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Coordinated Scheduling of Distribution Networks and PEDF Microgrids via a Stackelberg Game with Wasserstein-DRO Reserve Requirements

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Abstract

High photovoltaic penetration increases reserve needs in active distribution networks, while PEDF microgrid flexibility is seldom coordinated as a reserve product. This study develops a complete-information Stackelberg framework in which a distribution system operator posts bounded prices for energy exchange, reactive support, and directional reserve. A Wasserstein ambiguity set centered on the empirical source–load error distribution supports a worst-case conditional value-at-risk calculation of hourly requirements. PEDF reserve offers are constrained by available battery energy, flexible-load headroom, recoverable photovoltaic curtailment, shared converter capacity, and activated feeder states. KKT conditions and Big-M linearization yield a mixed-integer second-order cone program. On a modified IEEE 33-bus feeder, joint PEDF and external reserve procurement reduces system cost by 2.455% and aggregate PEDF net cost by 10.402%, with zero reserve shortages and a minimum voltage of 0.951284 p.u. Fixed-ratio, SAA-CVaR, Box-RO, and WDRO-CVaR comparisons reveal an economy–reliability tradeoff. Rolling out-of-sample tests identify 0.1 as the smallest tested radius multiplier meeting a 5% violation target in all three training windows. An IEEE 69-bus extension reduces system cost by 4.233%, and all 144 nonlinear AC power-flow states converge. The results support one-interval steady-state reserve coordination under the tested assumptions.

Keywords: active distribution network; PEDF microgrid; reserve coordination; Stackelberg game; Wasserstein distributionally robust optimization; out-of-sample validation

1. Introduction

Distributed photovoltaic generation, power-electronic interfaces, and flexible demand are turning buildings into active resources for distribution-system operation. The PEDF concept integrates local photovoltaic generation, energy storage, direct-current demand, and flexible loads within one building-energy architecture [1]. Subsequent reviews have summarized its resource composition, development status, and unresolved technical requirements [2]. Device-level studies have quantified the regulation potential of air-conditioning demand under photovoltaic direct-driven operation [3]. Hierarchical microgrid management has also combined energy storage with demand response, while polygeneration studies have coordinated active and reactive power [4,5]. Photovoltaic–storage coordination has further been examined as a source of distribution-network voltage support [6]. These studies establish the physical basis of PEDF flexibility, but its value as a verified distribution-level reserve product remains insufficiently characterized.



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Network-constrained coordination requires a feeder representation that preserves operational meaning without making the scheduling problem intractable. Baran and Wu established the radial branch-flow formulation that underpins many distribution-system optimization models [7]. Conditions for exact second-order-cone relaxation in radial networks were later derived by Gan et al. [8]. Multi-period active–reactive coordination has since been formulated as a mixed-integer second-order cone program [9]. Renewable-aware voltage control has also been studied through network partitioning [10]. Adjustable robust optimization has been used to coordinate active distribution networks with multiple microgrids, and recent cluster-level scheduling has incorporated active–reactive coordination with phase-imbalance mitigation [11,12]. Although these models embed distributed resources in feeder constraints, they rarely combine energy exchange, reactive support, reserve procurement, and post-activation PEDF feasibility within one market framework.

Price-based mechanisms provide an economic interface between distribution-system requirements and autonomous microgrid decisions. A Stackelberg game has been used to coordinate coupled heat–electricity demand response in grid-connected microgrids [13]. Market-oriented flexibility-ramping products have been developed for multi-microgrid systems [14]. Equilibrium-constrained formulations have linked distribution-system operation with transmission-market clearing [15]. Transactive energy frameworks and network-aware distribution markets have also represented microgrid flexibility through hierarchical prices [16,17]. Related work has addressed photovoltaic-oriented scheduling, prosumer reserve coordination, and price-responsive voltage regulation [18–20]. These studies generally model generic microgrids or prosumer groups. The PEDF combination of direct-current demand, photovoltaic curtailment, battery energy, flexible-load comfort, and shared converter capacity creates additional coupling when the same resources participate in energy, reactive-power, and reserve transactions.

Reserve procurement also depends on how forecast uncertainty is translated into an operational requirement. Conditional value-at-risk provides a convex measure of adverse-tail loss [21]. Wasserstein distributionally robust optimization expands the empirical distribution into a neighborhood of plausible distributions and admits tractable reformulations under suitable assumptions [22]. Power-system applications include unit commitment and tie-line scheduling under distributional uncertainty [23,24]. More recent studies have used Wasserstein ambiguity for cyber-physical resilience under uncertain attack resources and for short-term cascade hydropower–wind–solar scheduling with distributionally robust chance constraints [25,26]. These applications demonstrate the versatility of the ambiguity-set approach, but neither addresses a distribution-level reserve market in which data-driven demand is matched with price-responsive PEDF supply.

Research closer to the present scheduling problem further clarifies the remaining gap. Wasserstein distributionally robust joint chance constraints have been applied to multi-energy microgrid management [27]. An exact and physically bounded chance-constrained formulation has been developed for centralized energy and reserve dispatch [28]. Wasserstein ambiguity has also supported resilience-oriented grid-forming storage scheduling and storage-based wind-ramp management [29,30]. Across these formulations, ambiguity-set size, risk approximation, support information, and validation procedure materially affect reliability and cost. The uncertainty model alone, however, cannot determine whether a customer-side reserve offer remains deliverable after energy scheduling, battery energy reservation, flexible-demand limits, converter sharing, and feeder constraints are imposed.

This study links the demand and supply sides of a distribution-level reserve market. Historical source–load errors are converted into hourly upward and downward requirements through a Wasserstein ambiguity set and worst-case CVaR. PEDF reserve supply emerges from follower optimization and is limited by battery power and energy, flexible-

load headroom, recoverable photovoltaic curtailment, and converter capacity. External reserve provides an explicit fallback, while separate activated feeder states verify voltage and current security after a full one-interval UP or DOWN deployment. The resulting structure differs from centralized energy-and-reserve dispatch because each PEDF retains its own objective and responds to bounded prices. It also extends conventional Stackelberg energy trading by coupling market response with a physical deliverability test.

Table 1 reveals three unresolved links. PEDF studies seldom value energy, reactive-power, and reserve flexibility in one distribution-level interaction. Market-oriented microgrid models often omit explicit post-activation checks of battery energy, converter headroom, and feeder security. Wasserstein-DRO reserve studies generally optimize centralized decisions and provide limited evidence for a leader–follower market in which risk-based demand and customer-side response are cleared jointly. The resulting research question is whether a DSO can procure physically constrained PEDF reserve through bounded prices while maintaining feeder security under limited source–load samples.

Table 1. Positioning of the proposed formulation relative to representative studies. The comparison records each study’s primary modeling focus and does not imply that omitted features could not be added in future extensions.

Study	Coordination Setting	Uncertainty Treatment	Reserve Modeling	Hierarchical Prices	Main Distinction from This Work
Liu et al. [13]	Grid-connected heat–electricity microgrids	Deterministic scheduling	Not the primary product	Stackelberg interaction	Coordinates coupled demand response, without a data-driven reserve requirement or activation-based PEDF deliverability check.
Hua et al. [16]	Transactive energy among multiple microgrids	Deterministic flexibility trading	Demand-side flexibility	Stackelberg interaction	Establishes transactive flexibility provision, but does not couple feeder reserve demand with PEDF stored-energy and converter reservation.
Zhang et al. [17]	Network-aware distribution market with microgrids	Deterministic market clearing	Not the primary product	Leader–follower prices and DLMP	Embeds microgrids in network-aware clearing, while uncertainty-driven reserve procurement is outside its main scope.
Zhong et al. [19]	Distribution network, prosumer group, and prosumer	Flexible-reserve uncertainty model	Flexible reserve	Hierarchical coordination	Represents generic prosumers rather than an integrated PEDF structure with Wasserstein-CVaR reserve demand.
Du et al. [25]	Cyber-physical power-system resilience	Wasserstein ambiguity for uncertain attack resources	Not the primary product	No market-price hierarchy	Models power–communication interdependence and resilient defense, rather than source–load reserve procurement from customer-side resources.
Liu et al. [26]	Cascade hydropower–wind–solar scheduling	Wasserstein distributionally robust chance constraints	Reliability and flexibility constraints in centralized generation	No DSO–PEDF price hierarchy	Coordinates hybrid generation under renewable uncertainty, without PEDF follower autonomy or activation-based battery and converter reserve checks.
Zhai et al. [27]	Multi-energy microgrid management	Wasserstein-DRO joint chance constraints with CVaR approximation	Flexibility used for uncertainty mitigation	No DSO–PEDF price hierarchy	Provides data-driven risk-aware microgrid scheduling, without a customer-side reserve market or radial-feeder price interaction.

Table 1. *Cont.*

Study	Coordination Setting	Uncertainty Treatment	Reserve Modeling	Hierarchical Prices	Main Distinction from This Work
Arrigo et al. [28]	Centralized energy and reserve dispatch	Exact physically bounded Wasserstein DR chance constraints	System-level energy and reserve	No customer-side follower model	Provides a rigorous reserve-dispatch benchmark, while the present work studies bounded price guidance and PEDF response.
This study	DSO and PEDF microgrids in a radial feeder	Wasserstein-DRO worst-case CVaR with rolling OOS calibration	Upward and downward reserve with stored-energy and converter checks	Complete-information Stackelberg pricing	Integrates PEDF energy, reactive power, reserve supply, data-driven reserve demand, and network security in one KKT-MISOCP framework.

The contribution is an integrated decision and validation chain. The follower model coordinates PV utilization, battery operation, flexible-load energy conservation, reactive support, power exchange, and reserve revenue. Upward and downward reserve are decomposed by resource and linked to stored-energy reservation, converter loading, and activated network states. The leader procures PEDF and external reserve against a Wasserstein-CVaR requirement, and total-system accounting removes internal payment transfers. KKT conditions embed the continuous convex follower response in an MISOCP. The numerical study combines alternative uncertainty treatments, radius and sample-size tests, CVaR-confidence and safety-factor sensitivities, Big-M diagnostics, a larger IEEE 69-bus feeder, chronological out-of-sample validation, and independent nonlinear AC power flows.

The scope is deliberately bounded. The market is a prospective coordination design rather than a claim that present distribution utilities simultaneously set every modeled price. The benchmarks cover a fixed-ratio rule, SAA-CVaR, Box-RO, and WDRO-CVaR; KL-divergence DRO and exact chance-constrained variants are left for separate reformulations. The equilibrium assumes complete information and truthful parameters, so incentive compatibility is not established. Reserve activation is evaluated as a separate one-hour UP or DOWN state and does not represent frequency dynamics, converter controls, repeated deployment, or battery electrochemistry. The data contain 30 July days; rolling chronological tests assess finite-sample reliability within that period, while cross-season and extreme-event validation remain open.

The remainder of the paper is organized as follows. Section 2 describes the coordination architecture and market interpretation. Section 3 formulates the DSO problem, the PEDF response, activated reserve deliverability, Wasserstein-DRO requirements, and the single-level reformulation. Section 4 presents the comparative, sensitivity, out-of-sample, scalability, and nonlinear AC results. Section 5 summarizes the principal findings and limitations.

2. Coordinated Optimization Framework for the Distribution Network and PEDF Microgrids

The coordinated system comprises the upstream grid, a distribution system operator, and multiple PEDF microgrids. The DSO is responsible for feeder security and acts as the Stackelberg leader. It posts bounded prices for electricity purchased by and sold from each PEDF, reactive-power support, and upward and downward reserve. Each PEDF acts as a follower and schedules PV utilization, battery charging and discharging, fixed and flexible demand, reactive support, and feeder exchange. Figure 1 connects the full decision chain: historical errors determine reserve demand, prices induce local responses, external reserve covers residual demand, base and activated network states enforce security, and the bilevel interaction is reformulated for solution.

The mechanism should be interpreted as a prospective local-flexibility market rather than a direct representation of existing retail tariffs. The EcoGrid 2.0 field trial demonstrated that aggregated residential resources can participate when communication, verification, aggregation, and settlement are available [31]. Reviews of current projects nevertheless identify governance, DSO coordination, strategic gaming, and entry barriers as persistent challenges [32]. Baseline-dependent products may also create conflicts when distributed resources participate simultaneously in local and wider markets [33], while long-term flexibility prices can influence network-planning decisions [34]. Reserve-oriented microgrid sizing provides a complementary planning perspective by linking installed flexibility to operating-reserve needs [35]. Practical deployment of the present framework would therefore require regulator-approved reserve and reactive-power products, interval metering, capability verification, communication standards, settlement rules, and penalties for non-delivery.

The equilibrium is evaluated under complete information. The DSO knows the modeled follower objectives and feasible regions, and each PEDF implements the optimizer-predicted response using reported cost and capability parameters. A no-worse-off participation constraint is imposed relative to the external-reserve benchmark. This condition supports participation in the tested setting, but it does not guarantee truthful revelation. Private costs, false capacity declarations, strategic withholding, bounded rationality, communication failures, and settlement deviations require a mechanism-design extension.

