

Article

Optimization of Active Power Losses for High-Voltage Distribution Grids Using Mixed-Integer Quadratic Programming

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Abstract

The energy transition is leading to increased loading of high-voltage distribution grids (HVDGs), which in turn results in higher active power losses. In addition, the meshed network structure typical for German 110 kV grids enables transit power flows originating from the superimposed transmission grid (TG). These transit power flows may further increase grid loading and active power losses. This paper presents a sensitivity-based mixed-integer quadratic programming (MIQP) approach for active power loss minimization in meshed HVDGs. The proposed methodology achieves near-real-time computation times while maintaining high accuracy by means of a piecewise linearized alternating current (AC) formulation. In addition, losses of static synchronous compensators (STATCOMs) and transformer-reactance variations are explicitly modeled. Reactive power provision by distributed energy resources (DERs) and STATCOMs, as well as discrete tap positions of power transformers (PTs) and phase-shifting transformers (PSTs), are considered as control variables. The proposed approach is validated using hourly snapshots of a real German HVDG provided by a distribution system operator (DSO). The results demonstrate a significant reduction in active power losses together with computation times suitable for near-real-time applications.

Keywords: active power losses; distribution grids; optimization; grid operation

1. Introduction

Due to the energy transition, conventional power plants located near industrial centers in Germany are increasingly being decommissioned, while the expansion of generation capacity of DERs is primarily taking place in rural areas. Offshore and onshore wind turbines are predominantly installed in northern Germany, whereas photovoltaic capacity is more prevalent in the southern federal states [1].

As a consequence of this changing generation structure, distance-weighted electricity transmission is expected to increase substantially in Germany as well as across Europe [2]. Already today, largely driven by the geographical imbalance in wind power generation, significant north-to-south power flows occur within the German TG. This demand for long-distance power transmission is expected to continue to grow in the future [3].

Due to the meshed coupling of the TG and the HVDGs, transit power flows occur within these grids (see Figure 1). As the transport demand in the TG increases, the transit power flows in the HVDGs increase accordingly. Under normal operating conditions, transit power flows can cause loading levels of up to 30% [4] of the rated capacity of individual lines and transformers within HVDGs. In contingency situations in the upstream TG, even higher transit-induced loadings may occur. Depending on their direction relative



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to demand- and generation-driven power flows, these transit flows may in some cases reduce the loading of HVDG assets [5]. Consequently, transit power flows can significantly influence asset loading in HVDGs and may increase both the need for grid reinforcement and active power losses.

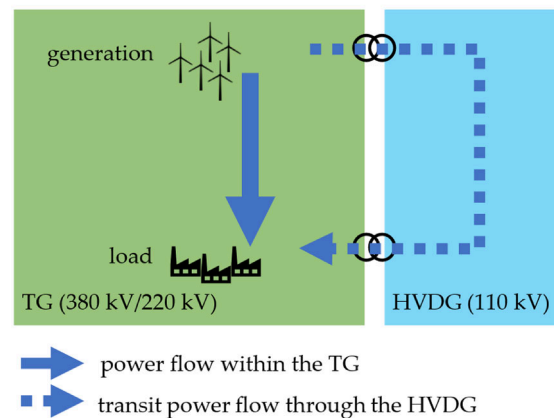


Figure 1. Illustration of transit power flows through an HVDG resulting from the meshed coupling of the TG and the HVDG as well as a geographical mismatch between generation and demand.

In Germany, energy transmission and distribution losses account for approximately 4% of total electricity demand, with about one fifth attributable to the HV level [6]. Moreover, reducing grid losses contributes to lower greenhouse gas emissions, as less electrical energy needs to be supplied. Therefore, reducing active power losses represents a suitable objective for the economic optimization of HVDG operation.

Against this background, controllable assets capable of influencing active power flows become increasingly important. PSTs are particularly suitable for controlling transit power flows and reducing active power losses in HVDGs, as they enable active power flow control at the interconnections between HVDGs and the TG [7]. Alternatively, unified power flow controllers (UPFCs) can be employed. In Germany, PSTs are predominantly used in the TG, e.g., for loop flow reduction and control of cross-border flows [8,9], whereas their use in HVDGs is currently restricted to individual cases.

To further reduce losses and branch loading, a global optimization of all assets available in HVDGs represents an option. These assets include the on-load tap changers of PTs for voltage control, the setpoints of reactive power compensation systems, and the reactive power setpoints of generation units, particularly DERs [10]. To obtain an appropriate solution for operating HVDGs with a large number of nodes and branches, the optimization algorithm must be computationally efficient and capable of near-real-time computation.

A large number of algorithms already exist for the technical optimization of distribution grids, e.g., to avoid congestion by using flexibilities [11–14]. There are also several approaches for loss-optimized grid operation, including grid reconfiguration, flexibility provided by DERs, flexible AC transmission systems (FACTS) and transformer tap changers as control variables [15–26]. The aforementioned algorithms can be broadly classified into linear programming (LP) [15,16], mixed-integer quadratic programming (MIQP) and related mixed-integer formulations [23–26], nonlinear programming (NLP) [21], and heuristic approaches [17–20,22]. Depending on the specific optimization problem, LP and quadratic programming can achieve significantly shorter computation times than NLP and heuristic methods, while yielding comparable optimization results [27,28]. For optimization problems with convex objective functions, QP can achieve computation times comparable to those of LP [29]. However, the existing approaches differ significantly in terms of the considered control variables, modeling accuracy and computational effort. Furthermore, only

a limited number of studies focus on meshed 110 kV distribution grids with characteristics typical of German HVDGs, such as meshed coupling with the TG and the occurrence of transit power flows.

In [15], an AC-based LP formulation is proposed, but the primary focus is generation cost minimization rather than loss minimization, while comparatively long computation times are reported. The sensitivity-based LP approach presented in [16] considers PT tap changers, voltage-controlled generators, and switchable reactive power devices. However, PSTs are not considered, and the dependence of transformer reactance on the tap position is not modeled. Furthermore, the formulation approximates the impact of the control variables on active power losses by superimposing their individual loss contributions, which may reduce accuracy.

The NLP-based approach presented in [21] achieves comparatively short computation times and additionally penalizes control variable changes to avoid unnecessary adjustments. However, only reactive power is considered as a control variable. The heuristic approaches in [17–20,22] also mainly focus on subsets of controllable assets, such as shunt devices, PT tap changers, or reactive power control.

Several MIQP-based approaches previously mentioned address grid reconfiguration problems. For example, ref. [23] uses a linear power-flow model to optimize switchable capacitors, tap changers of PTs, and grid reconfiguration decisions. Active power loss minimization through grid reconfiguration is investigated in [25], while [26] combines reactive power optimization and grid reconfiguration in radial distribution grids using a linear branch flow model. Accordingly, the control scope of these studies is largely characterized by topology-related decisions, partly combined with reactive power control or PT tap changers. A different focus is taken in [24], where the optimal dispatch of PV units is considered while, among other aspects, accounting for active power loss costs. Overall, these works demonstrate the applicability of MIQP-based formulations to distribution system optimization, predominantly in radial grid structures, but address different operational problems or control scopes than those considered in this paper.

