

Article

Fault Section Location Based on Transient Zero-Sequence Current Waveform Dissimilarity for Single-Phase Grounding Faults in Distribution Networks

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Abstract

A fault-section location method based on waveform-difference analysis of transient zero-sequence currents is proposed for distribution networks with small-current grounding systems. The method addresses weak single-phase-to-ground fault-current amplitudes, indistinct fault features, and limited adaptability of steady-state-quantity-based location criteria under arc-suppression-coil compensation. Steady-state and transient characteristics of single-phase-to-ground faults were analyzed in neutral-point ungrounded systems and arc-suppression-coil-grounded systems. Transient zero-sequence currents exhibited more pronounced amplitude, polarity, and waveform differences than steady-state zero-sequence currents, with lower sensitivity to compensation effects. These characteristics provide effective features for fault-section discrimination. The discrete Fréchet distance was used to quantify the overall waveform difference between transient zero-sequence currents at both ends of each line section. The coefficient of variation was introduced to characterize the dispersion of waveform differences among different sections. A fault-section location criterion was then constructed. The location performance was verified through PSCAD/EMTDC–MATLAB co-simulation under different neutral-point grounding modes, transition resistances, and fault inception angles. The results show that the proposed method accurately identifies fault sections and maintains good adaptability under both distinct and weak fault-feature conditions. Compared with the correlation-coefficient method, grey relational analysis, and empirical mode decomposition, the proposed method more comprehensively characterizes waveform differences in transient zero-sequence currents at both ends of a line section. It shows good section-discrimination capability under the representative fault conditions considered in this study.



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Keywords: distribution network; single-phase grounding fault; fault section location; transient zero-sequence current; discrete Fréchet distance; coefficient of variation

1. Introduction

The distribution network is a critical link between the power system and end users. Its fault-location capability directly affects fault-isolation speed, service-restoration efficiency,

and system-operating reliability [1–3]. In medium-voltage distribution networks, single-phase-to-ground faults occur relatively frequently and constitute one of the main fault types affecting safe network operation [4–6]. In small-current grounding systems with an ungrounded neutral point or arc-suppression-coil grounding, the line voltages can remain approximately symmetrical for a certain period after a single-phase-to-ground fault. The fault-current amplitude is relatively small, and the fault features are less pronounced than those of short-circuit faults [7]. If the fault is not identified and isolated in time, it may develop into an interphase short circuit, insulation breakdown, or a wider power outage [8]. Rapid and accurate identification of the fault section is essential for reducing the inspection scope, improving fault-handling efficiency, and ensuring safe operation of the distribution network [9,10].

Compared with transmission networks, distribution networks are characterized by numerous branch lines, complex topologies, non-uniform line parameters, and frequent changes in operating modes [3,11–13]. The characteristics of single-phase-to-ground faults are also affected by neutral-point grounding mode, transition resistance, fault inception angle, line distributed capacitance, and noise interference [1,5,14–16]. In arc-suppression-coil-grounded systems, the compensation current reduces the steady-state zero-sequence current difference between faulty and healthy lines. Conventional location criteria based on steady-state power-frequency quantities may suffer from insufficient sensitivity and limited adaptability [17,18].

Existing methods for locating single-phase-to-ground fault sections mainly include steady-state-electrical-quantity-based methods, transient-electrical-quantity-based methods, and artificial-intelligence-based methods. Steady-state-electrical-quantity-based methods usually construct location criteria using zero-sequence current, zero-sequence voltage, phase-current variation, equivalent admittance, or phase-angle differences before and after fault occurrence. Chen et al. proposed a single-phase-to-ground fault-location method based on the equivalent-admittance distortion rate in 2021. The method locates faults by measuring equivalent-admittance variations in each phase, feeder, and line section before and after fault occurrence, improving the location performance for high-resistance grounding faults [19]. In 2024, Su et al. analyzed single-phase-to-ground faults in active distribution networks with arc-suppression-coil grounding. Fault-line identification was achieved by comparing the ratio between the feeder zero-sequence current and the neutral-point zero-sequence current and enhancing fault features through grounding-mode adjustment [4]. These methods have clear theoretical principles and low engineering implementation complexity. Their performance still depends mainly on steady-state or quasi-steady-state electrical-quantity variations. When the fault current is weak, the transition resistance is high, or the compensation effect is strong, steady-state feature differences may become indistinct, reducing location reliability.

Transient-electrical-quantity-based methods mainly use zero-sequence current, zero-sequence voltage, and their time–frequency characteristics during the initial fault stage for section discrimination. In 2020, Wan et al. proposed a single-phase-to-ground fault-section location method based on synchronous transient energy analysis. The fault feeder section was identified by analyzing the energy distribution and polarity relationship of transient zero-sequence currents within a specific frequency band. The effectiveness of the method was verified through simulation and field-test results [6]. In 2021, Li et al. proposed a fault-section location method based on amplitude characteristics and an intelligent distance algorithm, addressing the effects of neutral-point grounding mode, transition resistance, and location blind zones on section location. The fault section was determined by comparing the matching degree of transient zero-sequence current amplitudes at both ends of each section. The method showed good adaptability under different grounding

modes and transition-resistance conditions [20]. Some studies have also employed wavelet decomposition, Hausdorff distance, variational mode decomposition, energy features, and waveform-matching methods to extract transient fault information. Transient-electrical-quantity-based methods can exploit richer electrical information during the initial fault stage and are less affected by arc-suppression-coil compensation. Existing methods still mainly rely on amplitude, polarity, energy, or local time–frequency features. The overall morphological differences between transient waveforms at both ends of a line section remain insufficiently utilized.

More specifically, existing transient waveform matching methods usually evaluate the similarity between fault records using scalar similarity indices or locally extracted features. Hausdorff-distance-based methods measure the maximum deviation between two point sets, but they do not strictly preserve the temporal order of waveform evolution and may be sensitive to isolated extreme points. Dynamic time warping (DTW) emphasizes elastic alignment along the time axis and is effective for sequences with temporal shifts or unequal local stretching; however, excessive temporal warping may weaken the physical correspondence between simultaneously measured transient currents at both ends of a line section. Grey relational analysis mainly evaluates the relational degree between sequences after normalization, but its discrimination capability depends on the selected resolution coefficient and may decrease when different sections have close waveform trends. Energy-based methods compress transient information into energy indicators within selected time or frequency bands, which may lose polarity and detailed waveform-shape information. Traveling-wave-based methods rely mainly on high-frequency wavefront arrival-time differences and require high sampling rates and accurate time synchronization. In contrast, the proposed method treats the transient zero-sequence currents measured at both ends of each section as ordered waveform curves and uses the discrete Fréchet distance to evaluate their overall morphological dissimilarity while preserving point-order constraints. Therefore, the proposed method focuses on section-level waveform discrepancy caused by the spatial distribution of transient zero-sequence currents, rather than only on local amplitude, energy, arrival-time, or scalar similarity features.

In recent years, artificial-intelligence-based methods have also been introduced into single-phase-to-ground fault identification and location. In 2024, Wang et al. proposed a fault-section location method based on improved empirical wavelet transform and graph isomorphism network to address indistinct zero-sequence current features and noise susceptibility [11]. These methods can adaptively extract complex fault features and offer advantages in non-linear modeling and multi-condition adaptability. Artificial-intelligence-based methods usually depend on sample size, training-condition coverage, and data quality. In practical distribution networks, real fault samples are limited, and feeder topologies and operating conditions vary considerably. The generalization capability and physical interpretability of these models still require further improvement.

Existing studies have advanced single-phase-to-ground fault-location techniques from the perspectives of steady-state quantities, transient quantities, and data-driven modeling. Several limitations remain. Steady-state-quantity-based methods are susceptible to arc-suppression-coil compensation, transition resistance, and changes in operating mode. Some transient-quantity-based methods mainly focus on amplitude, polarity, energy, or local time–frequency features. Their ability to characterize the overall morphological differences between transient zero-sequence current waveforms at both ends of a line section remains insufficient. Artificial-intelligence-based methods have strong feature-learning capabilities. Their performance depends heavily on sample quality and training-condition coverage, and their physical interpretability remains limited. Transient zero-sequence currents at both ends of a faulty section usually exhibit pronounced differences in amplitude, frequency

components, polarity, and waveform shape. Waveforms at both ends of healthy sections are more similar. Fault-section location can be formulated as a transient-waveform difference measurement problem between adjacent measurement points. This formulation better exploits the spatial distribution information of transient zero-sequence currents.

This paper proposes a single-phase-to-ground fault-section location method for distribution networks based on the waveform-difference degree of transient zero-sequence currents. Transient zero-sequence current characteristics of single-phase-to-ground faults were analyzed in neutral-point ungrounded systems and arc-suppression-coil-grounded systems. Transient zero-sequence current was selected as the location feature because it contains richer fault information and is less affected by compensation effects. The proposed method uses the discrete Fréchet distance to quantify the overall difference between transient zero-sequence current waveforms at both ends of each section. The coefficient of variation is introduced to characterize the dispersion of waveform-difference indicators among sections. A location criterion applicable to both terminal and non-terminal fault sections is then constructed. Simulation analyses were performed under different neutral-point grounding modes, transition resistances, fault inception angles, end-zone fault conditions, and noise conditions. The proposed method was compared with the correlation-coefficient method, grey relational analysis, and empirical mode decomposition to verify its effectiveness. The novelty of this study lies not in using a general curve-distance metric alone, but in constructing a physically interpretable section-location criterion from the spatial distribution characteristics of transient zero-sequence currents. The faulty section is identified by measuring the ordered waveform dissimilarity between adjacent measurement points, and the coefficient of variation is further introduced to evaluate the dispersion of section-level dissimilarity indicators. This design enables the method to distinguish between non-terminal and terminal fault sections within a unified criterion. The main contributions of this study are summarized as follows.

1. A fault-section location framework based on ordered waveform dissimilarity of transient zero-sequence currents is proposed. The method uses the spatial distribution differences of transient zero-sequence currents at both ends of line sections and transforms fault-section location into a section-level waveform-dissimilarity measurement problem.
2. A section identification criterion combining the discrete Fréchet distance and the coefficient of variation is constructed. The discrete Fréchet distance is used to quantify the overall ordered morphological difference between the transient zero-sequence current waveforms at both ends of each section, while the coefficient of variation is used to evaluate the dispersion of the section-level dissimilarity indicators.
3. The adaptability of the proposed method was verified through simulations under representative fault conditions. The tested cases include different neutral-point grounding modes, transition resistances, fault inception angles, terminal-section faults, and noise levels. Compared with the correlation-coefficient method, grey relational analysis, and empirical mode decomposition, the proposed method provides better section-discrimination capability within the tested simulation cases.