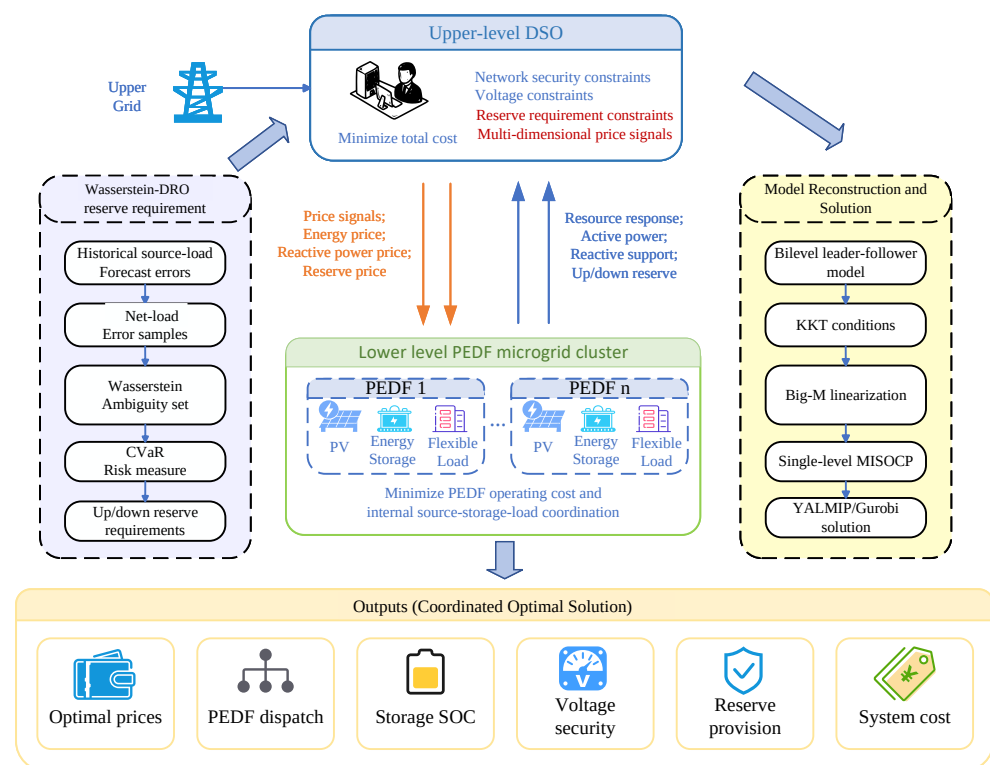


Figure 1. Coordination framework and solution procedure for the DSO–PEDF Stackelberg model.

Figure 1 summarizes the operational loop: uncertainty defines reserve demand, the DSO coordinates prices and external procurement, PEDF followers schedule local flexibility, and the reformulated model returns prices, dispatch, stored energy, voltage, reserve, and system cost.

3. Stackelberg Game Model for the Distribution Network and PEDF Microgrids Considering Wasserstein-DRO-Based Reserve Requirements

The model represents a complete-information Stackelberg interaction between a distribution system operator (DSO) and multiple photovoltaic–energy storage–direct current–flexibility (PEDF) microgrids. The DSO acts as the leader and selects bounded prices for active-power exchange, reactive-power support, and upward and downward reserve. Each PEDF microgrid acts as a follower and minimizes its local operating cost by scheduling PV utilization, battery charging and discharging, flexible demand, reactive-power support, and reserve provision. This hierarchy preserves local scheduling autonomy while allowing the DSO to coordinate aggregate behavior through economically interpretable signals.

The mathematical formulation closes five linked decisions. Historical source–load errors determine hourly upward and downward reserve requirements. The DSO then chooses price signals together with external reserve procurement and feeder operating variables. PEDF followers respond by creating reserve from battery, flexible-load, and recoverable-PV headroom. Base and activated network states test whether the resulting schedules satisfy voltage, current, stored-energy, and converter limits. Finally, the convex follower problems are embedded through their KKT conditions, yielding a single mixed-integer second-order cone program (MISOCP). The reformulation is valid under follower convexity, feasibility, strong duality, finite Big-M bounds, and the adopted branch-flow relaxation.

3.1. Sets, Variables, and Model Structure

The principal sets and indices are listed in the Nomenclature. For node j , Ω_j^{in} and Ω_j^{out} denote the incoming and outgoing branch sets used in the nodal balance equations.

The principal upper- and lower-level symbols are summarized in the Nomenclature. The upper-level block contains market prices, external reserve, shortage, and feeder-state decisions, whereas the lower-level block contains local energy, reactive-power, storage, flexible-load, and reserve schedules. The local data $P_{n,t}^{\text{PV}}$, $P_{n,t}^{\text{fixed}}$, and $P_{n,t}^{\text{flex,ref}}$ denote available PV power, fixed demand, and the flexible-load reference profile, respectively, and $P_{n,t}^{\text{PV,use}}$ denotes scheduled PV utilization. The optimization tracks stored energy in kWh, while the reported SOC ratio is obtained from the relation given in the Nomenclature.

This classification makes the direction of causality explicit: the DSO controls market and network variables, the PEDF controls local schedules, and the reserve-adequacy equations couple the two levels. The distinction is important for interpreting the model. The DSO does not directly dispatch customer-side devices; it changes the economic conditions under which each PEDF selects its own schedule. The reserve-adequacy and activated-state constraints then translate that commercial response into a physically checkable feeder commitment. Without this separation, a reduction in the leader’s objective could be produced by an administratively imposed customer schedule rather than by a feasible market response.

3.2. Upper-Level DSO Optimization Model

The upper-level problem converts feeder-security and reserve-adequacy requirements into bounded market signals. Its objective includes upstream energy purchases, network losses, external reserve procurement, transactions with PEDF microgrids, and a high penalty for any remaining reserve shortfall. The leader therefore compares customer-side reserve with an explicit external alternative instead of treating uncovered demand as the only fallback. Total-system cost is defined separately for case comparison so that internal market transfers are not counted as social cost.

3.2.1. Objective Function

The DSO's objective is

$$\min C^{\text{DSO}} = C^{\text{grid}} + C^{\text{loss}} + C^{\text{ext}} - R^{\text{PEDF}} + C^{\text{short}}. \quad (1)$$

The component terms are

$$\begin{aligned} C^{\text{grid}} &= \sum_{t \in \mathcal{T}} \lambda_t^{\text{grid}} p_t^{\text{sub}} S_{\text{base}} \Delta t, \\ C^{\text{loss}} &= \sum_{t \in \mathcal{T}} c^{\text{loss}} p_t^{\text{loss}} \Delta t, \\ C^{\text{ext}} &= \sum_{t \in \mathcal{T}} \left(c_{\text{ext}}^{\text{up}} R_t^{\text{ext,up}} + c_{\text{ext}}^{\text{dn}} R_t^{\text{ext,dn}} \right) \Delta t, \\ R^{\text{PEDF}} &= \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} \left[\lambda_t^{\text{sell}} p_{n,t}^{\text{buy}} - \lambda_t^{\text{buy}} p_{n,t}^{\text{sell}} - \lambda_t^Q \left(Q_{n,t}^+ + Q_{n,t}^- \right) \right. \\ &\quad \left. - \lambda_t^{\text{up}} F_{n,t}^{\text{up}} - \lambda_t^{\text{dn}} F_{n,t}^{\text{dn}} \right] \Delta t, \\ C^{\text{short}} &= \sum_{t \in \mathcal{T}} \left(c_{\text{short}}^{\text{up}} \Delta R_t^{\text{up}} + c_{\text{short}}^{\text{dn}} \Delta R_t^{\text{dn}} \right) \Delta t. \end{aligned} \quad (2)$$

Here, C^{grid} , C^{loss} , C^{ext} , and C^{short} denote upstream-energy, network-loss, external-reserve, and reserve-shortage costs, respectively. λ_t^{grid} is the upstream energy price, Δt is the scheduling-interval duration, c^{loss} is the unit value assigned to active-power loss, and c_{ext}^s and c_{short}^s are the external-reserve price and shortage penalty for direction s . R^{PEDF} is the DSO's net revenue from PEDF transactions under the stated sign convention. The per-unit substation import is converted through S_{base} , while p_t^{loss} is valued in physical power units, as indicated in the Nomenclature. Because the shortage penalty exceeds the external-reserve price, the model uses available external or PEDF reserve before accepting ΔR_t^s .

The reported total-system-cost metric is

$$C^{\text{sys}} = C^{\text{DSO}} + \sum_{n \in \mathcal{N}} C_n^{\text{PEDF}}. \quad (3)$$

Because the PEDF transaction terms enter the DSO and PEDF accounting expressions with opposite signs, they cancel in C^{sys} . The resulting metric measures external energy and reserve costs, network-loss cost, shortage cost, and PEDF physical operating costs without counting internal transfers as social cost. The distinction is important because a price change can redistribute settlement payments between the DSO and PEDF participants without producing an equal change in physical resource use. The case study therefore reports total-system cost as the aggregate economic indicator and PEDF net cost as the participant-level indicator, while the Stackelberg leader continues to minimize C^{DSO} .

3.2.2. Price Constraints

The price bounds maintain economic interpretability and prevent the leader from forcing a follower response through arbitrarily large numerical incentives:

$$\begin{aligned} \lambda_t^{\text{grid}} &\leq \lambda_t^{\text{sell}} \leq \lambda_t^{\text{grid}} + \Delta \lambda^{\text{sell}}, \\ \lambda_t^{\text{grid}} - \Delta \lambda^{\text{buy}} &\leq \lambda_t^{\text{buy}} \leq \lambda_t^{\text{sell}}, \\ 0 &\leq \lambda_t^Q \leq \lambda^Q, \text{max}, \\ 0 &\leq \lambda_t^{\text{up}} \leq \lambda^{\text{up}, \text{max}}, \\ 0 &\leq \lambda_t^{\text{dn}} \leq \lambda^{\text{dn}, \text{max}}. \end{aligned} \quad (4)$$

Here, $\Delta\lambda^{\text{sell}}$ and $\Delta\lambda^{\text{buy}}$ are the admissible selling-price markup and purchasing-price markdown relative to the upstream tariff. $\lambda^{Q,\text{max}}$, $\lambda^{\text{up},\text{max}}$, and $\lambda^{\text{dn},\text{max}}$ are the upper bounds on reactive-power, upward-reserve, and downward-reserve prices.

The price structure is a prospective coordination design. It assumes that the DSO can procure verified reserve and reactive-power support through regulator-approved products. Price bounds restrict the numerical search space, but they do not establish truthful revelation or strategy-proof participation under private information.

3.2.3. Distribution-Network Power-Flow and Security Constraints

The radial DistFlow model follows the branch-flow formulation of Baran and Wu [7], while the conic relaxation is interpreted in light of the radial-network conditions discussed by Gan et al. [8]. For each non-slack node j , the active- and reactive-power balances are

$$\begin{aligned} \sum_{i \in \Omega_j^{\text{in}}} (P_{ij,t} - R_{ij} I_{ij,t}^2) - \sum_{k \in \Omega_j^{\text{out}}} P_{jk,t} + P_{j,t}^{\text{PEDF}} &= P_{j,t}^{\text{load}}, \\ \sum_{i \in \Omega_j^{\text{in}}} (Q_{ij,t} - X_{ij} I_{ij,t}^2) - \sum_{k \in \Omega_j^{\text{out}}} Q_{jk,t} + Q_{j,t}^{\text{PEDF}} &= Q_{j,t}^{\text{load}}. \end{aligned} \quad (5)$$

If PEDF n is connected at node j , its base-state injections are

$$P_{j,t}^{\text{PEDF}} = \frac{p_{n,t}^{\text{sell}} - p_{n,t}^{\text{buy}}}{S_{\text{base}}}, \quad Q_{j,t}^{\text{PEDF}} = \frac{Q_{n,t}^{\text{PEDF}}}{S_{\text{base}}}. \quad (6)$$

The branch voltage-drop equation and second-order-cone constraint are

$$\begin{aligned} V_{j,t}^2 &= V_{i,t}^2 - 2(R_{ij}P_{ij,t} + X_{ij}Q_{ij,t}) + (R_{ij}^2 + X_{ij}^2)I_{ij,t}^2, \\ \left\| \begin{pmatrix} 2P_{ij,t}, 2Q_{ij,t}, V_{i,t}^2 - I_{ij,t}^2 \end{pmatrix} \right\|_2 &\leq V_{i,t}^2 + I_{ij,t}^2. \end{aligned} \quad (7)$$

Voltage and current limits, together with the fixed slack-bus voltage, are

$$\begin{aligned} (V_i^{\text{min}})^2 &\leq V_{i,t}^2 \leq (V_i^{\text{max}})^2, \quad 0 \leq I_{ij,t}^2 \leq (I_{ij}^{\text{max}})^2, \\ V_{r,t}^2 &= (V_r^{\text{set}})^2. \end{aligned} \quad (8)$$

Here, R_{ij} and X_{ij} are the resistance and reactance of branch (i, j) ; $P_{j,t}^{\text{load}}$ and $Q_{j,t}^{\text{load}}$ are the active and reactive demands at node j ; V_i^{min} and V_i^{max} are nodal-voltage limits; I_{ij}^{max} is the branch-current limit; and V_r^{set} is the fixed slack-bus voltage. Branch powers, squared currents, and squared voltages are expressed on the selected per-unit base.

These constraints ensure that the economic coordination remains embedded in feeder physics. In the base state, they verify the energy and reactive-power schedule produced before reserve activation. The activated states introduced later reuse the same topology, voltage limits, current limits, and branch-flow relationships after the full UP or DOWN commitment is applied. Thus, a PEDF offer is not accepted solely because local equipment has unused capacity; it must also remain compatible with the electrical location of the resource. This feature is especially relevant for the remote buses used in the case study, where the same active-power adjustment can have different voltage consequences from an equivalent adjustment near the substation. Equivalence to nonlinear AC power flow requires a sufficiently tight conic relaxation, so the numerical study reports relaxation residuals and independent AC checks rather than assuming exactness.