Accordingly, the contribution of this work is not the use of an MIQP formulation alone, but the development of a sensitivity-based MIQP framework tailored to active power loss minimization in meshed 110 kV HVDGs with significant transit power flows. Compared with the discussed MIQP-based approaches, the proposed formulation jointly coordinates DER and STATCOM reactive power setpoints with the discrete tap positions of PTs and PSTs within a piecewise-linearized AC-based OPF framework suitable for near-real-time operation. In addition, STATCOM losses and tap-position-dependent transformer reactance are explicitly modeled, and setpoint changes are penalized to avoid unnecessary control actions and tap operations.

The proposed approach is intended for loss-minimizing operation under normal, feasible grid operating conditions. In abnormal or emergency operating states, operational security and the restoration of secure operation have priority over loss minimization and may require further actions such as topology changes, redispatch, and load shedding, which are outside the scope of the present loss-minimization framework.

2. Proposed Algorithm

In order to obtain an MIQP formulation with convex constraints and computationally efficient solution times, the nonlinear AC power flow equations must be linearized. Therefore, this paper proposes a sensitivity-based piecewise linearization approach for the AC power flow equations. Section 2.1 introduces the proposed sensitivity-based linearization approach. Section 2.1.1 first considers variations in nodal reactive power, followed by Section 2.1.2, which addresses changes in the tap positions of PSTs and PTs. Section 2.2

then formulates the developed OPF problem, which aims at reducing active power losses and branch loadings. The formulation includes STATCOM losses, tap-position-dependent transformer reactance variations, and a penalty term for setpoint changes. Since all calculations are performed in the per-unit system, the corresponding symbols used in the equations are denoted by lower-case letters.

2.1. Linearization of Power Flow Equations

The linearization presented below extends the distribution factor-based methodology introduced in [30]. In addition to capturing the effects of nodal power variations on branch power flows, the proposed approach also accounts for their impact on nodal voltages. Furthermore, the methodology is extended to incorporate transformer tap changers. In contrast to [30], the linearization is performed in a piecewise manner. Hence, the following linear models are derived separately for positive and negative deviations Δx_{lin} of the control variables from the linearization point.

2.1.1. Sensitivities for Changes in Nodal Reactive Power

A change in nodal reactive power affects both voltage magnitudes and voltage angles. In HVDCs, variations in reactive power predominantly influence voltage magnitudes. Therefore, the impact of reactive power on voltage angles is neglected in some studies [30]. In this work, however, the coupling of both state variables—voltage magnitude and voltage angle—is considered to improve accuracy.

To quantify the effect of changes in nodal reactive power Δq_i on nodal voltage angles $\Delta \vartheta_{q,i}$ and voltage magnitudes $\Delta u_{q,i}$, (1) is applied. The equation contains the partial derivatives of the nodal voltage magnitude ∂u_i and voltage angle $\partial \vartheta_i$ with respect to the nodal reactive power ∂q_i [10].

$$\begin{bmatrix} \Delta \vartheta_{q,i} \\ \Delta u_{q,i} \end{bmatrix} = \begin{bmatrix} \vartheta_i \\ u_i \end{bmatrix} - \begin{bmatrix} \vartheta_i \\ u_i \end{bmatrix}_{x_0} = \begin{bmatrix} \frac{\partial \vartheta_i}{\partial q_i} \\ \frac{\partial u_i}{\partial q_i} \end{bmatrix} \cdot [\Delta q_i] = [S_{bus,q}] [\Delta q_i] \quad (1)$$

The sensitivities contained in $S_{bus,q}$ are approximated for each node at which a flexible reactive power injection is connected by performing two Newton–Raphson load flow (NR-LF) calculations. This approach is based on the method proposed in [30]. The sensitivity of the voltage magnitude at node k with respect to a change in reactive power at node i is given in (2). Here, u_{k,x_0} denotes the voltage magnitude at node k at the linearization point x_0 and u_k represents the corresponding voltage magnitude resulting from a reactive power change Δq_i at node i .

$$\frac{\partial u_k}{\partial q_i} \approx \frac{u_k - u_{k,x_0}}{\Delta q_i} \quad (2)$$

To reduce the maximum linearization error, the linearization point for the reactive power injection at node i is selected at 0 p.u., and the sensitivities are evaluated for a reactive power variation of $\Delta q_i = 84\%$ of the respective maximum capacitive and inductive capability. In contrast to [30], this value was determined empirically based on a parameter study using the test grid described in Section 3. Choosing too small step sizes leads to poor accuracy for large setpoint changes, while too large linearization step sizes reduce the accuracy for typical small- to medium-range adjustments. The selected value of 84% minimized the maximum linearization error over the operating range from 0% to 100% and therefore represents a compromise between local accuracy and the accurate representation of larger setpoint changes. An illustrative example of the underlying parameter study is provided in Appendix A.2.

In order to incorporate the maximum branch loading as a constraint in the OPF, the branch active power flow p_{ij} and the branch reactive power flow q_{ij} must be linearized.

To determine the impact of a nodal reactive power adjustment on the branch power flows, (3) is applied. For the derivation of the sensitivity matrix $S_{br,q}$, the approximate calculation method described above is used. The two required NR-LF calculations are already performed for the determination of $S_{bus,q}$ in (1).

$$\begin{bmatrix} \Delta p_{q,ij} \\ \Delta q_{q,ij} \end{bmatrix} = \begin{bmatrix} p_{ij} \\ q_{ij} \end{bmatrix} - \begin{bmatrix} p_{ij} \\ q_{ij} \end{bmatrix}_{x_0} = \begin{bmatrix} \frac{\partial p_{ij}}{\partial q_i} \\ \frac{\partial q_{ij}}{\partial q_i} \end{bmatrix} [\Delta q_i] = [S_{br,q}] [\Delta q_i] \quad (3)$$

Equation (4) illustrates an example of the approximate calculation of the sensitivity of a nodal reactive power change at node i on the active power flow from node i to node j .

$$\frac{\partial p_{ij}}{\partial q_i} \approx \frac{p_{ij} - p_{ij,x_0}}{\Delta q_i} \quad (4)$$

2.1.2. Sensitivities for Changes in Transformer Tap Position

After presenting the linearization for variations in reactive power injections, the same procedure is applied in the following to changes in the tap positions of PSTs and PTs. By changing the complex transformer ratio, the nodal voltages and the active and reactive branch power flows can be controlled. The transformer ratio in (5) is characterized by its magnitude t and the phase-shift angle θ . In this paper, PSTs are modeled as ideal PSTs with respect to the voltage magnitude, i.e., only the phase-shift angle is affected by the tap position, while the magnitude of the transformer ratio remains constant. Accordingly, for each tap step $tap_{PST,ij}$, the phase-shift angle changes by a constant value $\Delta\theta_{PST,ij}$.