2. Materials and Methods

2.1. Distribution Network Model and Fault Scenario

2.1.1. Neutral-Point Ungrounded System

In a neutral-point ungrounded small-current grounding system, the single-phase-to-ground fault current is mainly the phase-to-ground capacitive current of the network and is usually relatively small. In some engineering guidelines and studies on non-effective grounding systems, 10 A is often used as a reference limit for selecting the neutral grounding

mode [21]. Under normal operating conditions, the three-phase voltages remain balanced, and the phase-to-ground capacitive currents remain symmetrical. Under symmetrical three-phase load and line-parameter conditions, no zero-sequence component exists in the system. The corresponding voltage and current phasor relationship is shown in Figure 1.

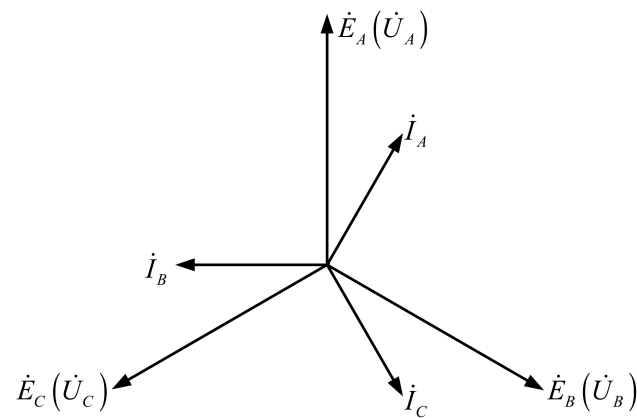


Figure 1. Phasor relationship between voltage and current under normal system operating conditions.

When the system is operating normally, the zero-sequence voltage ($\dot{U}_0$) and zero-sequence current ($\dot{I}_0$) can be expressed by Equation (1):

$$\begin{cases} \dot{U}_0 = \frac{1}{3}(\dot{U}_A + \dot{U}_B + \dot{U}_C) = 0 \\ \dot{I}_0 = \frac{1}{3}(\dot{I}_A + \dot{I}_B + \dot{I}_C) = 0 \end{cases} \quad (1)$$

When phase C of line L₃ experiences a grounding fault, the faulty-phase voltage decreases to zero, while the voltages of the two healthy phases increase to line-voltage levels. At this stage, zero-sequence components are generated in the system. Figure 2 shows the voltage and current phasor relationship under a phase-C grounding fault.

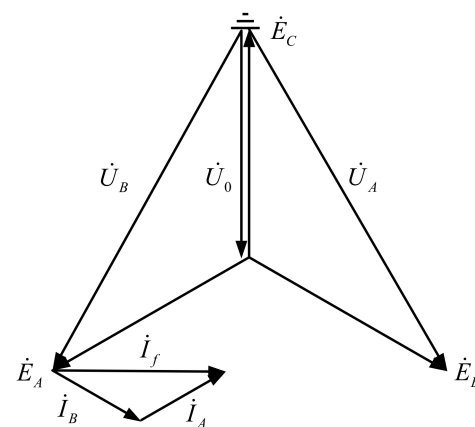


Figure 2. Phasor diagram of voltage and current under phase-C ground fault conditions.

At this stage, the voltages of phases A, B, and C can be expressed by Equation (2):

$$\begin{cases} \dot{U}_A = \dot{E}_A - \dot{E}_C = \sqrt{3}\dot{E}_C e^{-j150} \\ \dot{U}_B = \dot{E}_B - \dot{E}_C = \sqrt{3}\dot{E}_C e^{j150} \\ \dot{U}_C = 0 \end{cases} \quad (2)$$

According to the symmetrical component method, the zero-sequence voltage at the neutral point can be expressed by Equation (3):

$$\dot{U}_0 = \frac{1}{3}(\dot{U}_A + \dot{U}_B + \dot{U}_C) = -\dot{E}_C \quad (3)$$

When a fault occurs in the system, the capacitive currents of each line also change accordingly. Figure 3 shows the corresponding capacitive-current distribution.

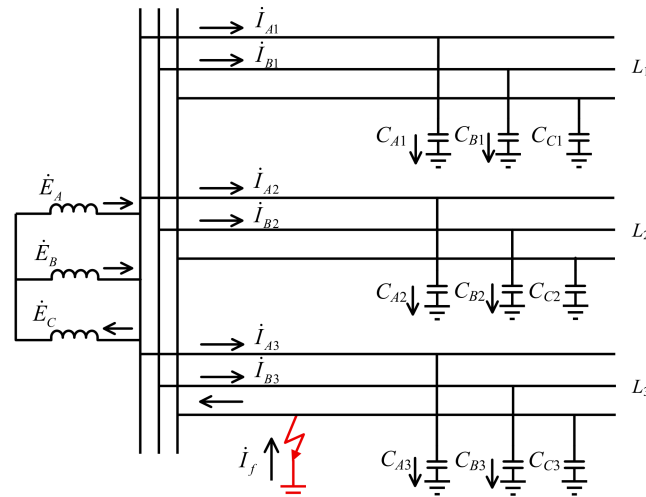


Figure 3. Capacitive-current distribution in ungrounded neutral system.

For healthy lines, taking line L_1 as an example, the phase-to-ground capacitive currents can be expressed by Equation (4):

$$\begin{cases} \dot{I}_{A1} = j\omega C_1 \dot{U}_A \\ \dot{I}_{B1} = j\omega C_1 \dot{U}_B \\ \dot{I}_{C1} = 0 \end{cases} \quad (4)$$

The zero-sequence currents detected at the beginning of lines L_1 and L_2 can be expressed by Equation (5):

$$\begin{cases} 3\dot{I}_{01} = \dot{I}_{A1} + \dot{I}_{B1} + \dot{I}_{C1} = j\omega C_1 (\dot{U}_A + \dot{U}_B) = j3\omega C_1 \dot{U}_0 \\ 3\dot{I}_{02} = \dot{I}_{A2} + \dot{I}_{B2} + \dot{I}_{C2} = j\omega C_2 (\dot{U}_A + \dot{U}_B) = j3\omega C_2 \dot{U}_0 \end{cases} \quad (5)$$

According to Equation (5), the zero-sequence current at the beginning of a healthy line has the same amplitude as the line-to-ground capacitive current of that line, and its direction is from the busbar side to the line side. The phase-to-ground capacitive currents of the faulty line can be expressed by Equation (6):

$$\begin{cases} \dot{I}_{A3} = j\omega C_3 \dot{U}_A \\ \dot{I}_{B3} = j\omega C_3 \dot{U}_B \\ \dot{I}_{C3} = -j3\omega U_0 (C_1 + C_2 + C_3) \end{cases} \quad (6)$$

For the faulty line L_3 , the zero-sequence current detected at its starting point can be expressed by Equation (7):

$$\begin{aligned} 3\dot{I}_{03} &= \dot{I}_{A3} + \dot{I}_{B3} + \dot{I}_{C3} \\ &= \dot{I}_{A3} + \dot{I}_{B3} - (3\dot{I}_{01} + 3\dot{I}_{02} + \dot{I}_{A3} + \dot{I}_{B3}) \\ &= -3(\dot{I}_{01} + \dot{I}_{02}) \end{aligned} \quad (7)$$

According to Equation (7), the zero-sequence current at the beginning of the faulty line has an amplitude equal to the sum of the zero-sequence currents at the beginnings of all healthy lines, and its direction is from the line side to the busbar side.

According to the current distribution relationship, the current flowing through the fault point is supplied by the phase-to-ground capacitive branches of the healthy phases. As expressed in Equation (8):

$$I_f = (\dot{I}_{A1} + \dot{I}_{B1}) + (\dot{I}_{A2} + \dot{I}_{B2}) + (\dot{I}_{A3} + \dot{I}_{B3}) = j3\omega C_{\Sigma} \dot{U}_0 \quad (8)$$

According to the above analysis, when a single-phase-to-ground fault occurs in an ungrounded system, the steady-state zero-sequence current differs markedly between the faulty line and healthy lines. Specifically, the faulty line has a higher zero-sequence current amplitude, and its current direction differs from those of the healthy lines. Based on this characteristic, fault location under this grounding mode can be achieved by comparing steady-state zero-sequence current amplitudes or determining current directions.

2.1.2. Neutral-Point Grounded System with Arc-Suppression Coil

After a single-phase-to-ground fault occurs, a large phase-to-ground capacitive current may sustain an arc at the fault point. This condition can induce intermittent arc overvoltage, posing a serious threat to system insulation and power-supply reliability. In engineering applications, an inductive coil is usually connected between the transformer neutral point and ground to inject inductive current into the system. This current is opposite in phase to the capacitive current, compensating for and suppressing the fault current and promoting rapid arc extinction. This grounding mode is referred to as arc-suppression-coil grounding.

Under normal operating conditions, the voltage and current distributions are similar to those in an ungrounded system. The neutral-point potential remains zero, and no zero-sequence component exists in any line. Figure 4 shows the capacitive-current distribution of each line after a single-phase-to-ground fault occurs in the system.

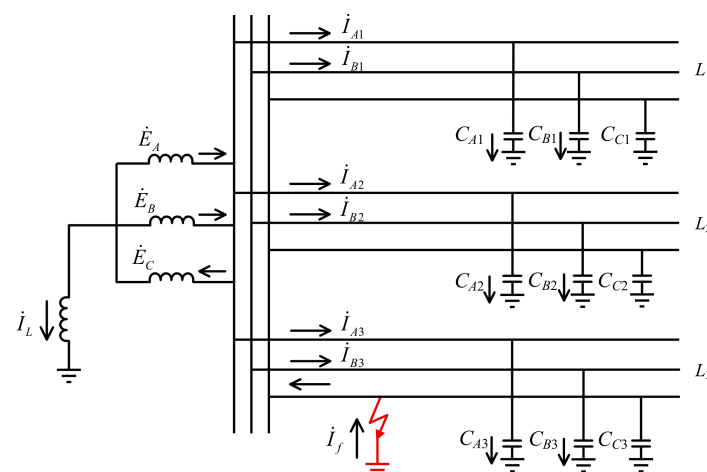


Figure 4. Capacitive-current distribution in arc-suppression-coil-grounded system.