3.3. Lower-Level PEDF Optimization Model

The follower model represents the least-cost local response to the DSO prices. Battery cycling, flexible-load movement, PV curtailment, and reactive-power support carry explicit costs or physical limits, which gives reserve provision an opportunity cost instead of treating flexibility as free capacity.

3.3.1. PEDF Objective and Participation Condition

For PEDF n , the operating objective is

$$\begin{aligned} C_n^{\text{PEDF}} = \sum_{t \in \mathcal{T}} & \left[\lambda_t^{\text{sell}} P_{n,t}^{\text{buy}} - \lambda_t^{\text{buy}} P_{n,t}^{\text{sell}} - \lambda_t^Q (Q_{n,t}^+ + Q_{n,t}^-) - \lambda_t^{\text{up}} F_{n,t}^{\text{up}} - \lambda_t^{\text{dn}} F_{n,t}^{\text{dn}} \right. \\ & + c^{\text{cut}} P_{n,t}^{\text{cut}} + c^{\text{deg}} (P_{n,t}^{\text{ch}} + P_{n,t}^{\text{dis}}) + c^{\text{flex}} (P_{n,t}^{\text{flex},+} + P_{n,t}^{\text{flex},-}) \\ & + c^Q (Q_{n,t}^+ + Q_{n,t}^-) + c_{\text{ES}}^{\text{up}} (F_{n,t}^{\text{dis},\text{up}} + F_{n,t}^{\text{ch},\text{up}}) + c_{\text{ES}}^{\text{dn}} (F_{n,t}^{\text{ch},\text{dn}} + F_{n,t}^{\text{dis},\text{dn}}) \\ & \left. + c_{\text{flex}}^{\text{up}} F_{n,t}^{\text{flex},\text{up}} + c_{\text{flex}}^{\text{dn}} F_{n,t}^{\text{flex},\text{dn}} + c_{\text{PV}}^{\text{up}} F_{n,t}^{\text{cut},\text{up}} \right] \Delta t. \end{aligned} \quad (9)$$

The absolute flexible-load deviation is represented by

$$\begin{aligned} P_{n,t}^{\text{flex}} - P_{n,t}^{\text{flex},\text{ref}} &= P_{n,t}^{\text{flex},+} - P_{n,t}^{\text{flex},-}, \\ P_{n,t}^{\text{flex},+} &\geq 0, \quad P_{n,t}^{\text{flex},-} \geq 0. \end{aligned} \quad (10)$$

Here, $P_{n,t}^{\text{flex},+}$ and $P_{n,t}^{\text{flex},-}$ are nonnegative positive and negative deviations from the flexible-load reference profile. With $c^{\text{flex}} > 0$, minimization makes their sum equal to the absolute deviation at the optimum.

The reported market case also enforces a no-worse-off participation condition relative to the external-reserve benchmark:

$$C_n^{\text{PEDF}} \leq C_n^{\text{ref}}, \quad \forall n \in \mathcal{N}. \quad (11)$$

Here, C_n^{ref} is the PEDF operating cost in the external-reserve benchmark used as the participation reference.

This constraint supports individual participation under the complete-information benchmark. Economically, the reserve and reactive-power payments must compensate the PEDF for battery degradation, altered flexible-load operation, retained energy, and any opportunity to sell electricity that is forgone when headroom is reserved. The reference case therefore provides a transparent participation benchmark rather than an arbitrary lower bound on the objective. It does not prove incentive compatibility because the reference cost and reported parameters are assumed to be observable and truthful. A participant could still benefit from misreporting private costs or capability if auditing and settlement rules were absent.

3.3.2. Power Balance, Flexible Demand, and Battery Operation

The PEDF active-power balance and transaction domains are

$$\begin{aligned} P_{n,t}^{\text{buy}} - P_{n,t}^{\text{sell}} + P_{n,t}^{\text{PV},\text{use}} + P_{n,t}^{\text{dis}} &= P_{n,t}^{\text{fixed}} + P_{n,t}^{\text{flex}} + P_{n,t}^{\text{ch}}, \\ 0 \leq P_{n,t}^{\text{buy}} &\leq P_n^{\text{ex},\text{max}}, \quad 0 \leq P_{n,t}^{\text{sell}} \leq P_n^{\text{ex},\text{max}}. \end{aligned} \quad (12)$$

Reactive exchange is represented with nonnegative directional components,

$$Q_{n,t}^{\text{PEDF}} = Q_{n,t}^+ - Q_{n,t}^-, \quad Q_{n,t}^+ \geq 0, \quad Q_{n,t}^- \geq 0, \quad Q_{n,t}^+ + Q_{n,t}^- \leq Q_n^{\text{max}}. \quad (13)$$

Here, $Q_n^{\max}$ is the absolute reactive-power capability of PEDF n .

The magnitude $Q_{n,t}^+ + Q_{n,t}^-$ enters both the service payment and the local provision cost. This convention compensates capacitive and inductive support symmetrically while retaining the signed injection required by the network equations. PV utilization and curtailment satisfy

$$0 \leq P_{n,t}^{\text{cut}} \leq P_{n,t}^{\text{PV}}, \quad P_{n,t}^{\text{PV,use}} = P_{n,t}^{\text{PV}} - P_{n,t}^{\text{cut}}. \quad (14)$$

Flexible demand is bounded in each period and conserves its total scheduled energy:

$$0 \leq P_{n,t}^{\text{flex}} \leq P_n^{\text{flex,max}}, \quad \sum_{t \in \mathcal{T}} P_{n,t}^{\text{flex}} \Delta t = E_n^{\text{req}}. \quad (15)$$

The formal implementation tracks stored battery energy in kWh:

$$\begin{aligned} E_{n,0}^{\text{sto}} &= \text{SOC}_n^0 E_n^{\text{bat}}, & E_{n,|\mathcal{T}|}^{\text{sto}} &= E_{n,0}^{\text{sto}}, \\ E_{n,t}^{\text{sto}} &= E_{n,t-1}^{\text{sto}} + \eta^{\text{ch}} P_{n,t}^{\text{ch}} \Delta t - \frac{P_{n,t}^{\text{dis}} \Delta t}{\eta^{\text{dis}}}, \\ \text{SOC}_n^{\min} E_n^{\text{bat}} &\leq E_{n,t}^{\text{sto}} \leq \text{SOC}_n^{\max} E_n^{\text{bat}}, \\ 0 \leq P_{n,t}^{\text{ch}} &\leq P_n^{\text{ch,max}}, & 0 \leq P_{n,t}^{\text{dis}} &\leq P_n^{\text{dis,max}}. \end{aligned} \quad (16)$$

Here, $P_n^{\text{flex,max}}$ is the period-wise flexible-load ceiling, E_n^{req} is its required daily energy, SOC_n^0 is the initial state-of-charge ratio, $\text{SOC}_n^{\min}$ and $\text{SOC}_n^{\max}$ are the allowable SOC limits, η^{ch} and η^{dis} are charging and discharging efficiencies, and $P_n^{\text{ch,max}}$ and $P_n^{\text{dis,max}}$ are battery power limits.

The reported dimensionless state of charge follows the relation stated in the Nomenclature. The initial condition maps the stated SOC ratio into stored energy, and the cyclic terminal condition prevents the day-ahead optimization from improving its objective by depleting the battery at the horizon boundary. Writing the optimization state in energy units matches the computational output and makes the charging and discharging terms dimensionally explicit. The flexible-load equality and the battery-energy dynamics serve different purposes: the former prevents net demand deletion, while the latter prevents reserve or arbitrage from using unavailable stored energy. Under the reported positive cycling cost and non-unit efficiencies, simultaneous charging and discharging does not occur in the obtained schedules; the follower remains continuous so that the KKT reformulation is applicable. At hours for which the buy and sell tariffs are identical, the continuous directional decomposition can admit offsetting gross transaction components. Such pairs are interpreted through their net exchange and can be netted without changing nodal injection or settlement value; no exclusivity binary is introduced because it would destroy follower convexity.

3.4. PEDF Reserve Capability and Activated-State Deliverability

Reserve is decomposed by resource because unused rated power alone does not establish deliverability. Battery power and energy, flexible-load headroom, recoverable PV curtailment, converter loading, and feeder security can all become binding. The formulation evaluates upward and downward activation separately over one dispatch interval.

3.4.1. Upward and Downward Reserve Components

Upward reserve increases PEDF net injection or reduces net demand:

$$F_{n,t}^{\text{up}} = F_{n,t}^{\text{dis,up}} + F_{n,t}^{\text{ch,up}} + F_{n,t}^{\text{cut,up}} + F_{n,t}^{\text{flex,up}}, \quad (17)$$

with

$$\begin{aligned} 0 &\leq F_{n,t}^{\text{dis,up}} \leq p_{n,t}^{\text{dis,max}} - p_{n,t}^{\text{dis}}, \\ 0 &\leq F_{n,t}^{\text{ch,up}} \leq p_{n,t}^{\text{ch}}, \quad 0 \leq F_{n,t}^{\text{cut,up}} \leq p_{n,t}^{\text{cut}}, \\ 0 &\leq F_{n,t}^{\text{flex,up}} \leq p_{n,t}^{\text{flex}}. \end{aligned} \quad (18)$$

Here, $F_{n,t}^{\text{dis,up}}$, $F_{n,t}^{\text{ch,up}}$, $F_{n,t}^{\text{cut,up}}$, and $F_{n,t}^{\text{flex,up}}$ denote upward reserve from additional discharge, cancelled charging, recovered curtailed PV, and flexible-load reduction.

These terms represent additional battery discharge, cancellation of scheduled charging, recovery of curtailed PV, and temporary load reduction.

Downward reserve increases net absorption or reduces net injection:

$$F_{n,t}^{\text{dn}} = F_{n,t}^{\text{ch,dn}} + F_{n,t}^{\text{dis,dn}} + F_{n,t}^{\text{flex,dn}}, \quad (19)$$

with

$$\begin{aligned} 0 &\leq F_{n,t}^{\text{ch,dn}} \leq p_{n,t}^{\text{ch,max}} - p_{n,t}^{\text{ch}}, \\ 0 &\leq F_{n,t}^{\text{dis,dn}} \leq p_{n,t}^{\text{dis}}, \\ 0 &\leq F_{n,t}^{\text{flex,dn}} \leq p_{n,t}^{\text{flex,max}} - p_{n,t}^{\text{flex}}. \end{aligned} \quad (20)$$

Here, $F_{n,t}^{\text{ch,dn}}$, $F_{n,t}^{\text{dis,dn}}$, and $F_{n,t}^{\text{flex,dn}}$ denote downward reserve from additional charging, cancelled discharge, and flexible-load increase.

Downward reserve is supplied by additional charging, cancellation of scheduled discharge, and load increase. Deliberate PV curtailment is excluded as a primary downward product because it conflicts with the PEDF accommodation objective. The directional decomposition also avoids assuming that a resource has symmetric regulation capability. A battery near its lower SOC limit can retain charging headroom for downward reserve but may have little sustainable upward capability; the opposite condition occurs near the upper limit. Flexible demand is similarly asymmetric because reduction depends on the scheduled load level, whereas increase depends on the remaining load ceiling. These distinctions explain why the numerical results show different PEDF coverage rates for upward and downward reserve.

3.4.2. Stored-Energy and Converter Reservation

A full directional activation sustained for one interval must keep stored energy within the limits implied by the SOC range. Upward activation lowers stored energy when scheduled discharge is increased or charging is curtailed. Downward activation raises stored energy when charging is increased or scheduled discharge is curtailed:

$$\begin{aligned} E_{n,t}^{\text{sto}} - \frac{F_{n,t}^{\text{dis,up}} \Delta t}{\eta^{\text{dis}}} - \eta^{\text{ch}} F_{n,t}^{\text{ch,up}} \Delta t &\geq \text{SOC}_n^{\text{min}} E_n^{\text{bat}}, \\ E_{n,t}^{\text{sto}} + \eta^{\text{ch}} F_{n,t}^{\text{ch,dn}} \Delta t + \frac{F_{n,t}^{\text{dis,dn}} \Delta t}{\eta^{\text{dis}}} &\leq \text{SOC}_n^{\text{max}} E_n^{\text{bat}}. \end{aligned} \quad (21)$$

The interconnection converter is shared by active exchange, reactive support, and reserve. The implementation uses a polygonal linear approximation of the apparent-power boundary. For each direction angle θ_k ,

$$\begin{aligned} (p_{n,t}^{\text{sell}} - p_{n,t}^{\text{buy}} + F_{n,t}^{\text{up}}) \cos \theta_k + Q_{n,t}^{\text{PEDF}} \sin \theta_k &\leq S_n^{\text{AFR}}, \\ (-p_{n,t}^{\text{sell}} + p_{n,t}^{\text{buy}} + F_{n,t}^{\text{dn}}) \cos \theta_k - Q_{n,t}^{\text{PEDF}} \sin \theta_k &\leq S_n^{\text{AFR}}. \end{aligned} \quad (22)$$

Here, S_n^{AFR} is the apparent-power rating of the shared PEDF interconnection converter, and θ_k is the k th supporting angle of its polygonal capacity approximation.

