

Article

Scaling Laws and Thermodynamic Limits of Modular Thermoelastic Energy Harvesting from Low-Grade Heat

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Abstract

Low-grade thermal energy is widely available in industrial waste-heat streams and natural temperature fluctuations, yet its utilization remains limited because of weak thermodynamic driving forces and the complexity of conventional heat-engine technologies. This study presents a physics-based framework for modular thermoelastic energy harvesting using the reversible thermal expansion and contraction of structural materials. Analytical models are established to quantify thermoelastic work, structural constraints, thermodynamic and exergy efficiencies, and long-term energy production. Material selection and thermo-mechanical limitations are evaluated through parametric analysis and finite-element verification. The results indicate that extractable work is fundamentally constrained by yield strength, buckling resistance, temperature swing, and the limited exergy content of low-grade heat. Scaling laws show that annual energy generation scales approximately linearly with active structural mass while remaining strongly dependent on column diameter, thermal-cycle frequency, and material performance indices. Thermodynamic and exergy efficiencies remain well below the Carnot limit, highlighting the inherent limitations of solid-state thermoelastic conversion. A techno-economic assessment further indicates that economic viability depends primarily on multi-cycle operation and low-cost implementation. Although the achievable energy density remains modest compared with conventional renewable technologies, the proposed framework provides quantitative performance limits and practical design guidelines for evaluating thermoelastic energy harvesting from low-grade heat.

Keywords: thermoelastic energy harvesting; low-grade heat; waste heat recovery; scaling laws; thermodynamic limits; exergy analysis; techno-economic assessment



Academic Editor: Andrea De Pascale

Received: 12 June 2026

Revised: 28 July 2026

Accepted: 30 July 2026

Published: 4 August 2026

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1. Introduction

Low-grade thermal energy remains one of the largest underutilized energy resources due to weak thermodynamic driving forces and the complexity of conventional heat-engine technologies [1,2]. Beyond established renewable energy technologies, increasing attention is being directed toward unconventional approaches that utilize low-grade and widely distributed thermal resources [3]. A significant portion of primary energy is dissipated as low- or moderate-temperature waste heat, motivating the development of technologies capable of harvesting such diffuse energy streams [4,5].

Several approaches have been proposed for converting low-grade heat into useful work. Thermoelectric generators provide direct solid-state conversion [6], while Organic Rankine and Kalina cycles rely on working-fluid-based thermodynamic processes optimized for low temperature differences [7,8]. Despite their maturity, these systems typically require working fluids, pressure vessels, and rotating components, thereby limiting their applicability in small-scale or maintenance-constrained environments. In general, thermoelectric generators offer relatively low conversion efficiency but benefit from compact solid-state operation, whereas ORC and Kalina systems generally achieve higher energy-conversion efficiencies at the expense of increased system complexity associated with working fluids, heat exchangers, and rotating machinery [6–8].

Solid-state thermal expansion provides an alternative pathway for harvesting low-grade thermal energy by directly converting the reversible expansion and contraction of structural materials into mechanical work. Existing research on thermoelastic energy harvesting has followed two principal directions. The first direction is based on shape-memory alloys (SMAs), which utilize reversible phase transformations to produce recoverable deformation and mechanical output from thermal cycling. Although SMA-based systems can achieve relatively high energy density, their practical application is constrained by material cost, thermal hysteresis, cyclic durability limitations, and the need for careful thermo-mechanical system optimization [9,10]. Recent studies have further demonstrated that the efficiency of thermoelastic harvesters strongly depends on the coupled optimization of material properties, thermal cycling, and system design parameters [10]. The second direction employs constrained thermal expansion of conventional structural materials governed by classical thermoelasticity and thermal-stress theory [11,12]. Within this framework, the present authors previously proposed a column-type actuator that converts reversible thermal elongation into mechanical output through constrained expansion [13]. However, existing studies have primarily focused on theoretical thermoelastic behavior or individual device concepts, while systematic analysis of modular thermoelastic energy harvesting, including scaling behavior, structural constraints, and techno-economic performance, remains limited. To address these gaps, this study develops a physically consistent modeling framework for modular thermoelastic energy harvesting based on constrained thermal expansion. The proposed framework extends previous device-level concepts by establishing system-level scaling relationships that explicitly account for coupled yield and buckling constraints in modular thermoelastic systems. Specifically, the study makes three principal contributions: (i) formulation of a physics-based framework linking thermoelastic work generation with coupled yield and buckling limits; (ii) derivation of scaling laws governing energy output, active mass, and cost within a modular architecture; and (iii) development of a techno-economic assessment supported by sensitivity and scenario analyses. A key engineering feature of the proposed architecture is the use of sleeve-coupled column segments, which enable scalable assembly while maintaining mechanical alignment and structural integrity.

The remainder of the paper is organized as follows. Section 2 presents the governing thermoelastic model and its finite-element verification. Section 3 develops the scaling framework and modular architecture. Section 4 evaluates structural feasibility and operating constraints. Section 5 presents the techno-economic assessment and sensitivity analysis. Section 6 discusses engineering implications and practical positioning, followed by the conclusions presented in Section 7.

2. Thermoelastic Governing Equations, Structural Limits, and FEM-Based Verification

For clarity and consistency, all symbols, abbreviations, and technical terms are defined at their first occurrence and are used consistently throughout the manuscript and appendices. The proposed concept converts reversible thermal expansion of solids into mechanical work under constrained conditions. The governing behaviour follows classical linear thermoelasticity [14–16]. For a uniform temperature change, the free thermal strain is

$$\varepsilon_T = \alpha \Delta T, \quad (1)$$

where α is the coefficient of linear thermal expansion and ΔT is the change in temperature.

When thermal expansion is constrained, the resulting strain generates internal stress. In the linear elastic regime, the corresponding axial stress is

$$\sigma_T = E \alpha \Delta T, \quad (2)$$

where E is Young's modulus. This thermoelastic stress generates the axial force available for mechanical work extraction under constrained conditions [15,16].

The admissible force is governed by material strength and structural stability. To ensure elastic operation and adequate fatigue resistance under cyclic loading [14,17], the axial force must remain below the yield limit, which depends only on material properties and cross-sectional area. According to Euler's buckling theory, the critical load decreases with increasing structural length and depends strongly on geometry and boundary conditions [18,19]. Beyond this limit, structural response becomes unstable [20]. The maximum usable force is therefore governed by the more restrictive of the yield and buckling limits, defining a force envelope for thermo-mechanical operation. The available displacement follows directly from thermal strain. For an element of length L , the thermal elongation is

$$\Delta L = \alpha L \Delta T. \quad (3)$$

This displacement represents the maximum stroke available per thermal cycle, independent of the specific conversion mechanism.

To verify the analytical thermoelastic formulation, a three-dimensional finite element model was developed in ANSYS Mechanical 2025 R2 using the Static Structural solver. A uniform temperature increase was applied to the entire steel rod, corresponding to the prescribed temperature difference adopted in the analytical model. The rod was modelled as an isotropic linear-elastic AISI 4340 steel member. Both end faces were axially constrained to reproduce the restrained thermal expansion assumed in the theoretical derivation. No external mechanical loads were applied, and thermal convection and radiation were neglected. Consequently, the computed stresses arose exclusively from restrained thermoelastic expansion. The objective of the finite-element model was to verify the analytical thermoelastic solution rather than to simulate the complete heat-transfer process. The numerical solution is based on the governing equations of small-strain linear thermoelasticity together with the analytical thermoelastic relations presented in Equations (1)–(3), while detailed derivations are provided in Appendix A. The corresponding thermal loading, boundary conditions, and reaction forces are illustrated in Figure 1.

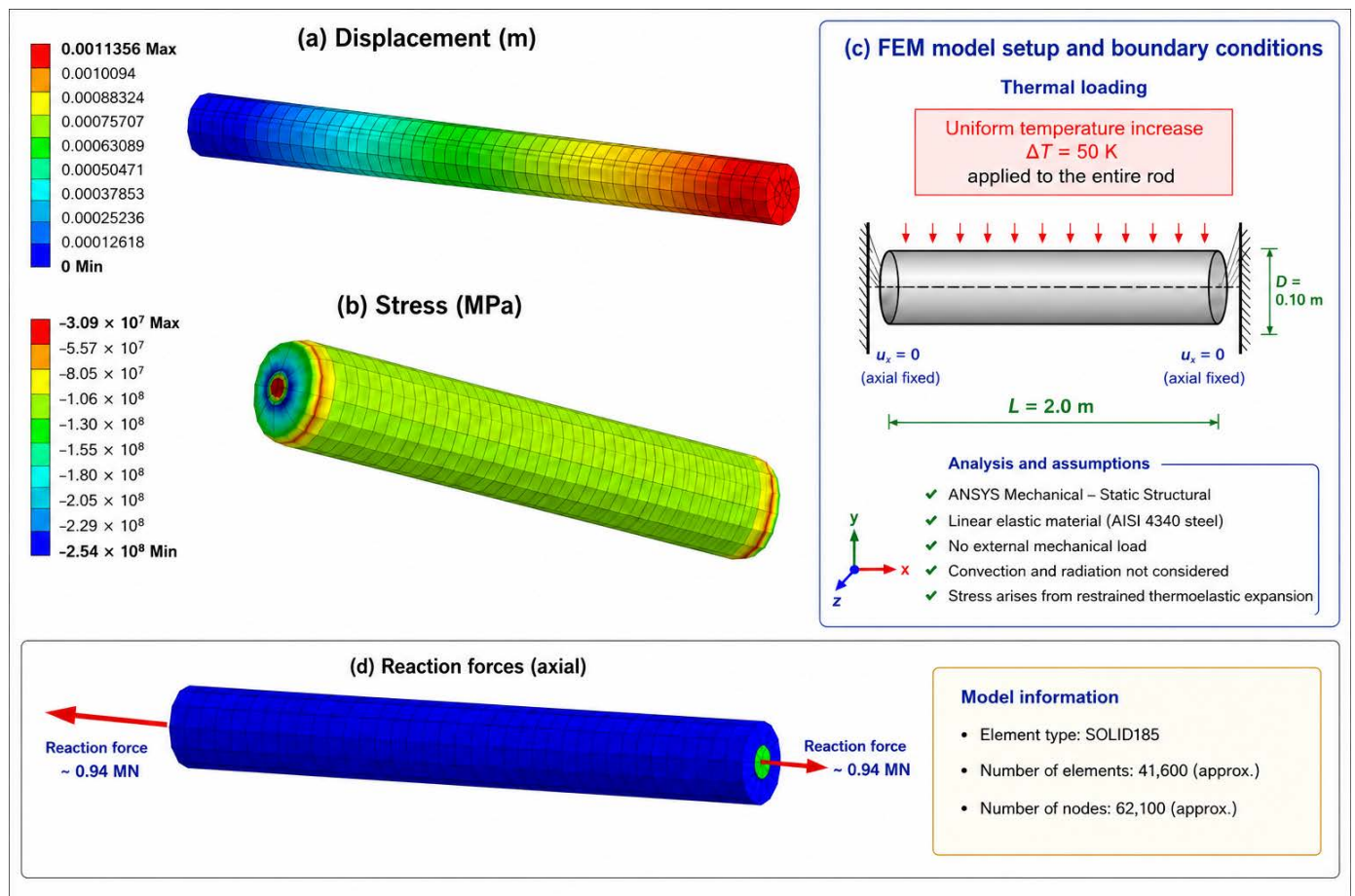


Figure 1. Finite element model, boundary conditions, and verification of the thermoelastic formulation. (a) Axial displacement distribution of the rod under free thermal expansion resulting from a uniform temperature increase. (b) Axial thermoelastic stress distribution under restrained thermal expansion. (c) Schematic of the finite element model showing the model geometry, applied uniform temperature increase ($\Delta T = 50$ K), boundary conditions, and principal modeling assumptions. (d) Axial reaction forces generated by restrained thermal expansion.