When a single-phase-to-ground fault occurs in such a system, the arc-suppression coil provides an inductive current ($\dot{I}_L$) from the neutral point to the fault point. This current is opposite in phase to the capacitive current ($\dot{I}_{C\Sigma}$), reducing the ground-fault current amplitude ($\dot{I}_f$).

If a phase-C grounding fault occurs on line L_3 , the inductive current flowing through the arc-suppression coil can be expressed by Equation (9):

$$\dot{I}_L = \frac{\dot{U}_0}{j\omega L} = -\frac{\dot{E}_C}{j\omega L} \quad (9)$$

For the faulty line L_3 , the zero-sequence current detected at its starting point can be expressed by Equation (10):

$$\begin{aligned} 3\dot{I}_{03} &= \dot{I}_{A3} + \dot{I}_{B3} + \dot{I}_{C3} + \dot{I}_L \\ &= -3(\dot{I}_{01} + \dot{I}_{02}) + \dot{I}_L \end{aligned} \quad (10)$$

At this stage, the ground-fault current consists of two components: the total phase-to-ground capacitive current of all lines in the system and the inductive current flowing through the arc-suppression coil. As expressed in Equation (11):

$$\dot{I}_f = \dot{I}_{C\Sigma} + \dot{I}_L \quad (11)$$

In arc-suppression-coil-grounded systems, different compensation levels lead to three operating states: full compensation, under-compensation, and overcompensation. The compensation state of the arc-suppression coil is usually described by the detuning degree (P). As expressed in Equation (12):

$$P = \frac{I_L - I_{C\Sigma}}{I_{C\Sigma}} \times 100\% \quad (12)$$

Full compensation may bring the system close to the resonance point, causing an abnormal increase in the neutral-to-ground voltage and posing a threat to insulation. In the under-compensation mode, disconnection of part of the lines during a fault may reduce the capacitive current and drive the system toward the full-compensation state. In engineering practice, overcompensation is often adopted. In practical arc-suppression-coil-grounded distribution networks, the arc-suppression coil is commonly operated in a slightly overcompensated state, and the overcompensation degree is generally set within 5–10% [22].

When the system operates in the overcompensation mode, the residual current of the faulty phase becomes inductive. The zero-sequence current measured at the beginning of the faulty line changes direction and becomes consistent with that of healthy lines, flowing from the busbar side to the line side. Arc-suppression-coil-grounded systems usually operate in the overcompensation state. Under this condition, the steady-state zero-sequence current difference between the faulty line and healthy lines becomes indistinct. Fault-location methods based on steady-state zero-sequence current amplitude or direction comparison are difficult to apply effectively.

It should be noted that Figures 3 and 4 are simplified current-distribution diagrams used to explain the dominant steady-state zero-sequence current paths. For clarity, the longitudinal line impedance, line leakage conductance, and the equivalent resistance of the arc-suppression coil are not explicitly shown in these diagrams. In small-current grounding systems, the active component of the zero-sequence current caused by line conductance and coil resistance is usually much smaller than the capacitive and inductive reactive

components. Therefore, the simplified diagrams focus on the main capacitive-current and compensation-current relationships. The effects of resistance, inductance, and capacitance are further considered in the transient equivalent circuit and in the PSCAD/EMTDC distributed-parameter simulation model.

2.2. Transient Zero-Sequence Current Characteristics

For distribution lines with single-phase-to-ground faults, transient zero-sequence currents in different line sections exhibit pronounced spatial distribution differences in amplitude, direction, and polarity. An arc-suppression-coil-grounded system is taken as the analysis object, and the corresponding fault transient equivalent circuit is shown in Figure 5.

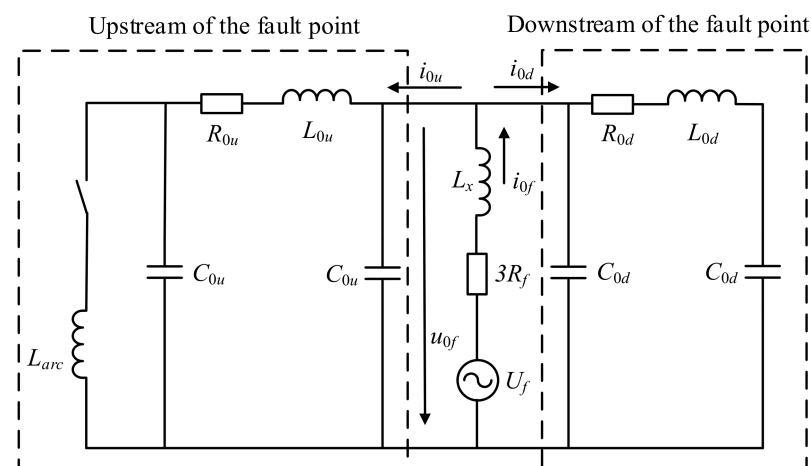


Figure 5. Transient equivalent circuit of faults in non-effectively grounded systems.

In this equivalent circuit, the line sections upstream and downstream of the fault point are represented by π -equivalent models. In Figure 5, the zero-sequence equivalent capacitances of the upstream and downstream sections from the fault point are denoted by (C_{0u}) and (C_{0d}) , respectively. The corresponding zero-sequence equivalent inductances are denoted by (L_{0u}) and (L_{0d}) , and the zero-sequence equivalent resistances are denoted by (R_{0u}) and (R_{0d}) . The grounding resistance is denoted by (R_f) . The inductance of the arc-suppression coil is denoted by (L_{arc}) , and the cumulative equivalent inductance in the line-mode network is denoted by (L_x) .

The equivalent zero-sequence inductance and capacitance parameters upstream of the fault point can be expressed by Equations (13) and (14):

$$L_{0u} = L_{01} // L_{02} // L_{0i} // \cdots // L_{0N} + L_{0bf} \quad (13)$$

$$C_{0u} = \frac{1}{2} \left(\sum_{i=1, i \neq k}^N C_{0i} + C_{0bf} \right) \quad (14)$$

In the equations, L_{0i} and C_{0i} denote the zero-sequence inductance and zero-sequence capacitance of the healthy section, respectively. L_{0bf} and C_{0bf} denote the zero-sequence inductance and zero-sequence capacitance from the fault point to the busbar, respectively.

The equivalent zero-sequence inductance and capacitance parameters downstream of the fault point can be expressed by Equations (15) and (16):

$$L_{0d} = L_{0fe} \quad (15)$$

$$C_{0d} = \frac{1}{2} C_{0fe} \quad (16)$$

In the equations, L_{0fe} and C_{0fe} denote the zero-sequence inductance and zero-sequence capacitance of the line section from the fault point to the line end, respectively.

Equations (13)–(16) indicate that practical distribution networks usually contain multiple feeders connected to the busbar. The phase-to-ground capacitance of the upstream network (C_{0u}) is much larger than that of the downstream section (C_{0d}). The principal resonant frequency upstream of the fault point is lower than that downstream of the fault point, producing pronounced differences in the frequency distribution of transient waveforms at both ends of the fault section. Healthy sections on the same side of the fault point have similar equivalent line parameters, and their principal resonant frequencies are nearly identical.

According to the zero-sequence equivalent-network analysis, the upstream section of the fault point acts as the main path for the system transient reactive current. The transient zero-sequence current flowing through this section consists of three components: the transient capacitive currents of all healthy lines in the network, the transient capacitive current of the downstream section from the fault point, and the neutral-point branch current. The transient zero-sequence current measured downstream of the fault point consists only of the capacitive current from the detection point to the line end. The phase-to-ground distributed capacitance in this section is relatively small. The amplitude of the transient zero-sequence current is much lower than that in the upstream section. For fault sections, the transient currents measured at both ends exhibit opposite polarities. For healthy sections, the transient currents at both ends have the same direction and consistent polarity characteristics. In small-current grounding systems, transient zero-sequence currents exhibit pronounced differences between faulty and healthy regions. The waveform similarity between both ends of each line section can be used to identify the fault region effectively.

2.3. Waveform Dissimilarity Measurement Using Discrete Fréchet Distance

During the fault transient process, the zero-sequence current measured in the fault section differs markedly from those measured at the ends of healthy sections. Based on this feature, the discrete Fréchet distance algorithm is introduced to quantify waveform similarity between both ends of each section, enabling accurate fault-section location.

The Fréchet distance is a distance-measurement method used to quantify the similarity between two curves, as expressed in Equation (17). It has been widely applied in trajectory matching, pattern recognition, and related fields. Unlike the traditional Hausdorff distance, the Fréchet distance considers the overall curve shape, point positions, and point-order information. It can effectively capture the global variation characteristics of a curve. The Fréchet distance between two curves is illustrated in Figure 6.

$$F(A, B) = \inf_{\alpha, \beta} \max_{t \in [0, 1]} \{d(A(\alpha(t)), B(\beta(t)))\} \quad (17)$$

In Equation (17), A and B denote the trajectories of points a and b , respectively. $A(\alpha(t))$ and $B(\beta(t))$ denote the positions at time t of points a and b , respectively. d denotes the distance metric between two points, and $\inf$ denotes the infimum.

In practical engineering applications, the collected signals are usually discrete time series, making direct calculation of the continuous Fréchet distance unsuitable. The discrete Fréchet distance is therefore adopted. It quantifies waveform-shape differences by constructing an optimal matching path between two discrete sequences and can be regarded as a discrete approximation of the continuous Fréchet distance. Its principle is described as follows:

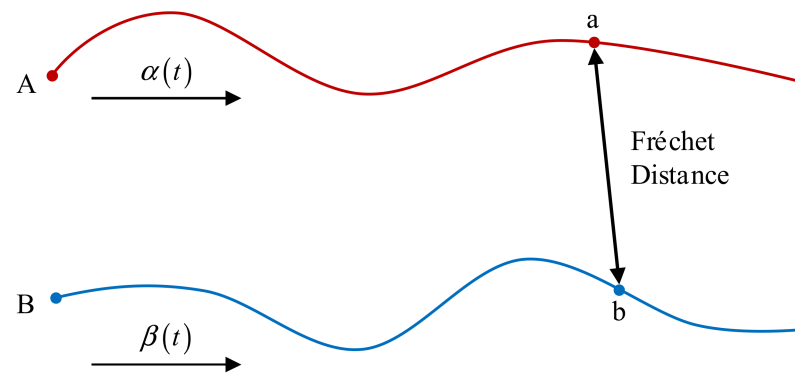


Figure 6. Schematic diagram of Fréchet distance model.