Together, these constraints prevent the same energy or converter headroom from being committed simultaneously to the base schedule and to a directional reserve activation. The converter constraint is not a separate nameplate check. Active exchange, reactive support, and reserve activation all use the same interface, so additional reactive-power service can reduce the active reserve that remains available in a given hour. Conversely, a high reserve commitment can restrict the reactive-power range needed for voltage support. This coupling is one reason why the market outcome cannot be inferred from battery power and energy ratings alone.

3.4.3. Network Feasibility Under Reserve Activation

Device-level headroom does not ensure that reserve remains feasible after it changes feeder injections.

Let $s \in \{\text{up}, \text{dn}\}$ denote the activated reserve direction, with $\sigma_{\text{up}} = +1$ and $\sigma_{\text{dn}} = -1$. For a PEDF connected at node j , the activated active-power injection is

$$P_{j,t}^{\text{PEDF},s} = \frac{P_{n,t}^{\text{sell}} - P_{n,t}^{\text{buy}} + \sigma_s F_{n,t}^s}{S_{\text{base}}}, \quad Q_{j,t}^{\text{PEDF},s} = \frac{Q_{n,t}^{\text{PEDF}}}{S_{\text{base}}}. \quad (23)$$

External reserve is activated at the upstream interface through

$$P_t^{\text{sub},s} = P_t^{\text{sub}} + \frac{\sigma_s R_t^{\text{ext},s}}{S_{\text{base}}}. \quad (24)$$

Here, $P_t^{\text{sub},s}$ is the substation active-power injection after direction s is activated, and $P_{ij,t}^s$, $Q_{ij,t}^s$, $I_{ij,t}^{2,s}$, and $V_{i,t}^{2,s}$ are the corresponding activated-state branch and voltage variables.

For each activated state, a separate set of branch variables ($P_{ij,t}^s$, $Q_{ij,t}^s$, $I_{ij,t}^{2,s}$, $V_{i,t}^{2,s}$) satisfies the balances, voltage-drop equation, conic constraint, and operating limits in Equations (5)–(8), with the base PEDF and root injections replaced by Equations (23) and (24). The three-state construction has a clear interpretation. BASE represents the scheduled operating point, UP applies the complete upward commitment for the selected hour, and DOWN applies the complete downward commitment. UP and DOWN are evaluated as separate contingencies because the two directions are not called simultaneously. This structure verifies that the promised capacity can be delivered without violating steady-state network limits at the activation endpoint. These constraints establish one-interval steady-state network deliverability for the scheduled UP and DOWN activations; they do not represent converter transients, frequency response, repeated calls, or recovery dynamics.

3.5. Reserve-Requirement Uncertainty and Wasserstein-DRO Modeling

Historical source–load forecast errors are converted into directional reserve requirements. CVaR provides a convex representation of adverse-tail loss [21], while a Wasserstein ambiguity set protects against nearby probability distributions around the empirical sample [22]. The resulting requirement is data-driven and tractable, although its conservatism depends on the confidence level, ambiguity radius, support assumptions, and additional safety multiplier.

3.5.1. Forecast Errors and Directional Risk

For period t , define

$$\tilde{\zeta}_t = \begin{bmatrix} \varepsilon_{L,t} \\ \varepsilon_{PV,t} \end{bmatrix}, \quad \Delta P_t(\tilde{\zeta}_t) = \varepsilon_{L,t} - \varepsilon_{PV,t}. \quad (25)$$

Here, $\varepsilon_{L,t}$ and $\varepsilon_{PV,t}$ are the load and PV forecast errors, and ΔP_t is their resulting net-load error. The vector $\tilde{\zeta}_t$ collects these error components as summarized in the Nomenclature.

Positive net-load error requires upward reserve, while negative error requires downward reserve. The directional loss variables are

$$\begin{aligned} r_t^{\text{up}}(\tilde{\zeta}_t) &= \max\{\Delta P_t(\tilde{\zeta}_t), 0\}, \\ r_t^{\text{dn}}(\tilde{\zeta}_t) &= \max\{-\Delta P_t(\tilde{\zeta}_t), 0\}. \end{aligned} \quad (26)$$

The separation is necessary because upward and downward reserve use different PEDF actions and face different physical limits. It also preserves the sign information that would be lost if the net-load error were summarized by one unsigned dispersion metric. A positive load error or negative PV error creates an upward requirement, whereas the opposite realization creates a downward requirement. Treating the two tails independently therefore connects the statistical model to the directional device constraints defined above.

3.5.2. Empirical Distribution and Wasserstein Ambiguity Set

With M historical observations $\hat{\zeta}_t^m$, the empirical distribution and first-order Wasserstein ambiguity set are

$$\hat{\mathbb{P}}_t = \frac{1}{M} \sum_{m=1}^M \delta_{\hat{\zeta}_t^m}, \quad \mathcal{P}_t = \left\{ \mathbb{P} : W_1(\mathbb{P}, \hat{\mathbb{P}}_t) \leq \rho_t \right\}. \quad (27)$$

Here, M is the number of historical observations, $\hat{\zeta}_t^m$ is sample m , $\delta_{\hat{\zeta}_t^m}$ is its Dirac probability measure, W_1 is the first-order Wasserstein distance, and $\mathbb{P}$ is a candidate probability distribution. The empirical center $\hat{\mathbb{P}}_t$ and radius ρ_t follow the Nomenclature.

For each hour, the base radius is estimated from 1000 bootstrap replications at a nominal confidence level of 0.95. A dimensionless multiplier then scales the base value. The formal dispatch uses multiplier 1.0; sensitivity tests cover 0 to 2.0, and chronological out-of-sample tests identify 0.1 as the smallest tested common multiplier meeting the 5% target across the selected training windows. The larger value retained in the base case is therefore a conservative case-study specification rather than an OOS-optimal claim.

3.5.3. Worst-Case CVaR Reserve Requirement

For direction $s \in \{\text{up}, \text{dn}\}$, the unscaled worst-case CVaR is

$$\hat{R}_t^s = \inf_{\eta_t^s} \left\{ \eta_t^s + \frac{1}{1-\beta} \sup_{\mathbb{P} \in \mathcal{P}_t} \mathbb{E}_{\mathbb{P}}[(r_t^s(\tilde{\zeta}_t) - \eta_t^s)_+] \right\}. \quad (28)$$

Here, $\hat{R}_t^s$ is the unscaled worst-case CVaR reserve requirement, $\mathbb{E}_{\mathbb{P}}$ denotes expectation under distribution $\mathbb{P}$, and $(u)_+ = \max\{u, 0\}$ is the positive-part operator. The confidence level β and threshold η_t^s are listed in the Nomenclature.

The scheduled requirement includes an optional engineering multiplier:

$$R_t^s \geq \kappa \hat{R}_t^s. \quad (29)$$

The formal case uses $\beta = 0.90$ and $\kappa = 1.00$. Thus, CVaR averages the upper 10% adverse tail without applying an additional margin. Values $\beta \in \{0.80, 0.90, 0.95\}$ and $\kappa \in \{1.00, 1.05, 1.10\}$ are examined separately. These settings are transparent risk choices for the test system, not universal standards or jointly optimized parameters.

3.5.4. Finite-Dimensional Reformulation and Reserve Adequacy

Let $\ell_t^s(\xi_t, \eta_t^s) = (r_t^s(\xi_t) - \eta_t^s)_+$. For the adopted Lipschitz loss, the finite-dimensional epigraph used in the scheduling model is

$$\begin{aligned} \sup_{\mathbb{P} \in \mathcal{P}_t} \mathbb{E}_{\mathbb{P}}[\ell_t^s] &\leq \alpha_t^s \rho_t + \frac{1}{M} \sum_{m=1}^M q_{t,m}^s, \\ \alpha_t^s &\geq L_t^s, \quad q_{t,m}^s \geq \ell_t^s(\hat{\xi}_t^m, \eta_t^s), \quad q_{t,m}^s \geq 0, \\ R_t^s &\geq \kappa \left[\eta_t^s + \frac{\alpha_t^s \rho_t + \frac{1}{M} \sum_{m=1}^M q_{t,m}^s}{1 - \beta} \right]. \end{aligned} \quad (30)$$

Here, α_t^s is the nonnegative Wasserstein-dual coefficient, $q_{t,m}^s$ is the sample-wise epigraph variable, and L_t^s is a valid Lipschitz bound for ℓ_t^s . The hourly adequacy constraints are

$$\begin{aligned} \sum_{n \in \mathcal{N}} F_{n,t}^{\text{up}} + R_t^{\text{ext,up}} + \Delta R_t^{\text{up}} &\geq R_t^{\text{up}}, \\ \sum_{n \in \mathcal{N}} F_{n,t}^{\text{dn}} + R_t^{\text{ext,dn}} + \Delta R_t^{\text{dn}} &\geq R_t^{\text{dn}}, \\ R_t^{\text{ext,up}}, R_t^{\text{ext,dn}}, \Delta R_t^{\text{up}}, \Delta R_t^{\text{dn}} &\geq 0. \end{aligned} \quad (31)$$

Case 1 fixes PEDF reserve to zero and meets the requirement with external reserve and, only if unavoidable, shortage. Case 2 allows PEDF and external reserve to be procured jointly. This equation is the direct link between risk-based reserve demand and price-responsive reserve supply. The ambiguity radius, CVaR confidence level, and safety multiplier act at different stages. The radius protects against distributional displacement from the empirical sample, β determines the adverse tail averaged by CVaR, and κ applies a transparent engineering margin after the risk calculation. Keeping these roles distinct avoids describing three different sources of conservatism as one tuning parameter and motivates the separate sensitivity studies.

3.6. Single-Level Reformulation and Solution

For fixed upper-level prices λ , PEDF n solves

$$\begin{aligned} \min_{x_n} \quad & C_n^{\text{PEDF}}(x_n, \lambda) \\ \text{s.t.} \quad & h_n(x_n) = 0, \\ & g_n(x_n) \leq 0. \end{aligned} \quad (32)$$

Here, x_n is the PEDF decision vector, λ collects the upper-level prices, $h_n(x_n) = 0$ denotes the follower equality constraints, and $g_n(x_n) \leq 0$ denotes its inequality constraints.

With unrestricted equality dual variables μ_n and nonnegative inequality dual variables ν_n , the Lagrangian is

$$\mathcal{L}_n = C_n^{\text{PEDF}}(x_n, \lambda) + \mu_n^\top h_n(x_n) + \nu_n^\top g_n(x_n). \quad (33)$$

If the follower is continuous, convex, feasible, and satisfies a constraint qualification such as Slater's condition, its KKT conditions are necessary and sufficient:

$$\begin{aligned} h_n(x_n) &= 0, & g_n(x_n) &\leq 0, & v_n &\geq 0, \\ \nabla_{x_n} \mathcal{L}_n &= 0, & v_n^\top g_n(x_n) &= 0. \end{aligned} \quad (34)$$

The complementarity pair for inequality $g_i(x) \leq 0$ is linearized as

$$0 \leq v_i \leq M_i^y z_i, \quad 0 \leq -g_i(x) \leq M_i^g (1 - z_i), \quad z_i \in \{0, 1\}. \quad (35)$$

Here, i indexes an individual follower inequality. The bounds M_i^y and M_i^g and binary variable z_i , listed in the Nomenclature, enforce the disjunction between a positive dual variable and a positive primal slack.

Finite, constraint-valid bounds are essential for the complementarity reformulation. A bound that is too small can exclude a valid follower optimum, whereas an unnecessarily large bound weakens the mixed-integer relaxation and may impair numerical stability [36]. The modeling section therefore states the validity requirement without treating a particular numerical constant as theoretically universal. The selected dual and slack bounds, their utilization, and the associated sensitivity diagnostics are reported in Section 4.4. The KKT block has a specific economic meaning: at the final leader decision, each PEDF schedule must remain optimal for the posted prices rather than merely feasible. Primal feasibility enforces device operation, dual feasibility values the active limits, stationarity balances marginal cost and revenue, and complementarity identifies which limits determine the response.

The final reformulated model is

$$\begin{aligned} \min \quad & C^{\text{DSO}} \\ \text{s.t.} \quad & x^{\text{DSO}} \in \Omega^{\text{DSO}}, \\ & x_n^{\text{PEDF}} \in \Omega_n^{\text{PEDF}}, \quad \forall n \in \mathcal{N}, \\ & \mathcal{K}_n(x_n, \mu_n, v_n), \quad \forall n \in \mathcal{N}, \\ & \mathcal{B}_n(g_n, v_n, z_n), \quad \forall n \in \mathcal{N}, \\ & \mathcal{W}_t(R_t^{\text{up}}, R_t^{\text{dn}}), \quad \forall t \in \mathcal{T}. \end{aligned} \quad (36)$$

Here, x^{DSO} and x_n^{PEDF} are the upper-level and follower decision vectors. Ω^{DSO} collects the price, external-reserve, base-network, and activated-network constraints, while Ω_n^{PEDF} contains the local operating, participation, and deliverability constraints. $\mathcal{K}_n$, $\mathcal{B}_n$, and $\mathcal{W}_t$ denote the KKT, Big-M complementarity, and Wasserstein-DRO blocks. The reformulated model retains the upper-level DSO operating-cost objective defined in Equation (1). Total-system cost in Equation (3) is evaluated after optimization for economic comparison and is not substituted for the leader's objective. A global mixed-integer solver certifies the reformulated MISOCP to the requested tolerance. The certificate applies to this reformulation with the selected Big-M bounds and conic relaxation, not to an unrestricted nonconvex AC bilevel problem.

