

Article

Variable-Horizon MPC-Based Energy Management for Battery–Supercapacitor Hybrid Power Supply of Contactless Rail Vehicles

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Abstract

The absence of overhead catenary systems in contactless trams imposes stringent requirements on onboard energy efficiency and real-time power management. Hybrid energy storage systems combining batteries and supercapacitors provide an effective solution; however, conventional energy management strategies often suffer from limited global optimality under frequent traction–braking conditions. To address this issue, this paper proposes a variable-horizon model predictive control (MPC)-based energy management strategy for a battery–supercapacitor hybrid power supply system in contactless trams. A power-level-matching method is first adopted for capacity configuration, and the MPC prediction horizon is then dynamically adjusted to cover the entire traction phase, enabling global energy loss optimization while satisfying voltage, current, and SOC constraints. Simulation results obtained in MATLAB/Simulink demonstrate that the proposed strategy effectively suppresses excessive battery current and premature supercapacitor depletion. Compared with the conventional single-step MPC, the total energy loss is reduced by 9.88%, indicating improved energy efficiency and operational performance.

Keywords: contactless tram; hybrid energy storage system; model predictive control; energy management

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1. Introduction

With the increasing demand for energy-efficient, low-emission, and flexible urban rail transit systems, contactless trams equipped with onboard energy storage systems have attracted growing attention in recent years. Unlike conventional catenary-based traction systems, contactless trams rely on onboard energy storage devices to supply traction and auxiliary loads, enabling operation in catenary-free sections and significantly improving urban landscape compatibility and system flexibility [1,2]. However, the

absence of external power supply also imposes stringent requirements on onboard energy utilization efficiency, energy storage capacity configuration, and real-time power management capability.

Hybrid energy storage systems (HESS), typically composed of lithium-ion batteries and supercapacitors, have been widely recognized as an effective solution for contactless rail vehicles [3]. Batteries provide high energy density to satisfy long-term energy demand, while supercapacitors offer high power density and fast charge–discharge capability to handle transient power peaks during traction and regenerative braking [4,5]. Domínguez et al. [6] reviewed the application of energy storage systems in railway systems and pointed out that energy storage technologies are essential for improving energy efficiency, especially in regenerative braking energy recovery and onboard power supply applications. For tramway systems, Tostado-Véliz et al. [7] developed an optimization framework for planning wayside and onboard hybrid storage systems composed of batteries and supercapacitors, showing that the coordinated use of different storage technologies can improve energy utilization and reduce system operating costs. Fallah et al. [8] propose a battery–supercapacitor hybrid energy storage system for electric trains, in which a novel DC–DC converter and a sliding-mode-based energy management strategy are developed to enable regenerative braking energy recovery and traction power demand reduction, while an observer is introduced to estimate the internal voltage of the supercapacitor. Mansouri et al. [9] propose an advanced management strategy for adjusting battery charging and discharging cycles based on estimated demand, achieving optimal control of battery energy storage systems. Through coordinated utilization of these complementary characteristics, hybrid energy storage systems can significantly reduce battery current stress, enhance regenerative energy recovery, and extend the overall service life of energy storage devices. Nevertheless, the effectiveness of such systems strongly depends on rational capacity configuration and efficient energy management strategies, especially under frequent traction–braking cycles and highly dynamic operating conditions.

Extensive studies have been conducted on capacity sizing and energy management of hybrid energy storage systems in electric vehicles and rail transit applications. Conventional approaches often employ rule-based strategies or heuristic power allocation methods, which are easy to implement and computationally efficient [10,11]. However, these methods usually rely on predefined thresholds and lack global optimality, resulting in suboptimal energy efficiency and excessive battery degradation under varying operating conditions. More advanced optimization-based methods, such as dynamic programming (DP), equivalent consumption minimization strategy (ECMS), and model predictive control (MPC), have been introduced to overcome these limitations [12–14]. Han et al. [15] investigate a multi-stack fuel cell hybrid electric tracked vehicle and propose a multi-objective hierarchical energy management strategy reconstructed from dynamic programming results, achieving near-optimal fuel economy while ensuring battery SOC regulation and consistent aging among fuel cell systems. Zhang et al. [16] propose an adaptive energy management strategy based on a modified ECMS framework for automated hybrid electric vehicles, jointly optimizing torque distribution and gearshift commands to improve fuel efficiency while maintaining drivability. Xu et al. [17] proposed a real-time energy management strategy based on aging- and temperature-conscious MPC for hybrid electric trains, in which battery aging and thermal issues were considered to improve the long-term performance of the hybrid energy storage system. Xu et al. [18] further developed a two-stage coordinated power allocation strategy for onboard hybrid energy storage systems in urban rail transit, where traction energy consumption and lithium-ion battery degradation were jointly considered in a comprehensive operating cost. These studies show that MPC-based methods have strong

potential for constrained real-time energy management of rail transit hybrid energy storage systems. Among them, MPC has gained considerable attention due to its ability to explicitly handle system constraints and predict future system behavior, making it particularly suitable for real-time energy management of hybrid energy storage systems.

Despite its advantages, conventional MPC-based energy management strategies often adopt a fixed and short prediction horizon to reduce computational burden. While such single-step or short-horizon MPC schemes can ensure real-time feasibility, they tend to focus on local optimization and may lead to undesirable behaviors, such as premature depletion of supercapacitor energy and large battery current peaks during traction phases [19]. These issues are especially pronounced in contactless tram applications, where the energy management strategy must coordinate energy utilization over an entire inter-station traction process rather than individual time steps. Meanwhile, recent studies have significantly improved the reliability of contactless power transfer systems through advanced magnetic-coupler design, enhanced misalignment tolerance, and closed-loop multi-physics co-design, providing a more robust hardware foundation for electrified transportation [20,21]. However, improvements in wireless power transfer hardware do not eliminate the need for advanced energy management strategies, since efficient coordination of the onboard battery and supercapacitor remains critical for completing each inter-station traction process.

Although variable-horizon and look-ahead MPC methods have been investigated in energy management applications, their direct use in contactless tram systems still requires further adaptation. For a contactless tram powered by an onboard battery–supercapacitor hybrid power supply system, the vehicle must complete each inter-station traction process without continuous external power supply. Therefore, the energy management strategy should not only satisfy the instantaneous traction power demand, but also coordinate the utilization of the battery and supercapacitor over the entire remaining traction phase. If only single-step or short-horizon optimization is adopted, the supercapacitor may be depleted prematurely, forcing the battery to provide large current during the later traction stage.

In this study, the contribution is therefore not a generic MPC theory, but a tram-oriented variable-horizon MPC energy management framework. The prediction horizon is explicitly determined according to the remaining duration of the current inter-station traction phase. At station departure, the controller considers a longer future traction process; as the tram approaches the next station, the prediction horizon gradually shortens. This design enables the supercapacitor energy to be distributed more reasonably over the whole traction phase while maintaining voltage, current, and SOC constraints. Furthermore, the proposed control strategy is combined with a power-level-matching-based capacity configuration method and the operating-mode analysis of traction, braking, and station-stop conditions.

The rest of this paper is organized as follows. Section 2 presents the modeling of the battery and supercapacitor for the contactless tram traction system. Section 3 discusses the energy configuration method and analyzes energy flow characteristics under different operating conditions. Section 4 proposes the variable-horizon MPC-based energy management strategy in detail. Section 5 provides a simulation-based verification and performance comparison.

2. Characteristics Analysis and Modeling of a Contactless Tram Traction System

This paper takes a contactless tram as the research object. The traction power supply adopts an onboard energy storage-based contactless scheme, enabling full-line operation in areas without overhead contact lines. The hybrid power supply system of the

contactless tram investigated in this paper is illustrated in Figure 1, where the traction system employs both batteries and supercapacitors as energy sources to provide the required power for the train.

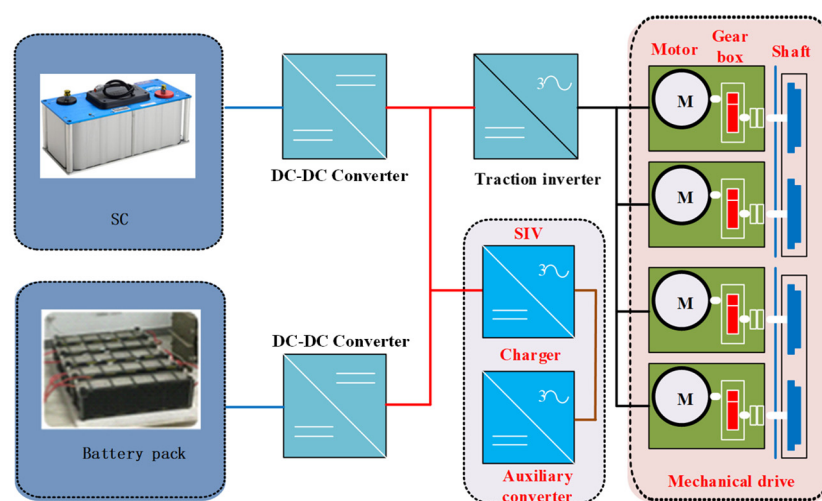


Figure 1. Schematic diagram of hybrid power supply system for contactless train.