The finite-element results confirm the analytical thermoelastic formulation. As shown in Figure 1a, the axial displacement increases approximately linearly along the rod length under a uniform temperature increase. The corresponding axial stress remains nearly uniform throughout the rod, with localized stress concentrations near the restrained end faces caused by the imposed axial constraints (Figure 1b). For $E = 210$ GPa, $\alpha = 11.3 \times 10^{-6}$ K $^{-1}$, and $\Delta T = 50$ K, Equation (2) predicts an axial thermoelastic stress of 118.65 MPa. The finite element model yielded an average axial compressive stress of 119.3 MPa in the uniform central region of the rod, corresponding to a relative difference of approximately 0.5% from the analytical prediction. This close agreement provides strong validation of the analytical thermoelastic formulation. The equal reaction forces at the restrained ends confirm static equilibrium and the consistency of the imposed boundary conditions. For numerical verification, AISI 4340 steel was selected as a representative high-strength material; the rationale for this material selection is presented in Section 4.1. The reaction forces shown in Figure 1d represent the resultant thermoelastic force generated by restrained thermal expansion. Their equal magnitudes at the two restrained ends confirms the expected static equilibrium of the finite element model and provides further verification of the adopted boundary conditions and thermoelastic formulation. The geometry of the finite element model, including the adopted dimensions, thermal loading, axial restraints, and principal modeling

assumptions, is summarized in Figure 1c. At the system level, the admissible thermoelastic force directly determines the upper bound of mechanical work per cycle, linking structural mechanics with system-level energy output. The extractable mechanical work per cycle is fundamentally bounded by the product of admissible force and thermal displacement, establishing an architecture-independent limit on thermoelastic energy conversion. This mechanical constraint forms the basis for the system-level scaling analysis developed in the next section. A detailed derivation of the governing force limits is provided in Appendix A. The derivation of the work expression and scaling relations is presented in Appendix B. The theoretical background follows classical stability theory and thermoelasticity [19,21].

2.1. Thermodynamic Consistency of Thermoelastic Work Extraction

Although the system is formulated in mechanical terms, its driving mechanism is inherently thermodynamic and requires a time-varying temperature field rather than a uniform thermal state. Mechanical work is generated during both heating and cooling phases. During heating, constrained thermal expansion produces force and displacement, enabling work extraction; during cooling, contraction generates additional displacement in the opposite direction, which can also be utilised. Consequently, the system operates over a complete thermal cycle, rather than relying on a single phase.

The temperature variation may originate from diurnal fluctuations, direct solar heating of exposed elements, or low-grade waste heat, thereby allowing significantly larger temperature excursions than ambient variation alone. From a thermodynamic perspective, the maximum extractable work is limited by the available temperature change and associated heat input [3]. At the same time, mechanical constraints such as yielding and buckling restrict admissible force and displacement. Consequently, system performance is governed by both thermodynamic and structural limits, with the more restrictive limit determining the attainable output. The thermodynamic implications of these limits are further quantified through efficiency and exergy analysis in Section 5. The present framework evaluates the theoretical upper bound of recoverable mechanical work under quasi-static thermoelastic loading. Accordingly, the work expression is formulated as an idealized limit based on the admissible thermoelastic force and corresponding thermal displacement. In practice, the recoverable energy may be lower because of transmission flexibility, mechanical compliance, friction, generator coupling, and other irreversible losses. These effects influence the actual conversion efficiency but do not alter the governing thermoelastic force limits or the scaling relationships derived in this study. Their quantitative evaluation requires detailed dynamic system modeling and experimental validation and is therefore beyond the scope of the present physics-based scaling framework. Accordingly, the energy-conversion pathway considered in this study comprises four successive stages: absorbed thermal energy, available thermal exergy, recoverable thermoelastic mechanical work stored as elastic strain energy, and the corresponding electrical energy after mechanical transmission and generator conversion.

2.2. Finite-Element Verification

To verify the finite-element implementation, the numerical results were compared with analytical solutions for a cylindrical steel rod subjected to a uniform temperature increase of $\Delta T = 50\text{ }^{\circ}\text{C}$. Two benchmark cases were considered.

In the first case, both ends of the rod were fixed, resulting in restrained thermal expansion and a nearly uniform axial compressive stress distribution throughout the rod. The maximum compressive stress predicted by the finite element model was 119.3 MPa, which is in close agreement with the analytical value of 118.7 MPa. The corresponding stress distribution is shown in Figure 2.

D: Static Structural

Normal Stress

Type: Normal Stress(Z Axis)

Unit: Pa

Global Coordinate System

Time: 1 s

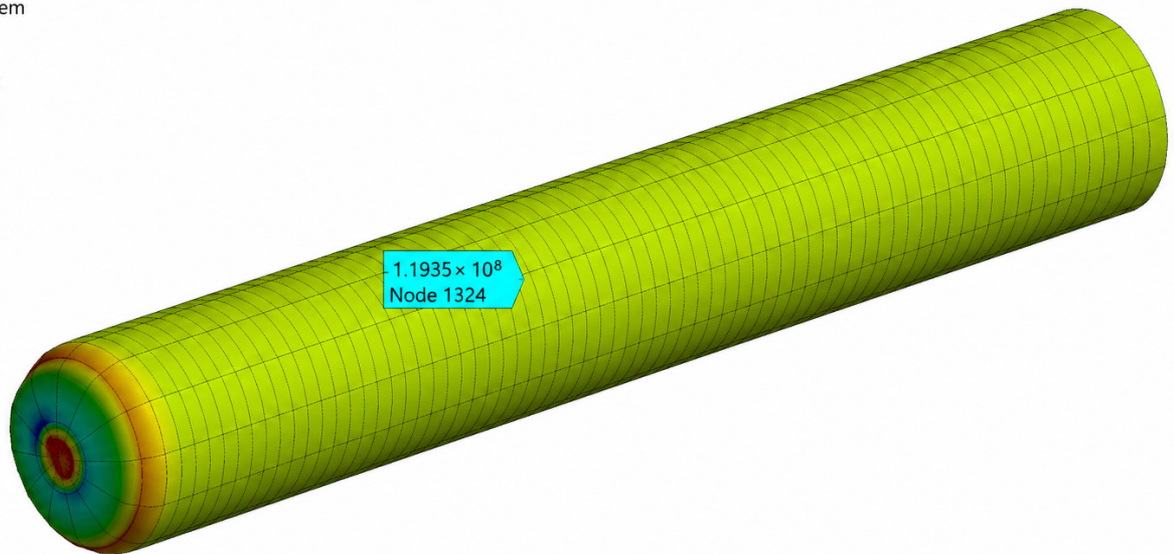
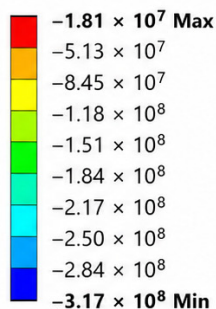


Figure 2. Finite element simulation of the restrained thermoelastic rod showing the axial thermoelastic stress distribution under a uniform temperature increase ($\Delta T = 50^\circ\text{C}$). The finite element model predicts an axial compressive stress of approximately 119.3 MPa in the uniform central region of the rod, in close agreement with the analytical prediction of 118.7 MPa.

In the second case, one end of the rod was left unconstrained to allow free thermal expansion. The finite element simulation predicted a maximum axial expansion of 1.135 mm, which is in excellent agreement with the analytical prediction of 1.130 mm. The resulting axial displacement field is presented in Figure 3.

B: Static Structural

Directional Deformation

Type: Directional Deformation (Z Axis)

Unit: m

Global Coordinate System

Time: 1 s

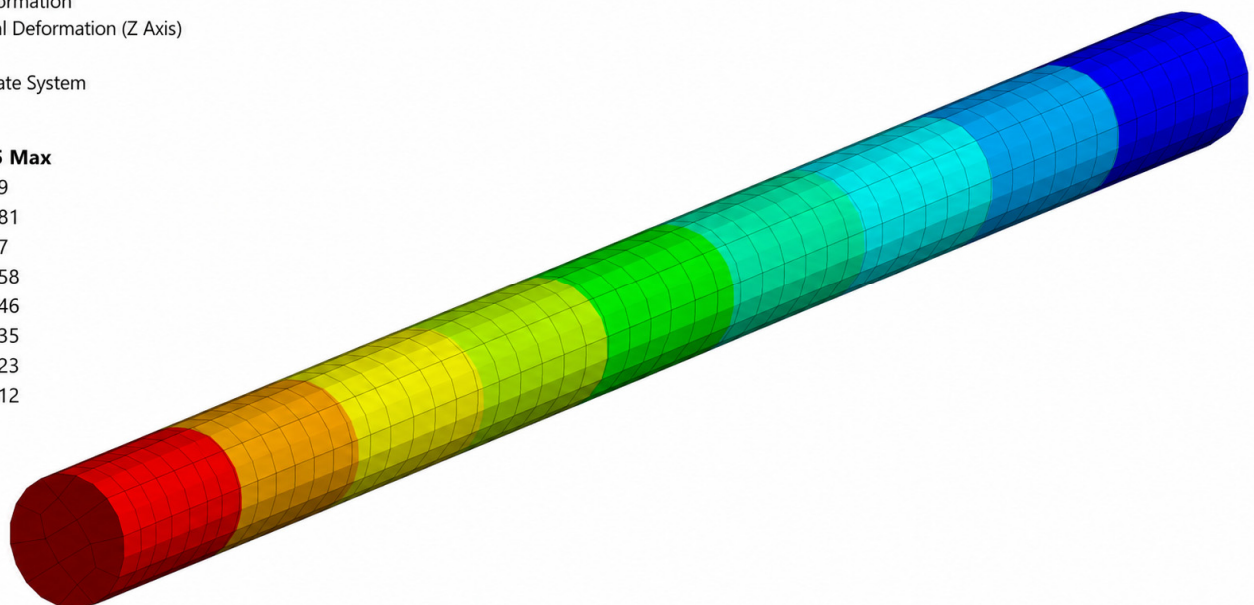
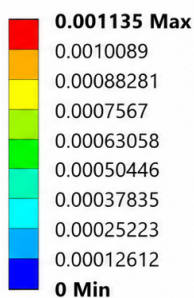


Figure 3. Finite element simulation of the freely expanding thermoelastic rod showing the axial directional deformation under a uniform temperature increase ($\Delta T = 50^\circ\text{C}$). The maximum axial expansion is 1.135 mm, in close agreement with the analytical solution (1.130 mm).

The close agreement between the analytical and numerical solutions confirms the correctness of the finite element model and demonstrates that the adopted modelling assumptions are suitable for the subsequent thermoelastic energy harvesting analysis. The analytical and numerical results are compared quantitatively in Table 1. The present finite-element model is intended to verify the fundamental thermoelastic response of the proposed concept rather than to simulate the complete modular energy-harvesting assembly. Detailed analyses of sleeve couplings, contact interfaces, cyclic loading, friction, and transmission components are beyond the scope of the present study and are reserved for future engineering optimisation studies.

Table 1. Comparison between analytical and finite element results used for model verification.