For PTs, the magnitude of the transformer ratio changes by $\Delta t_{PT,ij}$ for each tap step $tap_{PT,ij}$. Both variables $\Delta\theta_{PST,ij}$ and $\Delta t_{PT,ij}$ are multiplied by the sensitivity matrix $S_{bus,tap}$ to obtain the resulting changes in voltage magnitudes and angles relative to the linearization point in (6). The sensitivity matrix contains the partial derivatives of the nodal voltage magnitudes and voltage angles with respect to changes in the transformer ratio $\Delta t_{PT,ij}$ and the phase-shift angle $\Delta\theta_{PST,ij}$ of PTs and PSTs, respectively. As shown in Section 2.1.1, these partial derivatives are approximated using two NR-LF calculations.

$$t = te^{j\theta} \quad (5)$$

$$\begin{bmatrix} \Delta\theta_{tap,i} \\ \Delta u_{tap,i} \end{bmatrix} = \begin{bmatrix} \frac{\partial\theta_i}{\partial\theta_{PST,ij}} & \frac{\partial\theta_i}{\partial t_{PT,ij}} \\ \frac{\partial u_i}{\partial\theta_{PST,ij}} & \frac{\partial u_i}{\partial t_{PT,ij}} \end{bmatrix} \begin{bmatrix} \Delta\theta_{PST,ij} \\ \Delta t_{PT,ij} \end{bmatrix} = [S_{bus,tap}] \begin{bmatrix} \Delta\theta_{PST,ij} \\ \Delta t_{PT,ij} \end{bmatrix} \quad (6)$$

An analogous procedure is applied to the branch power flows using the sensitivity matrix $S_{br,tap}$ (7). The partial derivatives contained therein are also approximated. The linearization range used for the sensitivity calculation corresponds to ± 4 tap positions around the current operating point for PTs and ± 6 tap positions for PSTs. For PTs, larger ranges reduce the approximation accuracy for the comparatively small tap position changes that typically occur between two optimization intervals, whereas smaller ranges increase the error for larger tap changes. Since tap position changes exceeding ± 4 steps within a single optimization interval are uncommon for PTs and can already correspond to voltage variations of approximately $\pm 4\%$ of the nominal voltage, depending on the transformer design, a range of ± 4 tap positions represents a suitable compromise between local accuracy and the representation of larger tap changes. For PSTs, a larger range of ± 6 tap positions is selected to accurately capture situations with abrupt changes in transit power flows, e.g., caused by switching actions or contingencies in the upstream TG, which may require comparatively large PST tap adjustments. The impact of the selected ranges on the

approximation accuracy was evaluated by means of a parameter study using both the test grid and the model of the real HVDG described in Section 3. The results confirmed that the selected ranges provide a suitable compromise between approximation accuracy and the representation of larger tap position changes.

$$\begin{bmatrix} \Delta p_{\text{tap},ij} \\ \Delta q_{\text{tap},ij} \end{bmatrix} = \begin{bmatrix} \frac{\partial p_{ij}}{\partial \theta_{\text{PST},ij}} & \frac{\partial p_{ij}}{\partial t_{\text{PT},ij}} \\ \frac{\partial q_{ij}}{\partial \theta_{\text{PST},ij}} & \frac{\partial q_{ij}}{\partial t_{\text{PT},ij}} \end{bmatrix} \begin{bmatrix} \Delta \theta_{\text{PST},ij} \\ \Delta t_{\text{PT},ij} \end{bmatrix} = [S_{\text{br,tap}}] \begin{bmatrix} \Delta \theta_{\text{PST},ij} \\ \Delta t_{\text{PT},ij} \end{bmatrix} \quad (7)$$

Figure 2 illustrates the modeling of the impedance as well as the variable components of the transformer ratio of PSTs and PTs. In addition to the transformer ratio, the reactance of the transformers is assumed to depend on the tap position, whereas the resistance remains constant, as indicated in subfigure (c).

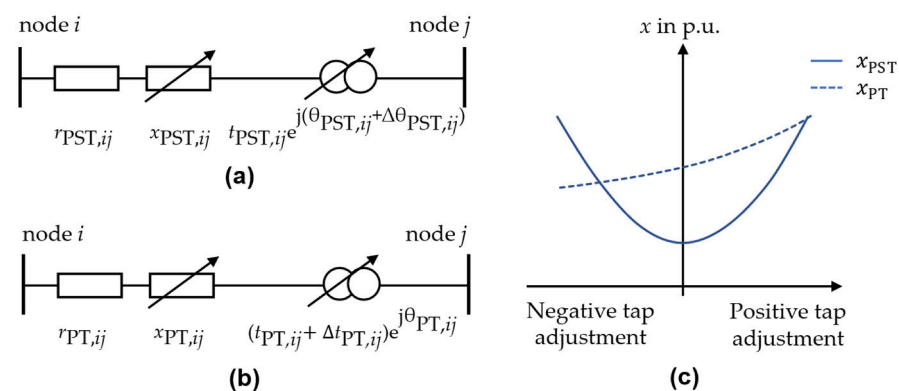


Figure 2. (a) Modeling of a PST with variable reactance and phase-shift angle. (b) Modeling of a PT with variable reactance and transformer ratio. (c) Qualitative representation of the dependence of the reactance of PSTs and PTs on the tap position [31].

For PSTs, the reactance is increased by 1.9% per tap step relative to the neutral position, following [31]. For PTs, the reactance changes by $\pm 0.3\%$ per tap step, based on characteristics of real transformers. Accordingly, a linear tap-dependent reactance model is adopted.

2.1.3. Superposition of Sensitivities

The changes in the nodal voltages caused by variations in nodal reactive power are superimposed on the changes resulting from the transformer tap steps, as given in (8). This yields the total voltage angle and magnitude variations $\Delta \theta_{\text{tot},i}$ and $\Delta u_{\text{tot},i}$, respectively. To obtain the linearized nodal voltages, the calculated variations are added to the values at the linearization point.

$$\begin{bmatrix} \Delta \theta_{\text{tot},i} \\ \Delta u_{\text{tot},i} \end{bmatrix} = \begin{bmatrix} \Delta \theta_{q,i} \\ \Delta u_{q,i} \end{bmatrix} + \begin{bmatrix} \Delta \theta_{\text{tap},i} \\ \Delta u_{\text{tap},i} \end{bmatrix} \quad (8)$$

The same applies to the variations in active and reactive branch power flows in (9). The linearized values are obtained by superimposing the values at the linearization point with the total variation $\Delta p_{\text{tot},ij}$ and $\Delta q_{\text{tot},ij}$ resulting from the adjusted tap positions and the updated reactive power setpoints.

$$\begin{bmatrix} \Delta p_{\text{tot},ij} \\ \Delta q_{\text{tot},ij} \end{bmatrix} = \begin{bmatrix} \Delta p_{q,ij} \\ \Delta q_{q,ij} \end{bmatrix} + \begin{bmatrix} \Delta p_{\text{tap},ij} \\ \Delta q_{\text{tap},ij} \end{bmatrix} \quad (9)$$

2.2. MIQP Formulation

In the following, the formulation of the MIQP is presented. First, the general definition of an MIQP problem is introduced, followed by the description of the employed objective function and the considered constraints.