2.1. Characteristics and Modeling of Power Battery Pack

At present, lithium batteries are widely used in catenary-free power supply systems in the rail transit field. When lithium batteries are applied to the onboard energy storage system of the contactless tram, they provide energy for both the traction system and auxiliary systems. Therefore, it is necessary to investigate and model the characteristics of the battery pack. Commonly used lithium batteries in energy storage systems include lithium titanate (LTO), lithium manganate (LMO), and lithium iron phosphate (LFP). A comparison of the performance of these three typical lithium batteries is presented in Table 1 [22].

Table 1. Typical lithium battery performance comparison.

Type	Lithium Manganate	Lithium Iron Phosphate	Lithium Titanate Battery
Voltage (V)	3.7	3.2	2.3
Energy density (Wh/kg)	120	110	80
Charge–discharge rate	0.5 C	2 C	10 C
Cycle life	1000	2000	10,000
Safety performance	Medium	High	High
Environmental performance	High	High	High

LTO batteries, as a novel type of lithium-ion battery, exhibit higher safety compared with other lithium-ion batteries. In addition, they feature a more stable structure and a longer cycle life. Considering the train operational requirements, energy-saving and environmental protection principles, as well as future development trends, the performance parameters of the selected LTO cells for the contactless tram power supply system are listed in Table 2.

Table 2. Lithium titanate battery cell parameters.

Performance	Value	Unit
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Working temperature range	−25~60	°C
Rated voltage	2.3	V
Nominal capacity	8.5	Ah
Mass energy density	73	Wh/kg
Volume energy density	100	Wh/L
Charging cut-off voltage	2.8	V
Discharge cut-off voltage	1.5	V
Maximum charging current	8 C	A
Maximum discharge current	12 C	A

With the increasing maturity of battery energy storage technologies, numerous studies have proposed and developed models for battery cells, including neural network models, electrochemical models, and equivalent circuit models [23–25]. To simplify the battery pack and facilitate subsequent analysis, an equivalent circuit model (ECM) is adopted in this paper to establish the LTO cell model, as illustrated in Figure 2.

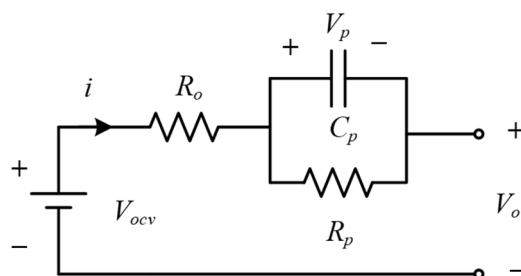


Figure 2. Model of battery cell.

According to the model shown in Figure 2, the equations for the output voltage and current of a single lithium titanate battery in continuous state can be derived as follows:

$$\begin{aligned} V_o(t) &= V_{ocv}(t) - i(t)R_o - V_p(t) \\ i(t) &= C_p \frac{dV_p(t)}{dt} + \frac{V_p(t)}{R_p} \end{aligned} \quad (1)$$

where V_o denotes the battery output voltage; i represents the battery output current; R_o is the internal resistance; V_{ocv} denotes the open-circuit voltage; V_p is the polarization voltage; C_p is the polarization capacitance; and R_p is the polarization resistance.

The output voltage equation in the discrete-time domain is given by:

$$\begin{aligned} V_o(k) &= V_{ocv}(k) - i(k)R_o - V_p(k) \\ V_p(k) &= (i(k-1) - \frac{V_p(k-1)}{R_p}) \frac{T_s}{C_p} + V_p(k-1) \end{aligned} \quad (2)$$

where T_s denotes the sampling period.

The state of charge (SOC) of the battery is calculated by the Coulomb counting method. The discharge current is defined as positive. Therefore, the continuous-time SOC equation is expressed as:

$$\frac{dSOC_{batt}(t)}{dt} = -\frac{i_{batt}(t)}{3600Q_{batt}} \quad (3)$$

equivalently,

$$SOC_{batt}(t) = SOC_{batt}(t_0) - \frac{1}{3600Q_{batt}} \int_{t_0}^t i_{batt}(\tau) d\tau \quad (4)$$

For discrete-time implementation, the SOC update equation is written as:

$$SOC_{batt}(k+1) = SOC_{batt}(k) - \frac{i_{batt}(k)T_s}{3600Q_{batt}} \quad (5)$$

where Q_{batt} denotes the nominal capacity of the battery. i_{batt} is the battery current. T_s is the sampling period.

The open-circuit voltage (OCV) of the LTO cell is correlated with its SOC, typically exhibiting a positive relationship. Figure 3 shows the relationship between the OCV and the SOC at room temperature (25 °C). It can be observed from the curve that when the SOC is higher than 90% or lower than 10%, the variation rate of the OCV is relatively large. When the SOC ranges from 10% to 90%, the curve is relatively smooth and the OCV fluctuation is small. In particular, when the SOC is between 10% and 40%, the OCV remains nearly constant and is barely affected by the SOC. Therefore, considering the objectives of extending battery lifetime, improving power supply quality, and reducing system cost, the depth of discharge should be minimized as much as possible while satisfying the train operational requirements, and the SOC operating range should be restricted accordingly. In this study, the SOC operating range of the battery is set to 20–80%.

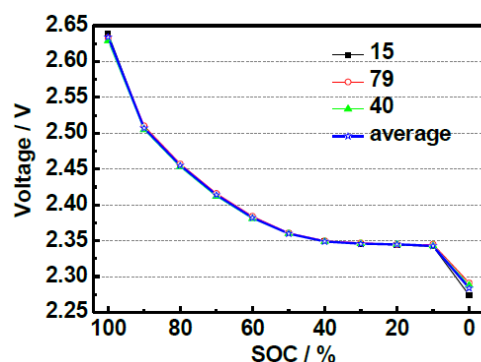


Figure 3. OCV–SOC curve of lithium titanate battery cells.

Figure 4 presents variations in the OCV, temperature, and SOC of the LTO cell under different charge–discharge rates at room temperature (25 °C). It can be observed that as the charge–discharge rate increases, the internal resistance of the battery becomes larger, leading to a corresponding increase in temperature rise. However, under high charge–discharge rates, the SOC operating range of the battery remains almost unchanged compared with that under a single-rate condition. This indicates that the LTO battery is capable of achieving high-rate charge and discharge within a short period, demonstrating its suitability as a high-power energy storage device.

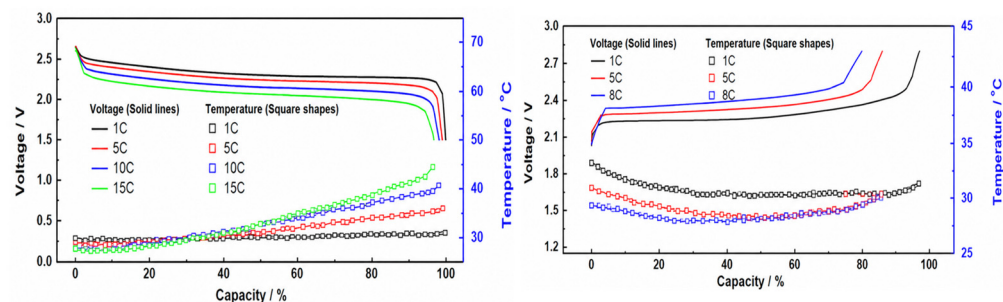


Figure 4. Voltage and temperature vs. SOC curve of lithium titanate cell with operating rate.

2.2. Characteristics and Modeling of Supercapacitor

Supercapacitors are energy storage devices positioned between conventional capacitors and batteries, featuring high power density, a short charge–discharge time, a long cycle life, and a wide operating temperature range. In recent years, supercapacitors have been increasingly applied in hybrid trains, mainly to address the following issues [26,27]:

- (1) To provide high transient power for the train during start-up, acceleration, and uphill conditions, thereby mitigating stress on the battery.
- (2) To address the high-current regenerative charging issue during braking conditions of hybrid trains.
- (3) To enhance the operational capability of hybrid trains under low-temperature conditions.

The performance parameters of the supercapacitor module selected for the contactless tram power supply system in this study are listed in Table 3.

