

Article

Flow-Tracing-Based Allocation of Congestion-Related Settlement Imbalances in Regional Electricity Markets with Priority-Based Implicit Transmission Rights

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Abstract

In transitional electricity markets that operate without explicit financial or physical transmission rights but retain priority generation schedules and rigidly executed medium- and long-term (MLT) contracts, network congestion gives rise to settlement imbalance funds that are difficult to allocate fairly. This paper proposes an allocation framework that couples proportional-sharing power-flow tracing with priority-based implicit transmission rights (PBITRs). Flow tracing quantifies each transaction's contribution to the realized flows on constrained lines, while the priority rank converts dispatch and contract-execution rules into a direction-dependent settlement weight. The clearing inputs are produced by a joint 24-period DC optimal power flow over a modified IEEE 30-bus system, and representative off-peak, peak, and flat periods are recalculated by AC optimal power flow to test the robustness of constrained-line identification, contribution ranking, and allocation shares. In the recalculated case study, the daily congestion-surplus pool is CNY 19,380 and the proposed method assigns 60.6% of it to the MLT A-to-C transaction that physically dominates the constrained corridor, compared with 21.1% under energy-proportional allocation. For the balancing congestion charge pool of CNY 42,492, the fund-specific responsibility set excludes the LMP-settled spot increment and the proposed method assigns 81.2% to the MLT A-to-C transaction. Across the three AC-OPF checks, the constrained-line sets coincide with the DC results at the 0.95 loading threshold, the transaction contribution rankings are identical, and the maximum DC-AC share differences are 1.01 percentage points for congestion surplus and 0.72 percentage points for balancing congestion charge. Sensitivity tests over the priority exponent and loading threshold show smooth and interpretable changes. The framework contributes an ex-post, settlement-neutral tool for markets that clear energy with LMPs before tradable transmission rights are introduced.



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Keywords: electricity market; congestion management; settlement imbalance funds; power-flow tracing; priority-based implicit transmission rights; DC optimal power flow; congestion surplus; balancing congestion charge

1. Introduction

Locational marginal pricing settles energy at prices that already embed the marginal value of transmission scarcity. In fully right-based markets, the resulting congestion rent funds the payouts to holders of financial or physical transmission rights (FTRs/PTRs)

auctioned ex ante, closing the loop between congestion charges and the parties entitled to them [1–3].

1.1. Background: Transitional Electricity Markets

Many markets, however, are still in transition toward this fully right-based architecture. China's regional and provincial markets are the clearest current instance and the focus of this study: in the Southern regional market anchored by Guangdong, inter-provincial exchanges are organized as network-to-network transactions along contractually designated corridors and provincial spot pilots publish nodal or zonal prices, yet no FTRs or PTRs are issued. The market simultaneously honors priority generation with consumption guarantees and large MLT contracts whose physical delivery is rigid and curtailed only as a last resort when a corridor binds [4–7]. Outside China, the Nordic electricity market provides a concrete zonal-market comparison. Nord Pool divides the market into multiple bidding areas; limited inter-area transmission capacity can produce different area prices, while an unconstrained Nordic system price is calculated as the reference for most financial contracts [8,9]. Area-price risk is hedged through instruments such as Electricity Price Area Differentials (EPADs), including official EPAD auctions, rather than by attributing each constrained-line flow to the individual transactions that caused it [8,10]. The Nordic arrangement is more mature than the transitional Chinese setting, but it illustrates the distinction between area-level congestion hedging and transaction-level physical responsibility. The present study addresses the narrower transitional case in which no explicit right holder exists for the two settlement pools considered.

1.2. Related Work, Research Gap, and Contributions

Four strands of the literature bear on this problem. First, work on transmission rights and congestion rent, including contract networks, rights design, and market-power effects, presumes that tradable FTRs or PTRs already exist in a mature market [1–3,11]. It therefore does not answer how a market should allocate congestion-related funds before rights exist. The Nordic zonal market offers an adjacent but distinct benchmark: congestion creates bidding-area price differentials and area-level hedging products, but these instruments address spatial price risk rather than transaction-specific physical responsibility on individual constrained lines [8–10]. Second, studies of imbalance, balancing, revenue adequacy, redispatch costs, and uplift document residual funds and their incentive effects [12–14], but usually treat them as market-settlement or efficiency questions rather than tracing each fund to the transactions that physically load the bottleneck.

Within the Chinese dual-track literature, the studies closest to this paper address different layers of the problem. Jing et al. [1] provide a systematic diagnosis: they classify imbalance funds by responsible entity, mechanism-design origin, product and market structure, and price-quantity manifestation; explain their formation through product conversion, price mismatch, quantity-price mismatch, and pricing rules that violate participation constraints; and discuss treatment principles from the perspectives of market efficiency and social welfare. Ding and Tan [5] couple contract and spot settlement through a day-ahead benchmark, real-time deviation, and contract-price difference. Their allocation and refund rules are volume-based: ancillary-service costs and generation-side deviation-assessment proceeds are assigned by users' actual metered consumption, while demand-side deviation-assessment proceeds are assigned to generators by government-authorized contract-energy shares, following a causer-responsibility principle. Zhou et al. [6] compare imbalance settlement in the United Kingdom, ERCOT, and the Nordic market. Rather than allocating a common fund pool, they settle each participant's metered deviation from contracted

or scheduled energy at the applicable imbalance price, and recommend finer contract decomposition and stronger contract-spot price linkage for China.

Long et al. [7] address the endogenous feedback between an allocation rule and market behavior. Their agent-based balancing-market model uses learning balancing service providers (BSPs) to compare four mechanisms: allocation to all market users through electricity-price adjustment, settlement-price adjustment for the relevant BSP, allocation by BSP market-participation scale, and allocation by BSP net market revenue with cost characteristics. The model therefore evaluates how an allocation rule changes BSP capacity and bidding decisions, balancing cost, imbalance-fund size, and social-welfare distribution. Its simulations show that a well-designed rule can increase BSP capacity committed to the balancing market by up to 30% and reduce imbalance funds by about 20%.

These contributions do not, however, construct the joint allocator required here. Reference [1] supplies a taxonomy and broad treatment principles; References [5,6] use metered energy, contract energy, deviation volume, and settlement prices; and Reference [7] evaluates participant-level incentive bases such as price adjustment, participation scale, and net revenue. None of these studies identifies how much each transaction contributes to each binding line, and none combines that physical contribution with the transaction's institutional priority in the curtailment order. A fourth strand, power-flow tracing, does decompose realized line flows into injection and withdrawal contributions and has been used for transmission-cost allocation, network-use pricing, congestion management, and inter-area transactions [15–21]. It supplies the physical component of responsibility but not the contract-priority component. The research gap therefore lies in forming one fund-specific responsibility weight from constrained-line contribution and contract priority, then applying it consistently to both congestion surplus and the balancing congestion charge in a transitional market without explicit transmission rights.

The contributions are threefold. First, the paper recasts priority dispatch and rigid MLT execution as PBITRs, making the transitional setting legible to readers familiar with explicit transmission-right markets. Second, it couples proportional-sharing flow tracing with a PBITR-derived priority weight into a single effective responsibility weight, applied consistently to both funds over explicitly defined responsibility sets. Third, it evaluates the framework by using allocation deviation, flow consistency, priority consistency, and settlement-balance indices on a modified IEEE 30-bus system over 24 clearing periods, with sensitivity checks for the priority exponent, congestion threshold, and demand level.