For two given discrete sequences with (m) and (n) sampling points, respectively, the discrete Fréchet distance can be calculated as follows: $A = \{a_1, \dots, a_i, \dots, a_m\}$ and $B = \{b_1, \dots, b_j, \dots, b_n\}$. The Euclidean distance between any two points on trajectories A and B is calculated to obtain the distance matrix. As expressed in Equations (18) and (19):

$$D_{ij} = \|a_i - b_j\| = \sqrt{(x_{a_i} - x_{b_j})^2 + (y_{a_i} - y_{b_j})^2} \quad (18)$$

$$D = \begin{bmatrix} D_{11} & D_{12} & \dots & D_{1j} & \dots & D_{1n} \\ D_{21} & D_{22} & \dots & D_{2j} & \dots & D_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ D_{i1} & D_{i2} & \dots & D_{ij} & \dots & D_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ D_{m1} & D_{m2} & \dots & D_{mj} & \dots & D_{mn} \end{bmatrix} \quad (19)$$

In the formula, D_{ij} represents the Euclidean distance between points a_i and b_j , while x_{a_i} , x_{b_j} , y_{a_i} , and y_{b_j} respectively denote the horizontal and vertical coordinates of a_i and b_j .

During the calculation of the discrete Fréchet distance, assume that there are w feasible paths from D_{11} to D_{mn} . The path set is denoted as: $X = \{x_1, x_2, \dots, x_k, \dots, x_w\}$. Taking one feasible path as an example, the number of distance values traversed along this path is denoted by z . These distance values can be represented as a set: $Y_k = \{y_{k1}, y_{k2}, \dots, y_{kz}\}$. This path must satisfy the following constraints:

- (1) $y_{k1} = D_{11}$ and $y_{kz} = D_{mn}$;
- (2) If there is a certain point in the path $y_{k(t-1)} = D_{ij}$, then, y_{kt} is satisfied: $d_k = \{D_{i,(j+1)}, D_{(i+1),j}, D_{(i+1),(j+1)}\}$.

The above constraints indicate that all valid paths in the distance matrix must satisfy monotonicity and directionality requirements. Starting from D_{11} and ending with D_{mn} , each path can move only to the right, downward, or diagonally to the lower right, forming a valid matching path.

The discrete Fréchet distance between two curves is defined as the minimum value among the maximum distances of all valid matching paths. The constraint conditions of the discrete Fréchet distance are illustrated in Figure 7.

The schematic diagrams of the discrete Fréchet distance under different numbers of sampling points are shown in Figure 8. According to the definition of the continuous Fréchet distance, the continuous Fréchet distances in the two cases shown in Figure 8 are both equal to the Euclidean distance between points O and a_2 . After discretization of the continuous curves, Figure 8a represents the case with fewer sampling points, in which the discrete Fréchet distance is equal to the Euclidean distance between points a_2 and b_2 .

Figure 8b represents the case with more sampling points, in which the discrete Fréchet distance is equal to the Euclidean distance between points a_2 and b .

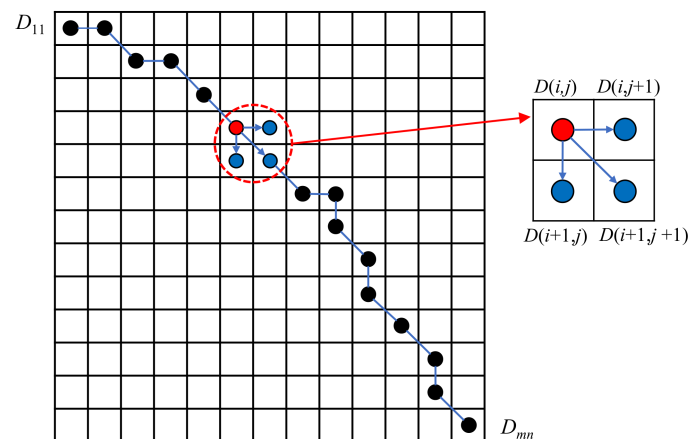


Figure 7. Constraints on discrete Fréchet distances.

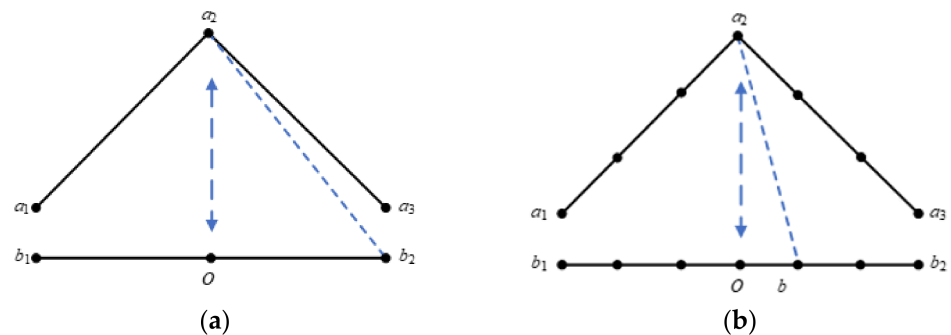


Figure 8. Discrete Fréchet distance under varying numbers of sampling points. (a) Discrete Fréchet distance with fewer sampling points. (b) Discrete Fréchet distance with more sampling points.

As the number of discrete trajectory points increases, the discrete Fréchet distance gradually approaches the continuous Fréchet distance. In this study, each transient zero-sequence current waveform contains several thousand sampling points, satisfying the application requirements of the discrete Fréchet distance.

According to the above analysis, a smaller waveform difference between two curves yields a smaller discrete Fréchet distance. A larger waveform difference yields a larger distance value. The discrete Fréchet distance can effectively reflect the waveform-difference degree between two curves. It is used to compare the similarity between transient zero-sequence current waveforms at both ends of each section, enabling reliable fault-section discrimination.

The proposed method differs from conventional transient-feature-based and waveform-matching-based methods in the following aspects. First, instead of using only amplitude, polarity, energy, or local time–frequency features, the proposed method treats the transient zero-sequence currents measured at both ends of a line section as ordered waveform curves and evaluates their overall morphological dissimilarity. Second, unlike the Hausdorff distance, which mainly reflects the maximum spatial deviation between point sets and does not strictly preserve the temporal order of waveform evolution, the discrete Fréchet distance considers both point positions and point-order constraints. This property is suitable for transient zero-sequence current waveforms whose amplitude, polarity, oscillation, and attenuation characteristics evolve with time after fault inception. Third, unlike DTW, which emphasizes elastic time-axis alignment and cumulative matching cost, the discrete

Fréchet distance used here emphasizes the maximum ordered waveform discrepancy between two section-end waveforms. Therefore, the proposed method is more consistent with the physical mechanism that the two ends of a faulty section show pronounced differences in transient current amplitude, polarity, frequency components, and waveform shape, whereas the two ends of a healthy section show higher waveform similarity. Finally, by introducing the coefficient of variation, the method further evaluates the dispersion of waveform-dissimilarity indicators among different sections, enabling discrimination between non-terminal and terminal fault-section cases.

2.4. Fault Section Identification Criterion Based on the Coefficient of Variation

The coefficient of variation (CV) is a dimensionless statistical indicator that reflects data dispersion and is commonly used to measure relative fluctuations among different datasets. Compared with standard deviation, the coefficient of variation more effectively represents data dispersion relative to the mean level.

For a given dataset $\{x_1, x_2, \dots, x_n\}$, its mean and standard deviation can be expressed by Equations (20) and (21):

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (20)$$

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2} \quad (21)$$

Then, the coefficient of variation of this dataset can be expressed by Equation (22):

$$CV = \frac{\sigma}{\bar{x}} \quad (22)$$

In the equations, $\bar{x}$ and σ denote the mean and standard deviation of the dataset, respectively. (CV) denotes the coefficient of variation. A larger (CV) indicates higher overall data dispersion and more pronounced differences among data values. A smaller (CV) indicates lower data dispersion and a more concentrated distribution.

In this study, the coefficient of variation is used to evaluate whether the discrete Fréchet distance values of different line sections contain a dominant outlier. Since all discrete Fréchet distance values are non-negative, $CV = \frac{\sigma}{\bar{x}} = 1$ means that the standard deviation of the section waveform-dissimilarity indicators reaches the same order as their mean value. Therefore, $CV \geq 1$ indicates that the distance distribution is highly dispersed and that at least one section has a significantly larger waveform-dissimilarity value than the others. In contrast, $CV < 1$ indicates that the distance values are relatively concentrated and that no obvious section-level outlier exists.

The $CV = 1$ threshold is used as an engineering dispersion boundary in this study. It is not regarded as a universally optimized threshold for all possible network structures. A systematic sensitivity analysis of the CV threshold under randomized network topologies, fault locations, transition resistances, inception angles, and noise levels will be considered in future work.

The above analysis indicates that the discrete Fréchet distance values of different sections can effectively reflect their waveform differences. The magnitudes of the discrete Fréchet distances alone are insufficient to determine whether abnormal sections are present. The coefficient of variation is further introduced to characterize the overall dispersion of the discrete Fréchet distance values calculated for different sections. This indicator is used to determine whether the fault section is located at the line end.

Assume that the discrete Fréchet distances of each section constitute a dataset $\{D_1, D_2, \dots, D_m\}$. The coefficient of variation can be calculated using Equations (20)–(22).

In traditional statistical analysis, a larger coefficient of variation indicates greater data dispersion relative to the mean. When obvious outliers deviate markedly from the overall distribution of a dataset, the standard deviation increases, leading to an increase in the coefficient of variation. The selection of $CV = 1$ is based on both the statistical meaning of the coefficient of variation and the physical distribution pattern of the section waveform-dissimilarity indicators. For non-terminal faults, the transient zero-sequence current waveforms at the two ends of the fault section differ significantly in amplitude, polarity, oscillation, and attenuation characteristics, resulting in a much larger discrete Fréchet distance than those of healthy sections. This produces an outlier-type distribution of the section distance values and increases the CV. For terminal-section faults, however, no two-sided fault-section measurement pair exists at the line end, and the distance values among the available adjacent measurement points are more evenly distributed. Therefore, $CV = 1$ is adopted as an engineering dispersion boundary to distinguish between a pronounced non-terminal fault-section outlier and a terminal-fault condition without an obvious section-level outlier. This threshold is not regarded as a universally optimized value for all network structures, but as a conservative criterion applicable to the tested system and parameter settings.