To facilitate the subsequent analysis of the charge–discharge characteristics of the supercapacitor, an equivalent model is first established. The RC equivalent circuit model of the supercapacitor cell is constructed, as illustrated in Figure 5.

Table 3. Main specifications of the supercapacitor module.

Performance	Value	Unit
Rated voltage	48	V
Rated capacitance	83	F
Capacity	26.56	Wh
Maximum current	1150	A
Internal resistance	0.01	Ω
Service life	10	Year
Operating temperature	−40~65	$^{\circ}\text{C}$

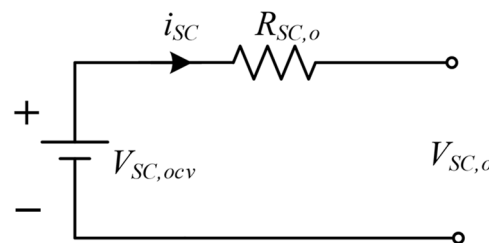


Figure 5. Supercapacitor cell model.

Based on the model shown in Figure 5, the continuous-time equations describing the open-circuit voltage, output voltage, and current of the supercapacitor cell can be derived as follows:

$$V_{SC,ocv}(t) = V_{SC,ocv}(0) - \frac{1}{C} \int_0^t i_{SC} dt \quad (6)$$

$$V_{SC,o}(t) = V_{SC,ocv}(t) - i_{SC}(t)R_{SC,o}$$

where $V_{SC,ocv}$ denotes the open-circuit voltage of the supercapacitor. C is the capacitance. i_{SC} represents the output current of the supercapacitor. $V_{SC,o}$ is the output voltage of the supercapacitor. $R_{SC,o}$ denotes the equivalent internal resistance of the supercapacitor.

The corresponding discrete-time equations are given by:

$$V_{SC,o}(k) = V_{SC,ocv}(k) - i_{SC}(k)R_{SC,o}$$

$$V_{SC,ocv}(k) = i_{SC}(k-1)\frac{T_s}{C} + V_{SC,ocv}(k-1) \quad (7)$$

The SOC of the supercapacitor is calculated as follows:

$$\text{SOC}_{SC}(t) = \left(\frac{V_{SC,ocv}(t)^2}{V_{SC,ocvmax}(t)^2} \right) \quad (8)$$

where $V_{SC,ocvmax}$ denotes the nominal open-circuit voltage of the supercapacitor.

When the supercapacitor is charged and discharged under constant current conditions, the variation of its terminal voltage is shown in Figure 6, where ΔV_1 , ΔV_2 , ΔV_3 and ΔV_4 denote the voltage drops across the internal resistance at the beginning of charging, the end of charging, the beginning of discharging, and the end of discharging, respectively.

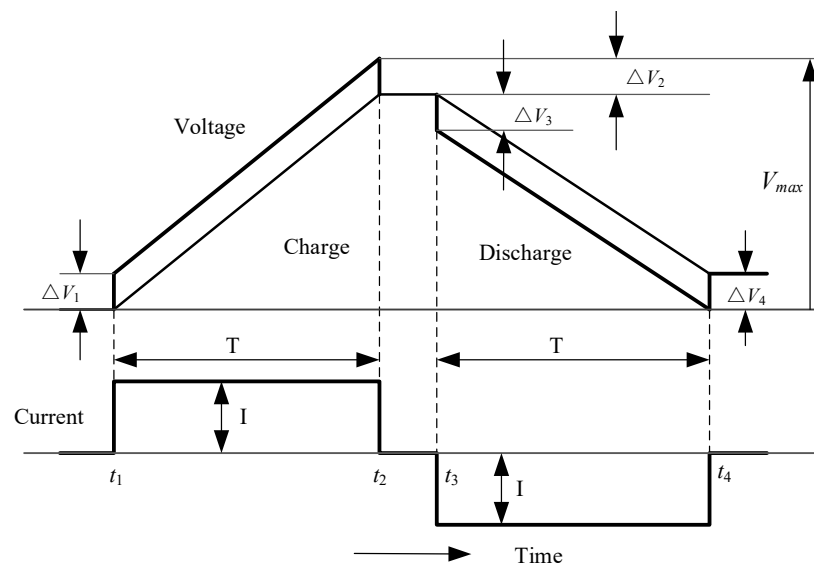


Figure 6. Supercapacitor charge and discharge waveforms.

3. Energy Configuration and Modeling of the Train Traction Power Supply System

To ensure efficient, safe, and economical operation of the hybrid energy storage-based traction power supply system over the entire operating cycle, it is necessary to systematically model and analyze the system energy configuration and its dynamic energy flow characteristics.

3.1. Capacity Configuration of Power Supply System

To ensure that the train traction system meets the energy demand of train operation while minimizing the number of battery cells and supercapacitor cells, thereby improving the overall economic performance, it is necessary to reasonably and effectively configure the two energy sources of the traction system. In this paper, a power-level-matching configuration method is adopted. The main idea is to fully exploit the performance of the supercapacitor: under traction conditions, the supercapacitor provides energy to the train at its maximum discharge capability, while the remaining traction energy demand is supplied by the battery, thereby determining the required battery capacity. Under braking conditions, the supercapacitor absorbs the regenerative braking energy at its maximum charging capability, thereby determining the required supercapacitor capacity. The schematic of the power-level-matching configuration method is shown in Figure 7.

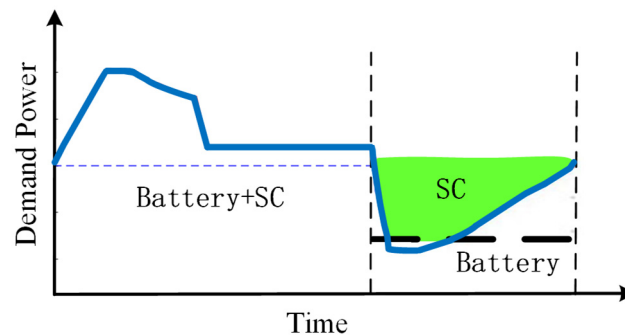


Figure 7. Power-level-estimation method.

Therefore, the capacity of the supercapacitor is determined as follows:

$$N_{p.SC} N_{s.SC} = \text{ceil} \left(\frac{2E_{SC,max}}{0.5C(V_{SC,max}^2 - V_{SC,min}^2)\eta_{DC/DC}} \right) \quad (9)$$

where $N_{s.SC}$ denotes the number of supercapacitor cells connected in series. $N_{p.SC}$ denotes the number of supercapacitor cells connected in parallel. $E_{SC,max}$ represents the maximum energy absorbed by the supercapacitor during the entire train operation. $V_{SC,max}$ and $V_{SC,min}$ denote the upper and lower voltage limits of a supercapacitor cell, respectively. $\eta_{DC/DC}$ denotes the converter efficiency. ceil represents the ceiling function.

Under traction conditions, the battery capacity is determined as follows:

$$N_{p.batt} N_{s.batt} = \text{ceil} \left(\frac{E_{tra.all} - E_{SC.all}}{QV_{1.batt} DOD_{batt} \eta_{DC/DC}} \right) \quad (10)$$

where $N_{s.batt}$ denotes the number of lithium-ion battery cells connected in series; $N_{p.batt}$ denotes the number of lithium-ion battery cells connected in parallel. $E_{tra.all}$ represents the total energy demand of the train. $E_{SC.all}$ denotes the total energy of the supercapacitor. Q is the capacity of a lithium-ion battery cell. $V_{1.batt}$ denotes the voltage of a lithium-ion battery cell. DOD_{batt} represents the depth of discharge of the lithium-ion battery.

Based on the energy requirements of the train under traction and braking conditions, the proposed method utilizes (7) and (8) for calculation. The resulting configuration yields 200 cells in series and 20 cells in parallel for the LTO battery pack, with a total energy of 78.20 kWh. For the supercapacitor pack, 10 cells are connected in series and 18 in parallel, resulting in a total energy of 4.78 kWh.

It should be noted that the power-level-matching method used in this study is an engineering-oriented preliminary sizing method rather than a comprehensive multi-objective capacity optimization. The obtained battery and supercapacitor capacities depend on the selected route power profile, regenerative braking energy, converter efficiency, allowable battery DOD, and initial energy state of the storage devices. Therefore, the configuration obtained from Equations (7) and (8) should be regarded as a feasible baseline design for the studied contactless tram operating condition. If the route power profile, converter efficiency, allowable DOD, or initial SOC changes significantly, the required capacity should be recalculated accordingly.

3.2. Energy Flow Distribution of the Hybrid Power Supply System Under Different Operating Conditions

When the train operates under traction conditions, the supercapacitor is preferentially utilized to supply power due to its high power density and fast charge–discharge capability. As the supercapacitor reaches its maximum discharge capability, i.e., when its voltage drops to the lower threshold, the remaining traction energy demand is

supplied by the battery pack. The energy flow of the system under traction conditions is illustrated in Figure 8.

