA (DAOA)	52.45	24 h Profile	✓	✗	High
This Study (Case 4)	GA (5-Objective)	64.57	Realistic (R pkg)	✓	LFE	—
This Study (Case 5)	GA (5-Objective)	60.87	Realistic (R pkg)	✓	LFE + PUR	—

Note: ✓ indicates that the feature is explicitly included in the cited study, whereas ✗ indicates that it is not explicitly included. ^a Comparability assesses methodological alignment: low denotes fundamentally different assumptions (e.g., static load); medium denotes shared features without key temporal constraints; high denotes a comparable framework (e.g., 24 h analysis). The LFE/PUR column identifies an engineering deployability benefit rather than a direct loss-reduction benefit: It shows whether the study explicitly screens daily generation–load mismatch and diesel phase-power imbalance before reporting the final allocation. Direct APLRI comparisons are approximate due to varying load profiles and objective functions. Case 4 and Case 5 values from this study represent the best optimization run. Abbreviations: AOA = arithmetic optimization algorithm; GTO = gorilla troops optimizer; GWO = grey wolf optimizer; SCO = sine cosine optimization.

The theoretical basis for the differences in Table 11 is the modeling burden imposed on the optimizer. Static-load studies can place DGs near peak-loss locations and report high APLRI values because the solution is not repeatedly tested against off-peak overgeneration, profile mismatch, or diesel phase-power imbalance. Stochastic or 24 h studies improve temporal realism, but if they do not include a feeder-level coordination metric or diesel PUR screen, a high loss-reduction solution may still be difficult to deliver in practice. If an optimized dispatch requires phase-power imbalance beyond the adopted PUR threshold, the diesel control or protection layer may prevent that operating point from being supplied as calculated. The present comparison therefore distinguishes numerical performance from engineering deployability: a lower APLRI under stricter screening can represent a more implementable solution than a higher APLRI obtained under simplified assumptions.

Studies reporting higher APLRI values (80% to 94%) [11,43,44] are generally based on less restrictive settings, including static peak-loading assumptions, fewer optimization objectives, and no explicit diesel phase-power screening. By contrast, the present framework combines realistic 24 h profiles, a five-objective formulation, the LFE metric, PUR screening, and algorithm-level reproducibility checks. Within that more demanding setting, the 60.87% APLRI obtained in Case 5 still exceeds the 52.45% reported by Eid and Alsafrani [16], whose study also considered a 24 h horizon but did not incorporate temporal coordination or diesel phase-imbalance screening. The more relevant comparison is therefore the level of operational realism retained while achieving the reported loss reduction.

4.8. Practical Deployment Considerations

From a computational standpoint, the adopted GA exhibits $\mathcal{O}(N_{pop} \times N_{gen} \times N_{bus}^2)$ complexity, where the dominant burden arises from repeated unbalanced load-flow evaluations within the fitness loop [45]. For the IEEE 37-bus system with a population size of 1000 and 200 generations, the average execution time was approximately 2.8 h on the workstation used in this study. For larger feeders, parallel fitness evaluation would be the most direct route to maintaining practical runtimes. To indicate scalability, Table 12 gives an order-of-magnitude runtime projection using the measured IEEE 37-bus runtime as the baseline and the above N_{bus}^2 dependence. The estimate is not a substitute for a full larger-feeder optimization study, but it clarifies the expected computational growth and the role of parallel fitness evaluation. Thus, the expected growth is polynomial with feeder size for the load-flow component rather than exponential; however, the absolute runtime can still become substantial because each candidate solution is evaluated over 24 hourly operating points.

Table 12. Indicative scalability estimate based on the measured IEEE 37-bus runtime.

Feeder Size	N_{bus}	Relative Load-Flow Effort	Estimated Serial Runtime	Idealized 16-Worker Runtime ^a
IEEE 37-bus baseline	37	1.00	2.8 h	0.18 h
IEEE 123-bus scale	123	11.05	30.9 h	1.93 h
250-bus scale	250	45.65	127.8 h	8.0 h

^a Idealized estimate based on independent fitness evaluations; actual speedup depends on load-balancing overhead, memory bandwidth, and parallel-pool efficiency.

From a deployment perspective, utility DG screening often still relies on fixed penetration heuristics, trial placement guided mainly by feeder topology, balanced approximations that ignore phase-specific behavior, or static snapshots that do not reflect daily coordination. The LFE profile, PUR screening, repeatability statistics, and branch-thermal verification developed here provide quantitative alternatives to these simplified checks, establishing a transparent basis for screening hybrid DG allocations on unbalanced feeders. Before field implementation, feeder-specific protection review and operating-policy validation would still be required.

The LFE profile also indicates when the hybrid PV-WT output departs appreciably from feeder demand and therefore when complementary measures such as storage, curtailment, or demand response may be warranted. The PUR screening criterion, in turn, distinguishes PUR-noncompliant from PUR-compliant diesel-backup dispatches, making the technical cost of the adopted screening requirement explicit. The earlier discussion has therefore been revised to avoid assigning numerical I_2/I_1 equivalents to the bus 712 PUR values. The reported reduction from 11.35% to 7.55% should be interpreted as a directly computed reduction in phase-power unbalance under the adopted PUR screening metric and not as a universal conversion to negative-sequence current. A detailed I_2/I_1 assessment would require post-processing the phase-current phasors from the load-flow solution and is outside the present planning-level constraint formulation.