The model is closed in both economic and physical terms. Forecast-error samples generate R_t^s ; Equation (31) allocates that requirement among PEDF reserve, external reserve, and shortage; the PEDF reserve components are constrained by local power, stored-energy, participation, and converter limits; activated DistFlow states verify feeder security after deployment; and the KKT system ensures that every PEDF schedule is optimal for the posted prices. The resulting chain answers four practical questions in one optimization: how much directional reserve is needed, which resources should supply it, whether each

PEDF can honor the commitment after its energy schedule is fixed, and whether the feeder remains secure when the reserve is activated. The case studies test each link through method comparisons, risk sensitivities, solver diagnostics, activated-state AC power flow, and a larger feeder.

4. Case Studies

4.1. Test System, Data, and Parameter Calibration

The main study uses a modified IEEE 33-bus radial feeder with 32 energized branches. Five normally open tie branches are excluded from the operating topology. PEDF1 and PEDF2 are connected at buses 18 and 33, respectively, and the slack-bus voltage is fixed at 1.04 p.u. rather than optimized. Figure 2 shows the feeder topology. The two connection points are electrically remote from the substation and provide a demanding setting for checking the combined effect of active power, reactive power, and reserve activation on voltage. Table 2 summarizes the device ratings, access buses, and operating limits used for PEDF1 and PEDF2.

The coefficients are expressed in CNY per kWh for energy-throughput terms and equivalently in CNY per kW over the one-hour dispatch interval for reserve-capacity terms. Table 3 reports the corresponding physical operating-cost coefficients. These coefficients reproduce the physical-cost accounting used in the reported computational results; reserve revenue therefore competes with resource-specific opportunity and provision costs rather than treating all flexibility as free.

Table 2. PEDF equipment parameters used in the main IEEE 33-bus study.

Parameter	PEDF1	PEDF2
Access bus	18	33
Peak load (kW)	60	55
PV peak (kW)	65	77
Flexible-load proportion (%)	25	20
Battery energy capacity (kWh)	48	40
Allowed SOC ratio	0.20–1.00	0.20–1.00
Initial SOC ratio	0.55	0.55
Charge/discharge limit (kW)	20/20	16/16
Reactive-power limit (kvar)	40	35
Converter rating (kVA)	150	150
Gross active-power transaction limit (kW)	150	150

Table 3. PEDF physical operating-cost coefficients used in the formal base case.

Coefficient	Value	Costed Action
c^{deg}	0.05	Battery charging and discharging throughput
c^Q	0.08	Absolute reactive-power support
c^{cut}	0.80	Photovoltaic curtailment
c^{flex}	0.02	Flexible-load deviation from the reference profile
$c_{\text{ES}}^{\text{up}}/c_{\text{ES}}^{\text{dn}}$	0.08/0.08	Battery reserve provision
$c_{\text{flex}}^{\text{up}}/c_{\text{flex}}^{\text{dn}}$	0.12/0.12	Flexible-load reserve provision
$c_{\text{PV}}^{\text{up}}$	0.05	Recovery of curtailed PV for upward reserve

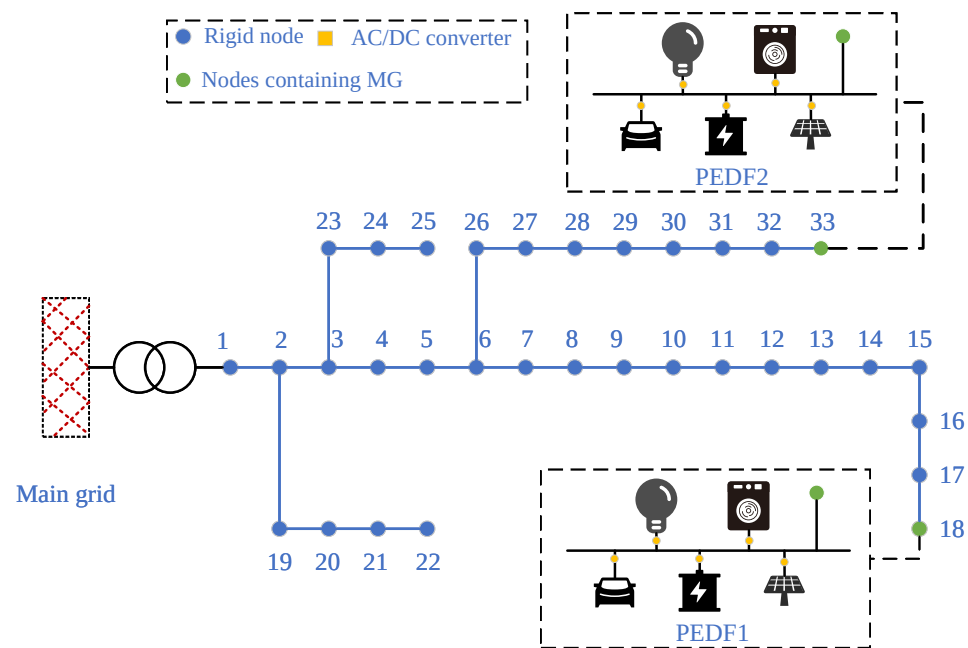


Figure 2. Modified IEEE 33-bus feeder with PEDF microgrids connected at buses 18 and 33. The blue numbers denote the bus indices.

Load and photovoltaic data are taken from the Elia Open Data platform [37]. Data from 1 to 30 July 2025 form a 30×24 historical net-load error sample, and 31 July 2025 is the dispatch day. The normalized daily bases are 10,227.9850 for load and 3,375.0023 for photovoltaic generation. Missing 15 min observations are linearly completed before hourly aggregation. The use of 30 summer days keeps the empirical model transparent but does not capture seasonal transitions, prolonged cloud events, heat waves, or abnormal demand. Sample-size sensitivity and rolling holdout tests are therefore reported rather than treating the empirical distribution as fully representative.

The parameter roles are separated to avoid double counting conservatism. The confidence level β determines which adverse tail is averaged by CVaR, whereas κ multiplies the resulting reserve requirement after the WDRO-CVaR calculation. The formal base uses $\beta = 0.90$ and $\kappa = 1.00$. The former represents an intermediate tail-risk preference within the tested range, and the latter avoids adding an uncalibrated margin on top of the ambiguity-set and CVaR protection. A dedicated sensitivity study evaluates $\beta = 0.80, 0.90$, and 0.95 and $\kappa = 1.00, 1.05$, and 1.10 . Neither parameter is presented as a universal standard or a globally optimized setting.

All runs were performed on a Windows 11 workstation with an Intel Core i7-1260P processor, 12 physical cores, and 16 logical processors. The target relative MIP gap for the formal IEEE 33-bus cases was 0.1%. Case 1 disables PEDF reserve and satisfies the requirement through external procurement, whereas Case 2 allows PEDF and external reserve to be cleared jointly. In both cases, the shortage variables remain available only as a heavily penalized feasibility safeguard. This design isolates the value of customer-side participation without changing the forecast-error sample, reserve requirement, or feeder limits. Table 4 summarizes the risk, market, and numerical parameters used for the formal base case.

Table 4. Risk, market, and numerical parameters for the formal base case.

Parameter	Value	Role in the Model
Historical sample size	30 days	Hour-specific source–load error distribution
CVaR confidence level β	0.90	Formal upper-tail preference; tested at 0.80, 0.90, and 0.95
Bootstrap replications/confidence	1000/0.95	Empirical Wasserstein radius construction
Main radius multiplier	1.0	Conservative base dispatch; distinct from the OOS-selected minimum
Reserve safety factor κ	1.00	No additional multiplication in the formal base; 1.05 and 1.10 are tested as optional margins
External reserve prices (CNY/kW)	1.20 upward; 1.00 downward	Case 1 and residual external procurement in Case 2
Reserve-price upper bounds (CNY/kW)	0.90 upward; 0.75 downward	Bounds on DSO price guidance
Shortage penalty (CNY/kW)	100 upward/downward	Discourages avoidable uncovered reserve
Big-M dual/slack bounds	100/500	Complementarity linearization
Solver	Gurobi 13.0.0	MISOCP solution through MATLAB and YALMIP

4.2. Base-Case Economic and Physical Performance

The formal WDRO-CVaR base case requires 924.3584 kW of upward reserve and 845.4638 kW of downward reserve over the 24 h horizon. Figure 3 shows their hourly distribution together with the dispatch-day net-load error. Reserve demand becomes larger during periods with greater historical dispersion and tail exposure. The requirements represent the selected 30-day sample, $\beta = 0.90$, radius multiplier 1.0, and $\kappa = 1.00$; they are not universal reserve standards.

Total-system cost is evaluated separately from the DSO leader objective so that internal market transfers cancel at the system level. It decreases from CNY 60,123.3928 in Case 1 to CNY 58,647.2865 in Case 2, corresponding to a 2.455% reduction. This accounting metric is used only for case comparison; the Stackelberg formulation retains the DSO operating-cost objective defined in Section 3.2.1.

Allowing PEDF participation reduces external upward reserve by 866.2070 kW and external downward reserve by 601.4664 kW. The associated cost reduction is not merely a redistribution of market payments. Battery schedules, flexible demand, recoverable PV curtailment, reactive support, network losses, and residual external procurement all change in the coordinated solution. Reserve revenue compensates the PEDF microgrids for retaining energy and adjustment headroom, while the DSO avoids most of the external upward-reserve purchase and a substantial share of the downward purchase.

Figure 4 confirms that the scheduled combination of PEDF and external reserve satisfies every hourly upward and downward requirement, with zero shortfall throughout the day. PEDF resources cover most of the upward requirement, while a larger residual external contribution remains in the downward direction. This directional difference motivates the resource-level decomposition shown next.

Figure 5 clarifies why the PEDF coverage rates differ by direction. Upward reserve can draw on battery discharge, cancellation of charging, recovery of curtailed photovoltaic generation, and flexible-load reduction. In the reported solution, battery discharge and

flexible-load reduction provide the dominant upward contributions, charging reduction is used only in selected hours, and the recovered-PV-curtailment contribution is zero over the reported horizon. Downward reserve relies more strongly on increased charging and flexible-load increase, which become restricted when batteries approach their upper energy range or converter capacity is shared with reactive power. The market result therefore reflects coupled physical headroom rather than a fixed fraction of installed capacity.

Figure 6 shows that the voltage trajectories of the two cases remain within the 0.95–1.05 p.u. operating band. The minimum value in Case 2 is 0.951284 p.u., indicating that coordinated reserve procurement changes feeder loading without violating the lower-voltage limit. The independent nonlinear AC checks reported later provide an additional steady-state feasibility assessment.

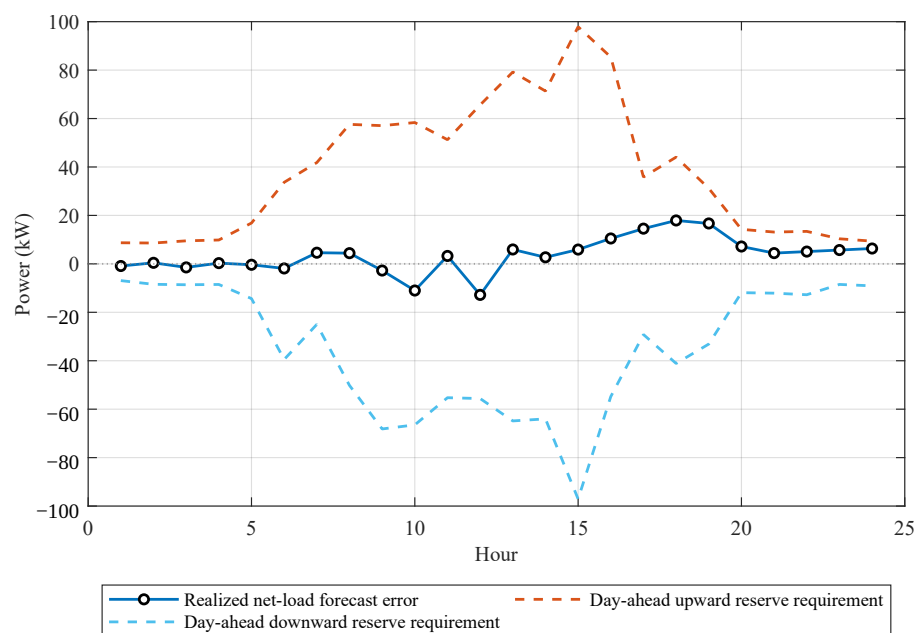


Figure 3. Hourly net-load forecast error and WDRO-CVaR upward/downward reserve requirements in the base case.