2. Materials and Methods

2.1. Terminology and Scope

This paper solves an ex-post allocation problem, not a market-clearing or pricing problem. The market outcome is taken as given; the task is to distribute two congestion-related settlement funds among the participating transactions according to physical responsibility and institutional priority.

Inputs: the network topology, reactances, and line ratings; the cleared generator outputs, nodal LMPs, and line flows; transaction schedules, executed quantities, contract prices, source and destination areas, and supplying generators; and each transaction's priority rank and the congestion threshold *etc.*

Outputs: each transaction's share of the congestion surplus and balancing congestion charge, per period and over the study horizon, plus the diagnostic indices used in Section 3.

Scope: the method redistributes realized funds. It does not auction rights, alter dispatch, set energy prices, or replace the market operator's clearing model.

Priority generation and MLT contracts are treated as firm energy commitments over the scheduling window rather than rigid per-period curves. Under congestion these commitments are curtailed, not inflated; any additional energy delivered beyond firm commitments is a separate spot increment settled at the LMP. This is why curtailed firm energy, rather than over-delivery, enters the balancing congestion charge.

Because the institutional context of transitional markets does not map cleanly onto the vocabulary of mature right-based markets, the main terms used in the allocation model are fixed before the mathematical formulation. The terms below (Table 1) follow the wording used in the revised introduction and are used consistently in the remainder of the paper.

Table 1. Terminology adopted in this paper and the rationale for each term.

Concept	Adopted Term	Rationale
Residual congestion rent	congestion surplus	LMP spread multiplied by active-power flow on constrained corridors.
Contract imbalance from curtailment	balancing congestion charge	Contract-versus-spot price gap on curtailed firm energy.
Guaranteed administrative schedules	priority generation	Highest-priority firm energy commitment, assigned $\omega = 1$.
Rigid bilateral energy contracts	MLT contracts	Medium- and long-term fixed-price firm contracts, assigned $\omega = 2$.
Pure LMP-settled increment	spot increment/pure spot transaction	LMP-settled energy with no fixed contract price, assigned $\omega = 3$.
Administrative firmness	PBITR	De facto corridor-capacity claim embedded in priority rules.
Eligible transactions	responsibility set	Transactions included in a fund-specific allocation denominator.

The framework remains an ex-post settlement tool. PBITR is not a tradable property right and does not replace a future FTR/PTR mechanism; it formalizes the priority order already embedded in current dispatch and contract-execution rules.

2.2. Formation and Classification of Congestion-Related Settlement Imbalances

Let line l connect buses $i(l)$ and $j(l)$, oriented along the cleared active-power flow. The congestion surplus is the price spread across constrained corridors multiplied by the active-power flow:

$$CS_t = \sum_{l \in \mathcal{L}} \left(\lambda_{j(l),t} - \lambda_{i(l),t} \right) f_{l,t} \quad (1)$$

In the lossless DC market used here, the price difference is nonzero only when a transmission constraint binds. The signed flow $f_{l,t}$ records direction, while the product is nonnegative along the congested direction because the receiving-end LMP is above the sending-end LMP.

The balancing congestion charge is the contract-versus-spot price gap on scheduled firm energy that could not be delivered:

$$BC_t = \sum_{c \in \mathcal{R}} (q_{c,t} - \hat{q}_{c,t}) \max(0, \lambda_{c,t}^{\text{spot}} - \pi_c) \quad (2)$$

Here $q_{c,t}$ is the contract price and $\mathcal{R}$ contains only fixed-price transactions whose scheduled quantity is curtailed. By construction, BC_t is nonnegative. Pure spot transactions are excluded from this charge because they settle entirely at the LMP and have no contract reference price.

2.3. Market Clearing Model and DC Approximation

The allocation model requires as inputs the cleared active-power dispatch, nodal LMPs, and line flows. In the numerical study these inputs are generated by a multi-period security-constrained DC optimal power flow (DC-OPF), solved jointly over 24 hourly periods so that ramping constraints are represented. The full formulation is given in Appendix A.

The DC approximation is used because the settlement funds studied here are active-power congestion and LMP-based monetary quantities. DC-OPF is a standard transparent approximation for congestion pricing, active-power transmission limits, and FTR-style congestion-rent studies when voltage magnitudes are close to 1 p.u., angle differences are moderate, resistance and losses are small, and reactive-power constraints are not binding [22,23]. These assumptions match the purpose of this paper: to evaluate a fund-allocation layer, not to propose a new AC stability or security assessment.

This modeling choice does not imply that frequency stability, voltage stability, reactive-power balance, or AC security constraints are unimportant in real operation. Those issues are handled by the operator's AC security assessment, reserve procurement, voltage/reactive-power control, and post-contingency security checks. The proposed allocation rule is model-agnostic: if a real market uses an AC security-constrained OPF, the same responsibility-weighting layer can take the resulting dispatch, prices, and active-power line flows as inputs.

To test whether the DC approximation materially changes the allocation conclusions, periods 8, 12, and 20 are selected as representative off-peak, peak, and flat conditions and recalculated with AC-OPF in MATPOWER [24]. The AC test preserves the modified IEEE 30-bus topology, line ratings, and branch reactances, retains the original branch R/X ratios and charging data, assumes a 0.97 lagging load power factor, imposes voltage limits of 0.94–1.06 p.u., and enforces apparent-power branch limits. Thus, the validation introduces voltage magnitudes, reactive-power balance, resistance, and active-power losses without changing the transaction schedules or responsibility rules.

For line-set comparison, the DC loading ratio is $\rho_{l,t}^{\text{DC}} = \frac{|P_{l,t}|}{F_l^{\text{max}}}$ and the AC loading ratio is $\rho_{l,t}^{\text{AC}} = \frac{\max(|S_{l,t}^f|, |S_{l,t}^t|)}{S_l^{\text{max}}}$. The Jaccard index compares constrained-line sets at loading thresholds of 0.98 and 0.95. For AC flow tracing, the line active power is represented by $P_{l,t}^{\text{mid}} = \frac{P_{l,t}^f + P_{l,t}^t}{2}$, while one half of each line's active-power loss is assigned to the equivalent load at each terminal bus. Spearman rank correlation and the maximum absolute difference in allocation shares are then used to compare DC and AC results. These checks assess steady-state consistency only; they are not frequency-stability, transient-stability, or $N - 1$ security studies.

2.4. Proportional-Sharing Power-Flow Tracing

Power-flow tracing is applied to the cleared dispatch of the interconnected system as a single connected network; the zones are not treated as islanded systems. If an external tie

or priority import is represented in the test system, it is modeled as an equivalent generator so that nodal active-power balance closes exactly.

$$[\mathbf{A}_u]_{ij} = \begin{cases} 1, & i = j, \\ -\frac{|f_{j \rightarrow i,t}|}{P_{j,t}}, & \text{if line } j \rightarrow i \text{ carries flow into } i, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

$$F_{l,t}^g = \frac{|f_{l,t}|}{P_{i(l),t}} [\mathbf{A}_u^{-1}]_{i(l),g} P_{Gg,t} \quad (4)$$

$$\begin{aligned} \sum_{g \in \mathcal{G}} F_{l,t}^g &= \frac{|f_{l,t}|}{P_{i(l),t}} [\mathbf{A}_u^{-1} \mathbf{P}_G]_{i(l)} \\ &= |f_{l,t}| \end{aligned} \quad (5)$$

$$[\mathbf{A}_d]_{ij} = \begin{cases} 1, & i = j, \\ -\frac{|f_{i \rightarrow j,t}|}{P_{j,t}}, & \text{if line } i \rightarrow j \text{ carries flow from } i \text{ to } j, \\ 0, & \text{otherwise.} \end{cases} \quad (6a)$$

$$\begin{aligned} w_{i,t}^Z &= \sum_{m \in \mathcal{Z}} [\mathbf{A}_d^{-1}]_{i,m} \frac{D_{m,t}}{P_{i,t}}, Z \in \{B, C\}, \\ F_{l,t}^{A \rightarrow Z} &= |f_{l,t}| u_{i(l),t}^A w_{j(l),t}^Z \end{aligned} \quad (6b)$$

The upstream matrix $\mathbf{A}_u$ is not a PTDF. A PTDF is a signed marginal sensitivity of a line flow to an injection-withdrawal pair, whereas $\mathbf{A}_u$ contains realized nonnegative inflow shares that decompose the actual flow. Therefore PTDF-weighted terms do not generally sum to the realized line flow, while proportional-sharing contributions do so by construction.