The presented OPF is implemented as a linearly constrained MIQP in Python 3.10.13 using the Gurobi Optimizer 11.0 [32]. The objective function f is minimized subject to the constraints given in (10), with respect to the state vector x defined in (11). Both linear equality constraints (a) and inequality constraints (b) can be considered. The constraints are formulated using the matrices A_{eq} and A_{ineq} , and the vectors x , b_{eq} and b_{ineq} . The state vector x is bounded by its lower and upper limits, lb and ub , respectively, as given in (c).

$$\min f(x) \text{ subject to } \begin{cases} A_{\text{eq}} x = b_{\text{eq}} & (a) \\ A_{\text{ineq}} x \leq b_{\text{ineq}} & (b) \\ lb \leq x \leq ub & (c) \end{cases} \quad (10)$$

The state vector x includes the nodal voltage magnitudes and voltage angles, the nodal reactive power injections and the tap positions of PSTs and PTs. The reactive power injections are modeled as continuous variables, whereas the tap positions of PSTs and PTs are modeled as discrete variables. The proposed formulation targets steady-state operation. Contingency analysis could be incorporated by extending the model, for example, using the approach presented in [30], but this would increase computational complexity and solution times. The computational performance of the proposed MIQP formulation is discussed in the case study in Section 3.

$$x = \begin{pmatrix} \vartheta \\ u \\ q \\ tap_{\text{PST}} \\ tap_{\text{PT}} \end{pmatrix} \quad (11)$$

2.2.1. Objective Function

The objective function aims to minimize the active power losses $p_L(x)$ as a function of transformer tap positions and nodal reactive power injections, as defined in (12). To this end, the current-dependent losses $p_{L,I}(x)$ and the voltage-dependent losses $p_{L,V}(x)$ are considered. Losses associated with STATCOMs are accounted for as other losses $p_{L,\text{oth}}(x)$. In addition, the term $c(x)$ is introduced to penalize deviations of the control variables from their current operating point. This prevents unnecessary actuation and reduces stress on components such as transformer tap changers.

$$\min f(x) = \min p_L(x) = p_{L,I}(x) + p_{L,V}(x) + p_{L,\text{oth}}(x) + c(x) \quad (12)$$

Figure 3 illustrates the π -equivalent circuit of a line connecting nodes i and j , including a shunt element connected at node i , e.g., a shunt reactor. The current-dependent losses are caused by the real part of the series impedance $\underline{z}_{ij}$. The voltage-dependent losses result from the shunt conductance $g_{\text{sh},i}$. In this paper, voltage-dependent losses are considered for shunt capacitors, shunt reactors and transformers.

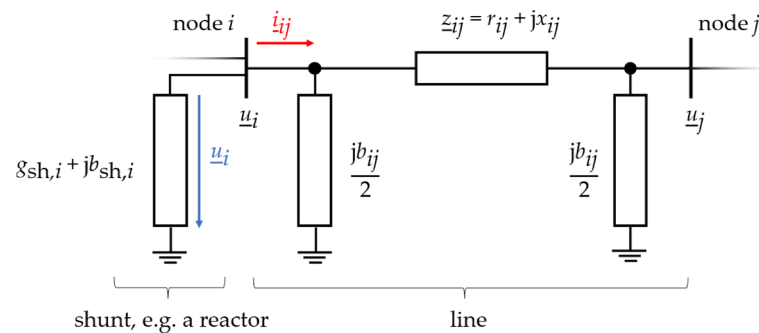


Figure 3. Line (modeled as π -equivalent) and shunt connected to node i .

The first term of the objective function consists of a quadratic approximation of the current-dependent grid losses [10]. Equation (13) includes the conductance g_{ij} between nodes i and j , as well as the voltage magnitudes u and voltage angles ϑ of the nodes connected to the branch. The cosine term in (13) is approximated by the second-order Taylor series expansion given in (14). As the angle differences between electrically adjacent nodes in HVDGs are typically small [33], the resulting approximation error is negligible. Even for an angle difference of 10° , which is already large in this context, the relative approximation error is only about 0.004%.

In addition, the product $u_i u_j$ is approximated by a constant value of 1.1, as proposed in [10]. This results in (15). This choice reflects typical operating conditions in HVDGs, where transformer tap changers are commonly set to voltage levels above the nominal value in order to reduce active power losses and branch loadings. Assuming a voltage setpoint of 1.05 p.u., which is close to the voltage setpoints in the considered case study (see Section 3), and similar voltage magnitudes at neighboring nodes, this yields $u_i u_j \approx 1.05^2$, which is close to the adopted approximation.

$$p_{L,I} = \sum_{(i,j)} g_{ij} \left[u_i^2 + u_j^2 - 2u_i u_j \cos(\vartheta_i - \vartheta_j) \right] \quad (13)$$

$$\cos(\vartheta_i - \vartheta_j) \approx 1 - 0.5(\vartheta_i - \vartheta_j)^2 \quad (14)$$

$$p_{L,I} \approx \sum_{(i,j)} g_{ij} \left[u_i^2 + u_j^2 + u_i u_j (\vartheta_i - \vartheta_j)^2 - 2u_i u_j \right] \approx \sum_{(i,j)} g_{ij} \left[(u_i - u_j)^2 + 1.1(\vartheta_i - \vartheta_j)^2 \right] \quad (15)$$

Depending on the operating strategy of a grid, other values may be used to approximate the voltage product $u_i u_j$. However, assuming nominal voltages ($u_i = u_j = 1.0$ p.u.), as done for instance in [34], systematically underestimates the voltage product for grids operated above nominal voltage and therefore leads to a larger approximation error in this case.

Equations (13) and (15) are valid for lines and for transformers operated with a constant per-unit transformer ratio $\underline{t} = 1$, where t is defined according to (5). It should be noted that if the transformer ratio differs from unity, the complex voltage difference between the HV and low-voltage transformer nodes does not directly determine the active power losses. In this case, the voltage change caused by the phase-shift angle and the transformer ratio must be considered in (15). Therefore, (16) is applied instead. In (16), i denotes the HV node of the transformer and j the low-voltage node. Furthermore, the tap changer is assumed to be located on the HV side. The resulting phase-shift angle $\theta_{PST,ij}$ is composed of the

shift angle at the linearization point $\theta_{\text{PST},x0,ij}$ and the change $\Delta\theta_{\text{PST},ij}$ caused by the setpoint adjustment, as given in (17).

$$p_{L,ij} \approx g_{ij} \left[\left(\frac{u_i}{t_{\text{PT},ij}} - u_j \right)^2 + 1.1(\vartheta_i - \vartheta_j - \theta_{\text{PST},ij})^2 \right] \quad (16)$$

$$\theta_{\text{PST},ij} = \theta_{\text{PST},x0,ij} + \Delta\theta_{\text{PST},ij} \quad (17)$$

The transformer ratio $t_{\text{PT},ij}$ in (16) consists of the ratio at the linearization point $t_{\text{PT},x0,ij}$ and the change $\Delta t_{\text{PT},ij}$ caused by the setpoint adjustment, as defined in (18). To avoid dividing by a control variable, the effect of the tap changer is incorporated by subtracting the resulting open-circuit voltage change $\Delta u_{\text{PT},ij}$. Substituting (17) and (18) into (16) yields (19).

$$\frac{u_i}{t_{\text{PT},ij}} = \frac{u_i}{t_{\text{PT},x0,ij} + \Delta t_{\text{PT},ij}} \approx \frac{u_i}{t_{\text{PT},x0,ij}} - \Delta u_{\text{PT},ij} \quad (18)$$

$$p_{L,ij} \approx g_{ij} \left[\left(\frac{u_i}{t_{\text{PT},x0,ij}} - \Delta u_{\text{PT},ij} - u_j \right)^2 + 1.1(\vartheta_i - \vartheta_j - \theta_{\text{PST},x0,ij} - \Delta\theta_{\text{PST},ij})^2 \right] \quad (19)$$

Furthermore, the voltage-dependent losses $p_{L,V}$ of all shunt elements, such as the no-load losses of transformers, are calculated based on their nominal losses. First, the nominal losses $p_{\text{sh},i}$ at the nominal voltage $u_{\text{sh},N,i}$ are determined for each element considering the shunt conductance $g_{\text{sh},i}$. The nominal losses are then scaled with the squared per-unit voltage u_i^2 to obtain the actual losses given in (20).

$$p_{L,V} = \sum_i p_{\text{sh},N,i} u_i^2 = \sum_i g_{\text{sh},i} u_{\text{sh},N,i}^2 u_i^2 \quad (20)$$

While the current- and voltage-dependent losses of grid assets were discussed above, the losses of reactive power control devices are considered in more detail in the following. Accordingly, Figure 4 illustrates the considered assets for reactive power provision as well as the corresponding losses taken into account in the proposed formulation. The operating-point-dependent losses of DERs (a) are not considered in the optimization, as these losses are typically attributed to the plant operator rather than to the grid. However, they should be included in an overall economic assessment. The losses of passive shunt elements (b) are taken into account via the voltage-dependent loss model. The losses of STATCOMs (c) are included using an approximate model.