Parameter	Analytical	FEM
Maximum compressive stress	118.7 MPa	119.3 MPa
Maximum axial expansion	1.130 mm	1.135 mm

The relative differences between the analytical and finite-element solutions are 0.55% for the restrained thermoelastic stress and 0.44% for the free thermal expansion, demonstrating excellent agreement and confirming the predictive capability of the proposed analytical model within the adopted assumptions.

The finite-element results presented in this study are validated against the corresponding analytical thermoelastic solutions rather than experimental measurements. Direct quantitative comparison with published experimental studies is not appropriate because the available thermoelastic energy-harvesting concepts generally employ different materials, structural configurations, boundary conditions, and energy-conversion mechanisms. Consequently, the present numerical verification is intended to confirm the correctness of the proposed analytical framework. Experimental validation of the modular thermoelastic architecture using a dedicated laboratory prototype constitutes the next stage of this research and will enable direct assessment of the complete thermo-mechanical and electromechanical energy-conversion process.

The scope of the present numerical analysis should be clearly emphasised. The finite-element simulations presented in this study are intended solely to verify the governing thermoelastic formulation by evaluating the restrained and free thermal response of a representative AISI 4340 cylindrical rod under linear-elastic conditions. Accordingly, the numerical model does not represent the complete modular energy-harvesting system and therefore does not include sleeve-coupled joints, rack-and-pinion transmission, cyclic contact behavior, friction losses, generator dynamics, or electromechanical energy conversion. These components involve additional contact mechanics, structural nonlinearity, and multi-physics coupling that are beyond the objectives of the present physics-based scaling study. Although these components are not explicitly modelled in the present study, they are recognised as important engineering design considerations and will be investigated in detail during subsequent numerical and experimental studies.

3. Scaling Laws and Modular Architecture

The theoretical bounds derived in Section 2 provide the basis for understanding the scaling behavior of thermo-mechanical energy harvesting systems based on constrained thermal expansion. The extractable work per cycle is governed by the combined influence of admissible force and thermal displacement, leading to two distinct operating regimes: a yield-controlled regime and a buckling-controlled regime [16,19]. The transition between these regimes defines a characteristic structural length. For sufficiently long and slender ele-

ments, elastic instability dominates because of its strong dependence on structural geometry, whereas for shorter elements, the admissible force is limited by material strength [19].

To overcome this limitation, a modular architecture is introduced. The system is divided into N identical segments of length l_m , such that the total deployed length is the sum of individual modules. By selecting an appropriately short module length, buckling is suppressed and yield-controlled operation is ensured. Under these conditions, the admissible force becomes independent of structural length and is governed solely by material strength. The total displacement remains proportional to system length. As a result, modular segmentation preserves the total thermal stroke while suppressing elastic instability at the element level.

An essential outcome of this analysis is that, once buckling is structurally suppressed, the extractable mechanical work scales linearly with total active length and material cross-section. This establishes an approximately linear scaling of extractable work with system size under yield-controlled operation [19]. Buckling is therefore a structural, not thermodynamic, limitation and can be mitigated through modular design.

System Architecture and Working Principle

The configuration and operating principle of the proposed system are illustrated in Figure 4. The system utilises diurnal temperature variations to generate mechanical work through constrained thermal expansion of modular steel elements, consistent with prior thermo-mechanical concepts [13].

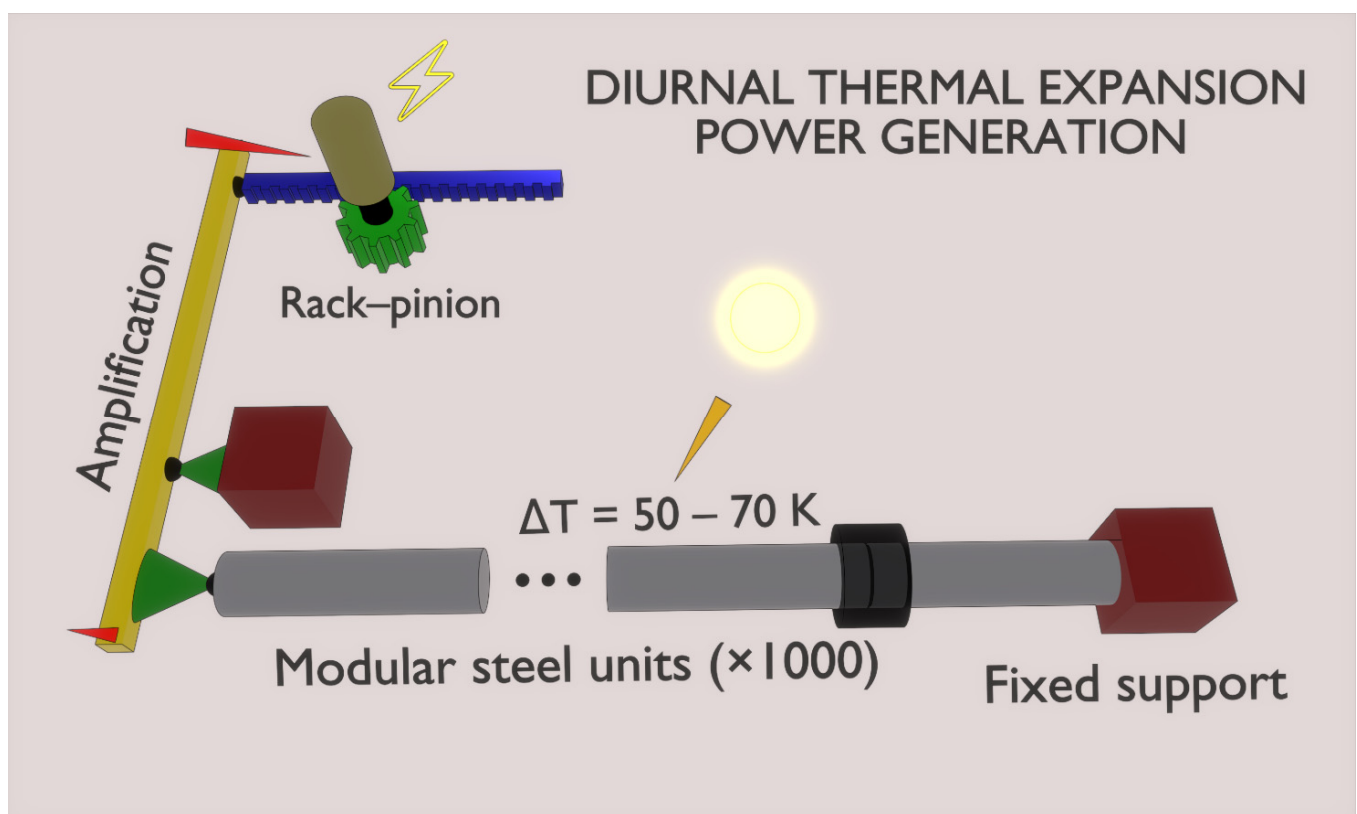


Figure 4. Schematic of the proposed thermoelastic energy harvesting architecture based on threaded sleeve-coupled modular columns. Individual column segments are interconnected via threaded sleeve couplings to form a scalable assembly that maintains axial alignment and suppresses buckling. Cyclic temperature variations induce constrained thermal expansion and contraction, generating axial force and displacement, which are converted into electrical energy through a mechanical transmission system. The modular configuration enables yield-controlled operation and preserves linear scalability with increasing system size.

As illustrated in Figure 4, the system consists of horizontally arranged column modules connected in series and anchored to a fixed support. Thermal expansion and contraction generate axial displacement, which is amplified and converted into mechanical work through a rack-and-pinion mechanism coupled to an electric generator.

The structural elements are implemented as modular segments connected via sleeve-coupled joints, allowing the total length to be adjusted through assembly. The sleeve-coupled joints maintain alignment and stable load transfer across the assembly. Each module undergoes cyclic expansion and contraction in response to cyclic temperature variations ($\Delta T \approx 50\text{--}70\text{ K}$). Under constrained boundary conditions, thermal deformation generates internal stress while producing axial displacement, forming the basis of energy conversion. Possible thermal sources, including diurnal fluctuations, solar heating, and low-grade industrial waste heat, are discussed in Section 2.1. The displacements of individual modules accumulate along the chain and are converted into rotational energy through a rack-and-pinion transmission. One end of the assembly is anchored to a fixed support, whereas the opposite end is connected to a displacement transmission mechanism. This configuration provides partial constraint of thermal expansion, allowing force development while avoiding fully free deformation. The modular design enables scalable increases in total displacement and extractable work while maintaining yield-controlled operation. The primary energy-conversion process relies on solid-state thermo-mechanical deformation and does not require working fluids or thermodynamic cycles.

An important consequence of the proposed scaling formulation is that the obtained scaling laws are geometry-independent once the operating regime has been identified. After normalization with respect to the active structural mass and operation within the yield-controlled regime, the governing relations become independent of the absolute number of modules and depend primarily on material properties, temperature swing, cycle frequency, and conversion efficiency. This generalized formulation allows the derived relationships to be applied to modular systems of different sizes without repeating the structural derivation.

4. Structural Feasibility and Operating Constraints

The scaling relations established in Sections 2 and 3 are meaningful only when the assumed operating regime is physically realisable. In particular, repeated thermoelastic cycling requires verification of yield-controlled operation, thermal response adequacy, fatigue durability, and negligible auxiliary energy consumption. This section defines a physically consistent feasibility envelope using AISI 4340 as a representative high-strength material.

4.1. Material Selection and Module-Level Feasibility

Once elastic instability is suppressed, system performance is governed primarily by material properties and deployed mass. The combined performance index $\alpha \cdot \sigma_y$ serves as a first-order indicator of extractable work, but feasibility depends on additional factors including stiffness, cyclic durability, manufacturability, and cost. Structural steels and low-alloy steels (S355, AISI 4140, AISI 4340) offer a favorable balance of strength, stiffness, and cost [22,23], whereas aluminum alloys are limited by lower strength and stiffness [24], and shape-memory alloys by high cost and limited scalability. Material properties are listed in Table A1 [18,22,24] (Appendix C).

A first-order comparison of performance index values is shown in Figure 5, where steels occupy a favorable region when performance and cost are considered jointly. Material performance alone is insufficient; stability constraints must also be satisfied. As shown in Section 3, elastic buckling imposes a geometry-dependent limitation that defines the admissible module length.