For transactions that share the same source zone but serve different sink zones, the downstream matrix $\mathbf{A}_d$ separates A-to-B and A-to-C transfers according to their ultimate sink-side load shares. The receiving-node gross through-flow $P_{j,t}$ is used in the downstream normalization. Transactions in the same area pair are then split by executed quantity.

Proposition 1 (conservation and coverage). *For any directed line l and period t , proportional sharing is conservative when contributions are summed over the complete set of modeled injection sources or the complete set of modeled sinks: each complete sum equals the realized active-power flow $|f_{l,t}|$. At the transaction level, the same equality holds only when the tracked transaction set is a complete partition of the modeled source-to-sink flow components. If some injection, withdrawal, or external exchange is outside that set, the transaction sum recovers only the covered subflow, and the difference is the uncovered component.*

Proof. Let $\mathbf{P}$ be the vector of nodal gross through-flows and $\mathbf{P}_G$ the vector of modeled source injections. Nodal balance and the definition of $\mathbf{A}_u$ give $\mathbf{A}_u \mathbf{P} = \mathbf{P}_G$, so $\mathbf{P} = \mathbf{A}_u^{-1} \mathbf{P}_G$. Substituting the sending-node element of this identity into Equation (4) gives Equation (5). The downstream result follows analogously because the complete sink shares at the receiving node sum to one. If each source-to-sink component is then partitioned among transactions with fractions that sum to one, summing the transaction contributions reproduces the complete decomposition; otherwise it reproduces only the covered fraction.

$$\sum_{c \in \mathcal{C}} F_{l,t}^c = |f_{l,t}| \quad (7)$$

if and only if $\mathcal{C}$ completely partitions the modeled line-flow components.

The statement assumes a connected steady-state active-power network, closed nodal balances, and positive gross through-flow at every node where a sharing ratio is formed. The DC formulation neglects losses; in the representative AC check, line losses are assigned one half to the equivalent load at each terminal as described above. Across the 24-period numerical run, the maximum absolute residual of the complete upstream and downstream decompositions is below 2.6×10^{-13} MW, which is numerical zero at the reported precision. $\square$

2.5. Constrained-Line Set, Priority Rule, and Responsibility Weights

Only lines that are actually binding should drive the allocation. A line is included in the constrained-line set when its loading reaches the threshold η :

$$\mathcal{L}_{\text{cong},t} = \left\{ l \in \mathcal{L} : \frac{|f_{l,t}|}{F_l^{\text{max}}} \geq \eta \right\} \quad (8)$$

For transaction c , the physical responsibility term is the total absolute contribution of that transaction over the constrained-line set:

$$\Phi_{c,t} = \sum_{l \in \mathcal{L}_{\text{cong},t}} |F_{l,t}^c| \quad (9)$$

$\Phi_{c,t}$ is the sum of the absolute transaction contributions over the constrained-line set. The priority rank ω_c encodes the institutional ordering. Priority generation has $\omega = 1$, MLT contracts have $\omega = 2$, and the spot increment has $\omega = 3$. Smaller omega means a stronger de facto claim on scarce corridor capacity. When scheduled inter-area transfer exceeds deliverable transfer, the spot increment is curtailed first, MLT contracts of equal priority are curtailed pro rata, and priority generation is protected until lower-priority transactions have been exhausted.

The base case uses $k = 1$. Setting $k = 0$ recovers priority-neutral pure flow tracing; increasing k strengthens the priority signal relative to physical contribution:

$$\begin{aligned} \theta_{c,t}^+ &= \frac{I(c|\mathcal{R}_t^{\text{CS}})\Phi_{c,t}\omega_c^{-k}}{\sum_{c' \in \mathcal{R}_t^{\text{CS}}} \Phi_{c',t}\omega_{c'}^{-k}} \\ \theta_{c,t}^- &= \frac{I(c|\mathcal{R}_t^{\text{BC}})\Phi_{c,t}\omega_c^k}{\sum_{c' \in \mathcal{R}_t^{\text{BC}}} \Phi_{c',t}\omega_{c'}^k} \end{aligned} \quad (10)$$

2.6. Unified Allocation and Responsibility Sets

The congestion surplus is distributed over $\mathcal{R}_t^{\text{CS}}$ in Equation (11), which contains all transactions with nonzero contribution to constrained lines. The balancing congestion charge is distributed over $\mathcal{R}_t^{\text{BC}}$, which contains only fixed-price transactions because that charge is defined by a contract-versus-spot price gap. A pure spot transaction is therefore eligible for congestion-surplus credit but is assigned zero balancing-congestion responsibility. The same fund-specific eligibility sets are imposed on the proposed method and both benchmarks, preventing differences in eligibility from being mistaken for differences in allocation logic. If no constrained line is identified in a period, executed energy replaces $\Phi_{c,t}$ within the same responsibility set. The per-period weights sum to one, so both allocations are settlement-neutral by construction.

$$CS^{(c)} = \sum_t \theta_{c,t}^+ CS_t \quad (11)$$

$$BC^{(c)} = \sum_t \theta_{c,t}^- BC_t \quad (12)$$

2.7. Illustrative Three-Node Example

Consider three nodes A, B, and C connected as a triangle. Node A has a 100 CNY/MWh unit; B and C have 300 CNY/MWh backup units. The A–C corridor is rated at 55 MW and binds. Four transactions are scheduled: T1 priority A-to-B, T2 MLT A-to-B, T3 MLT A-to-C, and T4 spot A-to-C. Clearing gives $\lambda_A = 100$, $\lambda_B = 200$, and $\lambda_C = 300$. Tracing decomposes the 55 MW A–C flow into 53.3 MW from A-to-C and 1.2 MW from A-to-B, so the congestion surplus is 11,000 CNY and the allocation is driven mainly by physical A-to-C responsibility, with priority then separating the MLT and spot shares.