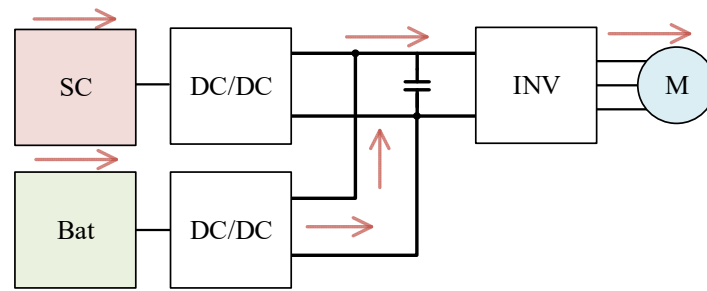


Figure 8. Energy flow direction under traction conditions.

According to the energy management strategy, the relationship among the traction power demand, the supercapacitor output power, and the battery pack output power under traction conditions is illustrated in Figure 9. The two dashed lines indicate the moments when the supercapacitor and battery pack reach their maximum output power, respectively.

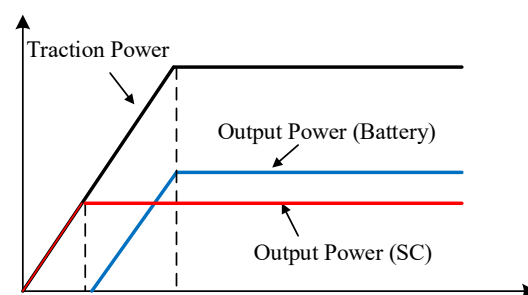


Figure 9. Power distribution under traction conditions.

When the train operates under braking conditions, according to the capacity configuration criteria in the previous section, the supercapacitor absorbs the regenerative braking energy at its maximum charging capability. The energy flow of the system under braking conditions is illustrated in Figure 10.

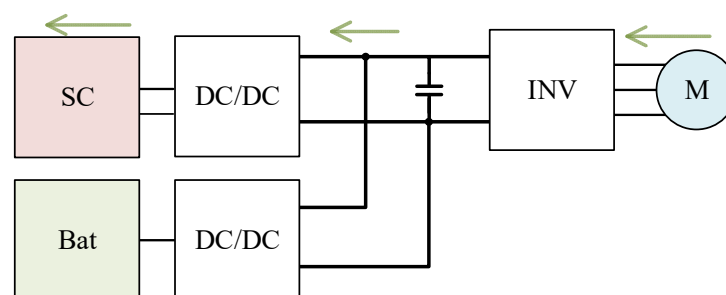


Figure 10. Energy flow direction under braking conditions.

When the train is at a station, the battery discharges to supply the auxiliary loads. If the SOC of both the supercapacitor and the battery pack cannot meet the train start-up requirements, both are charged simultaneously at the station. If the supercapacitor has absorbed sufficient regenerative braking energy to meet the start-up requirement while the battery pack SOC is relatively low, the supercapacitor can be used to charge the battery pack. The energy flow of the system under the station-stop condition is illustrated in Figure 11.

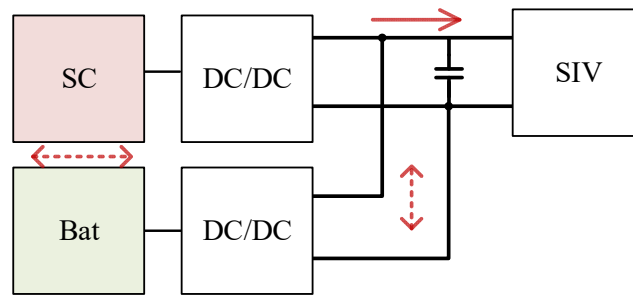


Figure 11. Energy flow direction under stop conditions.

From the above analysis, the energy flow distribution of the traction system under different operating conditions can be obtained. However, during actual train operation, the detailed energy flow control as well as the charge–discharge control of the supercapacitor and the battery pack must be realized through an energy management control algorithm.

4. Model Predictive Control-Based Energy Management for Trains

To realize energy management control of the self-guided train traction system, it is necessary to design a model predictive control (MPC) strategy by considering the train operational requirements as well as the characteristics of lithium-ion batteries and supercapacitors. The MPC framework mainly includes the optimization objective, constraints, and energy allocation strategy. The objectives of the energy management control system can be summarized as follows:

- (1) The energy storage-based power supply system should satisfy the operational requirements of the train under various operating conditions.
- (2) The battery and supercapacitor in the energy storage system must operate within reasonable limits, which are reflected as constraints on charge–discharge current, voltage, and SOC.
- (3) The energy loss of the energy storage system should be minimized to improve overall efficiency.
- (4) Considering that the battery and supercapacitor exhibit high energy density and high power density, respectively, the supercapacitor should be preferentially utilized to handle transient high-current demands, particularly by absorbing regenerative braking energy under braking conditions.

4.1. Optimization Objectives of MPC for Energy Management

MPC is employed to perform energy management of the train hybrid power system. While ensuring that the operational requirements of the train are satisfied, the optimization objective in this paper is to reduce the energy loss of the hybrid power supply system during train operation, mainly considering the energy losses of the battery and the supercapacitor. Compared with the conventional fixed-horizon MPC approach, an improved variable-horizon MPC algorithm is proposed. By assigning different initial prediction horizons, two control modes can be realized. In the variable-horizon MPC, the prediction horizon is designed to cover the entire inter-station traction phase from each station departure, thereby theoretically enabling optimal control over the full operating range.

In this study, the MPC objective is formulated as an energy-loss-oriented criterion based on the internal resistive losses of the battery and supercapacitor. This is because the main purpose of the proposed strategy is to determine the real-time power allocation between the two energy storage devices; for a given traction power demand, the allocation

coefficient directly determines the battery and supercapacitor currents, and thus their heat losses. Therefore, this objective provides a practical and computationally efficient formulation for system-level energy management. It should be noted that converter losses, battery aging cost, thermal effects, and DC-bus voltage quality are not explicitly included in the present objective. Converter efficiency is considered in the capacity configuration stage, while voltage, current, and SOC limits are imposed as MPC constraints. Therefore, the proposed formulation should be regarded as a simplified energy efficiency-oriented optimization rather than a comprehensive life-cycle or electro-thermal optimization. Future work will extend the objective by incorporating converter efficiency maps, battery aging and thermal models, and DC-bus voltage quality indices.

In the fixed-horizon predictive control, the initial prediction horizon is set to 1. The optimization objective of the control system is to minimize the energy consumption of the traction system at time step k , which can be expressed as follows:

$$W_{loss}(k) = W_{loss.batt}(k) + W_{loss.SC}(k) \quad (11)$$

$$W_{loss.batt}(k) = i_{batt}^2(k) R_{o.batt}(k) T_s = \left(\frac{P_{dmd}(k)(1-\alpha(k))}{V_{batt}(k)} \right)^2 R_{o.batt}(k) T_s \quad (12)$$

$$W_{loss.SC}(k) = i_{SC}^2(k) R_{o.SC}(k) T_s = \left(\frac{P_{dmd}(k)\alpha(k)}{V_{SC}(k)} \right)^2 R_{o.SC}(k) T_s \quad (13)$$

where W_{loss} denotes the total energy loss of the traction system. $W_{loss.batt}$ represents the energy loss of the lithium-ion battery. $W_{loss.SC}$ denotes the energy loss of the supercapacitor. P_{dmd} is the DC-side power. α is the power allocation coefficient, which also serves as the control variable of the control system.