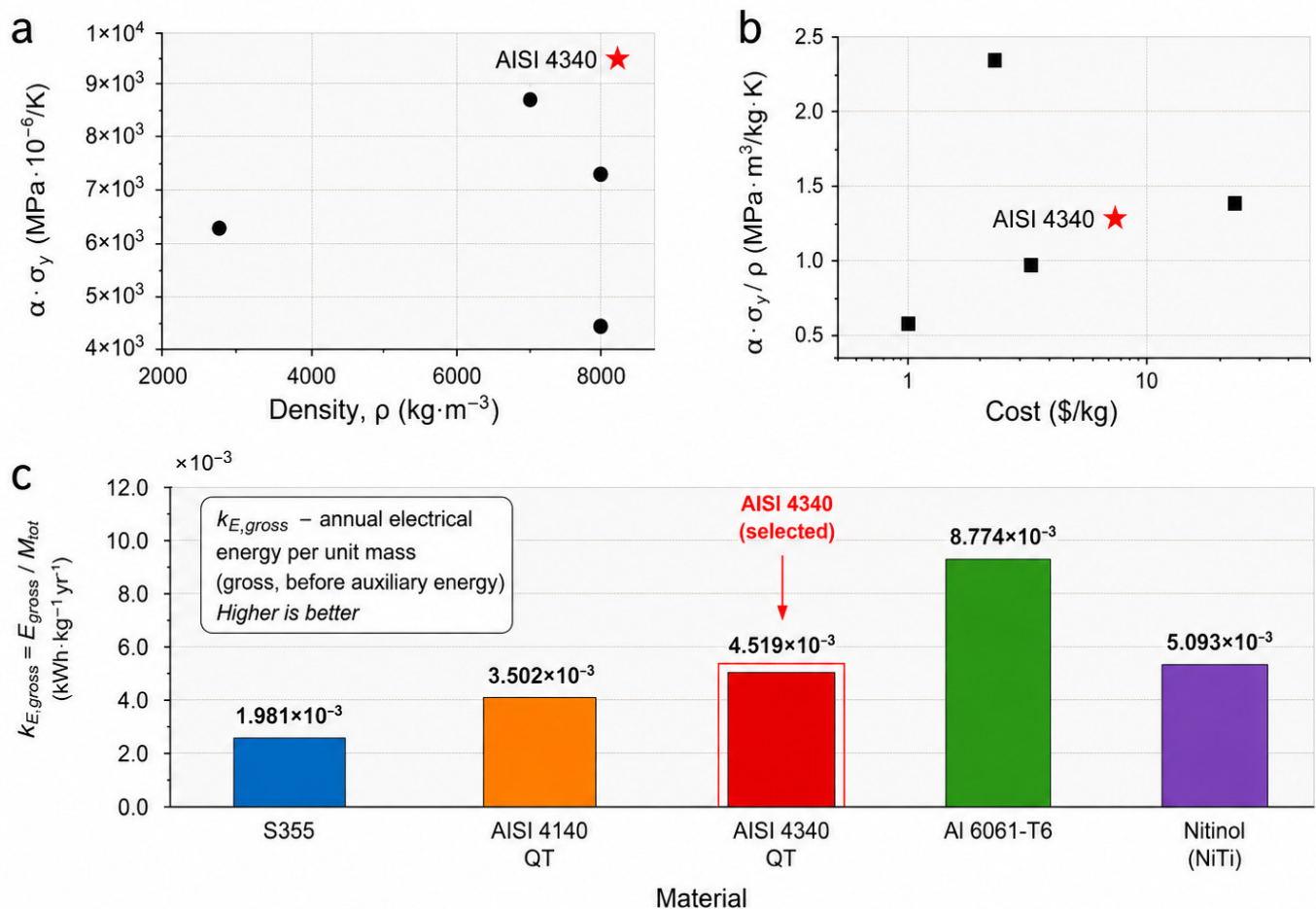


Figure 5. Material selection criteria for modular thermoelastic energy harvesting. (a) Thermo-mechanical performance index ($\sigma_y \alpha$) as a function of material density. (b) Cost-performance index ($\sigma_y \alpha / \rho$) as a function of material cost. (c) Annual recoverable electrical energy per unit mass, calculated using Equations (A15)–(A17) under the reference operating conditions ($\Delta T = 72$ K, $N_c = 1$ cycle/day, $\eta = 0.25$). The selected AISI 4340 steel is highlighted by a red five-pointed star, whereas the black circles and black squares represent the other candidate materials considered in the comparative analysis.

Figure 5 illustrates the trade-off between thermoelastic performance and material cost. As shown in Figure 5a, materials with higher yield strength exhibit a larger thermoelastic strength index ($\alpha \cdot \sigma_y$), indicating greater force-generation capability under constrained thermal expansion. High-strength steels therefore occupy the upper region of the performance map. Figure 5b further incorporates material cost, revealing a performance–cost trade-off among candidate materials. While some materials achieve high specific performance ($\alpha \cdot \sigma_y / \rho$), their elevated cost limits practical applicability. To complement the comparative material assessment, Figure 5c presents the annual recoverable electrical energy per unit mass, calculated using Equations (A15)–(A17). This metric reflects the intrinsic energy-generation capability of each material independently of the module dimensions and depends primarily on the material index ($\sigma_y \alpha / \rho$). As expected, Al 6061-T6 exhibits the highest mass-specific energy output because of its low density and relatively high thermal expansion coefficient. However, material selection for thermoelastic energy harvesting cannot be based solely on the mass-specific energy metric. Structural strength, elastic stiffness, buckling resistance, manufacturability, durability, and cost must also be considered. Considering all these criteria together, AISI 4340 provides the most favorable overall engineering compromise for modular thermoelastic energy harvesting systems.

Among the evaluated options, AISI 4340 provides a favourable combination of strength, density, and relative material cost, making it a practical engineering candidate for scalable thermoelastic energy harvesting systems.

To further illustrate the influence of structural geometry and material properties on the achievable annual energy generation, Figure 6 presents system-level scaling surfaces for the five representative structural materials over the investigated ranges of module diameter and length. Consistent with the governing scaling law (Equation (A8)), the predicted annual electrical energy increases approximately proportionally with both the active cross-sectional area and module length. Among the investigated materials, AISI 4340 (QT) consistently provides the highest annual energy output throughout the design space owing to its favorable combination of yield strength and thermal expansion characteristics. The figure therefore serves as a practical engineering design map for preliminary material selection and module sizing.

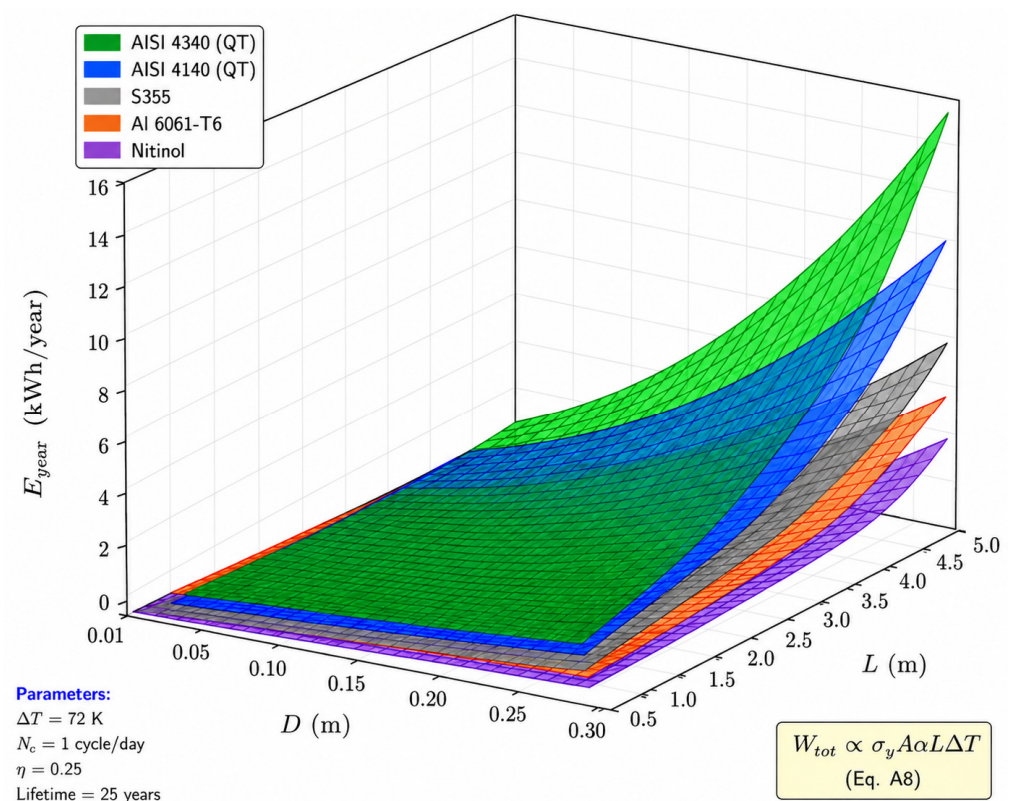


Figure 6. System-level scaling surfaces showing the predicted annual electrical energy generation as a function of module diameter (D) and module length (L) for representative structural materials under the reference operating conditions ($\Delta T = 72 \text{ K}$, $N_c = 1 \text{ cycle day}^{-1}$, $\eta = 0.25$). The surfaces visualize the governing scaling relationship expressed by Equation (A8) and provide a comparative engineering design map for material selection, module geometry optimization, and preliminary system design.

The maximum admissible axial force is governed by the lower of the material yield limit, F_{yield} , and the Euler buckling limit, F_{buckling} ,

$$F_{\text{max}} = \min(F_{\text{yield}}, F_{\text{buckling}}), \quad (4)$$

which defines the transition between yield-controlled and stability-controlled regimes and supports the basis for the subsequent parametric analysis. For the selected geometry ($D = 0.1 \text{ m}$) and restrained boundary conditions ($K = 0.5$), the transition between yield and buckling occurs at approximately $l_m \approx 2.46 \text{ m}$. The adopted module length ($l_m = 2.0 \text{ m}$) ensures operation within the yield-controlled regime, as indicated in Figure 7.

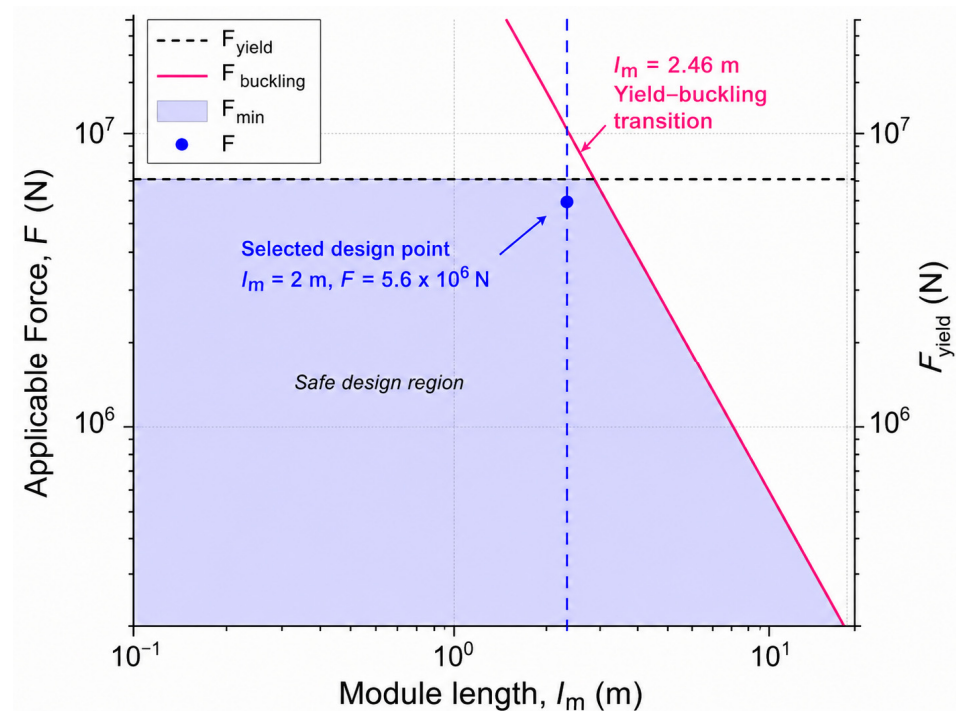


Figure 7. Regime map defining the structural partition boundary for modular thermoelastic columns. Yield force (F_{yield}) and Euler buckling force (F_{buckling}) are plotted as functions of module length (l_m) for $D = 0.1$ m and $K = 0.5$. Their intersection ($l_m \approx 2.46$ m) defines the transition between the yield-controlled and buckling-controlled regimes, while the shaded region represents the feasible design domain adopted in this study.