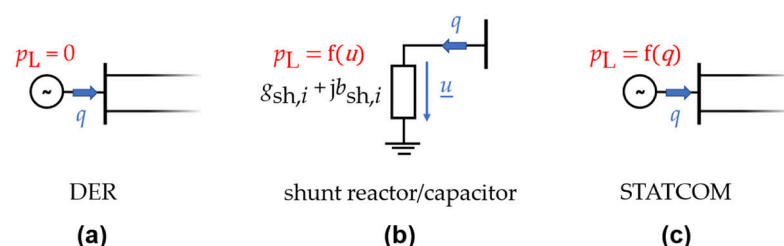


Figure 4. Reactive power control devices and losses considered in the proposed formulation: (a) DER, for which only the controllable reactive power contribution is shown and active power injections are assumed fixed; (b) shunt reactor/capacitor with voltage-dependent losses; (c) STATCOM with approximate reactive-power-dependent losses.

The STATCOM losses are modeled using an approximation based on [35], as given in (21). Here, $q_{\text{STATCOM},i}$ denotes the reactive power injection of the STATCOM at node i and $q_{\text{STATCOM,max},i}$ its maximum reactive power capacity. To ensure that a STATCOM disconnected from the grid does not contribute to the active power losses, the binary

variable $b_{\text{STATCOM},i}$ is introduced according to (22). If detailed information on the STATCOM technology and its loss characteristics is available, the quadratic approximation in (21) can be parameterized individually for each STATCOM using device-specific coefficients.

$$p_{L,oth}(x) = \sum_i \left[0.008 \left(\frac{q_{\text{STATCOM},i}}{q_{\text{STATCOM,max},i}} \right)^2 + 0.002 b_{\text{STATCOM},i} \right] |q_{\text{STATCOM,max},i}| \quad (21)$$

$$b_{\text{STATCOM},i} = \begin{cases} 1, & |q_{\text{STATCOM},i}| \neq 0 \\ 0, & |q_{\text{STATCOM},i}| = 0 \end{cases} \quad (22)$$

Finally, the last term of the objective function is introduced, which aims at minimizing setpoint changes. For this purpose, the absolute values of the control variable changes are considered in (23). This includes the tap changer adjustments of PSTs $\Delta tap_{\text{PST},ij}$ and PTs $\Delta tap_{\text{PT},ij}$ as well as the nodal reactive power injections Δq_i . The weighting factors w_1 to w_3 are set to 0.01. These values were determined empirically based on a parameter study using both the test grid and the real HVDG considered in the case study. Their magnitude is chosen such that the contribution of $c(x)$ remains smaller than the loss terms in the objective function, ensuring that loss minimization remains the primary optimization goal. Depending on the grid characteristics and the desired trade-off between loss minimization and control effort, these weighting factors can be adjusted accordingly.

$$c(x) = w_1 \sum |\Delta tap_{\text{PT},ij}| + w_2 \sum |\Delta tap_{\text{PST},ij}| + w_3 \sum |\Delta q_i| \quad (23)$$

2.2.2. Constraints

In addition to the objective function, the constraints required for the formulation of the MIQP are specified in the following. These constraints are primarily based on the linearized load flow equations derived in Section 2.1.

To implement the piecewise linearization introduced above, the quantities to be linearized, denoted by y_{lin} in (24), are approximated around the linearization point x_0 . Here, c_{x0} represents the value of the corresponding quantity calculated at the linearization point.

The linearization interval is divided into a positive setpoint deviation and a negative setpoint deviation Δx_{lin} . Changes in the tap position of a PST represent an example of a setpoint variation. The coefficients $a_{\text{lin},1}$ and $a_{\text{lin},2}$ denote the sensitivity factors valid for the respective linearization intervals, obtained as described in Section 2.1. The piecewise linearization is implemented using the binary variable b_{lin} , as given in (25).

$$y_{\text{lin}} = a_{\text{lin},1} b_{\text{lin}} \Delta x_{\text{lin}} + a_{\text{lin},2} (1 - b_{\text{lin}}) \Delta x_{\text{lin}} + c_{x0} \quad (24)$$

$$b_{\text{lin}} = \begin{cases} 1, & \Delta x_{\text{lin}} < 0 \\ 0, & \Delta x_{\text{lin}} \geq 0 \end{cases} \quad (25)$$

In addition, voltage limits (26), maximum branch apparent power flow limits (27), nodal reactive power limits (28), and minimum and maximum transformer tap changer positions (29) are imposed. Constraint (28) is also used to ensure compliance with the contractually agreed reactive power limits at the TSO–DSO interconnection points.

$$u_{\min,i} \leq u_i \leq u_{\max,i}; i \in \text{nodes} \quad (26)$$

$$s_{ij} = \sqrt{p_{ij}^2 + q_{ij}^2} \leq s_{\max,ij}; i, j \in \text{nodes} \quad (27)$$

$$q_{\min,i} \leq q_i \leq q_{\max,i}; i \in \text{nodes} \quad (28)$$

$$tap_{\min,k} \leq tap_k \leq tap_{\max,k}; k \in \text{tap changers} \quad (29)$$

The branch flow limit in (27) contains nonlinear terms. To approximate the apparent power constraint, it is linearized following the approach in [10,11], resulting in a rectangular polygonal approximation.

3. Case Study

To evaluate the methodology presented in Section 2, two case studies are considered. The first case is a static, synthetic 8-node test grid. The second case is a time-series-based investigation using snapshots of a real German HVDG. All simulations were performed on a workstation equipped with two Intel Xeon Gold 6242 CPUs running at 2.80 GHz and the Windows operating system.

3.1. Eight-Node Test Case

The test grid, shown in Figure 5, includes PTs at branches 1–3 and 2–6, which connect the 110 kV level to the 380 kV level. To account for PSTs, they are modeled as ideal elements connected in series with the PTs. Substations supplying the downstream medium-voltage grids are located at nodes 5 and 8 and are represented as equivalent power injections. In addition, a STATCOM with a rated power of 40 MVar is installed at node 8. The wind farm connected at node 7 provides reactive power flexibility within the limits specified in VDE-AR-N 4120 [36].