To make the construction of Equation (1) explicit, index A, B, and C as 1, 2, and 3. Let the directed nonzero flows from A be $f_{A \rightarrow B,t}$ and $f_{A \rightarrow C,t}$, and define the gross through-flow $P_{A,t} = f_{A \rightarrow B,t} + f_{A \rightarrow C,t}$. If no additional incoming-flow term is present in this matrix illustration, the upstream matrix and its inverse are:

$$A_u = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{f_{A \rightarrow B,t}}{P_{A,t}} & 1 & 0 \\ -\frac{f_{A \rightarrow C,t}}{P_{A,t}} & 0 & 1 \end{pmatrix}$$

$$A_u^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{f_{A \rightarrow B,t}}{P_{A,t}} & 1 & 0 \\ \frac{f_{A \rightarrow C,t}}{P_{A,t}} & 0 & 1 \end{pmatrix}$$

For any transaction c injected at node A, Equation (4) gives the following parameterized contribution on line A–C:

$$F_{A \rightarrow C,t}^c = \frac{|f_{A \rightarrow C,t}|}{P_{A,t}} [A_u^{-1}]_{A,A} P_{Gc,t} = \frac{|f_{A \rightarrow C,t}| P_{Gc,t}}{P_{A,t}} \quad (13)$$

The calculation is therefore transparent at the transaction level. The validated T3 injection at A is $P_{G,T3} = 32.40$ MW. Because $[A_u^{-1}]_{A,A} = 1$, Equation (4) gives:

$$F_{A \rightarrow C}^{T3} = \frac{55}{56.80} \times 1 \times 32.40 = 31.37 \text{ MW}$$

Figure 1 summarizes the research strategy and implementation sequence. For each period in the horizon, the joint DC-OPF provides dispatch, LMPs, and line flows. The allocation layer identifies the constrained-line set using Equation (7), traces transaction-level contributions using Equations (3)–(6), forms the physical responsibility term using Equation (8), applies the PBITR priority adjustment in Equation (9), and distributes the two settlement pools over their fund-specific responsibility sets using Equations (12) and (13). Tracing requires one upstream and one downstream sparse-matrix solve per period, which is negligible relative to the market-clearing solve. The representative-period AC procedure then repeats the tracing and weighting calculations on AC-OPF outputs and reports line-set, ranking, and share consistency.

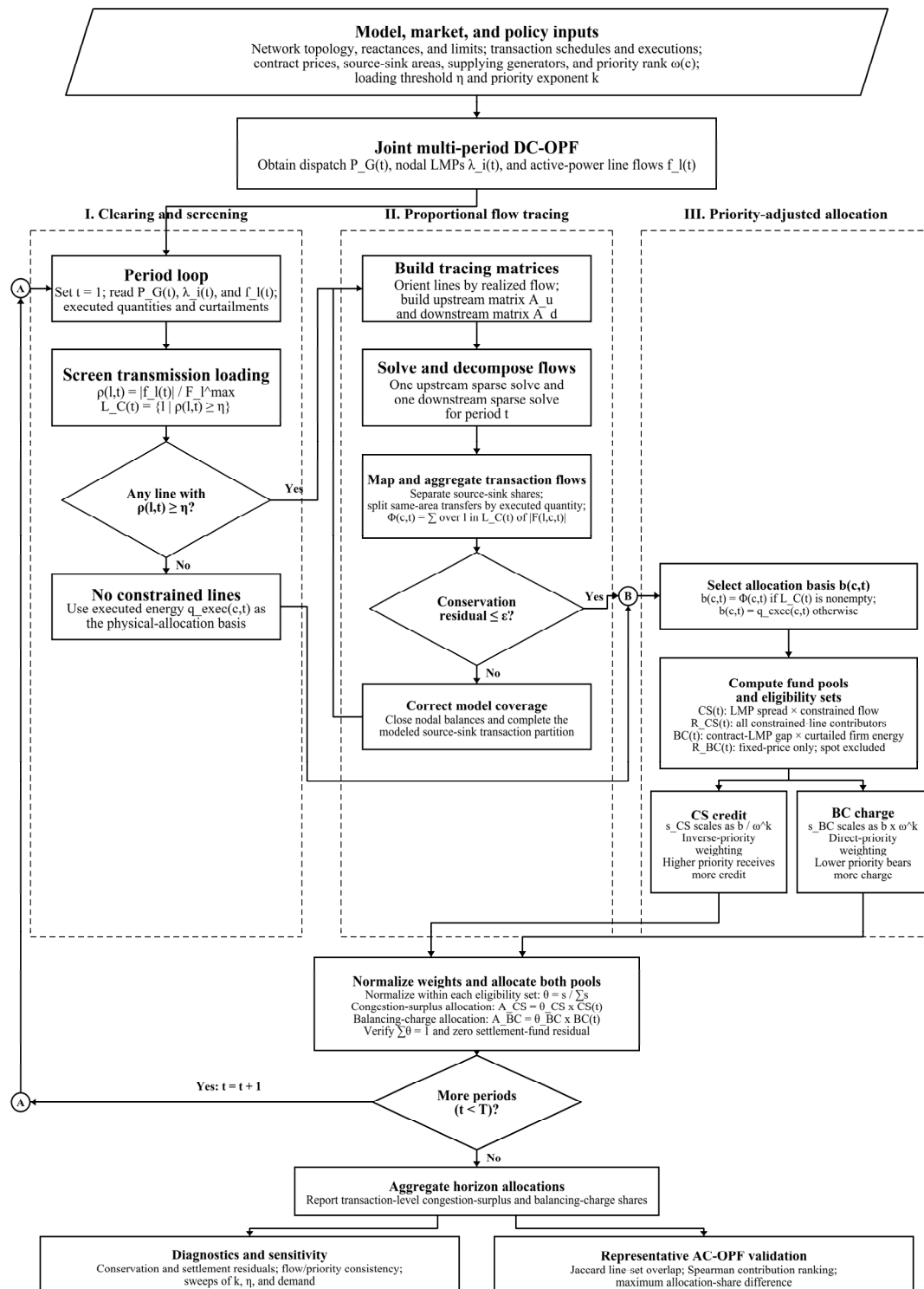


Figure 1. Research strategy and implementation steps of the proposed allocation framework.

3. Results

3.1. Test System and Transactions

The framework is validated on a modified IEEE 30-bus system partitioned into three zones: A is generation-rich and exports, while B and C are net importers, which is displayed in Figure 2. The system has 41 branches and seven dispatchable generators, including a low-cost hydro unit at bus 11, and is cleared over a 24-period daily horizon. Four settlement components are modeled: priority generation on A-to-B, an MLT contract on A-to-B, an

MLT contract on A-to-C, and a spot increment on A-to-C. And the settlement result is shown in Table 2.