As shown in Figure 7, the intersection of the yield and buckling curves occurs at approximately $l_m \approx 2.46$ m, marking the transition between strength-limited and stability-limited behavior. For module lengths below this threshold, the admissible force is governed by material yielding, whereas for longer elements, buckling becomes the limiting factor. The selected module length ($l_m = 2.0$ m) maintains operation within the yield-controlled regime, avoiding buckling-induced limitations while maintaining a high admissible force level. This result highlights that feasibility is governed by the coupled interaction between material strength, stiffness, and geometry. Structural steels provide a robust design space due to their high modulus and yield strength, whereas aluminum alloys exhibit a significantly reduced feasible domain when stability is enforced. Figure 7 therefore serves as a regime map that partitions the design space into yield-controlled and buckling-controlled operating regions, providing a practical design criterion for selecting module dimensions.

4.2. Thermal Response, Fatigue Durability, and Practical Operating Conditions

Thermoelastic operation is governed by coupled thermal, mechanical, and operational constraints. The feasibility of multiple daily cycles depends on the transient thermal response of the modules. For the adopted geometry ($D = 0.1$ m), the characteristic diffusion time is on the order of minutes, indicating rapid internal heat conduction.

The Biot number is estimated as

$$\text{Bi} = \frac{hL_c}{k}, \quad (5)$$

where $L_c \approx D/4$ for a cylinder. For typical values of $k \approx 40\text{--}50$ W/m·K and $h \approx 50\text{--}100$ W/m²·K, Bi remains below 0.1, confirming that internal temperature gradients within the rod are negligible and that lumped thermal behavior is valid. The transient response therefore follows an exponential approach to equilibrium, as shown in Figure A1 of Appendix C [21].

Consequently, multi-cycle daily operation (e.g., $N_c \approx 4$) is physically feasible for meter-scale modules, provided sufficient environmental or process-driven heat exchange is available. The value $N_c \approx 4$ is considered as an upper-bound illustrative scenario representing enhanced heat-transfer conditions rather than a universal operating assumption. The system operates under long-term elastic cyclic loading. For the adopted 25-year lifetime, the total number of cycles remains within the intermediate fatigue regime for structural steels, even at multiple daily cycles. Fatigue-resistant operation requires that the alternating stress amplitude remains below a fraction of the yield strength, typically in the range of 0.5–0.7 for alloy steels [14,25]. Under elastic limits, high-strength steels such as AISI 4340 are expected to sustain the required cycle counts [14].

For a 25-year lifetime and four cycles per day, the cumulative cycle count is approximately 3.65×10^4 , well below the very-high-cycle fatigue regime for structural steels. Elastic operation is expected to limit fatigue damage accumulation [14,23,25].

Because internal thermal response is rapid, the dominant limitation on cycle frequency is external heat transfer rather than conduction within the material. Under passive conditions, natural convection and radiation provide sufficient heat exchange for low-frequency operation, while higher cycle frequencies require enhanced heat transfer through environmental exposure or process integration.

Cycle frequency is therefore governed primarily by heat transfer rather than structural constraints. Accordingly, baseline operation assumes no continuous auxiliary energy consumption, and multi-cycle performance reflects utilization of naturally available temperature variations rather than externally driven heating or cooling.

To avoid treating auxiliary demand as universally negligible, three operating cases are distinguished. In the passive case, temperature cycling is driven by ambient variation or existing waste-heat streams, so auxiliary energy is negligible. In the assisted case, low-power airflow or heat-exchange enhancement may accelerate thermal equilibration, and its energy demand must be subtracted from gross output. In the forced case, active heating or cooling can increase cycle frequency, but net output depends strongly on auxiliary consumption. Accordingly, the baseline analysis considers only passive and low-assist operation, whereas forced operation requires a separate net-energy assessment that explicitly accounts for auxiliary energy consumption.

The present fatigue discussion is limited to the global thermoelastic response of the structural members operating within the elastic regime. Local fatigue phenomena associated with threaded sleeve couplings, contact interfaces, stress concentrations, fretting wear, corrosion, manufacturing tolerances, and surface finish were not explicitly analyzed. These effects depend on detailed joint geometry and contact mechanics and therefore require dedicated finite-element contact analysis together with experimental S–N characterization. Such investigations are beyond the scope of the present physics-based scaling study and represent important topics for future numerical and experimental investigations.

5. System-Level Techno-Economic Analysis

This section integrates the structural scaling relations (Sections 2 and 3) with a techno-economic framework based on standard discounted cash-flow methods widely used in energy system analysis [26,27].

5.1. System-Level Scaling and Cost Framework

The system-level performance and economic viability are intrinsically linked through the linear scaling of energy output and capital cost with deployed structural mass. For yield-controlled operation, the annual electrical output of the system scales linearly with the number of modules and, equivalently, with total active mass.

Similarly, total system mass scales linearly with the number of modules, leading to a direct proportionality between annual energy output and deployed material mass. In the material-dominated regime, capital cost can be approximated as a linear function of total mass, with an additional constant term representing balance-of-system components. As a result, both annual energy production and capital investment scale linearly with system size [26,28]. This linear relationship is illustrated in Figure 8, which shows a near-proportional increase in energy output with module count and total mass.

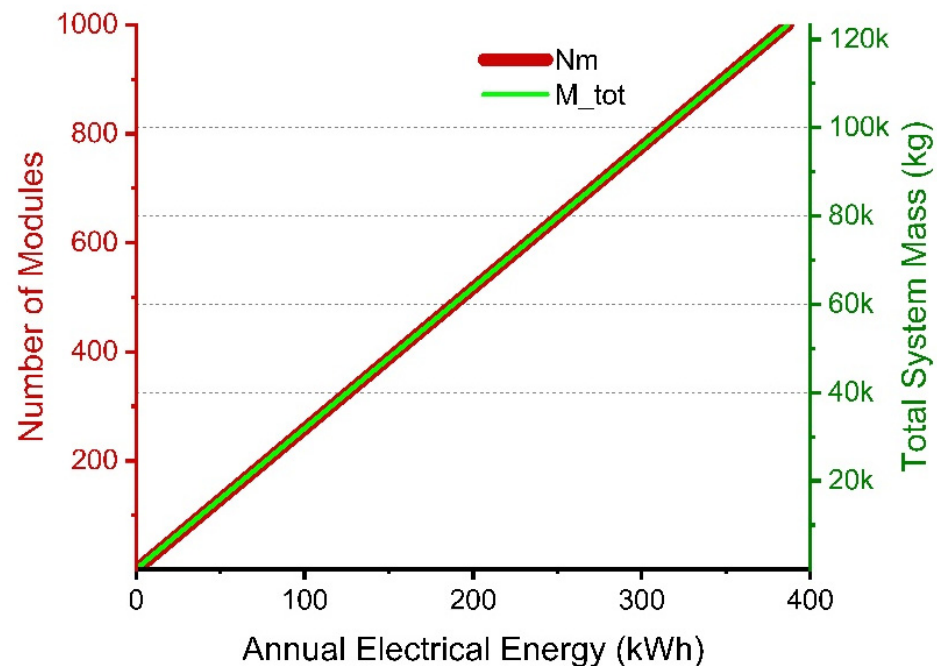


Figure 8. Linear scaling of annual electrical energy with module number and total active mass, illustrating proportional growth in system output under modular architecture.

The levelized cost of electricity (LCOE) is evaluated using the standard discounted cash-flow framework [26,27,29,30]. This value reflects a conservative baseline and illustrates the low energy density associated with thermoelastic energy conversion under the adopted assumptions. In early-stage systems with relatively simple mechanical design, operating costs are expected to be small compared to capital recovery and are accordingly neglected in first-order estimates. The baseline LCOE should therefore be interpreted as a first-order engineering estimate rather than a commercial benchmark, since the primary objective of this study is to identify physical scaling limits and design trends rather than to optimize a market-ready energy system.

Under these assumptions, LCOE reduces to a simplified form in which cost is primarily determined by the ratio of material cost to energy produced per unit mass. This highlights a key limitation of thermoelastic systems: the relatively low energy density associated with thermal expansion processes. For the baseline configuration (AISI 4340, $D = 0.1$ m, $L = 2.0$ m, $\Delta T = 72$ K, $N_c = 1$, $\eta = 0.25$, lifetime = 25 years, discount rate = 8%), the estimated electricity cost is approximately 10.4 USD/kWh. The assumed material cost (≈ 0.5 USD/kg) represents an illustrative large-scale structural steel fabrication cost adopted for the present techno-economic case study rather than the market price of specialty alloy steels. The simplified LCOE formulation, together with the case-study equations used for the Appendix D results, is provided in Table A2 and Appendix D.1 [26,27,29]. Additional thermodynamic performance metrics, including heat-input estimation, thermodynamic efficiency, exergy-based assessment, and comparison with the corresponding Carnot limit, are provided in Appendix D.2.

The present framework is intentionally simplified and is intended for comparative engineering assessment. Site-specific installation costs, maintenance logistics, financing conditions, and application-dependent balance-of-system costs are beyond the scope of the present analysis.

The present LCOE formulation intentionally represents a first-order, material-dominated engineering estimate intended for comparative evaluation of the proposed thermoelastic concept. Additional cost components, including fabrication, sleeve couplings, bearings, rack-and-pinion transmission, electrical generator, installation, maintenance, control systems, and heat-transfer enhancement, were not explicitly modeled because they are highly application-specific and depend on the final engineering implementation. These balance-of-system costs may increase the absolute LCOE but do not modify the governing scaling trends established in this study.

5.2. Sensitivity and Scenario Evaluation

System performance is primarily governed by three multiplicative parameters: cycle frequency, temperature variation, and conversion efficiency. Increasing any of these parameters improves annual energy output per unit mass and reduces LCOE, while coordinated improvements produce a compounded effect [4,5,31–33].

Baseline and advanced performance scenarios for different combinations are shown in Table 2. All performance metrics reported in Table 2 are evaluated using the thermo-mechanical and economic framework defined in Appendix D.

Table 2. Baseline and advanced performance scenarios for different combinations of temperature swing (ΔT), cycle frequency (N_c), and mechanical-to-electrical conversion efficiency (η), including the corresponding thermodynamic performance metrics.

Case	ΔT , K	N_c	η	E_{gross} , kWh/yr	E_{aux} , kWh/yr	E_{net} , kWh/yr	$k_{E,\text{net}}$, kWh/kg-yr	LCOE_{net} , USD/kWh	η_{th} , %	η_{ex} , %	η_{Carnot} , %
S_0 Baseline	72	1	0.25	0.557	0	0.557	4.53×10^{-3}	10.4	0.521	5.07	19.19
S_1 Increased cycle frequency	72	4	0.25	2.228	0.63	1.595	1.30×10^{-2}	3.6	0.521	5.07	19.19
S_2 Increased Temperature swing	120	1	0.25	0.928	0	0.928	7.54×10^{-3}	6.2	0.521	3.31	28.36
S_3 Combined enhanced operation	120	4	0.40	5.94	0.63	5.307	4.31×10^{-2}	1.1	0.521	3.31	28.36

Note: E_{gross} scales linearly with N_c , ΔT , and the mechanical-to-electrical conversion efficiency η . For $N_c > 1$ scenarios (S_1 , S_3), a conservative auxiliary energy consumption of $E_{\text{aux}} = 0.63 \text{ kWh} \cdot \text{yr}^{-1}$ per module is assumed. In the material-dominated regime, $\text{LCOE}_{\text{net}} \approx (C_m \cdot \text{CRF}) / k_{E,\text{net}}$, preserving the proportional scaling with total active mass. The thermodynamic efficiency, η_{th} , exergy efficiency, η_{ex} , and Carnot efficiency η_{Carnot} , are evaluated using the methodology described in Appendix D.2, assuming an ambient temperature of 30 °C.