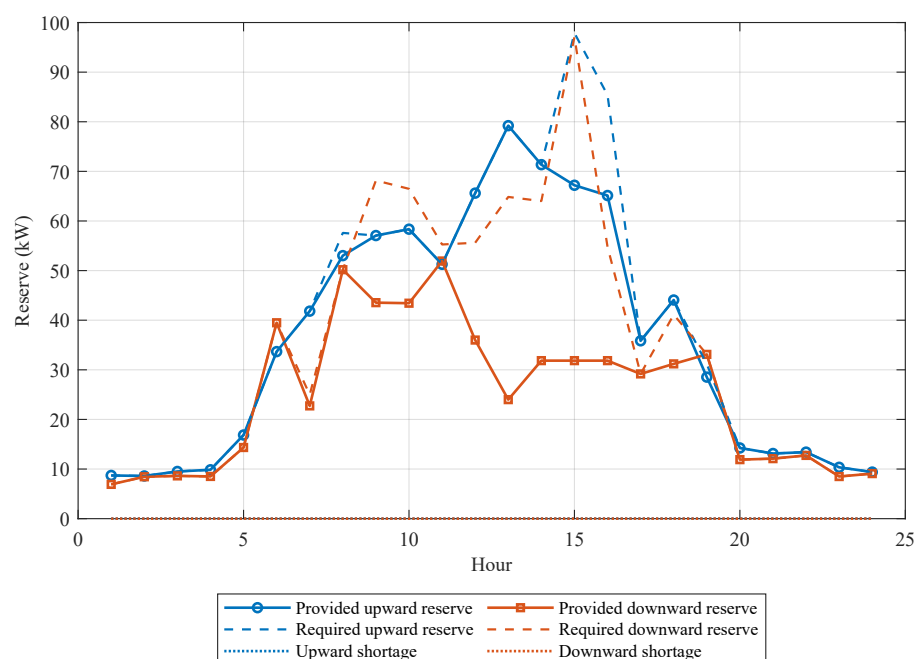


Figure 4. Hourly reserve balance and shortfall in Case 2.

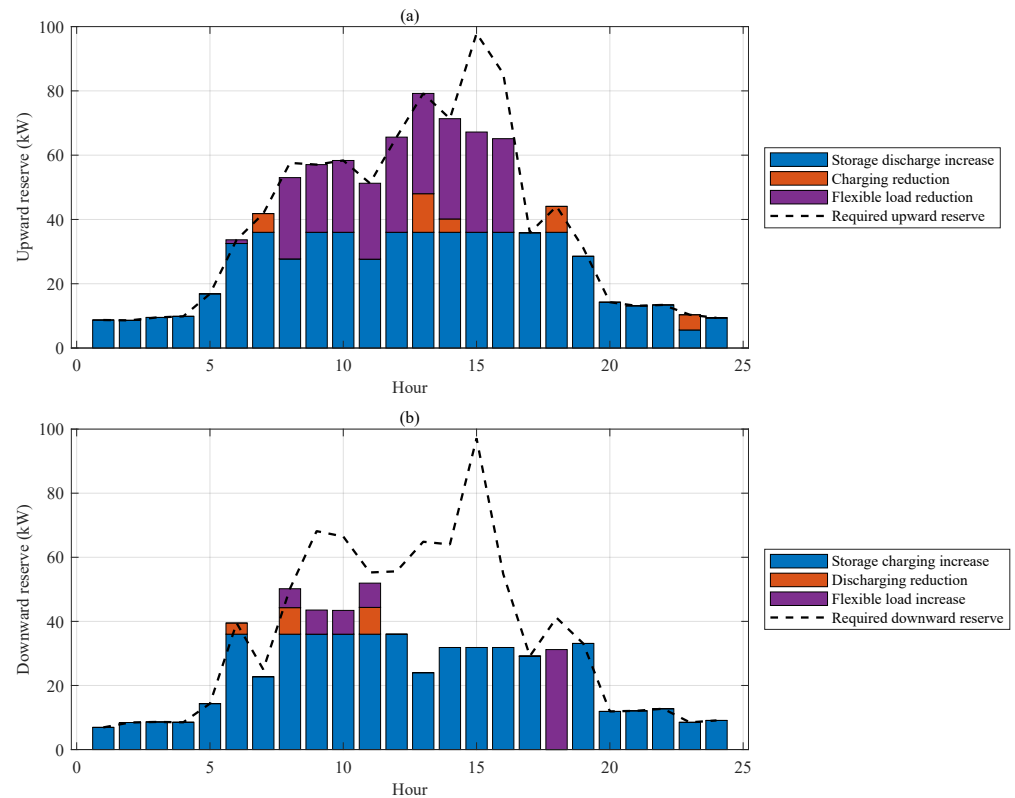


Figure 5. PEDF reserve contributions by resource type in Case 2: (a) upward reserve; (b) downward reserve.

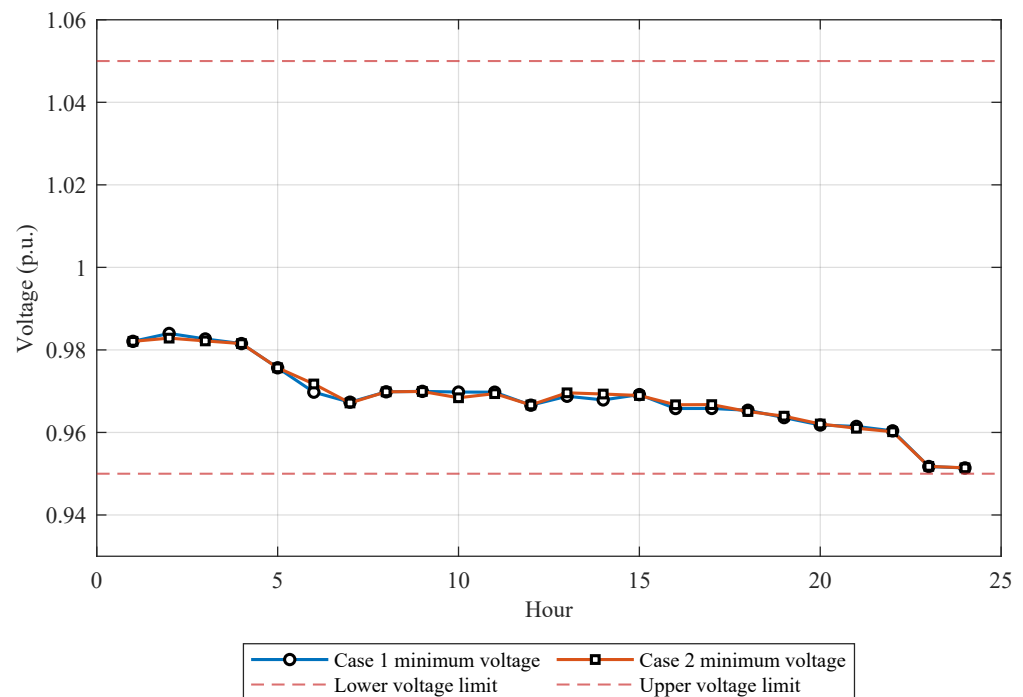


Figure 6. Minimum nodal voltage under the two reserve-procurement cases.

Figure 7 shows the intertemporal mechanism behind reserve provision. In Case 2, both batteries preserve energy before the main upward-risk periods, which reduces the freedom to pursue energy arbitrage alone. The stored-energy reservation constraints check a full scheduled activation for one hourly interval and assess upward and downward directions separately. These profiles establish steady-state one-interval deliverability under the mod-

eled schedule. They do not establish frequency response, sub-hourly converter tracking, repeated activation, rebound of flexible demand, or electrochemical battery behavior.

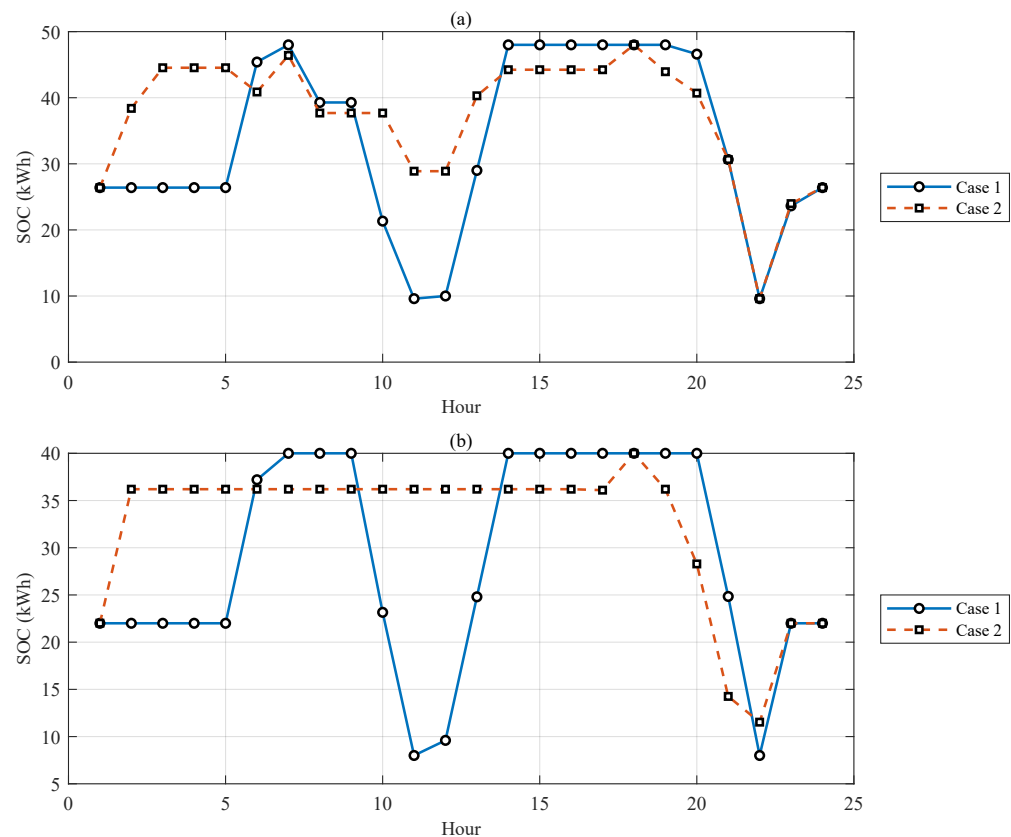


Figure 7. Battery stored-energy schedules in Cases 1 and 2: (a) PEDF1; (b) PEDF2.

4.3. Comparison with Alternative Uncertainty Treatments

The proposed WDRO-CVaR model is compared with a deterministic fixed-ratio reserve rule, sample-average CVaR, and box robust optimization. Table 5 reports the day-ahead economic results. The fixed-ratio model has the lowest absolute Case 2 cost, while WDRO-CVaR has the largest improvement relative to its own Case 1 baseline and the largest reduction in aggregate PEDF net cost. Absolute cost alone therefore does not establish superiority because each method purchases a different amount of reserve. Reliability must be considered with the economic result.

Rolling out-of-sample tests use 10-, 15-, and 20-day training windows and the remaining days as chronological validation observations. Table 6 reports the unweighted arithmetic mean of the three window-specific violation rates. The WDRO entry uses the smallest tested uniform radius multiplier, 0.1, that satisfies the 5% target in every training window. Fixed-ratio scheduling has the lowest in-sample reserve requirement and the highest violation rate. SAA-CVaR and Box-RO improve reliability, while WDRO-CVaR provides the lowest average upward and downward violations among the tested methods.

Table 5. Economic and computational comparison of the four reserve-requirement methods. Each percentage is calculated relative to the corresponding method's Case 1 benchmark.

Method	Case 2 Cost	System Improvement	PEDF Improvement	Runtime (s)	MIP Gap
Fixed ratio	58,213.4597	0.494%	0.549%	15.014	0.0340%
SAA-CVaR	58,248.7933	0.958%	8.077%	40.370	0.0444%
Box-RO	58,300.5048	1.177%	3.770%	23.746	0.0940%
WDRO-CVaR	58,647.2865	2.455%	10.402%	30.243	0.0494%

Table 6. Unweighted mean rolling out-of-sample reserve-violation rates across 10-, 15-, and 20-day training windows.

Method	Upward Violation	Downward Violation	Interpretation
Fixed ratio	24.0972%	9.0509%	Lowest reserve procurement, but the 5% target is not met.
SAA-CVaR	5.0463%	4.9074%	Downward target is met on average; upward performance is marginal.
Box-RO	4.0741%	4.3519%	Both mean rates are below 5%, with higher conservatism than SAA.
WDRO-CVaR, scale 0.1	3.0324%	2.5463%	Lowest mean violation rates among the tested alternatives.

The formal dispatch retains radius multiplier 1.0, although the chronological test identifies 0.1 as the smallest tested value meeting the selected reliability target. The two values answer different questions. Multiplier 0.1 is a data-driven lower bound within the tested grid, whereas multiplier 1.0 preserves the conservative specification used in the market, sensitivity, and network studies. The comparison therefore documents a reliability–economy tradeoff rather than universal optimality. Likewise, the four-method benchmark spans representative deterministic, sample-based stochastic, interval-robust, and Wasserstein-DRO treatments; KL-divergence DRO and exact chance-constrained counterparts require separate reformulations and remain future comparisons.

The comparison also separates reserve-design performance from market-design performance. Fixed ratio, SAA-CVaR, Box-RO, and WDRO-CVaR determine different reserve requirements before the Stackelberg market is solved. A lower Case 2 cost may therefore reflect a smaller requirement rather than a more efficient allocation of the same reliability target. The chronological violation rates provide the missing reliability context, while the Case 1-to-Case 2 improvement measures how effectively the PEDF market uses the requirement produced by each method. Interpreting both layers avoids ranking uncertainty models solely by day-ahead cost.

4.4. Wasserstein Radius, Sample Size, and Big-M Diagnostics

Table 7 shows the effect of the Wasserstein radius multiplier. Reserve requirements and external procurement generally increase as the ambiguity set expands, while total system cost rises from CNY 58,286.6478 at scale 0 to CNY 59,632.2087 at scale 2.0. PEDF net cost is nonmonotonic because the Stackelberg equilibrium changes both physical dispatch and payment allocation. A larger ambiguity set therefore improves protection against distributional error at an economic cost, but it does not imply a monotonic change in every participant’s net benefit.