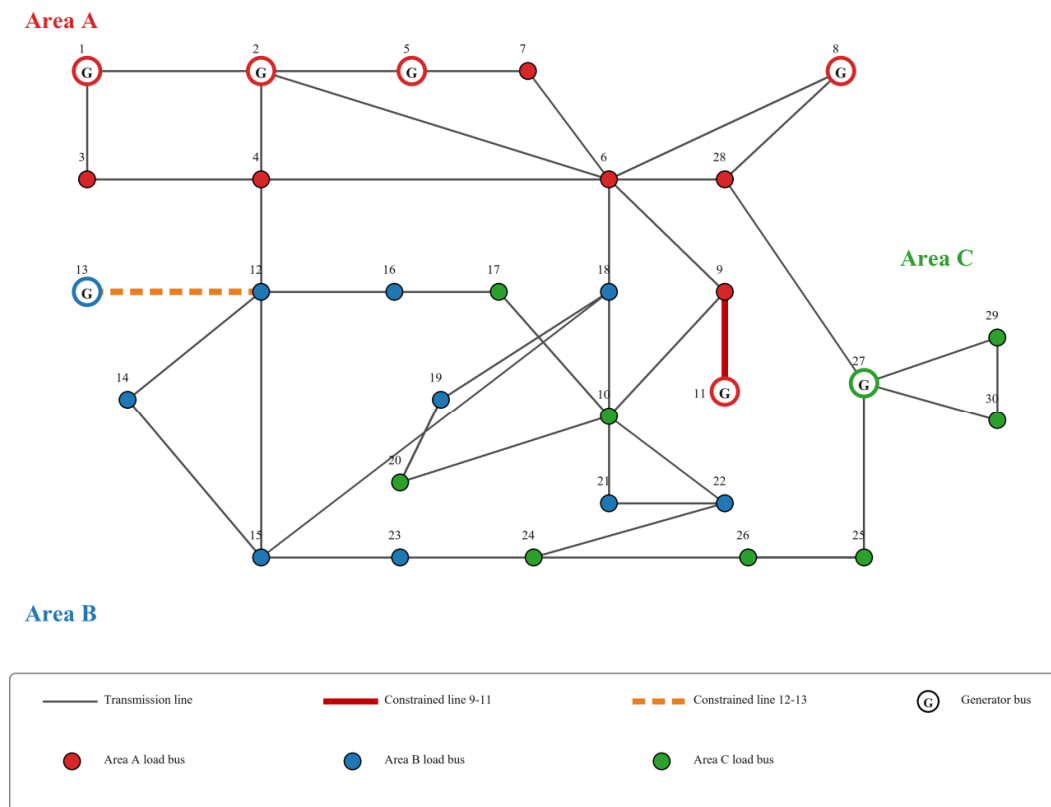


Figure 2. Zonal one-line diagram of the modified IEEE 30-bus system.

Table 2. Daily scheduled and executed energy of the four settlement components (MWh).

Settlement Component	Priority ω	Scheduled (MWh)	Executed (MWh)	Exec. Rate	Curtailed (MWh)
Priority generation	1	1320.0	1320.0	100.0%	0.0
MLT contract A $\rightarrow$ B	2	1540.0	1337.3	86.8%	202.7
MLT contract A $\rightarrow$ C	2	880.0	764.2	86.8%	115.8
Spot increment A $\rightarrow$ C	3	440.0	152.7	34.7%	287.3
Total	-	4180.0	3574.2	85.5%	605.8

Note: $\omega = 1$ denotes the highest priority. The spot increment is curtailed first under the priority order.

3.2. Market Clearing and Congestion Identification

Solving the joint 24-period DC-OPF reproduces the intended congestion pattern. With $\eta = 0.98$, line 9–11 is binding in all 24 periods and line 12–13 is binding in periods 9–22. At $\eta = 0.95$, line 12–13 also enters in periods 8 and 23. No other branch reaches 90% of its rating. Figure 3 shows the 24-period execution rates of the settlement components, while Figure 4 presents the zonal LMP profiles. The receiving zones price at or above the sending zone, and the price separation is concentrated in the A-to-C direction; Table 3 summarizes the average zonal LMPs and interzonal spreads by period block.

The identified congestion is not an artifact of a single loading threshold. Sweeping η from 0.90 to 1.00 leaves the same two dominant constrained lines: line 9–11 remains binding in every period, while line 12–13 remains binding throughout the central high-transfer block, with only the two edge periods changing when η is raised above 0.95. Table 4 summarizes the constrained-line identification across the tested loading thresholds.

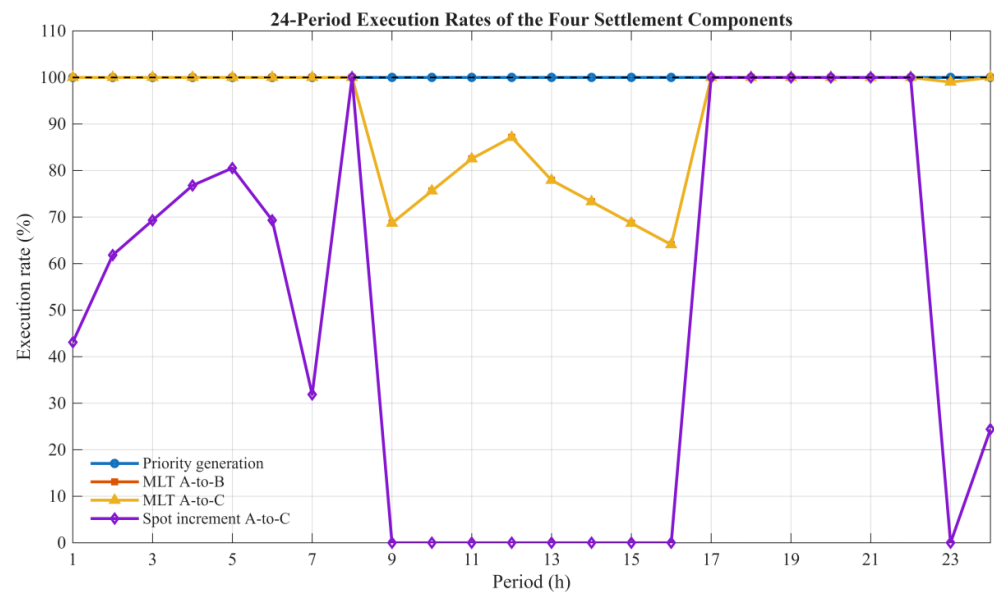


Figure 3. The 24-period execution rates of the four settlement components.

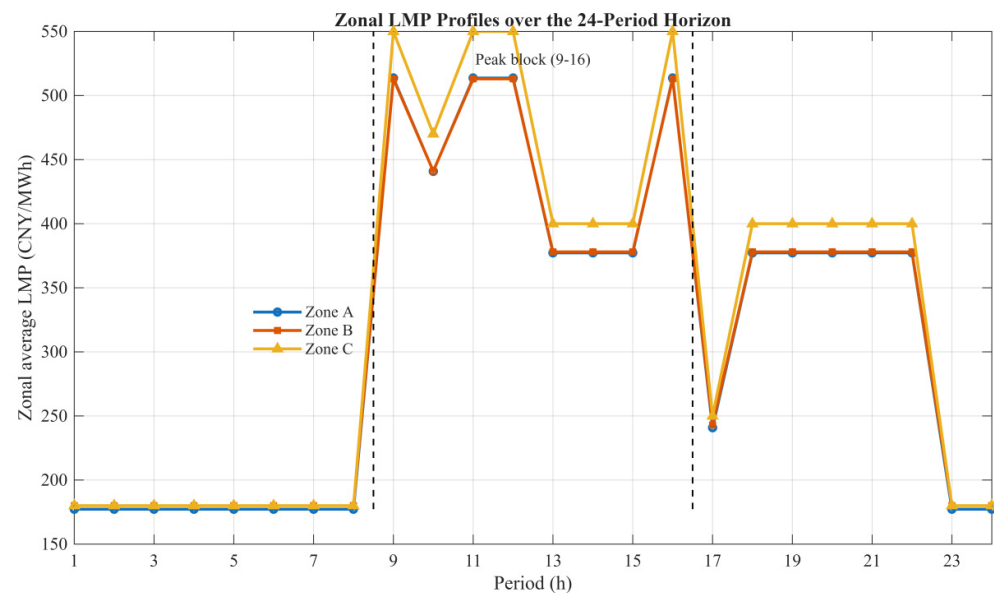


Figure 4. Zonal LMP profiles over the 24-period horizon.

Table 3. Average zonal LMPs and interzonal price spreads by period block (MWh).

Period Block	A	B	C	A-to-B Spread	A-to-C Spread
Peak (9–16)	453.4	453.4	483.8	approx. 0.0	30.3
Off-peak (1–8)	177.3	180.0	180.0	2.7	2.7
Flat (17–24)	310.2	311.6	326.3	1.4	16.0

Table 4. Robustness of constrained-line identification to the loading threshold η .