Accordingly, the evaluation function of the MPC optimization objective can be written as follows:

$$G(k) = W_{loss}(k) = \left(\frac{P_{dmd}(k)(1-\alpha(k))}{V_{batt}(k)} \right)^2 R_{o.batt} T_s + \left(\frac{P_{dmd}(k)\alpha(k)}{V_{SC}(k)} \right)^2 R_{o.SC}(k) T_s \quad (14)$$

In the proposed variable-horizon predictive control, the initial prediction horizon is set to 100. The optimization objective of the control system is to minimize the total energy consumption of the traction system from the current time step to the end of the traction phase, which can be expressed as follows:

$$W_{loss}(k) = \sum_{k=1}^i (W_{loss.batt}(k) + W_{loss.SC}(k)) \quad (15)$$

$$W_{loss.batt}(k) = \sum_{k=1}^i \left(\frac{P_{dmd}(k)(1-\alpha(k))}{V_{batt}(k)} \right)^2 R_{o.batt} T_s \quad (16)$$

$$W_{loss.SC}(k) = \sum_{k=1}^i \left(\frac{P_{dmd}(k)\alpha(k)}{V_{SC}(k)} \right)^2 R_{o.SC}(k) T_s \quad (17)$$

Accordingly, the evaluation function of the variable-horizon MPC optimization objective can be written as follows:

$$G(k) = W_{loss}(k) = \sum_{k=1}^i \left(\frac{P_{dmd}(k)(1-\alpha(k))}{V_{batt}(k)} \right)^2 R_{o.batt} T_s + \sum_{k=1}^i \left(\frac{P_{dmd}(k)\alpha(k)}{V_{SC}(k)} \right)^2 R_{o.SC}(k) T_s \quad (18)$$

4.2. Constraints of Model Predictive Control

In the fixed-horizon predictive control, the constraint is defined as the predicted SOC of the energy storage system at the next time step. Based on the established lithium-ion

battery cell model and supercapacitor cell model, the voltages of the lithium-ion battery and the supercapacitor at the next time step can be predicted as follows:

$$SOC(k+1) = SOC(k) - \frac{i_{batt}(k)T_s}{Q} \quad (19)$$

$$V_{ocv.batt}(k+1) = f_{ocv}(SOC(k+1)) \quad (20)$$

$$R(k+1) = f_R(SOC(k+1)) \quad (21)$$

$$V_{batt}(k+1) = V_{ocv.batt}(k+1) - i_{batt}(k)R_{o.batt}(k+1) - i_p R_p(k+1) \approx V_{ocv.batt} - i_{batt}R(k+1) \quad (22)$$

$$V_{sc}(k+1) = V_{ocv.sc}(k+1) - i_{sc}(k)R_{o.sc} = \frac{i_{sc}(k)T_s}{C} + V_{ocv.sc}(k) - i_{sc}(k)R_{o.sc} \quad (23)$$

The corresponding constraints of the model predictive control can be expressed as follows:

$$\begin{cases} V_{batt.min} \leq V_{batt}(k+1) \leq V_{batt.max} \\ V_{SC.min} \leq V_{SC}(k+1) \leq V_{SC.max} \\ I_{batt.min} \leq I_{batt}(k+1) \leq I_{batt.max} \\ I_{SC.min} \leq I_{SC}(k+1) \leq I_{SC.max} \\ SOC_{batt.min} \leq SOC_{batt}(k+1) \leq SOC_{batt.max} \\ SOC_{SC.min} \leq SOC_{SC}(k+1) \leq SOC_{SC.max} \\ \alpha \in [0,1] \end{cases} \quad (24)$$

In the variable-horizon predictive control, the constraints are defined based on the predicted values from the current time step to the end of the traction phase, which can be expressed as follows:

$$\begin{cases} V_{batt.min} \leq V_{batt}(k) \leq V_{batt.max}, k=1,2\dots i \\ V_{SC.min} \leq V_{SC}(k) \leq V_{SC.max}, k=1,2\dots i \\ I_{batt.min} \leq I_{batt}(k) \leq I_{batt.max}, k=1,2\dots i \\ I_{SC.min} \leq I_{SC}(k) \leq I_{SC.max}, k=1,2\dots i \\ SOC_{batt.min} \leq SOC_{batt}(k) \leq SOC_{batt.max}, k=1,2\dots i \\ SOC_{SC.min} \leq SOC_{SC}(k) \leq SOC_{SC.max}, k=1,2\dots i \\ \alpha \in [0,1] \end{cases} \quad (25)$$

where $V_{batt.min}$ and $V_{batt.max}$ denote the lower and upper voltage limits of the lithium-ion battery, respectively. $V_{SC.min}$ and $V_{SC.max}$ denote the lower and upper voltage limits of the supercapacitor, respectively. $i_{batt.min}$ and $i_{batt.max}$ denote the minimum and maximum allowable currents of the lithium-ion battery, respectively. $i_{SC.min}$ and $i_{SC.max}$ denote the minimum and maximum allowable currents of the supercapacitor, respectively. $SOC_{batt.min}$ and $SOC_{batt.max}$ denote the lower and upper bounds of the battery SOC, respectively. $SOC_{SC.min}$ and $SOC_{SC.max}$ denote the lower and upper bounds of the supercapacitor SOC, respectively. $\alpha(k)$ is the distribution coefficient.

4.3. Model Predictive Control-Based Energy Management Strategy

The principle of the fixed-horizon MPC algorithm is illustrated in Figure 12. The blue dashed line denotes zero power demand, and the green area represents the energy absorbed by the supercapacitor under braking conditions. The traction phase of the train is discretized, and at each discrete time step, the total energy loss of the traction system is obtained by evaluating the optimization objective. Meanwhile, the SOC of the supercapacitor at the next time step is predicted and incorporated into the constraint

equations, where $\alpha(k)$ denotes the power allocation ratio applied to the battery/supercapacitor model at the current time step, $U_m(k)$ represents the output of the controlled system model, $U_m(k+1)$ is the predicted output at the next time step, and $U_r(k+1)$ denotes the reference trajectory at the next time step. For the single-step predictive control, the predicted output $U_m(k+1)$ of the controlled system at the next time step is the result of multiple inputs, including both past inputs and the current input. The past inputs are known, while the current input needs to be determined.

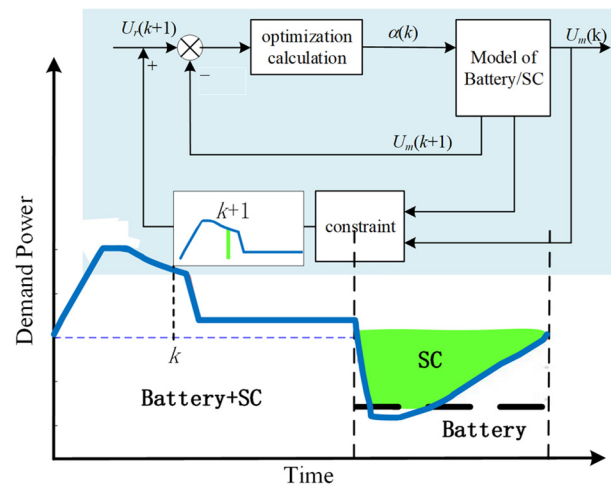


Figure 12. Schematic diagram of single-step predictive control.

The variable-horizon MPC is a control approach that aims to achieve global optimization over the entire traction phase. Its control principle is illustrated in Figure 13. In this study, the prediction horizon is defined in terms of the number of discrete sampling steps. If the control sampling period is T_s , a prediction horizon of N_p steps corresponds to a look-ahead time of $N_p T_s$. The maximum prediction horizon is set to $N_{p,\max} = 100$, and therefore, the maximum prediction time is $100T_s$. Similar to the fixed-horizon case, the traction phase of the train is discretized. At each discrete time step, the total energy loss of the traction system from the current time step to the end of the traction phase is obtained by evaluating the optimization objective, and the allocation coefficient is determined to realize energy management control of the traction system. Meanwhile, the SOC of the supercapacitor is predicted up to the end of the traction phase and incorporated into the constraints. The traction phase is discretized according to the control sampling period. At time step k , the prediction horizon is defined as $n = 1$ to $n = s$. At the subsequent time step $k+1$, the prediction horizon becomes $n = 1$ to $n = s-1$. Therefore, as the traction phase progresses, the prediction horizon decreases gradually at each control step, resulting in reduced computational burden.

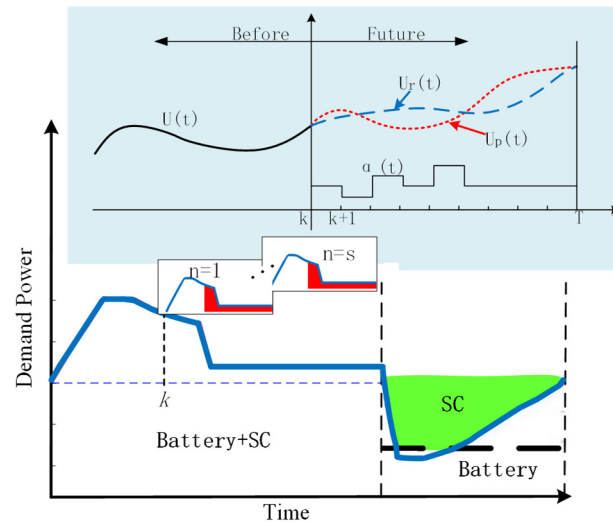


Figure 13. Schematic diagram of variable-horizon predictive control.