Specifically, the gross annual electrical energy, E_{gross} , is calculated using Equation (A15), the net annual electrical energy, E_{net} , using Equation (A16), and the mass-specific net annual energy yield, $k_{E,\text{net}}$, using Equation (A17). The levelized cost of electricity is evaluated using Equations (A18)–(A21). This formulation ensures that all scenarios are evaluated using a consistent analytical and economic framework, allowing direct comparison of the governing parameters. The selected scenarios are not intended to represent successive development stages but rather independent parametric cases illustrating the individual and combined influence of temperature swing, cycle frequency, and conversion efficiency on system performance.

Net electrical output must account for auxiliary energy consumption associated with heat exchange. However, this contribution remains relatively small and does not alter the fundamental scaling behavior. From a thermodynamic perspective, performance may also be evaluated relative to the available thermal exergy. Exergy provides a complementary thermodynamic measure of performance, and the corresponding metrics presented in Appendix D.2 exhibit trends consistent with those obtained from energy-based indicators. The results summarized in Table 2 demonstrate that increasing cycle frequency and temperature swing significantly improves performance, while combined optimization produces a compounded improvement because the governing parameters contribute multiplicatively to annual energy generation. In the most advanced scenario (S_3), the estimated electricity cost decreases to approximately 1.1 USD/kWh, illustrating the theoretical upper-bound potential of the concept under favorable operating conditions. The S_3 scenario should therefore be interpreted as an illustrative upper-bound scenario rather than an expected operating condition. The thermodynamic and exergy efficiencies associated with the analyzed scenarios remain substantially below the corresponding Carnot limit, consistent with the limited exergy content of low-grade heat sources. This behavior reflects the intrinsic limits of thermoelastic energy conversion rather than a limitation of the modular architecture.

Importantly, the architecture exhibits strict linear mass scalability: increasing system size raises both energy output and capital cost proportionally. As a result, geometric scaling alone cannot reduce LCOE. These observations provide the engineering basis for the improvement pathways discussed in Section 6.2. Meaningful cost reduction requires increasing energy extracted per unit mass through improved utilization, higher temperature gradients, and enhanced conversion efficiency.

The sensitivity analysis further indicates a clear hierarchy among the governing parameters. Cycle frequency provides the largest practical improvement because it directly increases annual energy production without increasing structural mass. Increasing temperature swing provides a secondary benefit, although it is constrained by the availability of suitable heat sources and material operating limits. Improvements in conversion efficiency contribute proportionally but remain bounded by the relatively small amount of available thermoelastic work. Consequently, practical performance enhancement should prioritize increased utilization of naturally available thermal cycles before pursuing more aggressive thermal or conversion-system optimization.

Although the present analysis is deterministic, the principal sources of engineering uncertainty arise from variability in material properties (Young's modulus, thermal expansion coefficient, and yield strength), operating temperature swing, cycle frequency, and practical conversion efficiency. The governing scaling relations indicate that annual energy generation varies approximately linearly with temperature swing, cycle frequency, and conversion efficiency, whereas the admissible thermoelastic force depends primarily on the material properties. Consequently, moderate uncertainties in these parameters mainly affect the quantitative magnitude of the predicted energy output without changing the governing scaling behavior, operating regimes, or the principal engineering conclusions presented in this study.

6. Discussion

6.1. Practical Performance and Energy Density Considerations

Previous studies have demonstrated stable and repeatable force generation under constrained thermoelastic cycling, confirming the physical realisability of the operating principle [34]. A conservative temperature swing of $\Delta T = 60$ K is considered, representative of typical industrial waste-heat streams (≈ 80 – 100 °C source and ≈ 20 – 30 °C ambient). A system with 1000 modules (total mass $\approx 1.23 \times 10^5$ kg) operating at two cycles per day

produces approximately 928 kWh/yr (see Table A2 in Appendix D). This result highlights the low energy density characteristic of thermoelastic energy conversion and should be interpreted as a quantitative upper-bound estimate rather than a target for commercial deployment. Although modest, the output reflects intrinsically low thermoelastic work density. The system should be interpreted as a distributed solid-state heat-recovery technology rather than a primary power-generation system. The absence of fluids and pressure vessels enables operation where conventional heat engines may be impractical.

System performance is governed by intrinsic thermoelastic work density, independent of geometric scale. Energy output should be normalised by active material mass. For the reference configuration, the net annual energy density is on the order of 0.009–0.018 kWh/kg, consistent with the fundamental limits of elastic strain energy storage. This value is significantly lower than flux-driven renewable systems such as photovoltaics and wind, which are governed by high natural energy flux [35,36].

Unlike PV and wind systems, thermoelastic devices rely on internally stored strain energy rather than high natural energy flux. Accordingly, the proposed concept should be interpreted as a structurally integrated, solid-state heat-recovery solution and not as a direct competitor to flux-driven renewable technologies.

6.2. Scaling Implications and System-Level Improvement Pathways

Compared with previously reported thermoelastic energy harvesting concepts, which primarily focus on demonstrating individual thermo-mechanical actuators or material-level behavior, the present work extends the analysis to the system level by establishing analytical scaling laws, identifying the transition between yield-controlled and buckling-controlled regimes, introducing a modular sleeve-coupled architecture, and integrating structural, thermodynamic, exergy, and techno-economic analyses within a unified framework. Rather than proposing a new thermoelastic material or actuator, this study provides a comprehensive engineering methodology for evaluating the physical limits, scalability, and practical feasibility of modular thermoelastic energy harvesting systems. The principal distinctions between representative previous studies and the present work are summarized in Table 3.

Table 3. Comparison of representative thermoelastic energy harvesting studies with the present work. The symbols ✓ and ✗ denote the presence and absence, respectively, of the corresponding feature.

Feature	Representative Previous Thermoelastic Studies	Present Work
Demonstration of thermoelastic actuation	✓	✓
Modular architecture	✗	✓
System-level scaling laws	✗	✓
Yield–buckling regime analysis	✗	✓
Finite-element verification	Limited	✓
Thermodynamic efficiency evaluation	Limited	✓
Exergy analysis	Rare	✓
Techno-economic assessment	Rare	✓
Material selection framework	Limited	✓
Practical engineering design guidelines	Limited	✓

The parameter hierarchy identified in Section 5.2 provides a practical roadmap for future system development and optimization. The observed scaling behavior defines both the fundamental limitations and the available pathways for system-level performance improvement. The analysis establishes robust system-level scaling laws: both annual energy output and capital cost scale linearly with active structural mass, leading to an LCOE that is approximately independent of system size [26,27]. This behavior arises from

the fundamental relation between thermoelastic force and displacement, both of which are bounded by material properties and temperature variation. Consequently, increasing system size proportionally increases both cost and energy output without improving energy density.

Meaningful performance improvement must come from increasing energy extracted per unit mass. As shown in Section 5, this can be achieved through higher cycle frequency, larger temperature variation, and improved conversion efficiency, which act multiplicatively [32,33]. Key challenges include fatigue, wear, and thermal-management trade-offs [25].

Further improvements may be achieved through material optimisation [37], improved mechanical transmission efficiency and electromechanical energy conversion [38–41], enhanced thermal management [42], lightweight structural design [43,44], balance-of-system optimization [26,27], and higher utilization of transient thermal gradients [45,46]. These conclusions are expected to remain valid irrespective of absolute system size because the governing scaling relations are normalized with respect to active structural mass.

6.3. Positioning Relative to Solar PV and Onshore Wind

Photovoltaic and wind technologies achieve low LCOE due to high natural energy flux and technological maturity [47]. However, they are optimized for applications characterized by abundant solar or wind resources, whereas thermoelastic systems target fundamentally different operating environments [48,49]. The proposed thermoelastic system is not intended to compete with PV or wind in bulk generation. Instead, it targets niche applications characterized by low-grade heat availability, modular deployment requirements, and harsh operating conditions. As summarized in Table 4, the concept offers a solid-state, structurally integrated alternative with potentially high robustness and low maintenance, but inherently limited by low energy density.

The objective of the comparison in Table 4 is not to position thermoelastic harvesting as an alternative to utility-scale photovoltaic or wind generation. Instead, the comparison highlights the fundamentally different operating domain of the proposed concept. Thermoelastic harvesting is most suitable where low-grade thermal energy is already available but remains unused, particularly on inaccessible industrial waste-heat surfaces, pipelines, pressure vessels, furnaces, and other high-temperature structural components where conventional renewable technologies cannot directly recover thermal energy. Additional niche applications include structurally integrated self-powered sensing, distributed condition monitoring, remote industrial infrastructure, and auxiliary low-power systems operating in harsh environments requiring high mechanical robustness and minimal maintenance.

This work establishes a physics-based framework for thermoelastic energy harvesting and provides quantitative scaling relations, thermodynamic performance limits, and engineering design guidance for evaluating modular solid-state heat-recovery systems.

The temperature swings and daily operating cycles considered in this study represent scenario-based engineering assumptions rather than predictions for a specific installation. Their practical realization depends on the characteristics of the available heat source, ambient conditions, thermal inertia, heat-transfer coefficients, and the selected passive or active cooling strategy. Consequently, the present results should be interpreted as generalised performance envelopes intended for preliminary system design rather than site-specific performance predictions. Detailed transient thermal analyses using representative industrial heat-source profiles constitute an important subject for future work.

Table 4. Engineering positioning of thermoelastic harvesting relative to utility-scale photovoltaic and wind technologies, emphasizing complementary application domains rather than direct competition.

Criterion	Solid-State Thermal Expansion (This Work)	Utility-Scale Solar PV	Land-Based Wind
Primary energy resource	Temperature gradients/low-grade heat	Solar irradiance	Wind kinetic energy
Typical natural energy flux	Intrinsically low (thermoelastic work density limited)	High surface irradiance (hundreds of W/m ²) [35,50]	Site-dependent wind power density (high-resource sites) [47,49]
Grid-scale LCOE (favorable sites)	Not targeted for bulk generation	Among lowest-cost bulk generation technologies [35,36]	Among lowest-cost bulk generation technologies [47,50]
Structural complexity	Solid-state; minimal moving parts	No moving parts	Rotating machinery with drivetrain
Operation & Maintenance (O & M) cost contribution	Architecture-dependent; potentially low	Generally modest share of LCOE [51]	O & M represents a significant life-cycle cost component [49]
Environmental sensitivity	Architecture-dependent; potentially robust	Soiling and degradation may reduce output in dusty regions [48]	Mechanical wear and access constraints affect O & M [49]
Modularity and scaling	Linear scaling with active mass (Sections 5.1 and 5.2)	Area-based modular scaling	Larger unit scale; nonlinear project scaling
Best-fit applications	Niche: low-grade heat, harsh/remote environments	Grid-connected and distributed generation	Utility-scale in high-wind regions
Fundamental constraint	Low thermoelastic energy density	Intermittency	Intermittency and mechanical wear

6.4. Study Limitations and Experimental Validation Roadmap

The present study is intentionally limited to a physics-based analytical framework supported by finite-element verification. Although the analytical predictions are internally consistent and the finite-element simulations closely reproduce the governing thermoelastic response, no experimental prototype has yet been constructed. Consequently, practical issues such as transmission compliance, friction losses, backlash, cyclic durability of sleeve couplings, generator matching, thermal contact resistance, and long-term environmental effects remain to be quantified experimentally.