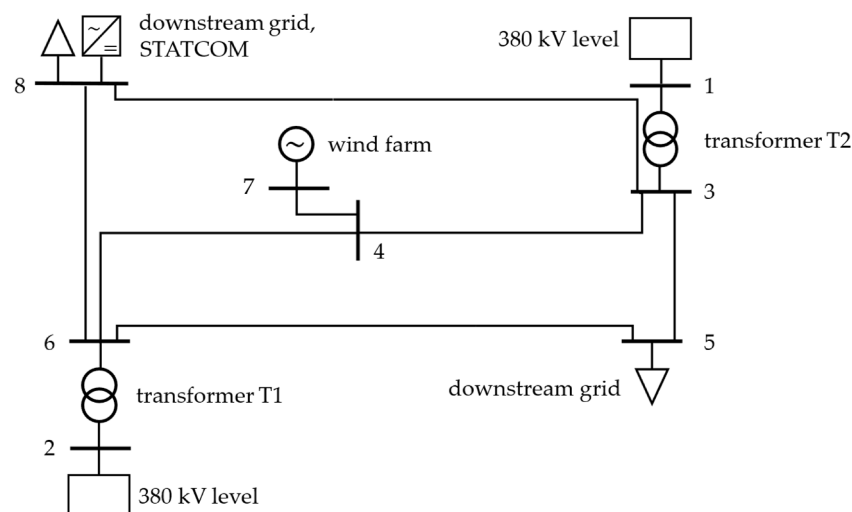


Figure 5. Grid topology of the 8-node test case [10].

Nodes 1 and 2 are modeled as slack buses with fixed voltages of $u_1 = 1.080e^{-j7^\circ}$ p.u. and $u_2 = 1.078e^{j0^\circ}$ p.u. The different voltage setpoints are selected to account for transit power flows from the TG through the HVDG. Further parameters of the test grid, such as line lengths, transformer parameters, and nodal power injections prior to optimization, are provided in Appendix A.1. For the test grid analysis, PST-related losses are neglected, whereas they are considered in the real-grid case study.

In the following, a base case without optimization (Scenario 0) and three scenarios employing different control variables in the optimization are analyzed. For all scenarios, the upper limit of the voltage magnitude at nodes in the HV-level is set to 1.08 p.u.

- Scenario 1: Optimization of reactive power injections (wind farm and STATCOM).
- Scenario 2: Optimization of reactive power injections as well as PT and PST tap changer positions.
- Scenario 3: Optimization of PT and PST tap changer positions without optimization of reactive power injections.

Since the test grid is heavily loaded by transit power flows, the largest loss reduction is achieved by adjusting the PST tap position (see Figure 6a, Scenarios 2 and 3). In these scenarios, the active power losses are reduced by approximately 37% compared to the initial operating state. If only the reactive power injection of the wind farm and the STATCOM, as well as the tap position of the PT, are optimized, the losses decrease by about 2%. Notably, in none of the investigated scenarios does the STATCOM inject reactive power. This is because the STATCOM losses considered in the optimization exceed the reduction in grid losses that could be achieved through reactive power injection.

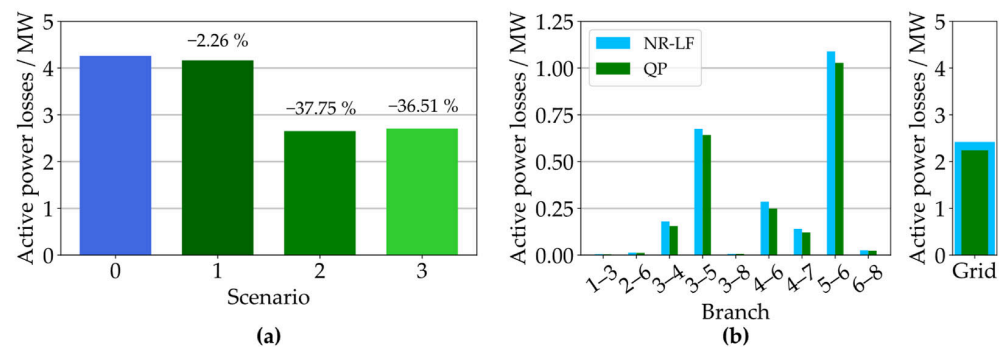


Figure 6. (a) Total active power losses and corresponding loss reduction for the considered scenarios. (b) Current-dependent active power losses calculated within the optimization compared to the results of an NR-LF performed after the optimization. Results are shown for Scenario 2.

Overall, depending on the selected control variables and the initial grid condition, the algorithm can achieve a substantial reduction in active power losses.

To evaluate the accuracy of the loss approximation, the current-dependent losses determined within the optimization according to Section 2.2.1 are compared with those obtained from an NR-LF using the control variables after optimization. Scenario 2 is considered as an example, as it leads to the largest change in the power flow and, consequently, in the active power losses compared to the initial state.

Figure 6b presents the calculated losses for Scenario 2, showing the branch-wise losses on the left and the total losses of the complete grid on the right. The losses resulting from the QP are slightly underestimated for all branches. One reason for this deviation is the constant approximation of the product of the nodal voltage magnitudes $u_i u_j$ in (15). Since, after optimization, the product $u_i u_j$ exceeds the approximated value of 1.1 for all branches in the test case, the losses obtained from the NR-LF are correspondingly higher.

The nodal voltages before optimization and the values obtained from the QP and NR-LF approaches after optimization are shown in Figure 7. For nodes 1 and 2, the voltage remains identical in all cases, as these nodes operate as slack buses. For the remaining nodes in the grid, the optimization results in higher voltage magnitudes compared to the initial state. In this test case, the linearized formulation slightly overestimates the voltage magnitude relative to the NR-LF solution. This behavior is preferable to underestimation, as the specified voltage limits can be satisfied without introducing an additional safety margin. The maximum linearization error is <0.001 p.u. in terms of nodal voltage magnitude.

Due to the presence of PSTs in Scenario 2, a significant change in the voltage angles at the 110 kV level can be observed between the pre- and post-optimization states. Nevertheless, the absolute linearization error in the post-optimization state remains below 0.033° for all nodes, which is negligible in comparison with the voltage angle changes resulting from the optimized PST tap positions.

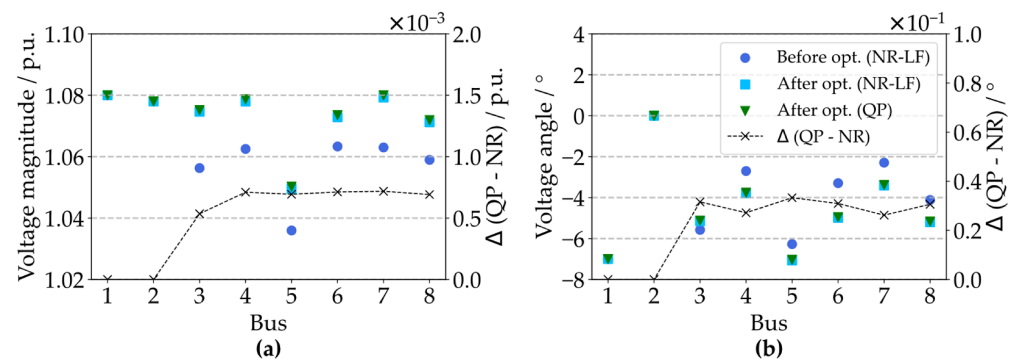


Figure 7. (a) Nodal voltage magnitude and (b) nodal voltage angle before and after optimization for Scenario 2. Post-optimization values obtained from the NR-LF and the QP approach are shown, together with the differences between both methods.