Since the supercapacitor operates at its maximum discharge capability during the traction phase, a rule-based constrained control strategy is adopted during the braking phase to maximize the absorption of regenerative braking energy and maintain the energy level of the supercapacitor. The energy management strategy of the supercapacitor during the braking phase is as follows:

$$P_{SC}(k) = \begin{cases} |P_d(k)| & |P_d(k)| \leq P_{dc}^{max}(k) \text{ and } SOC_{SC}(k) \leq SOC_{SC,max} \\ P_{dc}^{max}(k) & |P_d(k)| > P_{dc}^{max}(k) \text{ and } SOC_{SC}(k) \leq SOC_{SC,max} \\ 0 & SOC_{SC}(k) > SOC_{SC,max} \end{cases} \quad (26)$$

The energy management strategy of the battery is as follows:

$$P_{batt}(k) = \begin{cases} 0 & |P_d(k)| \leq P_{dc}^{max}(k) \\ |P_d(k)| - P_{dc}^{max}(k) & |P_d(k)| > P_{dc}^{max}(k) \end{cases} \quad (27)$$

where P_{SC} denotes the absorbed power of the supercapacitor. P_{batt} denotes the absorbed power of the lithium-ion battery. P_d represents the regenerative braking power of the train. P_{dc}^{max} denotes the maximum allowable power of the bidirectional DC/DC converter.

To make the MPC implementation explicit, the decision variable in the traction phase is defined as the power allocation coefficient $\alpha(k)$, which determines the power sharing between the battery and the supercapacitor. According to Equations (10) and (11), the supercapacitor power and battery power at the predicted step can be expressed as αP_{dmd} and $(1-\alpha)P_{dmd}$, respectively. At each sampling instant k , the prediction horizon is calculated as $N_p(k) = \min[N_{rem}(k), N_{p,max}]$, where $N_{p,max} = 100$. The optimization problem is then formulated as:

$$\alpha^*(k) = \arg \min_{\alpha \in A} \sum_{j=0}^{N_p(k)-1} \left[\left(\frac{(1-\alpha)P_{dmd}(k+j)}{V_{batt}(k+j)} \right)^2 R_{batt} T_s + \left(\frac{\alpha P_{dmd}(k+j)}{V_{SC}(k+j)} \right)^2 R_{SC} T_s \right] \quad (28)$$

subject to the voltage, current, and SOC constraints of the battery and supercapacitor. Here, A denotes the finite candidate set of the allocation coefficient within $[0, 1]$. In the simulation implementation, each candidate α is evaluated over the prediction horizon, and candidates that violate the constraints are discarded. The feasible candidate with the minimum accumulated loss is selected as $\alpha^*(k)$. Only the current power command corresponding to $\alpha^*(k)$ is applied, and the same procedure is repeated at the next sampling instant according to the receding-horizon principle.

Based on the energy management control strategy developed in the previous sections, the control flow of the traction system energy management is illustrated in Figure 14. The proposed strategy formulates an optimization problem over the prediction horizon to meet the power demand, with the objective of minimizing the energy losses of the battery and the supercapacitor. A receding-horizon scheme is adopted to iteratively compute the current and power allocation at each time step. By solving a nonlinear programming problem in real time, the optimal power allocation coefficient satisfying the constraints is obtained, and the reference currents for the battery and the supercapacitor are generated, thereby achieving coordinated energy management that balances efficiency, lifetime, and safety. As a model-based control strategy, the variable-horizon MPC updates the power allocation according to the real-time operating states of the battery and supercapacitor and the measured power demand, thereby providing good adaptability to operating condition variations within the specified operating range.

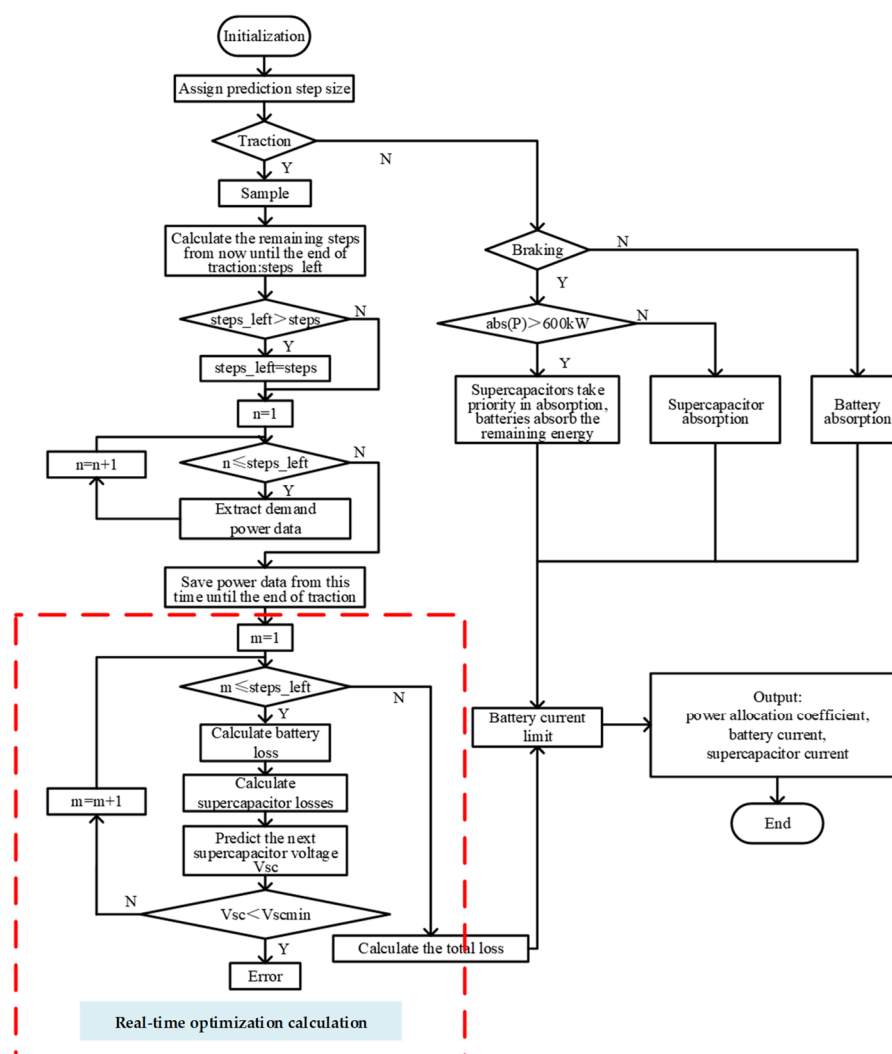


Figure 14. Flow chart of energy management control strategy improvement.

5. Verification

The purpose of the validation is to evaluate the influence of the prediction-horizon design on the energy management performance of the battery–supercapacitor hybrid power supply system. Therefore, the proposed variable-horizon MPC is compared with a single-step MPC baseline under the same operating profile, system parameters, optimization objective, and constraints. This controlled comparison is used to isolate the

effect of extending the prediction horizon from one sampling step to the remaining traction phase.

In this study, the conventional single-step MPC is selected as the baseline strategy. This baseline is chosen because it uses the same battery–supercapacitor model, optimization objective, constraints, and decision variable as the proposed variable-horizon MPC. Therefore, the performance difference between the two methods mainly results from the prediction-horizon design. Specifically, the single-step MPC optimizes the power allocation only at the next sampling step, whereas the proposed method optimizes the accumulated energy loss over the remaining traction phase with a dynamically changing prediction horizon. This controlled comparison enables the effect of the proposed horizon-adaptation mechanism to be directly evaluated under the same operating profile and system parameters.

To verify the real-time optimization performance of the proposed variable-horizon MPC-based energy management strategy for the hybrid power supply system of the contactless tram, a simulation model of the train hybrid power system is established in MATLAB/Simulink 2020, and corresponding validation analyses are carried out. In the simulation implementation, the control sampling period is $T_s = 0.1$ s. The power allocation coefficient is discretized from 0 to 1 with an interval of 0.01. The parameters of the studied hybrid power train system are listed in Table 4.

Table 4. System parameters.

Supercapacitor	Value	Battery	Value
Series number	10	Series number	200
Parallel number	18	Parallel number	20
Nominal capacity	83 F	Nominal capacity	8.5 Ah
Maximum voltage	48 V	Rated voltage	2.3 V
Minimum voltage	24 V	Maximum magnification	12 C
Internal resistance	0.01 Ω	Maximum Voltage	2.8 V
Maximum current	1900 A	Minimum voltage	1.5 V
Maximum SOC	100%	Maximum SOC	80%
Minimum SOC	25%	Minimum SOC	20%

The basic parameters of the vehicles studied in this article are shown in Table 5.

Table 5. Basic vehicle data.

Variable	Parameter
Vehicle weight	70 t (AW3)
Maximum speed	70 km/h
Wheel diameter	600 mm
Number of groups	5
Axle count	6
Efficiency of motor side inverter η_i	0.98
DC/DC converter efficiency η_d	0.95
Motor efficiency η_m	0.94

The line parameters are shown in Table 6.