η	Line 9–11	Line 12–13	Other Lines	Allocation Implication
0.90	24 periods	16 periods (8–23)	none	same two-line dominant set
0.95	24 periods	16 periods (8–23)	none	same two-line dominant set
0.98	24 periods	14 periods (9–22)	none	base case
1.00	24 periods	14 periods (9–22)	none	same dominant set

3.3. Physical Flow Contribution

Proportional-sharing tracing shows that congestion responsibility is concentrated in the A-to-C direction. Over the day, the MLT A-to-C and spot A-to-C components account for 63.2% and 12.9% of traced contribution on the constrained lines, respectively, although their scheduled-energy shares are only 21.1% and 10.5%. Conversely, the two A-to-B components account for 24.0% of traced contribution despite representing 68.4% of scheduled energy. This result is obtained from realized-flow decomposition and not from a marginal PTDF approximation. Figure 5 compares these traced contributions with the corresponding scheduled-energy shares.

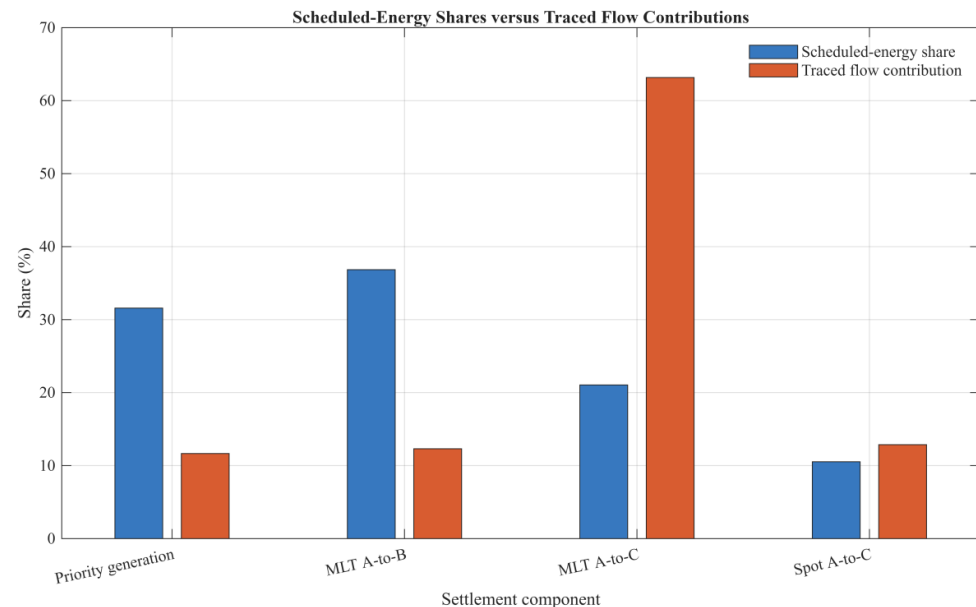


Figure 5. Comparison of each settlement component’s scheduled-energy share with its traced flow contribution on the constrained lines.

This reverses the ranking implied by traded energy: the MLT A-to-C transaction is only the third largest by scheduled volume but is the dominant constrained-line user. Therefore, energy-proportional allocation understates its physical congestion role and overstates the role of the high-volume A-to-B contract.

3.4. Allocation of the Congestion Surplus

The daily congestion-surplus pool totals 19,380 CNY. It is allocated under energy-proportional allocation, pure flow tracing, and the proposed method. All three sum exactly to the pool. Table 5 reports the resulting allocation shares and monetary amounts.

Table 5. Allocation of the congestion surplus.

Settlement Component	Energy-Prop.	Flow Tracing	Proposed	Proposed (CNY)
Priority generation	31.6%	12.0%	22.1%	4279.7
MLT contract A → B	36.8%	12.0%	11.1%	2144.3
MLT contract A → C	21.1%	66.3%	60.6%	11,742.9
Spot increment A → C	10.5%	9.7%	6.3%	1212.9
Total	100%	100%	100%	19,379.8

Energy-proportional allocation gives the largest shares to the A-to-B components because of their volume, even though their combined traced contribution is much smaller than that of the A-to-C components. Pure flow tracing shifts 76.0% of the surplus to A-to-C

transactions. The proposed method retains that physical dominance while applying the priority rule: the higher-priority MLT A-to-C transaction receives 60.6%, whereas the lower-priority spot increment receives 6.3%. Figure 6 visualizes the congestion-surplus allocation under the three methods.

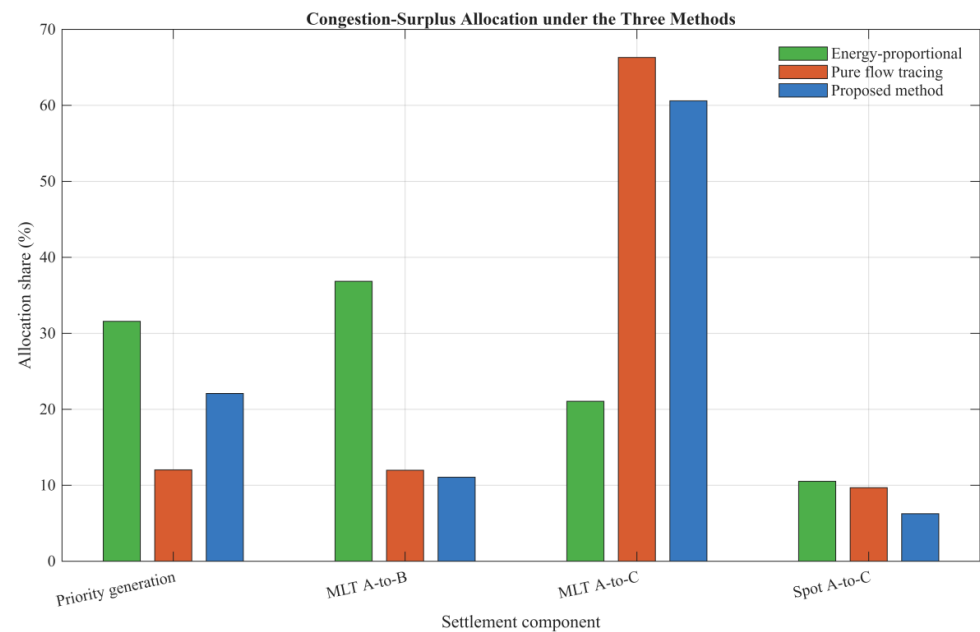


Figure 6. Congestion-surplus allocation under the three methods, shown as grouped bars by settlement component.

3.5. Allocation of the Balancing Congestion Charge

The balancing congestion charge pool totals 42,492 CNY. The spot increment settles at the LMP and has no contract price, so it is excluded from the balancing-congestion responsibility set. Table 6 reports the charge inputs, and Table 7 compares the allocations under the three methods.

Table 6. Inputs to the balancing congestion charge.

Settlement Component	Contract Price (CNY/MWh)	Curtailed (MWh)	Raw Charge Input (CNY)
Priority generation	230	0.0	0
MLT A-to-B	280	202.7	34,540.4
MLT A-to-C	420	115.8	7951.8
Spot A-to-C	LMP	287.3	not applicable

Table 7. Allocation of the balancing congestion charge.