Future work will therefore focus on a staged experimental validation programme. The first stage will involve laboratory-scale verification of thermoelastic displacement, force generation, and recoverable mechanical work using a single constrained modular element under controlled thermal cycling. The second stage will evaluate complete electromechanical energy conversion, including the rack-and-pinion transmission, generator coupling, and electrical output. The final stage will investigate long-term cyclic durability, thermo-mechanical reliability, and field validation under representative industrial waste-heat conditions. These experiments will provide quantitative validation of the analytical scaling laws developed in the present study and support future engineering optimization of the proposed system.

While the baseline fatigue assessment presented in this study evaluates the global elastic cyclic loading of the primary AISI 4340 thermoelastic rod, localized fatigue behavior at critical structural junctions also requires explicit engineering consideration. In particular, threaded sleeve couplings and rack-and-pinion transmission interfaces are expected to represent the most fatigue-sensitive regions because geometric discontinuities and repeated contact loading may generate localized stress concentrations. Consequently, the local fatigue strength of these components may be lower than that of the smooth cylindrical members forming the primary thermoelastic structure. From an engineering perspective, these risks should be mitigated

through appropriate thread geometry, generous root radii, high manufacturing quality, suitable surface engineering, effective lubrication of moving contacts, and periodic inspection during service. Although these local mechanisms are not explicitly simulated in the present finite-element model, they are recognized as practical design constraints that must be considered in the engineering design and long-term reliability assessment of modular thermoelastic energy-harvesting systems.

7. Conclusions

This study developed a physics-based framework for modular thermoelastic energy harvesting based on the reversible thermal expansion and contraction of structural materials. The proposed methodology integrates analytical thermoelastic modeling, finite-element verification, thermodynamic assessment, and techno-economic analysis to establish physically consistent performance limits for solid-state energy harvesting from low-grade heat.

The results demonstrate that the maximum recoverable mechanical work is fundamentally constrained by material yield strength, elastic buckling, and the available temperature difference. Modular segmentation enables yield-controlled operation while preserving approximately linear scaling of energy output with deployed structural mass. The finite-element simulations showed excellent agreement with the analytical thermoelastic predictions, thereby validating the governing formulation adopted in this study.

The thermodynamic and exergy analyses indicate that the achievable conversion efficiency remains substantially below the Carnot limit because of the intrinsic characteristics of thermoelastic energy conversion. Consequently, the proposed concept is best suited for applications where structural simplicity, modular scalability, robustness, and low maintenance are more important than high energy density, particularly for recovering low-grade industrial waste heat and other slowly varying thermal resources.

The present framework represents a first-order physics-based assessment rather than a complete engineering design. Local contact mechanics, threaded sleeve couplings, fatigue of mechanical interfaces, transmission losses, and complete electromechanical conversion were not explicitly modeled, while the techno-economic assessment was intentionally formulated as a simplified material-dominated scaling analysis. These aspects require detailed numerical investigation and experimental validation in future work.

Overall, the proposed framework establishes quantitative scaling laws, structural feasibility limits, and engineering design guidelines that provide a foundation for the future optimization, prototyping, and experimental development of modular thermoelastic energy-harvesting systems.

Author Contributions: Conceptualization, A.G.P.; methodology, A.G.P. and O.U.N.; software, A.G.P. and A.A.o.A.; validation, A.G.P. and O.U.N.; formal analysis, A.G.P.; investigation, A.G.P., A.A.o.A. and O.N.O.; resources, A.A.o.A. and O.U.N.; data curation, A.G.P.; visualization, A.G.P.; writing—original draft preparation, A.G.P.; writing—review and editing, all authors; supervision, A.G.P. and G.A.K.; project administration, G.A.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The original contributions presented in this study are included in the article and Appendices A–D. Further inquiries can be directed to the corresponding author.

Acknowledgments: During the preparation of this manuscript, the authors used AI-assisted language-editing tools solely to improve the clarity and readability of the text. The authors carefully reviewed and edited all AI-assisted suggestions and take full responsibility for the content of the manuscript.

Conflicts of Interest: The authors declare no conflicts of interest.

Nomenclature

Symbol	Definition	Unit
A	Cross-sectional area	m ²
Bi	Biot number	–
C _m	Material cost per unit mass	USD·kg ^{−1}
C _{tot}	Total capital cost	USD
c _p	Specific heat capacity	J·kg ^{−1} ·K ^{−1}
CRF	Capital recovery factor	–
D	Module diameter	m
E	Young's modulus	GPa
E _{aux}	Auxiliary annual energy consumption	kWh·yr ^{−1}
E _{gross}	Gross annual electrical energy	kWh·yr ^{−1}
E _{net}	Net annual electrical energy	kWh·yr ^{−1}
Ex _{in}	Available thermal exergy input	J
F _{buckling}	Euler buckling load	N
F _{max}	Maximum admissible force	N
F _{yield}	Yield-limited axial force	N
h	Convective heat-transfer coefficient	W·m ^{−2} ·K ^{−1}
I	Second moment of area	m ⁴
k	Thermal conductivity	W·m ^{−1} ·K ^{−1}
k _{E,net}	Mass-specific net annual electrical energy yield	kWh·kg ^{−1} ·yr ^{−1}
K	Effective length factor	–
L	Module length	m
L _c	Characteristic length	m
m	Module mass	kg
m ₁	Mass of one module	kg
M _{tot}	Total active mass	kg
N _c	Number of thermal cycles per day	–
N _{tot}	Total number of modules	–
n	System lifetime	yr
Q _{in}	Thermal energy input	J
r	Discount rate	–
T _{amb}	Ambient temperature	K
T _{hot}	Heat-source temperature	K
T ₀	Initial temperature	K
T _∞	Ambient (surrounding) temperature	K
W _{cyc}	Mechanical work per thermal cycle	J
W _{hc}	Mechanical work per half-cycle	J
W _{max}	Maximum recoverable mechanical work	J
α	Coefficient of linear thermal expansion	K ^{−1}
α _{th}	Thermal diffusivity	m ² ·s ^{−1}
ΔL	Thermal elongation	m
ΔL _m	Thermal elongation of one module	m
ΔL _{tot}	Total thermal elongation	m
ΔT	Temperature difference	K
ε _T	Free thermal strain	–
η	Mechanical-to-electrical conversion efficiency	–
η _{th}	Thermodynamic efficiency	%
η _{ex}	Exergy efficiency	%
η _{Carnot}	Carnot efficiency	%
ρ	Density	kg·m ^{−3}
σ _T	Thermoelastic stress	MPa
σ _y	Yield strength	MPa
τ	Thermal time constant	s

Appendix A. Mechanical Derivation of Force Limits

Under conditions of constrained thermal expansion, the thermoelastic stress follows Equation (2) in the main text, which represents the fully constrained stress state in linear thermoelasticity [21]. The maximum admissible axial force is governed by the combined effect of material strength and structural stability.

To ensure reversible operation under cyclic loading, the axial stress must remain below the material yield strength σ_y . The corresponding yield-limited force is

$$F_{\text{yield}} = \sigma_y A, \quad (\text{A1})$$

where A is the cross-sectional area.

Slender members are additionally limited by elastic instability. According to Euler buckling theory [19], the critical load is

$$F_{\text{buckling}} = \frac{\pi^2 EI}{(KL)^2} \quad (\text{A2})$$

where E is the Young's modulus, I is the second moment of area, L is the structural length, and K is the effective length factor determined by boundary conditions.

The admissible force is therefore

$$F_{\text{max}} = \min(F_{\text{yield}}, F_{\text{buckling}}), \quad (\text{A3})$$

and the transition between regimes is obtained from

$$\sigma_y A = \frac{\pi^2 EI}{(KL)^2}. \quad (\text{A4})$$

This formulation is consistent with standard stability–strength interaction frameworks used in structural mechanics [19,21].

Appendix B. Derivation of Work and Scaling Relations

For a structural element subjected to temperature variation, the maximum mechanical work per cycle, W_{max} , is bounded by

$$W_{\text{max}} \leq F_{\text{max}} \Delta L, \quad (\text{A5})$$

where the thermal elongation ΔL is defined in the main text.

In the yield-controlled regime, this becomes

$$W_{\text{max}} \leq \sigma_y A \alpha L \Delta T. \quad (\text{A6})$$

For a modular system with N elements connected in series, the total displacement, ΔL_{tot} , accumulates linearly,

$$\Delta L_{\text{tot}} = N \Delta l_m = \alpha L_{\text{tot}} \Delta T, \quad (\text{A7})$$

(where Δl_m is the thermal elongation of a single module, L_{tot} is the total length of the modular assembly) leading to total work, W_{tot} , scaling as

$$W_{\text{tot}} \propto \sigma_y A \alpha L_{\text{tot}} \Delta T. \quad (\text{A8})$$

Since total mass is $M_{\text{tot}} = \rho A L_{\text{tot}}$, the scaling relation becomes

$$W_{\text{tot}} \propto M_{\text{tot}}. \quad (\text{A9})$$

This linear scaling behavior is discussed in the main text (Section 3) and is consistent with structural energy scaling principles [19,21].

Appendix C

As shown in Figure A1, more than 95% of the temperature change occurs within a short time window ($\sim 3\tau \approx 10\text{--}12$ min). Since practical cycle durations are on the order of hours, internal heat diffusion does not limit operation. Instead, cycle frequency is governed primarily by external heat transfer conditions.

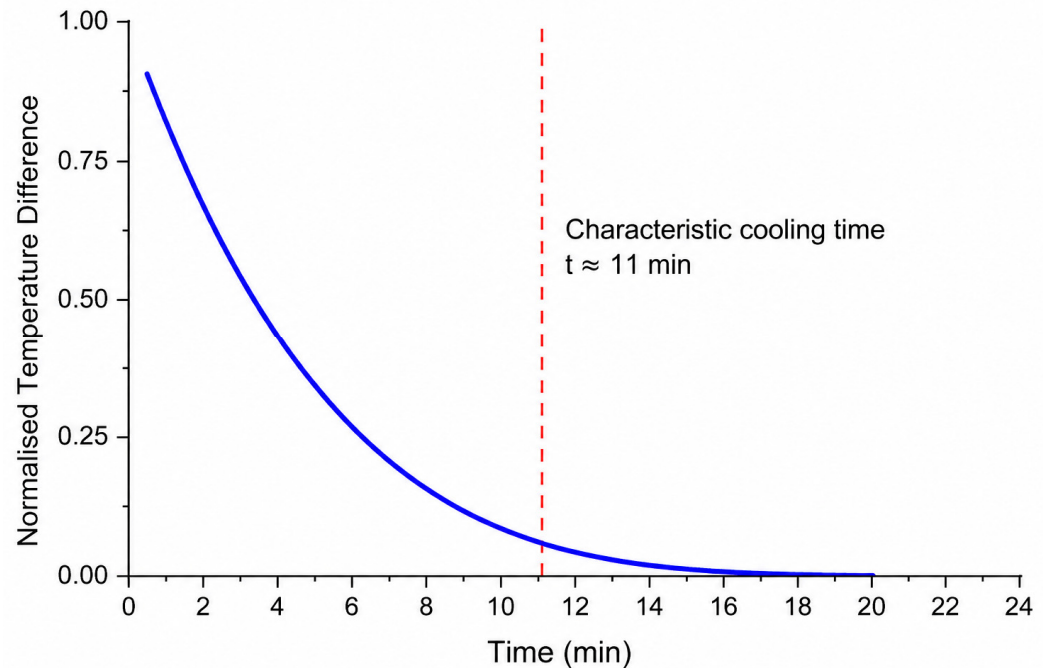


Figure A1. Transient thermo-mechanical response of a representative module subjected to cyclic temperature variation. The temperature evolution follows a first-order lumped thermal model of the form $T(t) = T_{\infty} + (T_0 - T_{\infty}) \times 10^{(-t/\tau)}$, where the characteristic time is $\tau \approx L_c^2 / \alpha_{th}$, with $L_c \approx D/4$. For the adopted geometry ($D = 0.1$ m) and typical thermal diffusivity $\alpha_{th} \approx 1.2 \times 10^{-5}$ m²/s, τ is on the order of a few minutes, and more than 95% of the temperature change is achieved within approximately $3\tau \approx 10\text{--}12$ min. This confirms that internal heat conduction does not limit cycle frequency [21].