Overall, the results confirm the accuracy of the proposed QP formulation for the considered test case. In the following, the method is applied to snapshots of a real German 110 kV grid to further assess its practical performance.

3.2. Snapshots of a German HVDG

In the following, the results obtained from the analysis of a real HVDG in Germany (see Figure 8) are presented. For this purpose, both the achievable loss reduction potentials and the required computation times are evaluated. The considered grid is identical to the one analyzed in [10]. A one-week period is investigated, with snapshots available at an hourly time resolution. These data originate from a Common Grid Model Exchange Standard (CGMES) export provided by the control center of a German DSO. This represents a significant advantage of the study, as the analysis is based on real operational data rather than on synthetic grid models.

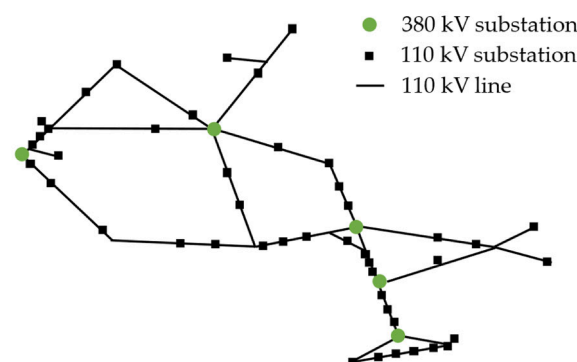


Figure 8. Simplified illustration of the considered German HVDG.

Depending on the switching state, the grid comprises approximately 170 nodes and 190 branches, including overhead lines as well as two-winding and three-winding transformers. Currently, only PTs are installed, while PSTs are not present.

The HVDG is meshed and strongly interconnected with the upstream TG. The superimposed grid itself is not explicitly modeled. Instead, it is represented by distributed slack nodes at the interconnection points between HVDG and TG. For a comprehensive assessment, particularly under $(n-1)$ contingency conditions, explicit modeling of the TG would be preferable. However, this was not possible, as the required snapshots of the TG were not available.

As controllable flexibilities, 12 DERs are modeled, providing a reactive power range from approximately -100 MVar (underexcited operation) to $+200$ MVar (overexcited operation).

tion). STATCOM units are not included in this study, as they would also contribute to voltage control in the upstream TG, which, as previously mentioned, is not explicitly modeled.

Within the optimization algorithm, very lightly loaded branches and very short lines (e.g., internal substation connections) are excluded. This improves numerical stability and reduces computation time, while the impact on the optimization results is negligible.

For the case study, the following scenarios are considered in accordance with the previous section:

- Scenario 1a: Optimization of reactive power injections (DERs).
- Scenario 1b: Optimization of reactive power injections as well as PT tap changer positions.
- Scenario 2: Optimization of reactive power injections as well as PT and PST tap changer positions.

Figure 9a presents the analysis of the loss reduction potential when applying Scenario 1b. The normalized active power losses before optimization are shown in red. The losses are normalized to the maximum value occurring within the considered period. The simulated loss reduction is depicted by a dashed line. Depending on the time step, the losses are reduced by approximately 1% to 16%. Over the entire period, an overall reduction of approximately 7% is achieved.

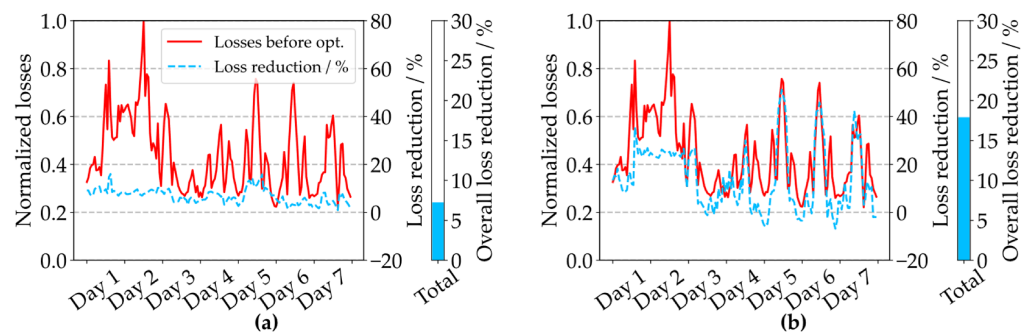


Figure 9. (a) Scenario 1b; (b) Scenario 2. Active power losses before optimization and percentage loss reduction per time step after optimization, including the overall loss reduction for the entire time period.

In Scenario 2, PSTs are added in the simulations at two selected 380/110 kV substations connected to the TG. Since PSTs are currently not installed in the investigated grid, this scenario represents a potential study rather than an optimization of the current grid configuration. The selected substations were identified as particularly effective locations for reducing transit power flows. The additional control flexibility provided by the PSTs leads to a significantly higher loss reduction, as the considered grid is substantially loaded by transit power flows resulting from the meshed coupling with the TG (see Figure 9b). These transit power flows can be mitigated by the use of PSTs. In this case, the losses are reduced by up to 51%, depending on the time step. At certain operating points, however, the additional PST losses outweigh the achievable loss reduction, resulting in an increase in total losses of up to 7%. Nevertheless, an overall loss reduction of 18% is achieved over the entire study period.

Already with PSTs installed in one substation in the northern part of the grid area, the losses can be reduced by approximately 12%. This demonstrates that even a limited number of strategically placed PSTs can reduce losses within the grid area.

The proposed method is not limited to grids with significant transit power flows or to grids equipped with PSTs. If PSTs are not available, the optimization can still be applied to the remaining control variables, as demonstrated in Scenarios 1a and 1b. In grids with less significant transit power flows, the additional loss reduction potential of PSTs is expected to be lower. Nevertheless, PSTs may still reduce losses by shifting power flows within the

110 kV grid, for example when overhead lines and cables with different impedances are operated in parallel.

Figure 10 shows the computation time required to apply the algorithm in Scenarios 1a, 1b, and 2. The total computation time includes not only the optimization time but also the time required for linearization, retrieving the snapshots from the database, and post-processing. The median computation time for Scenario 1a is approximately 3 s. For Scenario 1b, the computation time increases when PT tap changers are taken into account, resulting in a median of about 5 s. The runtime further increases in Scenario 2 when PSTs are additionally considered, reaching a median of approximately 6 s. For some outliers, the total computation time is up to almost 8 s.

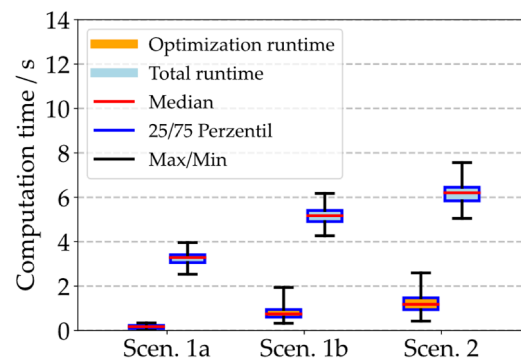


Figure 10. Comparison of optimization runtime and total runtime (linearization, optimization, pre- and post-processing) for Scenarios 1a, 1b, and 2.

The increase in computation time results from both the additional linearization steps required when tap changer positions are considered and the longer optimization times observed in Scenarios 1b and 2. The median optimization time increases from approximately 0.2 s in Scenario 1a to about 1.2 s in Scenario 2, partly due to the higher number of discrete variables introduced by the tap changer positions.