Settlement Component	Energy-Prop.	Flow Tracing	Proposed	Proposed (CNY)
Priority generation	35.3%	13.1%	7.0%	2975.2
MLT A-to-B	41.2%	11.0%	11.8%	5005.7
MLT A-to-C	23.5%	75.9%	81.2%	34,511.3
Spot A-to-C	0.0%	0.0%	0.0%	0.0
Total	100%	100%	100%	42,492.2

Two features are important. First, all three methods apply the same balancing-charge responsibility set, so the LMP-settled spot increment receives zero under every method; the comparison therefore isolates the effect of the allocation basis rather than participant eligibility. Second, energy proportionality assigns 76.5% of the charge to A-to-B transactions

because of their contract volumes, whereas traced responsibility shows that A-to-C is the dominant user of the constrained corridor. The proposed method assigns 81.2% to the MLT A-to-C transaction and 11.8% to the MLT A-to-B transaction, combining congestion-causer-pays logic with the declared priority order. Figure 7 visualizes the balancing-congestion-charge allocation under the three methods.

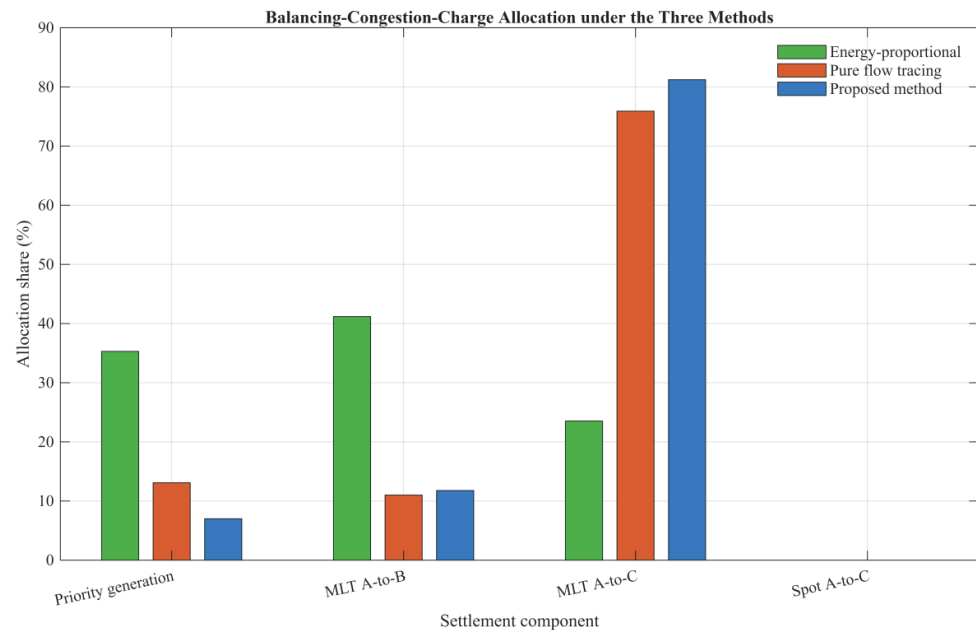


Figure 7. Balancing-congestion-charge allocation under the three methods.

3.6. Quantitative Performance and Sensitivity

The proposed method is the only one that is both strongly flow-consistent and priority-consistent. Energy-proportional allocation is negatively correlated with traced constrained-line use, pure flow tracing is almost perfectly flow-consistent but priority-neutral, and the proposed method retains a 0.96 flow-consistency coefficient while separating the higher-priority MLT transaction from the lower-priority spot increment. Table 8 reports the quantitative performance indices, and Table 9 reports the sensitivity of congestion-surplus shares to the priority exponent k .

Table 8. Quantitative performance indices for the congestion-surplus allocation.

Index	Energy-Prop.	Flow Tracing	Proposed
Allocation deviation (pp)	0.0	22.6	19.8
Flow consistency	−0.24	+1.00	+0.96
Priority consistency on A-to-C set	inconsistent	neutral	consistent
Settlement residual (CNY)	0	0	0

Table 9. Sensitivity of congestion-surplus shares (%) to the priority exponent k .

k	Priority Gen.	MLT A-to-B	MLT A-to-C	Spot A-to-C
0.0	12.0	12.0	66.3	9.7
0.5	16.5	11.6	64.0	7.9
1.0	22.1	11.1	60.6	6.3
1.5	28.9	10.3	56.0	4.8
2.0	36.8	9.3	50.4	3.6

The k-sweep shifts shares smoothly and monotonically: increasing k strengthens the claim of priority generation and progressively reduces the spot share, while the MLT A-to-C transaction remains the largest recipient throughout the tested range. The threshold sweep in Section 3.2 confirms that the dominant constrained corridor is stable.

3.7. Representative-Period AC Consistency Validation

The representative periods $t = 8$, $t = 12$, and $t = 20$ were recalculated by AC-OPF using the same transaction quantities, line ratings, and fund-specific responsibility sets. All three AC-OPF cases converged. Table 10 compares voltage ranges, constrained-line sets, transaction contribution rankings, and allocation shares with the DC results.

Table 10. Representative-period DC-AC consistency results.

Condition	Period	AC Voltage Range (p.u.)	Jaccard 0.98	Jaccard 0.95	Spearman	Same Top Transaction	Max CS Share Diff. (pp)	Max BCF Share Diff. (pp)
Off-peak	8	0.9756–1.0600	0.50	1.00	1.00	yes	0.12	0.14
Peak	12	0.9400–1.0600	1.00	1.00	1.00	yes	1.01	0.72
Flat	20	0.9490–1.0600	1.00	1.00	1.00	yes	0.05	0.05

At the 0.95 loading threshold, the DC and AC constrained-line sets are identical in all three periods. At the stricter 0.98 threshold, the peak and flat sets are identical, while the off-peak Jaccard index is 0.50 because line 12–13 is loaded to 95.1% in the DC active-power model but crosses the 0.98 threshold when AC apparent power is used. The dominant two-line corridor is therefore unchanged; the difference is a threshold-edge effect caused by reactive flow and losses.

The Jaccard index measures the overlap between the DC- and AC-identified constrained-line sets, with 1 indicating identical sets, whereas Spearman’s rank correlation measures the agreement between the corresponding transaction-contribution rankings, with 1 indicating identical rankings. The transaction contribution ranking is identical in every period, with a Spearman coefficient of 1.00 and the same leading transaction. The largest difference between DC- and AC-based shares is 1.01 percentage points for congestion surplus and 0.72 percentage points for balancing congestion charge. These results support the use of the DC model for the present ex-post allocation experiment while preserving the boundary that no dynamic or contingency-security conclusion is implied.

3.8. Numerical Conservation Residuals

As a direct numerical check of Proposition 1, the transaction contributions were summed on the two dominant lines included in the representative-period contribution basis for the off-peak, peak, and flat conditions. Across the six line-period checks, the largest absolute tracing residual is 1.05×10^{-6} MW in the flat period, while the off-peak and peak residuals remain below 4.27×10^{-9} MW. The contribution sums therefore reproduce the realized DC line flows at the reported precision. Table 11 reports all six representative line-period conservation checks.

Table 11. Numerical conservation checks for representative periods and dominant lines.

Period	Line	Line Flow (MW)	Sum of Transaction Contributions (MW)	Residual (MW)
8 (off-peak)	9–11	100.00	100.00	$<1.85 \times 10^{-9}$
8 (off-peak)	12–13	114.16	114.16	$<1.85 \times 10^{-9}$
12 (peak)	9–11	100.00	100.00	$<4.27 \times 10^{-9}$
12 (peak)	12–13	120.00	120.00	$<4.27 \times 10^{-9}$

Table 11. *Cont.*

Period	Line	Line Flow (MW)	Sum of Transaction Contributions (MW)	Residual (MW)
20 (flat)	9–11	100.00	100.00	$<1.05 \times 10^{-6}$
20 (flat)	12–13	120.00	120.00	$<1.05 \times 10^{-6}$

4. Discussion

The numerical results reveal the magnitude of the bias introduced by energy-proportional settlement. The two A-to-B transactions account for 68.4% of scheduled energy but only 24.0% of traced contribution to constrained lines. Conversely, the MLT A-to-C transaction accounts for only 21.1% of scheduled energy but 63.2% of traced constrained-line use. This 42.1-percentage-point gap between its scheduled-energy share and physical contribution explains why transaction volume alone is a poor proxy for congestion responsibility in the studied network.