Based on this criterion, fault-section location can be classified into the following two scenarios:

(1) When $CV \geq 1$, the waveform-similarity values calculated for different sections exhibit high dispersion, and obvious outliers deviate markedly from the overall level. This condition indicates that the faulty section differs prominently from the other sections. The section with the largest distance value is identified as the faulty section.

(2) When $CV < 1$, the waveform-similarity values calculated for different sections are evenly distributed, and no obvious outliers deviate from the overall level. The difference between the faulty section and healthy sections is not pronounced. The fault is then determined to be located in the terminal section. In this case, the zero-sequence current amplitude at the terminal measurement point is further used for discrimination.

2.5. Overall Fault Section Location Procedure

This section analyzes the amplitude, frequency-characteristic, and polarity differences between transient zero-sequence currents at both ends of each line section. The discrete Fréchet distance algorithm is used to calculate zero-sequence current waveform similarity at both ends of each section, and the coefficient of variation is combined to achieve single-phase-to-ground fault-section location. The specific process for locating the faulty section based on discrete Fréchet distance and coefficient of variation is as follows:

(1) The instantaneous neutral-point voltage u_0 is monitored in real time. When a single-phase-to-ground fault occurs, the condition $u_0(t) > kU_N$ is satisfied, and the protection device is activated. Here, U_N denotes the rated phase voltage of the system, and k is the reliability coefficient, usually set to 0.15.

(2) Transient zero-sequence current data within the first cycle after fault occurrence are collected. All line sections are traversed, and the waveform-difference degree of each section is calculated.

(3) Based on the waveform differences between both ends of each section, $\bar{x}$, σ , and CV are calculated to quantify the overall dispersion of the waveform-difference indicators among sections.

(4) When $CV \geq 1$, the waveform-dissimilarity indicators of different sections show a highly dispersed distribution. This indicates that a dominant outlier exists among the section distance values. The section with the largest discrete Fréchet distance is therefore identified as the faulty section.

(5) When $CV < 1$, the waveform-dissimilarity indicators of different sections are relatively concentrated, and no obvious section-level outlier exists. The fault is then determined to be located in a terminal section. The amplitudes of transient zero-sequence currents at all effective terminal measurement points are further compared, and the terminal section corresponding to the measurement point with the largest amplitude is identified as the fault section.

The flowchart of the fault section location method proposed in this paper is shown in Figure 9.

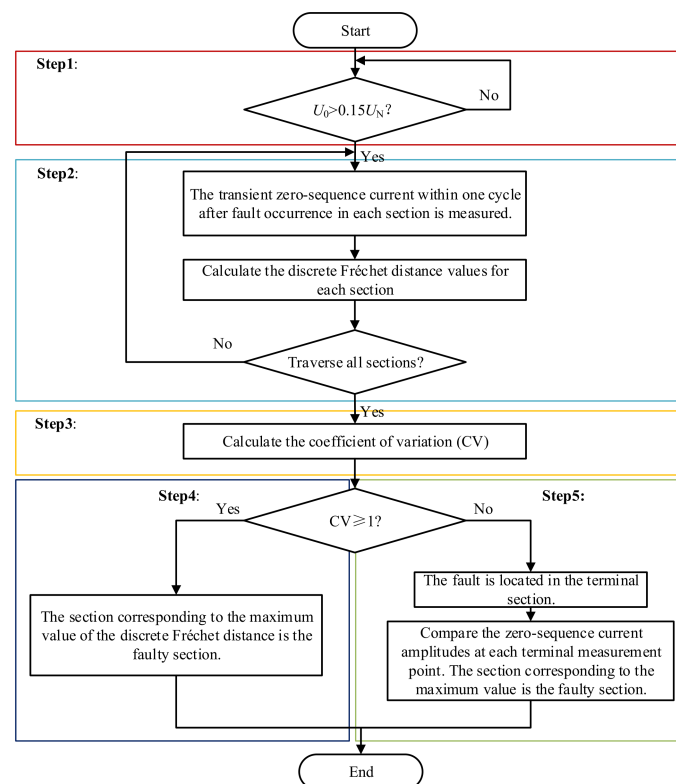


Figure 9. Flowchart of fault section location.

For engineering implementation, the proposed method requires zero-sequence current measurement units at both ends of each monitored line section, such as zero-sequence current transformers, residual-current transformers, or wideband current sensors. A zero-sequence voltage measurement unit installed at the busbar or neutral point is used for fault triggering. To ensure waveform comparability between both ends of a section, the measurement terminals should be time-synchronized using GPS/BeiDou, IEEE 1588, or equivalent synchronization methods. The transient zero-sequence current data within the first cycle after fault inception can be collected by feeder terminal units or intelligent electronic devices and transmitted to a local computing unit or master station, where the discrete Fréchet distance, coefficient of variation, and final fault-section criterion are calculated.

3. Results and Discussions

3.1. Validation Under Different Neutral Grounding Modes

3.1.1. Establishment of the Simulation Model

To verify the single-phase-to-ground fault characteristics described in this section, a simulation model of a 110/10 kV small-current grounding system was constructed in PSCAD/EMTDC, as shown in Figure 10. The transformation ratio of the main transformer T1 is 110/10.5 kV, with a YNd11 connection group and a rated capacity of 80 MVA. T2

is a grounding transformer connected to the arc-suppression coil through a switch. The neutral-point grounding mode can be changed by switching the operating state of the switching device. The rated capacity of each distribution transformer is set to 0.5 MVA, and the voltage ratio is set to 10/0.4 kV. Each line in the system is modeled using a distributed-parameter model. The line parameters of the simulation model are listed in Table 1.

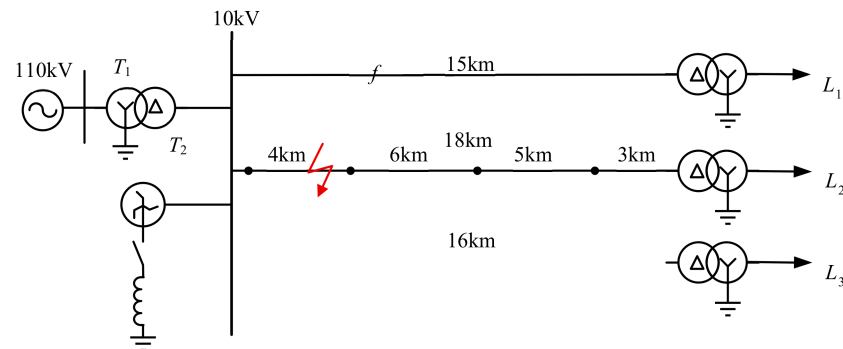


Figure 10. Simulation model and measurement-point arrangement of the non-effectively grounded distribution system. T1: main transformer; T2: grounding transformer; L1–L3: feeders; *f*: single-phase-to-ground fault point.

Table 1. Distribution network line parameters.

Parameter Name and Unit	Resistance/(Ω /km)	Inductance/(mH/km)	Capacity (μ F/km)
Ascending-order parameter	0.170	1.210	0.104
Zero-sequence parameters	0.230	5.470	0.007

3.1.2. Simulation Results and Analysis

A phase-A grounding fault is assumed to occur on line L2. The fault point is located 3 km from the busbar, and the fault time is set to 0.1 s. The fault inception angles are set to 0° and 90° , respectively, and the transition resistance is set to 10Ω . For the neutral-point ungrounded system, the waveforms of the busbar three-phase voltages, zero-sequence voltage, zero-sequence currents of each line, and zero-sequence current of the faulty line are shown in Figures 11 and 12.

The simulation results show that the zero-sequence voltage of the neutral-point ungrounded system remains at 0 during normal operation. When a phase-A grounding fault occurs, the faulty-phase voltage decreases to 0, while the voltages of phases B and C increase to line-voltage levels. The busbar zero-sequence voltage rises to the phase-voltage level. The zero-sequence current amplitude measured at the beginning of the faulty line is markedly higher than that of healthy lines, with opposite polarity. The transient zero-sequence current amplitude at the beginning of the faulty line is much higher than its steady-state value, often reaching several times or even tens of times the steady-state level. These results are consistent with the theoretical analysis.

A phase-A grounding fault is assumed to occur on line L2. The fault point is located 3 km from the busbar, and the fault time is set to 0.1 s. The fault inception angles are set to 0° and 90° , respectively, and the transition resistance is set to 10Ω . For the arc-suppression-coil-grounded system, the waveforms of the busbar three-phase voltages, zero-sequence voltage, zero-sequence currents of each line, and zero-sequence current of the faulty line are shown in Figures 13 and 14.

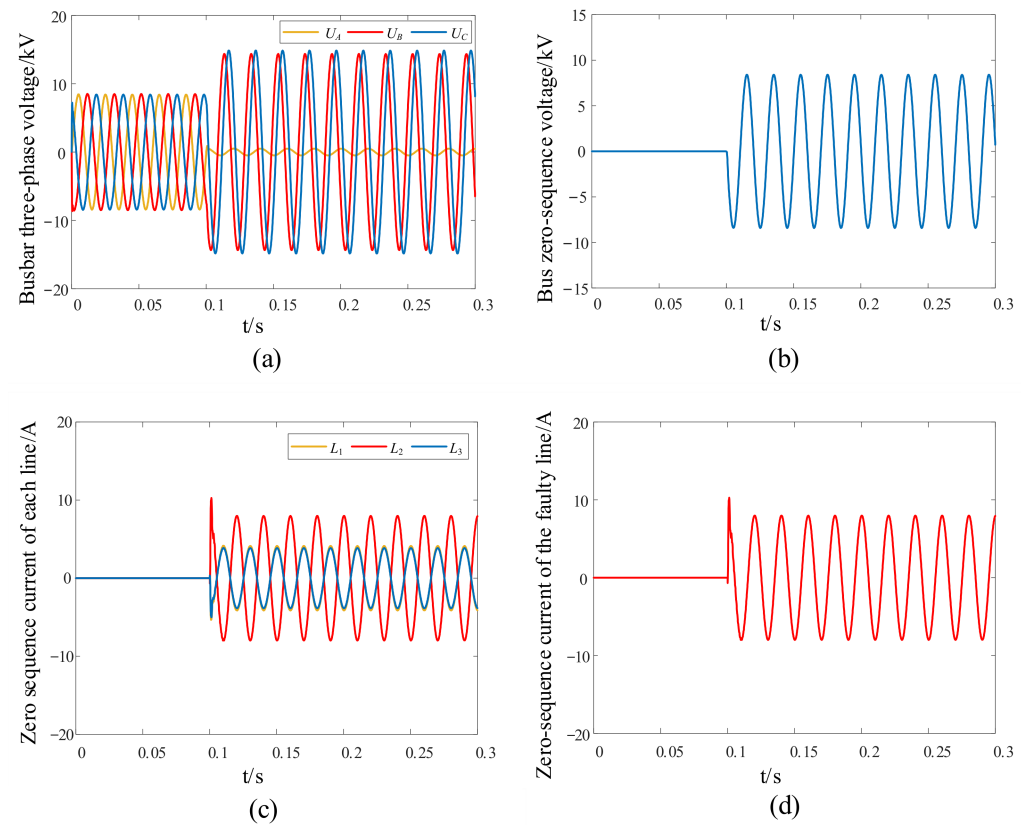


Figure 11. Simulation results at an initial phase angle of 0° . (a) Busbar three-phase voltage. (b) Bus zero-sequence voltage. (c) Zero-sequence current of each line. (d) Zero-sequence current of the faulty line.