Table 7. Sensitivity of the formal IEEE 33-bus Case 2 result to the Wasserstein radius multiplier.

Scale	System Cost	PEDF Net Cost	Reserve Up/Down (kW)	External Reserve Cost	Minimum Voltage
0.0	58,286.6478	1265.2676	328.3070/249.4124	72.5151	0.951364
0.5	58,382.8453	1268.9494	626.3327/547.4381	74.7462	0.951376
1.0	58,647.2865	1160.4838	924.3584/845.4638	313.7791	0.951284
1.5	59,094.2757	1242.8609	1222.3841/1143.4895	754.5782	0.951155
2.0	59,632.2087	1195.1015	1520.4098/1441.5152	1258.7545	0.951381

The CVaR confidence level and reserve safety factor affect different layers of conservatism and are therefore varied separately. Table 8 summarizes the two one-factor sweeps. The β sweep holds $\kappa = 1.05$ and the radius multiplier at 1.0, while the κ sweep holds $\beta = 0.90$ and the radius multiplier at 1.0. The first sweep evaluates tail-risk preference under a fixed moderate planning margin. The second sweep contains the formal $\kappa = 1.00$

base and measures the effect of adding 5% and 10% margins. This one-factor design isolates local effects; it is not a joint parameter optimization.

Table 8. Sensitivity to the CVaR confidence level and reserve safety factor. Reserve and coverage entries are reported as upward/downward pairs. The β sweep fixes $\kappa = 1.05$, while the κ sweep fixes $\beta = 0.90$.

Parameter	Value	Reserve Requirement (kW)	Case 2 System Cost (CNY)	PEDF Net Cost (CNY)	External Reserve (kW)	PEDF Coverage
β at $\kappa = 1.05$	0.80	574.0101/501.1507	58,331.5343	1247.3291	16.5931/19.1281	0.97109/0.96183
	0.90	970.5763/887.7370	58,695.5832	1232.4591	94.3494/250.6483	0.90279/0.71765
	0.95	1646.1038/1574.4143	59,875.9728	1201.7921	625.3731/756.5610	0.62009/0.51947
κ at $\beta = 0.90$	1.00	924.3584/845.4638	58,647.2865	1160.4838	58.1514/243.9974	0.93709/0.71140
	1.05	970.5763/887.7370	58,695.5832	1232.4591	94.3494/250.6483	0.90279/0.71765
	1.10	1016.7942/930.0102	58,759.6427	1245.1471	116.0232/283.2122	0.88589/0.69547

Raising β from 0.80 to 0.95 increases the 24 h upward/downward requirements from 574.0101/501.1507 kW to 1,646.1038/1,574.4143 kW and raises the Case 2 system cost from CNY 58,331.5343 to CNY 59,875.9728. PEDF reserve coverage declines as the tail requirement approaches the physical headroom of the two microgrids, so marginal reserve is increasingly supplied externally. The lower PEDF net cost at larger β reflects equilibrium payments and should not be interpreted as a reduction in total system burden.

At $\beta = 0.90$, increasing κ from 1.00 to 1.10 raises the reserve requirement by 10%, increases the Case 2 system cost by CNY 112.3563, and shifts additional upward and downward procurement to the external source. Across all six reported parameter points, both reserve shortages remain zero, the minimum voltage lies between 0.951277 and 0.951330 p.u., the Case 2 MIP gap remains between 0.0102% and 0.0494%, and the maximum normalized SOCP residual is 2.342×10^{-5} . The results support the selected parameters as transparent local design choices, while providing no basis for claiming universal or globally optimal values.

The different slopes of the two sweeps are operationally informative. Changing β alters which historical errors dominate the tail average and can more than double the required reserve, whereas κ scales an already calculated requirement by a modest percentage. As the requirement grows, the PEDF cluster reaches local energy and converter limits, so the incremental obligation is increasingly assigned to external reserve. The sensitivity therefore shows both the statistical effect of a stronger tail preference and the physical point at which customer-side flexibility ceases to be the marginal source. Table 9 reports the window-specific OOS violation rates used in the rolling validation.

Table 9. Rolling OOS results for the selected radius multiplier of 0.1. The mean row is the unweighted arithmetic mean across the three windows.

Training Window	Validation Days	Upward Violation	Downward Violation
10 days	20	4.3750%	3.7500%
15 days	15	3.0556%	2.2222%
20 days	10	1.6667%	1.6667%
Unweighted mean	–	3.0324%	2.5463%

Historical sample size changes both the empirical error distribution and the bootstrap radius. Table 10 shows that the mean radius decreases from 3.2624 to 2.4835 as the sample expands from 10 to 30 days, while the maximum hourly radius and market outcomes do not follow a perfectly monotonic path. The composition of the added days matters as much as the sample count. This observation supports chronological OOS validation and also limits the strength of conclusions drawn from one summer month.

Table 10. Sensitivity to the number of historical days used in the WDRO calibration.

Days	Mean Radius	Maximum Radius	Case 2 System Cost	Case 2 PEDF Net Cost
10	3.2624	7.1025	58,838.0385	1240.4790
15	3.2230	9.3145	58,885.1538	1265.2988
20	2.9228	7.0821	58,814.3242	1198.4037
25	2.7204	6.6957	58,707.9293	1185.2068
30	2.4835	6.5039	58,647.2865	1160.4838

Big-M selection is evaluated here as a numerical property of the case study. A bound that is too small may remove a valid follower optimum, while an excessively large value can weaken the mixed-integer relaxation and increase numerical instability. The implementation uses a dual bound of 100 and a slack bound of 500. In the formal base case, the largest dual value is 2.67, corresponding to 2.67% utilization of the dual bound, and no variable is classified as near the bound. The maximum complementarity residual is 2.618×10^{-15} and the payment diagnostic error is 2.957×10^{-9} . Table 11 reports additional dual-bound tests at 50 and 200. All three values remain feasible with small residuals, although the PEDF payment allocation varies. The dual-bound sweep and the observed 45.61% maximum slack-bound utilization support the selected values for the reported solution. The slack bound was not independently swept, and the results do not imply that every Big-M choice yields an identical equilibrium.

Table 11. Sensitivity to the dual-variable Big-M bound.

M^v	Case 2 Cost	PEDF Net Cost	Max. Dual	Bound Use	Comp. Residual	Runtime/Gap
50	58,620.9883	1,175.5032	2.3317	4.663%	9.611×10^{-11}	128.671 s/0.0038%
100	58,647.2865	1,160.4838	2.6700	2.670%	2.618×10^{-15}	24.383 s/0.0494%
200	58,621.7917	1,239.5995	2.6700	1.335%	7.984×10^{-15}	107.044 s/0.0050%

The main economic, reserve, voltage, and solver outcomes are summarized in Table 12. Runtime values in Tables 5, 11, and 12 come from separate saved sessions and include session-specific model construction and solver conditions; they are therefore used as within-session diagnostics rather than identical repeated timings. These diagnostics qualify the optimality statement. Gurobi supplies an objective bound for the reformulated MISOCP within the reported MIP gap. Equivalence to the follower optimum additionally requires continuous convex follower problems, constraint qualification, strong duality, and valid complementarity bounds. The branch-flow SOCP is also a relaxation of nonlinear AC power flow. The reported solver certificate therefore applies to the reformulated model and does not constitute an unconditional global-optimality proof for the original nonconvex AC bilevel system.

The Big-M tests also explain why participant payments are more sensitive than system cost. Several price and dual combinations can support physically similar schedules when constraints are weakly active or multiple follower optima are close. Changing the complementarity bound can alter which equilibrium is selected without materially changing reserve balance or voltage. Reporting the PEDF net cost, payment diagnostic, complementarity residual, and physical schedule together is therefore more informative than treating one objective value as evidence of numerical invariance.

Table 12. Main economic, reserve, voltage, and solver results for the IEEE 33-bus base case.

Indicator	Case 1	Case 2	Change or Interpretation
Total system cost (CNY)	60,123.3928	58,647.2865	−1476.1064 (−2.455%)
Aggregate PEDF net cost (CNY)	1295.2088	1160.4838	−134.7250 (−10.402%)
PEDF1 net-cost improvement	–	12.105%	Individual improvement
PEDF2 net-cost improvement	–	8.190%	Individual improvement
External upward reserve (kW)	924.3584	58.1514	PEDF coverage 93.709%
External downward reserve (kW)	845.4638	243.9974	PEDF coverage 71.140%
Upward/downward reserve shortage (kW)	0/0	0/0	Requirement fully scheduled
Minimum nodal voltage (p.u.)	0.951428	0.951284	Both above 0.95 p.u.
Maximum normalized SOCP residual	–	9.756×10^{-6}	Small for the formal 33-bus run
Maximum complementarity residual	–	2.618×10^{-15}	KKT diagnostic
Payment diagnostic error	–	2.957×10^{-9}	Primal–dual settlement check
Solve time (s)	1.31	16.20	Same computer and solver settings
Actual MIP gap	0.0017%	0.0494%	Below the 0.1% target

4.5. Sensitivity to Market and PEDF Parameters

Five additional parameter groups are varied by multipliers of 0.8, 1.0, and 1.2. The formal session contains 15 successful tasks. Every task has zero upward and downward reserve shortage, a Case 2 MIP gap below 0.1%, and a minimum voltage above 0.951239 p.u. Table 13 reports the complete economic and reserve results.

Table 13. Sensitivity of Case 2 to reserve-shortage penalties, reserve-price upper bounds, battery energy capacity, converter rating, and flexible-load proportion. External reserve and PEDF coverage are reported as upward/downward pairs.

Parameter	Scale	System Cost	System Imp.	PEDF Net Cost	PEDF Imp.	External Reserve	PEDF Coverage	Min. V
Shortage penalty	0.8	58,647.2865	2.455%	1160.4838	10.402%	58.1514/243.9974	0.93709/0.71140	0.951284
	1.0	58,647.2865	2.455%	1160.4838	10.402%	58.1514/243.9974	0.93709/0.71140	0.951284
	1.2	58,647.2865	2.455%	1160.4838	10.402%	58.1514/243.9974	0.93709/0.71140	0.951284
Reserve-price bounds	0.8	58,621.0322	2.499%	1221.2372	5.711%	52.1708/219.1694	0.94356/0.74077	0.951282
	1.0	58,647.2865	2.455%	1160.4838	10.402%	58.1514/243.9974	0.93709/0.71140	0.951284
	1.2	58,620.9798	2.499%	1258.0976	2.865%	52.1708/219.1694	0.94356/0.74077	0.951288
Battery capacity	0.8	58,635.7146	2.509%	1194.3670	8.992%	52.1708/219.1694	0.94356/0.74077	0.951459
	1.0	58,647.2865	2.455%	1160.4838	10.402%	58.1514/243.9974	0.93709/0.71140	0.951284
	1.2	58,626.8553	2.454%	1040.0014	17.566%	54.4145/240.7302	0.94113/0.71527	0.951301
Converter rating	0.8	58,633.2157	2.477%	1261.0154	1.977%	68.1699/223.7335	0.92625/0.73537	0.951281
	1.0	58,647.2865	2.455%	1160.4838	10.402%	58.1514/243.9974	0.93709/0.71140	0.951284
	1.2	58,643.2046	2.461%	1191.7593	5.743%	63.8218/235.5585	0.93096/0.72139	0.951281
Flexible-load proportion	0.8	58,732.1465	2.361%	1277.6303	3.301%	137.4544/232.7769	0.85130/0.72468	0.951378
	1.0	58,647.2865	2.455%	1160.4838	10.402%	58.1514/243.9974	0.93709/0.71140	0.951284
	1.2	58,552.7945	2.565%	1202.4723	4.095%	6.1209/213.6335	0.99338/0.74732	0.951239

The shortage-penalty results are identical over CNY 80 to 120/kW because shortage remains zero and the penalty variable is inactive. This finding indicates local insensitivity within the tested feasible range; it does not imply independence from every possible penalty. Reserve-price bounds change the payment allocation and equilibrium selection more than the physical reserve schedule, producing a nonmonotonic PEDF net cost. The 0.8 price-bound point has a maximum complementarity residual of 3.735×10^{-5} , a payment diagnostic error of 7.227×10^{-5} , and a normalized SOCP residual of 9.227×10^{-5} . These values remain small enough for a trend point, but this run is not used to claim machine-precision KKT agreement.

Increasing battery energy capacity from 0.8 to 1.2 of the base value lowers PEDF net cost from CNY 1194.3670 to CNY 1040.0014 and increases the PEDF improvement from 8.992% to 17.566%. The system-cost change is much smaller, indicating that additional energy capacity mainly enlarges intertemporal flexibility and participant benefit after the main system requirement is already feasible. Converter-rating results are nonmonotonic because active power, reactive power, reserve headroom, and price equilibrium share the same interface. A larger converter therefore does not guarantee a lower PEDF net cost.

Flexible-load proportion produces the clearest system-level effect. Increasing the proportion from 0.8 to 1.2 of the base value lowers Case 2 system cost from CNY 58,732.1465 to CNY 58,552.7945, reduces external upward reserve from 137.4544 kW to 6.1209 kW, and raises PEDF upward coverage from 85.130% to 99.338%. PEDF net-cost improvement remains nonmonotonic, which separates physical system efficiency from the distribution of market payments. This distinction is important when participation incentives and social operating cost are evaluated together.