Table 1 shows the problem size for selected scenarios of the investigated case study. As expected, the number of integer variables and constraints increases with the inclusion of additional controllable assets. This is caused by the additional discrete decision variables associated with the PT and PST tap changer positions considered in the respective scenarios, while the number of continuous variables remains unchanged. Nevertheless, for the investigated real 110 kV distribution grid comprising approximately 170 nodes and 190 branches, the resulting problem size remains moderate and enables computation times compatible with near-real-time operation.

Table 1. Size of the optimization problem for selected scenarios of the investigated case study, evaluated for a single snapshot.

Scenario	Continuous Variables	Integer Variables	Constraints
1a	1092	13	2952
2	1092	46	2998

The results indicate that the optimization of reactive power injections and PT tap changer positions alone can noticeably reduce active power losses using assets that are already available in the grid and can typically be integrated into existing operational and regulatory frameworks. Although larger loss reductions can be achieved by additionally using PSTs, this requires the installation of new grid assets associated with considerable investment costs and implementation efforts. A detailed economic assessment of PST installations is beyond the scope of this paper, as it would require a separate cost–benefit analysis

considering long-term operating conditions, grid development scenarios, assumptions on investment and operating costs, and alternative grid reinforcement and operational measures. Furthermore, the short computation times demonstrate the suitability of the proposed approach for near-real-time operation in HVDGs.

4. Conclusions

This paper presented a sensitivity-based MIQP formulation for a loss-minimizing OPF in HVDGs based on a piecewise linearized AC power flow model. The formulation considers reactive power control, PT and PST tap changers, STATCOM losses, and tap-position-dependent transformer reactance variations. The goal was not only to determine suitable operating points for minimizing active power losses but also to achieve computation times compatible with near-real-time grid operation.

The case studies demonstrated that substantial loss reductions can already be achieved by optimizing reactive power injections and PT tap positions using existing grid assets. This indicates that active power losses in HVDGs can be reduced through improved utilization of controllable assets and coordinated operational optimization strategies. Even larger loss reductions can be achieved when PSTs are additionally installed in grids with significant transit power flows. Therefore, the use of PSTs represents a promising option for reducing transit-related losses in highly transit-loaded HVDGs. However, the achievable loss reduction potential depends on the specific grid characteristics, in particular, on the local reactive power balance and the magnitude of prevailing transit power flows.

The methodological framework is applicable to various grid topologies, including radial and meshed structures, and is, in principle, suitable for different voltage levels. Although the focus of this work has been on meshed HVDGs with significant transit power flows, the approach itself is not restricted to this specific application.

When considering PSTs, substantial changes in power flows may result from the optimization. To prevent potential overloads under contingency conditions, $(n-1)$ contingency constraints could be incorporated into the optimization framework in linearized form following the approach proposed in [30]. Moreover, including snapshots of the TG would enable coordinated optimization across voltage levels, facilitating global loss minimization instead of grid-level-specific optimization and allowing the coordinated consideration of devices such as STATCOMs.

Furthermore, future work should investigate the economic feasibility of PST deployment by considering both loss reductions and potential impacts on grid reinforcement requirements based on long-term operational data.

Finally, minimizing the switching frequency of transformer tap changers is essential to extend their service life and improve practical applicability. Therefore, the incorporation of switching operations into the objective function represents an important extension of the proposed framework and will be investigated in future work, for example, by considering previous tap positions or by implementing forecast-based control strategies.

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Abbreviations

The following abbreviations are used in this manuscript:

AC	Alternating current
CGMES	Common Grid Model Exchange Standard
DERs	Distributed energy resources
DSO	Distribution system operator
FACTS	Flexible AC Transmission Systems
HV	High voltage
HVDG	High-voltage distribution grid
LP	Linear programming
MIQP	Mixed-integer quadratic programming
NLP	Nonlinear programming
NR-LF	Newton–Raphson load flow
OPF	Optimal power flow
PST	Phase-shifting transformer
PT	Power transformer
QP	Quadratic programming
STATCOM	Static synchronous compensator
TG	Transmission grid
TSO	Transmission system operator
UPFC	Unified power flow controller

Appendix A

Appendix A.1

The parameters of the 8-node test grid are presented as follows. The nominal line current and the nominal transformer apparent power are identical to those reported in [10]. The relative line impedance and the relative short-circuit voltage were also adopted from [10]. Each transformer has no-load losses of 100 kW. Additional transformer parameters, including the tap position prior to optimization and the change in open-circuit voltage per tap step, are given in Table A1. The line lengths of the test case are listed in Table A2. Furthermore, Table A3 lists the nodal power injections prior to optimization.

Table A1. Transformer data for the 8-node test case.

	Transformer Ratio t	Tap Position	Minimum Tap Position	Maximum Tap Position	Voltage Change per Tap Step
PTs	1.007 p.u.	0	−10	10	0.01 p.u.
PSTs	1.000 p.u.	0	−9	9	1.25°

Table A2. Line lengths for the 8-node test case.

Line	3–4	3–5	3–8	4–6	4–7	5–6	6–8
Length/km	40	30	20	20	4	20	8

Table A3. Nodal powers before optimization for the 8-node test case.

Node	P_i/MW	Q_i/MVar
5	110	20
7	−55	18
8	20	5

Appendix A.2

To illustrate the influence of the selected reactive power linearization range on the approximation accuracy, Figure A1 presents results for a reactive power variation at node 8 of the test system described in Section 3. Node 8 corresponds to the location of the STATCOM considered in the test system. The reactive power injection was varied between 0 MVar and 100 MVar. Figure A1a shows the minimum and maximum voltage magnitude errors obtained across all 110 kV nodes for different linearization ranges. Figure A1b illustrates the dependency of the maximum voltage magnitude error on the applied reactive power setpoint change for selected linearization ranges. Similar qualitative trends were observed for voltage angles as well as branch active and reactive power flows.

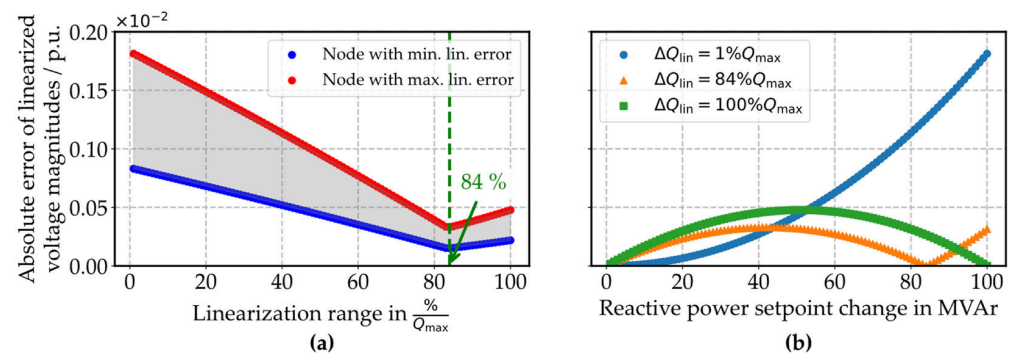


Figure A1. Voltage magnitude approximation errors resulting from a reactive power variation between 0 MVar and 100 MVar at the STATCOM node of the test system: (a) minimum and maximum errors for different linearization ranges, with the shaded area indicating the range between the minimum and maximum errors; (b) maximum errors for selected linearization ranges.

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