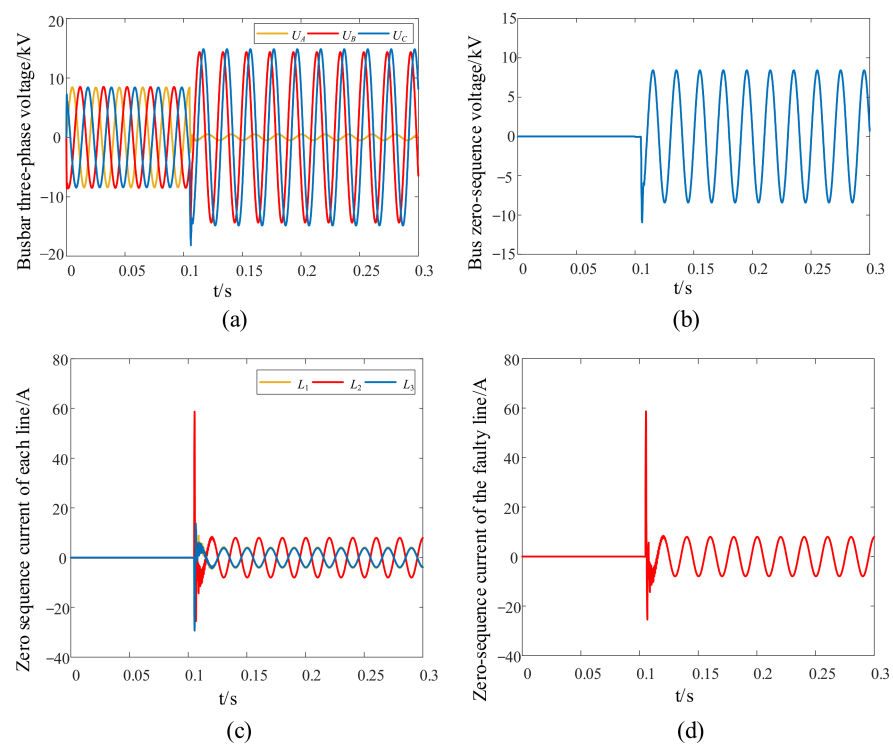


Figure 12. Simulation results at an initial phase angle of 90° . (a) Busbar three-phase voltage. (b) Bus zero-sequence voltage. (c) Zero-sequence current of each line. (d) Zero-sequence current of the faulty line.

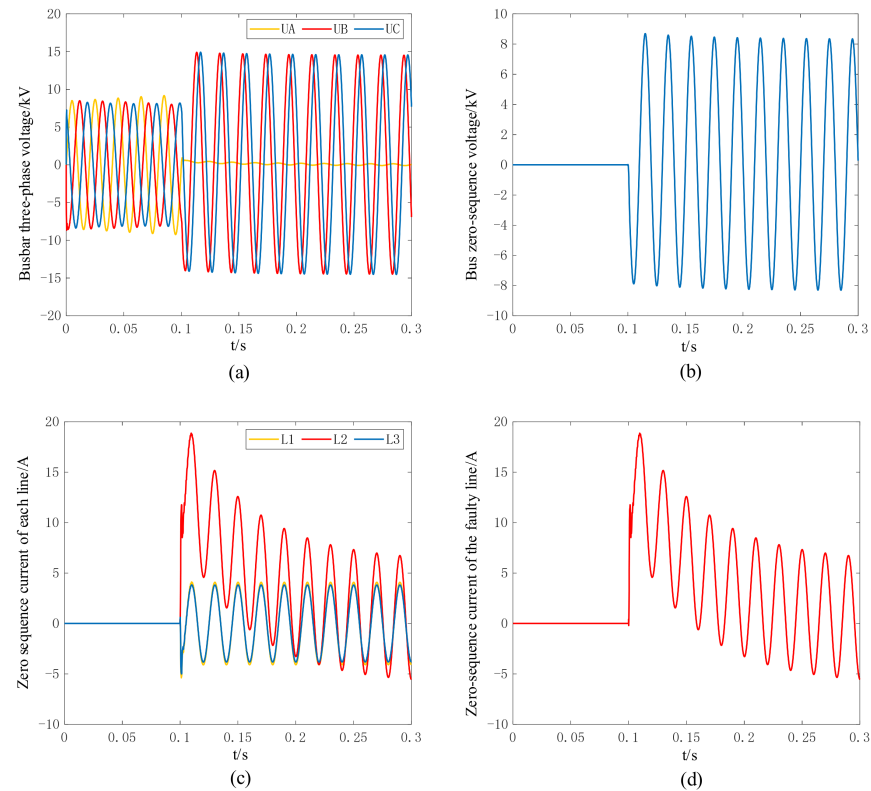


Figure 13. Simulation results at an initial phase angle of 0° . (a) Busbar three-phase voltage. (b) Bus zero-sequence voltage. (c) Zero-sequence current of each line. (d) Zero-sequence current of the faulty line.

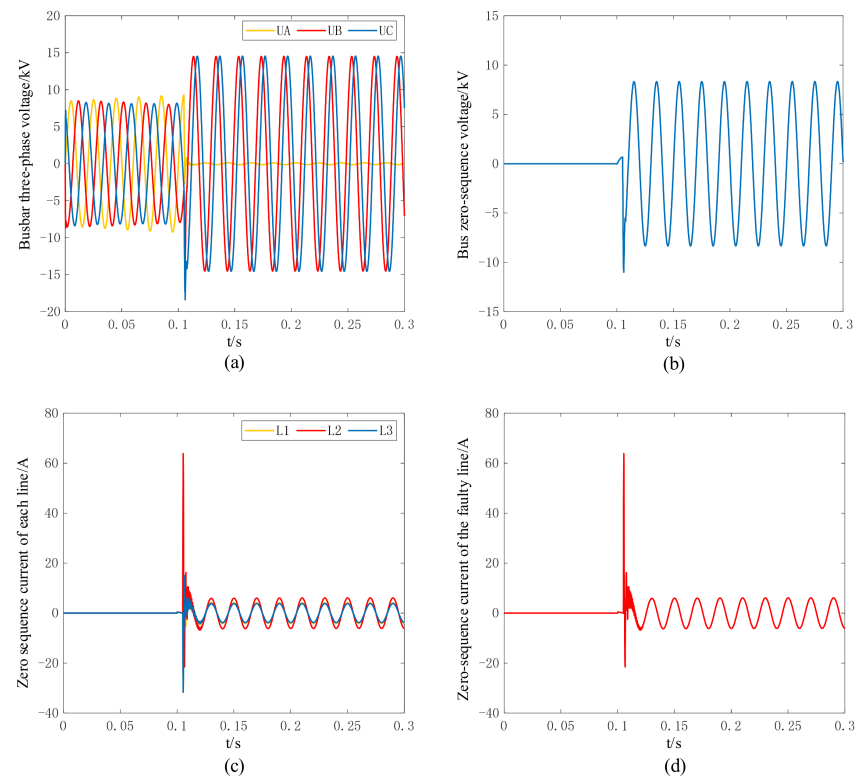


Figure 14. Simulation results at an initial phase angle of 90° . (a) Busbar three-phase voltage, (b) Bus zero-sequence voltage, (c) Zero-sequence current of each line, (d) Zero-sequence current of the faulty line.

The simulation results show that the fault characteristics of this system under normal operating conditions are consistent with those of the ungrounded system. When a phase-A grounding fault occurs, inductive-current compensation reduces the steady-state zero-sequence current difference between the faulty line and healthy lines. Their amplitudes become similar, and their polarities are no longer opposite. Fault location is difficult when only steady-state characteristics are used. Compared with steady-state quantities, transient zero-sequence currents have higher amplitudes and are less affected by the arc-suppression coil. The transient zero-sequence currents of the faulty line and healthy lines exhibit pronounced differences in amplitude and polarity. These results are consistent with the theoretical analysis.

To present the differences in transient zero-sequence current waveforms among line sections more intuitively, a simplified simulation model was established, as shown in Figure 10. The line-parameter settings are the same as those listed in Table 1. Taking line L_2 as an example, a phase-A grounding fault is set at a location 3 km from the busbar. The fault occurs at 0.1 s, and the simulation time step is set to 100 μ s. Transient zero-sequence current waveforms on both sides of the fault point were comparatively analyzed under the two grounding modes. The corresponding simulation results are shown in Figure 15.