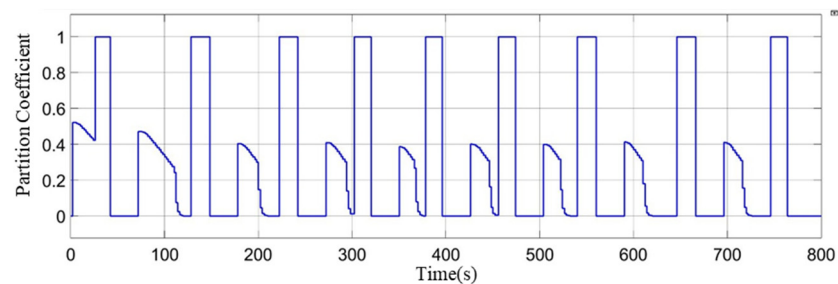
Table 6. Rail line parameters.

Station	Distance (m)	Runtime (s)	Arrival Time (s)
1–2	470	42	42
2–3	1147	75	147

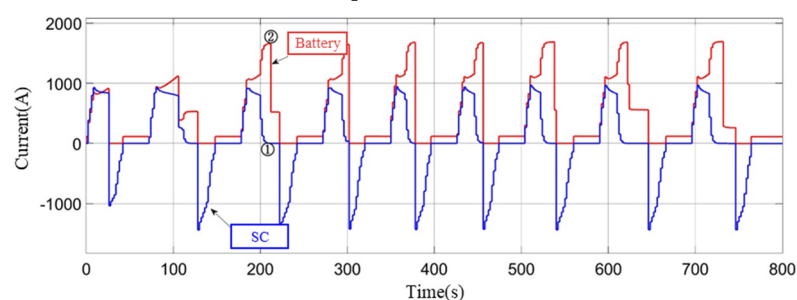
3–4	926	64	241
4–5	567	47	318
5–6	575	47	396
6–7	573	47	473
7–8	736	56	559
8–9	1158	75	665
9–10	1031	69	764

(1) The simulation results based on the fixed-horizon MPC energy management strategy (with the initial prediction horizon set to 1) are as follows.

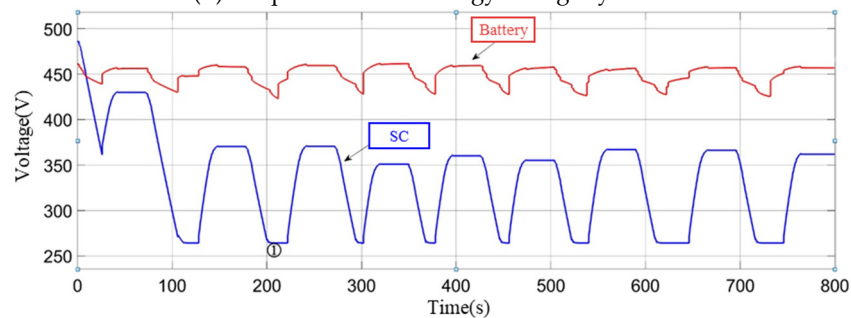
From the simulation results shown in Figure 15, it can be observed that under the energy management control strategy, the battery and the supercapacitor achieve effective coordination, maintaining their respective performance while satisfying the train power demand. The SOC of the battery varies within 80–20%, and the SOC of the supercapacitor remains above 25%, both operating within the prescribed constraint limits. During the traction phase, the supercapacitor discharges at its maximum capability, while the remaining energy demand is supplied by the battery. During the braking phase, the supercapacitor effectively absorbs regenerative braking energy, maintaining its energy level within a reasonable range. This not only reduces the system energy consumption but also ensures that the supercapacitor does not require additional charging at stations. Over the entire simulation, the total energy loss of the system is 1.872 kWh.



(a) Power partition coefficient



(b) Output current of energy storage system



(c) Output voltage of energy storage system

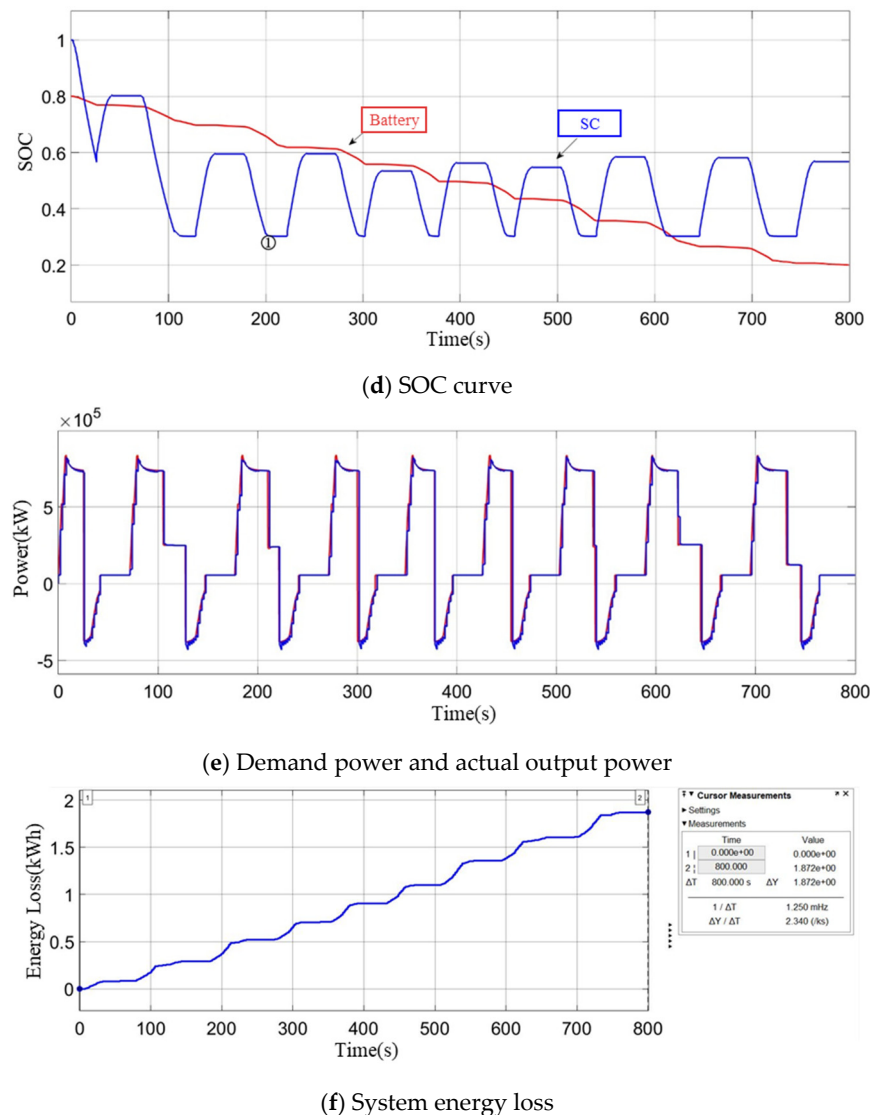


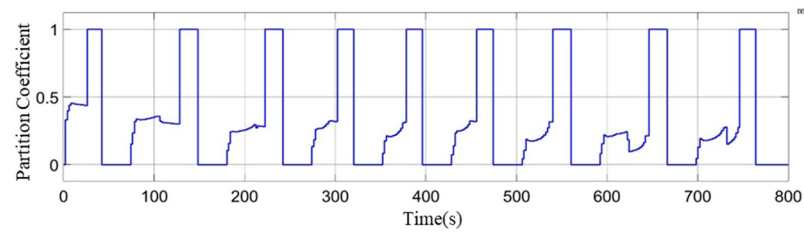
Figure 15. Simulation results of single-step model predictive energy control.

However, since the prediction horizon is limited to one step, the optimization objective and constraints do not account for global optimality. As can be observed in region ① of Figure 15, the supercapacitor tends to discharge to its lower limit in advance, which results in the battery supplying a larger current in region ② to meet the remaining energy demand of the train, thereby accelerating battery degradation.

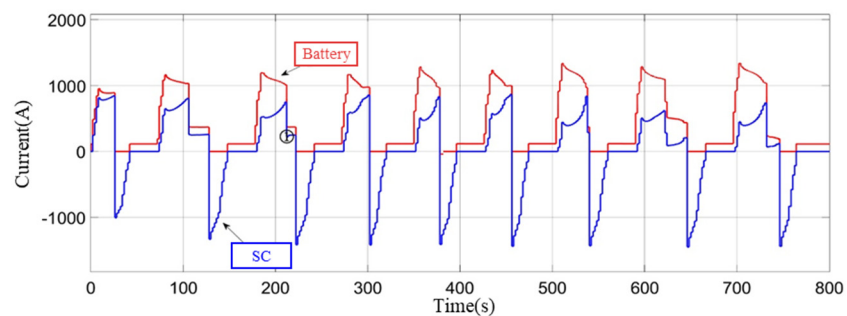
(2) The simulation results of the variable-horizon predictive control (with the initial prediction horizon set to 100) are as follows.