Table A1. Material properties of candidate alloys used in the thermoelastic performance assessment, including density (ρ), Young's modulus (E), coefficient of thermal expansion (α), and yield strength (σ_y). Values are representative of engineering-grade materials and are used consistently throughout the thermo-mechanical and techno-economic analysis [18,19,21].

Material	E (GPa)	σ_y (MPa)	α ($10^{-6}/K$)	ρ (kg/m ³)	Cost (USD/kg)
S355	210	355	12.0	7850	1.0
AISI 4140	205	655	11.5	7850	3.0
AISI 4340	210	860	11.3	7850	4.5
Al 6061-T6	69	275	23.6	2700	2.5
Nitinol	70	500	18.0	6450	20.0

Appendix D

The industrial waste-heat case study presented in Appendix D is independent of the baseline scenario used in Table 1 and serves only as an illustrative application example based on representative waste-heat conditions.

Table A2. Operating conditions and calculated performance metrics for the industrial waste-heat case study [29,31,32]. The reported values are obtained using the thermo-mechanical relations in Equations (A10)–(A18), the economic expressions in Equations (A19)–(A21), and the thermodynamic performance metrics defined in Equations (A22)–(A25).

Parameter	Symbol	Value	Unit	Notes/Definition
Waste heat temperature	T_{hot}	90	°C	Typical low-grade industrial source
Ambient temperature	T_{amb}	30	°C	Representative operating condition
Temperature swing	ΔT	60	K	$T_{\text{hot}} - T_{\text{amb}}$
Young's modulus	E	210	GPa	AISI 4340
Yield strength	σ_y	860	MPa	AISI 4340
Thermal expansion coefficient	α	1.13×10^{-5}	K^{-1}	AISI 4340
Density	ρ	7850	$\text{kg}\cdot\text{m}^{-3}$	AISI 4340
Module diameter	D	0.10	m	Reference geometry
Module length	L	2.0	m	Yield-controlled design
Cross-sectional area	A	7.85×10^{-3}	m^2	From Equation (A10)
Mass per module	m_1	123.3	kg	From Equation (A11)
Total modules	N_{tot}	1000	–	Modular system
Total active mass	M_{tot}	1.23×10^5	kg	From Equation (A12)
Mechanical–electrical efficiency	η	0.25	–	Conservative assumption
Cycles per day	N_c	2	–	Waste-heat scenario
Work per cycle (per module)	W_{cyc}	1.83×10^4	J	From Equation (A14)
Gross annual energy	E_{gross}	928	kWh/yr	From Equation (A15)
Auxiliary energy	E_{aux}	0	kWh/yr	Passive baseline
Net annual energy	E_{net}	928	kWh/yr	From Equation (A16)
Specific energy yield	$k_{E,\text{net}}$	7.54×10^{-3}	kWh/kg·yr	From Equation (A17)
Thermodynamic efficiency	η_{th}	0.521	%	From Equation (A23)
Exergy efficiency	η_{ex}	5.95	%	From Equation (A25)
Carnot efficiency	η_{Carnot}	16.5	%	From the Carnot relation

Note: All results are evaluated under yield-controlled operation, assuming fully constrained thermoelastic response and symmetric energy extraction during heating and cooling phases. The reported thermodynamic and exergy efficiencies correspond to the reference industrial waste-heat case defined in Appendix D.2.

Appendix D.1. Calculation Methodology for the Case-Study Results in Table A2

The results reported in Table A2 are evaluated using the thermo-mechanical framework presented in the main text, combined with a consistent mass-based scaling formulation grounded in classical thermoelasticity and structural mechanics [19,21].

For a cylindrical module of diameter D and length L , the cross-sectional area is

$$A = (\pi D^2)/4, \quad (\text{A10})$$

and the corresponding module mass is

$$m_1 = \rho AL. \quad (\text{A11})$$

For a system consisting of N_{tot} identical modules, the total active mass becomes

$$M_{\text{tot}} = N_{\text{tot}} m_1. \quad (\text{A12})$$

Under yield-controlled operation, the admissible axial force of a single module is limited by the material yield strength [19]. The thermal elongation ΔL is obtained from the linear thermoelastic relation given in the main text (Equation (3)), consistent with classical thermoelasticity theory [21].

Assuming quasi-static loading, the mechanical work generated during each loading half-cycle is approximated by the product of the admissible axial force and the corresponding thermoelastic displacement:

$$W_{hc} = F_{yield} \Delta L, \quad (A13)$$

and assuming identical recoverable work during the heating and cooling half-cycles, the work generated per complete thermal cycle becomes

$$W_{cyc} = 2F_{yield} \Delta L. \quad (A14)$$

If the system operates at N_c cycles per day, the gross annual electrical energy is

$$E_{gross} = \frac{365 N_{tot} N_c \eta W_{cyc}}{3.6 \cdot 10^6}, \quad (A15)$$

where η is the overall mechanical-to-electrical efficiency and the denominator converts joules to kWh. The net annual energy production accounts for auxiliary consumption,

$$E_{net} = E_{gross} - E_{aux}. \quad (A16)$$

The mass-specific net annual energy yield is defined as

$$k_{E,net} = E_{net} / M_{tot}. \quad (A17)$$

In the material-dominated approximation, the total capital cost is expressed as

$$C_{tot} \approx C_m M_{tot}, \quad (A18)$$

where C_m is the effective cost per unit mass. The levelized cost of electricity (LCOE) is first expressed in its conventional form as

$$LCOE = \frac{C_{tot} \cdot CRF}{E_{net}} \quad (A19)$$

with the capital recovery factor

$$CRF = \frac{r(1+r)^n}{(1+r)^n - 1} \quad (A20)$$

Substituting Equation (A18) together with the identity $E_{net} = k_{E,net} \cdot M_{tot}$ obtained from Equation (A17) into Equation (A19) yields the equivalent mass-normalized expression

$$LCOE_{net} \approx \frac{C_m \cdot CRF}{k_{E,net}} \quad (A21)$$

which indicates that, within the assumptions of the present material-dominated cost model, the LCOE is primarily governed by the net annual electrical energy yield per unit mass, consistent with established energy-economic scaling approaches [31–36].

The present LCOE formulation represents a first-order material-dominated estimate intended to establish scaling relationships rather than to predict the complete cost of a commercial installation. Accordingly, the simplified capital-cost model does not explicitly

include fabrication, sleeve couplings, bearings, transmission components, generator, installation, maintenance, control systems, or heat-transfer enhancement. These contributions are expected to depend strongly on the final engineering design and deployment scenario and should therefore be incorporated in future prototype-based techno-economic assessments. The present formulation is intended to isolate the influence of material selection and thermoelastic performance on the achievable cost scaling.

For the industrial waste-heat scenario reported in Table A2, the adopted parameters are $T_{\text{hot}} = 90\text{ }^{\circ}\text{C}$, $T_{\text{amb}} = 30\text{ }^{\circ}\text{C}$, $\Delta T = 60\text{ K}$, $D = 0.10\text{ m}$, $L = 2.0\text{ m}$, $\rho = 7850\text{ kg}\cdot\text{m}^{-3}$, $\sigma_y = 860\text{ MPa}$, $\eta = 0.25$, $N_c = 2$, and $N_{\text{tot}} = 1000$. These values give $m \approx 123.3\text{ kg}$, $M_{\text{tot}} \approx 1.23 \times 10^5\text{ kg}$, and $E_{\text{gross}} \approx 928\text{ kWh/yr}$, consistent with Table A2. For the passive baseline considered here, $E_{\text{aux}} = 0$, and therefore $E_{\text{net}} = E_{\text{gross}}$. This formulation ensures full reproducibility of the case-study results reported in Table A2 using only the material properties, geometric parameters, and operating conditions specified in Appendix D.1.

Appendix D.2. Thermodynamic Performance Metrics

Heat Input Estimation. For a thermoelastic module subjected to a temperature excursion ΔT , the corresponding thermal energy input may be approximated by the sensible heat absorbed by the material,

$$Q_{\text{in}} = mc_p \Delta T \quad (\text{A22})$$

where m is the module mass and c_p is the specific heat capacity of AISI 4340 steel. For the present calculations $c_p = 475\text{ J}/(\text{kg}\cdot\text{K})$, corresponding to its room-temperature value [18,22].

This expression provides a first-order estimate of the thermal energy associated with the operating temperature cycle.

Thermodynamic Efficiency. A first-order thermodynamic efficiency may be defined as the ratio between recoverable mechanical work and thermal energy input,

$$\eta_{\text{th}} = \frac{W_{\text{cyc}}}{Q_{\text{in}}} \quad (\text{A23})$$

where W_{cyc} is given by Equation (A14). Because thermoelastic energy conversion utilizes only a fraction of the absorbed thermal energy, the resulting thermodynamic efficiency remains inherently low.

Exergy Efficiency. Since only a portion of thermal energy is theoretically convertible into work, exergy provides a more meaningful measure of thermoelastic energy-conversion performance than thermal energy alone. For a solid module heated from the ambient temperature T_{amb} to the operating temperature T_{hot} , the available thermal exergy may be approximated by

$$Ex_{\text{in}} = mc_p \left[(T_{\text{hot}} - T_{\text{amb}}) - T_{\text{amb}} \ln \left(\frac{T_{\text{hot}}}{T_{\text{amb}}} \right) \right] \quad (\text{A24})$$

The corresponding exergy efficiency is then defined as

$$\eta_{\text{ex}} = \frac{W_{\text{cyc}}}{Ex_{\text{in}}}. \quad (\text{A25})$$

Relation to Carnot Limit. The efficiencies defined above should not be interpreted as direct heat-engine efficiencies. The maximum theoretical efficiency of any temperature-driven process is bounded by the Carnot limit. In thermoelastic systems, only a fraction of the available thermal energy is converted into recoverable elastic strain energy, resulting in efficiencies substantially below the corresponding Carnot value. This difference reflects the intrinsic characteristics of thermoelastic energy conversion rather than a deficiency of the proposed architecture. For the reference industrial waste-heat case considered in this study ($T_{\text{hot}} = 90\text{ }^{\circ}\text{C}$, $T_{\text{amb}} = 30\text{ }^{\circ}\text{C}$), the corresponding Carnot efficiency is $\eta_{\text{Carnot}} = 16.5\%$.

Using the reference case parameters $m = 123.31 \text{ kg}$, $c_p = 475 \text{ J/(kg}\cdot\text{K)}$, $\Delta T = 60 \text{ K}$, and $W_{\text{cyc}} = 18.32 \text{ kJ}$, the thermal energy input is $Q_{\text{in}} = 3.514 \text{ MJ}$, resulting in a thermodynamic efficiency of $\eta_{\text{th}} = 0.521\%$. The corresponding available thermal exergy is $Ex_{\text{in}} = 0.308 \text{ MJ}$, yielding an exergy efficiency of $\eta_{\text{ex}} = 5.95\%$. These values remain substantially below the corresponding Carnot efficiency of 16.5% .

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