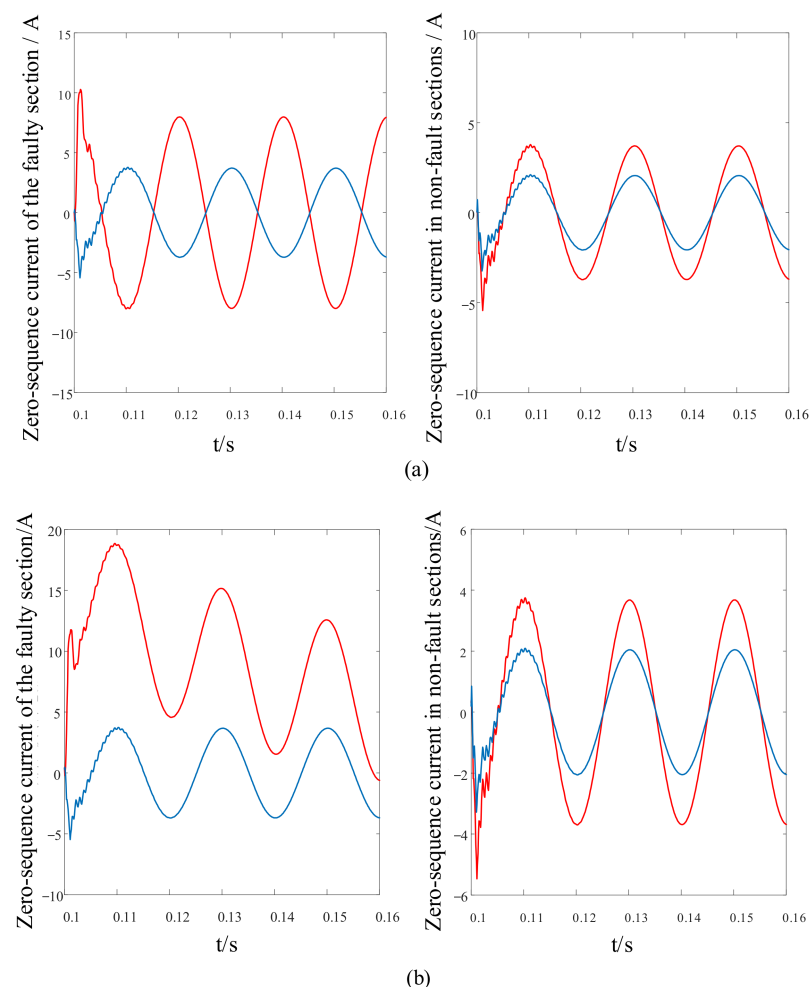


Figure 15. Zero-sequence current waveforms at both ends of the section under different grounding methods. (a) Zero-sequence current at both ends of the section in an ungrounded system. (b) The zero-sequence current at both ends of the section connected through the arc-suppression coil.

The simulation results show that, under both neutral-point ungrounded and arc-suppression-coil-grounded modes, the transient zero-sequence current waveforms at both

ends of fault sections and healthy sections exhibit markedly different characteristics. For the fault section, the transient zero-sequence currents measured at both ends differ markedly in amplitude and frequency components. Their waveform variations show inconsistent trends and opposite polarities. For healthy sections, the amplitudes and frequency components at both ends are relatively close, the waveform trends are similar, and the polarities remain consistent. Further analysis shows that changes in the neutral-point grounding mode do not alter the basic pattern of waveform differences. The transient zero-sequence current waveforms at both ends of a fault section remain markedly different, while those at both ends of a healthy section remain highly similar. To evaluate the influence of neutral-point grounding mode on the proposed method, simulation analyses were conducted under two typical grounding modes. In Figure 10, a single-phase-to-ground fault is set at point f_1 . The fault inception angle is set to 60° , and the transition resistance is set to $100\ \Omega$. The corresponding location results are listed in Table 2.

Table 2. Location results under different neutral grounding modes.

Grounding Method	Discrete Fréchet Distance of Zero-Sequence Current Between Adjacent Measurement Points					Coefficient of Variation	Positioning Result
	(1,2)	(2,3)	(4,5)	(5,6)	(6,7)		
Not grounded	0.2121	0.3888	0.3219	11.7699	0.7593	$2.0535 > 1$	(5,6)
Grounded through an arc-suppression coil	0.1632	0.3562	0.3265	10.2867	0.7708	$2.0265 > 1$	(5,6)

Table 2 shows that, under different grounding modes, the waveform-difference index of the fault section is markedly higher than those of the other sections, and the (CV) value exceeds the preset threshold. This indicates pronounced differences in waveform-difference degrees among sections. According to the location criterion, the section with the largest waveform-difference index is identified as the faulty section. The results indicate that the proposed method has good applicability under different neutral-point operating conditions and can accurately identify the fault section. Considering that practical distribution networks usually adopt arc-suppression-coil grounding, this grounding mode is used in the subsequent simulation verification under different fault conditions.

3.2. Adaptability to Transition Resistance and Fault Inception Angle

To analyze the adaptability of the proposed method under different transition resistances, simulation verification was conducted at points f_1 and f_2 . The fault inception angle was set to 60° , and the transition resistance was set to 0, 100, 500, 1000, and $1500\ \Omega$. The fault-section location results are listed in Table 3.

The simulation results show that the discrete Fréchet distance values of all sections decrease as the transition resistance increases. This trend occurs because the transient zero-sequence current amplitude is negatively correlated with the transition resistance. Under different transition resistances, the discrete Fréchet distance of the fault section remains the largest, and the coefficient of variation remains greater than 1. Pronounced differences still exist between the fault section and healthy sections. According to the established criterion, the fault section can be accurately located. The simulation results show that the proposed method achieves accurate location under different transition-resistance conditions and exhibits good robustness against transition-resistance variation.

To analyze the accuracy of the proposed method under different fault inception angles, single-phase-to-ground faults were set at points f_1 and f_2 . With the transition resistance set to $50\ \Omega$, simulations were conducted under fault inception angles of 0° , 30° , 60° , and 90° . The location results under different fault inception angles are listed in Table 4.

Table 3. Location results under different transition resistances.

Fault Location	Transition Resistance/ Ω	Discrete Fréchet Distance of Zero-Sequence Current Between Adjacent Measurement Points					Coefficient of Variation	Positioning Result
		(1,2)	(2,3)	(4,5)	(5,6)	(6,7)		
f_1	0	1.1204	1.1236	1.2351	95.7223	0.8983	2.2800 > 1	(5,6)
	100	0.1632	0.3562	0.3265	10.2867	0.7708	2.0265 > 1	(5,6)
	500	0.0605	0.1596	0.0858	2.5540	0.3491	1.7910 > 1	(5,6)
	1000	0.0293	0.0899	0.0503	1.3140	0.1678	1.8185 > 1	(5,6)
	1500	0.0126	0.0636	0.0350	0.8844	0.1093	1.8475 > 1	(5,6)
f_2	0	0.6506	0.7692	0.6128	0.9602	132.3265	2.3755 > 1	(6,7)
	100	0.1737	0.3432	0.3570	0.3948	16.2991	2.2067 > 1	(6,7)
	500	0.0831	0.1335	0.1297	0.3075	3.5829	1.9643 > 1	(6,7)
	1000	0.0409	0.0757	0.0916	0.1899	1.8131	1.8874 > 1	(6,7)
	1500	0.0272	0.0528	0.0693	0.1360	1.2137	1.8589 > 1	(6,7)

Table 4. Location results with different fault initial phase angles.

Fault Location	Initial Phase Angle/ $^\circ$	Discrete Fréchet Distance Values of Zero-Sequence Current at Adjacent Measurement Points					Coefficient of Variation	Positioning Result
		(1,2)	(2,3)	(4,5)	(5,6)	(6,7)		
f_1	0	0.1635	0.6214	0.5698	34.2860	0.9544	2.2431 > 1	(5,6)
	30	0.0628	0.0871	0.0625	25.9883	0.0605	2.4120 > 1	(5,6)
	60	0.4491	0.6426	0.5630	19.0580	0.8240	2.0770 > 1	(5,6)
	90	0.5845	1.1661	1.1455	35.1038	1.5083	2.0967 > 1	(5,6)
f_2	0	0.3621	0.4872	0.5775	0.4658	36.0958	2.2802 > 1	(6,7)
	30	0.0375	0.0378	0.0325	0.1016	32.9173	2.4267 > 1	(6,7)
	60	0.2750	0.4270	0.4640	0.4716	29.1299	2.2532 > 1	(6,7)
	90	0.5372	0.8690	0.8899	0.8153	32.5553	2.1426 > 1	(6,7)

The simulation results show that the discrete Fréchet distance values of different sections vary under different fault inception angles. The discrete Fréchet distance of the fault section remains the largest, and the coefficient of variation remains greater than 1. According to the criterion established in this section, the fault section can be accurately located. These results indicate that variations in the fault inception angle have little influence on the effectiveness of the proposed method. The fault section can still be accurately located even under small fault inception angles.

The localization results are presented in tabular form to retain the exact discrete Fréchet distance values, coefficient-of-variation values, and identified sections under each fault inception angle. This format allows direct comparison of the characteristic values among adjacent measurement sections. Graphical visualization of the localization results will be considered in future extended studies involving larger-scale statistical cases.

3.3. Terminal-Section Fault Location Performance

To analyze the accuracy of the proposed location method when a fault occurs in a terminal section, a single-phase-to-ground fault was set at point f_3 . The transition resistance was set to 0, 100, 500, 1000, and 1500 Ω , and the fault inception angle was set to 0°, 30°, 60°, and 90°. The final location results are listed in Tables 5 and 6.

Table 5. Location results for terminal section under different transition resistances.

Transition Resistance	Discrete Fréchet Distance Value of Zero-Sequence Current at Adjacent Measurement Points					Coefficient of Variation	Positioning Result
	(1,2)	(2,3)	(4,5)	(5,6)	(6,7)		
0	0.5129	0.8382	0.8657	0.8089	1.1666	0.2675 < 1	(7, -)
100	0.3032	0.3512	0.5178	0.5191	0.5834	0.2375 < 1	(7, -)
500	0.1224	0.0853	0.2760	0.2811	0.2537	0.4085 < 1	(7, -)
1000	0.0715	0.0415	0.1733	0.1740	0.1623	0.4582 < 1	(7, -)
1500	0.0506	0.0272	0.1263	0.1141	0.1200	0.4675 < 1	(7, -)

Table 6. Location results for terminal section under different fault initial phase angles.

Initial Phase Angle/ $^{\circ}$	Discrete Fréchet Distance Value of Zero-Sequence Current at Adjacent Measurement Points					Coefficient of Variation	Positioning Result
	(1,2)	(2,3)	(4,5)	(5,6)	(6,7)		
0	0.4089	0.4879	0.6856	0.7439	0.8570	$0.2635 < 1$	(7, -)
30	0.0381	0.0271	0.0842	0.0774	0.0919	$0.4757 < 1$	(7, -)
60	0.3800	0.5087	0.5644	0.5956	0.7715	$0.2273 < 1$	(7, -)
90	0.6431	0.8540	1.0716	1.1338	1.5939	$0.3117 < 1$	(7, -)

The simulation results show that, when the fault occurs in a terminal section, variations in transition resistance and fault inception angle affect the transient zero-sequence current characteristics of each section to some extent, but do not interfere with the fault-section location result. Unlike faults in non-terminal sections, the differences among the calculated discrete Fréchet distance values become less pronounced when a terminal-section fault occurs. Under different transition resistances and fault inception angles, the coefficient of variation remains below 1. The fault section ((7,-)) can then be identified by further comparing the amplitudes of transient zero-sequence currents at each terminal measurement point. The results indicate that the proposed method maintains good location performance under terminal-section fault conditions and can accurately identify the fault section even when only one measurement point is available in the terminal section.