Across the 15 tasks, the largest actual MIP gap is 0.0579%, the largest normalized SOCP residual is 1.030×10^{-4} , and the smallest voltage is 0.951239 p.u. The aggregate runtime is 1300.740 s, with individual runs between 44.486 s and 160.162 s. These diagnostics indicate stable feasibility over the tested local parameter range, while the economic nonmonotonicities show that Stackelberg market outcomes cannot be inferred from physical capacity changes alone.

The five parameter groups correspond to different planning and operating levers. Battery energy capacity changes intertemporal availability, converter rating changes instantaneous active–reactive sharing, and flexible-load proportion changes demand-side headroom. Reserve-price bounds primarily govern compensation and equilibrium selection, while the shortage penalty matters only after available PEDF and external reserve have been exhausted. The results therefore should not be read as five interchangeable flexibility investments. They indicate which model layer is affected and whether the benefit appears mainly in total operating cost, participant economics, or the source composition of reserve.

4.6. Scalability and Independent Nonlinear AC Validation

The larger-system test uses the IEEE 69-bus feeder with four PEDF microgrids. Table 14 compares model size and computational performance. The Case 2 formulation grows from 42,334 rows, 28,708 columns, and 3168 binary variables in the 33-bus study to 87,380 rows, 59,576 columns, and 6336 binary variables in the 69-bus study. Runtime rises from 16.20 s to 260.487 s, and peak private memory rises from approximately 5.00 GB to 8.34 GB. The increase reflects both the larger network and the doubled PEDF population. It should not be interpreted as a formal linear-complexity result.

Table 14. Model size, computational performance, and physical diagnostics for the two network studies.

Network	PEDFs	Rows	Columns	Binaries	Conic Constraints	Runtime	Peak Memory	System Imp.	PEDF Imp.	Min. Voltage/Max. Normalized SOCP Residual
IEEE 33	2	42,334	28,708	3168	2304	16.20 s	5.00 GB	2.455%	10.402%	$0.951284/9.756 \times 10^{-6}$
IEEE 69	4	87,380	59,576	6336	4896	260.487 s	8.34 GB	4.233%	4.330%	0.950924/0.006099

The IEEE 69-bus Case 2 reduces system cost by 4.233% and aggregate PEDF net cost by 4.330%, with all four PEDF microgrids improving. Its normalized SOCP residual reaches approximately 0.006 in activated states, which is larger than the formal 33-bus residual and precludes a claim of universal conic exactness. Independent nonlinear AC power flows are therefore solved for Case 1 and Case 2 under BASE, UP, and DOWN states for every hour. All 144 states converge and satisfy voltage, current, and power-balance checks. The largest AC-to-MISOCP voltage deviation is 1.6397×10^{-6} p.u., the maximum branch current is

0.4875 p.u., and the largest active and reactive balance residuals are 5.1024×10^{-11} MW and 1.3242×10^{-10} MVar. These results confirm steady-state feasibility for the tested activation states while remaining distinct from time-domain control validation.

The two validation layers answer different questions. The 69-bus optimization tests whether the formulation remains solvable when the feeder and PEDF population grow, while the nonlinear AC calculation checks whether the scheduled operating points remain feasible under the original steady-state equations. Neither test demonstrates millisecond-scale control performance, but together they reduce the risk that the reported benefit is an artifact of one small feeder or of a loose convex relaxation.

4.7. Discussion

The results are explained by the interaction of three mechanisms. The Wasserstein-CVaR block converts forecast uncertainty into an hourly reserve requirement. Each PEDF then exposes only the headroom that remains after its energy schedule, SOC limits, flexible-demand bounds, and converter loading are considered. The DSO compares this price-responsive offer with external reserve while maintaining feeder security in the base and activated states. Customer-side reserve is selected when it is physically available and economically competitive; external reserve remains when local energy or converter headroom is scarce. This interaction prevents installed capacity from being mistaken for deliverable reserve.

The economic improvement has a similarly layered interpretation. PEDF reserve revenue compensates resources for holding flexibility that could otherwise support energy arbitrage, PV accommodation, or demand comfort. The system benefit appears when that compensated local response avoids a larger external reserve purchase or reduces network-related operating cost. A lower PEDF net cost and a lower total-system cost can therefore occur together, but they are not the same quantity. The internal-payment cancellation in the system-cost metric is what allows the analysis to distinguish a real resource saving from a transfer between the DSO and the PEDF participants.

Among the tested resource parameters, the flexible-load share produces the clearest system-level response and nearly eliminates external upward reserve at the highest setting. Additional battery energy primarily improves intertemporal flexibility and participant benefit. Expanding the converter has a smaller, nonmonotonic effect because the base rating is nonbinding in many hours and remains shared with reactive support. Reserve-price bounds can change the distribution of payments even when the physical schedule changes little. System cost and participant net cost must therefore be reported separately.

Implementation would require institutional and digital infrastructure beyond the optimization model. Experience from local-flexibility projects highlights the importance of governance, DSO coordination, entry rules, baseline design, verification, and safeguards against strategic gaming [31–33]. A field deployment would also need verified forecasts and capability models, interval metering, secure communication, activation measurement, settlement, and penalties for non-delivery. Regulators must decide whether energy, reactive power, and reserve are cleared jointly or through linked products, and whether recurring flexibility costs should influence network planning [34]. The complete-information equilibrium can inform such design choices, but it does not demonstrate truthful bidding or strategy-proof operation.

The risk-parameter tests distinguish statistical tail emphasis from an engineering margin. Increasing β changes which adverse observations dominate the CVaR calculation and produces a much larger procurement response than the 0–10% multiplier represented by κ . As β rises, PEDF coverage declines because local power and energy limits are reached; external reserve then supplies the marginal requirement. The formal setting $\beta = 0.90$ is therefore an intermediate tested preference, while $\kappa = 1.00$ avoids layering an uncalibrated

margin on top of the ambiguity set and CVaR. The 1.05 and 1.10 cases show the cost of introducing such a margin explicitly.

The evidence remains bounded in four respects. The 30-day July sample supports radius, sample-size, and chronological out-of-sample tests within one summer period, but it cannot represent seasonal transitions, rare weather, equipment outages, or structural demand changes. The benchmark set spans deterministic, sample-based stochastic, interval-robust, and Wasserstein-DRO treatments; KL-divergence DRO, exact chance constraints, and multi-stage stochastic dispatch require separate formulations. KKT equivalence depends on follower convexity and strong duality, while the selected Big-M bounds can influence equilibrium allocation. Activated-state nonlinear AC power flow confirms steady-state feasibility for the tested schedules, but it does not replace frequency-domain, transient, or converter-control validation.

These boundaries define a focused research agenda. A multi-timescale extension should couple market clearing with converter controls, communication delay, frequency and voltage recovery, repeated reserve calls, flexible-load rebound, stored-energy restoration, and battery electrochemical limits. Strategic participation requires private-information models, bounded rationality, learning-based response, auditing, and incentive-compatible settlement. Multi-season and extreme-event data, alternative ambiguity sets, larger unbalanced feeders, and hardware-in-the-loop experiments are needed to determine whether the observed reliability–economy tradeoff persists beyond the present scheduling study.

5. Conclusions

This paper develops a day-ahead coordination framework that connects data-driven reserve demand with autonomous PEDF reserve supply. The DSO posts bounded prices and procures both PEDF and external reserve. Each PEDF schedules energy, reactive power, and reserve subject to battery stored-energy, flexible-demand, converter, and no-worse-off participation constraints. Separate activated feeder states test the one-interval UP and DOWN schedules. Under continuous convex followers and valid reformulation assumptions, KKT conditions and Big-M linearization yield an MISOCP.

In the IEEE 33-bus case, joint procurement reduces total system cost from CNY 60,123.3928 to CNY 58,647.2865, a decrease of 2.455%. Aggregate PEDF net cost falls by 10.402%, with individual improvements of 12.105% and 8.190%. PEDF resources cover 93.709% of upward reserve and 71.140% of downward reserve. External reserve supplies the remaining requirement, both shortage variables are zero, and the minimum nodal voltage is 0.951284 p.u. The Case 2 MIP gap is 0.0494%, the maximum complementarity residual is 2.618×10^{-15} , and the maximum normalized SOCP residual is 9.756×10^{-6} .

The sensitivity studies identify where robustness becomes costly. At fixed $\kappa = 1.05$, increasing β from 0.80 to 0.95 raises the aggregate upward/downward requirement from 574.0101/501.1507 kW to 1646.1038/1574.4143 kW and increases the Case 2 cost from CNY 58,331.5343 to CNY 59,875.9728. At fixed $\beta = 0.90$, increasing κ from 1.00 to 1.10 raises the requirement by 10% and the Case 2 cost by CNY 112.3563. All tested points retain zero shortage and secure voltage. The formal values $\beta = 0.90$ and $\kappa = 1.00$ should therefore be read as transparent case-study choices rather than universal optima.

Method comparisons and chronological validation qualify the economic result. The fixed-ratio rule achieves the lowest absolute day-ahead cost but the highest out-of-sample violation rates. WDRO-CVaR provides the lowest mean upward and downward violations among the four tested methods and the largest improvement relative to its own benchmark. A radius multiplier of 0.1 is the smallest tested value meeting the 5% target across 10-, 15-, and 20-day training windows; the formal market study retains the more conservative

multiplier 1.0. The IEEE 69-bus case reduces system cost by 4.233% and aggregate PEDF net cost by 4.330%, and all 144 nonlinear AC validation states converge.

The evidence supports steady-state, one-interval reserve deliverability for the two tested feeders. It does not establish universal cost dominance, incentive compatibility, transient or frequency support, repeated-activation performance, or cross-season robustness. Future work should address alternative ambiguity sets and chance constraints, multi-season and extreme-event data, strategic participation, unbalanced feeders, multi-timescale activation and recovery, battery dynamics, and real-time or hardware-in-the-loop validation.

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Nomenclature

The principal abbreviations and symbols used throughout the manuscript are listed below. Auxiliary variables that appear only in a local reformulation are defined at their first occurrence.

DSO	Distribution system operator
PEDF	Photovoltaic–energy storage–direct current–flexibility microgrid
DRO	Distributionally robust optimization
WDRO	Wasserstein distributionally robust optimization
SAA	Sample average approximation
OOS	Out-of-sample
CVaR	Conditional value-at-risk
KKT	Karush–Kuhn–Tucker
SOCP	Second-order cone programming
MISOCP	Mixed-integer second-order cone programming
AC	Alternating current
PV	Photovoltaic
SOC	State of charge
$\mathcal{T}, \mathcal{N}, \mathcal{V}, \mathcal{E}$	Sets of scheduling periods, PEDF microgrids, network nodes, and distribution branches
t, n, i, j, m, s, k, r	Indices for period, PEDF, sending/receiving branch nodes, historical sample, reserve direction, converter supporting direction, and slack bus
$\lambda_t^{\text{sell}}, \lambda_t^{\text{buy}}$	Electricity prices for power purchased by and sold from a PEDF microgrid
$\lambda_t^Q, \lambda_t^{\text{up}}, \lambda_t^{\text{dn}}$	Prices for reactive-power support, upward reserve, and downward reserve
$p_{n,t}^{\text{buy}}, p_{n,t}^{\text{sell}}$	Active power purchased and sold by PEDF microgrid n
$p_n^{\text{ex,max}}$	Gross active-power transaction limit of PEDF microgrid n
$p_{n,t}^{\text{ch}}, p_{n,t}^{\text{dis}}, E_{n,t}^{\text{sto}}$	Battery charging power, discharging power, and stored energy
$E_n^{\text{bat}}, \text{SOC}_{n,t}$	Rated battery energy capacity and reported state-of-charge ratio,

$SOC_{n,t} = E_{n,t}^{sto} / E_n^{bat}$	
$p_{n,t}^{flex}, p_{n,t}^{cut}$	Flexible-load power and curtailed photovoltaic power
$Q_{n,t}^{PEDF}, Q_{n,t}^{+}, Q_{n,t}^{-}$	Signed reactive-power exchange and its nonnegative directional components
$c^{deg}, c^Q, c^{cut}, c^{flex}$	Battery cycling, reactive-support, PV-curtailed, and flexible-load adjustment cost coefficients
$c_{ES}^s, c_{flex}^s, c_{PV}^{up}$	Resource-specific reserve provision cost coefficients for direction s
$F_{n,t}^{up}, F_{n,t}^{dn}$	Upward and downward reserve provided by PEDF microgrid n
R_t^{up}, R_t^{dn}	System upward and downward reserve requirements
$R_t^{ext,up}, R_t^{ext,dn}$	External upward and downward reserve procured at the upstream interface
$\Delta R_t^{up}, \Delta R_t^{dn}$	Uncovered upward and downward reserve
$\xi_t, \hat{\Pi}_t, \rho_t$	Forecast-error vector, empirical distribution, and Wasserstein radius
β, η_t^s	CVaR confidence level and threshold variable for reserve direction s
κ	Multiplicative reserve safety factor applied after the WDRO-CVaR calculation
$P_{ij,t}, Q_{ij,t}, I_{ij,t}^2, V_{i,t}^2$	Branch active power, reactive power, squared current, and squared nodal voltage
$p_t^{sub}, p_t^{loss}, S_{base}$	Upstream active-power injection, network loss, and power base
C^{sys}	Total-system cost used as the aggregate economic reporting metric
M_i^v, M_i^g, z_i	Big-M bounds for dual variables and primal slacks, and complementarity binary variable

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