From the simulation results shown in Figure 16, it can be observed that, compared with the fixed-horizon predictive control, the variable-horizon predictive control considers the global optimization objective over the entire traction phase. As a result, the SOC of the supercapacitor decreases gradually and only reaches its lower limit of 25% at the end of the traction phase, as indicated in region ①, thereby avoiding the large current discharge of the battery observed in the previous control strategy. By formulating the optimization objective from a global perspective, this strategy not only fully exploits the characteristics of the supercapacitor but also enables a more balanced power contribution between the two energy sources. Meanwhile, the overall energy consumption of the hybrid energy storage system is minimized. Under this energy management strategy, the total system energy loss is 1.687 kWh, which is reduced by 0.185 kWh compared with the fixed-horizon case. The comparison of energy consumption between the two strategies is

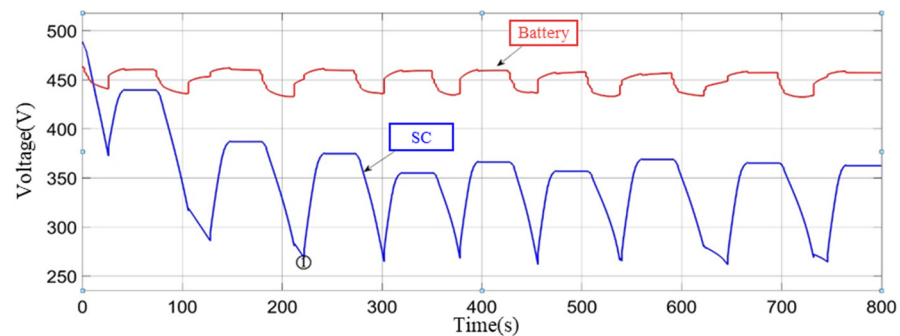
presented in Table 7. It can be seen that the proposed variable-horizon MPC-based energy management strategy achieves a 9.88% reduction in energy loss of the hybrid power system under the tested conditions compared with the conventional fixed-horizon MPC approach.



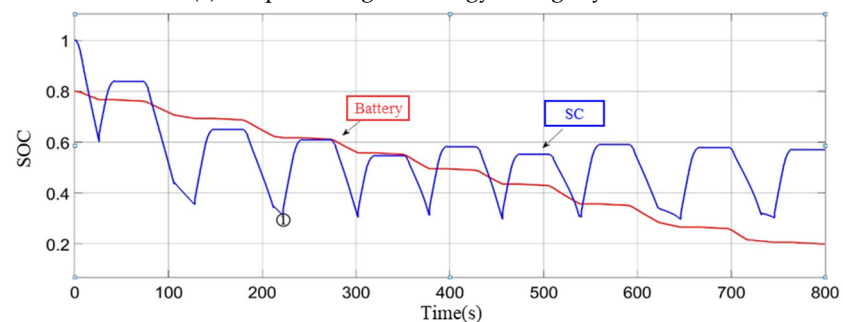
(a) Power partition coefficient



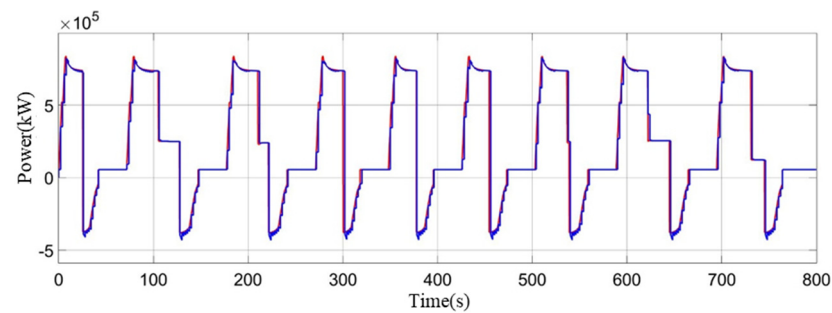
(b) Output current of energy storage system



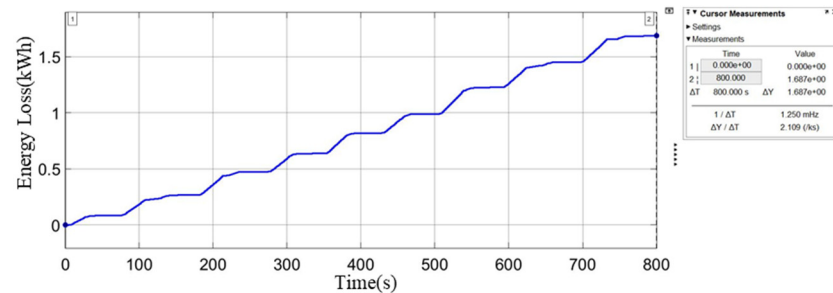
(c) Output voltage of energy storage system



(d) SOC curve



(e) Demand power and actual output power



(f) System energy loss

Figure 16. Simulation results of variable-horizon model predictive energy control.**Table 7.** Comparison of simulation results.

Control Method	Single-Step MPC	Variable-Horizon MPC
Energy loss (kWh)	1.872	1.687
Energy loss reduction (%)	—	9.88
Computational cost (average step time, ms)	0.117	0.785

6. Conclusions

The energy management performance of contactless tram traction systems is strongly affected by the coordinated utilization of hybrid energy storage units, especially under frequent traction–braking transitions and large power fluctuations. This paper investigated a battery–supercapacitor hybrid power supply system for contactless trams and proposed an energy management strategy based on variable-horizon model predictive control (MPC), aiming to improve overall energy efficiency while respecting operational constraints and extending battery lifetime. The main conclusions are summarized as follows:

- (1) A comprehensive modeling framework for the contactless tram hybrid power supply system was established, including equivalent circuit models for lithium titanate batteries and supercapacitors. By analyzing the energy demand characteristics under traction, braking, and stop conditions, a power-level-matching-based capacity configuration method was proposed, enabling rational sizing of both energy storage units while fully exploiting their complementary energy and power density characteristics.
- (2) To address the limitations of conventional single-step MPC, which tends to induce premature supercapacitor depletion and large battery current stress, a variable-horizon model predictive energy management strategy was developed. By dynamically adjusting the prediction horizon to cover the entire traction phase between stations, the proposed method enables global optimization of energy loss

while ensuring voltage, current, and SOC constraints for both the battery and supercapacitor.

- (3) Simulation results based on a detailed MATLAB/Simulink model demonstrate that the proposed variable-horizon MPC strategy achieves smoother power sharing and more balanced energy utilization compared with the traditional single-step MPC. In particular, the supercapacitor SOC is regulated to reach its lower limit only at the end of the traction phase, effectively suppressing excessive battery current peaks. Quantitative analysis shows that the total energy loss of the hybrid power system is reduced by 9.88% under the tested operating conditions.

Overall, the proposed modeling and energy management framework provides a simulation-based approach for improving the energy allocation of battery–supercapacitor hybrid power supply systems in contactless trams. The results under the studied representative operating condition indicate that the proposed variable-horizon MPC strategy can improve power sharing, reduce premature supercapacitor depletion, and decrease system energy loss compared with single-step MPC. However, the present study is limited to MATLAB/Simulink simulation. Hardware experiments, HIL validation, embedded real-time implementation, battery aging analysis, thermal constraints, and robustness verification under broader operating conditions have not yet been included. Therefore, practical deployment feasibility still requires further investigation. Although the proposed strategy improves the energy efficiency of the hybrid power supply system, the current optimization objective is still simplified. Converter loss characteristics, battery degradation cost, thermal behavior, and DC-bus voltage quality are not explicitly modeled in the cost function. Future work will focus on extending the proposed framework to a multi-objective MPC strategy incorporating converter efficiency maps, battery aging and thermal models, and DC-bus voltage regulation requirements. And the reported 9.88% reduction in energy losses was obtained under the representative operating cycle considered in this study. Therefore, this value should be understood as a case-specific improvement rather than a universal performance gain. Since the proposed strategy updates the power allocation according to the real-time power demand and the operating states of the battery and supercapacitor, it has adaptability to operating-condition variations within the modeled range. Nevertheless, the quantitative benefit may change under different station intervals, passenger loads, line gradients, initial SOC values, converter efficiencies, and parameter uncertainties. Future work will further investigate the robustness of the proposed strategy through multi-scenario simulations and sensitivity analysis.

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