4.9. Sensitivity to Objective Weight Selection

While this study adopts equal weighting ($k_i = 0.2$) for the reference Case 5 solution, alternative strategies can prioritize specific objectives. To illustrate these trade-offs, two

additional Case 5 variants were evaluated against the reference mean performance of 60.85% APLRI, 47.39% voltage improvement, and 1.37 LFE:

1. **Loss-focused weighting** (assigning $k_{APLRI} = 0.6$ and $k = 0.1$ for all other objectives) increased APLRI to 61.52% and voltage improvement to 48.15%, corresponding to gains of 0.67 pp and 0.76 pp, respectively, relative to the reference Case 5 solution. The corresponding LFE increased only slightly to 1.39.
2. **LFE-focused weighting** (assigning $k_{LFE} = 0.6$ and $k = 0.1$ for all others) reduced the best-run LFE to 1.10, improving load following relative to the 1.37 reference value. This improvement came at the expense of APLRI, which decreased to 55.26%, a reduction of 5.59 pp.

This analysis confirms the framework's flexibility and supports the equal-weight setting in Case 5 as a balanced compromise between technical performance, operational safety, and load following. At the same time, the weighted-sum formulation should not be interpreted as a complete Pareto-front search. It provides one transparent reference compromise and two local weighting checks, but it cannot reveal non-convex portions of the non-dominated trade-off surface among loss reduction, voltage quality, penetration, LFE, and DG count. Future work should employ multi-objective evolutionary algorithms (e.g., Non-Dominated Sorting Genetic Algorithm (NSGA-II/III)) to generate complete Pareto fronts for utility-specific trade-off selection [46,47].

4.10. Limitations

The present validation is limited to the IEEE 37-bus feeder and deterministic 24 h load and resource profiles. Although this benchmark is intentionally challenging because of its pronounced phase asymmetry, broader validation on additional feeders would strengthen the generality of the screening framework. In addition, the diesel-generator model remains planning-oriented: It captures dispatch limits and phase-unbalance safety, but it does not represent detailed fuel-consumption curves, thermal states, or start-stop cycling costs. The diesel-backup schedule is also fixed to the adopted 19:00–22:00 standby-service window, so the framework does not yet optimize backup timing jointly with DG siting and sizing. The added post-processing energy accounting reports the renewable-energy fraction, indicative diesel fuel use, and direct diesel CO₂ for the optimized cases, but it should not be interpreted as a complete emission-dispatch or techno-economic assessment. A further limitation is the computational cost: Because the fitness function repeatedly solves a three-phase load flow for each candidate and each hour, the reported implementation is more demanding than static single-snapshot allocation. Finally, the equal-weight objective is used as a transparent reference setting; a complete Pareto-front analysis would be needed before claiming that one weight set is universally optimal for all utility planning priorities. From a practical planning perspective, these limitations mean that the present results should be used as a safety-aware screening benchmark rather than as a final investment plan. In field studies, stochastic load and renewable scenarios would be needed to quantify seasonal and weather-driven uncertainty; the diesel schedule should be co-optimized with storage, curtailment, and demand-response options; techno-economic and environmental quantities, including capital and operating costs, fuel costs based on manufacturer-specific part-load curves, avoided-grid and lifecycle emissions, and avoided-loss value, should be incorporated before selecting a utility investment strategy. These extensions would allow the LFE/PUR screening concept to be integrated into a full techno-economic planning workflow.

5. Conclusions

This paper developed and validated an assessment framework for hybrid DG allocation in unbalanced distribution feeders, with explicit treatment of hourly generation–load

coordination and PUR-screened diesel backup operation. Applied to the IEEE 37-bus system with realistic 24 h load and resource profiles, the framework uses two post-load-flow quantities, LFE and PUR, to capture operational effects that conventional loss-based indices do not represent directly: hourly generation–load mismatch and phase-power asymmetry during diesel dispatch.

The results show that hybrid allocation can improve feeder performance while retaining compliance with the adopted diesel phase-power screening threshold. Enforcing the PUR screening criterion at 10%, used here as a study-specific planning–screening threshold, eliminates an infeasible diesel allocation while preserving most of the technical benefit of the unconstrained hybrid solution. Specifically, the feasible Case 5 solution (max PUR = 7.55%) retains 94.27% of the unconstrained loss-reduction benefit achieved by Case 4, with APLRI decreasing from 64.57% to 60.87%. The corresponding 3.7 pp performance penalty provides a direct measure of the cost of PUR compliance in this feeder.

Restricting diesel dispatch to the adopted 3 h evening window keeps the assessment centered on renewable supply rather than prolonged backup generation. Branch-thermal verification also confirmed that all studied scenarios remained within conductor ampacity limits, with peak utilization factors below 90%. This additional feasibility check supports interpreting the reported loss-reduction and voltage-regulation gains as operationally feasible on the studied feeder.

Across 30 independent runs and the GA/PSO/ACO benchmark, the adopted GA produced the lowest dispersion (CoV 0.07%), indicating that the reported Case 5 result is reproducible rather than an isolated stochastic outcome. Future work will extend the framework to larger feeders, stochastic renewable and load scenarios, and storage-integrated operation to reduce residual LFE. The framework therefore provides a practical basis for screening hybrid PV-WT integration in unbalanced distribution networks under explicit diesel phase-power screening limits.

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