3.4. Noise Robustness Analysis

To verify the applicability of the proposed method under noise interference, Gaussian white noise is superimposed on the fault transient zero-sequence current signals, followed by fault-section location analysis. The influence of noise on location performance is evaluated by comparing the location results before and after noise addition. The anti-noise capability of the proposed method is then verified. In the simulation, the fault inception angle was set to 60° , and the transition resistance was set to 10Ω . The location results are listed in Table 7.

Table 7. Location results under different noise interferences.

Fault Location	SNR/dB	Discrete Fréchet Distance Value of Zero-Sequence Current at Adjacent Measurement Points					Coefficient of Variation	Positioning Result
		(1,2)	(2,3)	(4,5)	(5,6)	(6,7)		
f_1	No noise	1.0818	1.0986	1.0243	56.6426	0.7599	$2.2013 > 1$	(5,6)
	30	1.9645	2.2469	2.5167	55.8603	2.0951	$1.9865 > 1$	(5,6)
	20	5.0710	5.5635	6.2664	60.9443	6.8922	$1.5217 > 1$	(5,6)
f_2	No noise	0.6449	0.6486	0.5826	0.8004	76.3682	$2.3340 > 1$	(6,7)
	30	3.4102	2.3385	2.1571	2.2492	75.9434	$2.0520 > 1$	(6,7)
	20	5.5442	5.6695	6.7665	5.5753	81.2153	$1.6919 > 1$	(6,7)
f_3	No noise	0.4794	0.7468	0.7670	0.7355	1.0631	$0.2574 < 1$	(7, -)
	30	1.7040	2.1448	2.7678	2.8850	3.2945	$0.2637 < 1$	(7, -)
	20	7.1269	5.6206	7.4220	9.3779	8.8601	$0.2147 < 1$	(7, -)

The simulation results in Table 7 show that, under noise-free conditions and Gaussian white noise levels of 30 dB and 20 dB, the (CV) calculated by the proposed method remains greater than 1 for non-terminal section faults. The waveform-difference index of the fault section remains the largest. When a terminal-section fault occurs, the CV value is less than 1. The proposed method still accurately identifies the fault section, satisfying the fault-section location criterion established in this study. No location blind zone is observed. The simulation results show that the location accuracy of the proposed method is not affected by a certain level of noise interference. The method exhibits good anti-noise capability and applicability.

These results provide a preliminary consistency check for the $CV = 1$ criterion under the tested noise conditions. For non-terminal faults, the fault section produces a dominant waveform-dissimilarity outlier, and CV remains greater than 1. For terminal faults, the section distance values remain relatively concentrated, and CV remains lower than 1.

3.5. Comparative Analysis with Existing Methods

To further verify the fault-section location performance of the proposed method, the correlation-coefficient method, grey relational analysis, and empirical mode decomposition were selected as comparison methods. Under arc-suppression-coil grounding, a single-phase-to-ground fault is set at point (f_1). Two typical scenarios are selected for simulation analysis. The fault-section location results for the two scenarios are listed in Tables 8 and 9. The detailed settings of the two fault scenarios are as follows:

Table 8. Fault location results of different location methods (Condition 1).

Location Method	Location Criterion	Calculation Results of Each Section					Location Result
		(1,2)	(2,3)	(4,5)	(5,6)	(6,7)	
Correlation coefficient method	Compare the size of the correlation coefficient and the threshold	1.0000	1.0000	1.0000	−0.7942	1.0000	(5,6)
Grey correlation degree	Compare the size of the grey correlation degree and the threshold	0.7702	0.7916	0.8163	0.5391	0.7424	(5,6)
Empirical mode decomposition	Positive and negative differences in adjacent energy weight coefficients	Positive	Positive	Positive	Positive	Positive	(5,6)
The proposed method	Compare the Fréchet distance and the coefficient of variation	0.0506	0.0691	0.0482	13.9978	0.0488	(5,6)

Table 9. Fault location results of different location methods (Condition 2).

Location Method	Location Criterion	Calculation Results of Each Section					Location Result
		(1,2)	(2,3)	(4,5)	(5,6)	(6,7)	
Correlation coefficient method	Compare the size of the correlation coefficient and the threshold	0.6732	0.7032	0.7824	−0.8489	0.7006	(5,6)
Grey correlation degree	Compare the size of the grey correlation degree and the threshold	0.6163	0.6454	0.6039	0.5453	0.6438	(5,6)
Empirical mode decomposition	Positive and negative differences in adjacent energy weight coefficients	Positive	negative	Positive	negative	Positive	/
The proposed method	Compare the Fréchet distance and the coefficient of variation	0.0103	0.0899	0.0555	1.6089	0.1950	(5,6)

(1) Scenario 1: The transition resistance is set to $100\ \Omega$, and the fault inception angle is set to 30° . Under this condition, the fault features are relatively distinct.

(2) Scenario 2: The transition resistance is set to $1500\ \Omega$, and the fault inception angle is set to 90° . Under this condition, the fault features are relatively weak.

The simulation results under Scenario 1 are listed in Table 8. The results show that all four selected location methods can accurately identify the fault section under this condition. For the correlation-coefficient method and grey relational analysis, the calculated values of the fault section differ markedly from those of healthy sections. The fault section can be identified using the preset threshold. The empirical mode decomposition method detects obvious polarity differences in the fault section and also achieves accurate location. For the proposed method, the discrete Fréchet distances of healthy sections remain at relatively

low levels, while that of the fault section increases to 13.9978. The coefficient of variation is greater than 1, indicating clear discrimination between the fault section and healthy sections. This indicates that, when the fault features are relatively distinct, the proposed method can effectively identify the fault section and provides a certain discrimination margin.

The simulation results under Scenario 2 are listed in Table 9. As the transition resistance increases, the fault transient signal becomes markedly weaker, and the location performance of all methods is affected to varying degrees. The criterion of the correlation-coefficient method is relatively sensitive to threshold settings, and its stability decreases under weak fault-feature conditions. For grey relational analysis, the discrimination margin between the fault section and healthy sections is markedly reduced, making the method prone to misjudgment when a fixed threshold is improperly selected. The empirical mode decomposition method exhibits the same polarity characteristics in multiple sections, making accurate fault-section discrimination difficult. In contrast, the proposed method still exhibits good discrimination capability under this condition. The discrete Fréchet distance of the fault section remains the largest, and the coefficient of variation is greater than 1. According to the established criterion, the fault section can still be accurately located.

Comparison of different fault-section location methods shows that the proposed method can accurately identify the fault section under both distinct and weak fault-feature conditions. By combining the discrete Fréchet distance with the coefficient of variation, the proposed method can characterize both the waveform differences between transient zero-sequence currents at both ends of each section and the dispersion degree of characteristic values among sections. The method thus proves effective under the tested operating conditions.

4. Conclusions

This study proposes a distribution-network single-phase-to-ground fault-section location method based on the waveform-difference degree of transient zero-sequence currents. The method is based on the spatial-distribution differences of transient zero-sequence currents at both ends of fault sections in small-current grounding systems. The discrete Fréchet distance is used to represent the overall waveform difference between both ends of each section, and the coefficient of variation is combined to construct the fault-section identification criterion. Through the joint simulation of PSCAD/EMTDC and MATLAB R2024a, the positioning performance under different neutral-point grounding methods, transition resistances, initial fault phase angles, end-zone faults, and noise interference conditions was verified. The main conclusions are as follows:

- (1) Transient zero-sequence current provides more effective fault-section information than steady-state zero-sequence current in arc-suppression-coil-grounded systems. Under compensation conditions, the steady-state zero-sequence current difference between faulty and healthy lines becomes less distinct, whereas transient zero-sequence current still retains differences in amplitude, polarity, oscillation, and waveform morphology.
- (2) For non-terminal faults, the fault section forms a dominant waveform-dissimilarity outlier. Under transition resistances from 0 Ω to 1500 Ω , the proposed method correctly identified fault section (5,6) for f1 and fault section (6,7) for f2. The CV values remained greater than 1, ranging from 1.7910 to 2.3755. Under fault inception angles from 0° to 90°, the fault-section Fréchet distance also remained the largest, and the correct section was identified.
- (3) For terminal-section faults, the section-level waveform-dissimilarity indicators were more concentrated. Under transition resistances from 0 Ω to 1500 Ω and fault inception angles from 0° to 90°, the CV values ranged from 0.2273 to 0.4757, all lower than 1.

The terminal fault section (7,-) was then correctly identified by comparing terminal transient zero-sequence current amplitudes.

- (4) Under noise-free, 30 dB, and 20 dB conditions, the proposed method still correctly identified the tested fault sections. The CV values of non-terminal faults remained greater than 1, while those of the terminal fault remained lower than 1. Specifically, the CV values were 2.2013, 1.9865, and 1.5217 for f1; 2.3340, 2.0520, and 1.6919 for f2; and 0.2574, 0.2637, and 0.2147 for f3.
- (5) In the weak-feature comparison case, the proposed method produced a discrete Fréchet distance of 1.6089 at the actual fault section (5,6), which was significantly larger than the values of the other sections. Compared with the correlation-coefficient method, grey relational analysis, and empirical mode decomposition, the proposed method provided clearer section discrimination under the tested weak-feature condition.

The proposed method combines the discrete Fréchet distance and the coefficient of variation to provide a physically interpretable solution for single-phase-to-ground fault-section location in small-current grounding systems. Since it evaluates the ordered waveform dissimilarity of transient zero-sequence currents at both ends of each section, it is not limited to a single frequency component or steady-state amplitude, and common harmonic distortion appearing similarly at adjacent measurement points is expected to have limited influence on the relative section-level dissimilarity distribution. In practical applications, non-fault switching disturbances should be distinguished using the neutral-point zero-sequence voltage trigger, disturbance duration, polarity relationship, and section-level CV distribution; events without sustained zero-sequence voltage rise or dominant waveform-dissimilarity outliers should not be directly classified as grounding faults. The present verification was conducted on a radial small-current grounding distribution-network model under representative deterministic scenarios, and the $CV = 1$ criterion was used as an engineering dispersion threshold rather than a universally optimized value. Future work will focus on larger-scale randomized verification, field-data validation, Monte Carlo statistical evaluation, comparisons with additional waveform-matching methods such as DTW and Hausdorff-distance-based methods, and extension to more complex networks and other fault types.

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