The monetary effects are correspondingly material. For the CNY 19,379.8 congestion-surplus pool, the proposed method assigns 60.6% to the MLT A-to-C transaction, compared with 21.1% under energy proportionality, an increase of 39.5 percentage points and an allocation of CNY 11,742.9. Relative to pure flow tracing, the priority adjustment reduces the lower-priority spot A-to-C share from 9.7% to 6.3% while increasing the protected priority-generation share from 12.0% to 22.1%. For the CNY 42,492.2 balancing congestion charge, the MLT A-to-C share rises from 23.5% under energy proportionality to 81.2% under the proposed method, a 57.7-percentage-point shift corresponding to CNY 34,511.3. The LMP-settled spot increment remains at zero because it has no contract-price-gap exposure, and the settlement residual is zero under all three methods.

The stronger priority signal does not eliminate physical consistency. The proposed allocation retains a flow-consistency coefficient of 0.96, compared with -0.24 for energy proportionality and 1.00 for pure flow tracing. As the priority exponent k increases from 0 to 2, the priority-generation share rises smoothly from 12.0% to 36.8% and the spot A-to-C share falls from 9.7% to 3.6%, while the MLT A-to-C transaction remains the largest recipient throughout, with a share between 50.4% and 66.3%. The same two dominant constrained lines persist for loading thresholds from 0.90 to 1.00. In the representative AC-OPF checks, the 0.95-threshold Jaccard index and the Spearman rank correlation are both 1.00 in all three periods; the maximum DC-AC share differences are 1.01 percentage points for congestion surplus and 0.72 percentage points for the balancing charge, and the largest tracing residual is only 1.05×10^{-6} MW. These results show that the principal allocation ranking and economic interpretation are numerically robust for the tested system.

The incremental contribution is therefore a settlement layer for the pre-rights stage of market reform, rather than a new OPF model or a transmission-right auction. It replaces volume-based sharing with a decomposable physical-responsibility term, adds the institutional curtailment priority that pure flow tracing omits, and separates the responsibility sets for congestion surplus and the balancing congestion charge. This decomposition gives the operator a reproducible explanation for each credit or debit and may also inform the later design of explicit FTRs/PTRs. Nevertheless, the numerical findings remain case-specific: the modified IEEE 30-bus system represents one topology, and the AC validation covers three representative periods rather than a full-day AC security-constrained co-optimization. Validation with real regional-grid data, contingency constraints, and alternative network topologies is still required before universal claims can be made. Because k is a regulatory design parameter rather than an empirically estimated quantity, future work should also

examine participant incentives, strategic scheduling, and the transition from PBITRs to explicit FTR/PTR instruments.

5. Conclusions

This paper developed an ex-post allocation framework for congestion-related settlement imbalance funds in transitional electricity markets that clear energy with LMPs but have not introduced tradable transmission rights. The central insight is that the missing right can be represented operationally as a priority-based implicit transmission right: a de facto claim on scarce corridor capacity embedded in dispatch and contract-execution rules.

By multiplying traced physical contribution by a priority-based weight within fund-specific responsibility sets, the method distinguishes between transactions that merely have large volume and transactions that actually load the constrained corridor. In the modified IEEE 30-bus case, the MLT A-to-C transaction represents 21.1% of scheduled energy but receives 60.6% of the congestion surplus and bears 81.2% of the balancing congestion charge under the proposed rule. The pure spot increment is excluded from the contract-price-gap charge, and all allocations preserve settlement balance.

The practical insight is that transitional markets do not need to rely on energy-proportional sharing while waiting for a mature transmission-right market. A transparent physical-and-institutional allocation can improve the fairness and explainability of residual fund settlement now, and the same measurements can later support explicit-right design. Representative AC-OPF checks preserve the constrained-corridor interpretation and transaction ranking, with maximum allocation-share deviations below 1.1 percentage points. Future work should extend the validation to full-day AC security-constrained market results, real regional grids, and incentive analysis under alternative priority parameters.

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Nomenclature

Symbol/Abbreviation	Description
$F_{l,t}$	Active power flow on line l in period t (MW)
$F_{l,t}^g$	Component of the flow on line l attributable to generator g
$F_{l,t}^c$	Component of the flow on line l attributable to transaction c
$F_l^{\max}$	Thermal capacity (rating) of line l (MW)
$P_{g,t}$	Total active output of generator g in period t (MW)
$P_{g,t}^{\text{gen}}$	Injection of generator g used in tracing (MW)
$P_{j,t}^{\text{thr}}$	Through-flow (sum of inflows) at node j in period t (MW)
A_u	Upstream distribution matrix (proportional-sharing)
$q_{c,t}^g$	Output of generator g allocated to transaction c (MW)
L^c	Set of congested (constrained) lines
η	Loading threshold defining a congested line (p.u.)

$\varphi_{c,t}$	Flow contribution of transaction c on congested lines (MW)
ω_c	Priority rank of transaction c (1 = highest priority)
$\theta_c^+ \theta_c^-$	Priority weights for surplus credit/charge debit
LMP_i	Locational marginal price at node i (CNY/MWh)
$Q_{c,t}^{sch}, E_{c,t}$	Scheduled and executed energy of transaction c (MWh)
$P_t^{spot}, P_c^{contract}$	Spot price and contract price (CNY/MWh)
CS, BC	Congestion surplus and balancing congestion charge pools (CNY)
$S_{c,t}^{CS}, S_{c,t}^{BC}$	Amounts of CS/BC allocated to transaction c (CNY)
FTR/PTR	Financial/physical transmission right
PBTR	Priority-based implicit transmission rights
LMP	Locational marginal price
MLT	Medium- and long-term (contract)
SCED	Security-constrained economic dispatch
DC-OPF	DC optimal power flow

Appendix A. Multi-Period DC Optimal Power Flow

The market is cleared by a security-constrained DC-OPF co-optimized over the daily horizon $T = \{1, \dots, 24\}$. Unlike a transport model that imposes only nodal balance and line-capacity bounds, the formulation explicitly links every branch flow to bus voltage angles. The variables are generator outputs $P_{g,t}$ and voltage angles $\theta_{n,t}$. A compact formulation is:

$$\min \sum_{t \in T} \sum_{g \in \mathcal{G}} P_{g,t} \quad (A1)$$

$$(B\theta_t)_n = \sum_{g \in \mathcal{G}_n} P_{g,t} - D_{n,t} \quad (A2)$$

$$f_{l,t} = b_l \left(\theta_{i(l),t} - \theta_{j(l),t} \right), -F_l^{max} \leq f_{l,t} \leq F_l^{max} \quad (A3)$$

$$P_g^{min} \leq P_{g,t} \leq P_g^{max}, \theta_{ref,t} = 0 \quad (A4)$$

$$-R_g \leq P_{g,t} - P_{g,t-1} \leq R_g, t = 2, \dots, 24 \quad (A5)$$

The LMP at node n and period t is the dual variable of the nodal active-power balance. Because the ramping constraints couple periods, the horizon is solved as one linear program rather than as 24 independent snapshots.

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